Problem 1-1

Represent each of the following combinations of units in the correct SI form using an appropriate prefix:
(a) m/ms **(b)** μkm **(c)** ks/mg **(d)** km· μN

Units Used:

$\mu N = 10^{-6}\ N$

$\mu km = 10^{-6}\ km$

$Gs = 10^{9}\ s$

$ks = 10^{3}\ s$

$mN = 10^{-3}\ N$

$ms = 10^{-3}\ s$

Solution:

(a) $\frac{m}{ms} = 1 \times 10^{3}\ \frac{m}{s}$

$\frac{m}{ms} = 1\ \frac{km}{s}$

(b) $\mu km = 1 \times 10^{-3}\ m$

$\mu km = 1\ mm$

(c) $\frac{ks}{mg} = 1 \times 10^{9}\ \frac{s}{kg}$

$\frac{ks}{mg} = 1\ \frac{Gs}{kg}$

(d) $km \cdot \mu N = 1 \times 10^{-3}\ m\,N$

$km \cdot \mu N = 1\ mm \cdot N$

Problem 1-2

Wood has a density *d*. What is its density expressed in SI units?

Units Used:

$\text{Mg} = 1000\ \text{kg}$

Given:

$d = 4.70\ \dfrac{\text{slug}}{\text{ft}^3}$

Solution:

$1\text{slug} = 14.594\ \text{kg}$

$d = 2.42\ \dfrac{\text{Mg}}{\text{m}^3}$

Problem 1-3

Represent each of the following combinations of units in the correct SI form using an appropriate prefix:

(a) Mg/mm **(b)** mN/μs **(c)** μm·Mg

Solution:

(a) $\dfrac{\text{Mg}}{\text{mm}} = \dfrac{10^3\ \text{kg}}{10^{-3}\ \text{m}} = \dfrac{10^6\ \text{kg}}{\text{m}} = \dfrac{\text{Gg}}{\text{m}}$

$\dfrac{\text{Mg}}{\text{mm}} = \dfrac{\text{Gg}}{\text{m}}$

(b) $\dfrac{\text{mN}}{\mu\text{s}} = \dfrac{10^{-3}\ \text{N}}{10^{-6}\ \text{s}} = \dfrac{10^3\ \text{N}}{\text{s}} = \dfrac{\text{kN}}{\text{s}}$

$\dfrac{\text{mN}}{\mu\text{s}} = \dfrac{\text{kN}}{\text{s}}$

(c) $\mu\text{m}\cdot\text{Mg} = \left(10^{-6}\ \text{m}\right)\left(10^3\ \text{kg}\right) = 10^{-3}\ \text{m}\cdot\text{kg}$

$\mu\text{m}\cdot\text{Mg} = \text{mm}\cdot\text{kg}$

Problem 1-4

Represent each of the following combinations of units in the correct SI form: **(a)** Mg/ms, **(b)** N/mm, **(c)** mN/(kg·μs).

Solution:

(a) $$\frac{\text{Mg}}{\text{ms}} = \frac{10^3\ \text{kg}}{10^{-3}\ \text{s}} = \frac{10^6\ \text{kg}}{\text{s}} = \frac{\text{Gg}}{\text{s}}$$

$$\frac{\text{Mg}}{\text{ms}} = \frac{\text{Gg}}{\text{s}}$$

(b) $$\frac{\text{N}}{\text{mm}} = \frac{1\ \text{N}}{10^{-3}\ \text{m}} = 10^3\frac{\text{N}}{\text{m}} = \frac{\text{kN}}{\text{m}}$$

$$\frac{\text{N}}{\text{mm}} = \frac{\text{kN}}{\text{m}}$$

(c) $$\frac{\text{mN}}{\text{kg}\cdot\mu\text{s}} = \frac{10^{-3}\ \text{N}}{10^{-6}\ \text{kg}\cdot\text{s}} = \frac{\text{kN}}{\text{kg}\cdot\text{s}}$$

$$\frac{\text{mN}}{\text{kg}\cdot\mu\text{s}} = \frac{\text{kN}}{\text{kg}\cdot\text{s}}$$

Problem 1-5

Represent each of the following with SI units having an appropriate prefix: **(a)** S_1, **(b)** S_2, **(c)** S_3.

Units Used:

$\text{kg} = 1000\ \text{g}$ $\quad \text{ms} = 10^{-3}\ \text{s}$ $\quad \text{kN} = 10^3\ \text{N}$

Given:

$S_1 = 8653\ \text{ms}$

$S_2 = 8368\ \text{N}$

$S_3 = 0.893\ \text{kg}$

Solution:

(a) $S_1 = 8.653\ \text{s}$

(b) $S_2 = 8.368\,\text{kN}$

(c) $S_3 = 893\,\text{g}$

Problem 1-6

Represent each of the following to three significant figures and express each answer in SI units using an appropriate prefix: **(a)** x, **(b)** y, and **(c)** z.

Units Used:

$\text{MN} = 10^6\ \text{N}$

$\mu\text{g} = 1 \times 10^{-6}\ \text{gm}$

$\text{kN} = 10^3\ \text{N}$

Given:

$x = 45320\ \text{kN}$

$y = \left(568 \times 10^5\right)\ \text{mm}$

$z = 0.00563\ \text{mg}$

Solution:

(a) $x = 45.3\,\text{MN}$

(b) $y = 56.8\,\text{km}$

(c) $z = 5.63\,\mu\text{g}$

Problem 1-7

Evaluate $(a \cdot b)/c$ to three significant figures and express the answer in SI units using an appropriate prefix.

Units Used:

$\mu\text{m} = 10^{-6}\ \text{m}$

Given:

$a = (204\ \text{mm})$

$b = (0.00457\ \text{kg})$

$c = (34.6\ \text{N})$

Solution:

$$l = \frac{ab}{c} \qquad l = 26.945\ \frac{\mu\text{m}\cdot\text{kg}}{\text{N}}$$

Problem 1-8

If a car is traveling at speed v, determine its speed in kilometers per hour and meters per second.

Given:

$$v = 55\ \frac{\text{mi}}{\text{hr}}$$

Solution:

$$v = 88.514\ \frac{\text{km}}{\text{hr}}$$

$$v = 24.6\ \frac{\text{m}}{\text{s}}$$

Problem 1-9

Convert: (a) S_1 to $\text{N}\cdot\text{m}$, (b) S_2 to kN/m^3, (c) S_3 to mm/s. Express the result to three significant figures. Use an appropriate prefix.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$S_1 = 200g\ \text{lb}\cdot\text{ft}$$

$$S_2 = 350g\ \frac{\text{lb}}{\text{ft}^3}$$

$$S_3 = 8\ \frac{\text{ft}}{\text{hr}}$$

Solution:

(a) $S_1 = 271\,\text{N}\cdot\text{m}$

(b) $S_2 = 55.0\,\dfrac{\text{kN}}{\text{m}^3}$

(c) $S_3 = 0.677\,\dfrac{\text{mm}}{\text{s}}$

Problem 1-10

What is the weight in newtons of an object that has a mass of: (a) m_1, (b) m_2, (c) m_3? Express the result to three significant figures. Use an appropriate prefix.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$

$\text{mN} = 10^{-3}\ \text{N}$

$\text{kN} = 10^3\ \text{N}$

Given:

$m_1 = 10\ \text{kg}$

$m_2 = 0.5\ \text{gm}$

$m_3 = 4.50\ \text{Mg}$

Solution:

(a) $W = m_1 g$

$W = 98.1\ \text{N}$

(b) $W = m_2 g$

$W = 4.90\,\text{mN}$

(c) $W = m_3 g$

$W = 44.1\ \text{kN}$

Problem 1-11

If an object has mass m, determine its mass in kilograms.

Given:

$m = 40 \text{ slug}$

Solution:

$m = 584 \text{ kg}$

Problem 1-12

The specific weight (wt./vol.) of brass is ρ. Determine its density (mass/vol.) in SI units. Use an appropriate prefix.

Units Used:

$\text{Mg} = 10^3 \text{ kg}$

Given:

$\rho = 520 \dfrac{\text{lb}}{\text{ft}^3}$

Solution:

$\rho = 8.33 \dfrac{\text{Mg}}{\text{m}^3}$

Problem 1-13

A concrete column has diameter d and length L. If the density (mass/volume) of concrete is ρ, determine the weight of the column in pounds.

Units Used:

$\text{Mg} = 10^3 \text{ kg}$

$\text{kip} = 10^3 \text{ lb}$

Given:

$d = 350 \text{ mm}$

$L = 2 \text{ m}$

$$\rho = 2.45\ \frac{\text{Mg}}{\text{m}^3}$$

Solution:

$$V = \pi\left(\frac{d}{2}\right)^2 L \qquad V = 192.423\ \text{L}$$

$$W = \rho\ V \qquad W = 1.04\ \text{kip}$$

Problem 1-14

The density (mass/volume) of aluminum is ρ. Determine its density in SI units. Use an appropriate prefix.

Units Used:

$$\text{Mg} = 1000\ \text{kg}$$

Given:

$$\rho = 5.26\ \frac{\text{slug}}{\text{ft}^3}$$

Solution:

$$\rho = 2.17\ \frac{\text{Mg}}{\text{m}^3}$$

Problem 1-15

Determine your own mass in kilograms, your weight in newtons, and your height in meters.

Solution:

Example

$$W = 150\ \text{lb}$$

$$m = W \qquad m = 68.039\ \text{kg}$$

$$W g = 667.233\ \text{N}$$

$$h = 72\ \text{in} \qquad h = 1.829\ \text{m}$$

Problem 1-16

Two particles have masses m_1 and m_2, respectively. If they are a distance d apart, determine the force of gravity acting between them. Compare this result with the weight of each particle.

Units Used:

$$G = 66.73 \times 10^{-12} \frac{m^3}{kg \cdot s^2}$$

$$nN = 10^{-9} N$$

Given:

$$m_1 = 8 \text{ kg}$$

$$m_2 = 12 \text{ kg}$$

$$d = 800 \text{ mm}$$

Solution:

$$F = \frac{G\, m_1\, m_2}{d^2}$$

$$F = 10.0 \text{ nN}$$

$$W_1 = m_1\, g \qquad W_1 = 78.5 \text{ N} \qquad \frac{W_1}{F} = 7.85 \times 10^9$$

$$W_2 = m_2\, g \qquad W_2 = 118 \text{ N} \qquad \frac{W_2}{F} = 1.18 \times 10^{10}$$

Problem 1-17

Using the base units of the SI system, show that $F = G(m_1 m_2)/r^2$ is a dimensionally homogeneous equation which gives F in newtons. Compute the gravitational force acting between two identical spheres that are touching each other. The mass of each sphere is m_1, and the radius is r.

Units Used:

$$\mu N = 10^{-6} N \qquad G = 66.73\ 10^{-12} \frac{m^3}{kg \cdot s^2}$$

Given:

$m_1 = 150 \text{ kg}$

$r = 275 \text{ mm}$

Solution:

$$F = \frac{G\, m_1^{\,2}}{(2r)^2}$$

$F = 4.96\ \mu\text{N}$

Since the force F is measured in Newtons, then the equation is dimensionally homogeneous.

Problem 1-18

Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) x, (b) y, (c) z.

Units Used:

$\text{MN} = 10^6\ \text{N}$

$\text{kN} = 10^3\ \text{N}$

$\mu\text{m} = 10^{-6}\ \text{m}$

Given:

$x = (200\ \text{kN})^2$

$y = (0.005\ \text{mm})^2$

$z = (400\ \text{m})^3$

Solution:

(a) $x = 0.040\,\text{MN}^2$

(b) $y = 25.0\,\mu\text{m}^2$

(c) $z = 0.0640\,\text{km}^3$

Problem 1-19

Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) a_1/b_1, (b) a_2b_2/c_2, (c) a_3b_3.

Units Used:

$\mu m = 10^{-6}\ m$ $\quad Mm = 10^{6}\ m$

$Mg = 10^{6}\ gm$ $\quad kg = 10^{3}\ gm$

$ms = 10^{-3}\ s$

Given:

$a_1 = 684\ \mu m$

$b_1 = 43\ ms$

$a_2 = 28\ ms$

$b_2 = 0.0458\ Mm$

$c_2 = 348\ mg$

$a_3 = 2.68\ mm$

$b_3 = 426\ Mg$

Solution:

(a) $\frac{a_1}{b_1} = 15.9\frac{mm}{s}$

(b) $\frac{a_2\ b_2}{c_2} = 3.69\,Mm\ \frac{s}{kg}$

(c) $a_3\ b_3 = 1.14\,km\cdot kg$

Problem 1-20

Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: **(a)** a_1/b_1^2 **(b)** $a_2^2b_2^3$.

Units Used:

$Mm = 10^{6}\ m$

Given:

$a_1 = 0.631\ \text{Mm}$

$b_1 = 8.60\ \text{kg}$

$a_2 = 35\ \text{mm}$

$b_2 = 48\ \text{kg}$

Solution:

(a) $$\frac{a_1}{b_1^2} = 8.532\,\frac{\text{km}}{\text{kg}^2}$$

(b) $$a_2^2\ b_2^3 = 135.48\,\text{kg}^3\cdot\text{m}^2$$

Problem 2-1

Determine the magnitude of the resultant force $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ and its direction, measured counterclockwise from the positive x axis.

Given:

$F_1 = 600\ \text{N}$

$F_2 = 800\ \text{N}$

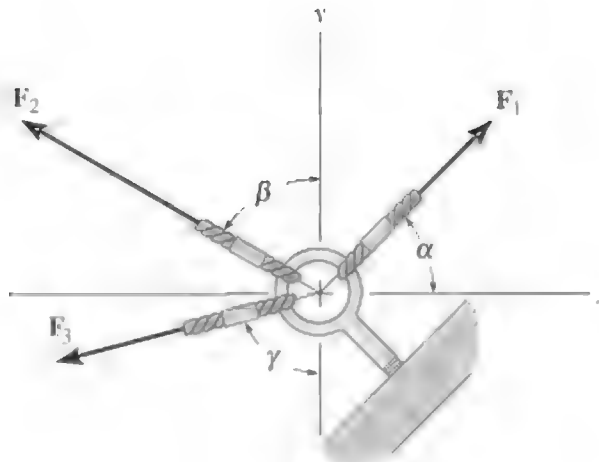
$F_3 = 450\ \text{N}$

$\alpha = 45\ \text{deg}$

$\beta = 60\ \text{deg}$

$\gamma = 75\ \text{deg}$

Solution:

$\psi = 90\ \text{deg} - \beta + \alpha$

$$F_R = \sqrt{F_1^2 + F_2^2 - 2\,F_1\,F_2\cos(\psi)}$$

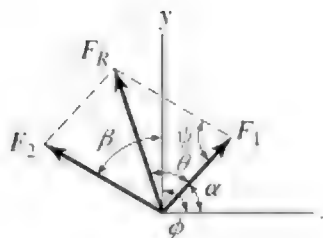
$F_R = 867\ \text{N}$

$$\frac{F_R}{\sin(\psi)} = \frac{F_2}{\sin(\theta)}$$

$$\theta = \operatorname{asin}\left(F_2\frac{\sin(\psi)}{F_R}\right)$$

$\theta = 63.05\ \text{deg}$

$\phi = \theta + \alpha$

$\phi = 108\ \text{deg}$

Problem 2-2

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.

Given:

$F_1 = 80\ \text{lb}$

$F_2 = 60\ \text{lb}$

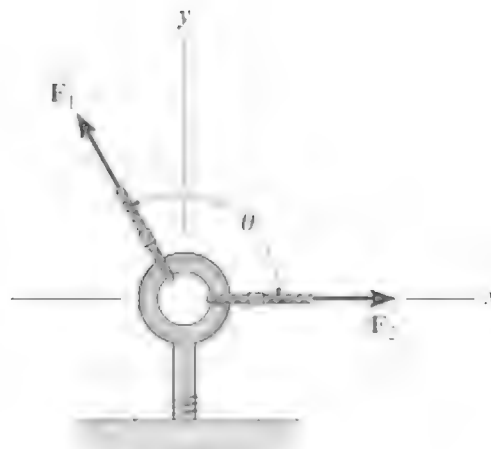
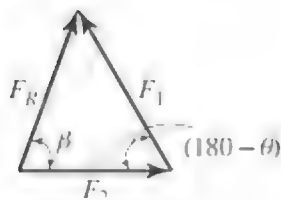
$\theta = 120 \text{ deg}$

Solution:

$$F_R = \sqrt{F_1^2 + F_2^2 - 2\, F_1\, F_2 \cos(180 \text{ deg} - \theta)}$$

$F_R = 72.1 \text{ lb}$

$$\beta = \operatorname{asin}\left(F_1 \frac{\sin(180 \text{ deg} - \theta)}{F_R}\right)$$

$\beta = 73.9 \text{ deg}$

Problem 2-3

Determine the magnitude of the resultant force $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ and its direction, measured counterclockwise from the positive x axis.

Given:

$F_1 = 250 \text{ lb}$

$F_2 = 375 \text{ lb}$

$\theta = 30 \text{ deg}$

$\phi = 45 \text{ deg}$

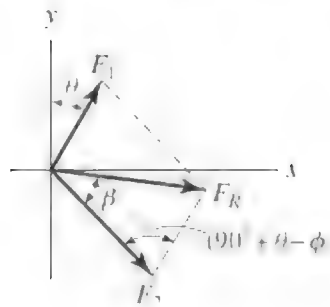
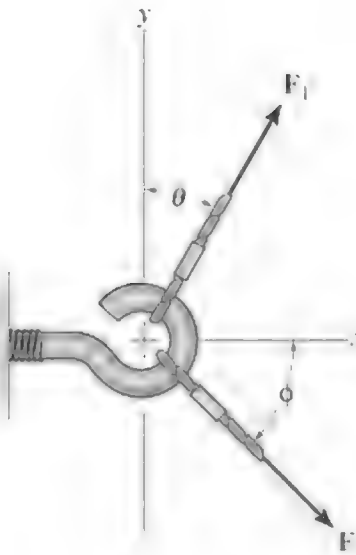
Solution:

$$F_R = \sqrt{F_1^2 + F_2^2 - 2\, F_1\, F_2 \cos(90 \text{ deg} + \theta - \phi)}$$

$F_R = 178 \text{ kg}$

$$\frac{F_R}{\sin(90 \text{ deg} + \theta - \phi)} = \frac{F_1}{\sin(\beta)}$$

$$\beta = \operatorname{asin}\left(\frac{F_1}{F_R} \sin(90 \text{ deg} + \theta - \phi)\right)$$

$\beta = 37.89 \text{ deg}$

Angle measured ccw from x axis

$360 \text{ deg} - \phi + \beta = 353 \text{ deg}$

Problem 2-4

Determine the magnitude of the resultant force $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ and its direction, measured counterclockwise from the positive u axis.

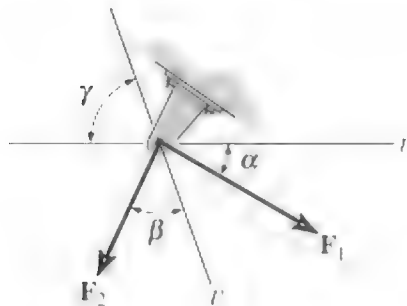
Given:

$F_1 = 300 \text{ N}$

$F_2 = 500 \text{ N}$

$\alpha = 30 \text{ deg}$

$\beta = 45 \text{ deg}$

$\gamma = 70 \text{ deg}$

Solution:

$$F_R = \sqrt{F_1^2 + F_2^2 - 2\,F_1\,F_2 \cos(180 \text{ deg} - \beta - \gamma + \alpha)}$$

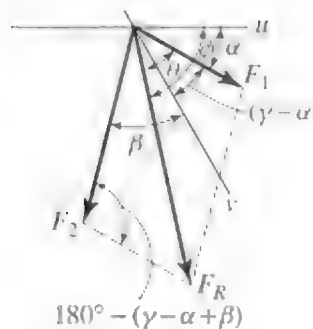
$F_R = 605 \text{ N}$

$$\frac{F_R}{\sin(180 \text{ deg} - \beta - \gamma + \alpha)} = \frac{F_2}{\sin(\theta)}$$

$$\theta = \operatorname{asin}\left(F_2 \frac{\sin(180 \text{ deg} - \beta - \gamma + \alpha)}{F_R}\right)$$

$\theta = 55.40 \text{ deg}$

$\phi = \theta + \alpha$

$\phi = 85.4 \text{ deg}$

Problem 2-5

Resolve the force $\mathbf{F_1}$ into components acting along the u and v axes and determine the magnitudes of the components.

Given:

$F_1 = 300 \text{ N}$ $\quad \alpha = 30 \text{ deg}$

$F_2 = 500\text{ N} \quad \beta = 45\text{ deg}$

$\gamma = 70\text{ deg}$

Solution:

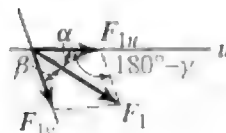
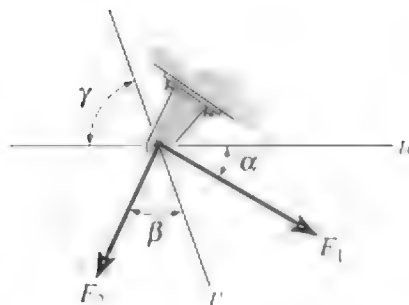
$$\frac{F_{1u}}{\sin(\gamma - \alpha)} = \frac{F_1}{\sin(180\text{ deg} - \gamma)}$$

$$F_{1u} = F_1 \frac{\sin(\gamma - \alpha)}{\sin(180\text{ deg} - \gamma)}$$

$F_{1u} = 205\text{ N}$

$$\frac{F_{1v}}{\sin(\alpha)} = \frac{F_1}{\sin(180\text{ deg} - \gamma)}$$

$$F_{1v} = F_1 \frac{\sin(\alpha)}{\sin(180\text{ deg} - \gamma)}$$

$F_{1v} = 160\text{ N}$

Problem 2-6

Resolve the force $\mathbf{F_2}$ into components acting along the u and v axes and determine the magnitudes of the components.

Given:

$F_1 = 300\text{ N}$

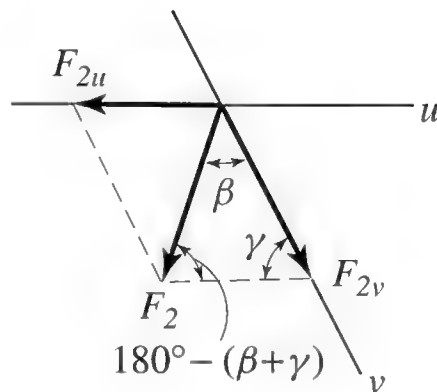
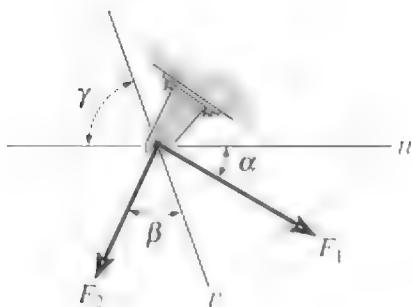
$F_2 = 500\text{ N}$

$\alpha = 30\text{ deg}$

$\beta = 45\text{ deg}$

$\gamma = 70\text{ deg}$

Solution:

$$F_{2u} = F_2\left(\frac{\sin(\beta)}{\sin(\gamma)}\right)$$

$F_{2u} = 376.2 \text{ N}$

$$F_{2v} = F_2\left[\frac{\sin[180 \text{ deg} - (\beta + \gamma)]}{\sin(\gamma)}\right]$$

$F_{2v} = 482.2 \text{ N}$

Problem 2-7

Determine the magnitude of the resultant force $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ and its direction measured counterclockwise from the positive u axis.

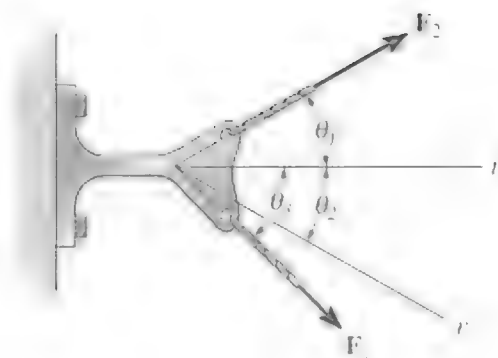
Given:

$F_1 = 25 \text{ lb}$

$F_2 = 50 \text{ lb}$

$\theta_1 = 30 \text{ deg}$

$\theta_2 = 30 \text{ deg}$

$\theta_3 = 45 \text{ deg}$

Solution:

$\alpha = 180 \text{ deg} - (\theta_3 + \theta_1)$

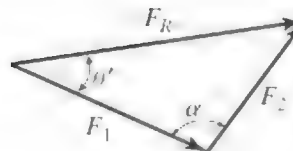
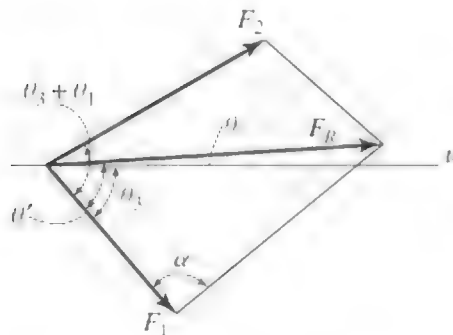
$$F_R = \sqrt{F_2^{\ 2} + F_1^{\ 2} - 2\ F_1\ F_2\ \cos(\alpha)}$$

$F_R = 61.4 \text{ lb}$

$$\frac{\sin(\theta')}{F_2} = \frac{\sin(\alpha)}{F_R} \qquad \theta' = \text{asin}\left(\sin(\alpha)\ \frac{F_2}{F_R}\right)$$

$\theta' = 51.8 \text{ deg}$

$\theta = \theta' - \theta_3$

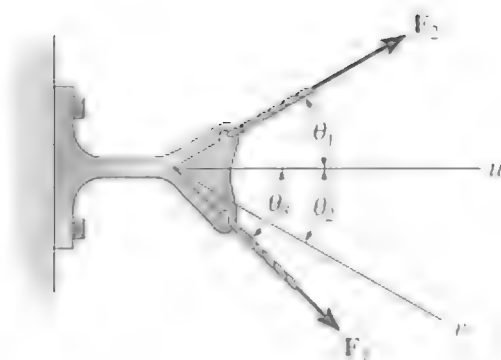
$\theta = 6.8 \text{ deg}$

Problem 2-8

Resolve the force $\mathbf{F_1}$ into components acting along the u and v axes and determine the components.

Given:

$F_1 = 25$ lb

$F_2 = 50$ lb

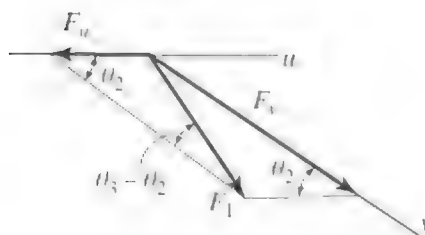
$\theta_1 = 30$ deg

$\theta_2 = 30$ deg

$\theta_3 = 45$ deg

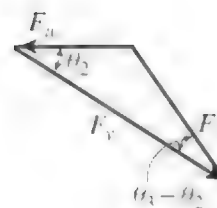
Solution:

$$\frac{-F_u}{\sin(\theta_3 - \theta_2)} = \frac{F_1}{\sin(\theta_2)}$$

$$F_u = \frac{-F_1 \sin(\theta_3 - \theta_2)}{\sin(\theta_2)}$$

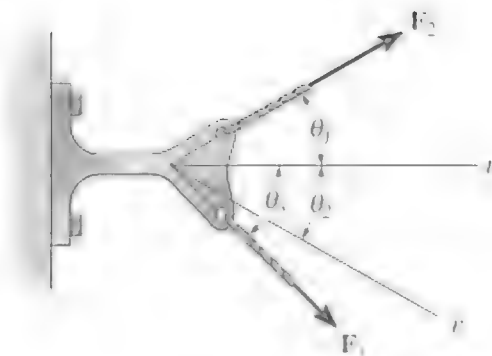
$F_u = -12.9$ lb

$$\frac{F_v}{\sin(180 \text{ deg} - \theta_3)} = \frac{F_1}{\sin(\theta_2)}$$

$$F_v = \frac{F_1 \sin(180 \text{ deg} - \theta_3)}{\sin(\theta_2)}$$

$F_v = 35.4$ lb

Problem 2-9

Resolve the force $\mathbf{F_2}$ into components acting along the u and v axes and determine the components.

Given:

$F_1 = 25$ lb

$F_2 = 50$ lb

$\theta_1 = 30$ deg

$\theta_2 = 30$ deg

$\theta_3 = 45$ deg

Solution:

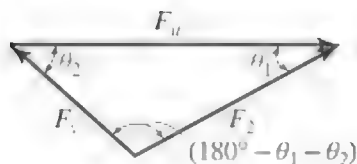
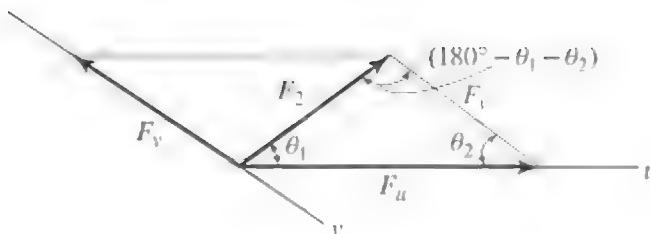
$$\frac{F_u}{\sin\left[180\text{ deg} - \left(\theta_1 + \theta_2\right)\right]} = \frac{F_2}{\sin\left(\theta_2\right)}$$

$$F_u = \frac{F_2 \sin\left[180\text{ deg} - \left(\theta_1 + \theta_2\right)\right]}{\sin\left(\theta_2\right)}$$

$F_u = 86.6$ lb

$$\frac{-F_v}{\sin\left(\theta_1\right)} = \frac{F_2}{\sin\left(\theta_2\right)}$$

$$F_v = \frac{-F_2 \sin\left(\theta_1\right)}{\sin\left(\theta_2\right)}$$

$F_v = -50$ lb

Problem 2-10

Determine the components of the **F** force acting along the u and v axes.

Given:

$\theta_1 = 70$ deg

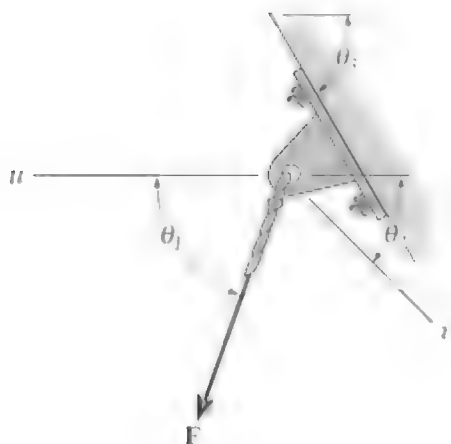
$\theta_2 = 45$ deg

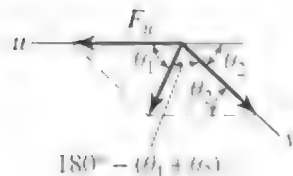
$\theta_3 = 60$ deg

$F = 250$ N

Solution:

$$\frac{F_u}{\sin\left[180\text{ deg} - \left(\theta_1 + \theta_2\right)\right]} = \frac{F}{\sin\left(\theta_2\right)}$$

$$F_u = \frac{F \sin\left[180\text{ deg} - \left(\theta_1 + \theta_2\right)\right]}{\sin\left(\theta_2\right)} \qquad F_u = 320\text{ N}$$

$$\frac{F_v}{\sin(\theta_1)} = \frac{F}{\sin(\theta_2)}$$

$$F_v = \frac{F\sin(\theta_1)}{\sin(\theta_2)} \qquad F_v = 332 \text{ N}$$

Problem 2-11

The force **F** acts on the gear tooth. Resolve this force into two components acting along the lines *aa* and *bb*.

Given:

$F = 20$ lb

$\theta_1 = 80$ deg

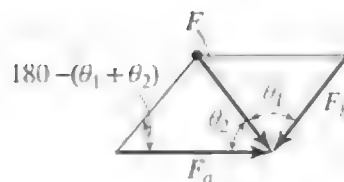
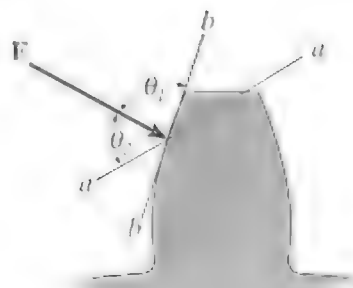
$\theta_2 = 60$ deg

Solution:

$$\frac{F}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]} = \frac{F_a}{\sin(\theta_1)}$$

$$F_a = \frac{F\sin(\theta_1)}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]} \qquad F_a = 30.6 \text{ lb}$$

$$\frac{F}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]} = \frac{F_b}{\sin(\theta_2)}$$

$$F_b = \frac{F\sin(\theta_2)}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]} \qquad F_b = 26.9 \text{ lb}$$

Problem 2-12

The component of force **F** acting along line *aa* is required to be F_a. Determine the magnitude of **F** and its component along line *bb*.

Given:

$F_a = 30 \text{ lb}$

$\theta_1 = 80 \text{ deg}$

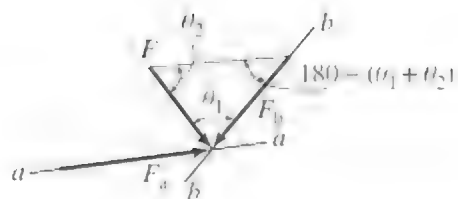
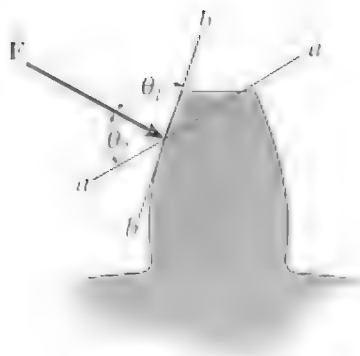
$\theta_2 = 60 \text{ deg}$

Solution:

$$\frac{F_a}{\sin(\theta_1)} = \frac{F}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]}$$

$$F = F_a \left(\frac{\sin(180 \text{ deg} - \theta_1 - \theta_2)}{\sin(\theta_1)} \right) \qquad F = 19.6 \text{ lb}$$

$$\frac{F_a}{\sin(\theta_1)} = \frac{F_b}{\sin(\theta_2)}$$

$$F_b = \frac{F_a \sin(\theta_2)}{\sin(\theta_1)} \qquad F_b = 26.4 \text{ lb}$$

Problem 2-13

A resultant force **F** is necessary to hold the ballon in place. Resolve this force into components along the tether lines *AB* and *AC*, and compute the magnitude of each component.

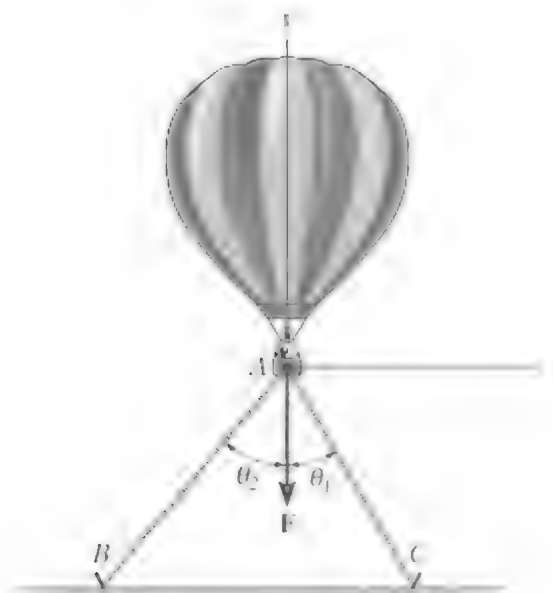
Given:

$F = 350 \text{ lb}$

$\theta_1 = 30 \text{ deg}$

$\theta_2 = 40 \text{ deg}$

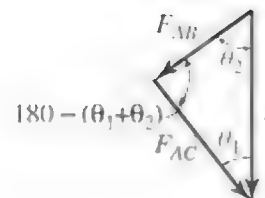
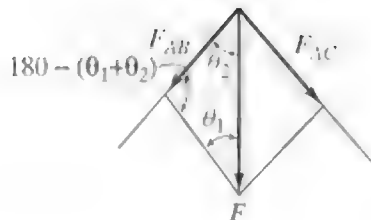
Solution:

$$\frac{F_{AB}}{\sin(\theta_1)} = \frac{F}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]}$$

$$F_{AB} = F\left[\frac{\sin(\theta_1)}{\sin[180\text{ deg} - (\theta_1 + \theta_2)]}\right]$$

$F_{AB} = 186\text{ lb}$

$$\frac{F_{AC}}{\sin(\theta_2)} = \frac{F}{\sin[180\text{ deg} - (\theta_1 + \theta_2)]}$$

$$F_{AC} = F\left[\frac{\sin(\theta_2)}{\sin[180\text{ deg} - (\theta_1 + \theta_2)]}\right]$$

$F_{AC} = 239\text{ lb}$

Problem 2-14

The post is to be pulled out of the ground using two ropes A and B. Rope A is subjected to force F_1 and is directed at angle θ_1 from the horizontal. If the resultant force acting on the post is to be F_R, vertically upward, determine the force **T** in rope B and the corresponding angle θ.

Given:

$F_R = 1200\text{ lb}$

$F_1 = 600\text{ lb}$

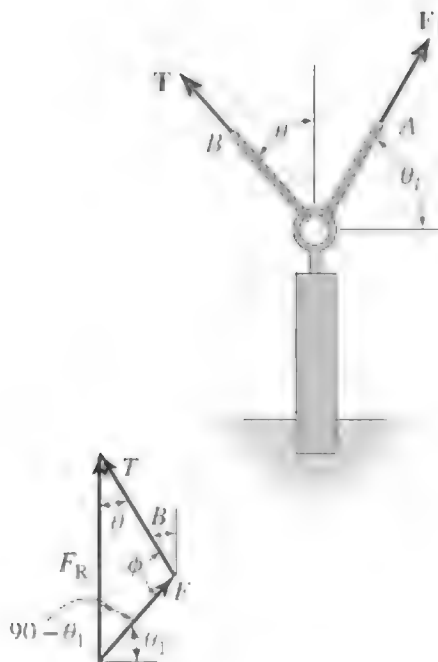
$\theta_1 = 60\text{ deg}$

Solution:

$$T = \sqrt{F_1^2 + F_R^2 - 2\,F_1\,F_R\cos(90\text{ deg} - \theta_1)}$$

$T = 744\text{ lb}$

$$\frac{\sin(\theta)}{F_R} = \frac{\sin(90 - \theta_1)}{T}$$

$$\theta = \operatorname{asin}\left(\sin(90\text{ deg} - \theta_1)\frac{F_1}{T}\right) \qquad \theta = 23.8\text{ deg}$$

Problem 2-15

Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.

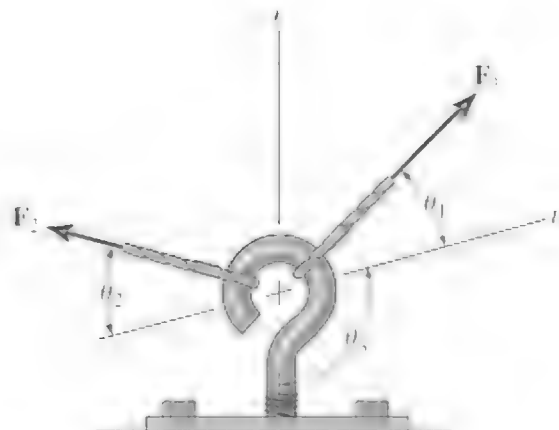
Given:

$F_1 = 250\text{ N}$

$F_2 = 150\text{ N}$

$\theta_1 = 30\text{ deg}$

$\theta_2 = 30\text{ deg}$

$\theta_3 = 105\text{ deg}$

Solution:

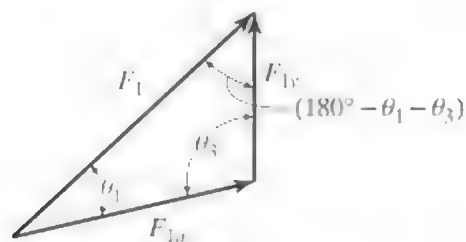
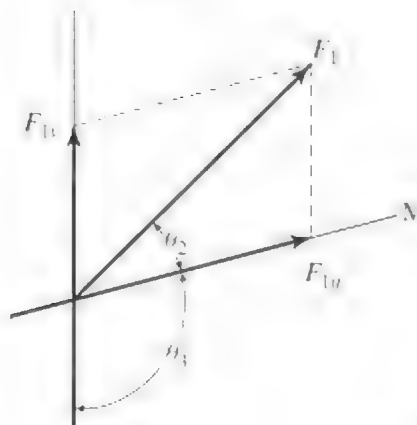
$$\frac{F_{1v}}{\sin(\theta_1)} = \frac{F_1}{\sin(\theta_3)}$$

$$F_{1v} = F_1\left(\frac{\sin(\theta_1)}{\sin(\theta_3)}\right)$$

$F_{1v} = 129\text{ N}$

$$\frac{F_{1u}}{\sin(180\text{ deg} - \theta_1 - \theta_3)} = \frac{F_1}{\sin(\theta_3)}$$

$$F_{1u} = F_1\left(\frac{\sin(180\text{ deg} - \theta_1 - \theta_3)}{\sin(\theta_3)}\right)$$

$F_{1u} = 183\text{ N}$

Problem 2-16

Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.

Given:

$F_1 = 250\text{ N}$

$F_2 = 150\text{ N}$

$\theta_1 = 30\text{ deg}$

$\theta_2 = 30\text{ deg}$

$\theta_3 = 105\text{ deg}$

Solution:

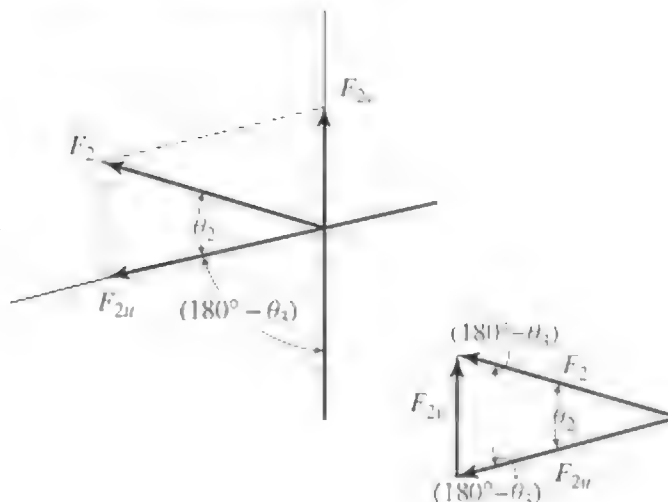
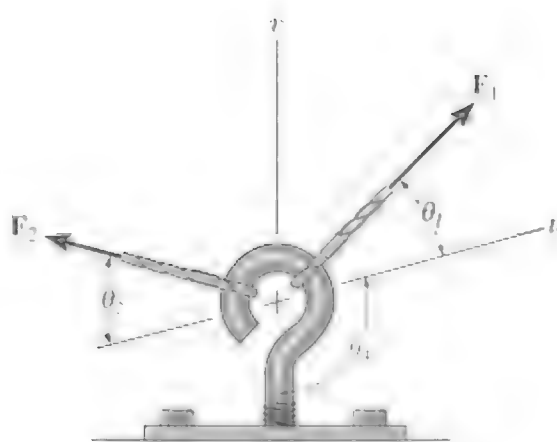
$$\frac{F_{1v}}{\sin(\theta_1)} = \frac{F_2}{\sin(180\text{ deg} - \theta_3)}$$

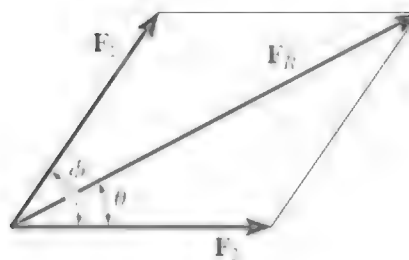
$$F_{1v} = F_2 \left(\frac{\sin(\theta_1)}{\sin(180\text{ deg} - \theta_3)}\right)$$

$F_{1v} = 77.6\text{ N}$

$$\frac{F_{2u}}{\sin(180\text{ deg} - \theta_3)} = \frac{F_2}{\sin(180\text{ deg} - \theta_3)}$$

$$F_{2u} = F_2 \left(\frac{\sin(180\text{ deg} - \theta_3)}{\sin(180\text{ deg} - \theta_3)}\right)$$

$F_{2u} = 150\text{ N}$

Problem 2-17

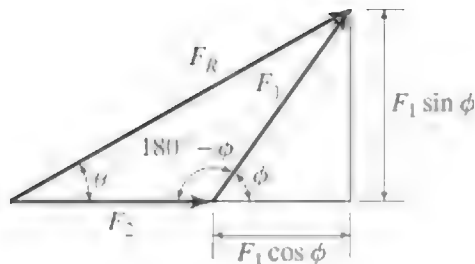
Determine the magnitude and direction of the resultant force $\mathbf{F_R}$. Express the result in terms of the magnitudes of the components $\mathbf{F_1}$ and $\mathbf{F_2}$ and the angle ϕ.

Solution:

$$F_R^2 = F_1^2 + F_2^2 - 2F_1\, F_2 \cos(180\text{ deg} - \phi)$$

Since $\cos(180\,\text{deg} - \phi) = -\cos(\phi)$,

$$F_R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2\,F_1\,F_2\,\cos(\phi)}$$

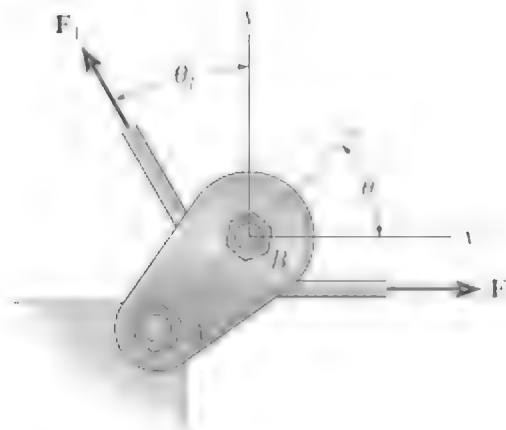
From the figure,

$$\tan(\theta) = \frac{F_1\,\sin(\phi)}{F_2 + F_1\,\cos(\phi)}$$

Problem 2-18

If the tension in the cable is $\mathbf{F_1}$, determine the magnitude and direction of the resultant force acting on the pulley. This angle defines the same angle θ of line AB on the tailboard block.

Given:

$F_1 = 400$ N

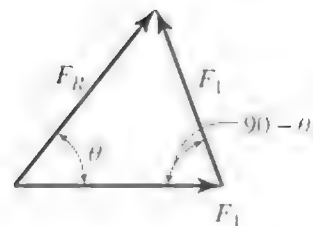
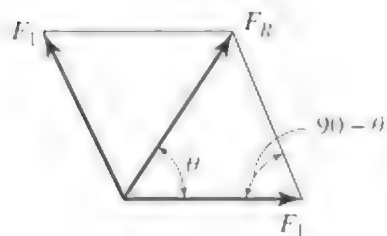
$\theta_1 = 30$ deg

Solution:

$$F_R = \sqrt{F_1^{\,2} + F_1^{\,2} - 2F_1\,F_1\cos(90\text{ deg} - \theta_1)}$$

$F_R = 400$ N

$$\frac{\sin(90\text{ deg} - \theta)}{F_R} = \frac{\sin(\theta_1)}{F_1}$$

$$\theta = 90\text{ deg} - \operatorname{asin}\left(\frac{F_R}{F_1}\sin(\theta_1)\right)$$

$\theta = 60\,\text{deg}$

Problem 2-19

The riveted bracket supports two forces. Determine the angle θ so that the resultant force is directed along the negative x axis. What is the magnitude of this resultant force?

Given:

$F_1 = 60 \text{ lb}$

$F_2 = 70 \text{ lb}$

$\theta_1 = 30 \text{ deg}$

Solution:

$$\frac{\sin(\theta)}{F_1} = \frac{\sin(\theta_1)}{F_2}$$

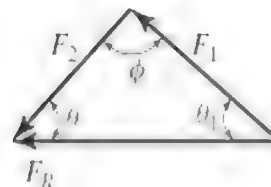
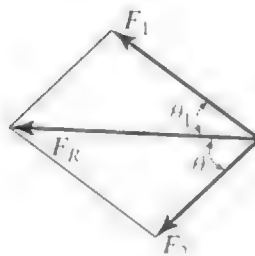
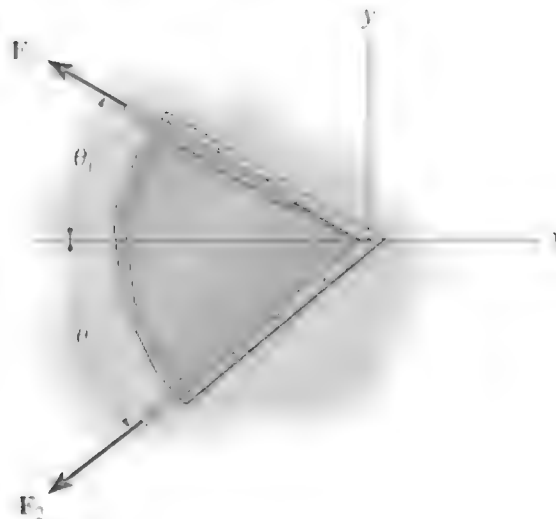
$$\theta = \operatorname{asin}\left(\sin(\theta_1)\frac{F_1}{F_2}\right)$$

$$\theta = 25.4 \text{ deg}$$

$$\phi = 180 \text{ deg} - \theta - \theta_1$$

$$\phi = 124.6 \text{ deg}$$

$$R = \sqrt{F_1^2 + F_2^2 - 2F_1 F_2 \cos(\phi)}$$

$$R = 115 \text{ lb}$$

Problem 2-20

The plate is subjected to the forces acting on members A and B as shown. Determine the magnitude of the resultant of these forces and its direction measured clockwise from the positive x axis.

Given:

$F_A = 400 \text{ lb}$

$F_B = 500 \text{ lb}$

$\theta_1 = 30$ deg

$\theta = 60$ deg

Solution:

Cosine law:

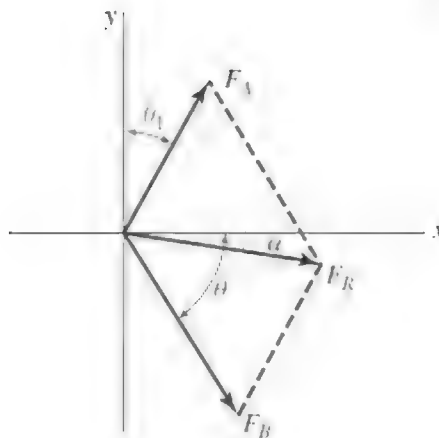
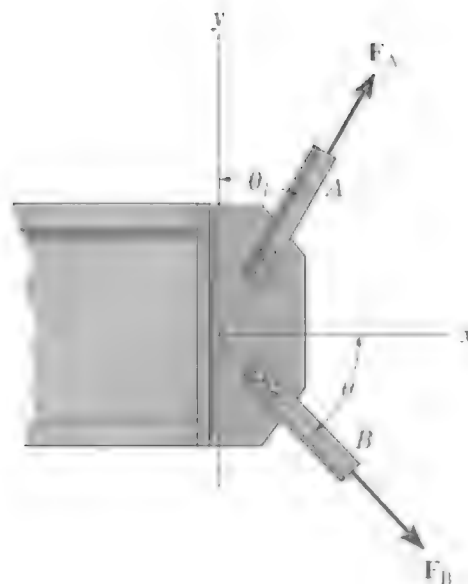
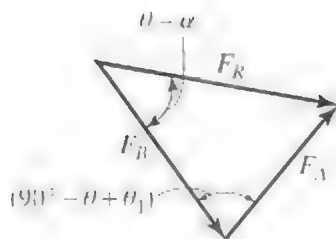
$$F_R = \sqrt{F_B^{\ 2} + F_A^{\ 2} - 2F_B F_A \cos(90 \text{ deg} - \theta + \theta_1)}$$

$F_R = 458\,\text{lb}$

Sine law:

$$\frac{\sin(90 \text{ deg} - \theta + \theta_1)}{F_R} = \frac{\sin(\theta - \alpha)}{F_A}$$

$$\alpha = \theta - \operatorname{asin}\left(\sin(90 \text{ deg} - \theta + \theta_1)\,\frac{F_A}{F_R}\right)$$

$\alpha = 10.9\,\text{deg}$

Problem 2-21

Determine the angle θ for connecting member B to the plate so that the resultant of $\mathbf{F_A}$ and $\mathbf{F_B}$ is directed along the positive x axis. What is the magnitude of the resultant force?

Given:

$F_A = 400$ lb

$F_B = 500$ lb

$\theta_1 = 30$ deg

Solution:

Sine law:

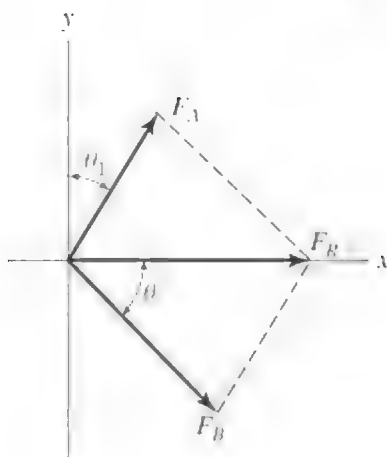
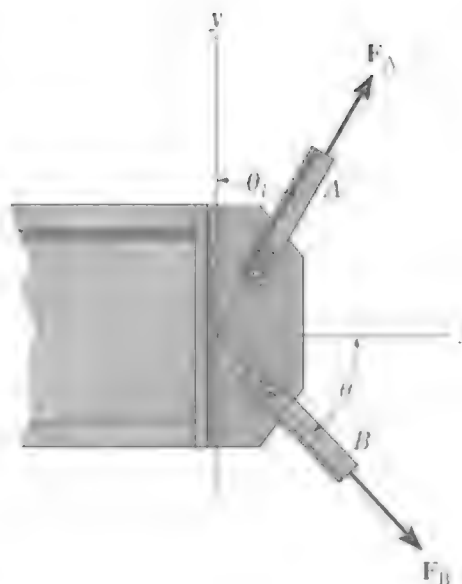
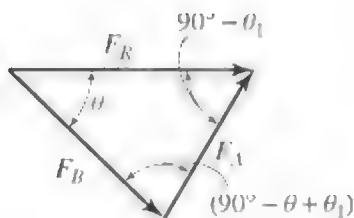
$$\frac{\sin(\theta)}{F_A} = \frac{\sin(90 \text{ deg} - \theta_1)}{F_B}$$

$$\theta = \text{asin}\left(\sin(90 \text{ deg} - \theta_1) \frac{F_A}{F_B}\right)$$

$\theta = 43.9 \text{ deg}$

$$\frac{F_R}{\sin(90 \text{ deg} + \theta_1 - \theta)} = \frac{F_A}{\sin(\theta)}$$

$$F_R = F_A \frac{\sin(90 \text{ deg} - \theta + \theta_1)}{\sin(\theta)}$$

$F_R = 561 \text{ lb}$

Problem 2-22

Determine the magnitude and direction of the resultant $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$ of the three forces by first finding the resultant $\mathbf{F'} = \mathbf{F_1} + \mathbf{F_2}$ and then forming $\mathbf{F_R} = \mathbf{F'} + \mathbf{F_3}$.

Given:

$F_1 = 30 \text{ N}$

$F_2 = 20 \text{ N}$

$F_3 = 50 \text{ N}$

$\theta = 20 \text{ deg}$

$c = 3$

$d = 4$

Solution:

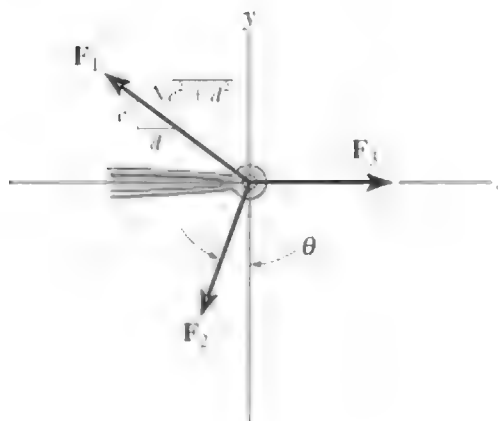
$$\alpha = \operatorname{atan}\left(\frac{c}{d}\right)$$

$$F' = \sqrt{F_1^{\,2} + F_2^{\,2} - 2F_1 F_2 \cos(90\ \text{deg} + \theta - \alpha)}$$

$$F' = 30.9\ \text{N}$$

$$\frac{F'}{\sin((90\ \text{deg} - \theta + \alpha))} = \frac{F_1}{\sin(90\ \text{deg} - \theta - \beta)}$$

$$\beta = 90\ \text{deg} - \theta - \operatorname{asin}\left(F_1\ \frac{\sin(90\ \text{deg} - \theta + \alpha)}{F'}\right)$$

$$\beta = 1.5\ \text{deg}$$

Now add in force F_3.

$$F_R = \sqrt{F'^2 + F_3^{\,2} - 2F'\ F_3 \cos(\beta)}$$

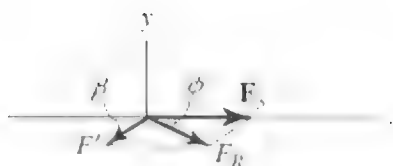
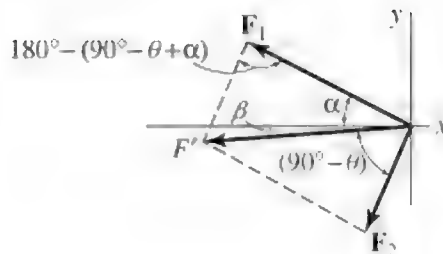
$$F_R = 19.2\ \text{N}$$

$$\frac{F_R}{\sin(\beta)} = \frac{F'}{\sin(\phi)}$$

$$\phi = \operatorname{asin}\left(F'\ \frac{\sin(\beta)}{F_R}\right)$$

$$\phi = 2.4\ \text{deg}$$

180° − (90° − θ + α) F₁ y α β x F′ (90° − θ) F₂

y β φ F₃ x F′ F_R

Problem 2-23

Determine the magnitude and direction of the resultant $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$ of the three forces by first finding the resultant $\mathbf{F'} = \mathbf{F_2} + \mathbf{F_3}$ and then forming $\mathbf{F_R} = \mathbf{F'} + \mathbf{F_1}$.

Given:

$$F_1 = 30\ \text{N}$$

$$F_2 = 20\ \text{N}$$

$F_3 = 50 \text{ N}$

$\theta = 20 \text{ deg}$

$c = 3$

$d = 4$

Solution:

$$F' = \sqrt{F_2^2 + F_3^2 - 2F_2 F_3 \cos((90 \text{ deg} - \theta))}$$

$$F' = 47.07 \text{ N}$$

$$\frac{F_2}{\sin(\beta)} = \frac{F'}{\sin(90 \text{ deg} - \theta)}$$

$$\beta = \operatorname{asin}\left(F_2 \frac{\sin(90 \text{ deg} - \theta)}{F'}\right)$$

$$\beta = 23.53 \text{ deg}$$

$$\alpha = \operatorname{atan}\left(\frac{c}{d}\right)$$

$$\gamma = \alpha - \beta$$

$$F_R = \sqrt{F'^2 + F_1^2 - 2F' F_1 \cos(\gamma)}$$

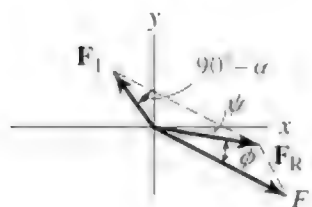
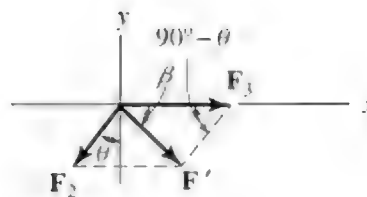
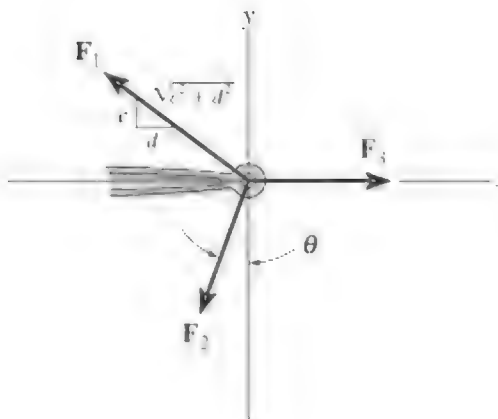
$$F_R = 19.2 \text{ N}$$

$$\frac{F_R}{\sin(\gamma)} = \frac{F_1}{\sin(\phi)}$$

$$\phi = \operatorname{asin}\left(F_1 \frac{\sin(\gamma)}{F_R}\right)$$

$$\phi = 21.16 \text{ deg}$$

$$\psi = \beta - \phi$$

$$\psi = 2.37 \text{ deg}$$

Problem 2-24

Resolve the force **F** into components acting along (a) the x and y axes, and (b) the x and y' axes.

Given:

$F = 50 \text{ lb}$

$\alpha = 65 \text{ deg}$

$\beta = 45 \text{ deg}$

$\gamma = 30 \text{ deg}$

Solution:

(a) $F_x = F\cos(\beta)$

$F_x = 35.4 \text{ lb}$

$F_y = F\sin(\beta)$

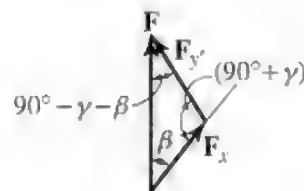
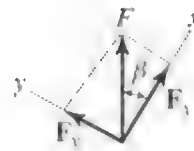
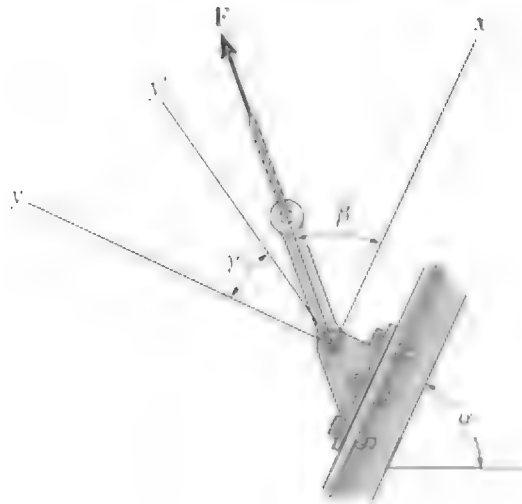
$F_y = 35.4 \text{ lb}$

(b) $$\frac{F_x}{\sin(90\text{ deg} - \beta - \gamma)} = \frac{F}{\sin(90\text{ deg} + \gamma)}$$

$$F_x = F\frac{\sin(90\text{ deg} - \beta - \gamma)}{\sin(90\text{ deg} + \gamma)}$$

$F_x = 14.9 \text{ lb}$

$$\frac{F_{y'}}{\sin(\beta)} = \frac{F}{\sin(90\text{ deg} + \gamma)}$$

$$F_{y'} = F\frac{\sin(\beta)}{\sin(90\text{ deg} + \gamma)}$$

$F_{y'} = 40.8\ \text{lb}$

Problem 2-25

The boat is to be pulled onto the shore using two ropes. Determine the magnitudes of forces **T** and **P** acting in each rope in order to develop a resultant force $\mathbf{F_1}$, directed along the keel axis *aa* as shown.

Given:

$\theta = 40\ \text{deg}$

$\theta_1 = 30\ \text{deg}$

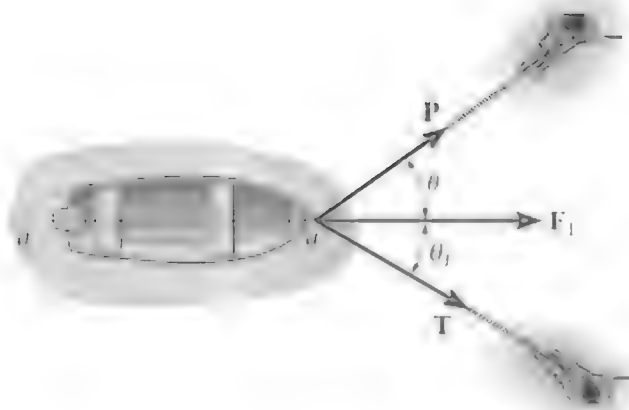
$F_1 = 80\ \text{lb}$

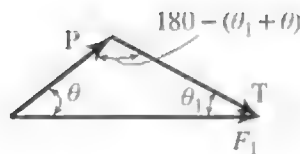
Solution:

$$\frac{F_1}{\sin[180\ \text{deg} - (\theta + \theta_1)]} = \frac{T}{\sin(\theta)}$$

$$T = F_1 \frac{\sin(\theta)}{\sin(180\ \text{deg} - \theta - \theta_1)}$$

$T = 54.7\ \text{lb}$

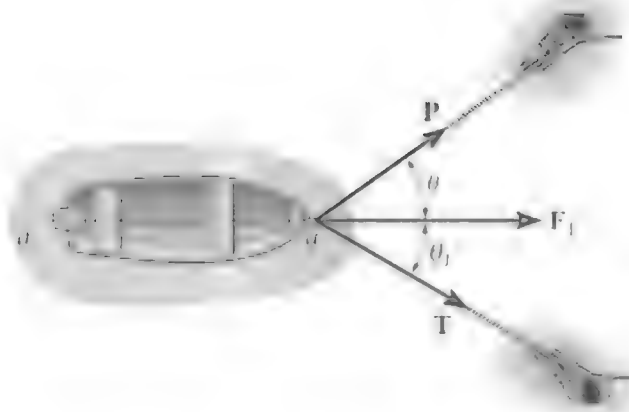
$$\frac{F_1}{\sin[180\ \text{deg} - (\theta + \theta_1)]} = \frac{P}{\sin(\theta_1)}$$

$$P = \sin(\theta_1) \frac{F_1}{\sin[180\ \text{deg} - (\theta + \theta_1)]}$$

$P = 42.6\ \text{lb}$

Problem 2-26

The boat is to be pulled onto the shore using two ropes. If the resultant force is to be $\mathbf{F_1}$, directed along the keel *aa* as shown, determine the magnitudes of forces **T** and **P** acting in each rope and the angle θ of **P** so that the magnitude of **P** is a *minimum*. **T** acts at θ from the keel as shown.

Given:

$\theta_1 = 30 \text{ deg}$

$F_1 = 80 \text{ lb}$

Solution:

From the figure, P is minimum when

$\theta + \theta_1 = 90 \text{ deg}$

$\theta = 90 \text{ deg} - \theta_1$

$\theta = 60 \text{ deg}$

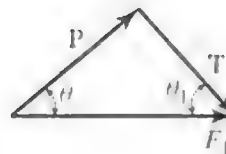
$$\frac{P}{\sin(\theta_1)} = \frac{F_1}{\sin(90 \text{ deg})}$$

$$P = \frac{F_1 \sin(\theta_1)}{\sin(90 \text{ deg})}$$

$P = 40 \text{ lb}$

$$\frac{T}{\sin(\theta)} = \frac{F_1}{\sin(90 \text{ deg})}$$

$$T = F_1 \frac{\sin(\theta)}{\sin(90 \text{ deg})}$$

$T = 69.3 \text{ lb}$

Problem 2-27

The beam is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F_A}$ and $\mathbf{F_B}$ acting on each chain in order to develop a resultant force $\mathbf{T}$ directed along the positive y axis.

Given:

$T = 600\ \text{N}$

$\theta_1 = 30\ \text{deg}$

$\theta = 45\ \text{deg}$

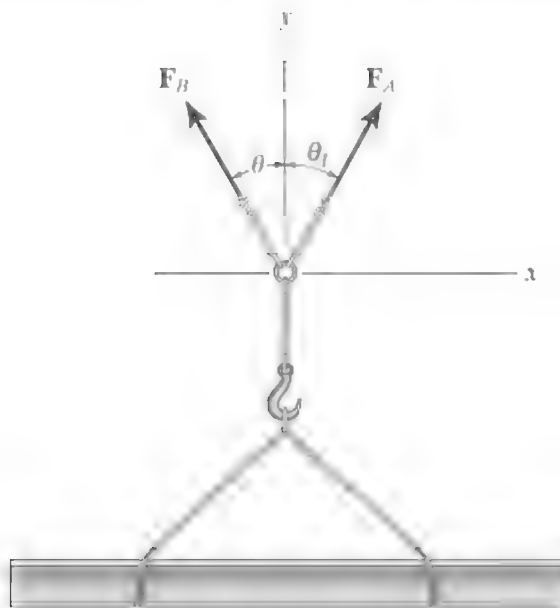
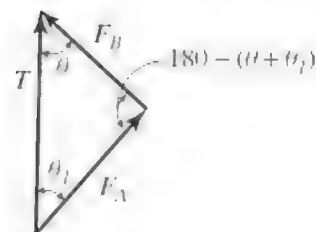
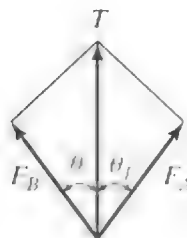
Solution:

$$\frac{F_A}{\sin(\theta)} = \frac{T}{\sin[180\ \text{deg} - (\theta + \theta_1)]}$$

$$F_A = \frac{T\sin(\theta)}{\sin[180\ \text{deg} - (\theta + \theta_1)]}$$

$$F_A = 439\ \text{N}$$

$$\frac{F_B}{\sin(\theta_1)} = \frac{T}{\sin[180\ \text{deg} - (\theta + \theta_1)]}$$

$$F_B = T\frac{\sin(\theta_1)}{\sin[180\ \text{deg} - (\theta + \theta_1)]}$$

$$F_B = 311\ \text{N}$$

Problem 2-28

The beam is to be hoisted using two chains. If the resultant force is to be **F**, directed along the positive *y* axis, determine the magnitudes of forces $\mathbf{F_A}$ and $\mathbf{F_B}$ acting on each chain and the orientation θ of $\mathbf{F_B}$ so that the magnitude of $\mathbf{F_B}$ is a *minimum*.

Given:

$F = 600 \text{ N}$

$\theta_1 = 30 \text{ deg}$

Solution:

For minimum $\mathbf{F_B}$, require

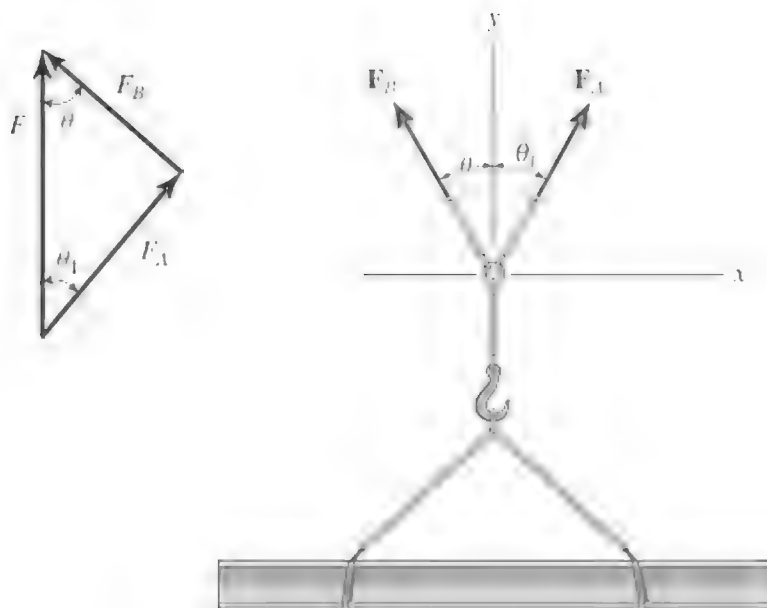
$\theta = 90 \text{ deg} - \theta_1$

$\theta = 60 \text{ deg}$

$F_A = F\cos(\theta_1)$

$F_A = 520 \text{ N}$

$F_B = F\sin(\theta_1)$

$F_B = 300 \text{ N}$

Problem 2-29

Three chains act on the bracket such that they create a resultant force having magnitude F_R. If two of the chains are subjected to known forces, as shown, determine the orientation θ of the third chain,measured clockwise from the positive *x* axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the *x-y* plane.What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.

Given:

$F_R = 500 \text{ lb}$

$F_1 = 200 \text{ lb}$

$F_2 = 300 \text{ lb}$

$\phi = 30 \text{ deg}$

Solution:

Cosine Law:

$$F_{R1} = \sqrt{F_1^2 + F_2^2 - 2 F_1 F_2 \cos(90 \text{ deg} - \phi)}$$

$F_{R1} = 264.6 \text{ lb}$

Sine Law: Make **F** parallel to $\mathbf{F}_{R1}$

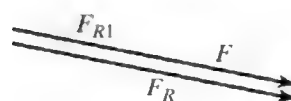
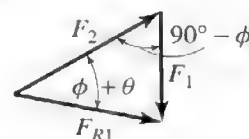
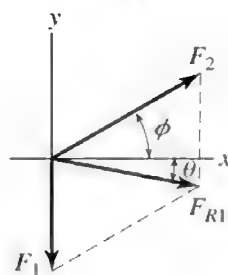
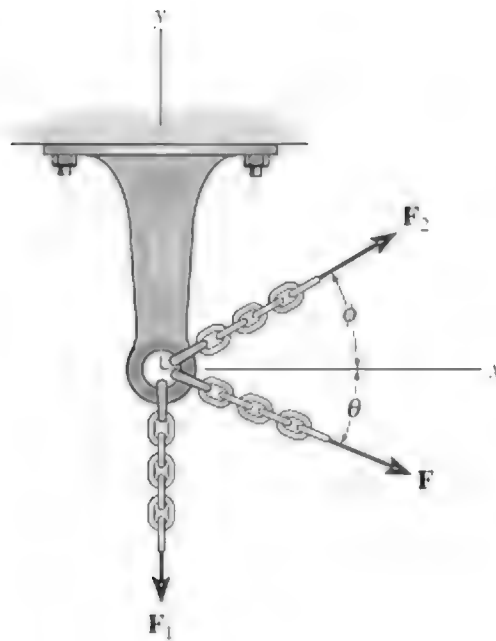
$$\frac{\sin(\phi + \theta)}{F_1} = \frac{\sin(90 \text{ deg} - \phi)}{F_{R1}}$$

$$\theta = -\phi + \operatorname{asin}\left(\sin(90 \text{ deg} - \phi) \frac{F_1}{F_{R1}}\right)$$

$\theta = 10.9 \text{ deg}$

When **F** is directed along $\mathbf{F}_{R1}$, **F** will be minimum to create the resultant forces.

$F = F_R - F_{R1}$

$F = 235 \text{ lb}$

Problem 2-30

Three cables pull on the pipe such that they create a resultant force having magnitude F_R. If two of the cables are subjected to known forces, as shown in the figure, determine the direction θ of the third cable so that the magnitude of force **F** in this cable is a *minimum*. All forces lie in the

x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces.

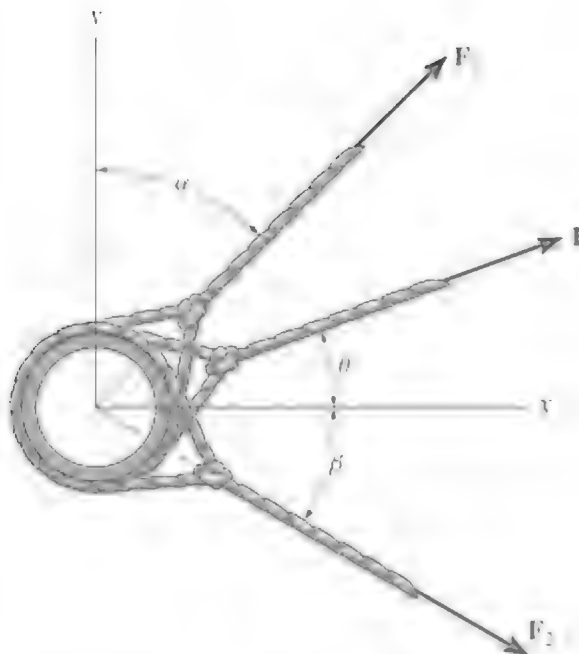
Given:

$F_R = 900$ lb

$F_1 = 600$ lb

$F_2 = 400$ lb

$\alpha = 45$ deg

$\beta = 30$ deg

Solution:

$$F' = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos(90 \text{ deg} + \alpha - \beta)}$$

$F' = 802.64$ lb

$F = F_R - F'$

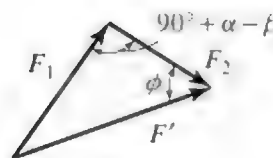
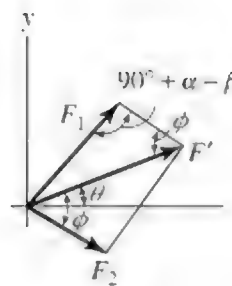
$F = 97.4$ lb

$$\frac{\sin(\phi)}{F_1} = \frac{\sin(90 \text{ deg} + \alpha - \beta)}{F'}$$

$$\phi = \operatorname{asin}\left(\sin(90 \text{ deg} + \alpha - \beta)\frac{F_1}{F'}\right)$$

$\phi = 46.22$ deg

$\theta = \phi - \beta$

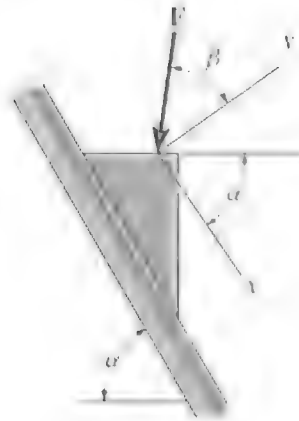
$\theta = 16.2$ deg

Problem 2-31

Determine the x and y components of the force **F**.

Given:

$F = 800 \text{ lb}$

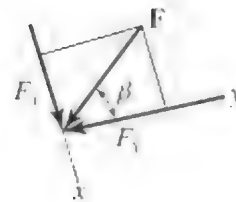
$\alpha = 60 \text{ deg}$

$\beta = 40 \text{ deg}$

Solution:

$F_x = F\sin(\beta)$

$F_y = -F\cos(\beta)$

$F_x = 514.2 \text{ lb}$

$F_y = -612.8 \text{ lb}$

Problem 2-32

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.

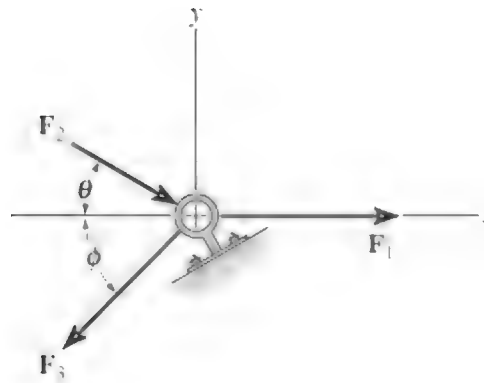
Given:

$F_1 = 70 \text{ N}$

$F_2 = 50 \text{ N}$

$F_3 = 65 \text{ N}$

$\theta = 30 \text{ deg}$

$\phi = 45 \text{ deg}$

Solution:

$\xrightarrow{+}$ $\quad F_{Rx} = \Sigma F_x; \qquad F_{RX} = F_1 + F_2\cos(\theta) - F_3\cos(\phi)$

$+\uparrow \quad F_{Ry} = \Sigma F_y; \qquad F_{RY} = -F_2 \sin(\theta) - F_3 \sin(\phi)$

$$F_R = \sqrt{F_{RX}^2 + F_{RY}^2}$$

$$\theta = \operatorname{atan}\left(\frac{|F_{RY}|}{|F_{RX}|}\right)$$

$F_R = 97.8$ N

$\theta = 46.5$ deg

Problem 2-33

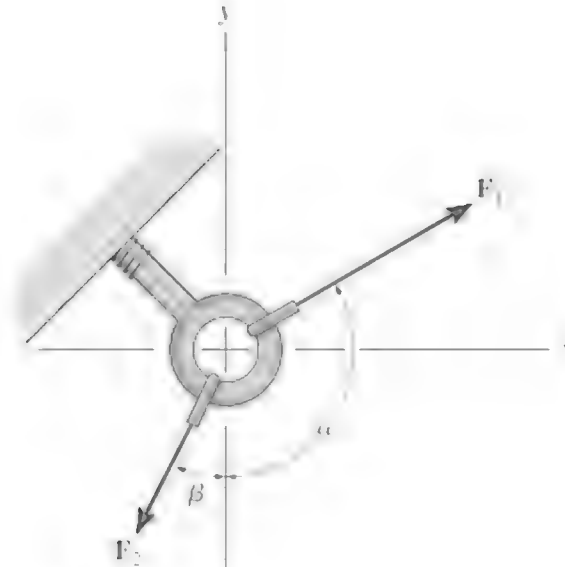
Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.

Given:

$F_1 = 50$ lb

$F_2 = 35$ lb

$\alpha = 120$ deg

$\beta = 25$ deg

Solution:

$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = F_1 \sin(\alpha) - F_2 \sin(\beta)$

$F_{Rx} = 28.5$ lb

$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -F_1 \cos(\alpha) - F_2 \cos(\beta)$

$F_{Ry} = -6.7\ \text{lb}$

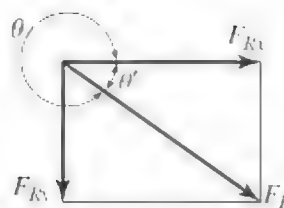
$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$

$F_R = 29.3\ \text{lb}$

$\theta' = \text{atan}\left(\frac{|F_{Ry}|}{|F_{Rx}|}\right)$

$\theta' = 13.3\ \text{deg}$

$\theta = 360\ \text{deg} - \theta'$

$\theta = 347\ \text{deg}$

Problem 2-34

Determine the magnitude of the resultant force and its direction , measured counterclockwise from the positive *x* axis.

Given:

$F_1 = 850\ \text{N}$

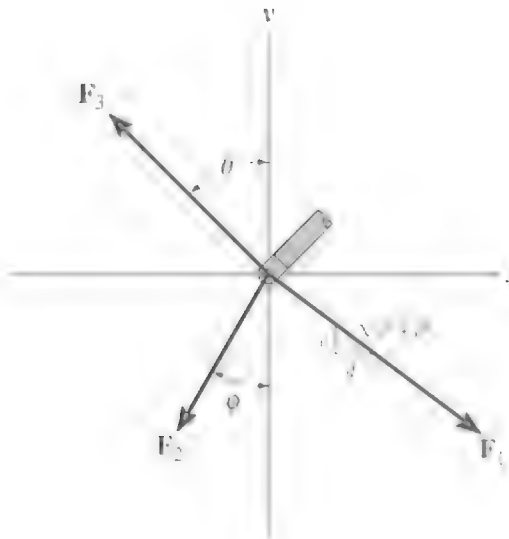
$F_2 = 625\ \text{N}$

$F_3 = 750\ \text{N}$

$\theta = 45\ \text{deg}$

$\phi = 30\ \text{deg}$

$c = 3$

$d = 4$

Solution:

$\xrightarrow{+}$ $F_{Rx} = SF_x;$ $\quad F_{RX} = F_1 \frac{d}{\sqrt{c^2 + d^2}} - F_2 \sin(\phi) - F_3 \sin(\theta)$

$+\uparrow \; F_{Ry} = SF_y;$ $\quad F_{RY} = -F_1 \dfrac{c}{\sqrt{c^2 + d^2}} - F_2 \cos(\phi) + F_3 \cos(\theta)$

$F_{RX} = -162.8 \text{ N}$ $\quad F_{RY} = -520.9 \text{ N}$

$F_R = \sqrt{F_{RX}^2 + F_{RY}^2}$ $\quad F_R = 546 \text{ N}$

$\alpha = \operatorname{atan}\left(\dfrac{F_{RY}}{F_{RX}}\right)$

$\alpha = 72.64 \deg$

$\beta = \alpha + 180 \deg$

$\beta = 252.6 \deg$

Problem 2-35

Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F_1}$ so that the resultant force is directed along the positive x' axis and has a magnitude of $\mathbf{F_R}$.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$F_R = 1 \text{ kN}$

$F_2 = 450 \text{ N}$

$F_3 = 200 \text{ N}$

$\alpha = 45 \deg$

$\beta = 30 \deg$

Solution:

$\overset{+}{\rightarrow} \; F_{Rx} = SF_x;$ $\quad F_R \cos(\beta) = F_3 + F_2 \cos(\alpha) + F_1 \cos(\theta + \beta)$

$+\uparrow \; F_{Ry} = SF_y;$ $\quad -F_R \sin(\beta) = F_2 \sin(\alpha) - F_1 \sin(\theta + \beta)$

$F_1 \cos(\theta + \beta) = F_R \cos(\beta) - F_3 - F_2 \cos(\alpha)$

$F_1 \sin(\theta + \beta) = F_2 \sin(\alpha) + F_R \sin(\beta)$

$$\theta = \operatorname{atan}\left(\frac{F_2 \sin(\alpha) + F_R \sin(\beta)}{F_R \cos(\beta) - F_3 - F_2 \cos(\alpha)}\right) - \beta$$

$\theta = 37 \text{ deg}$

$$F_1 = \sqrt{\left(F_R \cos(\beta) - F_3 - F_2 \cos(\alpha)\right)^2 + \left(F_2 \sin(\alpha) + F_R \sin(\beta)\right)^2}$$

$F_1 = 889 \text{ N}$

Problem 2-36

Determine the magnitude and direction, measured counterclockwise from the x' axis, of the resultant force of the three forces acting on the bracket.

Given:

$F_1 = 300 \text{ N}$

$F_2 = 450 \text{ N}$

$F_3 = 200 \text{ N}$

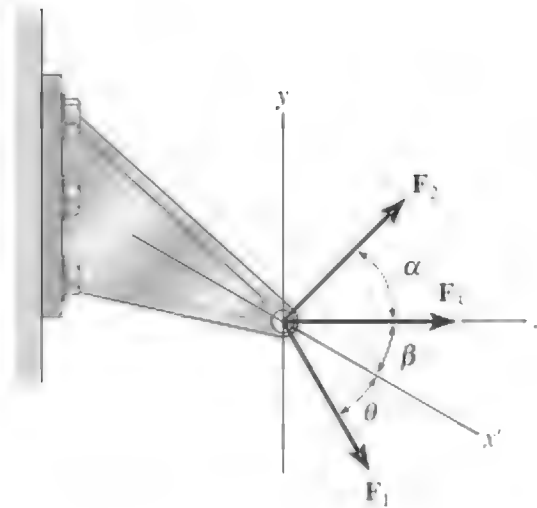
$\alpha = 45 \text{ deg}$

$\beta = 30 \text{ deg}$

$\theta = 20 \text{ deg}$

Solution:

$F_{Rx} = F_1 \cos(\theta + \beta) + F_3 + F_2 \cos(\alpha)$ $F_{Rx} = 711.03 \text{ N}$

$F_{Ry} = -F_1 \sin(\theta + \beta) + F_2 \sin(\alpha)$ $F_{Ry} = 88.38 \text{ N}$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $F_R = 717\ \text{N}$

ϕ (angle from x axis)

$\phi = \operatorname{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right)$ $\phi = 7.1\ \text{deg}$

ϕ' (angle from x' axis)

$\phi' = \beta + \phi$ $\phi' = 37.1\ \text{deg}$

Problem 2-37

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

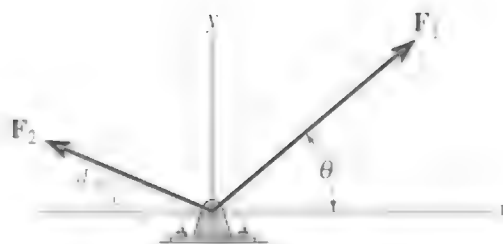
Given:

$F_1 = 800\ \text{N}$

$F_2 = 600\ \text{N}$

$\theta = 40\ \text{deg}$

$c = 12$

$d = 5$

Solution:

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $F_{Rx} = F_1 \cos(\theta) - F_2\left(\frac{c}{\sqrt{c^2 + d^2}}\right)$ $F_{Rx} = 59\ \text{N}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $F_{Ry} = F_1 \sin(\theta) + F_2\left(\frac{d}{\sqrt{c^2 + d^2}}\right)$ $F_{Ry} = 745\ \text{N}$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $F_R = 747\ \text{N}$

$$\theta = \operatorname{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \theta = 85.5\,\text{deg}$$

Problem 2-38

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

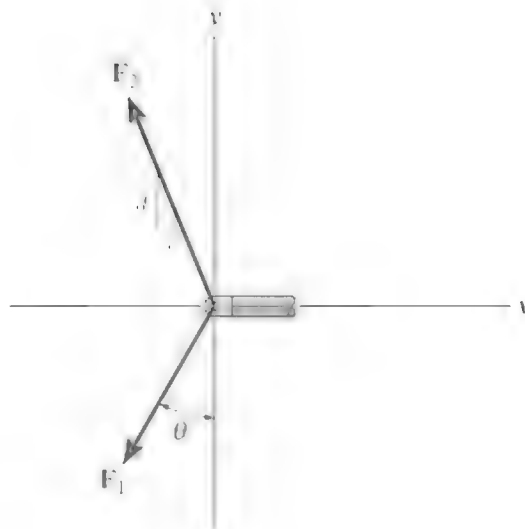
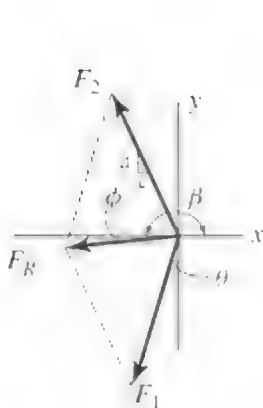
Given:

$F_1 = 30\ \text{kN}$

$F_2 = 26\ \text{kN}$

$\theta = 30\ \text{deg}$

$c = 5$

$d = 12$

Solution:

$\xrightarrow{+}\ F_{Rx} = \Sigma F_x;$ $$F_{Rx} = -F_1 \sin(\theta) - \left(\frac{c}{\sqrt{c^2 + d^2}}\right) F_2 \qquad F_{Rx} = -25\,\text{kN}$$

$+\uparrow\ F_{Ry} = \Sigma F_y;$ $$F_{Ry} = -F_1 \cos(\theta) + \left(\frac{d}{\sqrt{c^2 + d^2}}\right) F_2 \qquad F_{Ry} = -2\,\text{kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F_R = 25.1\,\text{kN}$$

$$\phi = \operatorname{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \phi = 4.5\,\text{deg}$$

$$\beta = 180\ \text{deg} + \phi \qquad \beta = 184.5\,\text{deg}$$

Problem 2-39

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.

Given:

$F_1 = 60\ \text{lb}$

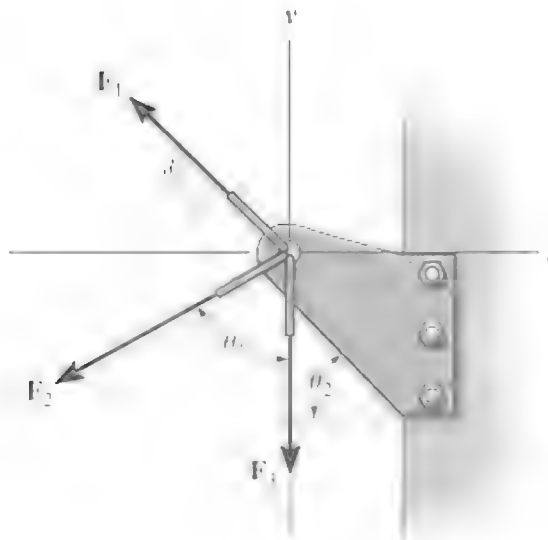
$F_2 = 70\ \text{lb}$

$F_3 = 50\ \text{lb}$

$\theta_1 = 60\ \text{deg}$

$\theta_2 = 45\ \text{deg}$

$c = 1$

$d = 1$

Solution:

$$\theta_3 = \text{atan}\left(\frac{d}{c}\right)$$

$$F_{Rx} = -F_1 \cos(\theta_3) - F_2 \sin(\theta_1) \qquad F_{Rx} = -103\ \text{lb}$$

$$F_{Ry} = F_1 \sin(\theta_3) - F_2 \cos(\theta_1) - F_3 \qquad F_{Ry} = -42.6\ \text{lb}$$

$$F_R = \sqrt{F_{Rx}^{\ 2} + F_{Ry}^{\ 2}} \qquad F_R = 111.5\ \text{lb}$$

$$\theta = 180\ \text{deg} + \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \theta = 202\ \text{deg}$$

Problem 2-40

Determine the magnitude of the resultant force $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ and its direction, measured counterclockwise from the positive x axis by summing the rectangular or x, y components of the forces to obtain the resultant force.

Given:

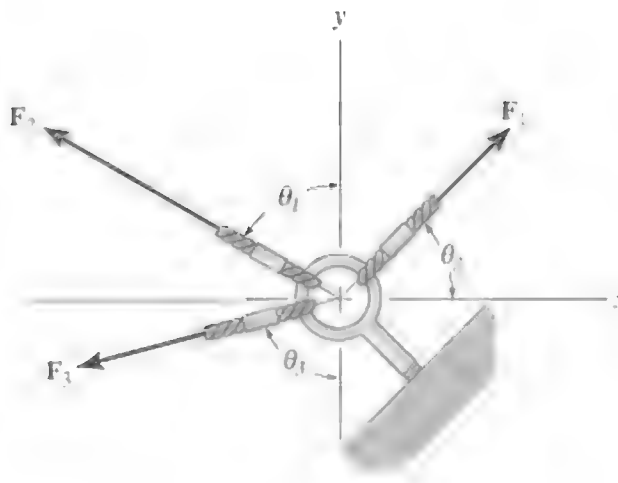
$F_1 = 600$ N

$F_2 = 800$ N

$F_3 = 450$ N

$\theta_1 = 60$ deg

$\theta_2 = 45$ deg

$\theta_3 = 75$ deg

Solution:

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $F_{Rx} = F_1 \cos(\theta_2) - F_2 \sin(\theta_1)$ $F_{Rx} = -268.556$ N

$+\uparrow F_{Ry} = \Sigma F_y;$ $F_{Ry} = F_1 \sin(\theta_2) + F_2 \cos(\theta_1)$ $F_{Ry} = 824.264$ N

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F_R = 867 \text{ N}$$

$$\theta = 180 \text{ deg} - \operatorname{atan}\left(\frac{|F_{Ry}|}{|F_{Rx}|}\right) \qquad \theta = 108 \text{ deg}$$

Problem 2-41

Determine the magnitude and direction of the resultant $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$ of the three forces by summing the rectangular or *x, y* components of the forces to obtain the resultant force.

Given:

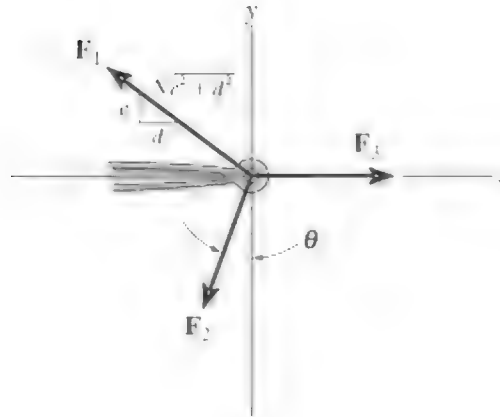
$F_1 = 30$ N

$F_2 = 20$ N

$F_3 = 50\ \text{N}$

$\theta = 20\ \text{deg}$

$c = 3$

$d = 4$

Solution:

$$F_{Rx} = -F_1\left(\frac{d}{\sqrt{c^2 + d^2}}\right) - F_2(\sin(\theta)) + F_3 \qquad F_{Rx} = 19.2\ \text{N}$$

$$F_{Ry} = F_1\left(\frac{c}{\sqrt{c^2 + d^2}}\right) - F_2\cos(\theta) \qquad F_{Ry} = -0.8\ \text{N}$$

$$F_R = \sqrt{F_{Rx}^{\ 2} + F_{Ry}^{\ 2}} \qquad F_R = 19.2\ \text{N}$$

$$\theta = \operatorname{atan}\left(\frac{-F_{Ry}}{F_{Rx}}\right) \qquad \theta = 2.4\ \text{deg}$$

Problem 2-42

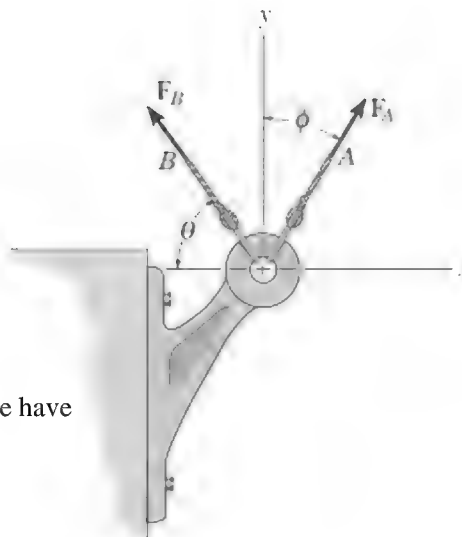
Determine the magnitude and orientation, measured counterclockwise from the positive *y* axis, of the resultant force acting on the bracket.

Given:

$F_A = 700\ \text{N}$

$F_B = 600\ \text{N}$

$\theta = 20\ \text{deg}$

$\phi = 30\ \text{deg}$

Solution:

Scalar Notation: Suming the force components algebraically, we have

$F_{Rx} = \Sigma F_x;\quad F_{Rx} = F_A \sin(\phi) - F_B \cos(\theta)$

$$F_{Rx} = -213.8\ \text{N}$$

$F_{Ry} = \Sigma F_y;\quad F_{Ry} = F_A \cos(\phi) + F_B \sin(\theta)$

$$F_{Ry} = 811.4\ \text{N}$$

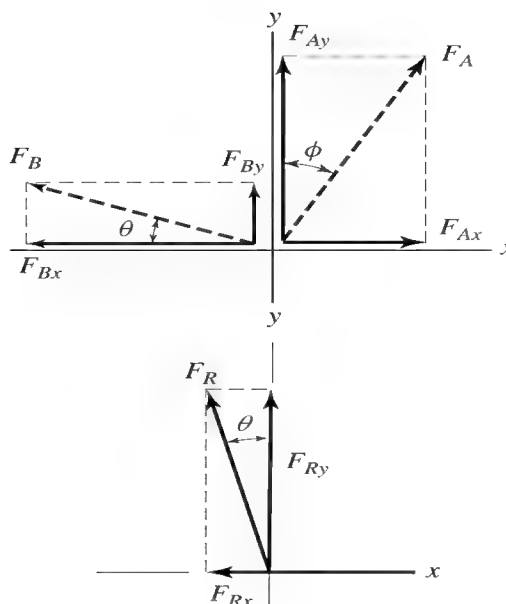
The magnitude of the resultant force F_R is

$$F_R = \sqrt{F_{Rx}^{\ 2} + F_{Ry}^{\ 2}}$$

$$F_R = 839\ \text{N}$$

The directional angle θ measured counterclockwise from the positive x axis is

$$\theta = \operatorname{atan}\left(\frac{|F_{Rx}|}{F_{Ry}}\right)$$

$$\theta = 14.8\ \text{deg}$$

Problem 2-43

Determine the magnitude and direction, measured counterclockwise from the positive x' axis, of the resultant force of the three forces acting on the bracket.

Given:

$F_1 = 300\ \text{N}$

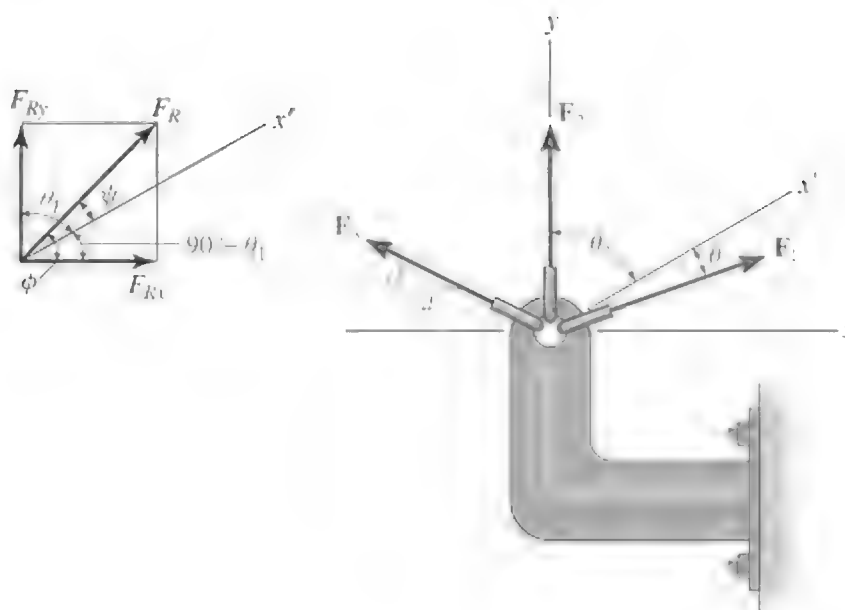
$F_2 = 200\ \text{N}$

$F_3 = 180\ \text{N}$

$\theta = 10\ \text{deg}$

$\theta_1 = 60\ \text{deg}$

$c = 5$

$d = 12$

Solution:

$\xrightarrow{+}\ F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = F_1 \sin(\theta_1 + \theta) - \left(\frac{d}{\sqrt{c^2 + d^2}}\right) F_3$ $\quad F_{Rx} = 115.8\ \text{N}$

$+\uparrow\ F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = F_1 \cos(\theta_1 + \theta) + F_2 + \left(\frac{c}{\sqrt{c^2 + d^2}}\right) F_3$ $\quad F_{Ry} = 371.8\ \text{N}$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $\quad F_R = 389\ \text{N}$

$\phi = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right)$ $\quad \phi = 72.7\ \text{deg}$

$\psi = \phi - (90\ \text{deg} - \theta_1)$ $\quad \psi = 42.7\ \text{deg}$

Problem 2-44

Determine the x and y components of $\mathbf{F_1}$ and $\mathbf{F_2}$.

Given:

$F_1 = 200$ N

$F_2 = 150$ N

$\theta = 45$ deg

$\phi = 30$ deg

Solution:

$$F_{1x} = F_1 \sin(\theta)$$

$$F_{1x} = 141.4 \text{ N}$$

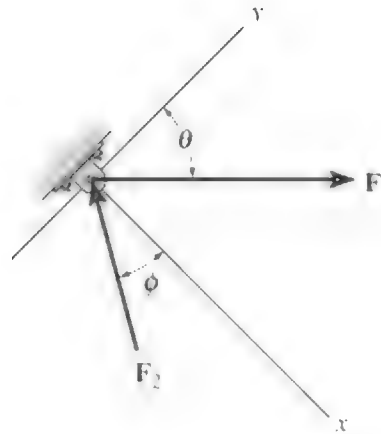
$$F_{1y} = F_1 \cos(\theta)$$

$$F_{1y} = 141.4 \text{ N}$$

$$F_{2x} = -F_2 \cos(\phi)$$

$$F_{2x} = -129.9 \text{ N}$$

$$F_{2y} = F_2 \sin(\phi)$$

$$F_{2y} = 75 \text{ N}$$

Problem 2-45

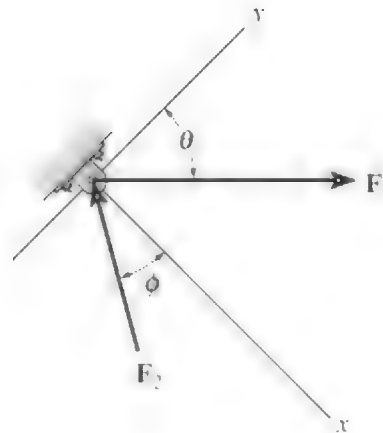
Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive *x* axis.

Given:

$F_1 = 200$ N

$F_2 = 150$ N

$\theta = 45$ deg

$\phi = 30$ deg

Solution:

$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $F_{Rx} = F_1 \sin(\theta) - F_2 \cos(\phi)$ $F_{Rx} = 11.5\text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $F_{Ry} = F_1 \cos(\theta) + F_2 \sin(\phi)$ $F_{Ry} = 216.4\text{ N}$

$$F = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F = 217\text{ N}$$

$$\beta = \operatorname{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \beta = 87\text{ deg}$$

Problem 2-46

Determine the *x* and *y* components of each force acting on the *gusset plate* of the bridge truss.

Given:

$F_1 = 200\text{ lb}$ $c = 3$

$F_2 = 400\text{ lb}$ $d = 4$

$F_3 = 300\text{ lb}$ $e = 3$

$F_4 = 300\text{ lb}$ $f = 4$

Solution:

$$F_{1x} = -F_1$$

$$F_{1x} = -200\text{ lb}$$

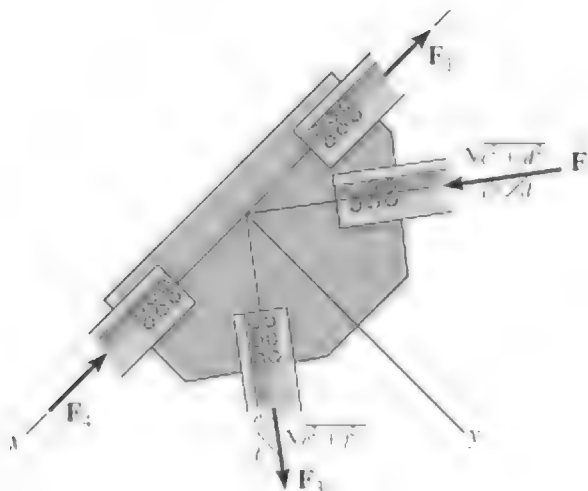
$$F_{1y} = 0\text{ lb}$$

$$F_{2x} = F_2\left(\frac{d}{\sqrt{c^2 + d^2}}\right)$$

$$F_{2x} = 320\text{ lb}$$

$$F_{2y} = -F_2\left(\frac{c}{\sqrt{c^2 + d^2}}\right)$$

$$F_{2y} = -240\text{ lb}$$

$$F_{3x} = F_3\left(\frac{e}{\sqrt{e^2 + f^2}}\right)$$

$F_{3x} = 180\,\text{lb}$

$F_{3y} = F_3\left(\dfrac{f}{\sqrt{e^2 + f^2}}\right)$

$F_{3y} = 240\,\text{lb}$

$F_{4x} = -F_4$

$F_{4x} = -300\,\text{lb}$

$F_{4y} = 0\ \text{lb}$

Problem 2-47

Determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F_1 = 20\ \text{kN}$

$F_2 = 40\ \text{kN}$

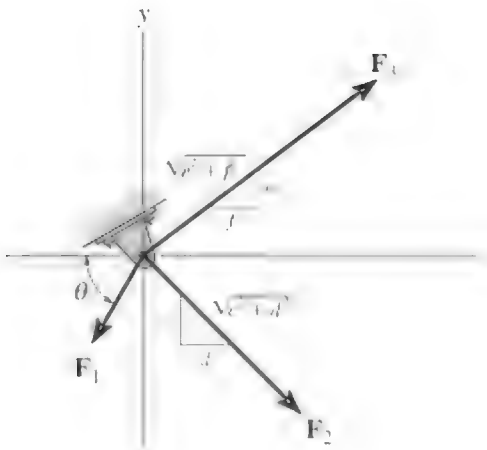
$F_3 = 50\ \text{kN}$

$\theta = 60\ \text{deg}$

$c = 1$

$d = 1$

$e = 3$

$f = 4$

Solution:

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = F_3\left(\dfrac{f}{\sqrt{e^2 + f^2}}\right) + F_2\left(\dfrac{d}{\sqrt{c^2 + d^2}}\right) - F_1\cos(\theta)$

$F_{Rx} = 58.28\,\text{kN}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = F_3\left(\frac{e}{\sqrt{e^2+f^2}}\right) - F_2\left(\frac{c}{\sqrt{c^2+d^2}}\right) - F_1 \sin(\theta)$

$F_{Ry} = -15.6\,\text{kN}$

$F = \sqrt{F_{Rx}{}^2 + F_{Ry}{}^2}$

$F = 60.3\,\text{kN}$

$\theta = \text{atan}\left(\frac{|F_{Ry}|}{F_{Rx}}\right)$

$\theta = 15\,\text{deg}$

Problem 2-48

Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F_1}$ so that the resultant force is directed along the positive x' axis and has magnitude $\mathbf{F_R}$.

Given:

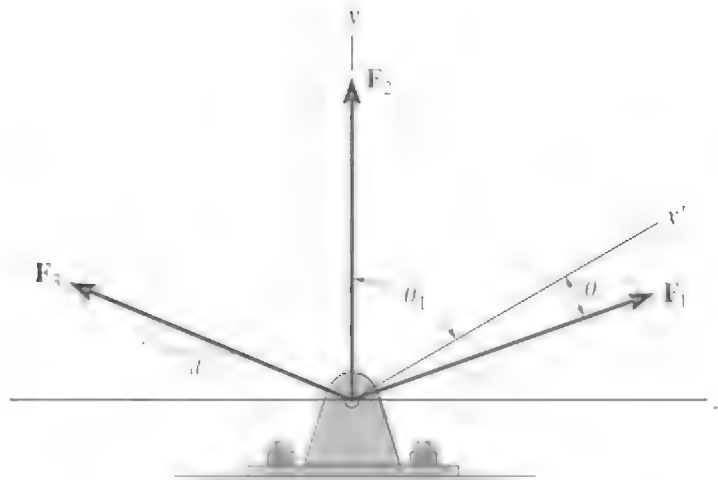
$F_2 = 200\text{ N}$

$F_3 = 180\text{ N}$

$\theta_1 = 60\text{ deg}$

$F_R = 800\text{ N}$

$c = 5$

$d = 12$

Solution:

Initial Guesses: $\quad F_1 = 100\text{ N} \qquad \theta = 10\text{ deg}$

Given

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_{x'};$ $\quad F_R \sin(\theta_1) = F_1 \sin(\theta_1 + \theta) - \left(\frac{d}{\sqrt{c^2+d^2}}\right)F_3$

$+\uparrow \quad F_{Ry} = \Sigma F_y; \qquad F_R(\cos(\theta_1)) = F_1 \cos(\theta_1 + \theta) + F_2 + \left(\frac{c}{\sqrt{c^2 + d^2}}\right) F_3$

$$\begin{pmatrix} F_1 \\ \theta \end{pmatrix} = \text{Find}(F_1, \theta)$$

$F_1 = 869\ \text{N} \qquad \theta = 21.3\,\text{deg}$

Problem 2-49

Determine the magnitude and direction, measured counterclockwise from the positive x' axis, of the resultant force acting on the bracket.

Given:

$F_1 = 300\ \text{N}$

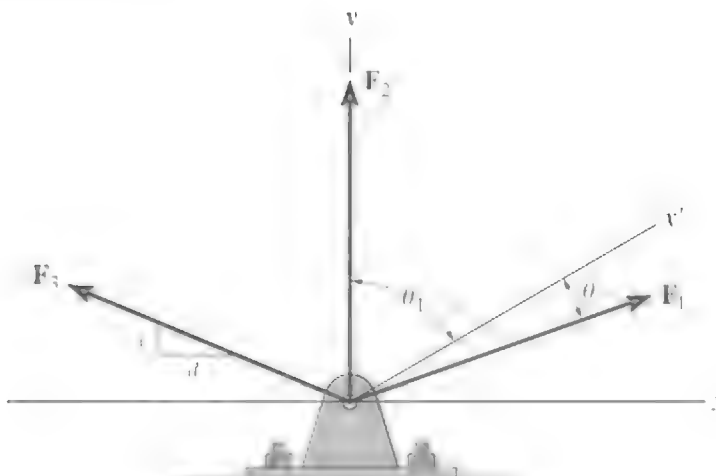
$F_2 = 200\ \text{N}$

$F_3 = 180\ \text{N}$

$\theta_1 = 60\ \text{deg}$

$\theta = 10\ \text{deg}$

$c = 5$

$d = 12$

Solution:

Guesses $\quad F_{Rx} = 100\ \text{N} \qquad F_{Ry} = 100\ \text{N}$

Given

$\overset{+}{\rightarrow} \quad F_{Rx} = \Sigma F_x; \qquad F_{Rx} = F_1 \sin(\theta_1 + \theta) - \left(\frac{d}{\sqrt{c^2 + d^2}}\right) F_3$

$+\uparrow \quad F_{Ry} = \Sigma F_y; \qquad F_{Ry} = F_1 \cos(\theta_1 + \theta) + F_2 + \left(\frac{c}{\sqrt{c^2 + d^2}}\right)(F_3)$

$$\begin{pmatrix} F_{Rx} \\ F_{Ry} \end{pmatrix} = \text{Find}(F_{Rx}, F_{Ry}) \qquad \begin{pmatrix} F_{Rx} \\ F_{Ry} \end{pmatrix} = \begin{pmatrix} 115.8 \\ 371.8 \end{pmatrix} \text{ N}$$

$$F_R = \sqrt{F_{Rx}^{\ 2} + F_{Ry}^{\ 2}} \qquad F_R = 389 \text{ N}$$

$$\phi = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \phi = 72.7 \text{ deg}$$

$$\phi' = [\phi - (90 \text{ deg} - \theta_1)] \qquad \phi' = 42.7 \text{ deg}$$

Problem 2-50

Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.

Given:

$F_1 = 150$ lb $\qquad \theta = 60$ deg

$F_2 = 275$ lb $\qquad c = 4$

$F_3 = 75$ lb $\qquad d = 3$

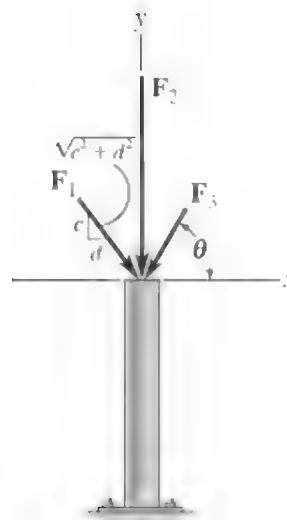
Solution:

Find the components of each force.

$$F_{1x} = F_1\left(\frac{d}{\sqrt{c^2 + d^2}}\right) \qquad F_{1y} = F_1\left(\frac{-c}{\sqrt{c^2 + d^2}}\right)$$

$$\mathbf{F_{1v}} = \begin{pmatrix} F_{1x} \\ F_{1y} \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 90 \\ -120 \end{pmatrix} \text{ lb}$$

$$F_{2x} = 0 \text{ lb} \qquad F_{2y} = -F_2$$

$$\mathbf{F_{2v}} = \begin{pmatrix} F_{2x} \\ F_{2y} \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 0 \\ -275 \end{pmatrix} \text{ lb}$$

$F_{3x} = -F_3 \cos(\theta)$ $\qquad F_{3y} = -F_3 \sin(\theta)$

$$\mathbf{F_{3v}} = \begin{pmatrix} F_{3x} \\ F_{3y} \end{pmatrix} \qquad \mathbf{F_{3v}} = \begin{pmatrix} -37.5 \\ -65 \end{pmatrix} \text{lb}$$

Now find the magnitude of the resultant force.

$$F_R = |\mathbf{F_{1v}} + \mathbf{F_{2v}} + \mathbf{F_{3v}}| \qquad F_R = 462.9 \text{ lb}$$

Problem 2-51

Determine the magnitude of force **F** so that the resultant $\mathbf{F_R}$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F_R}$?

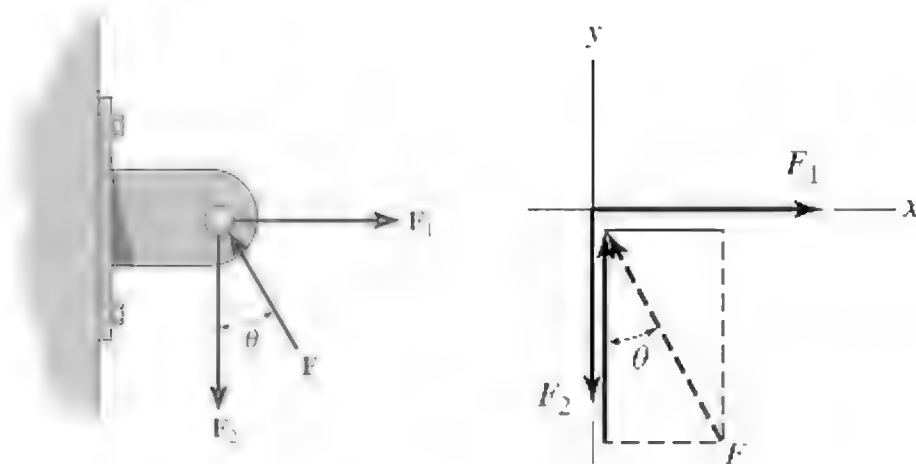
Units Used:

$\text{kN} = 1000 \text{ N}$

Given:

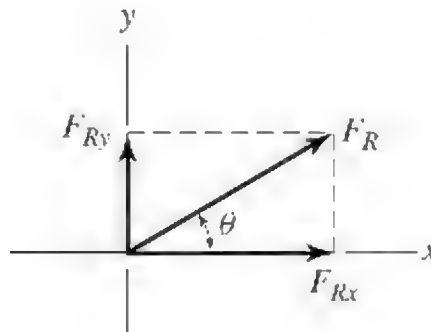
$F_1 = 5 \text{ kN}$

$F_2 = 4 \text{ kN}$

$\theta = 30 \text{ deg}$

Solution:

Scalar Notation: Suming the force components algebrically, we have

$\overset{+}{\rightarrow}$ $\quad F_{Rx} = \Sigma F_x; \quad F_{Rx} = F_1 - F \sin(\theta)$

$+\uparrow$ $\quad F_{Ry} = \Sigma F_y; \quad F_{Ry} = F \cos(\theta) - F_2$

The magnitude of the resultant force $\mathbf{F_R}$ is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(F_1 - F \sin(\theta))^2 + (F \cos(\theta) - F_2)^2}$$

$$F_R^2 = F_1^2 + F_2^2 + F^2 - 2FF_1\sin(\theta) - 2F_2F\cos(\theta)$$

$$2F_R\frac{dF_R}{dF} = 2F - 2F_1\sin(\theta) - 2F_2\cos(\theta)$$

If F is a minimum, then $\left(\frac{dF_R}{dF} = 0\right)$ $\quad F = F_1\sin(\theta) + F_2\cos(\theta)$ $\quad F = 5.96\,\text{kN}$

$$F_R = \sqrt{(F_1 - F\sin(\theta))^2 + (F\cos(\theta) - F_2)^2} \qquad F_R = 2.3\,\text{kN}$$

Problem 2-52

Express each of the three forces acting on the bracket in Cartesian vector form with respect to the x and y axes. Determine the magnitude and direction θ of $\mathbf{F_1}$ so that the resultant force is directed along the positive x' axis and has magnitude $\mathbf{F_R}$.

Units Used:

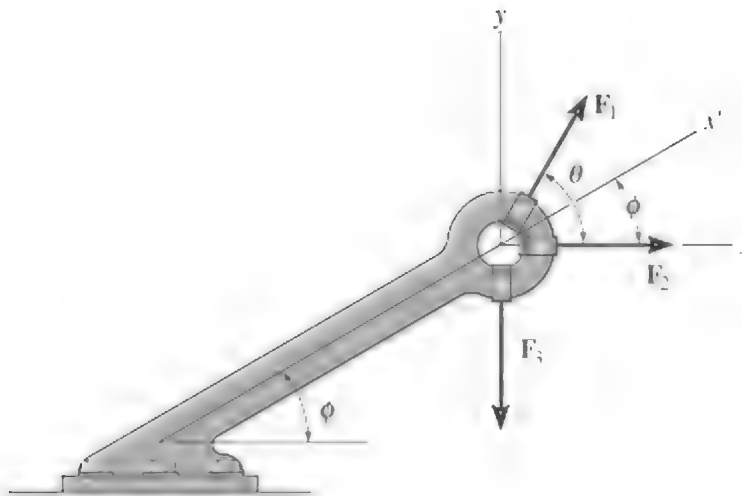
$\text{kN} = 1000\ \text{N}$

Given:

$F_R = 600\ \text{N}$

$F_2 = 350\ \text{N}$

$F_3 = 100\ \text{N}$

$\phi = 30\ \text{deg}$

Solution:

$$\mathbf{F_{2v}} = \begin{pmatrix} F_2 \\ 0 \end{pmatrix} \qquad \mathbf{F_{3v}} = \begin{pmatrix} 0 \\ -F_3 \end{pmatrix}$$

$$\mathbf{F_{1v}} = \begin{pmatrix} F_1\cos(\theta) \\ F_1\sin(\theta) \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 350 \\ 0 \end{pmatrix}\text{N} \qquad \mathbf{F_{3v}} = \begin{pmatrix} 0 \\ -100 \end{pmatrix}\text{N}$$

The initial guesses: $F_1 = 20\text{ N}$ $\theta = 10\text{ deg}$

Given

$$\begin{pmatrix} F_1 \cos(\theta) \\ F_1 \sin(\theta) \end{pmatrix} + \mathbf{F_{2v}} + \mathbf{F_{3v}} = \begin{pmatrix} F_R \cos(\phi) \\ F_R \sin(\phi) \end{pmatrix}$$

$$\begin{pmatrix} F_1 \\ \theta \end{pmatrix} = \text{Find}(F_1, \theta) \qquad F_1 = 434.5\text{ N} \qquad \theta = 67\text{ deg}$$

Problem 2-53

The three concurrent forces acting on the post produce a resultant force $\mathbf{F_R} = 0$. If $\mathbf{F_2} = (1/2)\mathbf{F_1}$, and $\mathbf{F_1}$ is to be 90° from $\mathbf{F_2}$ as shown, determine the required magnitude $\mathbf{F_3}$ expressed in terms of $\mathbf{F_1}$ and the angle θ.

Solution: Use the primed coordiates.

$\Sigma F_{Rx} = 0$ $\qquad F_3 \cos(\theta - 90\text{ deg}) - F_1 = 0$

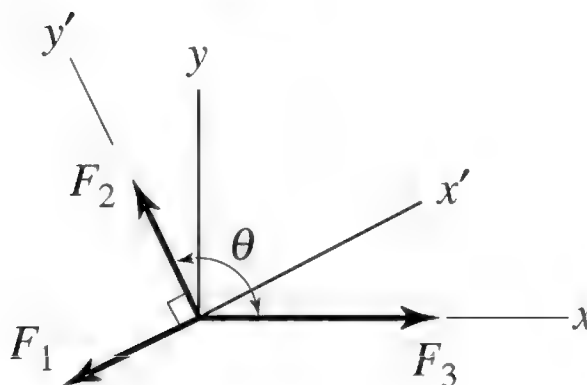
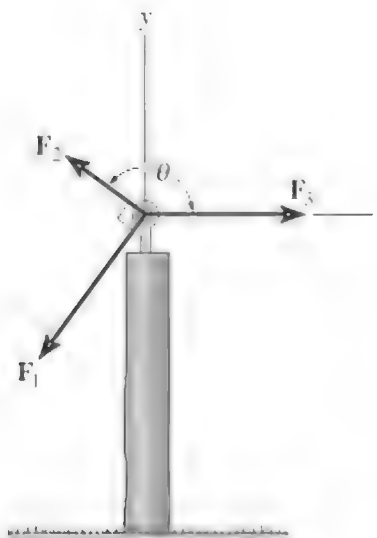
$\Sigma F_{Ry} = 0$ $\qquad -F_3 \sin(\theta - 90\text{ deg}) + F_2 = 0$

$$\tan(\theta - 90\text{ deg}) = \frac{F_2}{F_1} = \frac{1}{2}$$

$$\theta = 90\text{ deg} + \text{atan}\left(\frac{1}{2}\right)$$

$$\theta = 117\text{ deg}$$

$$k = \frac{1}{\cos(\theta - 90\text{ deg})}$$

$k = 1.1$ $\qquad F_3 = kF_1$

Problem 2-54

Three forces act on the bracket. Determine the magnitude and orientation θ of $\mathbf{F_2}$ so that the resultant force is directed along the positive u axis and has magnitude F_R.

Given:

$F_R = 50$ lb

$F_1 = 80$ lb

$F_3 = 52$ lb

$\phi = 25$ deg

$c = 12$

$d = 5$

Solution:

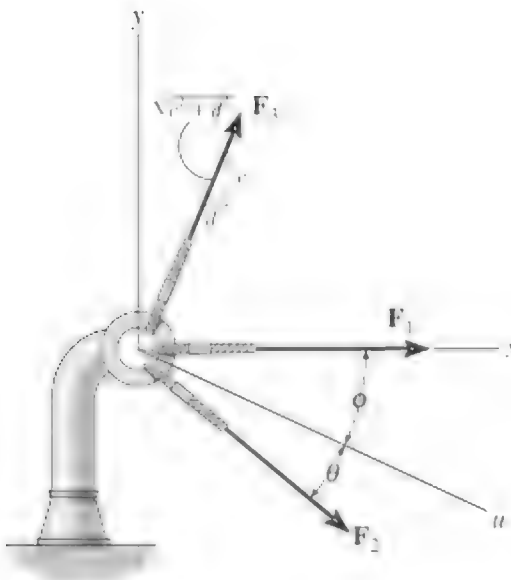
Guesses

$F_2 = 1$ lb $\quad \theta = 120$ deg

Given

$$F_R\cos(\phi) = F_1 + F_2\cos(\phi + \theta) + \left(\frac{d}{\sqrt{c^2 + d^2}}\right)F_3$$

$$-F_R\sin(\phi) = -F_2\sin(\phi + \theta) + \left(\frac{c}{\sqrt{c^2 + d^2}}\right)F_3$$

$$\begin{pmatrix} F_2 \\ \theta \end{pmatrix} = \text{Find}(F_2, \theta) \qquad F_2 = 88.1 \text{ lb} \qquad \theta = 103.3 \text{ deg}$$

Problem 2-55

Determine the magnitude and orientation, measured clockwise from the positive x axis, of the resultant force of the three forces acting on the bracket.

Given:

$F_1 = 80\text{ lb}$

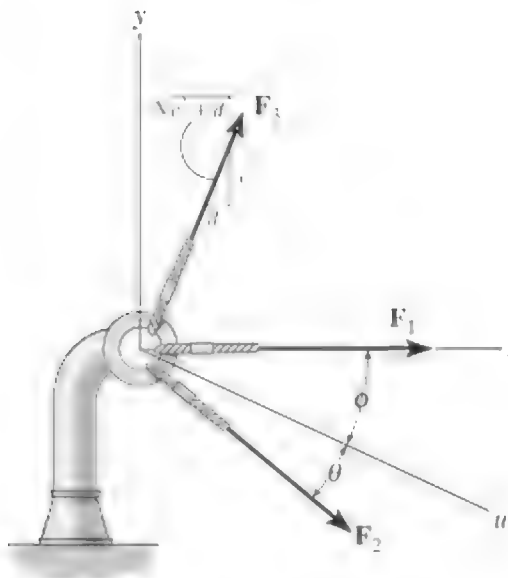
$F_2 = 150\text{ lb}$

$F_3 = 52\text{lb}$

$\theta = 55\text{ deg}$

$\phi = 25\text{ deg}$

$c = 12\text{ m}$

$d = 5\text{ m}$

Solution:

$$F_{Rx} = F_1 + F_3\left(\frac{d}{\sqrt{c^2 + d^2}}\right) + F_2\cos(\theta + \phi) \qquad F_{Rx} = 126.05\text{ lb}$$

$$F_{Ry} = F_3\left(\frac{c}{\sqrt{c^2 + d^2}}\right) - F_2\sin(\theta + \phi) \qquad F_{Ry} = -99.7\text{ lb}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F_R = 161\text{ lb}$$

$$\beta = \operatorname{atan}\left(\frac{|F_{Ry}|}{|F_{Rx}|}\right) \qquad \beta = 38.3\text{ deg}$$

Problem 2-56

Three forces act on the ring. Determine the range of values for the magnitude of **P** so that the magnitude of the resultant force does not exceed F. Force **P** is always directed to the right.

Units Used:

$kN = 10^3 \ N$

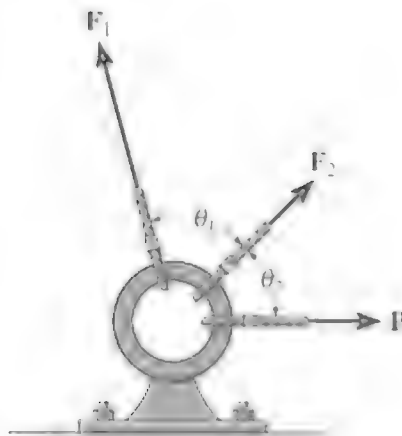
Given:

$F = 2500 \ N$

$F_1 = 1500 \ N$

$F_2 = 600 \ N$

$\theta_1 = 60 \ deg$

$\theta_2 = 45 \ deg$

Solution:

Initial Guesses: $F_{Rx} = 100 \ N$ $F_{Ry} = 100 \ N$ $P = 100 \ N$

Given

$\overset{+}{\rightarrow} \ F_{Rx} = \Sigma F_x;$ $F_{Rx} = P + F_2 \cos(\theta_2) + F_1 \cos(\theta_1 + \theta_2)$

$+\uparrow \ F_{Ry} = \Sigma F_y;$ $F_{Ry} = F_2 \sin(\theta_2) + F_1 \sin(\theta_1 + \theta_2)$

$$F = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\begin{pmatrix} F_{Rx} \\ F_{Ry} \\ P_{max} \end{pmatrix} = \text{Find}(F_{Rx}, F_{Ry}, P) \qquad P_{max} = 1.6 \ kN$$

Initial Guesses: $F_{Rx} = -100 \ N$ $F_{Ry} = 100 \ N$ $P = -2000 \ N$

Given

$\overset{+}{\rightarrow} \ F_{Rx} = \Sigma F_x;$ $F_{Rx} = P + F_2 \cos(\theta_2) + F_1 \cos(\theta_1 + \theta_2)$

$+\uparrow \ F_{Ry} = \Sigma F_y;$ $F_{Ry} = F_2 \sin(\theta_2) + F_1 \sin(\theta_1 + \theta_2)$

$$F = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\begin{pmatrix} F_{Rx} \\ F_{Ry} \\ P_{min} \end{pmatrix} = \text{Find}(F_{Rx}, F_{Ry}, P) \qquad P_{min} = -1.7 \ kN$$

Since $P > 0$ we conclude that $0 <= P <= P_{max} = 1.6\,\text{kN}$

Problem 2-57

Determine the magnitude and coordinate direction angles of $\mathbf{F_1}$ and $\mathbf{F_2}$. Sketch each force on an x, y, z reference.

Given:

$$\mathbf{F_1} = \begin{pmatrix} 60 \\ -50 \\ 40 \end{pmatrix} \text{N}$$

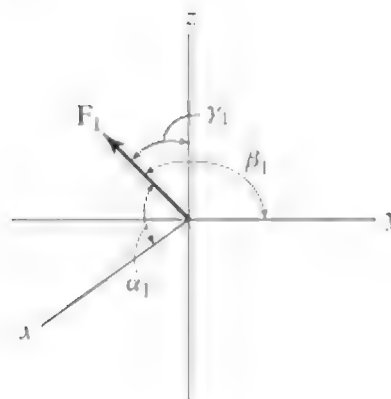
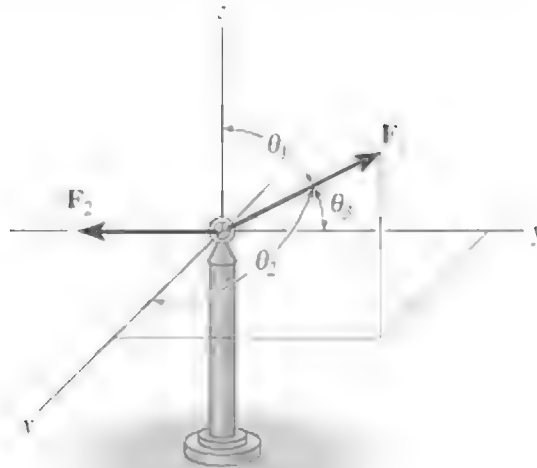
$$\mathbf{F_2} = \begin{pmatrix} -40 \\ -85 \\ 30 \end{pmatrix} \text{N}$$

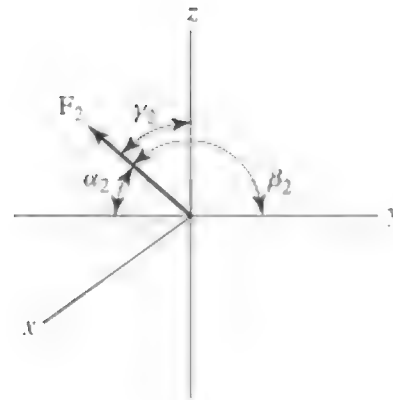
Solution:

$$|\mathbf{F_1}| = 87.7 \text{ N}$$

$$\begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{F_1}}{|\mathbf{F_1}|}\right)$$

$$\begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} 46.9 \\ 124.7 \\ 62.9 \end{pmatrix} \text{deg}$$

$$|\mathbf{F_2}| = 98.6 \text{ N}$$

$$\begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{F_2}}{|\mathbf{F_2}|}\right)$$

$$\begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 113.9 \\ 149.5 \\ 72.3 \end{pmatrix} \text{deg}$$

Problem 2-58

Express each force in Cartesian vector form.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F_1 = 5\ \text{kN}$

$F_2 = 2\ \text{kN}$

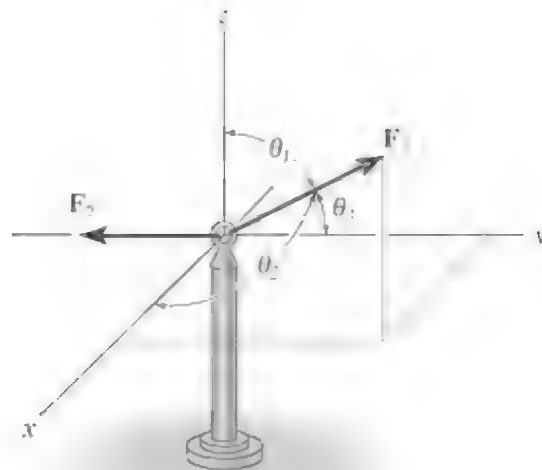
$\theta_1 = 60\ \text{deg}$

$\theta_2 = 60\ \text{deg}$

$\theta_3 = 45\ \text{deg}$

Solution:

$$\mathbf{F_{1v}} = F_1 \begin{pmatrix} \cos(\theta_2) \\ \cos(\theta_3) \\ \cos(\theta_1) \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 2.5 \\ 3.5 \\ 2.5 \end{pmatrix} \text{kN}$$

$$\mathbf{F_{2v}} = F_2 \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} \text{kN}$$

Problem 2-59

Determine the magnitude and coordinate direction angles of the force **F** acting on the stake.

Given:

$F_h = 40\ \text{N}$

$\theta = 70\ \text{deg}$

$c = 3$

$d = 4$

Solution:

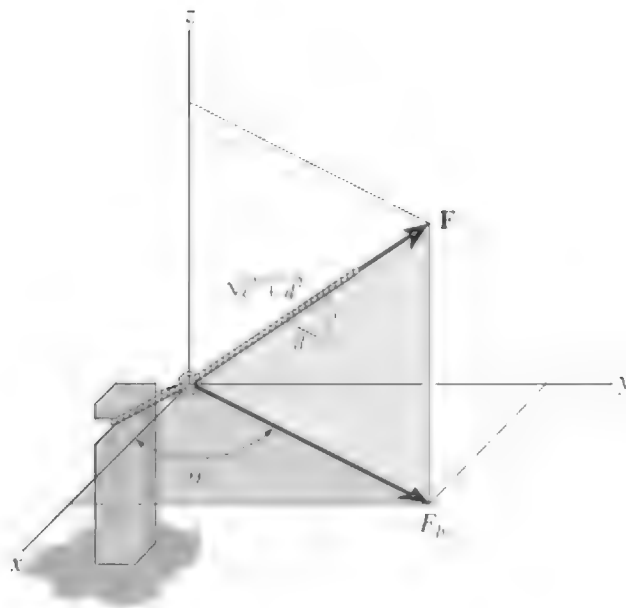
$$F = F_h\left(\frac{\sqrt{c^2 + d^2}}{d}\right)$$

$F = 50\ \text{N}$

$F_x = F_h \cos(\theta)$ $\quad F_y = F_h \sin(\theta)$ $\quad F_z = \left(\frac{c}{\sqrt{c^2 + d^2}}\right)F$

$F_x = 13.7\ \text{N}$ $\quad F_y = 37.6\ \text{N}$ $\quad F_z = 30\ \text{N}$

$\alpha = \text{acos}\left(\frac{F_x}{F}\right)$ $\quad \beta = \text{acos}\left(\frac{F_y}{F}\right)$ $\quad \gamma = \text{acos}\left(\frac{F_z}{F}\right)$

$\alpha = 74.1\ \text{deg}$ $\quad \beta = 41.3\ \text{deg}$ $\quad \gamma = 53.1\ \text{deg}$

Problem 2-60

Express each force in Cartesian vector form.

Given:

$F_1 = 400$ lb

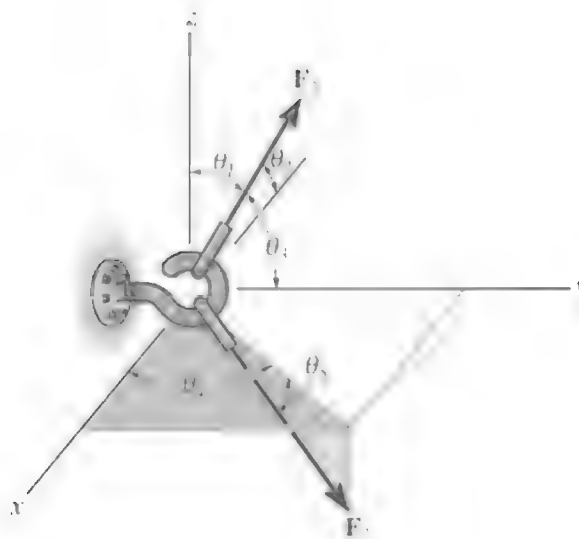
$F_2 = 600$ lb

$\theta_1 = 45$ deg

$\theta_2 = 60$ deg

$\theta_3 = 60$ deg

$\theta_4 = 45$ deg

$\theta_5 = 30$ deg

Solution:

$$\mathbf{F_{1v}} = F_1 \begin{pmatrix} -\cos(\theta_2) \\ \cos(\theta_3) \\ \cos(\theta_1) \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} -200 \\ 200 \\ 282.8 \end{pmatrix} \text{lb}$$

$$\mathbf{F_{2v}} = F_2 \begin{pmatrix} \cos(\theta_5)\cos(\theta_4) \\ \cos(\theta_5)\sin(\theta_4) \\ -\sin(\theta_5) \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 367.4 \\ 367.4 \\ -300 \end{pmatrix} \text{lb}$$

Problem 2-61

The stock *S* mounted on the lathe is subjected to a force **F**, which is caused by the die *D*. Determine the coordinate direction angle β and express the force as a Cartesian vector.

Given:

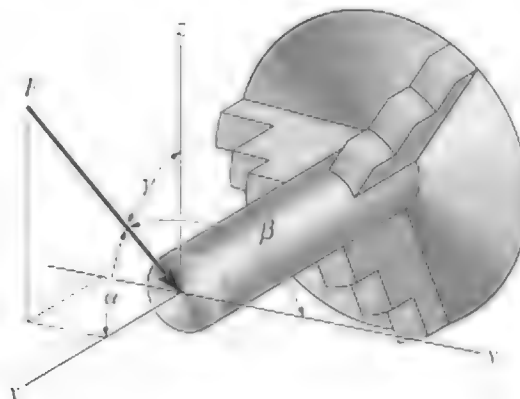
$F = 60$ N

$\alpha = 60$ deg

$\gamma = 30$ deg

Solution:

$$\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$$

$$\beta = \operatorname{acos}\left(1 - \cos(\alpha)^2 - \cos(\gamma)^2\right) \qquad \beta = 90\,\text{deg}$$

$$\mathbf{F_v} = F\begin{pmatrix} -\cos(\alpha) \\ -\cos(\beta) \\ -\cos(\gamma) \end{pmatrix} \qquad \mathbf{F_v} = \begin{pmatrix} -30 \\ 0 \\ -52 \end{pmatrix}\text{N}$$

Problem 2-62

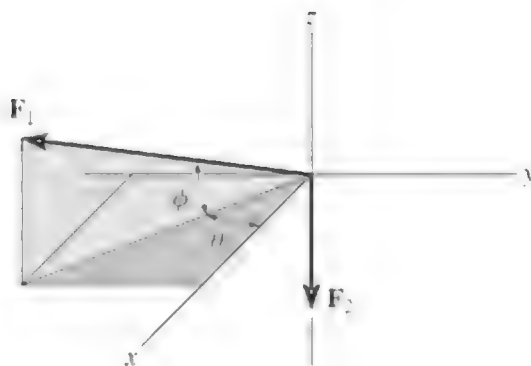
Determine the magnitude and coordinate direction angles of the resultant force.

Given:

$F_1 = 80$ lb

$F_2 = 130$ lb

$\theta = 40$ deg

$\phi = 30$ deg

Solution:

$$\mathbf{F_{1v}} = F_1\begin{pmatrix} \cos(\phi)\cos(\theta) \\ -\cos(\phi)\sin(\theta) \\ \sin(\phi) \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 53.1 \\ -44.5 \\ 40 \end{pmatrix}\text{lb}$$

$$\mathbf{F_{2v}} = F_2\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 0 \\ 0 \\ -130 \end{pmatrix}\text{lb}$$

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}} \qquad \left|\mathbf{F_R}\right| = 113.6\,\text{lb}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{F_R}}{\left|\mathbf{F_R}\right|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 62.1 \\ 113.1 \\ 142.4 \end{pmatrix}\text{deg}$$

Problem 2-63

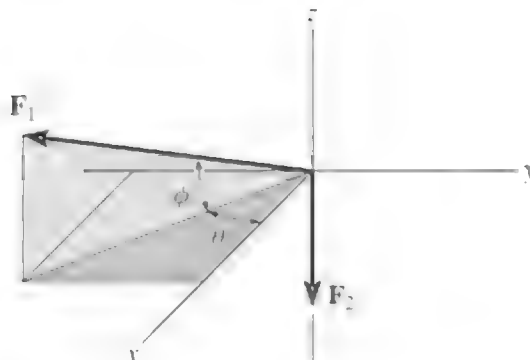
Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a cartesian vector.

Given:

$F_1 = 80 \text{ lb}$

$F_2 = 130 \text{ lb}$

$\phi = 30 \text{ deg}$

$\theta = 40 \text{ deg}$

Solution:

$$\mathbf{F_{1v}} = F_1 \begin{pmatrix} \cos(\phi)\cos(\theta) \\ -\cos(\phi)\sin(\theta) \\ \sin(\phi) \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 53.1 \\ -44.5 \\ 40 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_{1v}}}{F_1}\right) \qquad \begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} 48.4 \\ 123.8 \\ 60 \end{pmatrix} \text{deg}$$

$$\mathbf{F_{2v}} = F_2 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 0 \\ 0 \\ -130 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_{2v}}}{F_2}\right) \qquad \begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 90 \\ 90 \\ 180 \end{pmatrix} \text{deg}$$

Problem 2-64

The mast is subjected to the three forces shown. Determine the coordinate angles α_1, β_1, γ_1 of $\mathbf{F}_1$ so that the resultant force acting on the mast is $F_R\mathbf{i}$.

Given:

$F_R = 350 \text{ N}$

$F_1 = 500\ \text{N}$

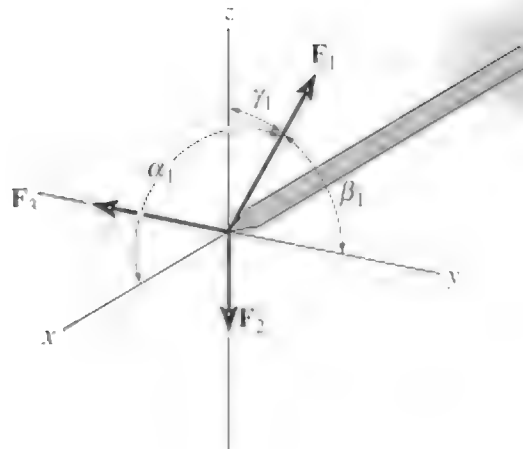
$F_2 = 200\ \text{N}$

$F_3 = 300\ \text{N}$

Solution:

Guesses $\alpha = 20\ \text{deg}$

$\beta = 20\ \text{deg}$

$\gamma = 20\ \text{deg}$

Given $$F_1\begin{pmatrix}\cos(\alpha)\\ \cos(\beta)\\ \cos(\gamma)\end{pmatrix} + F_2\begin{pmatrix}0\\0\\-1\end{pmatrix} + F_3\begin{pmatrix}0\\-1\\0\end{pmatrix} = F_R\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = \text{Find}(\alpha, \beta, \gamma) \qquad \begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = \begin{pmatrix}45.6\\53.1\\66.4\end{pmatrix}\text{deg}$$

Problem 2-65

The mast is subjected to the three forces shown. Determine the coordinate angles α_1, β_1, γ_1 of $\mathbf{F_1}$ so that the resultant force acting on the mast is zero.

Given:

$F_1 = 500\ \text{N}$

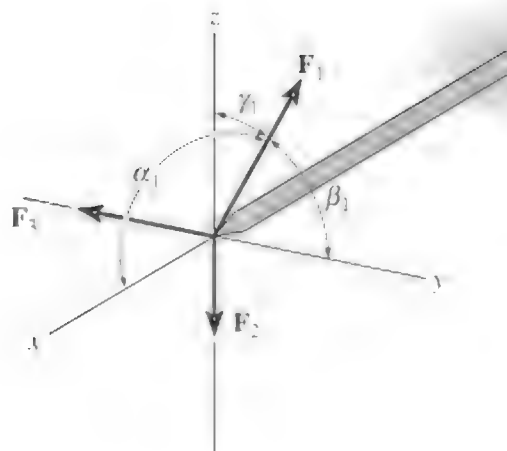
$F_2 = 200\ \text{N}$

$F_3 = 300\ \text{N}$

Solution:

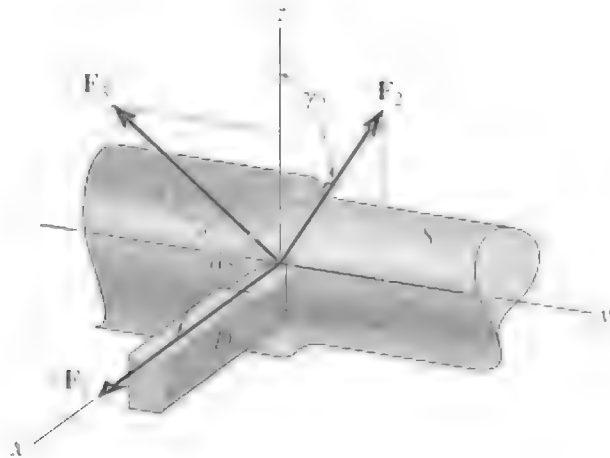
Guesses $\alpha = 20\ \text{deg}$

$\beta = 20\ \text{deg}$

$\gamma = 20\ \text{deg}$

Given $F_1\begin{pmatrix}\cos(\alpha)\\ \cos(\beta)\\ \cos(\gamma)\end{pmatrix} + F_2\begin{pmatrix}0\\ 0\\ -1\end{pmatrix} + F_3\begin{pmatrix}0\\ -1\\ 0\end{pmatrix} = 0$

$\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = \text{Find}(\alpha,\beta,\gamma)$ $\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} = \begin{pmatrix}90\\ 53.1\\ 66.4\end{pmatrix}\text{deg}$

Problem 2-66

The shaft S exerts three force components on the die D. Find the magnitude and direction of the resultant force. Force $\mathbf{F_2}$ acts within the octant shown.

Given:

$F_1 = 400\text{ N}$

$F_2 = 300\text{ N}$

$F_3 = 200\text{ N}$

$\alpha_2 = 60\text{ deg}$

$\gamma_2 = 60\text{ deg}$

$c = 3$

$d = 4$

Solution:

$\cos(\alpha_2)^2 + \cos(\beta_2)^2 + \cos(\gamma_2)^2 = 1$

Solving for the positive root,

$\beta_2 = \text{acos}\left(\sqrt{1 - \cos(\alpha_2)^2 - \cos(\gamma_2)^2}\right)$ $\beta_2 = 45\text{ deg}$

$\mathbf{F_{1v}} = \begin{pmatrix}F_1\\ 0\\ 0\end{pmatrix}$ $\mathbf{F_{2v}} = F_2\begin{pmatrix}\cos(\alpha_2)\\ \cos(\beta_2)\\ \cos(\gamma_2)\end{pmatrix}$ $\mathbf{F_{3v}} = \dfrac{F_3}{\sqrt{c^2+d^2}}\begin{pmatrix}0\\ -d\\ c\end{pmatrix}$

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}} + \mathbf{F_{3v}}$$ $$|\mathbf{F_R}| = 615 \text{ N}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_R}}{|\mathbf{F_R}|}\right)$$ $$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 26.6 \\ 85.1 \\ 64.0 \end{pmatrix} \text{deg}$$

Problem 2-67

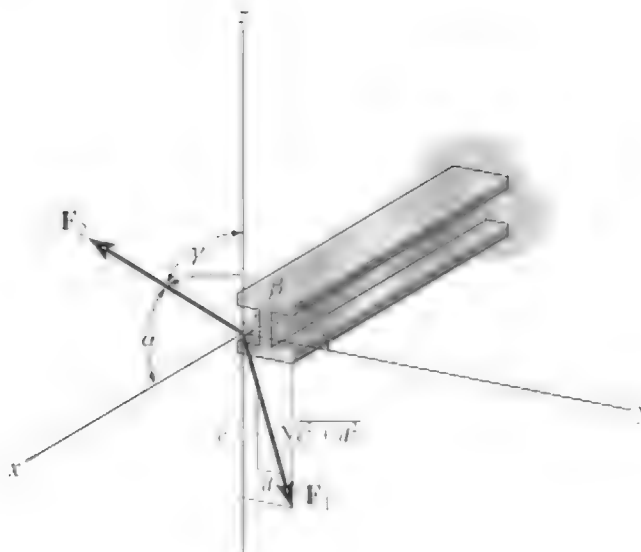
The beam is subjected to the two forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.

Given:

$F_1 = 630$ lb $\alpha = 60$ deg

$F_2 = 250$ lb $\beta = 135$ deg

$c = 24$ $\gamma = 60$ deg

$d = 7$

Solution:

$$\mathbf{F_{1v}} = \frac{F_1}{\sqrt{c^2 + d^2}}\begin{pmatrix} 0 \\ d \\ -c \end{pmatrix}$$ $$\mathbf{F_{1v}} = \begin{pmatrix} 0 \\ 176.4 \\ -604.8 \end{pmatrix} \text{lb}$$

$$\mathbf{F_{2v}} = F_2\begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix}$$ $$\mathbf{F_{2v}} = \begin{pmatrix} 125 \\ -176.8 \\ 125 \end{pmatrix} \text{lb}$$

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}}$$ $$|\mathbf{F_R}| = 495.8 \text{ lb}$$

$$\begin{pmatrix} \alpha_R \\ \beta_R \\ \gamma_R \end{pmatrix} = \mathrm{acos}\left(\frac{\mathbf{F_R}}{|\mathbf{F_R}|}\right) \qquad \begin{pmatrix} \alpha_R \\ \beta_R \\ \gamma_R \end{pmatrix} = \begin{pmatrix} 75.4 \\ 90 \\ 165.4 \end{pmatrix} \text{deg}$$

Problem 2-68

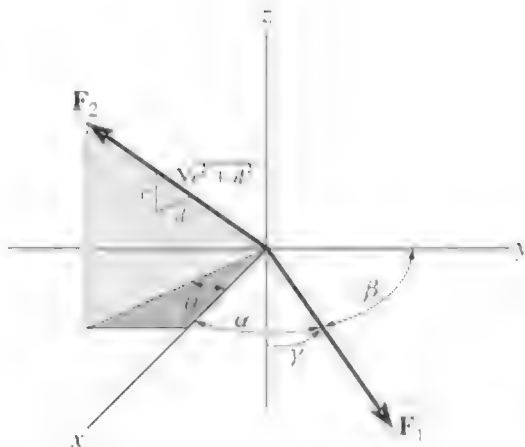
Determine the magnitude and coordinate direction angles of the resultant force.

Given:

$F_1 = 350$ N $\quad \alpha = 60$ deg

$F_2 = 250$N $\quad \beta = 60$ deg

$c = 3 \quad \gamma = 45$ deg

$d = 4 \quad \theta = 30$ deg

Solution:

$$\mathbf{F_{1v}} = F_1 \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ -\cos(\gamma) \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 175 \\ 175 \\ -247.5 \end{pmatrix} \text{N}$$

$$F_{2h} = F_2\left(\frac{d}{\sqrt{c^2 + d^2}}\right) \qquad F_{2y} = F_2\left(\frac{c}{\sqrt{c^2 + d^2}}\right)$$

$$\mathbf{F_{2v}} = \begin{pmatrix} F_{2h}\cos(\theta) \\ -F_{2h}\sin(\theta) \\ F_{2y} \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 173.2 \\ -100 \\ 150 \end{pmatrix} \text{N}$$

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}} \qquad |\mathbf{F_R}| = 369.3 \text{ N}$$

$$\begin{pmatrix} \alpha_R \\ \beta_R \\ \gamma_R \end{pmatrix} = \mathrm{acos}\left(\frac{\mathbf{F_R}}{|\mathbf{F_R}|}\right) \qquad \begin{pmatrix} \alpha_R \\ \beta_R \\ \gamma_R \end{pmatrix} = \begin{pmatrix} 19.5 \\ 78.3 \\ 105.3 \end{pmatrix} \text{deg}$$

Problem 2-69

Determine the magnitude and coordinate direction angles of $\mathbf{F_3}$ so that the resultant of the three

forces acts along the positive y axis and has magnitude F.

Given:

$F = 600$ lb

$F_1 = 180$ lb

$F_2 = 300$ lb

$\alpha_1 = 30$ deg

$\alpha_2 = 40$ deg

Solution:

Initial guesses:

$\alpha = 40$ deg $\quad \gamma = 50$ deg

$\beta = 50$ deg $\quad F_3 = 45$ lb

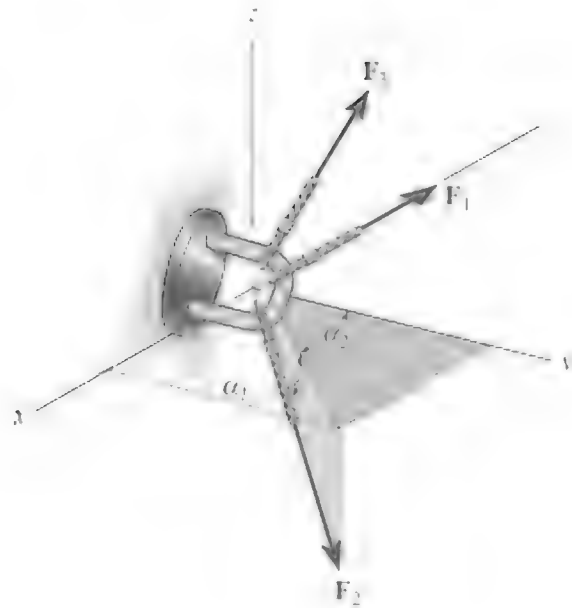
Given

$F_{Rx} = \Sigma F_x;\quad 0 = -F_1 + F_2\cos(\alpha_1)\sin(\alpha_2) + F_3\cos(\alpha)$

$F_{Ry} = \Sigma F_y;\quad F = F_2\cos(\alpha_1)\cos(\alpha_2) + F_3\cos(\beta)$

$F_{Rz} = \Sigma F_z;\quad 0 = -F_2\sin(\alpha_1) + F_3\cos(\gamma)$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma) \qquad F_3 = 428\,\text{lb} \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 88.3 \\ 20.6 \\ 69.5 \end{pmatrix}\text{deg}$$

Problem 2-70

Determine the magnitude and coordinate direction angles of $\mathbf{F_3}$ so that the resultant of the three forces is zero.

Given:

$F_1 = 180\ \text{lb} \quad \alpha_1 = 30\ \text{deg}$

$F_2 = 300\ \text{lb} \quad \alpha_2 = 40\ \text{deg}$

Solution:

Initial guesses:

$\alpha = 40\ \text{deg} \quad \gamma = 50\ \text{deg}$

$\beta = 50\ \text{deg} \quad F_3 = 45\ \text{lb}$

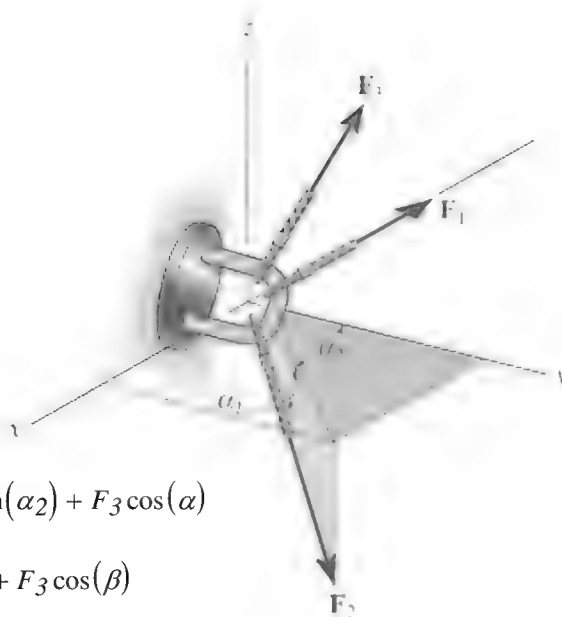
Given

$F_{Rx} = \Sigma F_x; \qquad 0 = -F_1 + F_2\cos(\alpha_1)\sin(\alpha_2) + F_3\cos(\alpha)$

$F_{Ry} = \Sigma F_y; \qquad 0 = F_2\cos(\alpha_1)\cos(\alpha_2) + F_3\cos(\beta)$

$F_{Rz} = \Sigma F_z; \qquad 0 = -F_2\sin(\alpha_1) + F_3\cos(\gamma)$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma) \qquad F_3 = 250\,\text{lb} \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 87.0 \\ 142.9 \\ 53.1 \end{pmatrix}\text{deg}$$

Problem 2-71

Specify the magnitude F_3 and directions α_3, β_3, and γ_3 of $\mathbf{F_3}$ so that the resultant force of the three forces is $\mathbf{F_R}$.

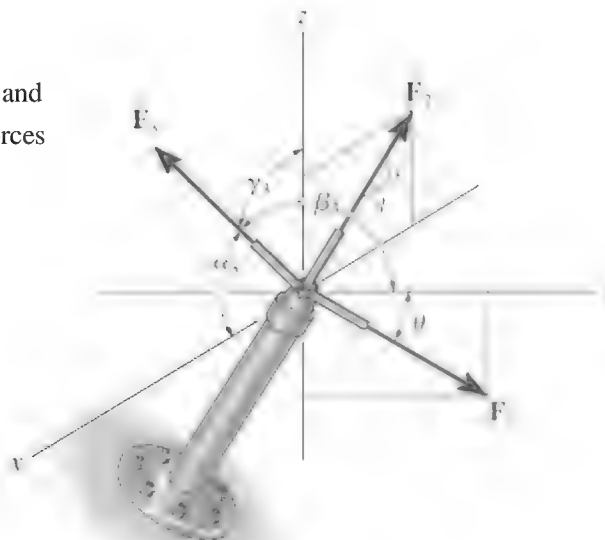
Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F_1 = 12\ \text{kN} \qquad c = 5$

$F_2 = 10\ \text{kN} \qquad d = 12$

$\theta = 30\ \text{deg}$

$$\mathbf{F_R} = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix} \text{kN}$$

Solution:

Initial Guesses: $F_{3x} = 1 \text{ kN}$ $F_{3y} = 1 \text{ kN}$ $F_{3z} = 1 \text{ kN}$

Given $$\mathbf{F_R} = \begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} + F_1 \begin{pmatrix} 0 \\ \cos(\theta) \\ -\sin(\theta) \end{pmatrix} + \frac{F_2}{\sqrt{c^2 + d^2}} \begin{pmatrix} -d \\ 0 \\ c \end{pmatrix}$$

$$\begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} = \text{Find}(F_{3x}, F_{3y}, F_{3z}) \qquad \mathbf{F_3} = \begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} \qquad \mathbf{F_3} = \begin{pmatrix} 9.2 \\ -1.4 \\ 2.2 \end{pmatrix} \text{kN} \qquad |\mathbf{F_3}| = 9.6 \text{ kN}$$

$$\begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_3}}{|\mathbf{F_3}|}\right) \qquad \begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 15.5 \\ 98.4 \\ 77.0 \end{pmatrix} \text{deg}$$

Problem 2-72

The pole is subjected to the force **F**, which has components acting along the *x*,*y*,*z* axes as shown. Given β and γ, determine the magnitude of the three components of **F**.

Units Used:

$\text{kN} = 1000 \text{ N}$

Given:

$F = 3 \text{ kN}$

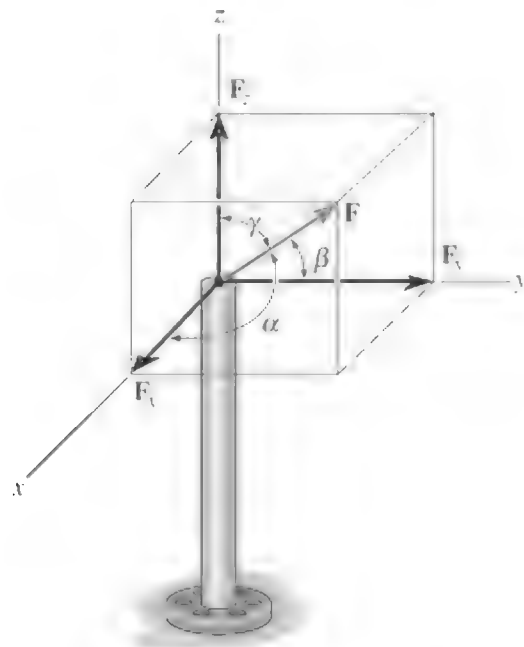
$\beta = 30 \text{ deg}$

$\gamma = 75 \text{ deg}$

Solution:

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

$\alpha = \mathrm{acos}\left(\sqrt{-\cos(\beta)^2 - \cos(\gamma)^2 + 1}\right)$

$\alpha = 64.67\ \mathrm{deg}$

$F_x = F\cos(\alpha) \qquad F_y = F\cos(\beta) \qquad F_z = F\cos(\gamma)$

$F_x = 1.28\ \mathrm{kN} \qquad F_y = 2.60\ \mathrm{kN} \qquad F_z = 0.8\ \mathrm{kN}$

Problem 2-73

The pole is subjected to the force **F** which has components $\mathbf{F_x}$ and $\mathbf{F_z}$. Determine the magnitudes of **F** and $\mathbf{F_y}$.

Units Used:

$\mathrm{kN} = 1000\ \mathrm{N}$

Given:

$F_x = 1.5\ \mathrm{kN}$

$F_z = 1.25\ \mathrm{kN}$

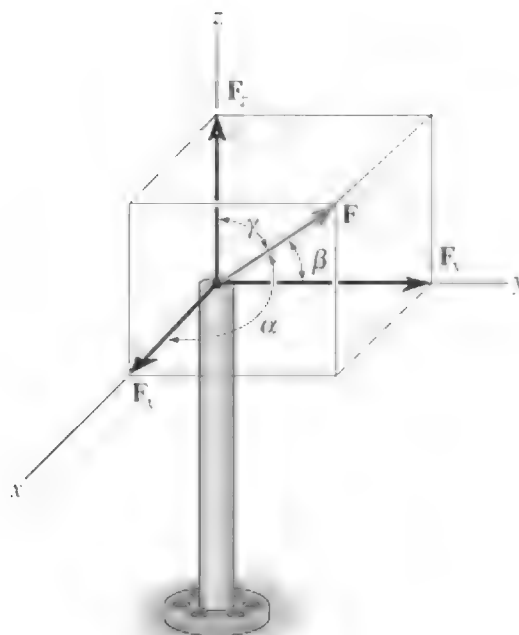
$\beta = 75\ \mathrm{deg}$

Solution:

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

$$\left(\frac{F_x}{F}\right)^2 + \cos(\beta)^2 + \left(\frac{F_z}{F}\right)^2 = 1$$

$$F = \sqrt{\frac{F_x^{\ 2} + F_z^{\ 2}}{1 - \cos(\beta)^2}} \qquad F = 2.02\ \mathrm{kN}$$

$$F_y = F\cos(\beta) \qquad F_y = 0.5\ \mathrm{kN}$$

Problem 2-74

The eye bolt is subjected to the cable force **F** which has a component F_x along the x axis, a component F_z along the z axis, and a coordinate direction angle β. Determine the magnitude of **F**.

Given:

$F_x = 60 \text{ N}$

$F_z = -80 \text{ N}$

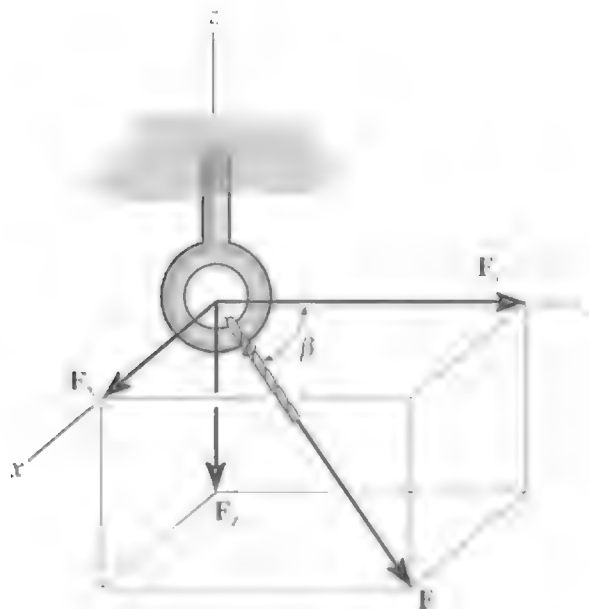
$\beta = 80 \text{ deg}$

Solution:

$F_y = F\cos(\beta)$

$F_y = \sqrt{F_x^{\,2} + F_z^{\,2} + F_y^{\,2}}\cos(\beta)$

$$F_y = \sqrt{\frac{F_x^{\,2} + F_z^{\,2}}{1 - \cos(\beta)^2}}\cos(\beta) \qquad F_y = 17.6 \text{ N}$$

$$F = \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}} \qquad F = 102 \text{ N}$$

Problem 2-75

Three forces act on the hook. If the resultant force $\mathbf{F_R}$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F_3}$.

Given:

$F_R = 120 \text{ N}$

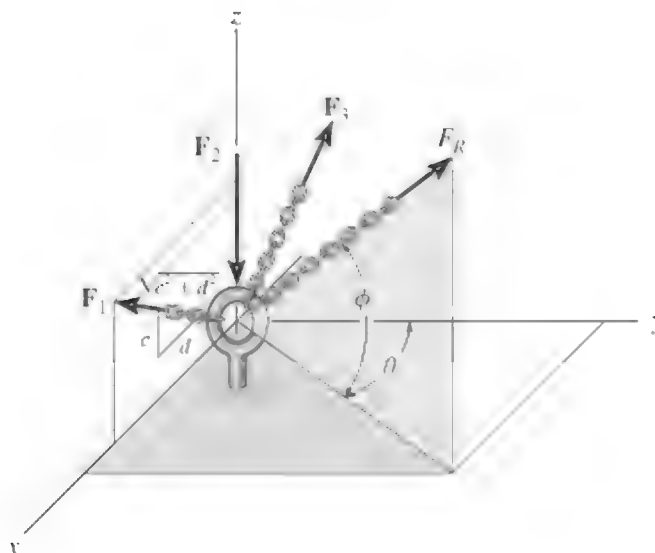
$F_1 = 80 \text{ N}$

$F_2 = 110 \text{ N}$

$c = 3$

$d = 4$

$\theta = 30 \text{ deg}$

$\phi = 45 \text{ deg}$

Solution:

$$\mathbf{F_{1v}} = \left(\frac{F_1}{\sqrt{c^2+d^2}}\right)\begin{pmatrix} d \\ 0 \\ c \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 64 \\ 0 \\ 48 \end{pmatrix} \text{N}$$

$$\mathbf{F_{2v}} = F_2\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 0 \\ 0 \\ -110 \end{pmatrix} \text{N}$$

$$\mathbf{F_{Rv}} = F_R\begin{pmatrix} \cos(\phi)\sin(\theta) \\ \cos(\phi)\cos(\theta) \\ \sin(\phi) \end{pmatrix} \qquad \mathbf{F_{Rv}} = \begin{pmatrix} 42.4 \\ 73.5 \\ 84.9 \end{pmatrix} \text{N}$$

$$\mathbf{F_{3v}} = \mathbf{F_{Rv}} - \mathbf{F_{1v}} - \mathbf{F_{2v}} \qquad \mathbf{F_{3v}} = \begin{pmatrix} -21.6 \\ 73.5 \\ 146.9 \end{pmatrix} \text{N} \qquad \left|\mathbf{F_{3v}}\right| = 165.6 \text{ N}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_{3v}}}{\left|\mathbf{F_{3v}}\right|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 97.5 \\ 63.7 \\ 27.5 \end{pmatrix} \text{deg}$$

Problem 2-76

Determine the coordinate direction angles of $\mathbf{F_1}$ and $\mathbf{F_R}$.

Given:

$F_R = 120$ N

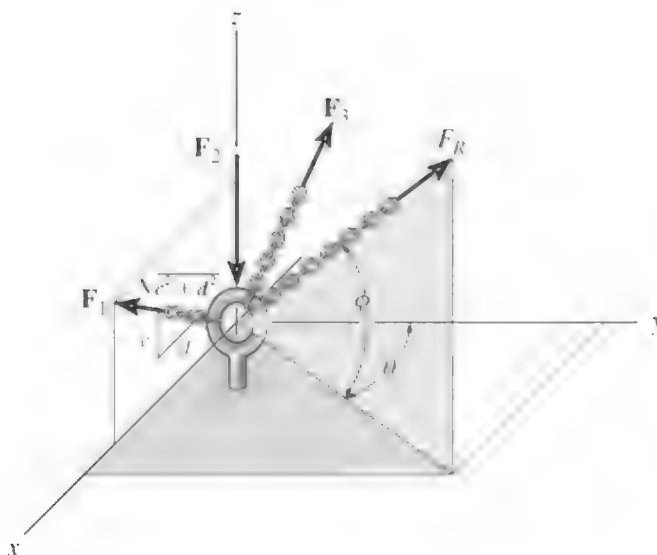
$F_1 = 80$ N

$F_2 = 110$ N

$c = 3$

$d = 4$

$\theta = 30$ deg

$\phi = 45$ deg

Solution:

$$\mathbf{F_{1v}} = \left(\frac{F_1}{\sqrt{c^2 + d^2}}\right)\begin{pmatrix} d \\ 0 \\ c \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 64 \\ 0 \\ 48 \end{pmatrix} \text{ N}$$

$$\begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_{1v}}}{|\mathbf{F_{1v}}|}\right) \qquad \begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} 36.9 \\ 90 \\ 53.1 \end{pmatrix} \text{deg}$$

$$\mathbf{F_{Rv}} = F_R\begin{pmatrix} \cos(\phi)\sin(\theta) \\ \cos(\phi)\cos(\theta) \\ \sin(\phi) \end{pmatrix} \qquad \mathbf{F_{Rv}} = \begin{pmatrix} 42.4 \\ 73.5 \\ 84.9 \end{pmatrix} \text{ N}$$

$$\begin{pmatrix} \alpha_R \\ \beta_R \\ \gamma_R \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_{Rv}}}{|\mathbf{F_{Rv}}|}\right) \qquad \begin{pmatrix} \alpha_R \\ \beta_R \\ \gamma_R \end{pmatrix} = \begin{pmatrix} 69.3 \\ 52.2 \\ 45 \end{pmatrix} \text{deg}$$

Problem 2-77

The pole is subjected to the force **F**, which has components acting along the x, y, z axes as shown. Given the magnitude of **F** and the angles α and γ, determine the magnitudes of the components of **F**.

Given:

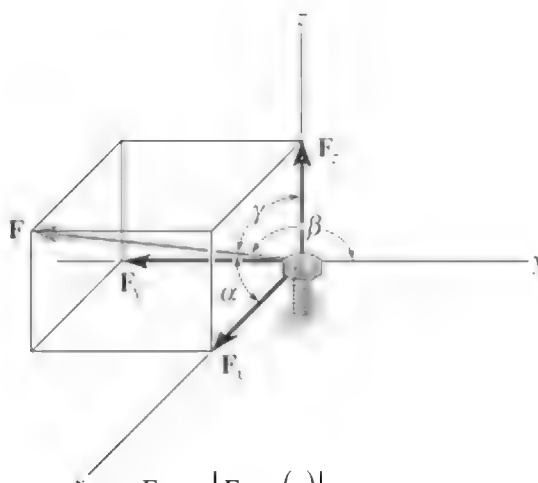
$F = 80 \text{ N} \quad \alpha = 60 \text{ deg} \quad \gamma = 45 \text{ deg}$

Solution:

$$\beta = \text{acos}\left(-\sqrt{1 - \cos(\alpha)^2 - \cos(\gamma)^2}\right)$$

$$\beta = 120 \text{ deg}$$

$F_x = |F\cos(\alpha)| \qquad F_y = |F\cos(\beta)| \qquad F_z = |F\cos(\gamma)|$

$F_x = 40 \text{ N} \qquad F_y = 40 \text{ N} \qquad F_z = 56.6 \text{ N}$

Problem 2-78

Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ act on the bolt. If the resultant force $\mathbf{F_R}$ has magnitude F_R and coordinate direction angles α and β, as shown, determine the magnitude of $\mathbf{F_2}$ and its coordinate direction angles.

Given:

$F_1 = 20 \text{ lb}$

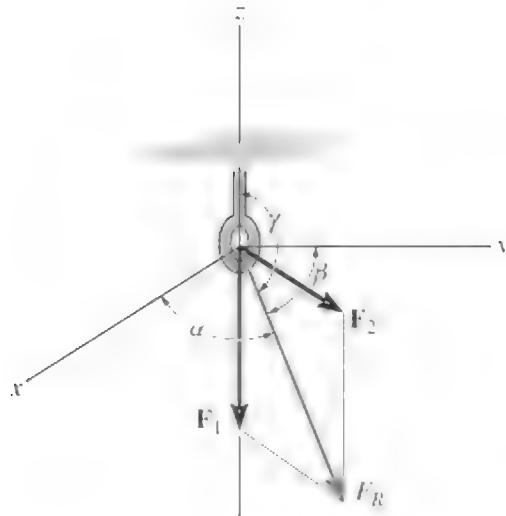
$F_R = 50 \text{ lb}$

$\alpha = 110 \text{ deg}$

$\beta = 80 \text{ deg}$

Solution:

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

$$\gamma = \operatorname{acos}\left(-\sqrt{1 - \cos(\alpha)^2 - \cos(\beta)^2}\right) \qquad \gamma = 157.44 \text{ deg}$$

Initial Guesses $\quad F_{2x} = 1 \text{ lb} \quad F_{2y} = 1 \text{ lb} \quad F_{2z} = 1 \text{ lb}$

Given $$F_R \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix} = F_1 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} F_{2x} \\ F_{2y} \\ F_{2z} \end{pmatrix} \qquad \begin{pmatrix} F_{2x} \\ F_{2y} \\ F_{2z} \end{pmatrix} = \text{Find}(F_{2x}, F_{2y}, F_{2z})$$

$$\mathbf{F_2} = \begin{pmatrix} F_{2x} \\ F_{2y} \\ F_{2z} \end{pmatrix} \qquad \mathbf{F_2} = \begin{pmatrix} -17.1 \\ 8.7 \\ -26.2 \end{pmatrix} \text{lb} \qquad |\mathbf{F_2}| = 32.4 \text{ lb}$$

$$\begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{F_2}}{|\mathbf{F_2}|}\right) \qquad \begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 121.8 \\ 74.5 \\ 143.8 \end{pmatrix} \text{deg}$$

Problem 2-79

Given $\mathbf{r_1}$, $\mathbf{r_2}$, and $\mathbf{r_3}$, determine the magnitude and direction of $r = 2\mathbf{r_1} - \mathbf{r_2} + 3\mathbf{r_3}$.

Given:

$$\mathbf{r_1} = \begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix} \text{m} \qquad \mathbf{r_2} = \begin{pmatrix} 4 \\ 0 \\ -5 \end{pmatrix} \text{m} \qquad \mathbf{r_3} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} \text{m}$$

Solution:

$$\mathbf{r} = 2\mathbf{r_1} - \mathbf{r_2} + 3\mathbf{r_3}$$

$$\mathbf{r} = \begin{pmatrix} 11 \\ -14 \\ 26 \end{pmatrix} \text{m} \qquad |\mathbf{r}| = 31.5 \text{ m}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 69.6 \\ 116.4 \\ 34.4 \end{pmatrix} \text{deg}$$

Problem 2-80

Represent the position vector **r** acting from point $A(a, b, c)$ to point $B(d, e, f)$ in Cartesian vector form. Determine its coordinate direction angles and find the distance between points A and B.

Given:

$a = 3$ m

$b = 5$ m

$c = 6$ m

$d = 5$ m

$e = -2$ m

$f = 1$ m

Solution:

$$\mathbf{r} = \begin{pmatrix} d-a \\ e-b \\ f-c \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 2 \\ -7 \\ -5 \end{pmatrix} \text{m} \qquad |\mathbf{r}| = 8.8 \text{ m}$$

$\alpha = \operatorname{acos}\left(\frac{d-a}{|\mathbf{r}|}\right)$ $\alpha = 76.9 \text{ deg}$

$\beta = \operatorname{acos}\left(\frac{e-b}{|\mathbf{r}|}\right)$ $\beta = 142 \text{ deg}$

$\gamma = \operatorname{acos}\left(\frac{f-c}{|\mathbf{r}|}\right)$ $\gamma = 124 \text{ deg}$

Problem 2-81

A position vector extends from the origin to point $A(a, b, c)$. Determine the angles α, β, γ which the tail of the vector makes with the x, y, z axes, respectively.

Given:

$a = 2 \text{ m}$ $b = 3 \text{ m}$ $c = 6 \text{ m}$

Solution:

$\mathbf{r} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \text{m}$

$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\right)$ $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 73.4 \\ 64.6 \\ 31.0 \end{pmatrix} \text{deg}$

Problem 2-82

Express the position vector $\mathbf{r}$ in Cartesian vector form; then determine its magnitude and coordinate direction angles.

Given:

$a = 4 \text{ m}$

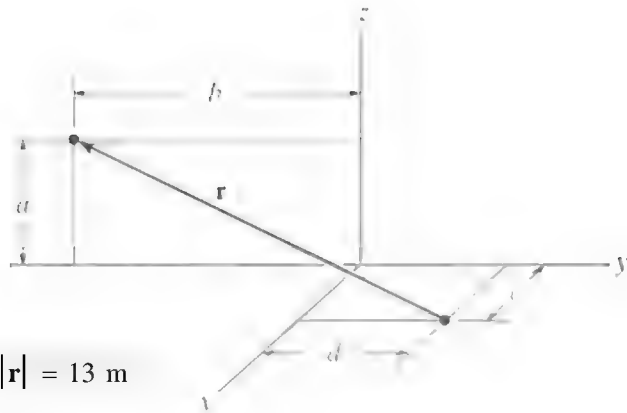
$b = 8\text{ m}$

$c = 3\text{ m}$

$d = 4\text{ m}$

Solution:

$$\mathbf{r} = \begin{pmatrix} -c \\ -d-b \\ a \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} -3 \\ -12 \\ 4 \end{pmatrix}\text{ m} \qquad |\mathbf{r}| = 13\text{ m}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 103.3 \\ 157.4 \\ 72.1 \end{pmatrix}\text{deg}$$

Problem 2-83

Express the position vector **r** in Cartesian vector form; then determine its magnitude and coordinate direction angles.

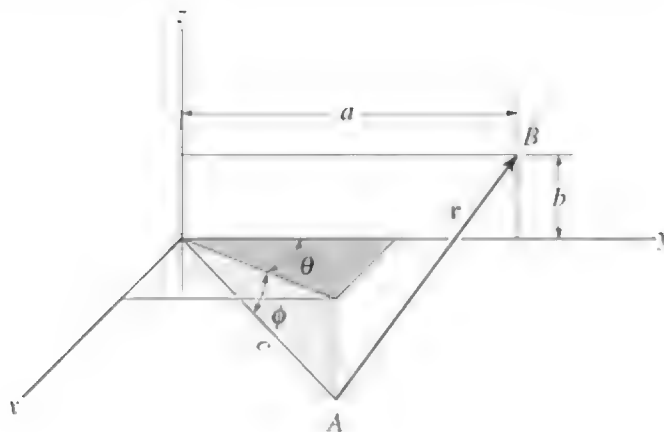
Given:

$a = 8\text{ ft}$

$b = 2\text{ ft}$

$c = 5\text{ ft}$

$\theta = 30\text{ deg}$

$\phi = 20\text{ deg}$

Solution:

$$\mathbf{r} = \begin{pmatrix} -c\cos(\phi)\sin(\theta) \\ a - c\cos(\phi)\cos(\theta) \\ b + c\sin(\phi) \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} -2.35 \\ 3.93 \\ 3.71 \end{pmatrix}\text{ft} \qquad |\mathbf{r}| = 5.89\text{ ft}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 113.5 \\ 48.2 \\ 51 \end{pmatrix} \text{deg}$$

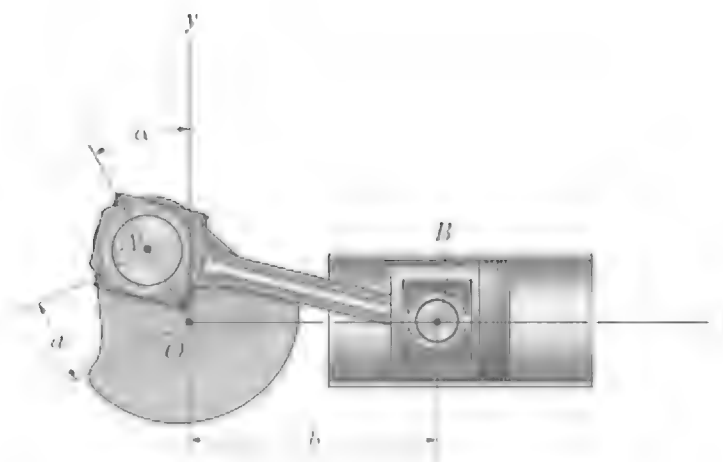
Problem 2-84

Determine the length of the connecting rod *AB* by first formulating a Cartesian position vector from *A* to *B* and then determining its magnitude.

Given:

$b = 16$ in

$a = 5$ in

$\alpha = 30$ deg

Solution:

$$\mathbf{r} = \begin{pmatrix} a\sin(\alpha) + b \\ -a\cos(\alpha) \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 18.5 \\ -4.3 \end{pmatrix} \text{in} \qquad |\mathbf{r}| = 19\,\text{in}$$

Problem 2-85

Determine the length of member *AB* of the truss by first establishing a Cartesian position vector from *A* to *B* and then determining its magnitude.

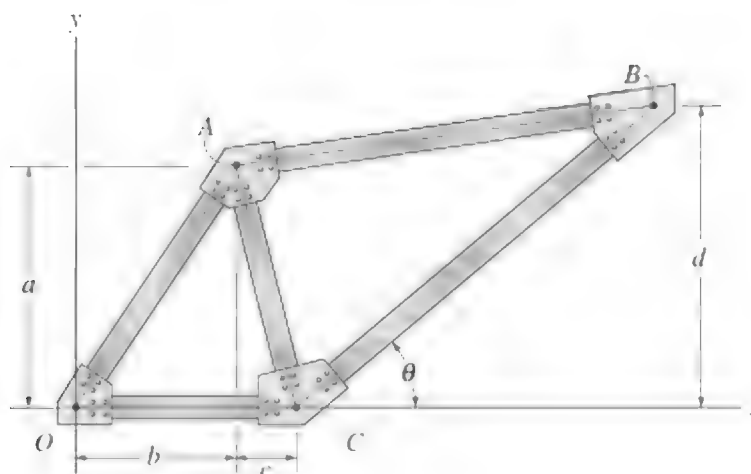
Given:

$a = 1.2$ m

$b = 0.8$ m

$c = 0.3$ m

$d = 1.5$ m

$\theta = 40$ deg

Solution:

$$\mathbf{r} = \begin{pmatrix} c + d\cot(\theta) \\ d - a \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 2.09 \\ 0.3 \end{pmatrix} \text{m} \qquad |\mathbf{r}| = 2.11 \text{ m}$$

Problem 2- 86

The positions of point *A* on the building and point *B* on the antenna have been measured relative to the electronic distance meter (EDM) at *O*. Determine the distance between *A* and *B*. *Hint:* Formulate a position vector directed from *A* to *B*; then determine its magnitude.

Given:

$a = 460$ m

$b = 653$ m

$\alpha = 60$ deg

$\beta = 55$ deg

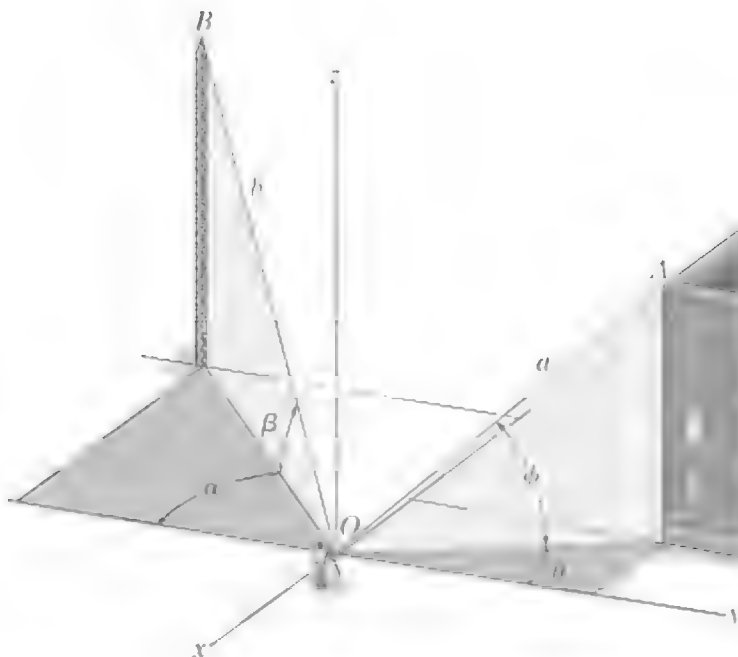
$\theta = 30$ deg

$\phi = 40$ deg

Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -a\cos(\phi)\sin(\theta) \\ a\cos(\phi)\cos(\theta) \\ a\sin(\phi) \end{pmatrix}$$

$$\mathbf{r_{OB}} = \begin{pmatrix} -b\cos(\beta)\sin(\alpha) \\ -b\cos(\beta)\cos(\alpha) \\ b\sin(\beta) \end{pmatrix}$$

$$\mathbf{r_{AB}} = \mathbf{r_{OB}} - \mathbf{r_{OA}} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -148.2 \\ -492.4 \\ 239.2 \end{pmatrix} \text{m} \qquad |\mathbf{r_{AB}}| = 567.2 \text{ m}$$

Problem 2-87

Determine the lengths of cords *ACB* and *CO*. The knot at *C* is located midway between *A* and *B*.

Given:

$a = 3$ ft

$b = 6$ ft

$c = 4$ ft

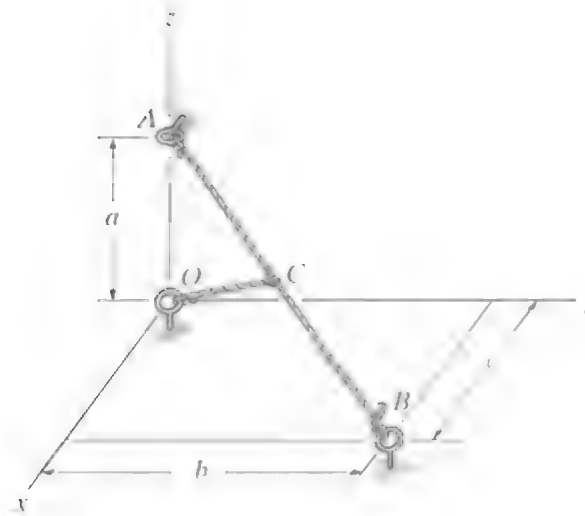
Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} c \\ b \\ -a \end{pmatrix} \qquad \mathbf{r_{OA}} = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}$$

$$\mathbf{r_{AC}} = \frac{\mathbf{r_{AB}}}{2}$$

$$\left|\mathbf{r_{AB}}\right| = 7.8 \text{ ft}$$

$$\mathbf{r_{OC}} = \mathbf{r_{OA}} + \mathbf{r_{AC}}$$

$$\left|\mathbf{r_{OC}}\right| = 3.91 \text{ ft}$$

Problem 2-88

Determine the length of the crankshaft *AB* by first formulating a Cartesian position vector from *A* to *B* and then determining its magnitude.

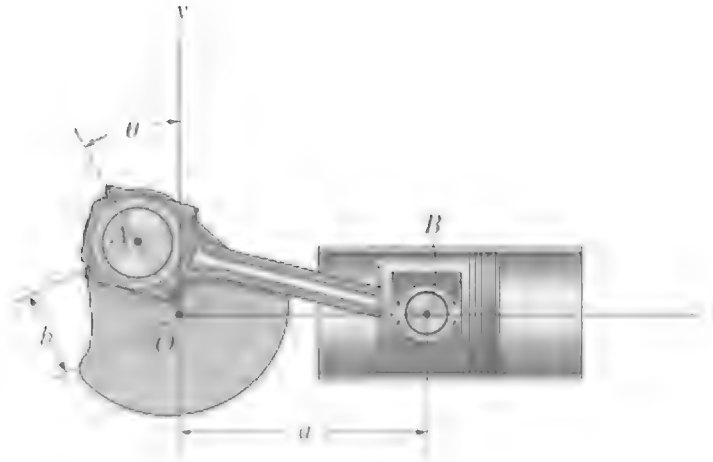
Given:

$a = 400$

$b = 125$

$\theta = 25 \text{ deg}$

Solution:

$$\mathbf{r_{AB}} = \begin{bmatrix} a + b\sin(\theta) \\ -(b\cos(\theta)) \\ 0 \end{bmatrix} \text{mm} \qquad |\mathbf{r_{AB}}| = 467 \text{ mm}$$

Problem 2-89

Determine the length of wires *AD*, *BD*, and *CD*. The ring at *D* is midway between *A* and *B*.

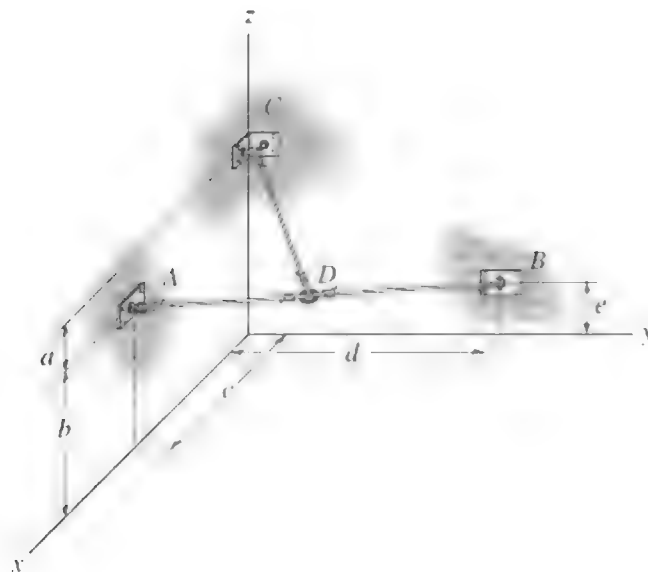
Given:

$a = 0.5 \text{ m}$

$b = 1.5 \text{ m}$

$c = 2 \text{ m}$

$d = 2 \text{ m}$

$e = 0.5 \text{ m}$

Solution:

$$\mathbf{r_{AD}} = \begin{pmatrix} \frac{-c}{2} \\ \frac{d}{2} \\ \frac{e}{2} - \frac{b}{2} \end{pmatrix} \qquad \mathbf{r_{AD}} = \begin{pmatrix} -1 \\ 1 \\ -0.5 \end{pmatrix} \text{ m} \qquad \left|\mathbf{r_{AD}}\right| = 1.5 \text{ m}$$

$$\mathbf{r_{BD}} = -\mathbf{r_{AD}}$$

$$\left|\mathbf{r_{BD}}\right| = 1.5 \text{ m}$$

$$\mathbf{r_{CD}} = \begin{pmatrix} \frac{c}{2} \\ \frac{d}{2} \\ a + \frac{b}{2} - \frac{e}{2} \end{pmatrix} \qquad \mathbf{r_{CD}} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ m} \qquad \left|\mathbf{r_{CD}}\right| = 1.7 \text{ m}$$

Problem 2-90

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.

Given:

$F = 600$ lb $\quad c = 3$ ft

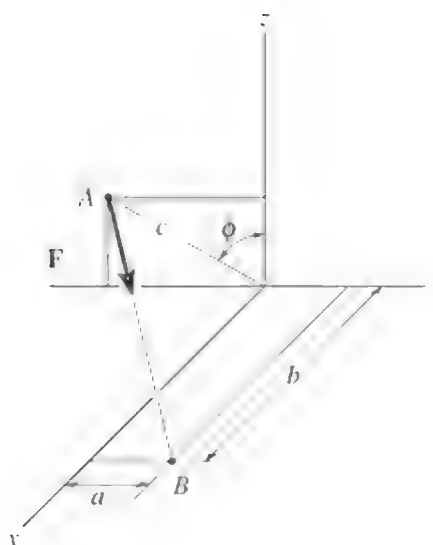
$a = 1.5$ ft $\quad \phi = 60$ deg

$b = 5$ ft

Solution:

$$r = b\mathbf{i} + (a + c\sin(\phi))\mathbf{j} + (0 - c\cos(\phi))\mathbf{k}$$

$$r = \sqrt{b^2 + (a + c\sin(\phi))^2 + (c\cos(\phi))^2}$$

$$r = 2 \text{ m}$$

$d = F\dfrac{b}{r}$ $e = F\left(\dfrac{a + c\sin(\phi)}{r}\right)$ $f = F\left[\dfrac{-(c\cos(\phi))}{r}\right]$

$d = 452\,\text{lb}$ $e = 370\,\text{lb}$ $f = -136\,\text{lb}$

$F = (d\mathbf{i} + e\mathbf{j} + f\mathbf{k})\ \text{lb}$

$\alpha = \operatorname{acos}\left(\dfrac{d}{F}\right)$ $\alpha = 41.1\ \text{deg}$

$\beta = \operatorname{acos}\left(\dfrac{e}{F}\right)$ $\beta = 51.9\ \text{deg}$

$\gamma = \operatorname{acos}\left(\dfrac{f}{F}\right)$ $\gamma = 103\ \text{deg}$

Problem 2-91

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.

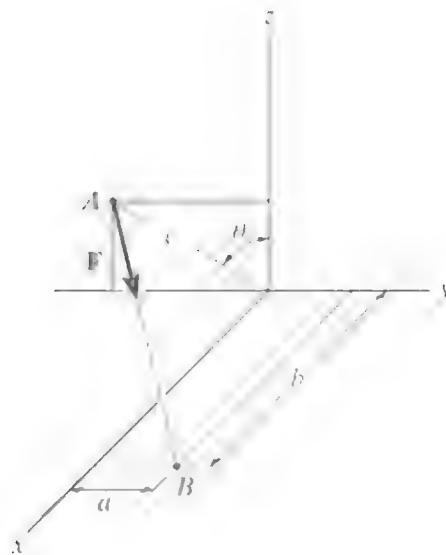
Given:

$a = 1.5\ \text{ft}$

$b = 5\ \text{ft}$

$c = 3\ \text{ft}$

$\theta = 60\ \text{deg}$

$F = 600\ \text{lb}$

Solution:

$$\mathbf{r} = \begin{pmatrix} b \\ a + c\sin(\theta) \\ -c\cos(\theta) \end{pmatrix} \qquad \mathbf{F} = F\frac{\mathbf{r}}{|\mathbf{r}|} \qquad \mathbf{F} = \begin{pmatrix} 452 \\ 370 \\ -136 \end{pmatrix}\text{lb}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{F}}{|\mathbf{F}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 41.1 \\ 51.9 \\ 103.1 \end{pmatrix}\text{deg}$$

Problem 2-92

Determine the magnitude and coordinate direction angles of the resultant force acting at point A.

Given:

$F_1 = 150$ N

$F_2 = 200$ N

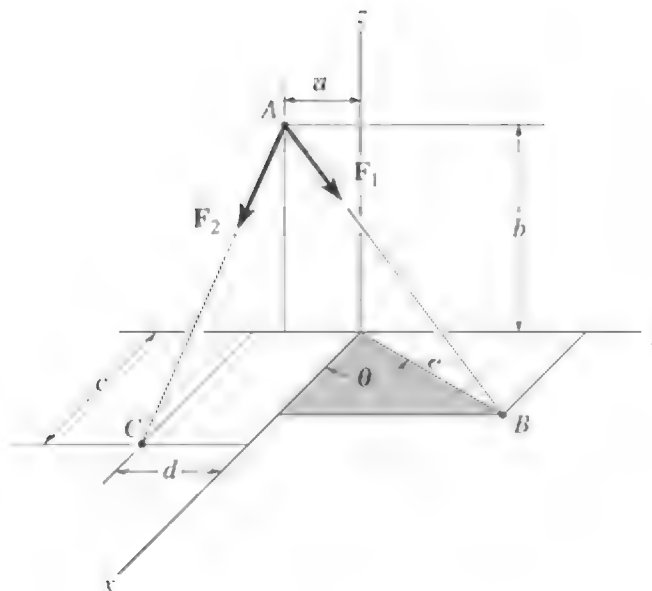
$a = 1.5$ m

$b = 4$ m

$c = 3$ m

$d = 2$ m

$e = 3$ m

$\theta = 60$ deg

Solution:

Define the position vectors and then the forces

$$\mathbf{r_{AB}} = \begin{pmatrix} e\cos(\theta) \\ a + e\sin(\theta) \\ -b \end{pmatrix} \qquad \mathbf{F_{1v}} = F_1 \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 38 \\ 103.8 \\ -101.4 \end{pmatrix} \text{N}$$

$$\mathbf{r_{AC}} = \begin{pmatrix} c \\ a - d \\ -b \end{pmatrix} \qquad \mathbf{F_{2v}} = F_2 \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 119.4 \\ -19.9 \\ -159.2 \end{pmatrix} \text{N}$$

Add the forces and find the magnitude of the resultant

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}} \qquad \mathbf{F_R} = \begin{pmatrix} 157.4 \\ 83.9 \\ -260.6 \end{pmatrix} \text{N} \qquad |\mathbf{F_R}| = 316 \text{ N}$$

Find the direction cosine angles

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_R}}{|\mathbf{F_R}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 60.1 \\ 74.6 \\ 145.6 \end{pmatrix} \text{deg}$$

Problem 2-93

The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.

Given:

$F_{BA} = 350$ lb

$F_{CA} = 500$ lb

$F_{DA} = 400$ lb

$a = 3$ ft

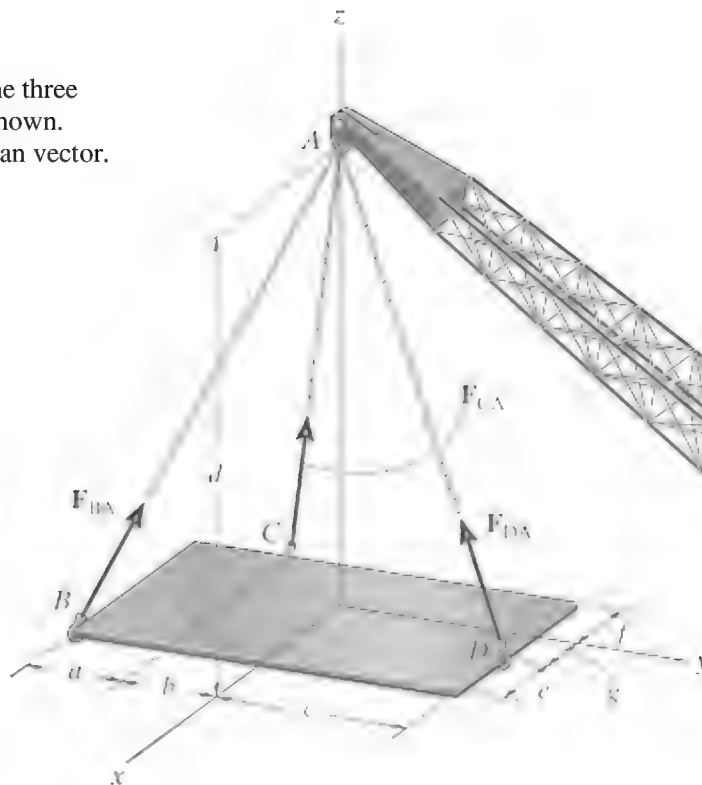
$b = 3$ ft

$c = 6$ ft

$d = 14$ ft

$e = 3$ ft

$f = 3$ ft

$g = 2$ ft

Solution:

$$\mathbf{r_{BA}} = \begin{pmatrix} -e-g \\ a+b \\ d \end{pmatrix} \qquad \mathbf{F_{BAv}} = F_{BA}\frac{\mathbf{r_{BA}}}{|\mathbf{r_{BA}}|} \qquad \mathbf{F_{BAv}} = \begin{pmatrix} -109.2 \\ 131 \\ 305.7 \end{pmatrix} \text{lb}$$

$$\mathbf{r_{CA}} = \begin{pmatrix} f \\ b \\ d \end{pmatrix} \qquad \mathbf{F_{CAv}} = F_{CA}\frac{\mathbf{r_{CA}}}{|\mathbf{r_{CA}}|} \qquad \mathbf{F_{CAv}} = \begin{pmatrix} 102.5 \\ 102.5 \\ 478.5 \end{pmatrix} \text{lb}$$

$$\mathbf{r_{DA}} = \begin{pmatrix} -g \\ -c \\ d \end{pmatrix} \qquad \mathbf{F_{DAv}} = F_{DA}\frac{\mathbf{r_{DA}}}{|\mathbf{r_{DA}}|} \qquad \mathbf{F_{DAv}} = \begin{pmatrix} -52.1 \\ -156.2 \\ 364.5 \end{pmatrix} \text{lb}$$

Problem 2-94

The engine of the lightweight plane is supported by struts that are connected to the space truss that makes up the structure of the plane. The anticipated loading in two of the struts is shown. Express each of these forces as a Cartesian vector.

Given:

$F_1 = 400 \text{ lb}$

$F_2 = 600 \text{ lb}$

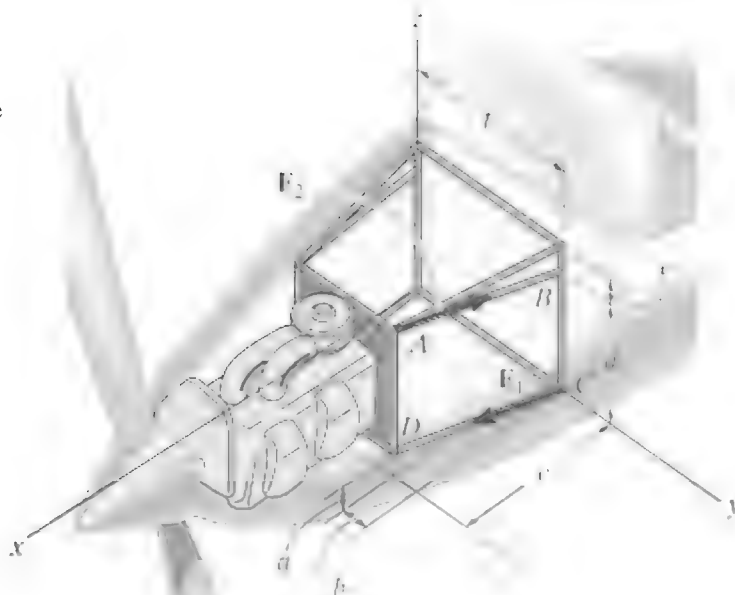
$a = 0.5 \text{ ft}$

$b = 0.5 \text{ ft}$

$c = 3.0 \text{ ft}$

$d = 2.0 \text{ ft}$

$e = 0.5 \text{ ft}$

$f = 3.0 \text{ ft}$

Solution:

$$\mathbf{r_{CD}} = \begin{pmatrix} c \\ -b \\ a \end{pmatrix} \qquad \mathbf{F_{1v}} = F_1 \frac{\mathbf{r_{CD}}}{|\mathbf{r_{CD}}|} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 389.3 \\ -64.9 \\ 64.9 \end{pmatrix} \text{lb}$$

$$\mathbf{r_{AB}} = \begin{pmatrix} -c \\ b \\ -e \end{pmatrix} \qquad \mathbf{F_{2v}} = F_2 \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{F_{2v}} = \begin{pmatrix} -584.0 \\ 97.3 \\ -97.3 \end{pmatrix} \text{lb}$$

Problem 2-95

The window is held open by cable *AB*. Determine the length of the cable and express the force **F** acting at *A* along the cable as a Cartesian vector.

Given:

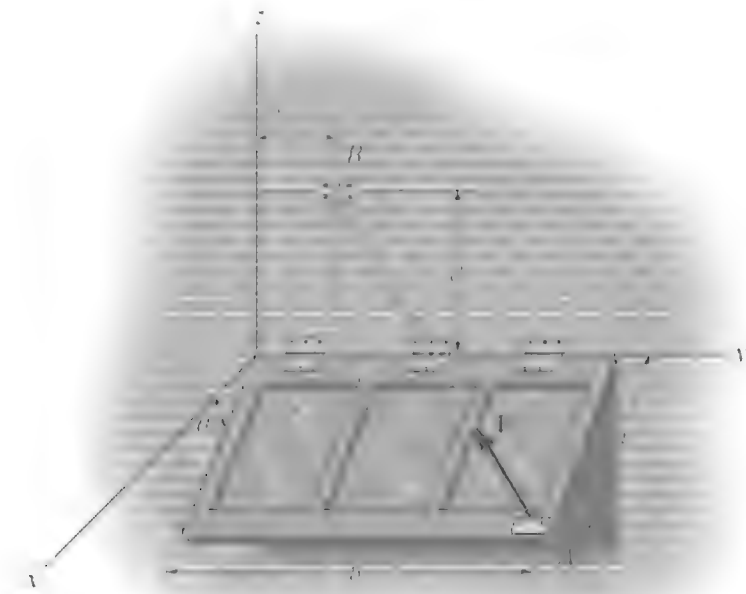
$a = 300\ \text{mm}$

$b = 500\ \text{mm}$

$c = 150\ \text{mm}$

$d = 250\ \text{mm}$

$\theta = 30\ \text{deg}$

$F = 30\ \text{N}$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} -a\cos(\theta) \\ c-b \\ d+a\sin(\theta) \end{pmatrix} \qquad |\mathbf{r_{AB}}| = 591.6\ \text{mm}$$

$$\mathbf{F_v} = F\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{F_v} = \begin{pmatrix} -13.2 \\ -17.7 \\ 20.3 \end{pmatrix} \text{N}$$

Problem 2-96

The force acting on the man, caused by his pulling on the anchor cord, is **F**. If the length of the cord is *L*, determine the coordinates *A*(*x*, *y*, *z*) of the anchor.

Given:

$$\mathbf{F} = \begin{pmatrix} 40 \\ 20 \\ -50 \end{pmatrix} \text{N}$$

$$L = 25 \text{ m}$$

Solution:

$$\mathbf{r} = L\frac{\mathbf{F}}{|\mathbf{F}|}$$

$$\mathbf{r} = \begin{pmatrix} 14.9 \\ 7.5 \\ -18.6 \end{pmatrix} \text{m}$$

Problem 2-97

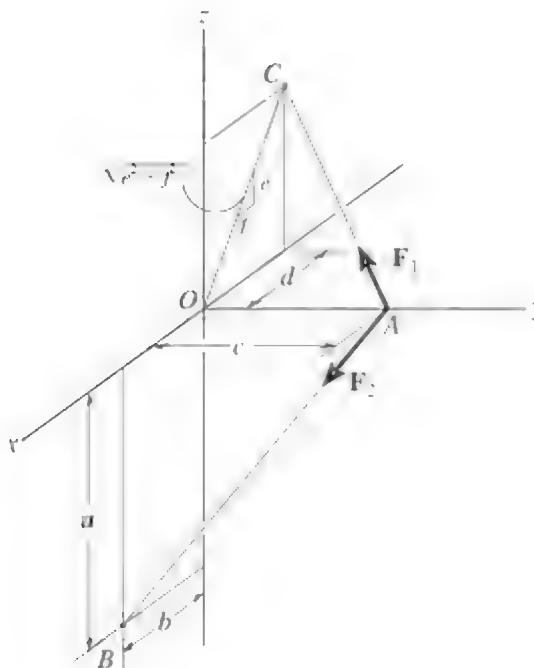
Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.

Given:

$F_1 = 80$ lb $\quad c = 4$ ft

$F_2 = 50\text{ lb}$ $d = 2.5\text{ ft}$

$a = 6\text{ ft}$ $e = 12$

$b = 2\text{ ft}$ $f = 5$

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} -d \\ -c \\ e\dfrac{d}{f} \end{pmatrix} \qquad \mathbf{F_{1v}} = F_1 \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad \mathbf{F_{1v}} = \begin{pmatrix} -26.2 \\ -41.9 \\ 62.9 \end{pmatrix} \text{lb}$$

$$\mathbf{r_{AB}} = \begin{pmatrix} b \\ -c \\ -a \end{pmatrix} \qquad \mathbf{F_{2v}} = F_2 \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 6.1 \\ -12.1 \\ -18.2 \end{pmatrix} \text{kg}$$

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}} \qquad \mathbf{F_R} = \begin{pmatrix} -12.8 \\ -68.7 \\ 22.8 \end{pmatrix} \text{lb} \qquad |\mathbf{F_R}| = 73.5\text{ lb}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_R}}{|\mathbf{F_R}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 1.7 \\ 2.8 \\ 1.3 \end{pmatrix}$$

Problem 2-98

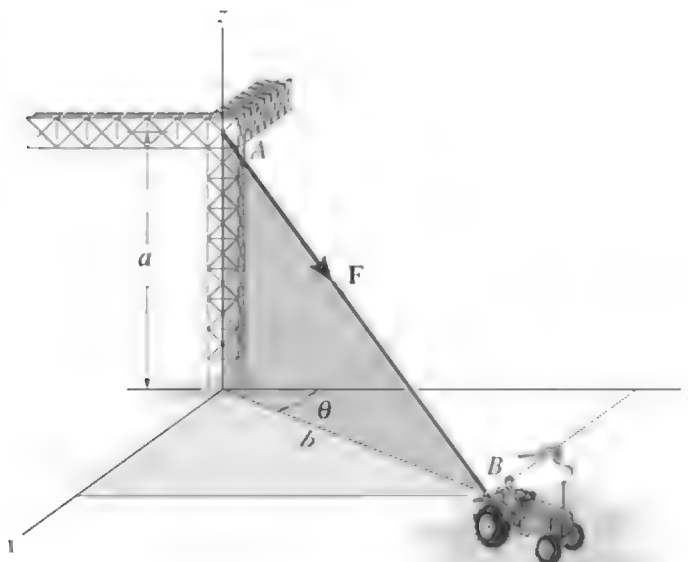
The cable attached to the tractor at B exerts force $\mathbf{F}$ on the framework. Express this force as a Cartesian vector

Given:

$F = 350 \text{ lb}$

$a = 35 \text{ ft}$

$b = 50 \text{ ft}$

$\theta = 20 \text{ deg}$

Solution:

Find the position vector and then the force

$$\mathbf{r_{AB}} = \begin{pmatrix} b\sin(\theta) \\ b\cos(\theta) \\ -a \end{pmatrix} \qquad \mathbf{F_v} = F\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{F_v} = \begin{pmatrix} 98.1 \\ 269.4 \\ -200.7 \end{pmatrix} \text{lb}$$

Problem 2-99

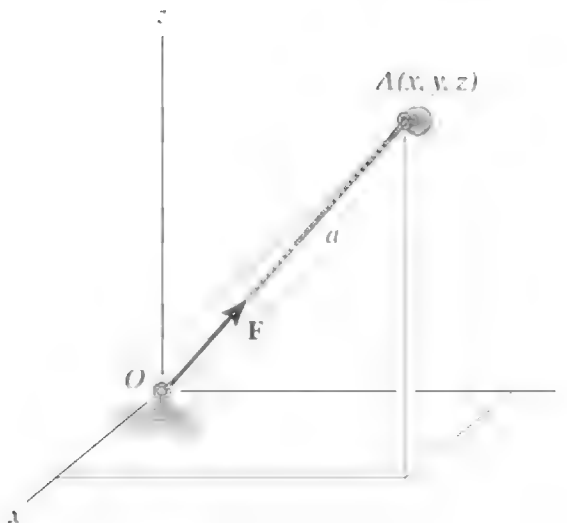
The cable *OA* exerts force **F** on point *O*. If the length of the cable is *L*, what are the coordinates (*x*, *y*, *z*) of point *A*?

Given:

$$\mathbf{F} = \begin{pmatrix} 40 \\ 60 \\ 70 \end{pmatrix} \text{N}$$

$L = 3 \text{ m}$

Solution:

$$\mathbf{r} = L\frac{\mathbf{F}}{|\mathbf{F}|}$$

$$\mathbf{r} = \begin{pmatrix} 1.2 \\ 1.8 \\ 2.1 \end{pmatrix} \text{ m}$$

Problem 2-100

Determine the position (x, y, 0) for fixing cable *BA* so that the resultant of the forces exerted on the pole is directed along its axis, from *B* toward *O,* and has magnitude F_R. Also, what is the magnitude of force $\mathbf{F_3}$?

Given:

$F_1 = 500$ N

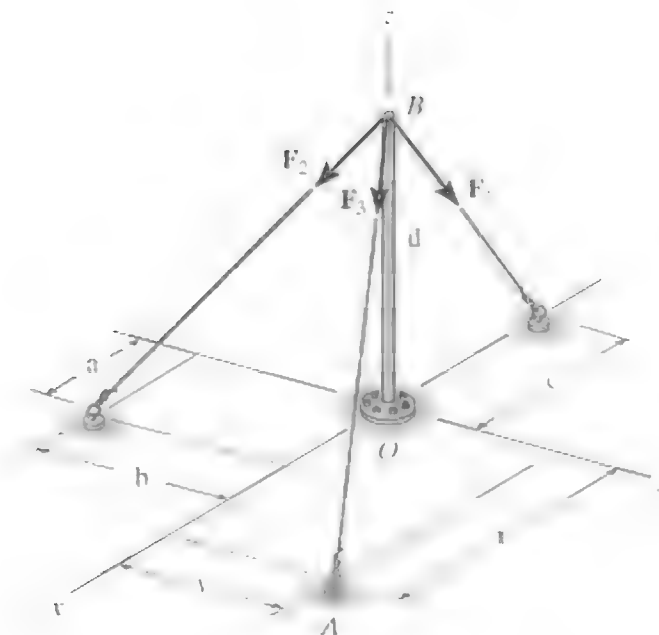
$F_2 = 400$ N

$F_R = 1000$ N

$a = 1$ m

$b = 2$ m

$c = 2$ m

$d = 3$ m

Solution:

Initial Guesses

$F_3 = 1$ N $\quad x = 1$ m $\quad y = 1$ m

Given

$$F_R \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \left(\frac{F_1}{\sqrt{c^2 + d^2}}\right)\begin{pmatrix} -c \\ 0 \\ -d \end{pmatrix} + \left(\frac{F_2}{\sqrt{a^2 + b^2 + d^2}}\right)\begin{pmatrix} a \\ -b \\ -d \end{pmatrix} + \left(\frac{F_3}{\sqrt{x^2 + y^2 + d^2}}\right)\begin{pmatrix} x \\ y \\ -d \end{pmatrix}$$

$$\begin{pmatrix} F_3 \\ x \\ y \end{pmatrix} = \text{Find}(F_3, x, y) \qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.4 \end{pmatrix} \text{ m} \qquad F_3 = 380 \text{ N}$$

Problem 2-101

The cord exerts a force **F** on the hook. If the cord is length L, determine the location x, y of the point of attachment B, and the height z of the hook.

Given:

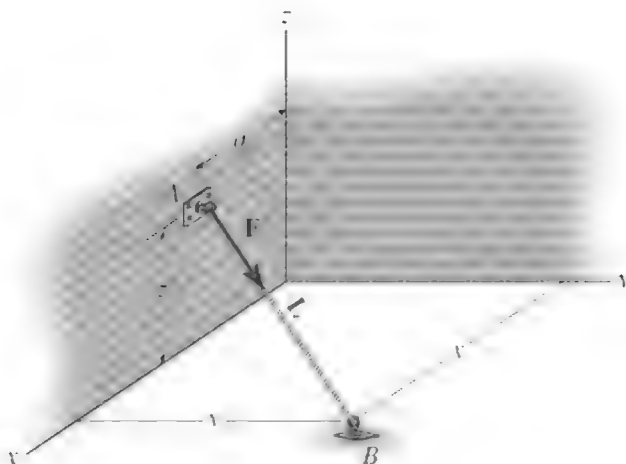
$$\mathbf{F} = \begin{pmatrix} 12 \\ 9 \\ -8 \end{pmatrix} \text{ lb}$$

$L = 8 \text{ ft}$

$a = 2 \text{ ft}$

Solution:

Initial guesses $\quad x = 1 \text{ ft} \quad y = 1 \text{ ft} \quad z = 1 \text{ ft}$

$$\text{Given} \quad \begin{pmatrix} x - a \\ y \\ -z \end{pmatrix} = L \frac{\mathbf{F}}{|\mathbf{F}|} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \text{Find}(x, y, z) \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7.65 \\ 4.24 \\ 3.76 \end{pmatrix} \text{ ft}$$

Problem 2-102

The cord exerts a force of magnitude F on the hook. If the cord length L, the distance z, and the x component of the force, F_x, are given, determine the location x, y of the point of attachment B of the cord to the ground.

Given:

$F = 30 \text{ lb}$

$L = 8 \text{ ft}$

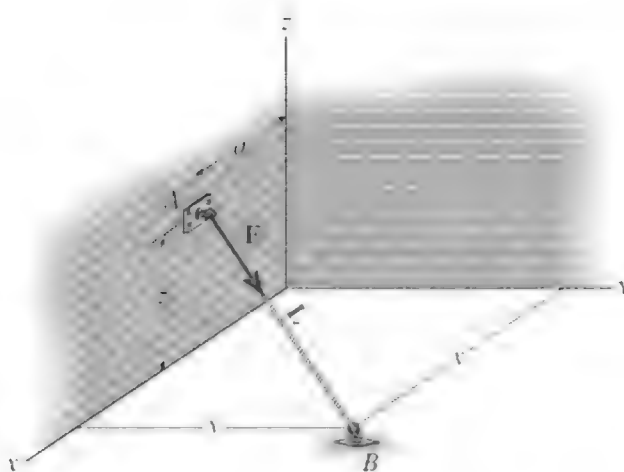
$z = 4 \text{ ft}$

$F_x = 25 \text{ lb}$

$a = 2 \text{ ft}$

Solution:

Guesses

$x = 1 \text{ ft}$

$y = 1\ \text{ft}$

Given

$F_x = \left(\dfrac{x-a}{L}\right)F \qquad L^2 = (x-a)^2 + y^2 + z^2$

$\begin{pmatrix} x \\ y \end{pmatrix} = \text{Find}(x, y) \qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8.67 \\ 1.89 \end{pmatrix} \text{ft}$

Problem 2-103

Each of the four forces acting at E has magnitude F. Express each force as a Cartesian vector and determine the resultant force.

Units used:

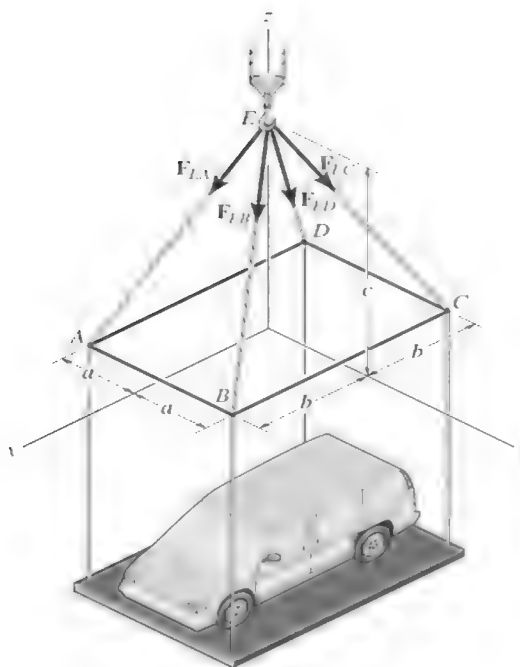
$\text{kN} = 10^3\ \text{N}$

Given:

$F = 28\ \text{kN}$

$a = 4\ \text{m}$

$b = 6\ \text{m}$

$c = 12\ \text{m}$

Solution:

Find the position vectors and then the forces

$\mathbf{r_{EA}} = \begin{pmatrix} b \\ -a \\ -c \end{pmatrix} \qquad \mathbf{F_{EA}} = F\dfrac{\mathbf{r_{EA}}}{|\mathbf{r_{EA}}|} \qquad \mathbf{F_{EA}} = \begin{pmatrix} 12 \\ -8 \\ -24 \end{pmatrix} \text{kN}$

$$\mathbf{r_{EB}} = \begin{pmatrix} b \\ a \\ -c \end{pmatrix} \qquad \mathbf{F_{EB}} = F\frac{\mathbf{r_{EB}}}{|\mathbf{r_{EB}}|} \qquad \mathbf{F_{EB}} = \begin{pmatrix} 12 \\ 8 \\ -24 \end{pmatrix} \text{kN}$$

$$\mathbf{r_{EC}} = \begin{pmatrix} -b \\ a \\ -c \end{pmatrix} \qquad \mathbf{F_{EC}} = F\frac{\mathbf{r_{EC}}}{|\mathbf{r_{EC}}|} \qquad \mathbf{F_{EC}} = \begin{pmatrix} -12 \\ 8 \\ -24 \end{pmatrix} \text{kN}$$

$$\mathbf{r_{ED}} = \begin{pmatrix} -b \\ -a \\ -c \end{pmatrix} \qquad \mathbf{F_{ED}} = F\frac{\mathbf{r_{ED}}}{|\mathbf{r_{ED}}|} \qquad \mathbf{F_{ED}} = \begin{pmatrix} -12 \\ -8 \\ -24 \end{pmatrix} \text{kN}$$

Find the resultant sum

$$\mathbf{F_R} = \mathbf{F_{EA}} + \mathbf{F_{EB}} + \mathbf{F_{EC}} + \mathbf{F_{ED}} \qquad \mathbf{F_R} = \begin{pmatrix} 0 \\ 0 \\ -96 \end{pmatrix} \text{kN}$$

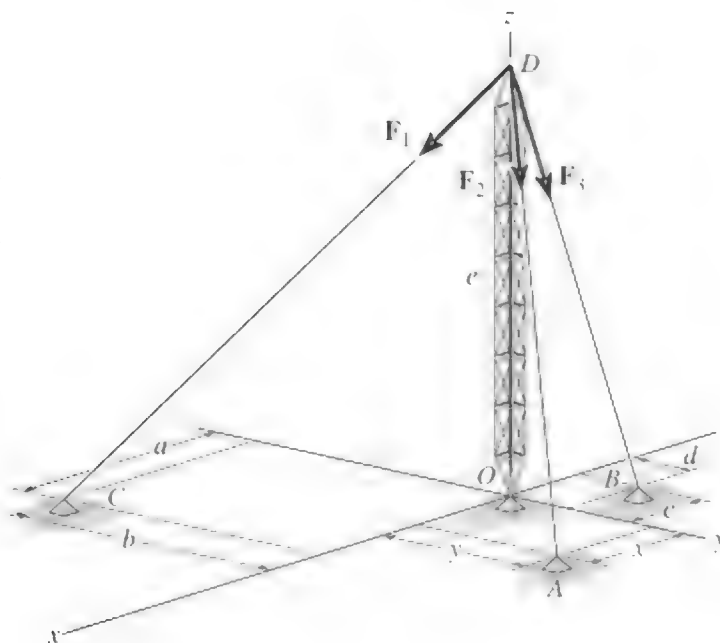
Problem 2-104

The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$x = 20 \text{ m}$	$a = 16 \text{ m}$
$y = 15 \text{ m}$	$b = 18 \text{ m}$
$F_1 = 600 \text{ N}$	$c = 6 \text{ m}$
$F_2 = 400 \text{ N}$	$d = 4 \text{ m}$
$F_3 = 800 \text{ N}$	$e = 24 \text{ m}$

Solution:

Find the position vectors, then the force vectors

$$\mathbf{r_{DC}} = \begin{pmatrix} a \\ -b \\ -e \end{pmatrix} \qquad \mathbf{F_{1v}} = F_1 \frac{\mathbf{r_{DC}}}{|\mathbf{r_{DC}}|} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 282.4 \\ -317.6 \\ -423.5 \end{pmatrix} \text{N}$$

$$\mathbf{r_{DA}} = \begin{pmatrix} x \\ y \\ -e \end{pmatrix} \qquad \mathbf{F_{2v}} = F_2 \frac{\mathbf{r_{DA}}}{|\mathbf{r_{DA}}|} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 230.8 \\ 173.1 \\ -277 \end{pmatrix} \text{N}$$

$$\mathbf{r_{DB}} = \begin{pmatrix} -c \\ d \\ -e \end{pmatrix} \qquad \mathbf{F_{3v}} = F_3 \frac{\mathbf{r_{DB}}}{|\mathbf{r_{DB}}|} \qquad \mathbf{F_{3v}} = \begin{pmatrix} -191.5 \\ 127.7 \\ -766.2 \end{pmatrix} \text{N}$$

Find the resultant, magnitude, and direction angles

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}} + \mathbf{F_{3v}} \qquad \mathbf{F_R} = \begin{pmatrix} 0.322 \\ -0.017 \\ -1.467 \end{pmatrix} \text{kN} \qquad |\mathbf{F_R}| = 1.502 \text{ kN}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F_R}}{|\mathbf{F_R}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 77.6 \\ 90.6 \\ 167.6 \end{pmatrix} \text{deg}$$

Problem 2-105

The chandelier is supported by three chains which are concurrent at point O. If the force in each chain has magnitude F, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.

Given:

$F = 60\ \text{lb}$

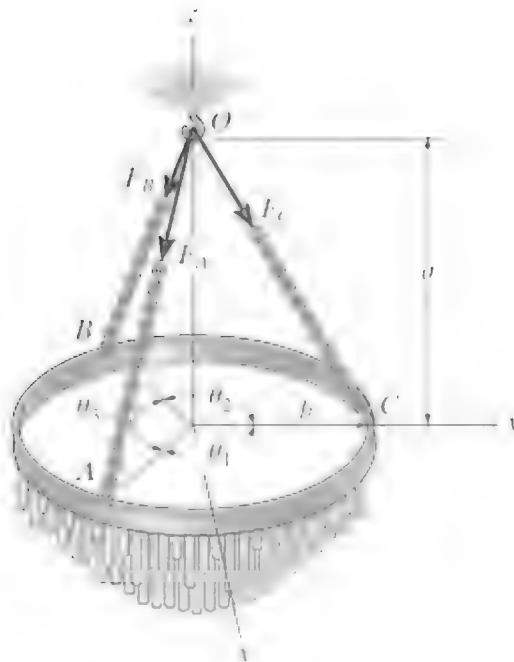
$a = 6\ \text{ft}$

$b = 4\ \text{ft}$

$\theta_1 = 120\ \text{deg}$

$\theta_2 = 120\ \text{deg}$

Solution:

$\theta_3 = 360\ \text{deg} - \theta_1 - \theta_2$

$$\mathbf{r_{OA}} = \begin{pmatrix} b\sin(\theta_1) \\ b\cos(\theta_1) \\ -a \end{pmatrix} \qquad \mathbf{F_A} = F\frac{\mathbf{r_{OA}}}{|\mathbf{r_{OA}}|} \qquad \mathbf{F_A} = \begin{pmatrix} 28.8 \\ -16.6 \\ -49.9 \end{pmatrix}\text{lb}$$

$$\mathbf{r_{OB}} = \begin{pmatrix} b\sin(\theta_1+\theta_2) \\ b\cos(\theta_1+\theta_2) \\ -a \end{pmatrix} \qquad \mathbf{F_B} = F\frac{\mathbf{r_{OB}}}{|\mathbf{r_{OB}}|} \qquad \mathbf{F_B} = \begin{pmatrix} -28.8 \\ -16.6 \\ -49.9 \end{pmatrix}\text{lb}$$

$$\mathbf{r_{OC}} = \begin{pmatrix} 0 \\ b \\ -a \end{pmatrix} \qquad \mathbf{F_C} = F\frac{\mathbf{r_{OC}}}{|\mathbf{r_{OC}}|} \qquad \mathbf{F_C} = \begin{pmatrix} 0 \\ 33.3 \\ -49.9 \end{pmatrix}\text{lb}$$

$$\mathbf{F_R} = \mathbf{F_A} + \mathbf{F_B} + \mathbf{F_C} \qquad |\mathbf{F_R}| = 149.8\ \text{lb}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}\left(\frac{\mathbf{F_R}}{|\mathbf{F_R}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 90 \\ 90 \\ 180 \end{pmatrix}\text{deg}$$

Problem 2-106

The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has magnitude F_R and is directed along the negative z axis, determine the force in each chain assuming $F_A = F_B = F_C = F$.

Given:

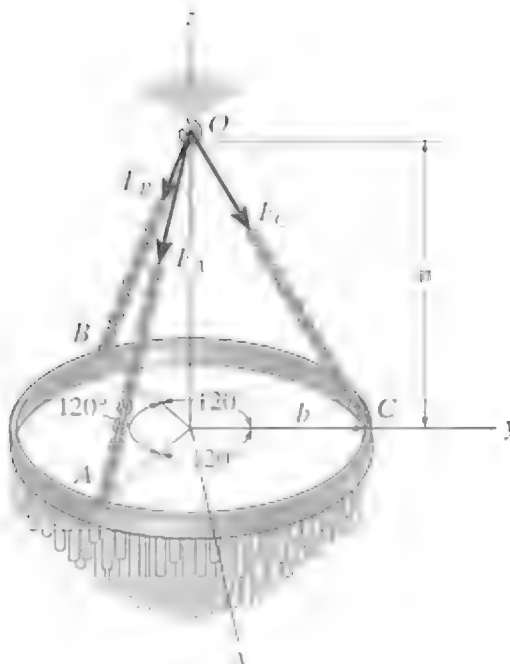
$a = 6 \text{ ft}$

$b = 4 \text{ ft}$

$F_R = 130 \text{ lb}$

Solution:

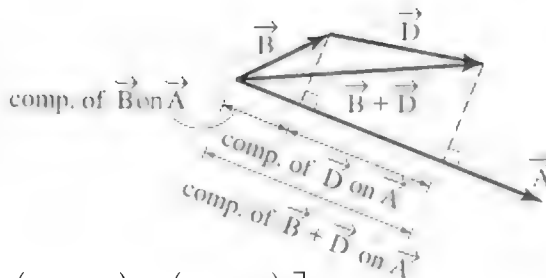
$$F = \frac{\sqrt{a^2 + b^2}}{3a} F_R$$

$F = 52.1 \text{ lb}$

Problem 2-107

Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$.

Solution:

Since the component of (**B** + **D**) is equal to the sum of the components of **B** and **D**, then

$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{D}$ (QED)

Also,

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\left[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}\right]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

$$= \left(A_x B_x + A_y B_y + A_z B_z\right) + \left(A_x D_x + A_y D_y + A_z D_z\right)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \text{(QED)}$$

Problem 2-108

Cable BC exerts force **F** on the top of the flagpole. Determine the projection of this force along the z axis of the pole.

Given:

$F = 28\ \text{N}$

$a = 12\ \text{m}$

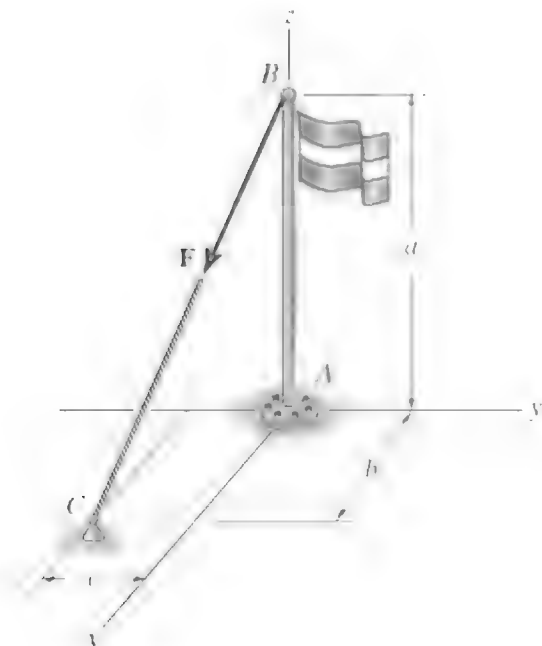
$b = 6\ \text{m}$

$c = 4\ \text{m}$

Solution:

$$\mathbf{r_{BC}} = \begin{pmatrix} b \\ -c \\ -a \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{F_v} = F\frac{\mathbf{r_{BC}}}{|\mathbf{r_{BC}}|}$$

$$F_z = -\mathbf{F_v}\,\mathbf{k} \qquad F_z = 24\ \text{N}$$

Problem 2-109

Determine the angle θ between the tails of the two vectors.

Given:

$r_1 = 9 \text{ m}$

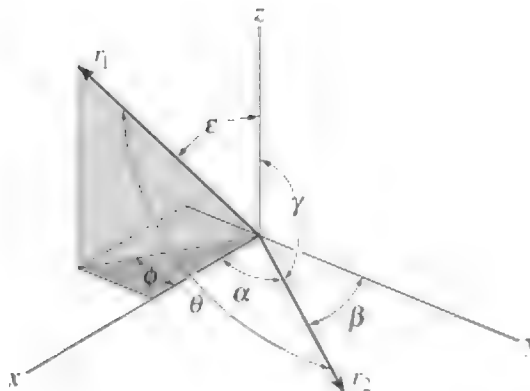
$r_2 = 6 \text{ m}$

$\alpha = 60 \text{ deg}$

$\beta = 45 \text{ deg}$

$\gamma = 120 \text{ deg}$

$\phi = 30 \text{ deg}$

$\varepsilon = 40 \text{ deg}$

Solution:

Determine the two position vectors and use the dot product to find the angle

$$\mathbf{r_{1v}} = r_1 \begin{pmatrix} \sin(\varepsilon)\cos(\phi) \\ -\sin(\varepsilon)\sin(\phi) \\ \cos(\varepsilon) \end{pmatrix} \qquad \mathbf{r_{2v}} = r_2 \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix}$$

$$\theta = \operatorname{acos}\left(\frac{\mathbf{r_{1v}} \cdot \mathbf{r_{2v}}}{|\mathbf{r_{1v}}|\ |\mathbf{r_{2v}}|}\right) \qquad \theta = 109.4 \text{ deg}$$

Problem 2-110

Determine the magnitude of the projected component of $\mathbf{r}_1$ along $\mathbf{r}_2$, and the projection of $\mathbf{r}_2$ along $\mathbf{r}_1$.

Given:

$r_1 = 9 \text{ m}$

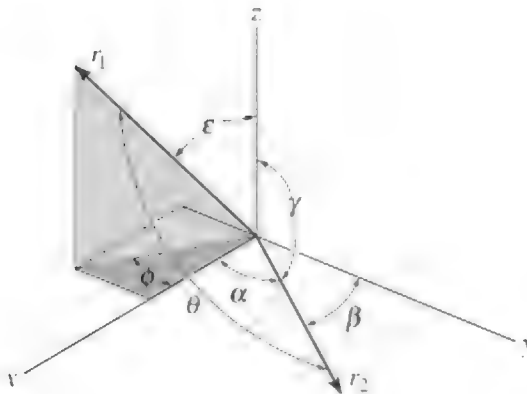
$r_2 = 6 \text{ m}$

$\alpha = 60 \text{ deg}$

$\beta = 45 \text{ deg}$

$\gamma = 120 \text{ deg}$

$\phi = 30 \text{ deg}$

$\varepsilon = 40 \text{ deg}$

Solution:

Write the vectors and unit vectors

$$\mathbf{r_{1v}} = r_1 \begin{pmatrix} \sin(\varepsilon)\cos(\phi) \\ -\sin(\varepsilon)\sin(\phi) \\ \cos(\varepsilon) \end{pmatrix} \qquad \mathbf{r_{1v}} = \begin{pmatrix} 5.01 \\ -2.89 \\ 6.89 \end{pmatrix} \text{m}$$

$$\mathbf{r_{2v}} = r_2 \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix} \qquad \mathbf{r_{2v}} = \begin{pmatrix} 3 \\ 4.24 \\ -3 \end{pmatrix} \text{m}$$

$$\mathbf{u_1} = \frac{\mathbf{r_{1v}}}{|\mathbf{r_{1v}}|} \qquad \mathbf{u_2} = \frac{\mathbf{r_{2v}}}{|\mathbf{r_{2v}}|} \qquad \mathbf{u_1} = \begin{pmatrix} 0.557 \\ -0.321 \\ 0.766 \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} 0.5 \\ 0.707 \\ -0.5 \end{pmatrix}$$

The magnitude of the projection of $\mathbf{r_1}$ along $\mathbf{r_2}$. $|\mathbf{r_{1v}} \cdot \mathbf{u_2}| = 2.99$ m

The magnitude of the projection of $\mathbf{r_2}$ along $\mathbf{r_1}$. $|\mathbf{r_{2v}} \cdot \mathbf{u_1}| = 1.99$ m

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Problem 2-111

Determine the angles θ and ϕ between the wire segments.

Given:

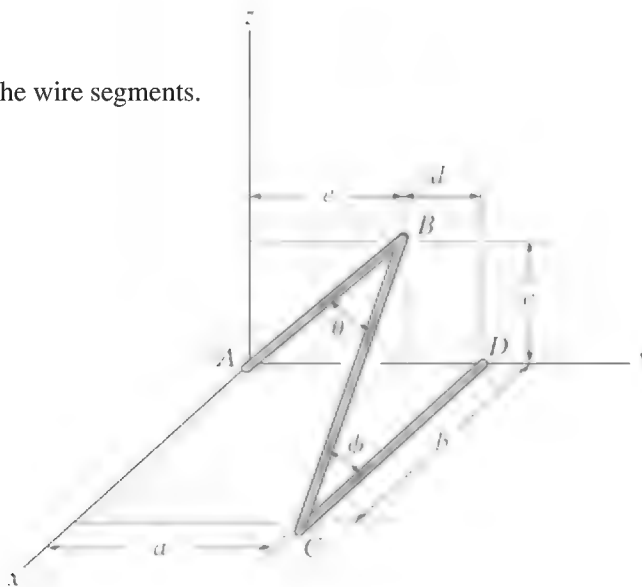
$a = 0.6$

$b = 0.8$

$c = 0.5$

$d = 0.2$

Solution:

$e = a - d$

$$\mathbf{r_{BA}} = \begin{pmatrix} 0 \\ -e \\ -c \end{pmatrix} \text{m} \qquad \mathbf{r_{BC}} = \begin{pmatrix} b \\ d \\ -c \end{pmatrix} \text{ft} \qquad \theta = \text{acos}\left(\frac{\mathbf{r_{BA}} \cdot \mathbf{r_{BC}}}{|\mathbf{r_{BA}}|\ |\mathbf{r_{BC}}|}\right) \qquad \theta = 74.0 \text{ deg}$$

$$\mathbf{r_{CB}} = \begin{pmatrix} -b \\ -d \\ c \end{pmatrix} \text{m} \qquad \mathbf{r_{CD}} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \text{ft} \qquad \phi = \text{acos}\left(\frac{\mathbf{r_{CB}} \cdot \mathbf{r_{CD}}}{|\mathbf{r_{CB}}|\ |\mathbf{r_{CD}}|}\right) \qquad \phi = 33.9 \text{ deg}$$

Problem 2-112

Determine the angle θ between the two cords.

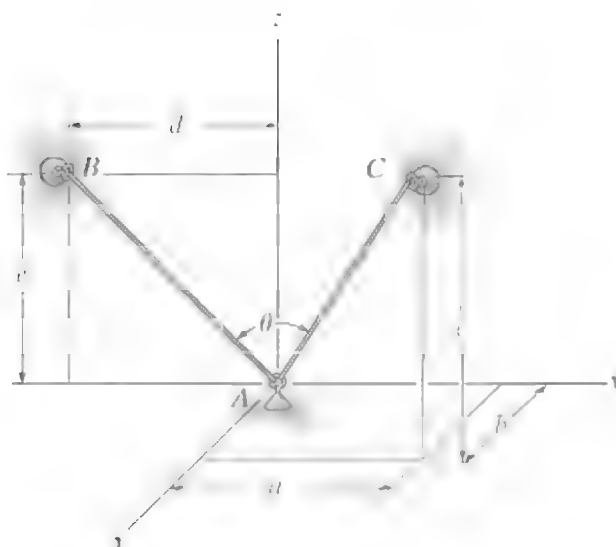
Given:

$a = 3$ m

$b = 2$ m

$c = 6$ m

$d = 3$ m

$e = 4$ m

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} b \\ a \\ c \end{pmatrix} \text{ft} \qquad \mathbf{r_{AB}} = \begin{pmatrix} 0 \\ -d \\ e \end{pmatrix} \text{ft} \qquad \theta = \text{acos}\left(\frac{\mathbf{r_{AC}} \cdot \mathbf{r_{AB}}}{|\mathbf{r_{AC}}|\ |\mathbf{r_{AB}}|}\right) \qquad \theta = 64.6 \text{ deg}$$

Problem 2-113

Determine the angle θ between the two cables.

Given:

$a = 8$ ft

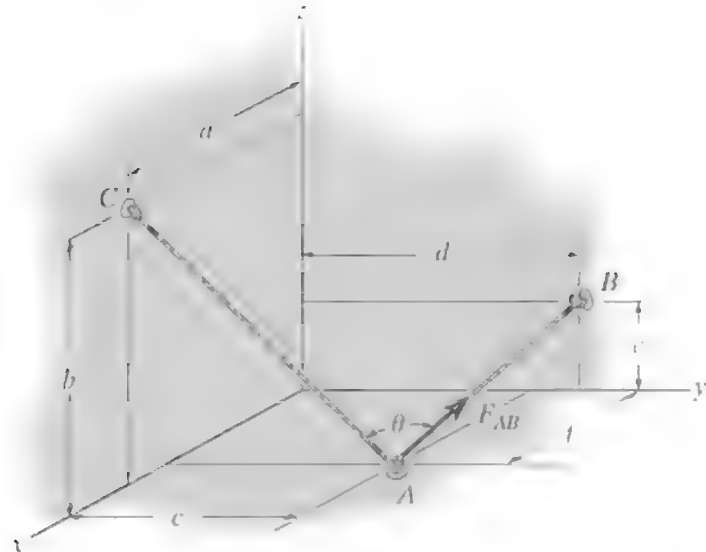
$b = 10$ ft

$c = 8 \text{ ft}$

$d = 10 \text{ ft}$

$e = 4 \text{ ft}$

$f = 6 \text{ ft}$

$F_{AB} = 12 \text{ lb}$

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} a-f \\ -c \\ b \end{pmatrix} \text{ft} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -f \\ d-c \\ e \end{pmatrix} \text{ft} \qquad \theta = \operatorname{acos}\left(\frac{\mathbf{r_{AC}} \cdot \mathbf{r_{AB}}}{|\mathbf{r_{AC}}|\ |\mathbf{r_{AB}}|}\right) \qquad \theta = 82.9 \text{ deg}$$

Problem 2-114

Determine the projected component of the force **F** acting in the direction of cable AC. Express the result as a Cartesian vector.

Given:

$F = 12 \text{ lb}$

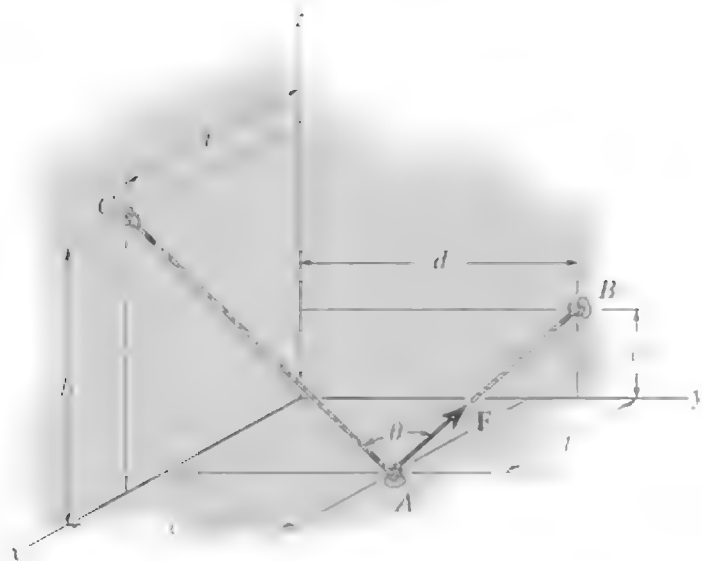
$a = 8$ ft

$b = 10$ ft

$c = 8$ ft

$d = 10$ ft

$e = 4$ ft

$f = 6$ ft

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} a-f \\ -c \\ b \end{pmatrix} \text{m} \qquad \mathbf{u_{AC}} = \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad \mathbf{u_{AC}} = \begin{pmatrix} 0.2 \\ -0.6 \\ 0.8 \end{pmatrix}$$

$$\mathbf{r_{AB}} = \begin{pmatrix} -f \\ d-c \\ e \end{pmatrix} \qquad \mathbf{F_{AB}} = F\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{F_{AB}} = \begin{pmatrix} -9.6 \\ 3.2 \\ 6.4 \end{pmatrix} \text{lb}$$

$$\mathbf{F_{AC}} = (\mathbf{F_{AB}} \cdot \mathbf{u_{AC}})\,\mathbf{u_{AC}} \qquad \mathbf{F_{AC}} = \begin{pmatrix} 0.229 \\ -0.916 \\ 1.145 \end{pmatrix} \text{lb}$$

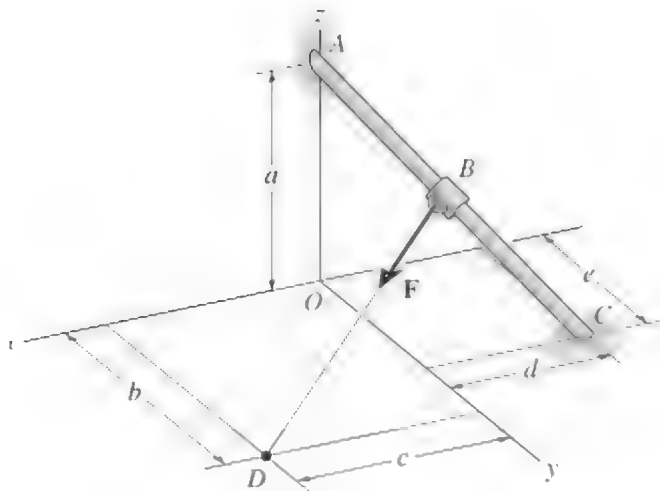
Problem 2-115

Determine the components of **F** that act along rod *AC* and perpendicular to it. Point *B* is located at the midpoint of the rod.

Given:

$F = 600$ N $\quad c = 4$ m

$a = 4$ m $\quad d = 3$ m

$b = 6$ m $\quad e = 4$ m

Solution:

Find the force vector and the unit vector $\mathbf{u}_{AC}$.

$$\mathbf{r}_{BD} = \begin{pmatrix} c + \frac{d}{2} \\ b - \frac{e}{2} \\ \frac{-a}{2} \end{pmatrix} \qquad \mathbf{r}_{BD} = \begin{pmatrix} 5.5 \\ 4 \\ -2 \end{pmatrix} \text{ m} \qquad \mathbf{F}_v = F\frac{\mathbf{r}_{BD}}{|\mathbf{r}_{BD}|} \qquad \mathbf{F}_v = \begin{pmatrix} 465.5 \\ 338.6 \\ -169.3 \end{pmatrix} \text{ N}$$

$$\mathbf{r}_{AC} = \begin{pmatrix} -d \\ e \\ -a \end{pmatrix} \qquad \mathbf{r}_{AC} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix} \text{ m} \qquad \mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} \qquad \mathbf{u}_{AC} = \begin{pmatrix} -0.5 \\ 0.6 \\ -0.6 \end{pmatrix}$$

Now find the component parallel to AC.

$$F_{parallel} = \mathbf{F}_v \cdot \mathbf{u}_{AC} \qquad F_{parallel} = 99.1 \text{ N}$$

The perpendicular component is now found

$$F_{perpendicular} = \sqrt{\mathbf{F}_v \cdot \mathbf{F}_v - F_{parallel}^{\;2}} \qquad F_{perpendicular} = 591.8 \text{ N}$$

Problem 2-116

Determine the components of **F** that act along rod AC and perpendicular to it. Point B is located a distance f along the rod from end C.

Given:

$F = 600\ \text{N}$ $c = 4\ \text{m}$

$a = 4\ \text{m}$ $d = 3\ \text{m}$

$b = 6\ \text{m}$ $e = 4\ \text{m}$

$f = 3\ \text{m}$

Solution:

$$r = \frac{f}{\sqrt{d^2 + e^2 + a^2}}$$

Find the force vector and the unit vector $\mathbf{u_{AC}}$.

$$\mathbf{r_{BD}} = \begin{bmatrix} c + d(1-r) \\ b - e(1-r) \\ -a\,r \end{bmatrix} \qquad \mathbf{r_{BD}} = \begin{pmatrix} 5.5944 \\ 3.8741 \\ -1.8741 \end{pmatrix} \text{m}$$

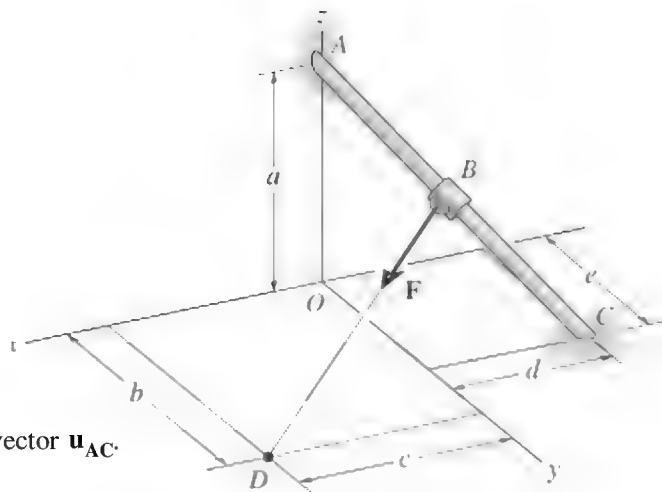
$$\mathbf{F_v} = F\frac{\mathbf{r_{BD}}}{|\mathbf{r_{BD}}|} \qquad \mathbf{F_v} = \begin{pmatrix} 475.6 \\ 329.3 \\ -159.3 \end{pmatrix} \text{N}$$

$$\mathbf{r_{AC}} = \begin{pmatrix} -d \\ e \\ -a \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix} \text{m} \qquad \mathbf{u_{AC}} = \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad \mathbf{u_{AC}} = \begin{pmatrix} -0.5 \\ 0.6 \\ -0.6 \end{pmatrix}$$

Now find the component parallel to AC.

$$F_{parallel} = \mathbf{F_v}\cdot\mathbf{u_{AC}} \qquad F_{parallel} = 82.4\ \text{N}$$

The perpendicular component is now found

$$F_{perpendicular} = \sqrt{\mathbf{F_v}\cdot\mathbf{F_v} - F_{parallel}^{\ 2}} \qquad F_{perpendicular} = 594.3\ \text{N}$$

Problem 2-117

Determine the magnitude of the projected component of the length of cord OA along the Oa axis.

Given:

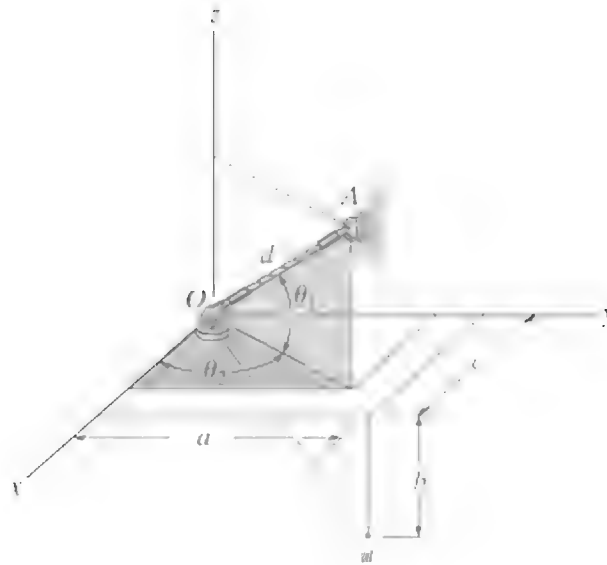
$a = 10\ \text{ft}$

$b = 5\ \text{ft}$

$c = 15\ \text{ft}$

$d = 5\ \text{ft}$

$\theta_1 = 45\ \text{deg}$

$\theta_2 = 60\ \text{deg}$

Solution:

$$\mathbf{r_{OA}} = d \begin{pmatrix} \cos(\theta_1)\cos(\theta_2) \\ \cos(\theta_1)\sin(\theta_2) \\ \sin(\theta_1) \end{pmatrix} \qquad \mathbf{r_{Oa}} = \begin{pmatrix} c \\ a \\ -b \end{pmatrix} \qquad \mathbf{u_{Oa}} = \frac{\mathbf{r_{Oa}}}{|\mathbf{r_{Oa}}|}$$

$$r_{Oa} = \mathbf{r_{OA}} \cdot \mathbf{u_{Oa}} \qquad r_{Oa} = 2.1\ \text{ft}$$

Problem 2-118

Force **F** acts at the end of the pipe. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which are directed along the pipe's axis and perpendicular to it.

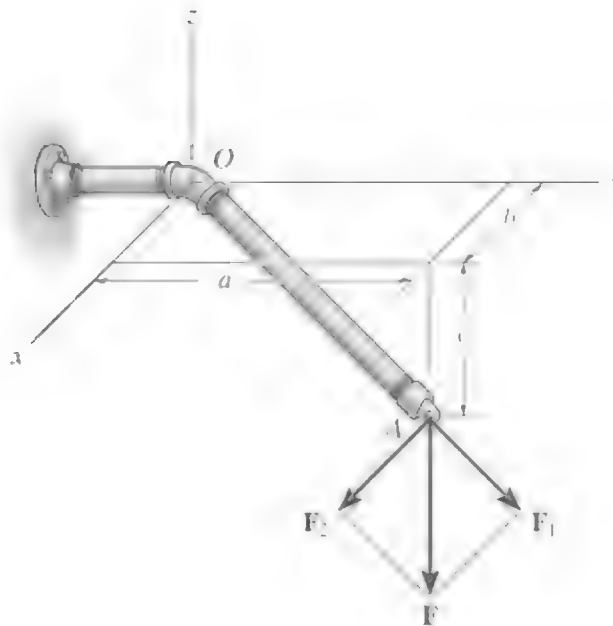
Given:

$$\mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ -40 \end{pmatrix} \text{lb} \qquad a = 5 \text{ ft} \qquad b = 3 \text{ ft} \qquad c = 3 \text{ ft}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} b \\ a \\ -c \end{pmatrix} \qquad \mathbf{u} = \frac{\mathbf{r}}{|\mathbf{r}|}$$

$$F_1 = \mathbf{F} \cdot \mathbf{u} \qquad F_1 = 18.3 \text{ lb}$$

$$F_2 = \sqrt{\mathbf{F} \cdot \mathbf{F} - F_1^2} \qquad F_2 = 35.6 \text{ lb}$$

Problem 2-119

Determine the projected component of the force **F** acting along the axis AB of the pipe.

Given:

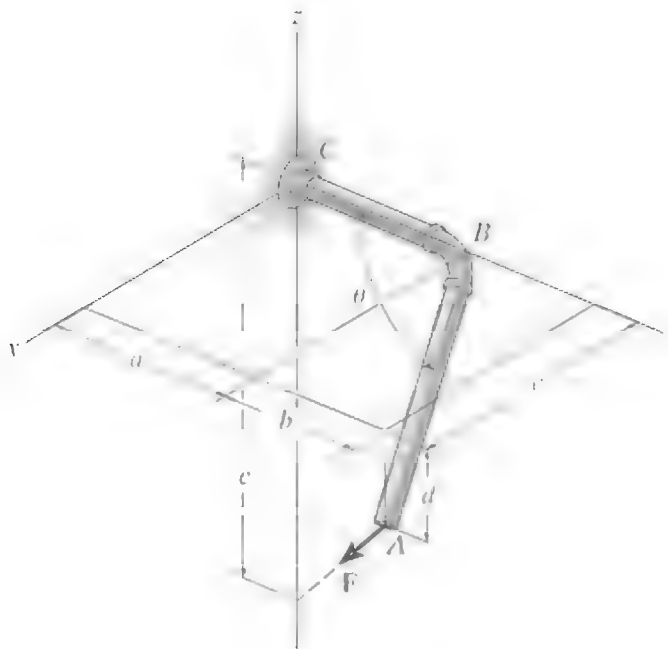
$F = 80\ \text{N}$

$a = 4\ \text{m}$

$b = 3\ \text{m}$

$c = 12\ \text{m}$

$d = 2\ \text{m}$

$e = 6\ \text{m}$

Solution:

Find the force and the unit vector

$$\mathbf{r_A} = \begin{pmatrix} -e \\ -a-b \\ d-c \end{pmatrix} \qquad \mathbf{r_A} = \begin{pmatrix} -6 \\ -7 \\ -10 \end{pmatrix} \text{m} \qquad \mathbf{F_v} = F\frac{\mathbf{r_A}}{|\mathbf{r_A}|} \qquad \mathbf{F_v} = \begin{pmatrix} -35.3 \\ -41.2 \\ -58.8 \end{pmatrix} \text{N}$$

$$\mathbf{r_{AB}} = \begin{pmatrix} -e \\ -b \\ d \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -6 \\ -3 \\ 2 \end{pmatrix} \text{m} \qquad \mathbf{u_{AB}} = \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{u_{AB}} = \begin{pmatrix} -0.9 \\ -0.4 \\ 0.3 \end{pmatrix}$$

Now find the projection using the Dot product.

$$F_{AB} = \mathbf{F_v}\cdot\mathbf{u_{AB}} \qquad F_{AB} = 31.1\ \text{N}$$

Problem 2-120

Determine the angles θ and ϕ between the axis OA of the pole and each cable, AB and AC.

Given:

$F_1 = 50\ \text{N}$

$F_2 = 35\ \text{N}$

$a = 1\ \text{m}$

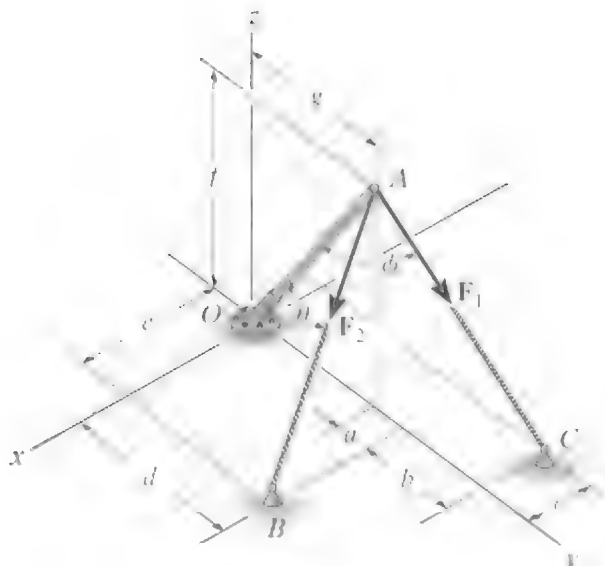
$b = 3\ \text{m}$

$c = 2\ \text{m}$

$d = 5\ \text{m}$

$e = 4\ \text{m}$

$f = 6\ \text{m}$

$g = 4\ \text{m}$

Solution:

$$\mathbf{r_{AO}} = \begin{pmatrix} 0 \\ -g \\ -f \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} e \\ a \\ -f \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -c \\ a+b \\ -f \end{pmatrix}$$

$$\theta = \operatorname{acos}\left(\frac{\mathbf{r_{AO}}\cdot\mathbf{r_{AB}}}{|\mathbf{r_{AO}}|\,|\mathbf{r_{AB}}|}\right) \qquad \theta = 52.4\ \text{deg}$$

$$\phi = \operatorname{acos}\left(\frac{\mathbf{r_{AO}}\cdot\mathbf{r_{AC}}}{|\mathbf{r_{AO}}|\,|\mathbf{r_{AC}}|}\right) \qquad \phi = 68.2\ \text{deg}$$

Problem 2-121

The two cables exert the forces shown on the pole. Determine the magnitude of the projected component of each force acting along the axis *OA* of the pole.

Given:

$F_1 = 50\ \text{N}$

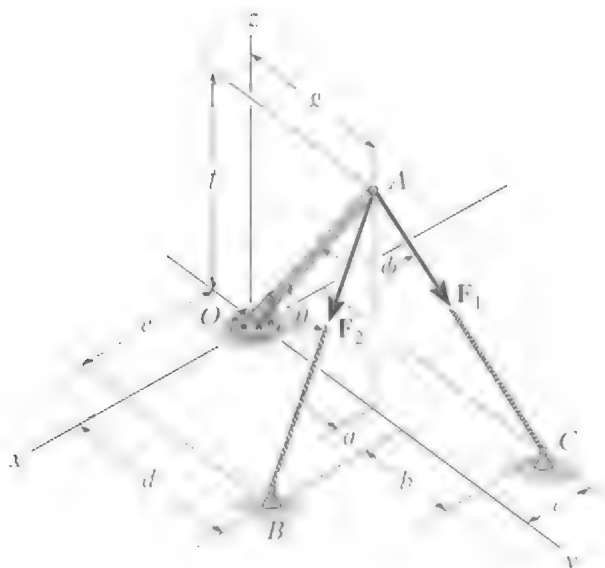
$F_2 = 35\ \text{N}$

$a = 1\ \text{m}$

$b = 3\ \text{m}$

$c = 2\ \text{m}$

$d = 5\ \text{m}$

$e = 4\ \text{m}$

$f = 6\ \text{m}$

$g = 4\ \text{m}$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} e \\ a \\ -f \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -c \\ a+b \\ -f \end{pmatrix} \qquad \mathbf{r_{AO}} = \begin{pmatrix} 0 \\ -g \\ -f \end{pmatrix} \qquad \mathbf{u_{AO}} = \frac{\mathbf{r_{AO}}}{|\mathbf{r_{AO}}|}$$

$$\mathbf{F_{1v}} = F_1 \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad F_{1AO} = \mathbf{F_{1v}} \cdot \mathbf{u_{AO}} \qquad F_{1AO} = 18.5\ \text{N}$$

$$\mathbf{F_{2v}} = F_2 \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad F_{2AO} = \mathbf{F_{2v}} \cdot \mathbf{u_{AO}} \qquad F_{2AO} = 21.3\ \text{N}$$

Problem 2-122

Force **F** is applied to the handle of the wrench. Determine the angle θ between the tail of the force and the handle AB.

Given:

$a = 300$ mm

$b = 500$ mm

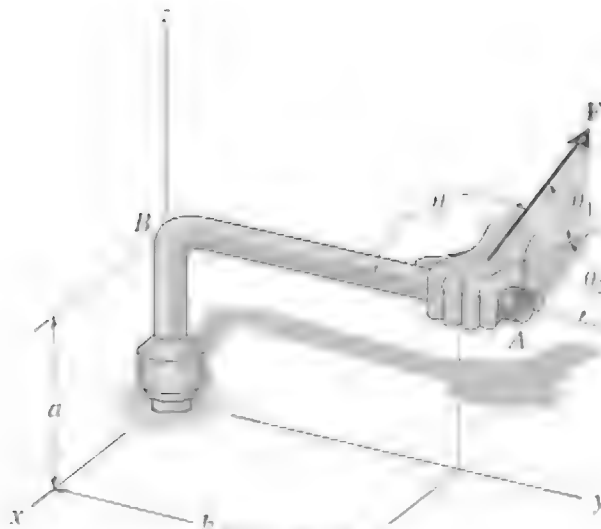
$F = 80$ N

$\theta_1 = 30$ deg

$\theta_2 = 45$ deg

Solution:

$$\mathbf{F_v} = F\begin{pmatrix} -\cos(\theta_1)\sin(\theta_2) \\ \cos(\theta_1)\cos(\theta_2) \\ \sin(\theta_1) \end{pmatrix} \qquad \mathbf{u_{ab}} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\theta = \operatorname{acos}\left(\frac{\mathbf{F_v}\cdot\mathbf{u_{ab}}}{F}\right) \qquad \theta = 127.8 \text{ deg}$$

Problem 2-123

Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F_1}$ along the line of action of $\mathbf{F_2}$.

Given:

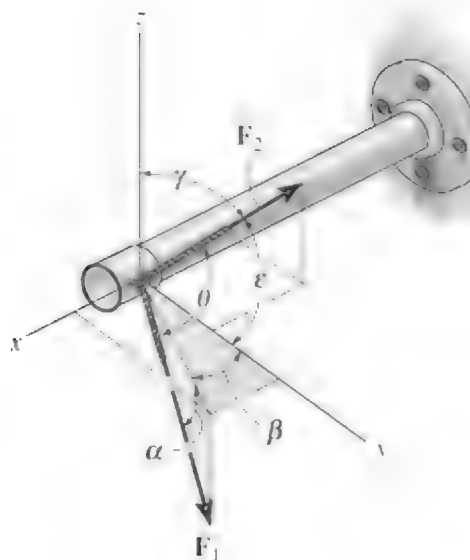
$F_1 = 30$ lb $\quad \beta = 30$ deg

$F_2 = 25$ lb $\quad \gamma = 60$ deg

$\alpha = 30$ deg $\quad \varepsilon = 60$ deg

Solution:

We first need to find the third angle (> 90 deg) that locates force $\mathbf{F_2}$.

Initial Guess: $\phi = 120$ deg

Given

$$\cos(\varepsilon)^2 + \cos(\gamma)^2 + \cos(\phi)^2 = 1$$

$$\phi = \text{Find}(\phi) \qquad \phi = 135\,\text{deg}$$

Find the force $\mathbf{F_{1v}}$ and the unit vector $\mathbf{u_2}$.

$$\mathbf{F_{1v}} = F_1 \begin{pmatrix} \cos(\alpha)\sin(\beta) \\ \cos(\alpha)\cos(\beta) \\ -\sin(\alpha) \end{pmatrix} \qquad \mathbf{F_{1v}} = \begin{pmatrix} 13 \\ 22.5 \\ -15 \end{pmatrix} \text{lb}$$

$$\mathbf{u_2} = \begin{pmatrix} \cos(\phi) \\ \cos(\varepsilon) \\ \cos(\gamma) \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} -0.7 \\ 0.5 \\ 0.5 \end{pmatrix}$$

Now find the projection $\quad F_{12} = \left|\mathbf{F_{1v}}\cdot\mathbf{u_2}\right| \qquad F_{12} = 5.4\,\text{lb}$

Problem 2-124

Determine the angle θ between the two cables attached to the pipe.

Given:

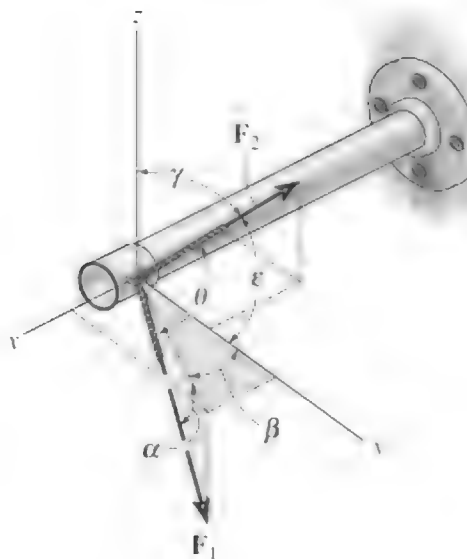
$F_1 = 30\text{ lb} \qquad \beta = 30\text{ deg}$

$F_2 = 25\text{ lb} \qquad \gamma = 60\text{ deg}$

$\alpha = 30\text{ deg} \qquad \varepsilon = 60\text{ deg}$

Solution:

We first need to find the third angle (> 90 deg) that locates force F_2.

Initial Guesses: $\quad \phi = 120\text{ deg}$

Given

$$\cos(\varepsilon)^2 + \cos(\gamma)^2 + \cos(\phi)^2 = 1$$

$\phi = \text{Find}(\phi)$ $\qquad\qquad \phi = 135\,\text{deg}$

Find the unit vectors $\mathbf{u_1}$ and $\mathbf{u_2}$.

$$\mathbf{u_1} = \begin{pmatrix} \cos(\alpha)\sin(\beta) \\ \cos(\alpha)\cos(\beta) \\ -\sin(\alpha) \end{pmatrix} \qquad \mathbf{u_1} = \begin{pmatrix} 0.4 \\ 0.8 \\ -0.5 \end{pmatrix}$$

$$\mathbf{u_2} = \begin{pmatrix} \cos(\phi) \\ \cos(\varepsilon) \\ \cos(\gamma) \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} -0.7 \\ 0.5 \\ 0.5 \end{pmatrix}$$

Find the angle using the dot product $\qquad \theta = \text{acos}(\mathbf{u_1}\cdot\mathbf{u_2}) \qquad \theta = 100.4\,\text{deg}$

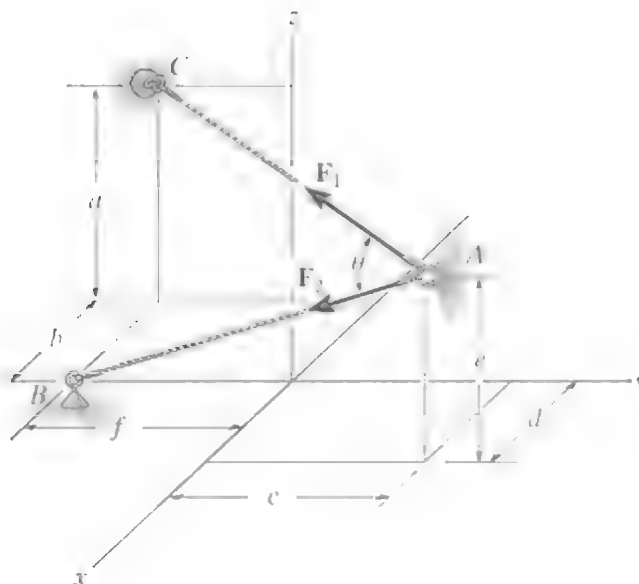
Problem 2-125

Determine the angle θ between the two cables.

Given:

$a = 7.5\text{ ft}$

$b = 2\text{ ft}$

$c = 3\text{ ft}$

$d = 2\text{ ft}$

$e = 3\text{ ft}$

$f = 3\text{ ft}$

$F_1 = 60\text{ lb}$

$F_2 = 30\text{ lb}$

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} -d-b \\ -c-f \\ a-e \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -d \\ -c-f \\ -e \end{pmatrix}$$

$$\theta = \operatorname{acos}\left(\frac{\mathbf{r_{AC}} \cdot \mathbf{r_{AB}}}{|\mathbf{r_{AC}}|\,|\mathbf{r_{AB}}|}\right) \qquad \theta = 59.2\,\text{deg}$$

Problem 2-126

Determine the projection of the force $\mathbf{F}_1$ along cable AB. Determine the projection of the force $\mathbf{F}_2$ along cable AC.

Given:

$a = 7.5$ ft

$b = 2$ ft

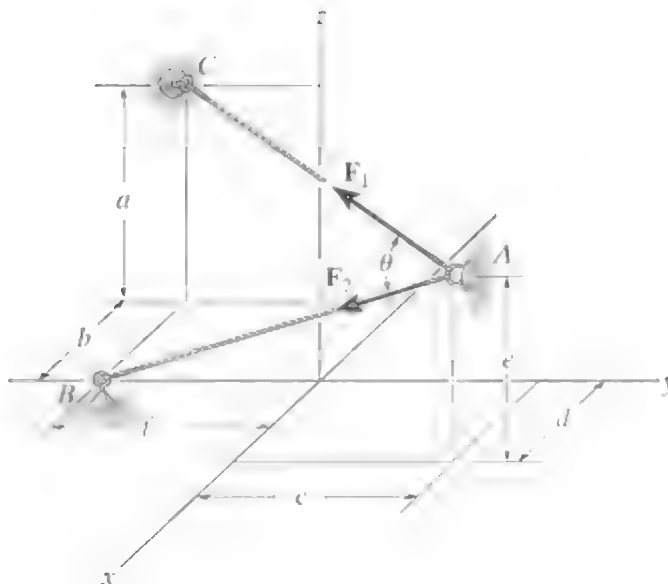
$c = 3$ ft

$d = 2$ ft

$e = 3$ ft

$f = 3$ ft

$F_1 = 60$ lb

$F_2 = 30$ lb

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} -d-b \\ -c-f \\ a-e \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -d \\ -c-f \\ -e \end{pmatrix} \qquad \mathbf{u_{AC}} = \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad \mathbf{u_{AB}} = \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|}$$

$$\mathbf{F_{1v}} = F_1\mathbf{u_{AC}} \qquad F_{1AB} = \mathbf{F_{1v}} \cdot \mathbf{u_{AB}} \qquad F_{1AB} = 30.8\,\text{lb}$$

$$\mathbf{F_{2v}} = F_2\mathbf{u_{AB}} \qquad F_{2AC} = \mathbf{F_{2v}} \cdot \mathbf{u_{AC}} \qquad F_{2AC} = 15.4\,\text{lb}$$

Problem 2-127

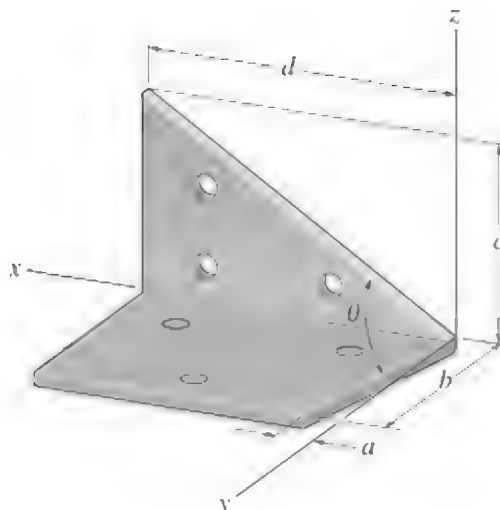
Determine the angle θ between the edges of the sheet-metal bracket.

Given:

$a = 50 \text{ mm}$

$b = 300 \text{ mm}$

$c = 250 \text{ mm}$

$d = 400 \text{ mm}$

Solution:

Find the unit vectors and use the dot product

$$\mathbf{r_1} = \begin{pmatrix} d \\ 0 \\ c \end{pmatrix} \qquad \mathbf{u_1} = \frac{\mathbf{r_1}}{|\mathbf{r_1}|}$$

$$\mathbf{r_2} = \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \qquad \mathbf{u_2} = \frac{\mathbf{r_2}}{|\mathbf{r_2}|}$$

$$\mathbf{u_1} = \begin{pmatrix} 0.848 \\ 0.000 \\ 0.530 \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} 0.164 \\ 0.986 \\ 0.000 \end{pmatrix} \qquad \theta = \operatorname{acos}(\mathbf{u_1} \cdot \mathbf{u_2}) \qquad \theta = 82 \text{ deg}$$

Problem 2-128

Determine the magnitude of the projected component of the force **F** acting along the axis *BC* of the pipe.

Given:

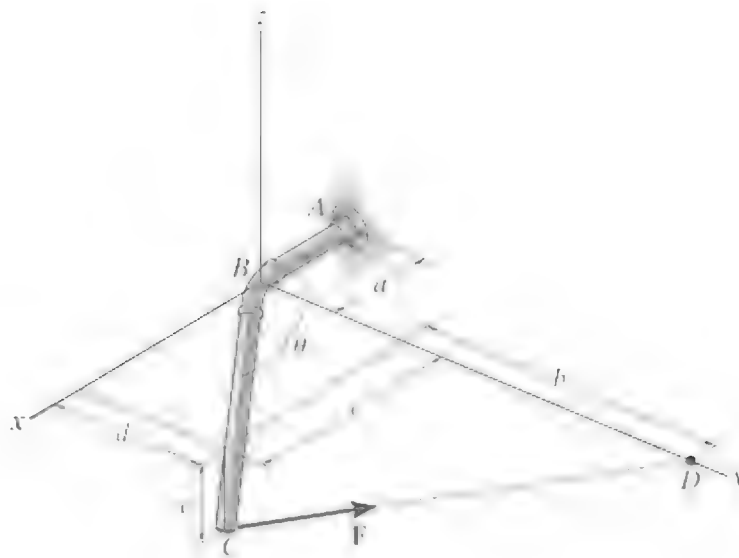
$F = 100$ lb

$a = 2$ ft

$b = 8$ ft

$c = 6$ ft

$d = 4$ ft

$e = 2$ ft

Solution:

$$\mathbf{r_{CD}} = \begin{pmatrix} -c \\ b \\ e \end{pmatrix} \qquad \mathbf{u_{CD}} = \frac{\mathbf{r_{CD}}}{|\mathbf{r_{CD}}|} \qquad \mathbf{r_{CB}} = \begin{pmatrix} -c \\ -d \\ e \end{pmatrix} \qquad \mathbf{u_{CB}} = \frac{\mathbf{r_{CB}}}{|\mathbf{r_{CB}}|}$$

$$F_{BC} = (F\mathbf{u_{CD}})\cdot\mathbf{u_{CB}} \qquad F_{BC} = 10.5 \text{ lb}$$

Problem 2-129

Determine the angle θ between pipe segments BA and BC.

Given:

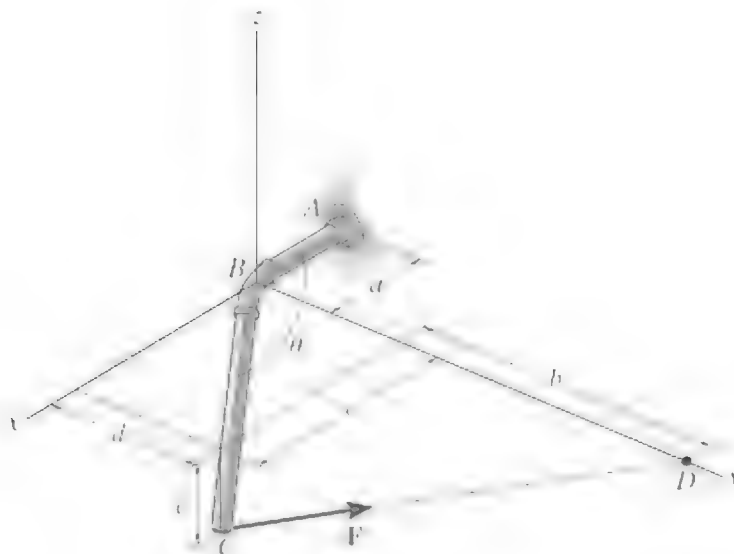
$F = 100$ lb

$a = 3$ ft

$b = 8$ ft

$c = 6$ ft

$d = 4$ ft

$e = 2$ ft

Solution:

$$\mathbf{r_{BC}} = \begin{pmatrix} c \\ d \\ -e \end{pmatrix} \qquad \mathbf{r_{BA}} = \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} \qquad \theta = \operatorname{acos}\left(\frac{\mathbf{r_{BC}}\cdot\mathbf{r_{BA}}}{|\mathbf{r_{BC}}|\,|\mathbf{r_{BA}}|}\right) \qquad \theta = 143.3 \text{ deg}$$

Problem 2-130

Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.

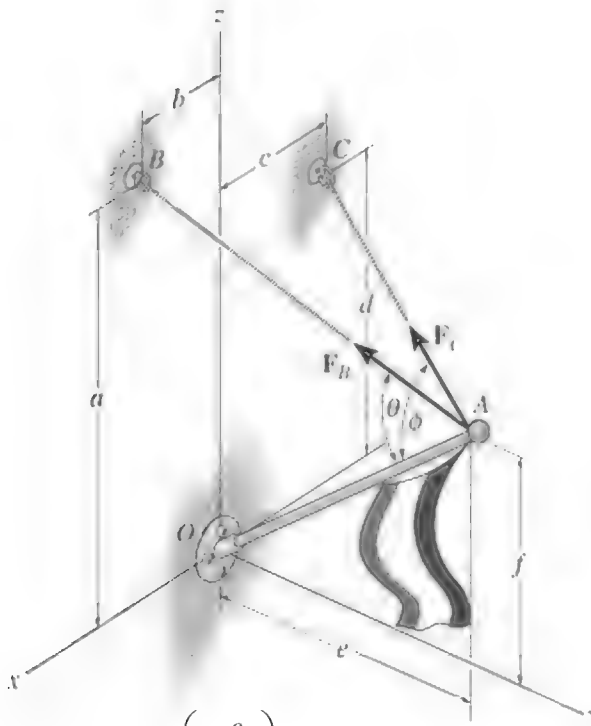
Given:

$F_B = 55$ N $\quad c = 2$ m

$F_c = 40$ N $\quad d = 4$ m

$a = 6$ m $\quad e = 4$ m

$b = 1.5$ m $\quad f = 3$ m

Solution:

$$\mathbf{r_{AO}} = \begin{pmatrix} 0 \\ -e \\ -f \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} b \\ -e \\ a - f \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -c \\ -e \\ d - f \end{pmatrix}$$

$$\theta = \text{acos}\left(\frac{\mathbf{r_{AB}} \cdot \mathbf{r_{AO}}}{|\mathbf{r_{AB}}|\,|\mathbf{r_{AO}}|}\right) \qquad \theta = 74.4\,\text{deg}$$

$$\phi = \text{acos}\left(\frac{\mathbf{r_{AC}} \cdot \mathbf{r_{AO}}}{|\mathbf{r_{AC}}|\,|\mathbf{r_{AO}}|}\right) \qquad \phi = 55.4\,\text{deg}$$

Problem 2-131

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that resultant of the three forces acts along the positive y axis and has magnitude F_R.

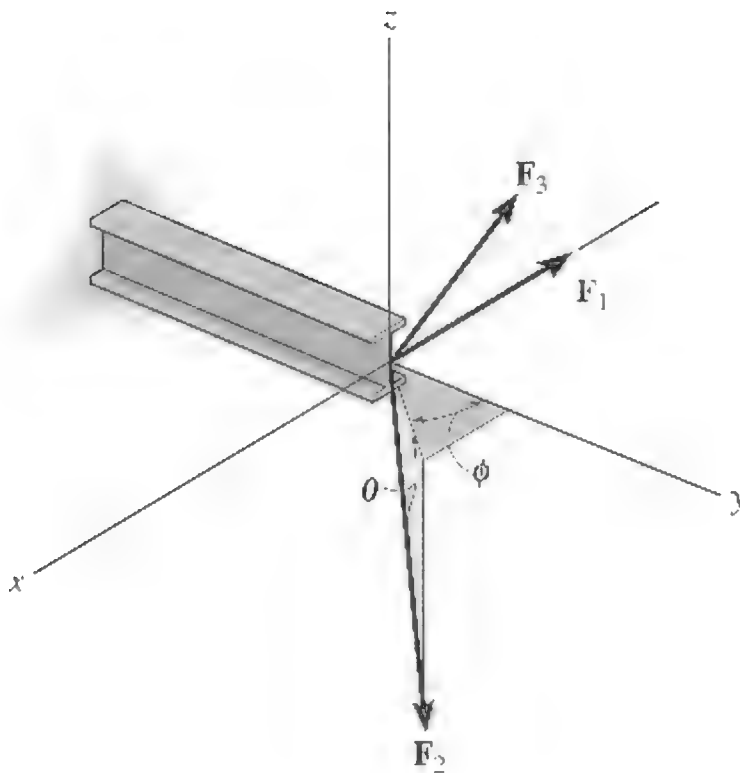
Given:

$F_R = 600\ \text{lb}$

$F_1 = 180\ \text{lb}$

$F_2 = 300\ \text{lb}$

$\phi = 40\ \text{deg}$

$\theta = 30\ \text{deg}$

Solution:

The initial guesses:

$F_3 = 100\ \text{lb}$ $\beta = 30\ \text{deg}$

$\alpha = 10\ \text{deg}$ $\gamma = 60\ \text{deg}$

Given

$F_{Rx} = \Sigma F_x;$ $-F_1 + F_2 \cos(\theta) \sin(\phi) + F_3 \cos(\alpha) = 0$

$F_{Ry} = \Sigma F_y;$ $F_2 \cos(\theta) \cos(\phi) + F_3 \cos(\beta) = F_R$

$F_{Rz} = \Sigma F_z;$ $-F_2 \sin(\theta) + F_3 \cos(\gamma) = 0$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

Solving:

$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 88.3 \\ 20.6 \\ 69.5 \end{pmatrix} \text{deg} \qquad F_3 = 428.3\ \text{lb}$$

Problem 2-132

Determine the magnitude and coordinate direction angles of $\mathbf{F_3}$ so that resultant of the three forces is zero.

Given:

$F_1 = 180 \text{ lb}$

$F_2 = 300 \text{ lb}$

$\phi = 40 \text{ deg}$

$\theta = 30 \text{ deg}$

Solution:

The initial guesses:

$\alpha = 10 \text{ deg}$

$\beta = 30 \text{ deg}$

$\gamma = 60 \text{ deg}$

$F_3 = 100 \text{ lb}$

Given

$F_{Rx} = \Sigma F_x;$ $\quad -F_1 + F_2 \cos(\theta) \sin(\phi) + F_3 \cos(\alpha) = 0$

$F_{Ry} = \Sigma F_y;$ $\quad F_2 \cos(\theta) \cos(\phi) + F_3 \cos(\beta) = 0$

$F_{Rz} = \Sigma F_z;$ $\quad -F_2 \sin(\theta) + F_3 \cos(\gamma) = 0$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

Solving:

$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 87 \\ 142.9 \\ 53.1 \end{pmatrix} \text{deg} \qquad F_3 = 249.6 \text{ lb}$$

Problem 2-133

Resolve the force **F** into two components, one acting parallel and the other acting perpendicular to the *u* axis.

Given:

$F = 600\ \text{lb}$

$\theta_1 = 60\ \text{deg}$

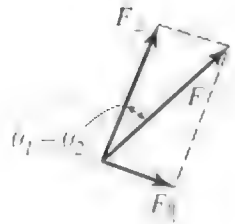
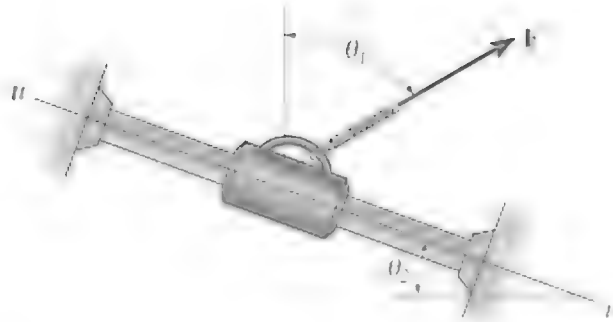
$\theta_2 = 20\ \text{deg}$

Solution:

$$F_{perpendicular} = F\cos(\theta_1 - \theta_2)$$

$$F_{perpendicular} = 460\ \text{lb}$$

$$F_{parallel} = F\sin(\theta_1 - \theta_2)$$

$$F_{parallel} = 386\ \text{lb}$$

Problem 2-134

The force **F** has a magnitude *F* and acts at the midpoint *C* of the thin rod. Express the force as a Cartesian vector.

Given:

$F = 80 \text{ lb}$

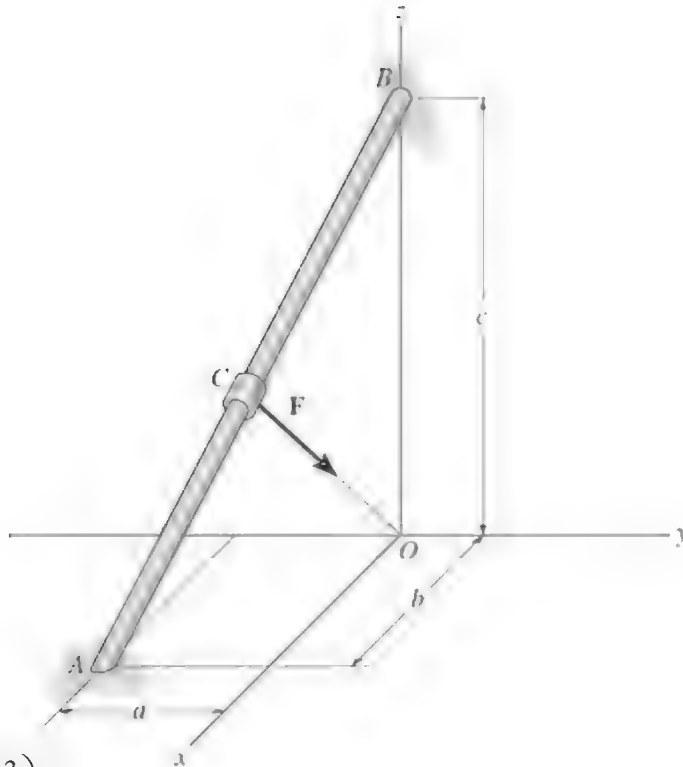
$a = 2 \text{ ft}$

$b = 3 \text{ ft}$

$c = 6 \text{ ft}$

Solution:

$$\mathbf{r_{CO}} = \begin{pmatrix} \frac{-b}{2} \\ \frac{a}{2} \\ \frac{-c}{2} \end{pmatrix}$$

$$\mathbf{F_v} = F\frac{\mathbf{r_{CO}}}{|\mathbf{r_{CO}}|} \qquad \mathbf{F_v} = \begin{pmatrix} -34.3 \\ 22.9 \\ -68.6 \end{pmatrix} \text{lb}$$

Problem 2-135

Determine the magnitude and direction of the resultant $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$ of the three forces by first finding the resultant $\mathbf{F'} = \mathbf{F_1} + \mathbf{F_3}$ and then forming $\mathbf{F_R} = \mathbf{F'} + \mathbf{F_2}$. Specify its direction measured counterclockwise from the positive x axis.

Given:

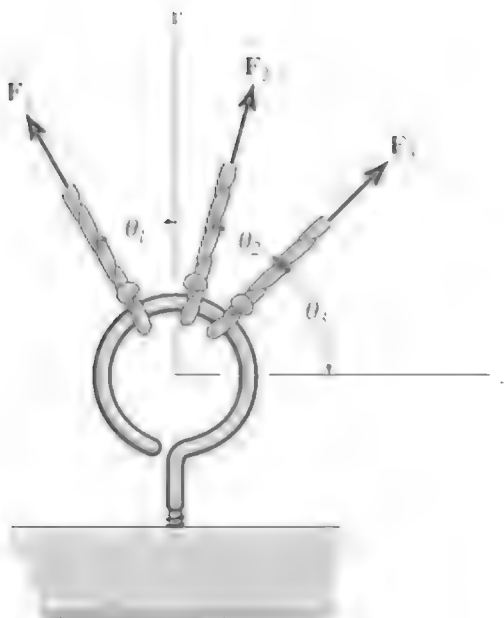
$F_1 = 80$ N

$F_2 = 75$ N

$F_3 = 50$ N

$\theta_1 = 30$ deg

$\theta_2 = 30$ deg

$\theta_3 = 45$ deg

Solution:

$$\mathbf{F_{1v}} = F_1\begin{pmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{pmatrix} \qquad \mathbf{F_{2v}} = F_2\begin{pmatrix} \cos(\theta_2+\theta_3) \\ \sin(\theta_2+\theta_3) \end{pmatrix} \qquad \mathbf{F_{3v}} = F_3\begin{pmatrix} \cos(\theta_3) \\ \sin(\theta_3) \end{pmatrix}$$

$$\mathbf{F'} = \mathbf{F_{1v}} + \mathbf{F_{3v}} \qquad \mathbf{F'} = \begin{pmatrix} -4.6 \\ 104.6 \end{pmatrix} \text{N} \qquad \mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{F_R} = \mathbf{F'} + \mathbf{F_{2v}} \qquad \mathbf{F_R} = \begin{pmatrix} 14.8 \\ 177.1 \end{pmatrix} \text{N} \qquad |\mathbf{F_R}| = 177.7 \text{ N}$$

$$\theta = \operatorname{atan}\left(\frac{\mathbf{F_R j}}{\mathbf{F_R i}}\right) \qquad \theta = 85.2 \text{ deg}$$

Problem 2-136

The leg is held in position by the quadriceps AB, which is attached to the pelvis at A. If the force exerted on this muscle by the pelvis is $\mathbf{F}$, in the direction shown, determine the stabilizing force component acting along the positive y axis and the supporting force component acting along the negative x axis.

Given:

$F = 85$ N

$\theta_1 = 55$ deg

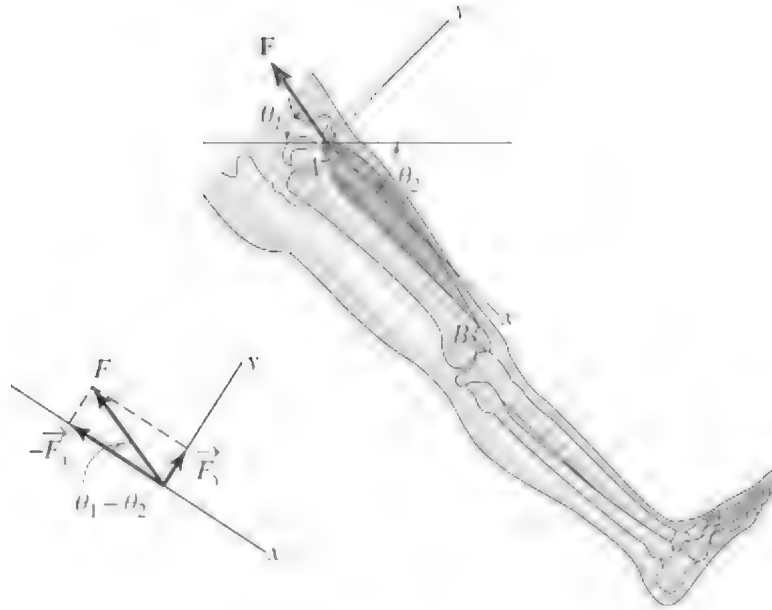
$\theta_2 = 45$ deg

Solution:

$F_x = F\cos(\theta_1 - \theta_2)$

$F_x = 83.7$ N

$F_y = F\sin(\theta_1 - \theta_2)$

$F_y = 14.8$ N

Problem 2-137

Determine the magnitudes of the projected components of the force **F** in the direction of the cables *AB* and *AC* .

Given:

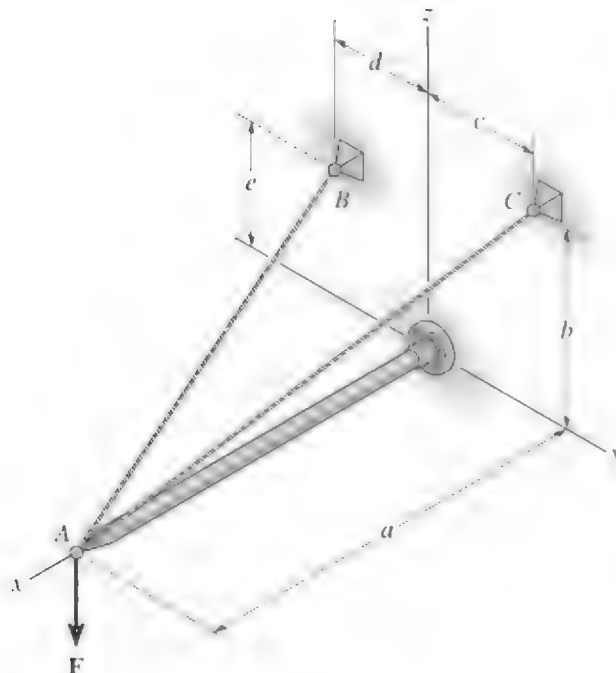
$$\mathbf{F} = \begin{pmatrix} 60 \\ 12 \\ -40 \end{pmatrix} \text{N}$$

$a = 3$ m

$b = 1.5$ m

$c = 1$ m

$d = 0.75$ m

$e = 1$ m

Solution:

Find the unit vectors, then use the dot product

$$\mathbf{r_{AB}} = \begin{pmatrix} -a \\ -d \\ e \end{pmatrix} \quad \mathbf{r_{AB}} = \begin{pmatrix} -3 \\ -0.8 \\ 1 \end{pmatrix} \text{m} \quad \mathbf{u_{AB}} = \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \quad \mathbf{u_{AB}} = \begin{pmatrix} -0.9 \\ -0.2 \\ 0.3 \end{pmatrix}$$

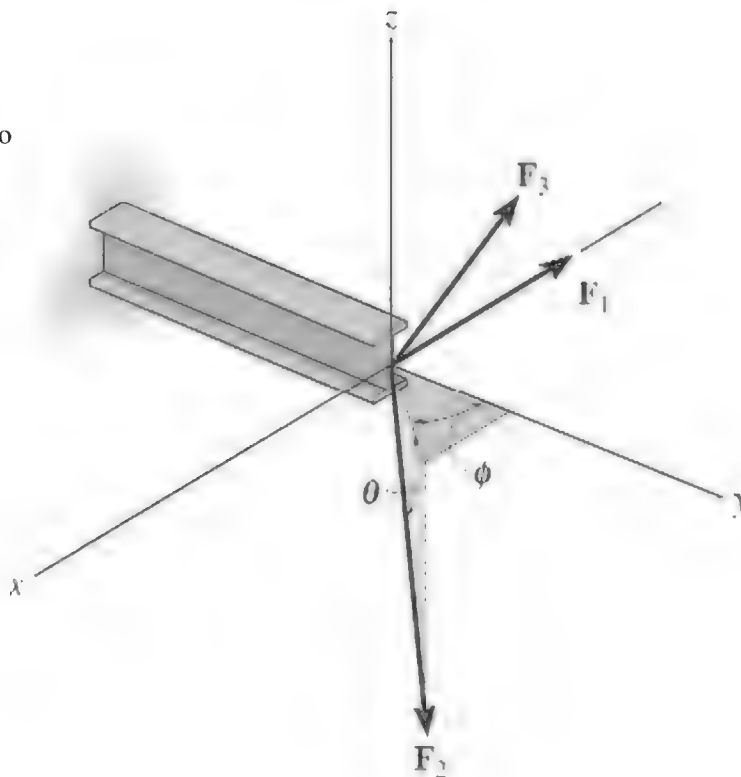
$$\mathbf{r_{AC}} = \begin{pmatrix} -a \\ c \\ b \end{pmatrix} \quad \mathbf{r_{AC}} = \begin{pmatrix} -3 \\ 1 \\ 1.5 \end{pmatrix} \text{m} \quad \mathbf{u_{AC}} = \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \quad \mathbf{u_{AC}} = \begin{pmatrix} -0.9 \\ 0.3 \\ 0.4 \end{pmatrix}$$

$$\mathbf{F_{AB}} = F\mathbf{u_{AB}} \quad \mathbf{F_{AB}} = \begin{pmatrix} -78.5 \\ -19.6 \\ 26.2 \end{pmatrix} \text{N} \quad \mathbf{F_{AC}} = F\mathbf{u_{AC}} \quad \mathbf{F_{AC}} = \begin{pmatrix} -72.9 \\ 24.3 \\ 36.4 \end{pmatrix} \text{N}$$

Problem 2-138

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that resultant of the three forces is zero.

Given:

$F_1 = 180 \text{ lb}$ $\quad \phi = 40 \text{ deg}$

$F_2 = 300 \text{ lb}$ $\quad \theta = 30 \text{ deg}$

Solution:

The initial guesses:

$\alpha = 10 \text{ deg}$

$\beta = 30 \text{ deg}$

$\gamma = 60 \text{ deg}$

$F_3 = 100 \text{ lb}$

Given

$F_{Rx} = \Sigma F_x;$ $\quad -F_1 + F_2 \cos(\theta) \sin(\phi) + F_3 \cos(\alpha) = 0$

$F_{Ry} = \Sigma F_y;$ $\quad F_2 \cos(\theta) \cos(\phi) + F_3 \cos(\beta) = 0$

$F_{Rz} = \Sigma F_z;$ $\qquad -F_2 \sin(\theta) + F_3 \cos(\gamma) = 0$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

Solving:

$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 87 \\ 142.9 \\ 53.1 \end{pmatrix} \text{deg} \qquad F_3 = 249.6 \text{ lb}$$

Problem 2-139

Determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has magnitude F_R.

Given:

$F_1 = 30$ lb

$F_2 = 30$ lb

$F_R = 20$ lb

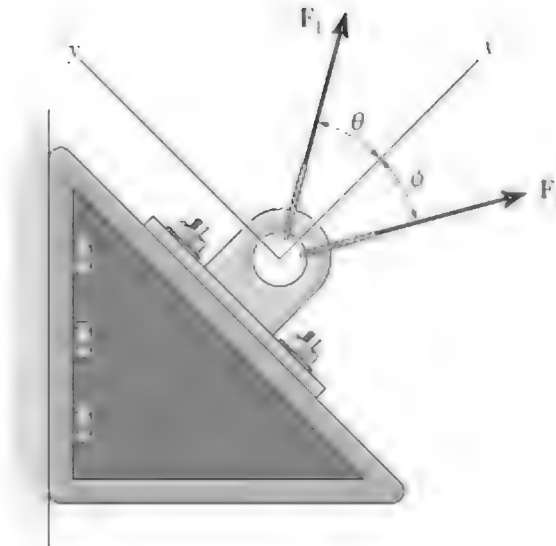
Solution:

Initial Guesses:

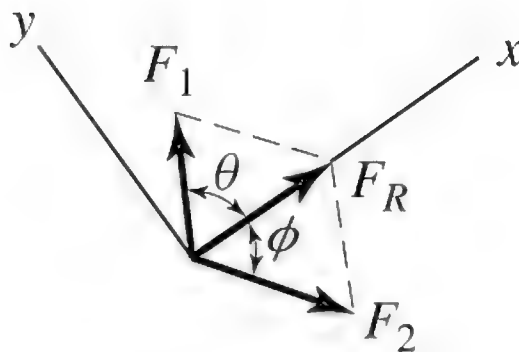
$\theta = 20$ deg $\qquad \phi = 20$ deg

Given

$$\frac{F_1}{\sin(\phi)} = \frac{F_2}{\sin(\theta)}$$

$$F_R^2 = F_1^2 + F_2^2 - 2\, F_1\, F_2 \cos(180 \text{ deg} - \theta - \phi)$$

$$\begin{pmatrix} \theta \\ \phi \end{pmatrix} = \text{Find}(\theta, \phi)$$

$\theta = 70.5\text{ deg}$

$\phi = 70.5\text{ deg}$

Problem 2-140

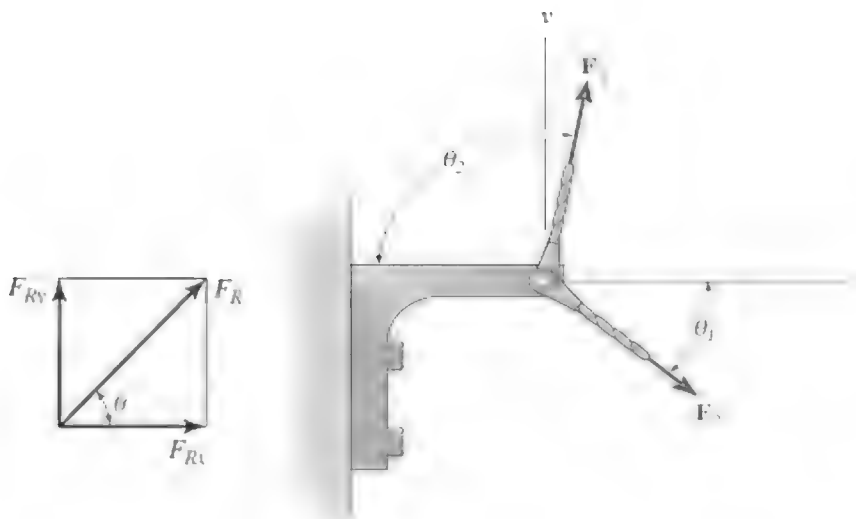
Determine the magnitude of the resultant force and its direction measured counterclockwise from the *x* axis.

Given:

$F_1 = 300\text{ lb}$

$F_2 = 200\text{ lb}$

$\theta_1 = 40\text{ deg}$

$\theta_2 = 100\text{ deg}$

Solution:

$$F_{Rx} = F_1 \cos(180\text{ deg} - \theta_2) + F_2 \cos(\theta_1) \qquad F_{Rx} = 205.3\text{ lb}$$

$$F_{Ry} = F_1 \sin(180\text{ deg} - \theta_2) - F_2 \sin(\theta_1) \qquad F_{Ry} = 166.9\text{ lb}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F_R = 265\text{ lb}$$

$$\theta = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \theta = 39.1\text{ deg}$$

Problem 3-1

Determine the magnitudes of $\mathbf{F_1}$ and $\mathbf{F_2}$ so that the particle is in equilibrium.

Given:

$F = 500\text{ N}$

$\theta_1 = 45\text{ deg}$

$\theta_2 = 30\text{deg}$

Solution:

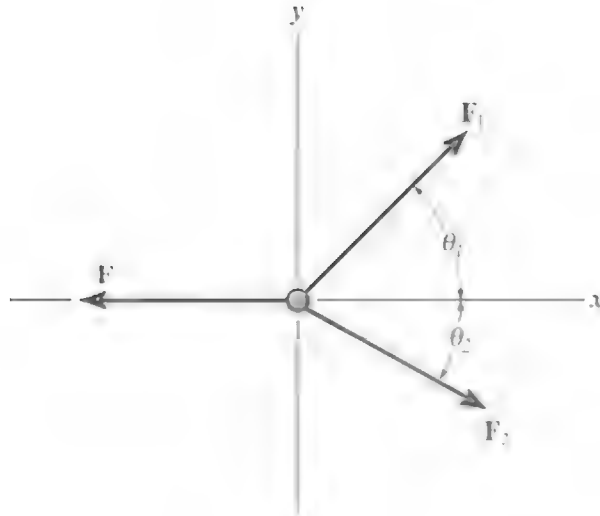
Initial Guesses

$F_1 = 1\text{N} \qquad F_2 = 1\text{N}$

Given

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_1\cos(\theta_1) + F_2\cos(\theta_2) - F = 0$

$+\uparrow \Sigma F_y = 0; \qquad F_1\sin(\theta_1) - F_2\sin(\theta_2) = 0$

$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \text{Find}(F_1, F_2) \qquad \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 259 \\ 366 \end{pmatrix}\text{ N}$

Problem 3-2

Determine the magnitude and direction θ of $\mathbf{F}$ so that the particle is in equilibrium.

Units Used:

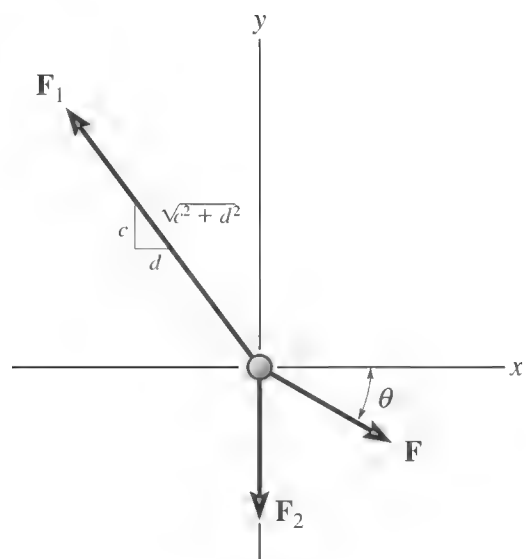
$\text{kN} = 10^3\text{ N}$

Given:

$F_1 = 7\text{ kN}$

$F_2 = 3\text{ kN}$

$c = 4$

$d = 3$

Solution:

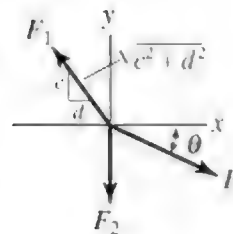
The initial guesses: $F = 1\text{kN}$ $\theta = 30\text{deg}$

Given

Equations of equilibrium:

$\xrightarrow{+} \Sigma F_x = 0;$ $\left(\frac{-d}{\sqrt{c^2 + d^2}}\right) F_1 + F\cos(\theta) = 0$

$+\uparrow \Sigma F_y = 0;$ $\left(\frac{c}{\sqrt{c^2 + d^2}}\right) F_1 - F_2 - F\sin(\theta) = 0$

$\begin{pmatrix} F \\ \theta \end{pmatrix} = \text{Find}(F, \theta)$ $F = 4.94\,\text{kN}$ $\theta = 31.8\,\text{deg}$

Problem 3-3

Determine the magnitude of **F** and the orientation θ of the force $\mathbf{F}_3$ so that the particle is in equilibrium.

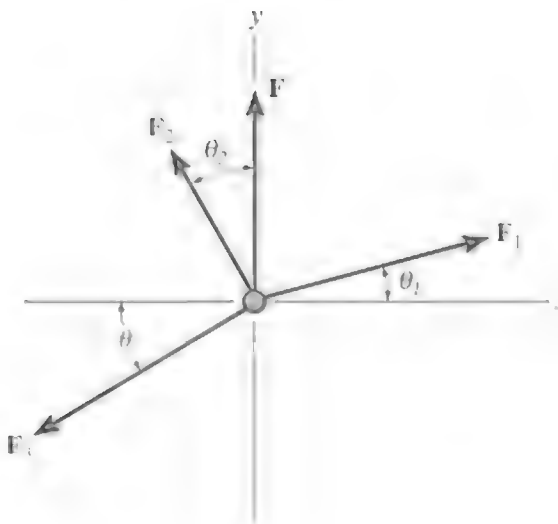
Given:

$F_1 = 700\ \text{N}$

$F_2 = 450\ \text{N}$

$F_3 = 750\ \text{N}$

$\theta_1 = 15\ \text{deg}$

$\theta_2 = 30\ \text{deg}$

Solution:

Initial Guesses: $F = 1\text{N}$ $\theta = 10\text{deg}$

Given

$\xrightarrow{+} \Sigma F_x = 0;$ $F_1\cos(\theta_1) - F_2\sin(\theta_2) - F_3\cos(\theta) = 0$

$+\uparrow\ \Sigma F_y = 0;\qquad F + F_2\cos(\theta_2) + F_1\sin(\theta_1) - F_3\sin(\theta) = 0$

$\begin{pmatrix} F \\ \theta \end{pmatrix} = \text{Find}(F, \theta)\qquad F = 28.25\ \text{N}\qquad \theta = 53.02\,\text{deg}$

Problem 3-4

Determine the magnitude and angle θ of **F** so that the particle is in equilibrium.

Units Used:

$\text{kN} = 10^3\ \text{N}$

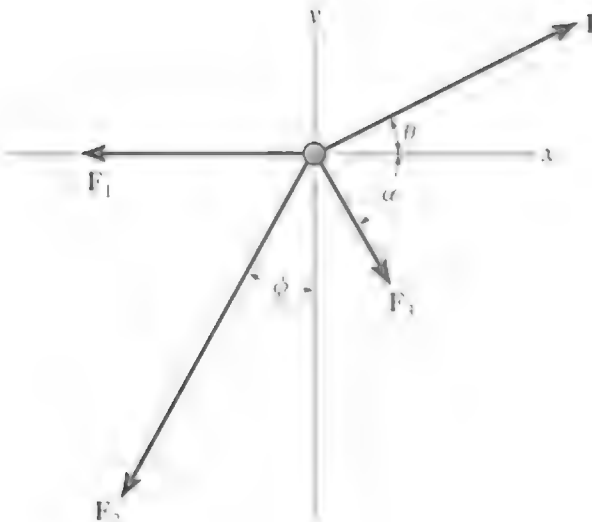
Given:

$F_1 = 4.5\ \text{kN}$

$F_2 = 7.5\ \text{kN}$

$F_3 = 2.25\ \text{kN}$

$\alpha = 60\ \text{deg}$

$\phi = 30\ \text{deg}$

Solution:

Guesses:

$F = 1\ \text{kN}\qquad \theta = 1$

Given

Equations of Equilibrium:

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;\qquad F\cos(\theta) - F_2\sin(\phi) - F_1 + F_3\cos(\alpha) = 0$

$+\uparrow\ \Sigma F_y = 0;\qquad F\sin(\theta) - F_2\cos(\phi) - F_3\sin(\alpha) = 0$

$\begin{pmatrix} F \\ \theta \end{pmatrix} = \text{Find}(F, \theta)\qquad F = 11.05\,\text{kN}\qquad \theta = 49.84\,\text{deg}$

Problem 3-5

The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of **F** and **T** for equilibrium.

Units Used:

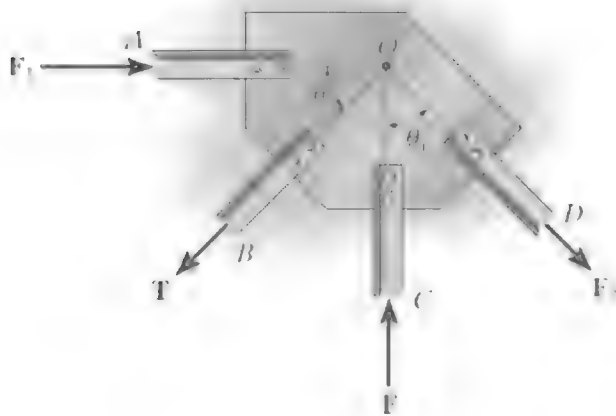
$$\text{kN} = 10^3\ \text{N}$$

Given:

$$F_1 = 8\ \text{kN}$$

$$F_2 = 5\ \text{kN}$$

$$\theta_1 = 45\ \text{deg}$$

$$\theta = 30\ \text{deg}$$

Solution:

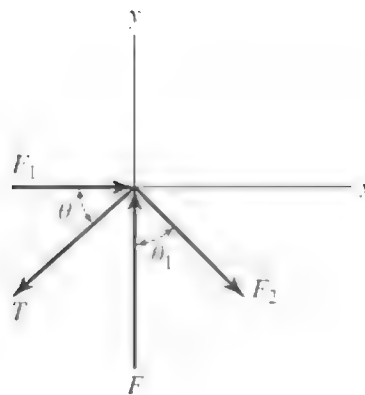
$\xrightarrow{+}\ \Sigma F_x = 0;\quad -T\cos(\theta) + F_1 + F_2\sin(\theta_1) = 0$

$$T = \frac{F_1 + F_2\sin(\theta_1)}{\cos(\theta)}$$

$$T = 13.3\,\text{kN}$$

$+\uparrow\ \Sigma F_y = 0;\quad F - T\sin(\theta) - F_2\cos(\theta_1) = 0$

$$F = T\sin(\theta) + F_2\cos(\theta_1)$$

$$F = 10.2\,\text{kN}$$

Problem 3-6

The gusset plate is subjected to the forces of four members. Determine the force in member B and its proper orientation θ for equilibrium. The forces are concurrent at point O.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$F = 12 \text{ kN}$

$F_1 = 8 \text{ kN}$

$F_2 = 5 \text{ kN}$

$\theta_1 = 45 \text{ deg}$

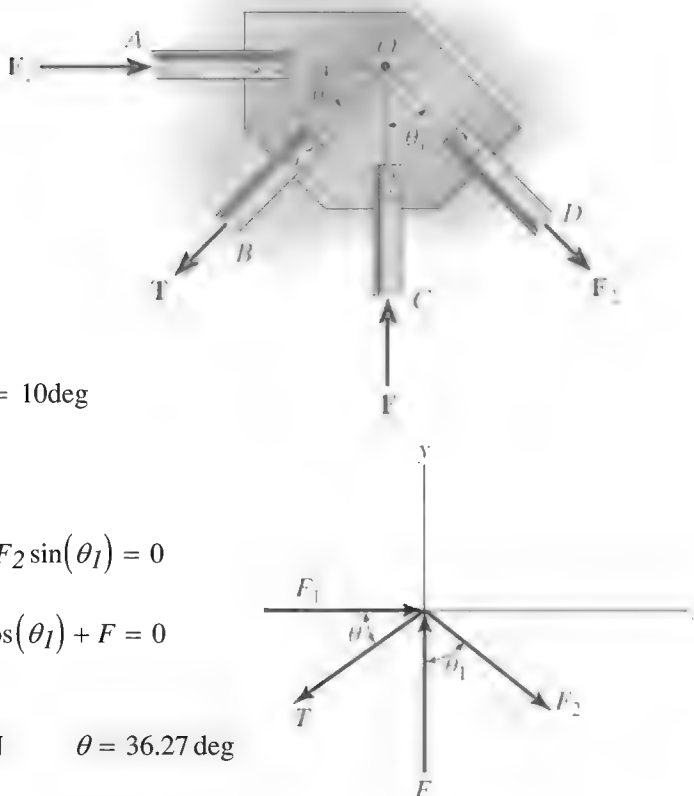
Solution:

Initial Guesses $T = 1\text{kN}$ $\theta = 10\text{deg}$

Given

$\xrightarrow{+} \Sigma F_x = 0;$ $F_1 - T\cos(\theta) + F_2\sin(\theta_1) = 0$

$+\uparrow \Sigma F_y = 0;$ $-T\sin(\theta) - F_2\cos(\theta_1) + F = 0$

$\begin{pmatrix} T \\ \theta \end{pmatrix} = \text{Find}(T, \theta)$ $T = 14.31 \text{ kN}$ $\theta = 36.27 \text{ deg}$

Problem 3-7

Determine the maximum weight of the engine that can be supported without exceeding a tension of $\mathbf{T_1}$ in chain *AB* and $\mathbf{T_2}$ in chain *AC*.

Given:

$\theta = 30 \text{ deg}$

$T_1 = 450 \text{ lb}$

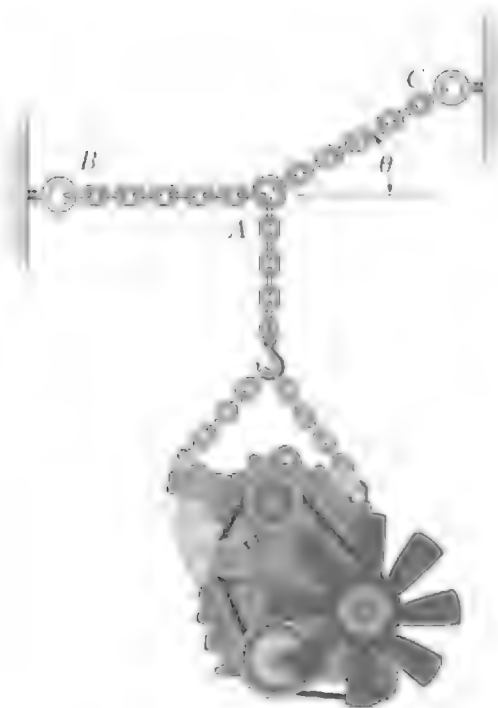
$T_2 = 480 \text{ lb}$

Solution:

Initial Guesses

$F_{AB} = T_1$

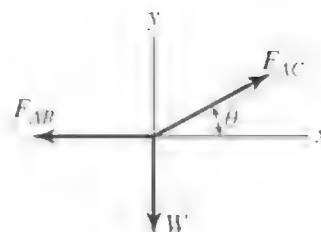
$F_{AC} = T_2$

$W = 1\text{lb}$

Given

Assuming cable AB reaches the maximum tension $F_{AB} = T_1$.

$\xrightarrow{+} \Sigma F_x = 0;$ $F_{AC}\cos(\theta) - F_{AB} = 0$

$+\uparrow \Sigma F_y = 0;$ $F_{AC}\sin(\theta) - W = 0$

$\begin{pmatrix} F_{AC1} \\ W_1 \end{pmatrix} = \text{Find}(F_{AC}, W)$ $W_1 = 259.81 \text{ lb}$

Given Assuming cable AC reaches the maximum tension $F_{AC} = T_2$.

$\xrightarrow{+} \Sigma F_x = 0;$ $F_{AC}\cos(\theta) - F_{AB} = 0$

$+\uparrow \Sigma F_y = 0;$ $F_{AC}\sin(\theta) - W = 0$

$\begin{pmatrix} F_{AB2} \\ W_2 \end{pmatrix} = \text{Find}(F_{AB}, W)$ $W_2 = 240.00 \text{ lb}$

$W = \min(W_1, W_2)$ $W = 240.00 \text{ lb}$

Problem 3-8

The engine of mass M is suspended from a vertical chain at A. A second chain is wrapped around the engine and held in position by the spreader bar BC. Determine the compressive force acting along the axis of the bar and the tension forces in segments BA and CA of the chain. *Hint:* Analyze equilibrium first at A, then at B.

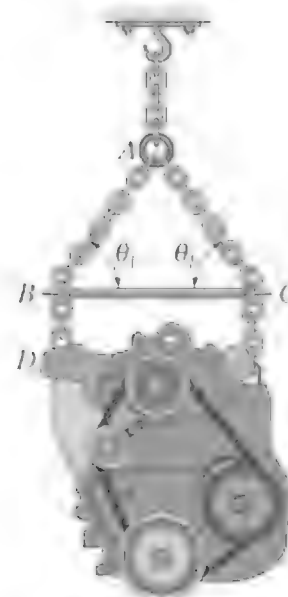
Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$M = 200 \text{ kg}$

$\theta_1 = 55 \text{ deg}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

Solution:

Initial guesses: $F_{BA} = 1\ \text{kN}$ $F_{CA} = 2\ \text{kN}$

Given Point A

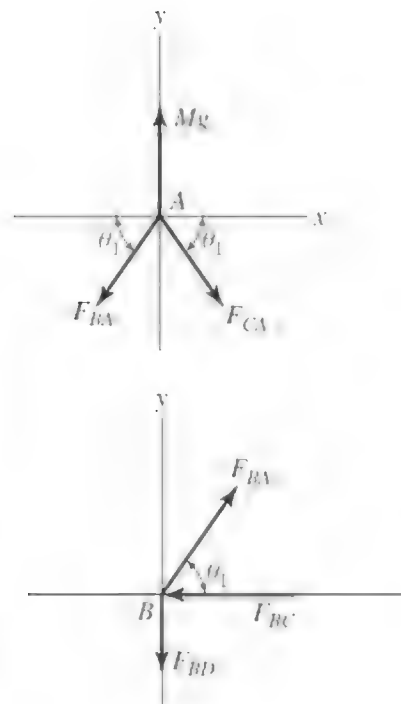
$\xrightarrow{+} \Sigma F_x = 0;$ $F_{CA}\cos(\theta_1) - F_{BA}\cos(\theta_1) = 0$

$+\uparrow \Sigma F_y = 0;$ $M(g) - F_{CA}\sin(\theta_1) - F_{BA}\sin(\theta_1) = 0$

$$\begin{pmatrix} F_{BA} \\ F_{CA} \end{pmatrix} = \text{Find}(F_{BA}, F_{CA}) \qquad \begin{pmatrix} F_{BA} \\ F_{CA} \end{pmatrix} = \begin{pmatrix} 1.20 \\ 1.20 \end{pmatrix} \text{kN}$$

At point B:

$\xrightarrow{+} \Sigma F_x = 0;$ $F_{BA}\cos(\theta_1) - F_{BC} = 0$

$F_{BC} = F_{BA}\cos(\theta_1)$ $F_{BC} = 687\ \text{N}$

Problem 3-9

Cords AB and AC can each sustain a maximum tension T. If the drum has weight W, determine the smallest angle θ at which they can be attached to the drum.

Given:

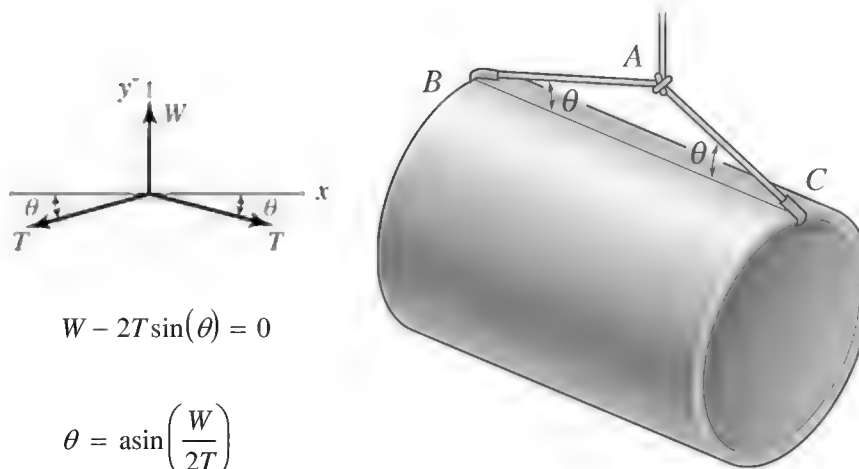
$T = 800\ \text{lb}$

$W = 900\ \text{lb}$

Solution:

$+\uparrow \Sigma F_y = 0;$ $W - 2T\sin(\theta) = 0$

$$\theta = \text{asin}\left(\frac{W}{2T}\right)$$

$\theta = 34.2\ \text{deg}$

Problem 3-10

The crate of weight W is hoisted using the ropes AB and AC. Each rope can withstand a maximum tension T before it breaks. If AB always remains horizontal, determine the smallest angle θ to which the crate can be hoisted.

Given:

$W = 500 \text{ lb}$

$T = 2500 \text{ lb}$

Solution:

Case 1: Assume $T_{AB} = T$

The initial guess $\theta = 30 \text{ deg}$ $T_{AC} = 2000 \text{ lb}$

Given

$\xrightarrow{+} \Sigma F_x = 0; \quad T_{AB} - T_{AC}\cos(\theta) = 0$

$+\uparrow \Sigma F_y = 0; \quad T_{AC}\sin(\theta) - W = 0$

$\begin{pmatrix} T_{AC1} \\ \theta_1 \end{pmatrix} = \text{Find}(T_{AC}, \theta) \quad \theta_1 = 11.31 \text{ deg} \quad T_{AC1} = 2550 \text{ lb}$

Case 1: Assume $T_{AC} = T$

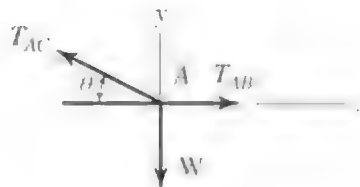
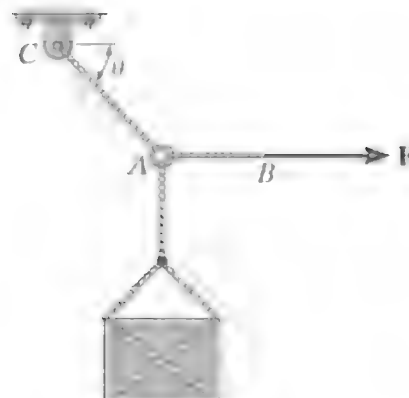
The initial guess $\theta = 30 \text{ deg}$ $T_{AB} = 2000 \text{ lb}$

Given

$\xrightarrow{+} \Sigma F_x = 0; \quad T_{AB} - T_{AC}\cos(\theta) = 0$

$+\uparrow \Sigma F_y = 0; \quad T_{AC}\sin(\theta) - W = 0$

$\begin{pmatrix} T_{AB2} \\ \theta_2 \end{pmatrix} = \text{Find}(T_{AB}, \theta) \quad \theta_2 = 11.54 \text{ deg} \quad T_{AB2} = 2449 \text{ lb}$

$\theta = \max(\theta_1, \theta_2) \quad \theta = 11.54 \text{ deg}$

Problem 3-11

Two electrically charged pith balls, each having mass M, are suspended from light threads of equal length. Determine the resultant horizontal force of repulsion, F, acting on each ball if the measured distance between them is r.

Given:

$M = 0.2$ gm

$r = 200$ mm

$l = 150$ mm

$d = 50$ mm

Solution:

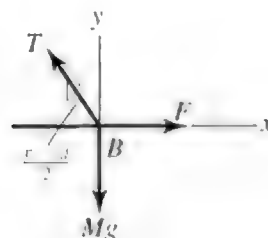
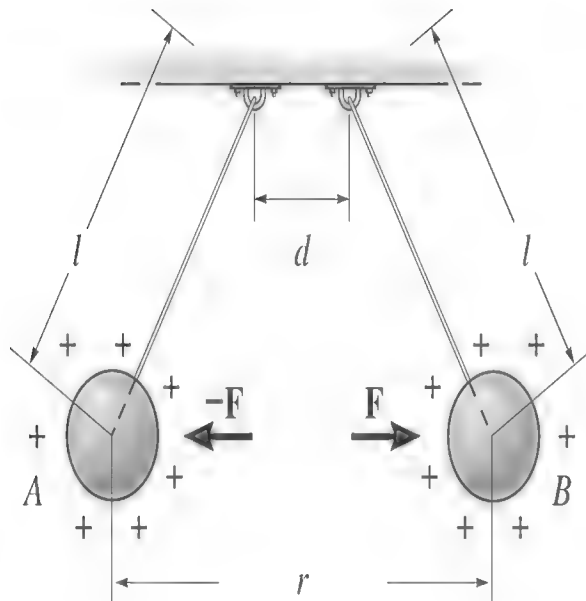
The initial guesses:

$T = 200$ N $\qquad F = 200$ N

Given

$$+\uparrow \Sigma F_x = 0; \quad F - T\left(\frac{r-d}{2l}\right) = 0$$

$$\overset{+}{\rightarrow} \Sigma F_y = 0; \quad T\left[\frac{\sqrt{l^2 - \left(\frac{r-d}{2}\right)^2}}{l}\right] - Mg = 0$$

$$\begin{pmatrix} T \\ F \end{pmatrix} = \text{Find}(T, F) \qquad F = 1.13 \times 10^{-3}\text{ N} \qquad T = 0.00\text{ N}$$

Problem 3-12

The towing pendant AB is subjected to the force **F** which is developed from a tugboat. Determine the force that is in each of the bridles, BC and BD, if the ship is moving forward with constant velocity.

Units Used:

$\text{kN} = 10^3$ N

Given:

$F = 50$ kN

$\theta_1 = 20$ deg

$\theta_2 = 30$ deg

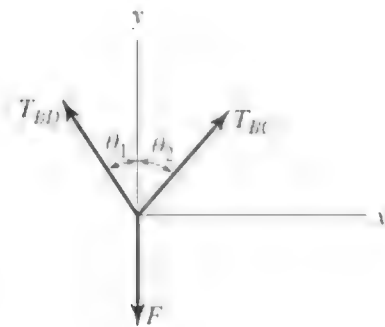
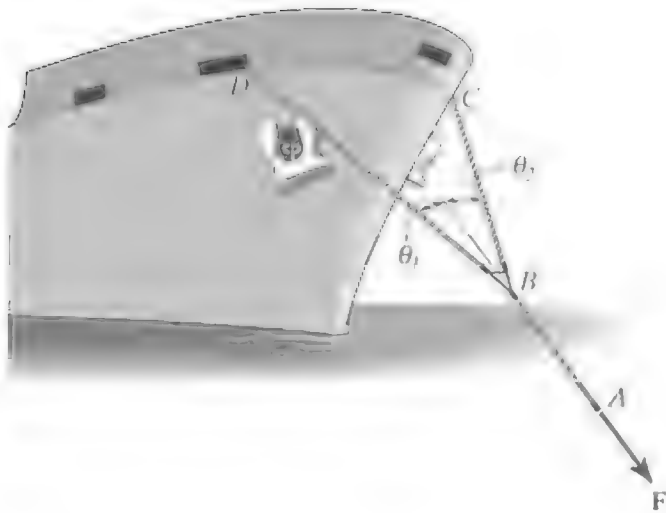
Solution:

Initial guesses: $T_{BC} = 1$ kN $\quad T_{BD} = 2$ kN

Given

$\xrightarrow{+} \Sigma F_x = 0; \quad T_{BC}\sin(\theta_2) - T_{BD}\sin(\theta_1) = 0$

$+\uparrow \Sigma F_y = 0; \quad T_{BC}\cos(\theta_2) + T_{BD}\cos(\theta_1) - F = 0$

$$\begin{pmatrix} T_{BC} \\ T_{BD} \end{pmatrix} = \text{Find}(T_{BC}, T_{BD}) \qquad \begin{pmatrix} T_{BC} \\ T_{BD} \end{pmatrix} = \begin{pmatrix} 22.32 \\ 32.64 \end{pmatrix} \text{kN}$$

Problem 3-13

Determine the stretch in each spring for equilibrium of the block of mass M. The springs are shown in the equilibrium position.

Given:

$M = 2\ \text{kg}$

$a = 3\ \text{m}$

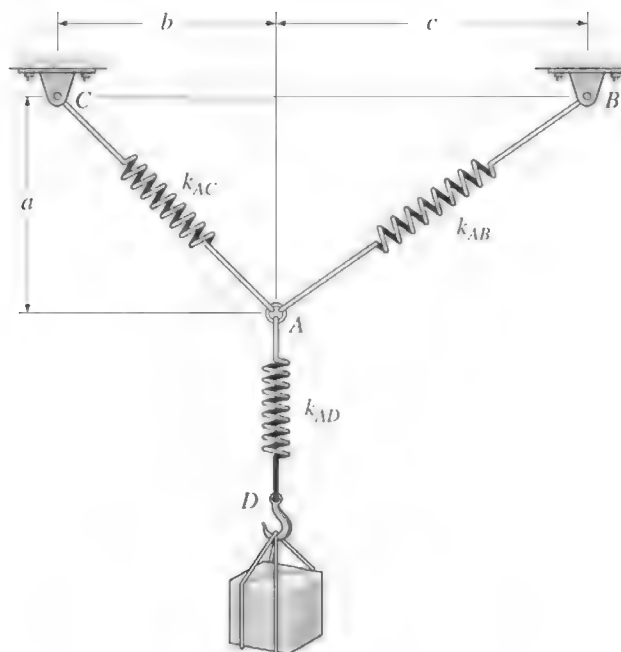
$b = 3\ \text{m}$

$c = 4\ \text{m}$

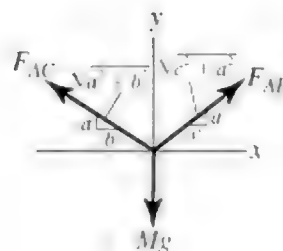
$k_{AB} = 30\ \frac{\text{N}}{\text{m}}$

$k_{AC} = 20\ \frac{\text{N}}{\text{m}}$

$k_{AD} = 40\ \frac{\text{N}}{\text{m}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

The initial guesses:

$F_{AB} = 1\ \text{N} \quad F_{AC} = 1\ \text{N}$

Given

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\left(\frac{c}{\sqrt{a^2+c^2}}\right) - F_{AC}\left(\frac{b}{\sqrt{a^2+b^2}}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{a}{\sqrt{a^2+b^2}}\right) + F_{AB}\left(\frac{a}{\sqrt{a^2+c^2}}\right) - M g = 0$$

$$\begin{pmatrix} F_{AC} \\ F_{AB} \end{pmatrix} = \text{Find}(F_{AC}, F_{AB}) \qquad \begin{pmatrix} F_{AC} \\ F_{AB} \end{pmatrix} = \begin{pmatrix} 15.86 \\ 14.01 \end{pmatrix} \text{N}$$

$$x_{AC} = \frac{F_{AC}}{k_{AC}} \qquad x_{AC} = 0.79\ \text{m}$$

$$x_{AB} = \frac{F_{AB}}{k_{AB}} \qquad x_{AB} = 0.47\ \text{m}$$

Problem 3-14

The unstretched length of spring AB is δ. If the block is held in the equilibrium position shown, determine the mass of the block at D.

Given:

$\delta = 2\ \text{m}$

$a = 3\ \text{m}$

$b = 3\ \text{m}$

$c = 4\ \text{m}$

$k_{AB} = 30\ \frac{\text{N}}{\text{m}}$

$k_{AC} = 20\ \frac{\text{N}}{\text{m}}$

$k_{AD} = 40\ \frac{\text{N}}{\text{m}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$F_{AB} = k_{AB}\left(\sqrt{a^2 + c^2} - \delta\right)$$

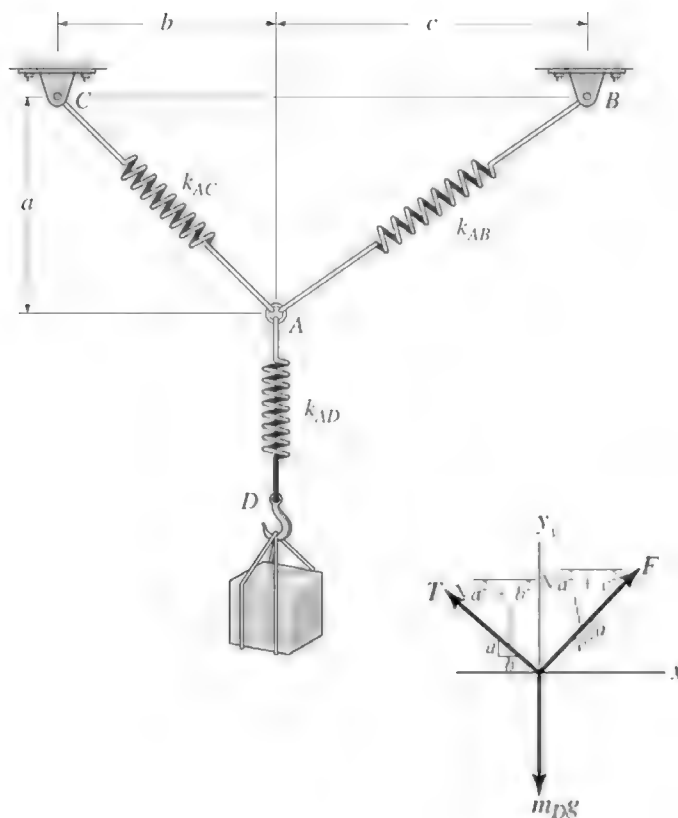
The initial guesses:

$m_D = 1\ \text{kg} \qquad F_{AC} = 1\ \text{N}$

Given

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB}\left(\frac{c}{\sqrt{a^2 + c^2}}\right) - F_{AC}\left(\frac{b}{\sqrt{a^2 + b^2}}\right) = 0$

$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{a}{\sqrt{a^2 + b^2}}\right) + F_{AB}\left(\frac{a}{\sqrt{a^2 + c^2}}\right) - m_D g = 0$

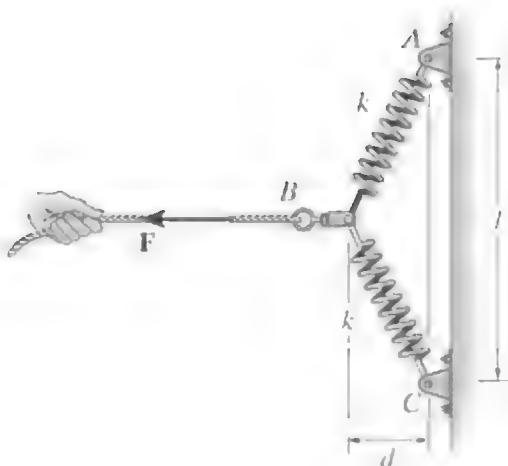
$$\begin{pmatrix} F_{AC} \\ m_D \end{pmatrix} = \text{Find}(F_{AC}, m_D) \qquad F_{AC} = 101.8\ \text{N} \qquad m_D = 12.8\ \text{kg}$$

Problem 3-15

The springs *AB* and *BC* have stiffness *k* and unstretched lengths *l*/2. Determine the horizontal force **F** applied to the cord which is attached to the *small* pulley *B* so that the displacement of the pulley from the wall is *d*.

Given:

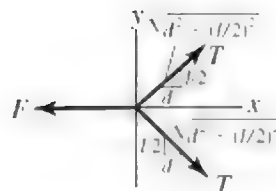
$l = 6\ \text{m}$

$k = 500\ \dfrac{\text{N}}{\text{m}}$

$d = 1.5\ \text{m}$

Solution:

$$T = k\left[\sqrt{\left(\frac{l}{2}\right)^2 + d^2} - \frac{l}{2}\right] \qquad T = 177.05\ \text{N}$$

$$\overset{+}{\rightarrow}\ \Sigma F_x = 0; \qquad \frac{d}{\sqrt{d^2 + \left(\frac{l}{2}\right)^2}}(2T) - F = 0$$

$$F = \frac{d}{\sqrt{\left(\frac{l}{2}\right)^2 + d^2}}(2T) \qquad F = 158.36\ \text{N}$$

Problem 3-16

The springs *AB* and *BC* have stiffness *k* and an unstretched length of *l*. Determine the displacement *d* of the cord from the wall when a force *F* is applied to the cord.

Given:

$l = 6$ m

$k = 500 \frac{N}{m}$

$F = 175$ N

Solution:

The initial guesses:

$d = 1$ m $\quad T = 1$ N

Given

$\xrightarrow{+} \Sigma F_x = 0; \quad -F + (2T)\frac{d}{\sqrt{d^2 + \left(\frac{l}{2}\right)^2}} = 0$

Spring $\quad T = k\left[\sqrt{d^2 + \left(\frac{l}{2}\right)^2} - \frac{l}{2}\right]$

$\begin{pmatrix} T \\ d \end{pmatrix} = \text{Find}(T, d) \quad T = 189.96 \text{ N} \quad d = 1.56 \text{ m}$

Problem 3-17

Determine the force in each cable and the force **F** needed to hold the lamp of mass M in the position shown. *Hint:* First analyze the equilibrium at B; then, using the result for the force in BC, analyze the equilibrium at C.

Given:

$M = 4$ kg

$\theta_1 = 30$ deg

$\theta_2 = 60$ deg

$\theta_3 = 30$ deg

Solution:

Initial guesses:

$T_{BC} = 1\ \text{N}$ $\quad T_{BA} = 2\ \text{N}$

Given

At B:

$\xrightarrow{+} \Sigma F_x = 0; \quad T_{BC}\cos(\theta_1) - T_{BA}\cos(\theta_2) = 0$

$+\uparrow \Sigma F_y = 0; \quad T_{BA}\sin(\theta_2) - T_{BC}\sin(\theta_1) - M\,g = 0$

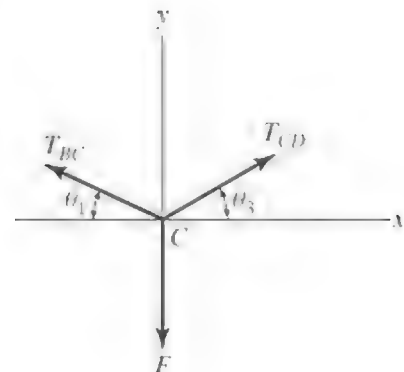
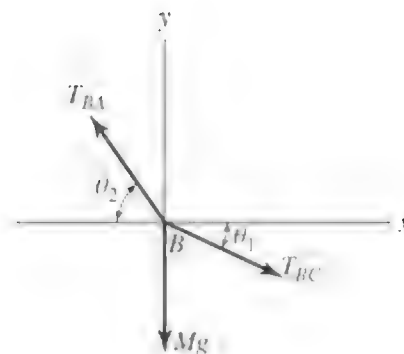
$$\begin{pmatrix} T_{BC} \\ T_{BA} \end{pmatrix} = \text{Find}(T_{BC}, T_{BA}) \qquad \begin{pmatrix} T_{BC} \\ T_{BA} \end{pmatrix} = \begin{pmatrix} 39.24 \\ 67.97 \end{pmatrix} \text{N}$$

At C: $\quad T_{CD} = 1\ \text{N}$ $\quad F = 2\ \text{N}$

Given

$\xrightarrow{+} \Sigma F_x = 0; \quad -T_{BC}\cos(\theta_1) + T_{CD}\cos(\theta_3) = 0$

$+\uparrow \Sigma F_y = 0; \quad T_{BC}\sin(\theta_1) + T_{CD}\sin(\theta_3) - F = 0$

$$\begin{pmatrix} T_{CD} \\ F \end{pmatrix} = \text{Find}(T_{CD}, F) \qquad \begin{pmatrix} T_{CD} \\ F \end{pmatrix} = \begin{pmatrix} 39.24 \\ 39.24 \end{pmatrix} \text{N}$$

Problem 3-18

The motor at B winds up the cord attached to the crate of weight W with a constant speed. Determine the force in cord CD supporting the pulley and the angle θ for equilibrium. Neglect the

size of the pulley at C.

Given:

$W = 65 \text{ lb} \qquad c = 12 \qquad d = 5$

Solution:

The initial guesses: $\theta = 100 \text{ deg} \qquad F_{CD} = 200 \text{ lb}$

Given

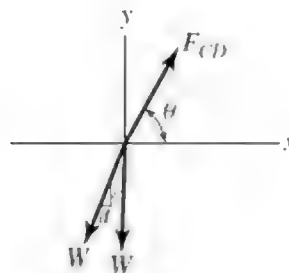
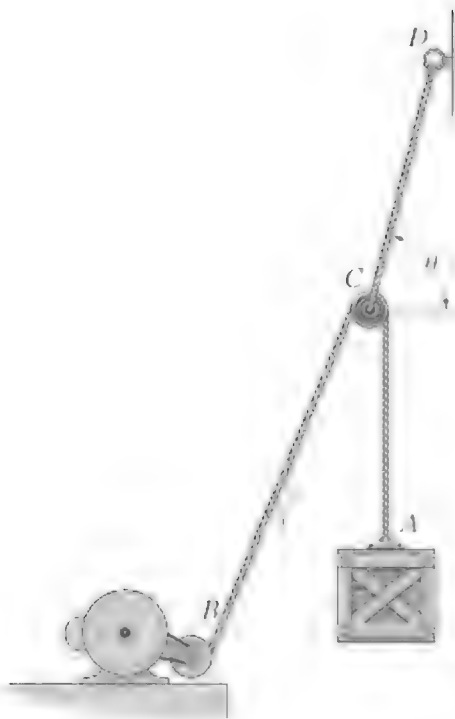
Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CD}\cos(\theta) - W\left(\frac{d}{\sqrt{c^2 + d^2}}\right) = 0$

$+\uparrow \Sigma F_y = 0; \quad F_{CD}\sin(\theta) - W\left(\frac{c}{\sqrt{c^2 + d^2}}\right) - W = 0$

$\begin{pmatrix} \theta \\ F_{CD} \end{pmatrix} = \text{Find}(\theta, F_{CD})$

$\theta = 78.69 \text{ deg}$

$F_{CD} = 127.5 \text{ lb}$

Problem 3-19

The cords BCA and CD can each support a maximum load T. Determine the maximum weight of the crate that can be hoisted at constant velocity, and the angle θ for equilibrium.

Given:

$T = 100 \text{ lb}$

$c = 12$

$d = 5$

The maximum will occur in *CD* rather than in *BCA*.

Solution :

The initial guesses: $\theta = 100 \text{ deg}$ $W = 200 \text{ lb}$

Given

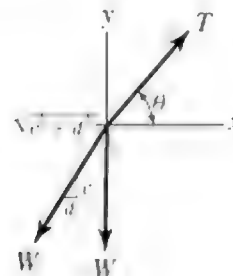
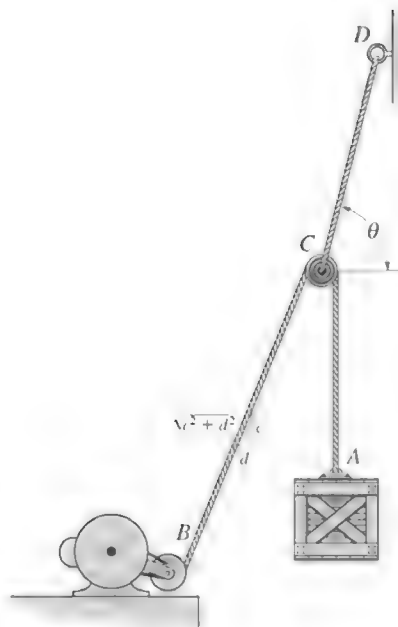
Equations of Equilibrium:

$$\xrightarrow{+} \Sigma F_x = 0; \quad T\cos(\theta) - W\left(\frac{d}{\sqrt{c^2 + d^2}}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad T\sin(\theta) - W\left(\frac{c}{\sqrt{c^2 + d^2}}\right) - W = 0$$

$$\begin{pmatrix} \theta \\ W \end{pmatrix} = \text{Find}(\theta, W)$$

$\theta = 78.69 \text{ deg}$

$W = 51.0 \text{ lb}$

Problem 3-20

The sack has weight W and is supported by the six cords tied together as shown. Determine the tension in each cord and the angle θ for equilibrium. Cord BC is horizontal.

Given:

$W = 15$ lb

$\theta_1 = 30$ deg

$\theta_2 = 45$ deg

$\theta_3 = 60$ deg

Solution:

Guesses $T_{BE} = 1$ lb $T_{AB} = 1$ lb

$\theta = 20\text{deg}$ $T_{BC} = 1$ lb $T_{AC} = 1$ lb

$T_{CD} = 1$ lb $T_{AH} = 1$ lb

Given

At H:

$+\uparrow \Sigma F_y = 0;$ $T_{AH} - W = 0$

At A:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $-T_{AB}\cos(\theta_2) + T_{AC}\cos(\theta_3) = 0$

$+\uparrow \Sigma F_y = 0;$ $T_{AB}\sin(\theta_2) + T_{AC}\sin(\theta_3) - W = 0$

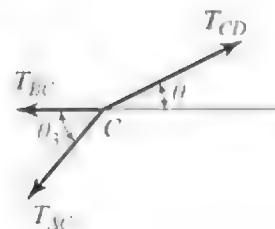
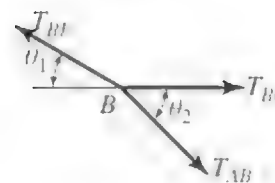
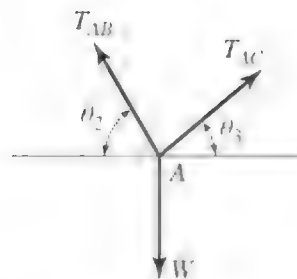
At B:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $T_{BC} - T_{BE}\cos(\theta_1) + T_{AB}\cos(\theta_2) = 0$

$+\uparrow \Sigma F_y = 0;$ $T_{BE}\sin(\theta_1) - T_{AB}\sin(\theta_2) = 0$

At C:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $T_{CD}\cos(\theta) - T_{BC} - T_{BE}\cos(\theta_3) = 0$

$+\uparrow \Sigma F_y = 0;$ $T_{CD}\sin(\theta) - T_{AC}\sin(\theta_3) = 0$

$$\begin{pmatrix} T_{BE} \\ T_{AB} \\ T_{BC} \\ T_{AC} \\ T_{CD} \\ T_{AH} \\ \theta \end{pmatrix} = \text{Find}(T_{BE}, T_{AB}, T_{BC}, T_{AC}, T_{CD}, T_{AH}, \theta)$$

$$\begin{pmatrix} T_{BE} \\ T_{AB} \\ T_{BC} \\ T_{AC} \\ T_{CD} \\ T_{AH} \end{pmatrix} = \begin{pmatrix} 10.98 \\ 7.76 \\ 4.02 \\ 10.98 \\ 13.45 \\ 15.00 \end{pmatrix} \text{lb}$$

$$\theta = 45.00 \text{ deg}$$

Problem 3-21

Each cord can sustain a maximum tension T. Determine the largest weight of the sack that can be supported. Also, determine θ of cord DC for equilibrium.

Given:

$T = 200$ lb

$\theta_1 = 30$ deg

$\theta_2 = 45$ deg

$\theta_3 = 60$ deg

Solution:

Solve for $W = 1$ and then scale the answer at the end.

Guesses $T_{BE} = 1$ $T_{AB} = 1$

$T_{BC} = 1$ $T_{AC} = 1$

$T_{CD} = 1$ $T_{AH} = 1$

$\theta = 20 \text{ deg}$

Given

At H:

$+\uparrow \Sigma F_y = 0; \quad T_{AH} - W = 0$

At A:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -T_{AB}\cos(\theta_2) + T_{AC}\cos(\theta_3) = 0$

$+\uparrow \Sigma F_y = 0; \quad T_{AB}\sin(\theta_2) + T_{AC}\sin(\theta_3) - W = 0$

At B:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad T_{BC} - T_{BE}\cos(\theta_1) + T_{AB}\cos(\theta_2) = 0$

$+\uparrow \Sigma F_y = 0; \quad T_{BE}\sin(\theta_1) - T_{AB}\sin(\theta_2) = 0$

At C:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad T_{CD}\cos(\theta) - T_{BC} - T_{BE}\cos(\theta_3) = 0$

$+\uparrow \Sigma F_y = 0; \quad T_{CD}\sin(\theta) - T_{AC}\sin(\theta_3) = 0$

$$\begin{pmatrix} T_{BE} \\ T_{AB} \\ T_{BC} \\ T_{AC} \\ T_{CD} \\ T_{AH} \\ \theta \end{pmatrix} = \text{Find}(T_{BE}, T_{AB}, T_{BC}, T_{AC}, T_{CD}, T_{AH}, \theta)$$

$$\begin{pmatrix} T_{BE} \\ T_{AB} \\ T_{BC} \\ T_{AC} \\ T_{CD} \\ T_{AH} \end{pmatrix} = \begin{pmatrix} 0.73 \\ 0.52 \\ 0.27 \\ 0.73 \\ 0.90 \\ 1.00 \end{pmatrix}$$

$$W = \frac{T}{\max(T_{BE}, T_{AB}, T_{BC}, T_{AC}, T_{CD}, T_{AH})}$$

$W = 200.00 \text{ lb}$

$\theta = 45.00 \text{ deg}$

Problem 3-22

The block has weight W and is being hoisted at uniform velocity. Determine the angle θ for equilibrium and the required force in each cord.

Given:

$W = 20$ lb

$\phi = 30$ deg

Solution:

The initial guesses:

$\theta = 10$ deg $\quad T_{AB} = 50$ lb

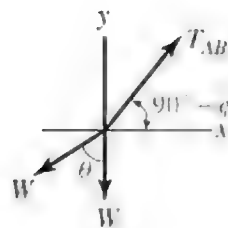
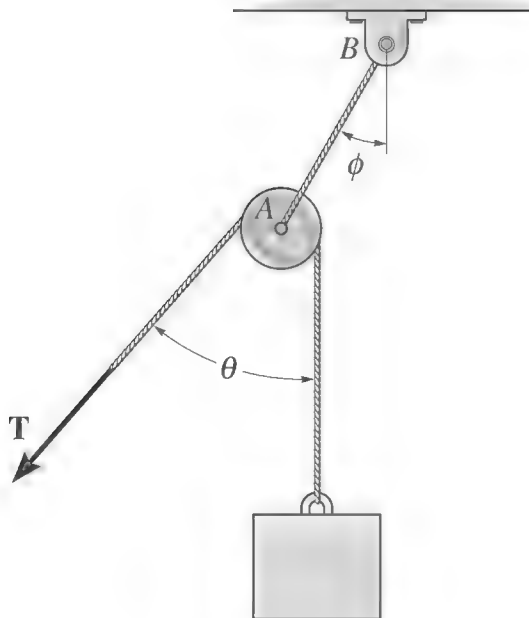
Given

$$T_{AB}\sin(\phi) - W\sin(\theta) = 0$$

$$T_{AB}\cos(\phi) - W - W\cos(\theta) = 0$$

$$\begin{pmatrix} \theta \\ T_{AB} \end{pmatrix} = \text{Find}(\theta, T_{AB})$$

$\theta = 60.00$ deg

$T_{AB} = 34.6$ lb

Problem 3-23

Determine the maximum weight W of the block that can be suspended in the position shown if each cord can support a maximum tension T. Also, what is the angle θ for equilibrium?

Given:

$T = 80\ \text{lb}$

$\phi = 30\ \text{deg}$

The maximum load will occur in cord *AB*.

Solution:

$T_{AB} = T$

The initial guesses:

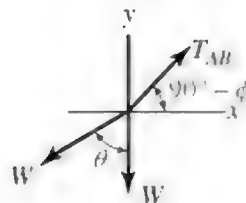
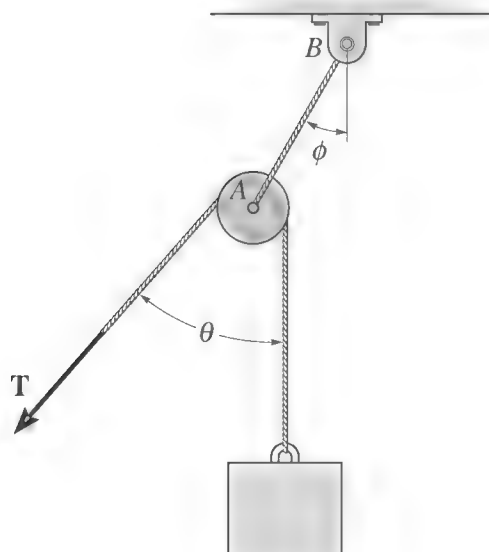
$\theta = 100\text{deg} \quad W = 200\text{lb}$

Given

$+\uparrow \Sigma F_y = 0; \qquad T_{AB}\cos(\phi) - W - W\cos(\theta) = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_{AB}\sin(\phi) - W\sin(\theta) = 0$

$$\begin{pmatrix} \theta \\ W \end{pmatrix} = \text{Find}(\theta, W)$$

$W = 46.19\ \text{lb} \qquad \theta = 60.00\ \text{deg}$

Problem 3-24

Two spheres *A* and *B* have an equal mass *M* and are electrostatically charged such that the repulsive force acting between them has magnitude *F* and is directed along line *AB*. Determine the angle θ, the tension in cords *AC* and *BC*, and the mass *M* of each sphere.

Unit used:

$mN = 10^{-3}\ N$

Given:

$F = 20\ mN$

$g = 9.81\ \frac{m}{s^2}$

$\theta_1 = 30\ deg$

$\theta_2 = 30\ deg$

Solution:

Guesses $T_B = 1\ mN$ $M = 1\ gm$

$T_A = 1\ mN$ $\theta = 30\ deg$

Given

For *B*:

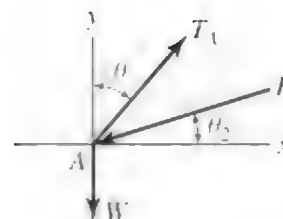
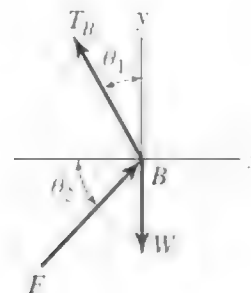
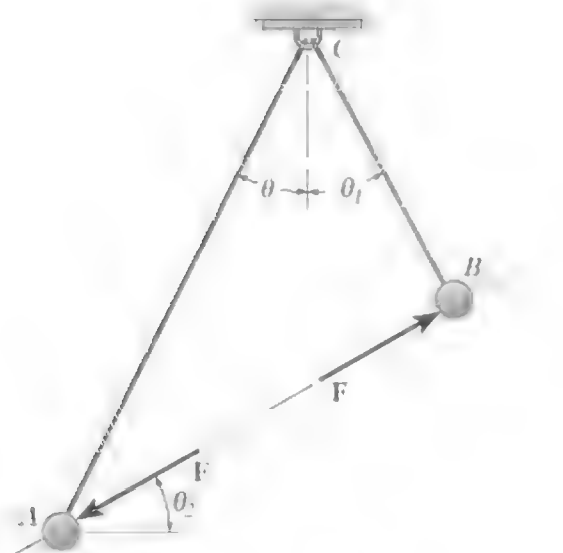
$\overset{+}{\rightarrow}\ \Sigma F_x = 0;\quad F\cos(\theta_2) - T_B\sin(\theta_1) = 0$

$+\uparrow\ \Sigma F_y = 0;\quad F\sin(\theta_2) + T_B\cos(\theta_1) - M g = 0$

For *A*:

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;\quad T_A\sin(\theta) - F\cos(\theta_2) = 0$

$+\uparrow\ \Sigma F_y = 0;\quad T_A\cos(\theta) - F\sin(\theta_2) - M g = 0$

$$\begin{pmatrix} T_A \\ T_B \\ \theta \\ M \end{pmatrix} = \text{Find}(T_A, T_B, \theta, M) \qquad \begin{pmatrix} T_A \\ T_B \end{pmatrix} = \begin{pmatrix} 52.92 \\ 34.64 \end{pmatrix} mN \qquad \theta = 19.11\ deg \qquad M = 4.08\ gm$$

Problem 3-25

Blocks *D* and *F* weigh W_1 each and block *E* weighs W_2. Determine the sag *s* for equilibrium. Neglect the size of the pulleys.

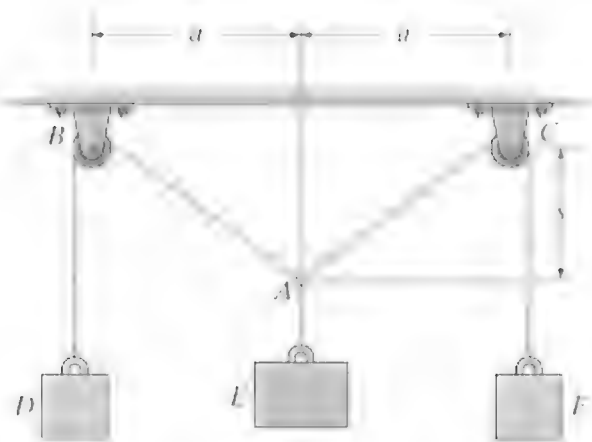
Given:

$W_1 = 5$ lb

$W_2 = 8$ lb

$a = 4$ ft

Solution:

Sum forces in the y direction

Guess $s = 1$ ft

Given $2\left(\frac{s}{\sqrt{s^2 + a^2}}\right)W_1 - W_2 = 0$ $s = \text{Find}(s)$ $s = 5.33$ ft

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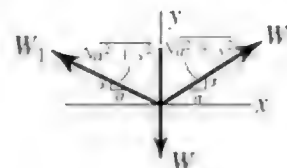

Problem 3-26

If blocks D and F each have weight W_1, determine the weight of block E if the sag is s. Neglect the size of the pulleys.

Given:

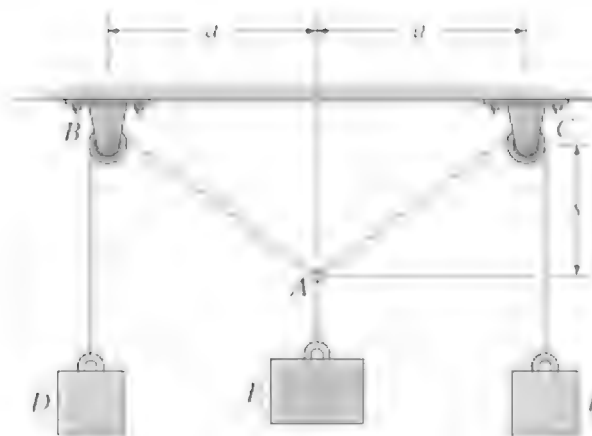
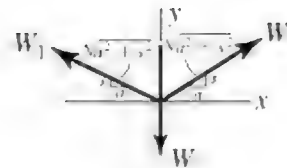
$W_1 = 5$ lb

$s = 3$ ft

$a = 4$ ft

Solution:

Sum forces in the y direction

$$2\left(\frac{s}{\sqrt{s^2 + a^2}}\right)W_1 - W = 0$$

$$W = 2\left(\frac{s}{\sqrt{s^2 + a^2}}\right)W_1 \qquad W = 6.00\,\text{lb}$$

Problem 3-27

The block of mass M is supported by two springs having the stiffness shown. Determine the unstretched length of each spring.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$M = 30\ \text{kg}$$

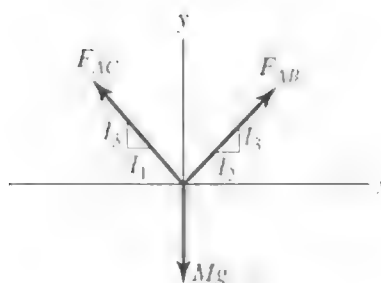
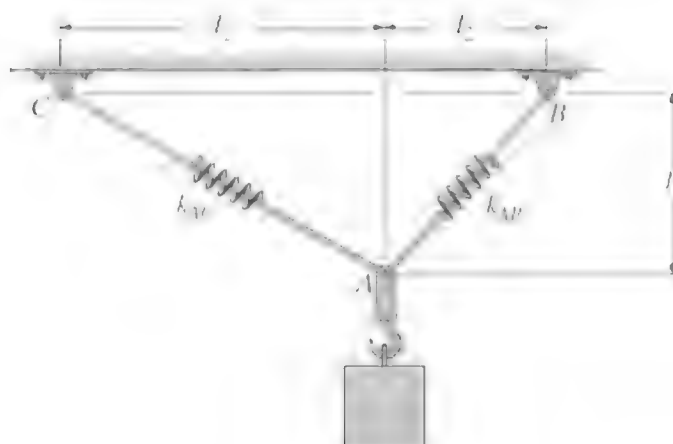
$$l_1 = 0.6\ \text{m}$$

$$l_2 = 0.4\ \text{m}$$

$$l_3 = 0.5\ \text{m}$$

$$k_{AC} = 1.5\ \frac{\text{kN}}{\text{m}}$$

$$k_{AB} = 1.2\ \frac{\text{kN}}{\text{m}}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

Solution:

Initial guesses: $F_{AC} = 20\ \text{N}$ $\quad F_{AB} = 30\ \text{N}$

Given

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{l_2 F_{AB}}{\sqrt{l_2^2 + l_3^2}} - \frac{l_1 F_{AC}}{\sqrt{l_1^2 + l_3^2}} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{l_3 F_{AB}}{\sqrt{l_2^2 + l_3^2}} + \frac{l_3 F_{AC}}{\sqrt{l_1^2 + l_3^2}} - M\,g = 0$$

$$\begin{pmatrix} F_{AC} \\ F_{AB} \end{pmatrix} = \text{Find}(F_{AC}, F_{AB}) \qquad \begin{pmatrix} F_{AC} \\ F_{AB} \end{pmatrix} = \begin{pmatrix} 183.88 \\ 226.13 \end{pmatrix} \text{N}$$

Then guess $L_{AB} = 0.1\ \text{m}$ $L_{AC} = 0.1\ \text{m}$

Given $F_{AC} = k_{AC}\left(\sqrt{l_1^2 + l_3^2} - L_{AC}\right)$

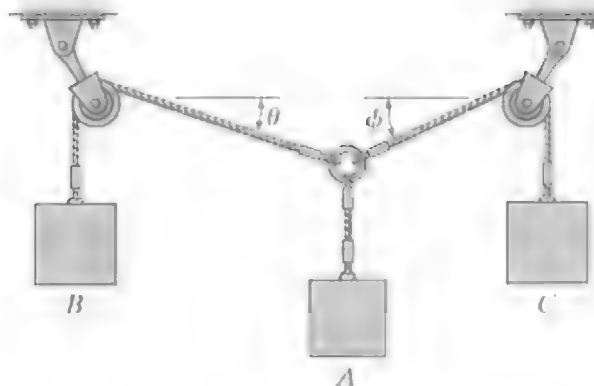
$$F_{AB} = k_{AB}\left(\sqrt{l_2^2 + l_3^2} - L_{AB}\right)$$

$$\begin{pmatrix} L_{AB} \\ L_{AC} \end{pmatrix} = \text{Find}(L_{AB}, L_{AC}) \qquad \begin{pmatrix} L_{AB} \\ L_{AC} \end{pmatrix} = \begin{pmatrix} 0.452 \\ 0.658 \end{pmatrix} \text{m}$$

Problem 3-28

Three blocks are supported using the cords and two pulleys. If they have weights of $W_A = W_C = W$, $W_B = kW$, determine the angle θ for equilibrium.

Given:

$k = 0.25$

Solution:

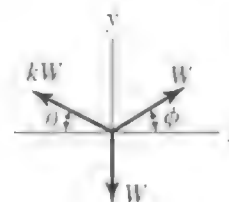
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $W\cos(\phi) - kW\cos(\theta) = 0$

$+\uparrow \Sigma F_y = 0;$ $W\sin(\phi) + kW\sin(\theta) - W = 0$

$$\cos(\phi) = k\cos(\theta)$$

$$\sin(\phi) = 1 - k\sin(\theta)$$

$$1 = k^2\cos(\theta)^2 + (1 - k\sin(\theta))^2 = 1 + k^2 - 2k\sin(\theta)$$

$$\theta = \operatorname{asin}\left(\frac{k}{2}\right) \qquad \theta = 7.18\ \text{deg}$$

Problem 3-29

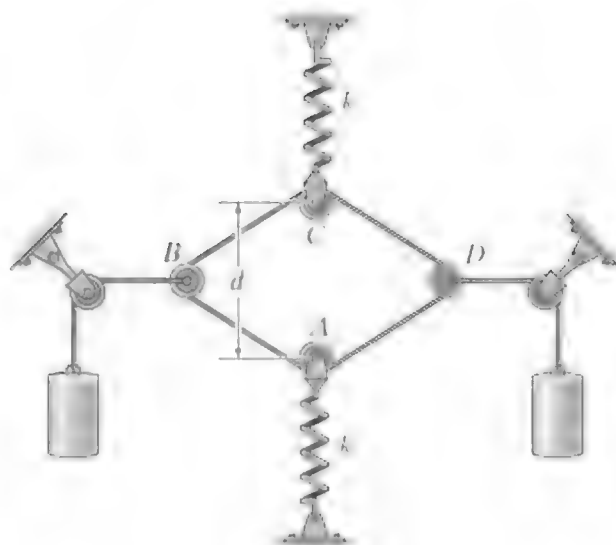
A continuous cable of total length l is wrapped around the *small* pulleys at *A, B, C*, and *D*. If each spring is stretched a distance b, determine the mass M of each block. Neglect the weight of the pulleys and cords. The springs are unstretched when $d = l/2$.

Given:

$l = 4\ \text{m}$

$k = 500\ \frac{\text{N}}{\text{m}}$

$b = 300\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$F_s = k\,b \qquad F_s = 150.00\ \text{N}$

Guesses

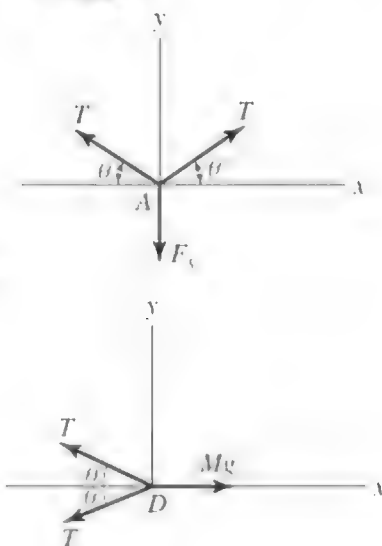
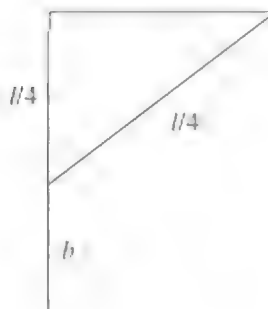
$T = 1\text{N} \qquad \theta = 10\ \text{deg} \qquad M = 1\ \text{kg}$

Given

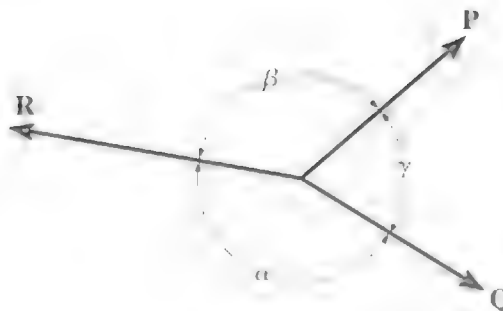
$$2T\sin(\theta) - F_s = 0$$

$$-2T\cos(\theta) + M\,g = 0$$

$$b + \frac{l}{4}\sin(\theta) = \frac{l}{4}$$

$$\begin{pmatrix} T \\ \theta \\ M \end{pmatrix} = \text{Find}(T, \theta, M)$$

$T = 107.14\ \text{N}$ $\theta = 44.43\ \text{deg}$ $M = 15.60\ \text{kg}$

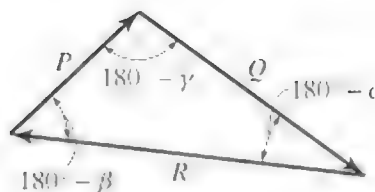
Problem 3-30

Prove *Lami's theorem*, which states that if three concurrent forces are in equilibrium, each is proportional to the sine of the angle of the other two; that is, $P/\sin\alpha = Q/\sin\beta = R/\sin\gamma$.

Solution:

Sine law:

$$\frac{R}{\sin(180\text{deg} - \gamma)} = \frac{Q}{\sin(180\text{deg} - \beta)} = \frac{P}{\sin(180\text{deg} - \alpha)}$$

However, in general $\sin(180\text{deg} - \phi) = \sin(\phi)$, hence

$$\frac{R}{\sin(\gamma)} = \frac{Q}{\sin(\beta)} = \frac{P}{\sin(\alpha)}$$ Q.E.D.

Problem 3-31

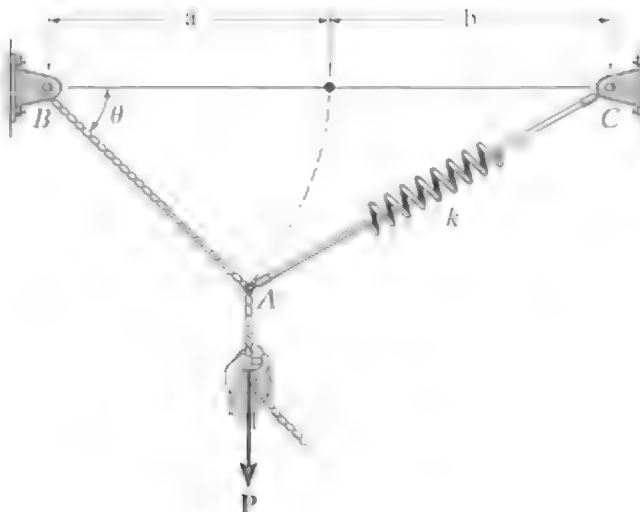
A vertical force **P** is applied to the ends of cord AB of length a and spring AC. If the spring has an unstretched length δ, determine the angle θ for equilibrium.

Given:

$P = 10\ \text{lb}$

$\delta = 2\ \text{ft}$

$k = 15\ \frac{\text{lb}}{\text{ft}}$

$a = 2\ \text{ft}$

$b = 2 \text{ ft}$

Guesses

$\theta = 10 \text{ deg}$ $\phi = 10 \text{ deg}$

$T = 1 \text{ lb}$ $F = 1 \text{ lb}$

$x = 1\text{ft}$

Given

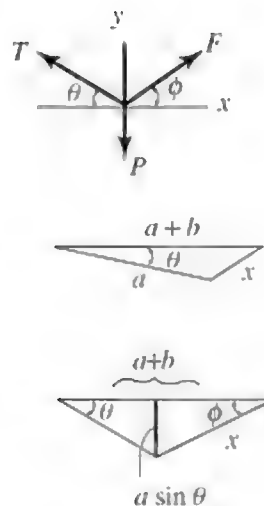
$-T\cos(\theta) + F\cos(\phi) = 0$

$T\sin(\theta) + F\sin(\phi) - P = 0$

$F = k(x - \delta)$

$a\sin(\theta) = x\sin(\phi)$

$a\cos(\theta) + x\cos(\phi) = a + b$

$$\begin{pmatrix} \theta \\ \phi \\ T \\ F \\ x \end{pmatrix} = \text{Find}(\theta, \phi, T, F, x) \qquad \begin{pmatrix} T \\ F \end{pmatrix} = \begin{pmatrix} 10.30 \\ 9.38 \end{pmatrix} \text{lb} \qquad \theta = 35\,\text{deg}$$

Problem 3-32

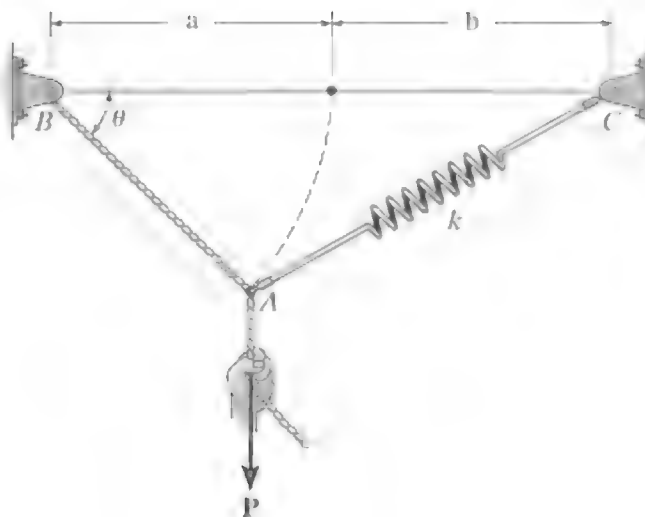
Determine the unstretched length δ of spring AC if a force **P** causes the angle θ for equilibrium. Cord AB has length a.

Given:

$P = 80 \text{ lb}$

$\theta = 60 \text{ deg}$

$k = 50 \dfrac{\text{lb}}{\text{ft}}$

$a = 2 \text{ ft}$

$b = 2 \text{ ft}$

Guesses

$\delta = 1\text{ft}$ $\qquad \phi = 10 \text{ deg}$

$T = 1 \text{ lb}$ $\qquad F = 1 \text{ lb}$

$x = 1\text{ft}$

Given

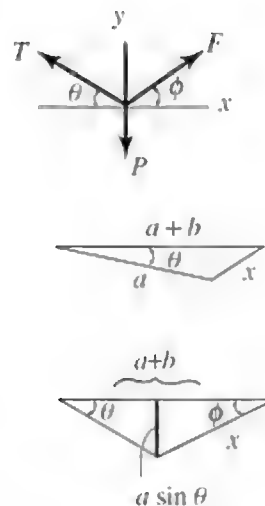
$$-T\cos(\theta) + F\cos(\phi) = 0$$

$$T\sin(\theta) + F\sin(\phi) - P = 0$$

$$F = k(x - \delta)$$

$$a\sin(\theta) = x\sin(\phi)$$

$$a\cos(\theta) + x\cos(\phi) = a + b$$

$$\begin{pmatrix} \delta \\ \phi \\ T \\ F \\ x \end{pmatrix} = \text{Find}(\delta, \phi, T, F, x) \qquad \begin{pmatrix} T \\ F \end{pmatrix} = \begin{pmatrix} 69.28 \\ 40.00 \end{pmatrix} \text{lb} \qquad \delta = 2.66 \text{ ft}$$

Problem 3-33

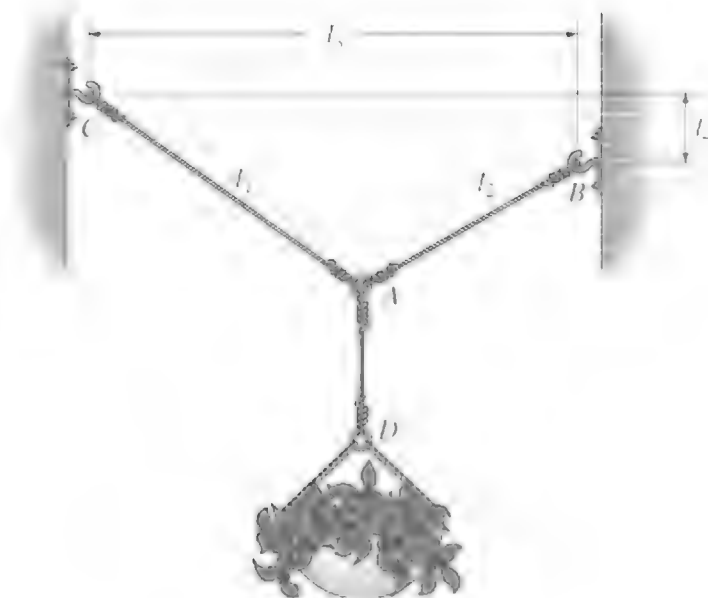
The flowerpot of mass M is suspended from three wires and supported by the hooks at B and C. Determine the tension in AB and AC for equilibrium.

Given:

$M = 20 \text{ kg}$

$l_1 = 3.5 \text{ m}$

$l_2 = 2 \text{ m}$

$l_3 = 4 \text{ m}$

$l_4 = 0.5\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

Initial guesses:

$T_{AB} = 1\ \text{N}$ $\quad T_{AC} = 1\ \text{N}$

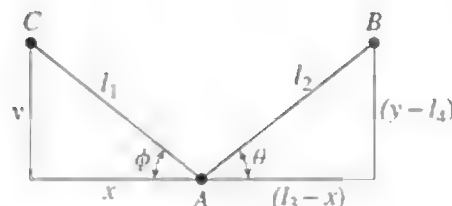
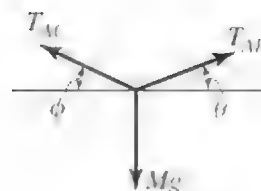
$\theta = 10\ \text{deg}$ $\quad \phi = 10\ \text{deg}$

Given

$-T_{AC}\cos(\phi) + T_{AB}\cos(\theta) = 0$

$T_{AC}\sin(\phi) + T_{AB}\sin(\theta) - M\,g = 0$

$l_1\cos(\phi) + l_2\cos(\theta) = l_3$

$l_1\sin(\phi) = l_2\sin(\theta) + l_4$

$$\begin{pmatrix} T_{AB} \\ T_{AC} \\ \theta \\ \phi \end{pmatrix} = \text{Find}(T_{AB}, T_{AC}, \theta, \phi) \qquad \begin{pmatrix} \theta \\ \phi \end{pmatrix} = \begin{pmatrix} 53.13 \\ 36.87 \end{pmatrix} \text{deg} \qquad \begin{pmatrix} T_{AB} \\ T_{AC} \end{pmatrix} = \begin{pmatrix} 156.96 \\ 117.72 \end{pmatrix} \text{N}$$

Problem 3-34

A car is to be towed using the rope arrangement shown. The towing force required is P. Determine the minimum length l of rope AB so that the tension in either rope AB or AC does not exceed T. *Hint:* Use the equilibrium condition at point A to determine the required angle θ for attachment, then determine l using trigonometry applied to triangle ABC.

Given:

$P = 600\ \text{lb}$

$T = 750\ \text{lb}$

$\phi = 30\ \text{deg}$

$d = 4 \text{ ft}$

Solution:

The initial guesses

$T_{AB} = T$

$T_{AC} = T$

$\theta = 30 \text{ deg}$

$l = 2 \text{ ft}$

Case 1: Assume $T_{AC} = T$

Given

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $T_{AC}\cos(\phi) - T_{AB}\cos(\theta) = 0$

$+\uparrow \Sigma F_y = 0;$ $P - T_{AC}\sin(\phi) - T_{AB}\sin(\theta) = 0$

$$\frac{l}{\sin(\phi)} = \frac{d}{\sin(180\text{deg} - \theta - \phi)}$$

$$\begin{pmatrix} T_{AB} \\ \theta \\ l_1 \end{pmatrix} = \text{Find}(T_{AB}, \theta, l) \qquad T_{AB} = 687.39 \text{ lb} \qquad \theta = 19.11 \text{ deg} \qquad l_1 = 2.65 \text{ ft}$$

Case 2: Assume $T_{AB} = T$

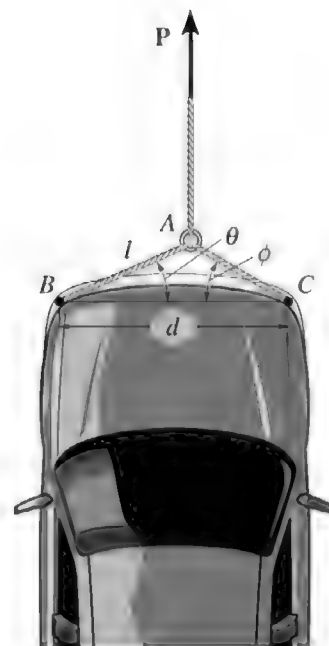
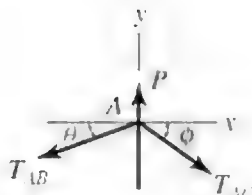
Given

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $T_{AC}\cos(\phi) - T_{AB}\cos(\theta) = 0$

$+\uparrow \Sigma F_y = 0;$ $P - T_{AC}\sin(\phi) - T_{AB}\sin(\theta) = 0$

$$\frac{l}{\sin(\phi)} = \frac{d}{\sin(180\text{deg} - \theta - \phi)}$$

$$\begin{pmatrix} T_{AC} \\ \theta \\ l_2 \end{pmatrix} = \text{Find}(T_{AC}, \theta, l) \qquad T_{AC} = 840.83 \text{ lb} \qquad \theta = 13.85 \text{ deg} \qquad l_2 = 2.89 \text{ ft}$$

$l = \min(l_1, l_2) \qquad l = 2.65 \text{ ft}$

Problem 3-35

Determine the mass of each of the two cylinders if they cause a sag of distance d when suspended from the rings at A and B. Note that $s = 0$ when the cylinders are removed.

Given:

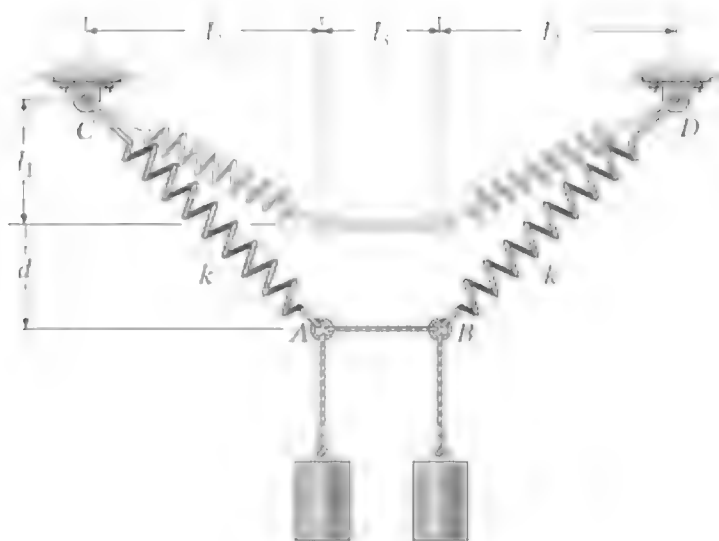
$d = 0.5\ \text{m}$

$l_1 = 1.5\ \text{m}$

$l_2 = 2\ \text{m}$

$l_3 = 1\ \text{m}$

$k = 100\ \frac{\text{N}}{\text{m}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

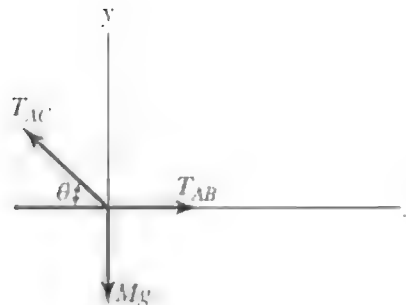
$$T_{AC} = k\left[\sqrt{(l_1 + d)^2 + l_2^2} - \sqrt{l_1^2 + l_2^2}\right]$$

$$T_{AC} = 32.84\ \text{N}$$

$$\theta = \text{atan}\left(\frac{l_1 + d}{l_2}\right)$$

$$\theta = 45\ \text{deg}$$

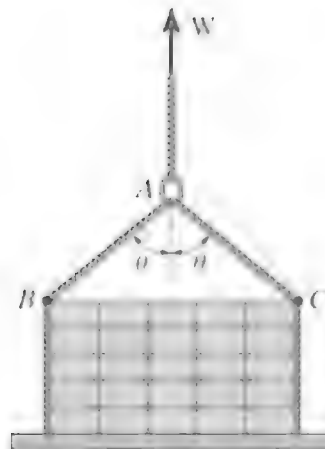
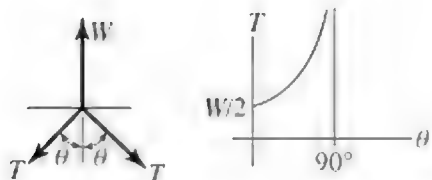
$$M = \frac{T_{AC}\sin(\theta)}{g}$$

$$M = 2.37\ \text{kg}$$

Problem 3-36

The sling BAC is used to lift the load W with constant velocity. Determine the force in the sling and plot its value T (ordinate) as a function of its orientation θ, *where* $0 \le \theta \le 90°$.

Solution:

$$W - 2T\cos(\theta) = 0$$

$$T = \frac{1}{2}\left(\frac{W}{\cos(\theta)}\right)$$

Problem 3-37

The lamp fixture has weight W and is suspended from two springs, each having unstretched length L and stiffness k. Determine the angle θ for equilibrium.

Units Used:

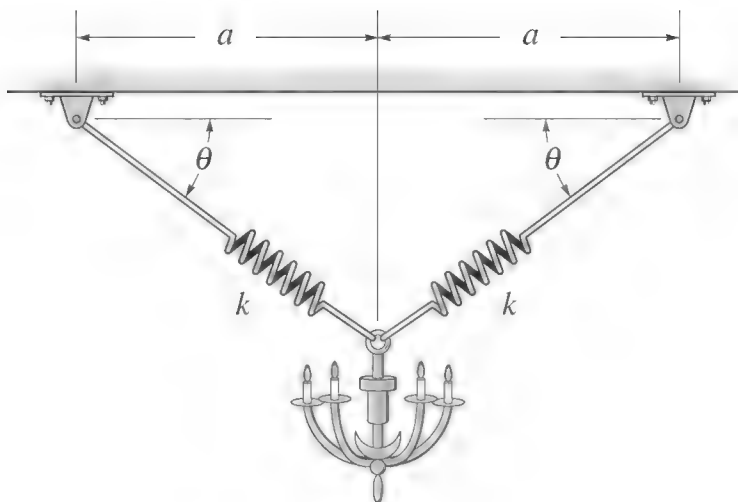
$$\mathrm{kN} = 10^3\ \mathrm{N}$$

Given:

$$W = 10\ \mathrm{lb}$$

$$L = 4\ \mathrm{ft}$$

$$k = 5\ \frac{\mathrm{lb}}{\mathrm{ft}}$$

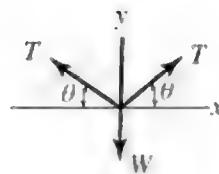
$$a = 4\ \mathrm{ft}$$

Solution:

The initial guesses: $T = 200$ lb $\theta = 10$ deg

Given

Spring $T = k\left(\frac{a}{\cos(\theta)} - L\right)$

$+\uparrow \Sigma F_y = 0;$ $2T\sin(\theta) - W = 0$

$\begin{pmatrix} T \\ \theta \end{pmatrix} = \text{Find}(T, \theta)$ $T = 7.34\,\text{lb}$ $\theta = 42.97\,\text{deg}$

Problem 3-38

The uniform tank of weight W is suspended by means of a cable, of length l, which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B, or C and D, determine which attachment produces the least amount of tension in the cable.What is this tension?

Given:

$W = 200$ lb

$l = 6$ ft

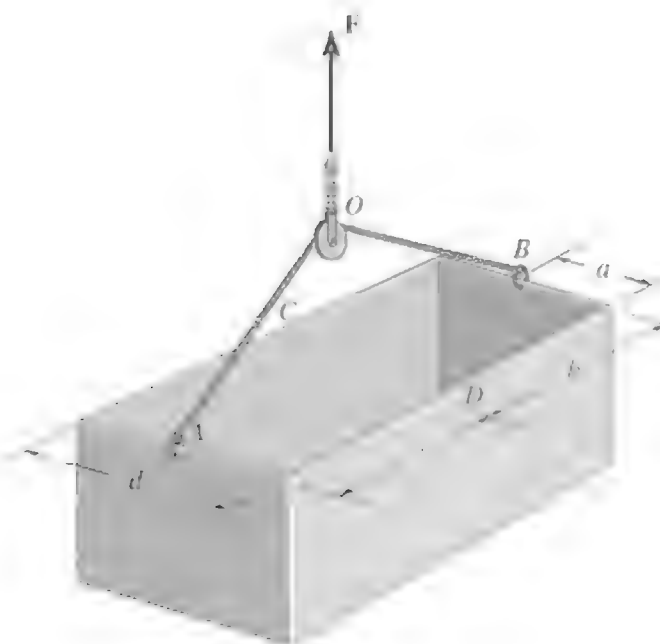
$a = 1$ ft

$b = 2$ ft

$c = b$

$d = 2a$

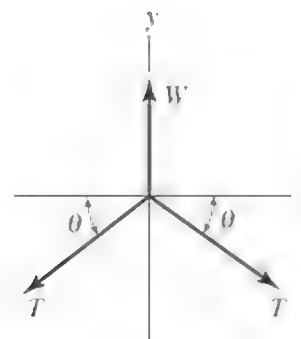
Solution:

Free Body Diagram: By observation, the force **F** has to support the entire weight of the tank. Thus, $F = W$. The tension in

cable is the same throughout the cable.

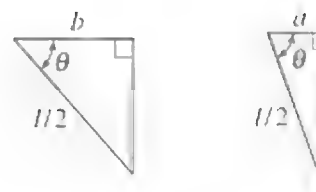
Equations of Equilibrium

$\Sigma F_y = 0;$ $W - 2T\sin(\theta) = 0$

Attached to CD $\theta_1 = \mathrm{acos}\left(\frac{2a}{l}\right)$ $\theta_1 = 70.53\,\mathrm{deg}$

Attached to AB $\theta_2 = \mathrm{acos}\left(\frac{2b}{l}\right)$ $\theta_2 = 48.19\,\mathrm{deg}$

We choose the largest angle (which will produce the smallest force)

$\theta = \max(\theta_1, \theta_2)$ $\theta = 70.53\,\mathrm{deg}$

$T = \frac{1}{2}\left(\frac{W}{\sin(\theta)}\right)$ $T = 106\,\mathrm{lb}$

Problem 3-39

A sphere of mass m_s rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass m_B of block B needed to hold it in the equilibrium position shown.

Given:

$m_s = 4\ \mathrm{kg}$

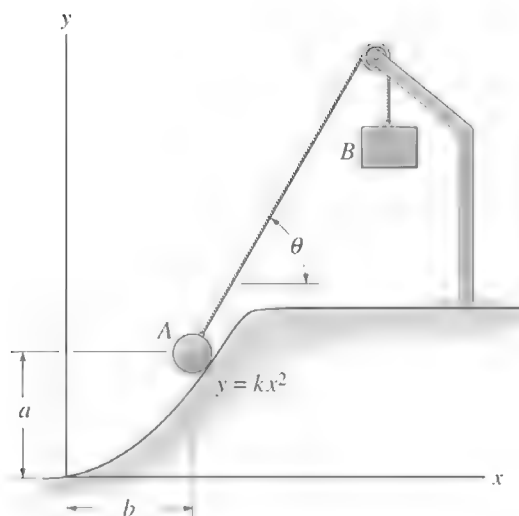
$a = 0.4\ \mathrm{m}$

$b = 0.4\ \mathrm{m}$

$\theta = 60\ \mathrm{deg}$

$g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$

Solution:

$k = \frac{a}{b^2}$

Geometry: The angle θ_1 which the surface make with the horizontal is to be determined first.

$\tan(\theta_1) = \frac{dy}{dx} = 2\ k\ x$ evaluated at $x = a$ $\theta_1 = \text{atan}(2\ k\ a)$ $\theta_1 = 63.43\,\text{deg}$

Free Body Diagram : The tension in the cord is the same throughout the cord and is equal to the weight of block B, $m_B g$.

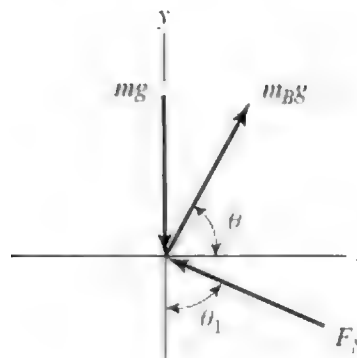
The initial guesses: $m_B = 200\ \text{kg}$ $F_N = 200\ \text{N}$

Given

$\xrightarrow{+} \Sigma\ F_x = 0;\quad m_B g \cos(\theta) - F_N \sin(\theta_1) = 0$

$+\uparrow \Sigma\ F_y = 0;\quad m_B g \sin(\theta) + F_N \cos(\theta_1) - m_s g = 0$

$\begin{pmatrix} m_B \\ F_N \end{pmatrix} = \text{Find}(m_B, F_N)$ $F_N = 19.66\ \text{N}$

$m_B = 3.58\ \text{kg}$

Problem 3-40

The pipe of mass M is supported at A by a system of five cords. Determine the force in each cord for equilibrium.

Given:

$M = 30\ \text{kg}$ $c = 3$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$ $d = 4$

$\theta = 60\ \text{deg}$

Solution:

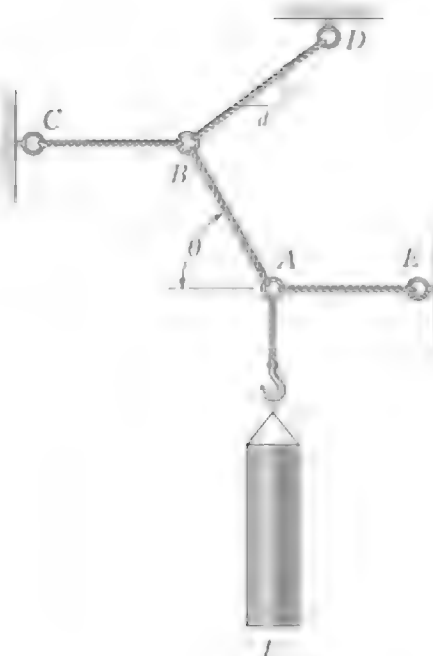
Initial guesses:

$T_{AB} = 1\ \text{N}$ $T_{AE} = 1\ \text{N}$

$T_{BC} = 1\ \text{N}$ $T_{BD} = 1\ \text{N}$

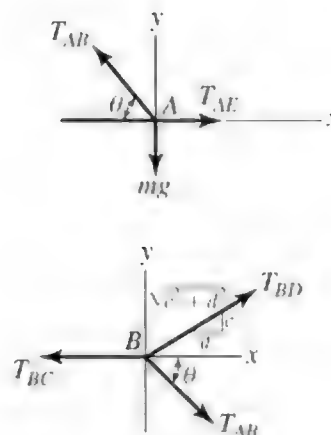
Given

$T_{AB} \sin(\theta) - M g = 0$

$T_{AE} - T_{AB} \cos(\theta) = 0$

$$T_{BD}\left(\frac{c}{\sqrt{c^2+d^2}}\right) - T_{AB}\sin(\theta) = 0$$

$$T_{BD}\left(\frac{d}{\sqrt{c^2+d^2}}\right) + T_{AB}\cos(\theta) - T_{BC} = 0$$

$$\begin{pmatrix} T_{AB} \\ T_{AE} \\ T_{BC} \\ T_{BD} \end{pmatrix} = \text{Find}(T_{AB}, T_{AE}, T_{BC}, T_{BD})$$

$$\begin{pmatrix} T_{AB} \\ T_{AE} \\ T_{BC} \\ T_{BD} \end{pmatrix} = \begin{pmatrix} 339.8 \\ 169.9 \\ 562.3 \\ 490.5 \end{pmatrix} \text{N}$$

Problem 3-41

The joint of a space frame is subjected to four forces. Strut *OA* lies in the *x-y* plane and strut *OB* lies in the *y-z* plane. Determine the forces acting in each of the three struts required for equilibrium.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

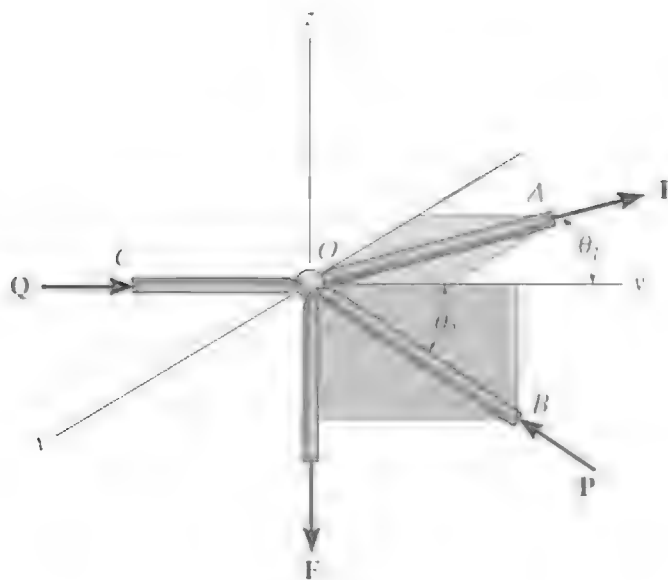
$F = 2\ \text{kN}$

$\theta_1 = 45\ \text{deg}$

$\theta_2 = 40\ \text{deg}$

Solution:

$\Sigma F_x = 0;\qquad -R\sin(\theta_1) = 0$

$R = 0$

$\Sigma F_z = 0;$ $P\ \sin(\theta_2) - F = 0$

$$P = \frac{F}{\sin(\theta_2)}$$

$P = 3.11\ \text{kN}$

$\Sigma F_y = 0;$ $Q - P\ \cos(\theta_2) = 0$

$$Q = P\ \cos(\theta_2)$$

$Q = 2.38\ \text{kN}$

Problem 3-42

Determine the magnitudes of $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$ for equilibrium of the particle.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

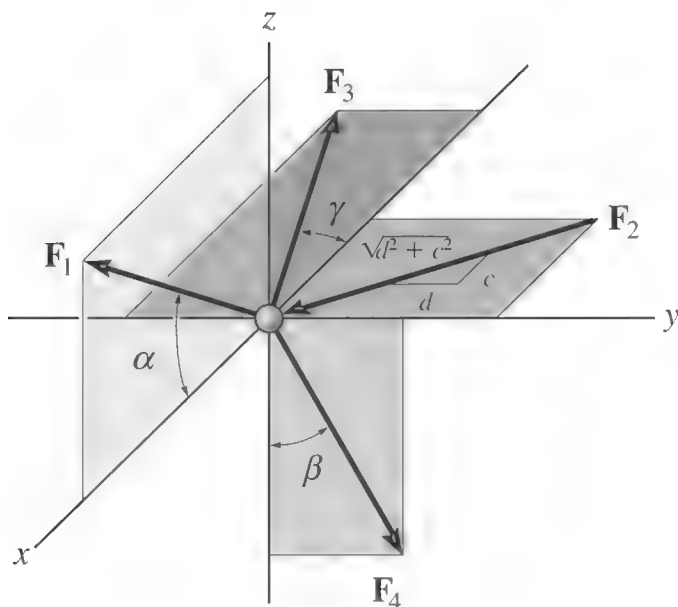
$F_4 = 800\ \text{N}$

$\alpha = 60\ \text{deg}$

$\beta = 30\ \text{deg}$

$\gamma = 30\ \text{deg}$

$c = 3$

$d = 4$

Solution:

The initial guesses: $F_1 = 100\ \text{N}$ $F_2 = 100\ \text{N}$ $F_3 = 100\ \text{N}$

Given

$$F_1\begin{pmatrix}\cos(\alpha)\\0\\\sin(\alpha)\end{pmatrix} + \frac{F_2}{\sqrt{c^2+d^2}}\begin{pmatrix}c\\-d\\0\end{pmatrix} + F_3\begin{pmatrix}-\cos(\gamma)\\-\sin(\gamma)\\0\end{pmatrix} + F_4\begin{pmatrix}0\\\sin(\beta)\\-\cos(\beta)\end{pmatrix} = 0$$

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \text{Find}(F_1, F_2, F_3) \qquad \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 800 \\ 147 \\ 564 \end{pmatrix} \text{N}$$

Problem 3-43

Determine the magnitudes of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ for equilibrium of the particle.

Units Used:

$\text{kN} = 1000\ \text{N}$

Given:

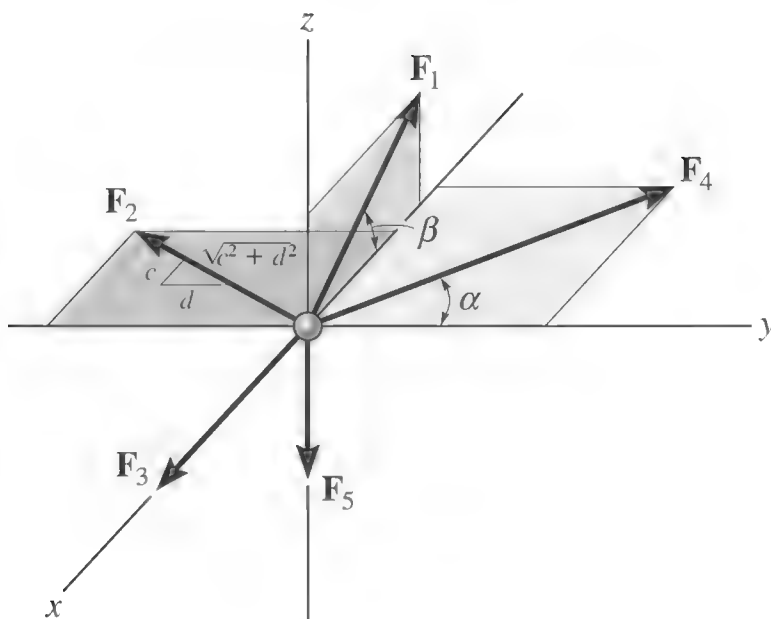
$F_4 = 8.5\ \text{kN}$

$F_5 = 2.8\ \text{kN}$

$\alpha = 15\ \text{deg}$

$\beta = 30\ \text{deg}$

$c = 7$

$d = 24$

Solution:

Initial Guesses: $F_1 = 1\ \text{kN} \qquad F_2 = 1\ \text{kN} \qquad F_3 = 1\ \text{kN}$

Given

$$F_1 \begin{pmatrix} -\cos(\beta) \\ 0 \\ \sin(\beta) \end{pmatrix} + \frac{F_2}{\sqrt{c^2 + d^2}} \begin{pmatrix} -c \\ -d \\ 0 \end{pmatrix} + F_3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + F_4 \begin{pmatrix} -\sin(\alpha) \\ \cos(\alpha) \\ 0 \end{pmatrix} + F_5 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \text{Find}(F_1, F_2, F_3) \qquad \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 5.60 \\ 8.55 \\ 9.44 \end{pmatrix} \text{kN}$$

Problem 3-44

Determine the magnitudes of $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ for equilibrium of the particle $\mathbf{F} = \{-9\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}\}$.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$F = \begin{pmatrix} -9 \\ -8 \\ -5 \end{pmatrix} \text{kN}$$

$$a = 4 \text{ m}$$

$$b = 2 \text{ m}$$

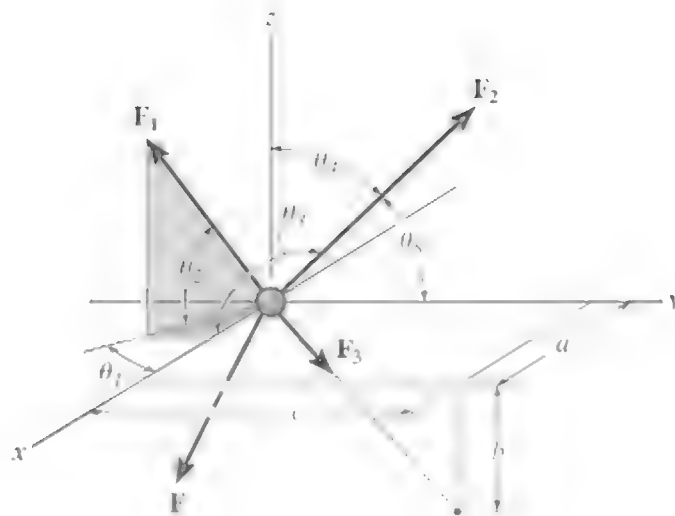
$$c = 4 \text{ m}$$

$$\theta_1 = 30 \text{ deg}$$

$$\theta_2 = 60 \text{ deg}$$

$$\theta_3 = 135 \text{ deg}$$

$$\theta_4 = 60 \text{ deg}$$

$$\theta_5 = 60 \text{ deg}$$

Solution:

Initial guesses: $F_1 = 8 \text{ kN} \quad F_2 = 3 \text{ kN} \quad F_3 = 12 \text{ kN}$

Given

$$F_1\begin{pmatrix}\cos(\theta_2)\cos(\theta_1)\\ -\cos(\theta_2)\sin(\theta_1)\\ \sin(\theta_2)\end{pmatrix} + F_2\begin{pmatrix}\cos(\theta_3)\\ \cos(\theta_5)\\ \cos(\theta_4)\end{pmatrix} + \frac{F_3}{\sqrt{a^2+b^2+c^2}}\begin{pmatrix}a\\ c\\ -b\end{pmatrix} + F = 0$$

$$\begin{pmatrix}F_1\\ F_2\\ F_3\end{pmatrix} = \text{Find}(F_1, F_2, F_3) \qquad \begin{pmatrix}F_1\\ F_2\\ F_3\end{pmatrix} = \begin{pmatrix}8.26\\ 3.84\\ 12.21\end{pmatrix}\text{kN}$$

Problem 3-45

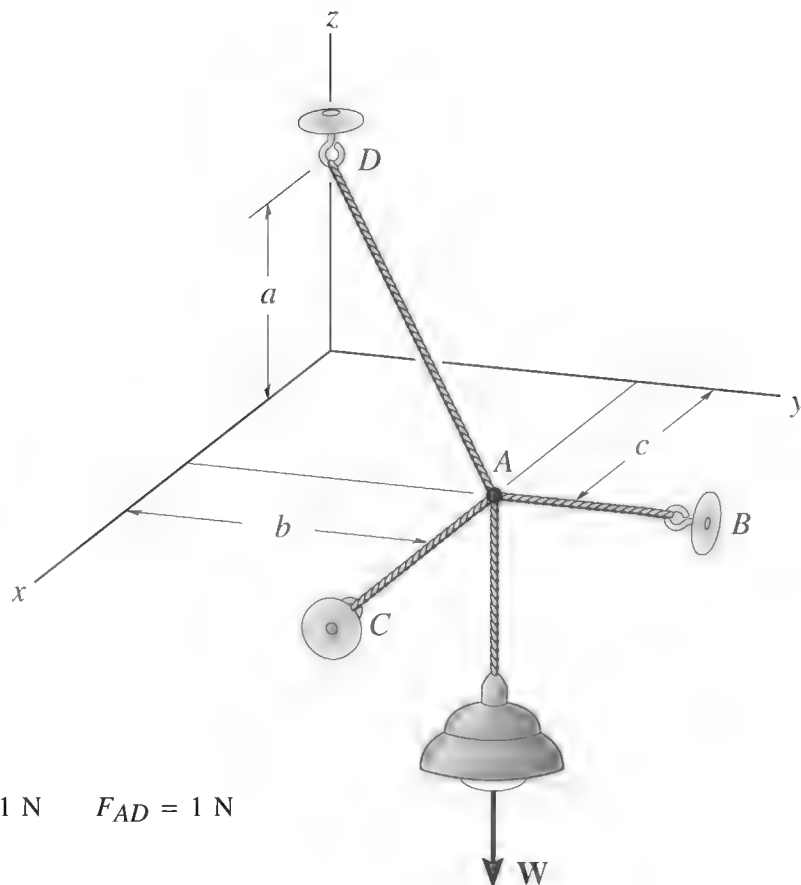
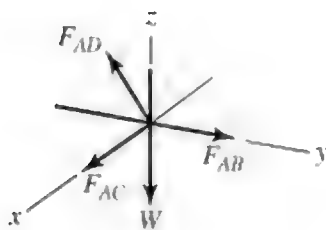
The three cables are used to support the lamp of weight W. Determine the force developed in each cable for equilibrium.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$W = 800\ \text{N} \qquad b = 4\ \text{m}$

$a = 4\ \text{m} \qquad c = 2\ \text{m}$

Solution:

Initial Guesses:

$F_{AB} = 1\ \text{N} \qquad F_{AC} = 1\ \text{N} \qquad F_{AD} = 1\ \text{N}$

Given

$$F_{AB}\begin{pmatrix}0\\1\\0\end{pmatrix} + F_{AC}\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{F_{AD}}{\sqrt{a^2+b^2+c^2}}\begin{pmatrix}-c\\-b\\a\end{pmatrix} + W\begin{pmatrix}0\\0\\-1\end{pmatrix} = 0$$

$$\begin{pmatrix}F_{AB}\\F_{AC}\\F_{AD}\end{pmatrix} = \text{Find}(F_{AB}, F_{AC}, F_{AD}) \qquad \begin{pmatrix}F_{AB}\\F_{AC}\\F_{AD}\end{pmatrix} = \begin{pmatrix}800\\400\\1200\end{pmatrix} \text{N}$$

Problem 3-46

Determine the force in each cable needed to support the load W.

Given:

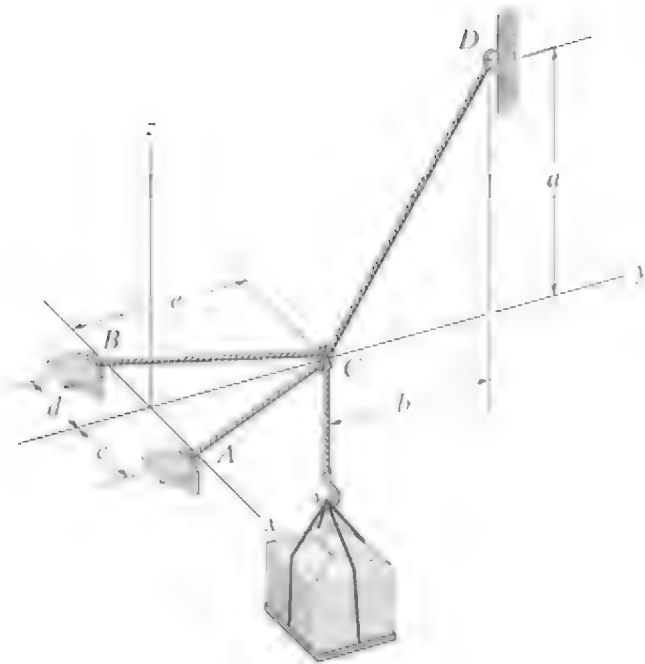
$a = 8$ ft

$b = 6$ ft

$c = 2$ ft

$d = 2$ ft

$e = 6$ ft

$W = 500$ lb

Solution:

Initial guesses:

$F_{CD} = 600$ lb $\qquad F_{CA} = 195$ lb $\qquad F_{CB} = 195$ lb

Given

$$\frac{F_{CA}}{\sqrt{c^2+e^2}}\begin{pmatrix} c \\ -e \\ 0 \end{pmatrix} + \frac{F_{CB}}{\sqrt{d^2+e^2}}\begin{pmatrix} -d \\ -e \\ 0 \end{pmatrix} + \frac{F_{CD}}{\sqrt{a^2+b^2}}\begin{pmatrix} 0 \\ b \\ a \end{pmatrix} + W\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$\begin{pmatrix} F_{CD} \\ F_{CA} \\ F_{CB} \end{pmatrix} = \text{Find}(F_{CD}, F_{CA}, F_{CB}) \qquad \begin{pmatrix} F_{CD} \\ F_{CA} \\ F_{CB} \end{pmatrix} = \begin{pmatrix} 625 \\ 198 \\ 198 \end{pmatrix} \text{lb}$$

Problem 3-47

Determine the stretch in each of the two springs required to hold the crate of mass m_c in the equilibrium position shown. Each spring has an unstretched length δ and a stiffness k.

Given:

$m_C = 20$ kg

$\delta = 2$ m

$k = 300 \frac{\text{N}}{\text{m}}$

$a = 4$ m

$b = 6$ m

$c = 12$ m

Solution:

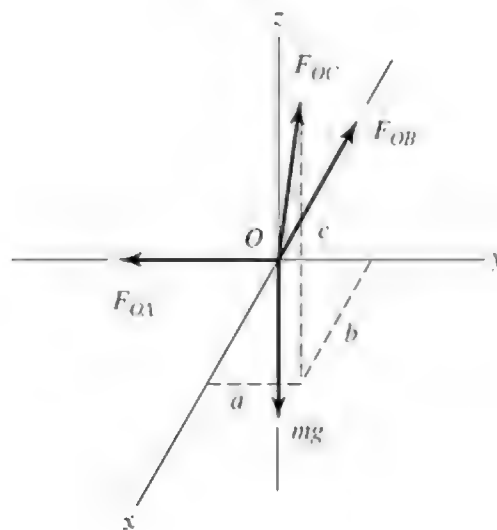
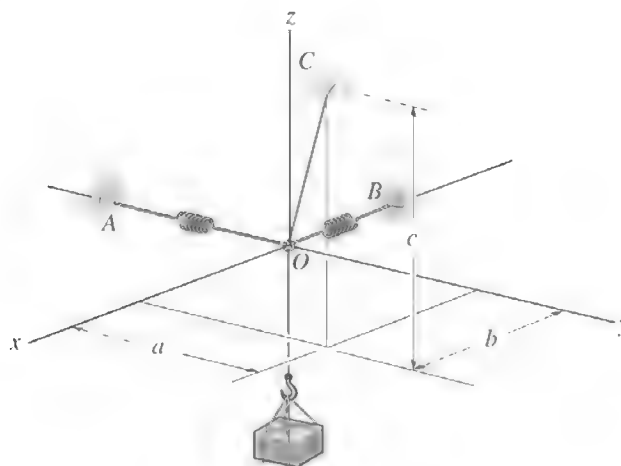
Initial Guesses

$F_{OA} = 1$ N

$F_{OB} = 1$ N

$F_{OC} = 1$ N

Given

$$F_{OA}\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + F_{OB}\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \frac{F_{OC}}{\sqrt{a^2+b^2+c^2}}\begin{pmatrix} b \\ a \\ c \end{pmatrix} + m_C g\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$\begin{pmatrix} F_{OA} \\ F_{OB} \\ F_{OC} \end{pmatrix} = \text{Find}(F_{OA}, F_{OB}, F_{OC}) \qquad \begin{pmatrix} F_{OA} \\ F_{OB} \\ F_{OC} \end{pmatrix} = \begin{pmatrix} 65.40 \\ 98.10 \\ 228.90 \end{pmatrix} \text{N}$$

$$\delta_{OA} = \frac{F_{OA}}{k} \qquad \delta_{OA} = 218\,\text{mm}$$

$$\delta_{OB} = \frac{F_{OB}}{k} \qquad \delta_{OB} = 327\,\text{mm}$$

Problem 3-48

If the bucket and its contents have total weight W, determine the force in the supporting cables DA, DB, and DC.

Given:

$W = 20$ lb

$a = 3$ ft

$b = 4.5$ ft

$c = 2.5$ ft

$d = 3$ ft

$e = 1.5$ ft

$f = 1.5$ ft

Solution:

The initial guesses:

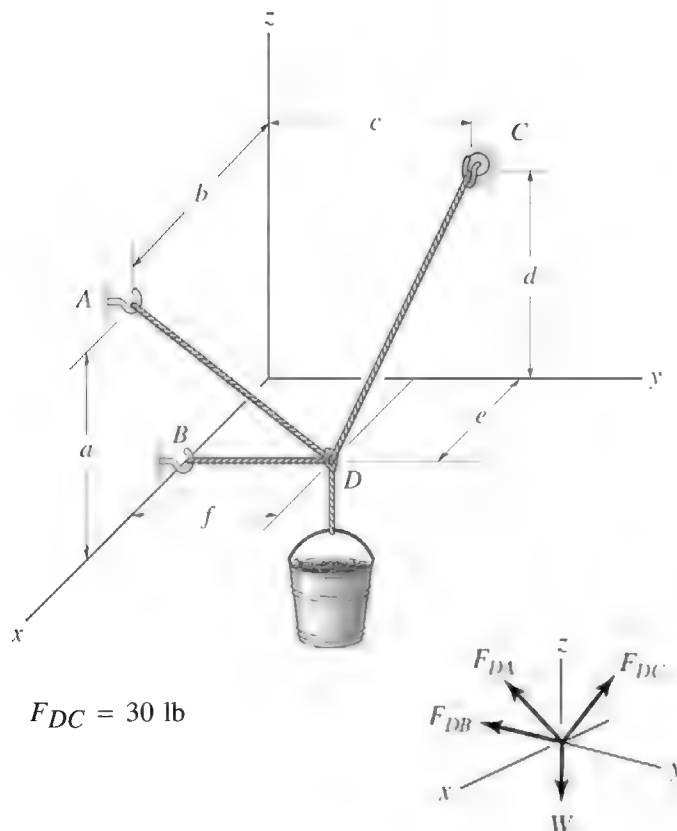
$F_{DA} = 40$ lb $\quad F_{DB} = 20$ lb $\quad F_{DC} = 30$ lb

Given

$\Sigma F_x = 0;$ $$\left[\frac{b-e}{\sqrt{(b-e)^2+f^2+a^2}}\right]F_{DA} - \left[\frac{e}{\sqrt{e^2+d^2+(c-f)^2}}\right]F_{DC} = 0$$

$\Sigma F_y = 0;$ $$\left[\frac{-f}{\sqrt{(b-e)^2+f^2+a^2}}\right]F_{DA} + \left[\frac{c-f}{\sqrt{e^2+d^2+(c-f)^2}}\right]F_{DC} - F_{DB} = 0$$

$\Sigma F_z = 0;$ $$\left[\frac{a}{\sqrt{(b-e)^2+f^2+a^2}}\right]F_{DA} + \left[\frac{d}{\sqrt{e^2+d^2+(c-f)^2}}\right]F_{DC} - W = 0$$

$$\begin{pmatrix} F_{DA} \\ F_{DB} \\ F_{DC} \end{pmatrix} = \text{Find}(F_{DA}, F_{DB}, F_{DC}) \qquad \begin{pmatrix} F_{DA} \\ F_{DB} \\ F_{DC} \end{pmatrix} = \begin{pmatrix} 10.00 \\ 1.11 \\ 15.56 \end{pmatrix} \text{lb}$$

Problem 3-49

The crate which of weight F is to be hoisted with constant velocity from the hold of a ship using the cable arrangement shown. Determine the tension in each of the three cables for equilibrium.

Units Used:

$kN = 10^3\ N$

Given:

$F = 2.5\ kN$

$a = 3\ m$

$b = 1\ m$

$c = 0.75\ m$

$d = 1\ m$

$e = 1.5\ m$

$f = 3\ m$

Solution:

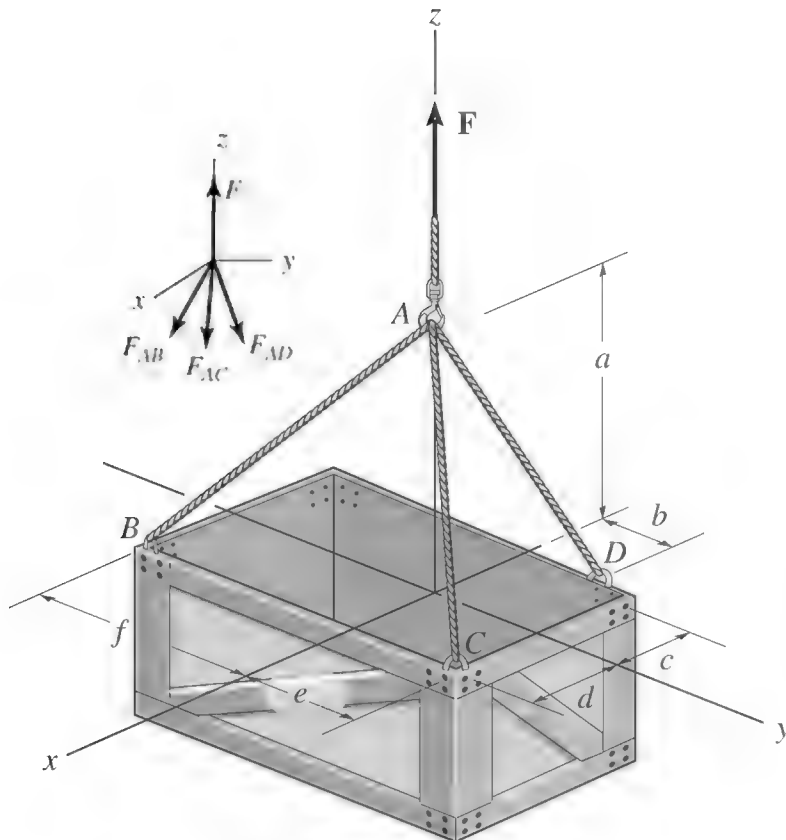
The initial guesses

$F_{AD} = 3\ kN \qquad F_{AC} = 3\ kN \qquad F_{AB} = 3\ kN$

Given
$$\frac{-c}{\sqrt{c^2+b^2+a^2}}F_{AD} + \frac{d}{\sqrt{d^2+e^2+a^2}}F_{AC} + \frac{d}{\sqrt{d^2+f^2+a^2}}F_{AB} = 0$$

$$\frac{b}{\sqrt{c^2+b^2+a^2}}F_{AD} + \frac{e}{\sqrt{d^2+e^2+a^2}}F_{AC} + \frac{-f}{\sqrt{d^2+f^2+a^2}}F_{AB} = 0$$

$$\frac{-a}{\sqrt{c^2+b^2+a^2}}F_{AD} + \frac{-a}{\sqrt{d^2+e^2+a^2}}F_{AC} + \frac{-a}{\sqrt{d^2+f^2+a^2}}F_{AB} + F = 0$$

$$\begin{pmatrix} F_{AD} \\ F_{AC} \\ F_{AB} \end{pmatrix} = \text{Find}(F_{AD}, F_{AC}, F_{AB}) \qquad \begin{pmatrix} F_{AD} \\ F_{AC} \\ F_{AB} \end{pmatrix} = \begin{pmatrix} 1.55 \\ 0.46 \\ 0.98 \end{pmatrix} kN$$

Problem 3-50

The lamp has mass m_l and is supported by pole AO and cables AB and AC. If the force in the pole acts along its axis, determine the forces in AO, AB, and AC for equilibrium.

Given:

$m_l = 15\ \text{kg}$ $d = 1.5\ \text{m}$

$a = 6\ \text{m}$ $e = 4\ \text{m}$

$b = 1.5\ \text{m}$ $f = 1.5\ \text{m}$

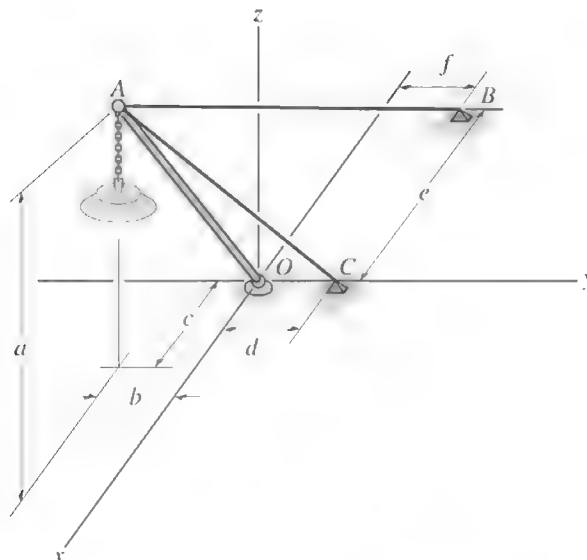
$c = 2\ \text{m}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

The initial guesses:

$F_{AO} = 100\ \text{N}$

$F_{AB} = 200\ \text{N}$

$F_{AC} = 300\ \text{N}$

Given

Equilibrium equations:

$$\frac{c}{\sqrt{c^2 + b^2 + a^2}}F_{AO} - \frac{c+e}{\sqrt{(c+e)^2 + (b+f)^2 + a^2}}F_{AB} - \frac{c}{\sqrt{c^2 + (b+d)^2 + a^2}}F_{AC} = 0$$

$$-\frac{b}{\sqrt{c^2 + b^2 + a^2}}F_{AO} + \frac{b+f}{\sqrt{(c+e)^2 + (b+f)^2 + a^2}}F_{AB} + \frac{b+d}{\sqrt{c^2 + (b+d)^2 + a^2}}F_{AC} = 0$$

$$\frac{a}{\sqrt{c^2 + b^2 + a^2}}F_{AO} - \frac{a}{\sqrt{(c+e)^2 + (b+f)^2 + a^2}}F_{AB} - \frac{a}{\sqrt{c^2 + (b+d)^2 + a^2}}F_{AC} - m_l g = 0$$

$$\begin{pmatrix} F_{AO} \\ F_{AB} \\ F_{AC} \end{pmatrix} = \text{Find}(F_{AO}, F_{AB}, F_{AC}) \qquad \begin{pmatrix} F_{AO} \\ F_{AB} \\ F_{AC} \end{pmatrix} = \begin{pmatrix} 318.82 \\ 110.36 \\ 85.84 \end{pmatrix} \text{N}$$

Problem 3-51

Cables *AB* and *AC* can sustain a maximum tension *Tmax*, and the pole can support a maximum compression *Pmax*. Determine the maximum weight of the lamp that can be supported in the position shown. The force in the pole acts along the axis of the pole.

Given:

$Tmax = 500\ \text{N}$ $c = 2\ \text{m}$

$Pmax = 300\ \text{N}$ $d = 1.5\ \text{m}$

$a = 6\ \text{m}$ $e = 4\ \text{m}$

$b = 1.5\ \text{m}$ $f = 1.5\ \text{m}$

Solution:

Lengths

$$AO = \sqrt{a^2 + b^2 + c^2}$$

$$AB = \sqrt{a^2 + (c+e)^2 + (b+d)^2}$$

$$AC = \sqrt{a^2 + c^2 + (b+d)^2}$$

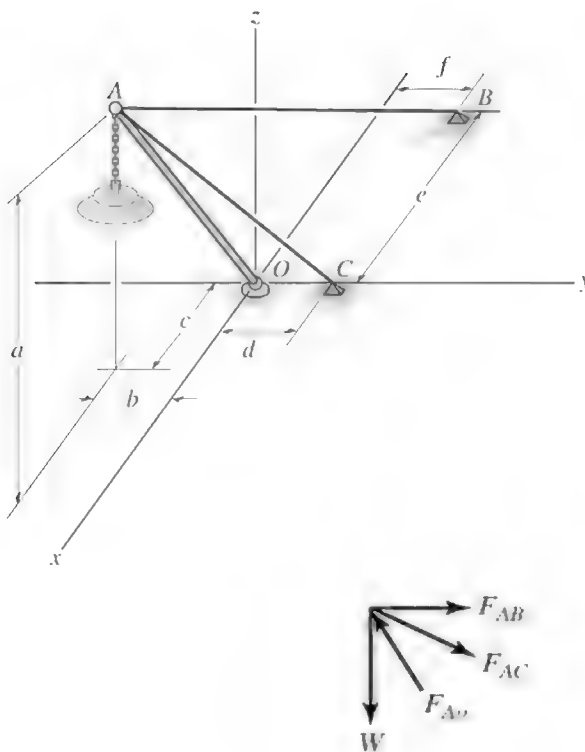
The initial guesses:

$F_{AO} = Pmax$ $F_{AB} = Tmax$ $F_{AC} = Tmax$ $W = 300\text{N}$

Case 1 Assume the pole reaches maximum compression

Given

$$\frac{F_{AO}}{AO}\begin{pmatrix} c \\ -b \\ a \end{pmatrix} + \frac{F_{AB}}{AB}\begin{pmatrix} -c-e \\ b+f \\ -a \end{pmatrix} + \frac{F_{AC}}{AC}\begin{pmatrix} -c \\ b+d \\ -a \end{pmatrix} + W\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$\begin{pmatrix} W_1 \\ F_{AB1} \\ F_{AC1} \end{pmatrix} = \text{Find}(W, F_{AB}, F_{AC}) \qquad \begin{pmatrix} W_1 \\ F_{AB1} \\ F_{AC1} \end{pmatrix} = \begin{pmatrix} 138.46 \\ 103.85 \\ 80.77 \end{pmatrix} \text{N}$$

Case 2 Assume that cable AB reaches maximum tension

Given

$$\frac{F_{AO}}{AO}\begin{pmatrix} c \\ -b \\ a \end{pmatrix} + \frac{F_{AB}}{AB}\begin{pmatrix} -c-e \\ b+f \\ -a \end{pmatrix} + \frac{F_{AC}}{AC}\begin{pmatrix} -c \\ b+d \\ -a \end{pmatrix} + W\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$\begin{pmatrix} W_2 \\ F_{AO2} \\ F_{AC2} \end{pmatrix} = \text{Find}(W, F_{AO}, F_{AC}) \qquad \begin{pmatrix} W_2 \\ F_{AO2} \\ F_{AC2} \end{pmatrix} = \begin{pmatrix} 666.67 \\ 1444.44 \\ 388.89 \end{pmatrix} \text{N}$$

Case 3 Assume that cable AC reaches maximum tension

Given

$$\frac{F_{AO}}{AO}\begin{pmatrix} c \\ -b \\ a \end{pmatrix} + \frac{F_{AB}}{AB}\begin{pmatrix} -c-e \\ b+f \\ -a \end{pmatrix} + \frac{F_{AC}}{AC}\begin{pmatrix} -c \\ b+d \\ -a \end{pmatrix} + W\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$\begin{pmatrix} W_3 \\ F_{AO3} \\ F_{AB3} \end{pmatrix} = \text{Find}(W, F_{AO}, F_{AB}) \qquad \begin{pmatrix} W_3 \\ F_{AO3} \\ F_{AB3} \end{pmatrix} = \begin{pmatrix} 857.14 \\ 1857.14 \\ 642.86 \end{pmatrix} \text{N}$$

Final Answer $W = \min(W_1, W_2, W_3)$ $W = 138.46$ N

Problem 3-52

Determine the tension in cables *AB, AC*, and *AD*, required to hold the crate of weight *W* in equilibrium.

Given:

$W = 60$ lb

$a = 6$ ft

$b = 12$ ft

$c = 8$ ft

$d = 9$ ft

$e = 4$ ft

$f = 6$ ft

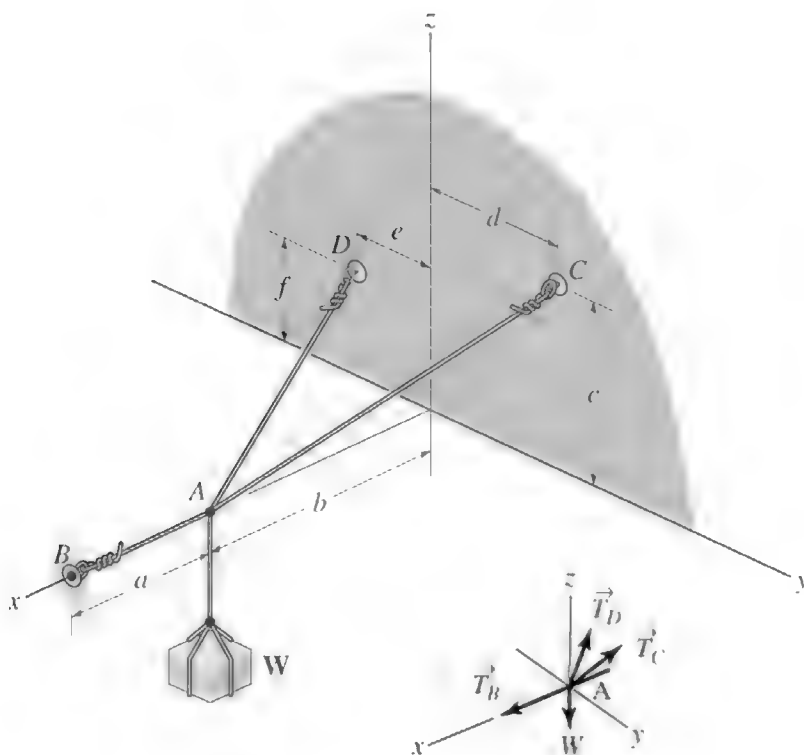
Solution:

The initial guesses:

$T_B = 100$ lb

$T_C = 100$ lb

$T_D = 100$ lb

Given

$\Sigma F_x = 0;$ $$T_B - \frac{b}{\sqrt{b^2 + c^2 + d^2}}T_C - \frac{b}{\sqrt{b^2 + e^2 + f^2}}T_D = 0$$

$\Sigma F_y = 0;$ $$\frac{d}{\sqrt{b^2 + c^2 + d^2}}T_C - \frac{e}{\sqrt{b^2 + e^2 + f^2}}T_D = 0$$

$\Sigma F_z = 0;$ $$-W + \frac{c}{\sqrt{b^2 + c^2 + d^2}}T_C + \frac{f}{\sqrt{b^2 + e^2 + f^2}}T_D = 0$$

Solving

$$\begin{pmatrix} T_B \\ T_C \\ T_D \end{pmatrix} = \text{Find}(T_B, T_C, T_D) \qquad \begin{pmatrix} T_B \\ T_C \\ T_D \end{pmatrix} = \begin{pmatrix} 108.84 \\ 47.44 \\ 87.91 \end{pmatrix} \text{lb}$$

Problem 3-53

The bucket has weight W. Determine the tension developed in each cord for equilibrium.

Given:

$W = 20$ lb

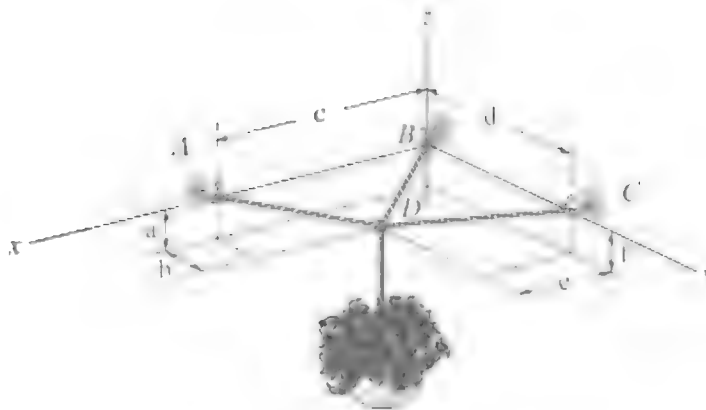
$a = 2 \text{ ft}$

$b = 2 \text{ ft}$

$c = 8 \text{ ft}$

$d = 7 \text{ ft}$

$e = 3 \text{ ft}$

$f = a$

Solution:

Initial Guesses: $F_{DA} = 20 \text{ lb}$ $F_{DB} = 10 \text{ lb}$ $F_{DC} = 15 \text{ lb}$

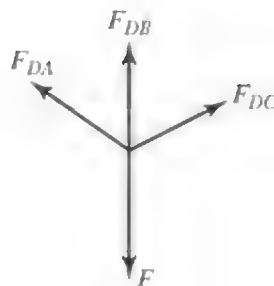
Given

$$\Sigma F_x = 0; \quad \frac{c-e}{\sqrt{(c-e)^2 + b^2 + a^2}} F_{DA} + \frac{-e}{\sqrt{e^2 + (d-b)^2 + f^2}} F_{DC} + \frac{-e}{\sqrt{e^2 + b^2 + f^2}} F_{DB} = 0$$

$$\Sigma F_x = 0; \quad \frac{-b}{\sqrt{(c-e)^2 + b^2 + a^2}} F_{DA} + \frac{d-b}{\sqrt{e^2 + (d-b)^2 + f^2}} F_{DC} + \frac{-b}{\sqrt{e^2 + b^2 + f^2}} F_{DB} = 0$$

$$\Sigma F_y = 0; \quad \frac{a}{\sqrt{(c-e)^2 + b^2 + a^2}} F_{DA} + \frac{f}{\sqrt{e^2 + (d-b)^2 + f^2}} F_{DC} + \frac{f}{\sqrt{e^2 + b^2 + f^2}} F_{DB} - W = 0$$

$$\begin{pmatrix} F_{DA} \\ F_{DB} \\ F_{DC} \end{pmatrix} = \text{Find}(F_{DA}, F_{DB}, F_{DC})$$

$$\begin{pmatrix} F_{DA} \\ F_{DB} \\ F_{DC} \end{pmatrix} = \begin{pmatrix} 21.54 \\ 13.99 \\ 17.61 \end{pmatrix} \text{lb}$$

Problem 3-54

The mast OA is supported by three cables. If cable AB is subjected to tension T, determine the tension in cables AC and AD and the vertical force $\mathbf{F}$ which the mast exerts along its axis on the collar at A.

Given:

$T = 500 \text{ N}$

$a = 6$ m

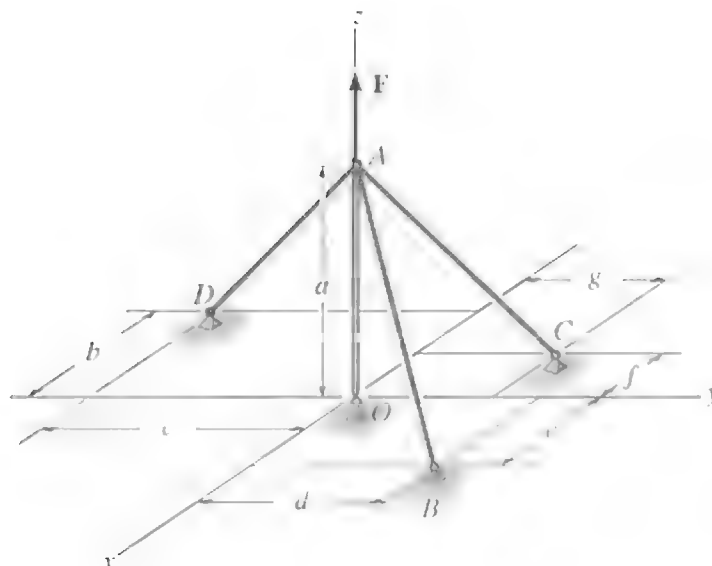
$b = 3$ m

$c = 6$ m

$d = 3$ m

$e = 2$ m

$f = 1.5$ m

$g = 2$ m

Solution:

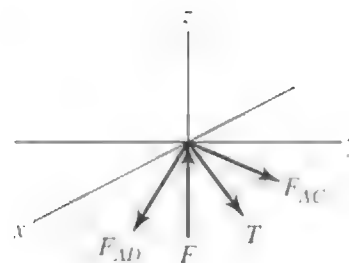
Initial Guesses: $F_{AC} = 90$ N $F_{AD} = 350$ N $F = 750$ N

Given

$\Sigma F_x = 0;$ $$\frac{e}{\sqrt{e^2+d^2+a^2}}T - \frac{f}{\sqrt{f^2+g^2+a^2}}F_{AC} - \frac{b}{\sqrt{b^2+c^2+a^2}}F_{AD} = 0$$

$\Sigma F_y = 0;$ $$\frac{d}{\sqrt{e^2+d^2+a^2}}T + \frac{g}{\sqrt{f^2+g^2+a^2}}F_{AC} - \frac{c}{\sqrt{b^2+c^2+a^2}}F_{AD} = 0$$

$\Sigma F_z = 0;$ $$\frac{-a}{\sqrt{e^2+d^2+a^2}}T - \frac{a}{\sqrt{f^2+g^2+a^2}}F_{AC} - \frac{a}{\sqrt{b^2+c^2+a^2}}F_{AD} + F = 0$$

$$\begin{pmatrix} F_{AC} \\ F_{AD} \\ F \end{pmatrix} = \text{Find}(F_{AC}, F_{AD}, F) \qquad \begin{pmatrix} F_{AC} \\ F_{AD} \\ F \end{pmatrix} = \begin{pmatrix} 92.9 \\ 364.3 \\ 757.1 \end{pmatrix} \text{ N}$$

Problem 3-55

The ends of the three cables are attached to a ring at A and to the edge of the uniform plate of mass M. Determine the tension in each of the cables for equilibrium.

Given:

$M = 150$ kg $e = 4$ m

$a = 2\ \text{m}$ $f = 6\ \text{m}$

$b = 10\ \text{m}$ $g = 6\ \text{m}$

$c = 12\ \text{m}$ $h = 6\ \text{m}$

$d = 2\ \text{m}$ $i = 2\ \text{m}$

$gravity = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

The initial guesses:

$F_B = 15\ \text{N}$

$F_C = 16\ \text{N}$

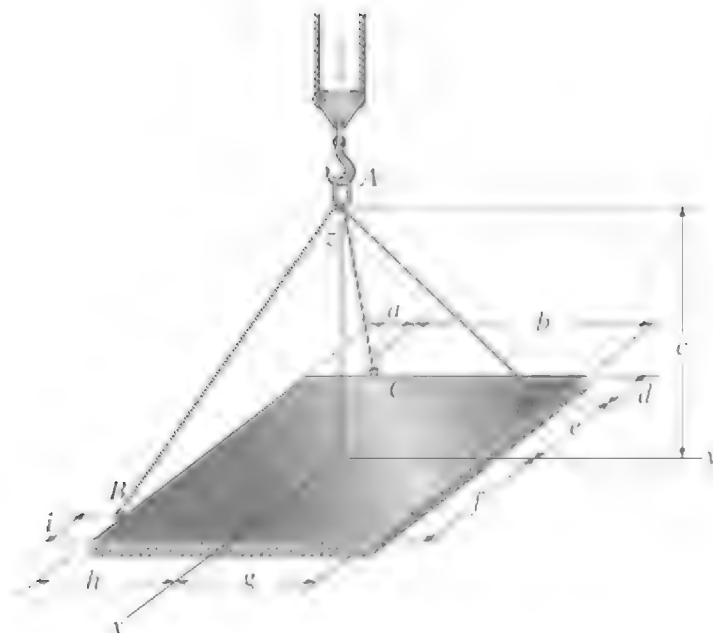
$F_D = 16\ \text{N}$

Given

$\Sigma F_x = 0;$ $$\frac{F_B(f-i)}{\sqrt{(f-i)^2+h^2+c^2}} + \frac{F_C(-d-e)}{\sqrt{(d+e)^2+(h-a)^2+c^2}} + \frac{F_D(-e)}{\sqrt{e^2+g^2+c^2}} = 0$$

$\Sigma F_y = 0;$ $$\frac{F_B(-h)}{\sqrt{(f-i)^2+h^2+c^2}} + \frac{F_C[-(h-a)]}{\sqrt{(d+e)^2+(h-a)^2+c^2}} + \frac{F_D(g)}{\sqrt{e^2+g^2+c^2}} = 0$$

$\Sigma F_z = 0;$ $$\frac{F_B(-c)}{\sqrt{(f-i)^2+h^2+c^2}} + \frac{F_C(-c)}{\sqrt{(d+e)^2+(h-a)^2+c^2}} + \frac{F_D(-c)}{\sqrt{e^2+g^2+c^2}} + M\,gravity = 0$$

$$\begin{pmatrix} F_B \\ F_C \\ F_D \end{pmatrix} = \text{Find}(F_B, F_C, F_D) \qquad \begin{pmatrix} F_B \\ F_C \\ F_D \end{pmatrix} = \begin{pmatrix} 858 \\ 0 \\ 858 \end{pmatrix}\ \text{N}$$

Problem 3-56

The ends of the three cables are attached to a ring at A and to the edge of the uniform plate. Determine the largest mass the plate can have if each cable can support a maximum tension of T.

$kN = 10^3\ N$

Given:

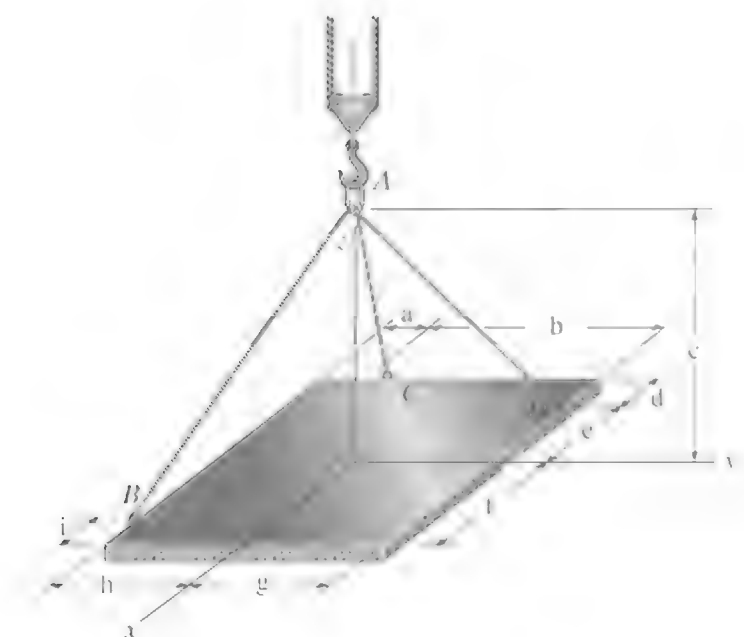
$T = 15\ kN$ $\quad e = 4\ m$

$a = 2\ m$ $\quad f = 6\ m$

$b = 10\ m$ $\quad g = 6\ m$

$c = 12\ m$ $\quad h = 6\ m$

$d = 2\ m$ $\quad i = 2\ m$

$gravity = 9.81\ \frac{m}{s^2}$

Solution:

The initial guesses: $F_B = T$ $\quad F_C = T$ $\quad F_D = T$ $\quad M = 1\ kg$

Case 1: Assume that cable B reaches maximum tension

Given

$$\Sigma F_x = 0; \quad \frac{F_B(f-i)}{\sqrt{(f-i)^2 + h^2 + c^2}} + \frac{F_C(-d-e)}{\sqrt{(d+e)^2 + (h-a)^2 + c^2}} + \frac{F_D(-e)}{\sqrt{e^2 + g^2 + c^2}} = 0$$

$$\Sigma F_y = 0; \quad \frac{F_B(-h)}{\sqrt{(f-i)^2 + h^2 + c^2}} + \frac{F_C[-(h-a)]}{\sqrt{(d+e)^2 + (h-a)^2 + c^2}} + \frac{F_D(g)}{\sqrt{e^2 + g^2 + c^2}} = 0$$

$$\Sigma F_z = 0; \quad \frac{F_B(-c)}{\sqrt{(f-i)^2 + h^2 + c^2}} + \frac{F_C(-c)}{\sqrt{(d+e)^2 + (h-a)^2 + c^2}} + \frac{F_D(-c)}{\sqrt{e^2 + g^2 + c^2}} + M\,gravity = 0$$

$$\begin{pmatrix} M_1 \\ F_{C1} \\ F_{D1} \end{pmatrix} = \text{Find}(M, F_C, F_D) \qquad \begin{pmatrix} F_{C1} \\ F_{D1} \end{pmatrix} = \begin{pmatrix} -0.00 \\ 15.00 \end{pmatrix} kN \qquad M_1 = 2621.23\ kg$$

Case 2: Assume that cable D reaches maximum tension

Given

$$\Sigma F_x = 0; \quad \frac{F_B(f-i)}{\sqrt{(f-i)^2+h^2+c^2}} + \frac{F_C(-d-e)}{\sqrt{(d+e)^2+(h-a)^2+c^2}} + \frac{F_D(-e)}{\sqrt{e^2+g^2+c^2}} = 0$$

$$\Sigma F_y = 0; \quad \frac{F_B(-h)}{\sqrt{(f-i)^2+h^2+c^2}} + \frac{F_C[-(h-a)]}{\sqrt{(d+e)^2+(h-a)^2+c^2}} + \frac{F_D(g)}{\sqrt{e^2+g^2+c^2}} = 0$$

$$\Sigma F_z = 0; \quad \frac{F_B(-c)}{\sqrt{(f-i)^2+h^2+c^2}} + \frac{F_C(-c)}{\sqrt{(d+e)^2+(h-a)^2+c^2}} + \frac{F_D(-c)}{\sqrt{e^2+g^2+c^2}} + M\,gravity = 0$$

$$\begin{pmatrix} M_2 \\ F_{B2} \\ F_{C2} \end{pmatrix} = \text{Find}(M, F_B, F_C) \qquad \begin{pmatrix} F_{B2} \\ F_{C2} \end{pmatrix} = \begin{pmatrix} 15.00 \\ 0.00 \end{pmatrix} \text{kN} \qquad M_2 = 2621.23 \text{ kg}$$

For this set of number $F_C = 0$ for any mass that is applied. For a different set of numbers it would be necessary to also check case 3: Assume that the cable C reaches a maximum.

$$M = \min(M_1, M_2) \qquad M = 2621.23 \text{ kg}$$

Problem 3-57

The crate of weight W is suspended from the cable system shown. Determine the force in each segment of the cable, i.e., AB, AC, CD, CE, and CF. *Hint:* First analyze the equilibrium of point A, then using the result for AC, analyze the equilibrium of point C.

Units Used:

$\text{kip} = 1000 \text{ lb}$

Given:

$W = 500 \text{ lb}$

$a = 10 \text{ ft}$

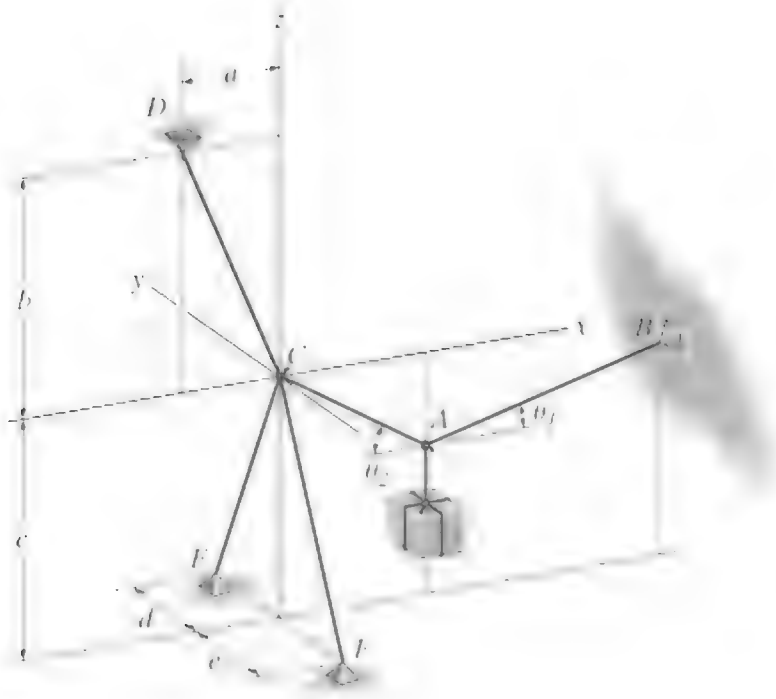
$b = 24 \text{ ft}$

$c = 24 \text{ ft}$

$d = 7$ ft

$e = 7$ ft

$\theta_1 = 20$ deg

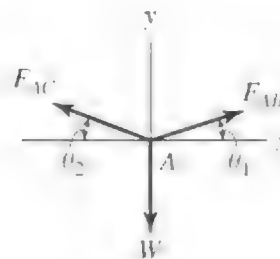
$\theta_2 = 35$ deg

Solution: At *A:*

Initial guesses: $F_{AC} = 570$ lb $F_{AB} = 500$ lb

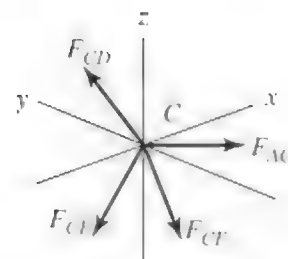
Given

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $F_{AB}\cos(\theta_1) - F_{AC}\cos(\theta_2) = 0$

$+\uparrow \Sigma F_y = 0;$ $F_{AB}\sin(\theta_1) + F_{AC}\sin(\theta_2) - W = 0$

$$\begin{pmatrix} F_{AC} \\ F_{AB} \end{pmatrix} = \text{Find}(F_{AC}, F_{AB}) \qquad \begin{pmatrix} F_{AC} \\ F_{AB} \end{pmatrix} = \begin{pmatrix} 574 \\ 500 \end{pmatrix} \text{lb}$$

At *C:* Initial Guesses

$F_{CD} = 1$ lb $F_{CE} = 1$ lb $F_{CF} = 1$ lb

Given

$$F_{AC}\begin{pmatrix} \cos(\theta_2) \\ 0 \\ -\sin(\theta_2) \end{pmatrix} + \frac{F_{CD}}{\sqrt{a^2+b^2}}\begin{pmatrix} -a \\ 0 \\ b \end{pmatrix} + \frac{F_{CE}}{\sqrt{c^2+d^2}}\begin{pmatrix} 0 \\ d \\ -c \end{pmatrix} + \frac{F_{CF}}{\sqrt{c^2+e^2}}\begin{pmatrix} 0 \\ -e \\ -c \end{pmatrix} = 0$$

$$\begin{pmatrix} F_{CD} \\ F_{CE} \\ F_{CF} \end{pmatrix} = \text{Find}(F_{CD}, F_{CE}, F_{CF}) \qquad F_{CD} = 1.22\text{ kip} \qquad \begin{pmatrix} F_{CE} \\ F_{CF} \end{pmatrix} = \begin{pmatrix} 416 \\ 416 \end{pmatrix} \text{lb}$$

Problem 3-58

The chandelier of weight *W* is supported by three wires as shown. Determine the force in each wire for equilibrium.

Given:

$W = 80\text{ lb}$

$r = 1\text{ ft}$

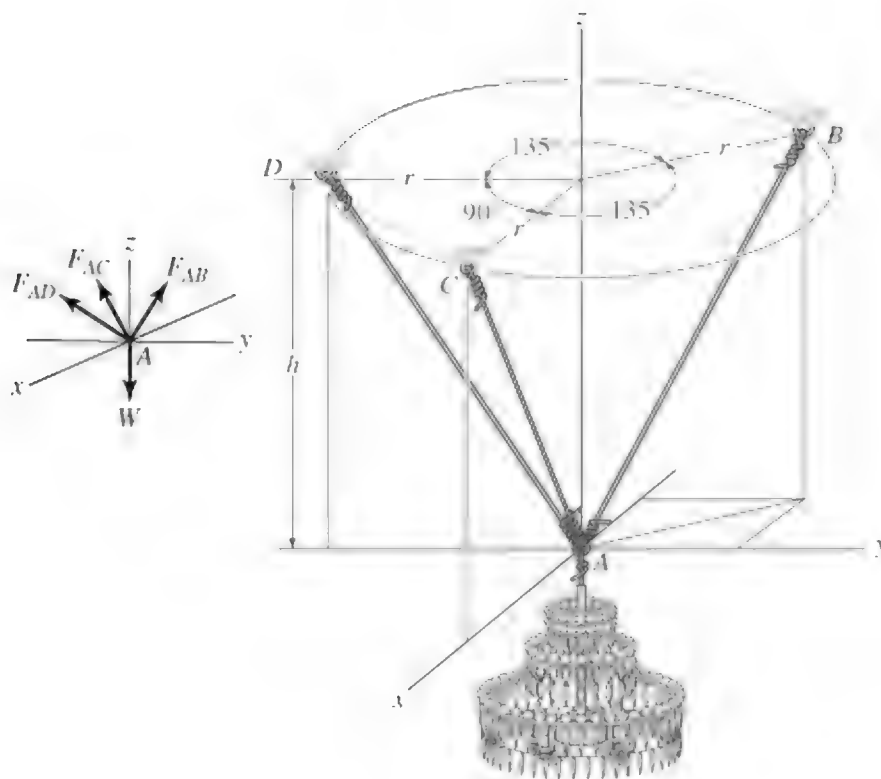
$h = 2.4\text{ ft}$

Solution:

The initial guesses:

$F_{AB} = 40\text{ lb}$

$F_{AC} = 30\text{ lb}$

$F_{AD} = 30\text{ lb}$

Given

$\Sigma\ F_x = 0;$ $$\frac{r}{\sqrt{r^2 + h^2}} F_{AC} - \frac{r\cos(45\text{ deg})}{\sqrt{r^2 + h^2}} F_{AB} = 0$$

$\Sigma\ F_y = 0;$ $$\frac{-r}{\sqrt{r^2 + h^2}} F_{AD} + \frac{r\cos(45\text{ deg})}{\sqrt{r^2 + h^2}} F_{AB} = 0$$

$\Sigma\ F_z = 0;\qquad \frac{h}{\sqrt{r^2+h^2}}F_{AC} + \frac{h}{\sqrt{r^2+h^2}}F_{AD} + \frac{h}{\sqrt{r^2+h^2}}F_{AB} - W = 0$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \text{Find}(F_{AB}, F_{AC}, F_{AD}) \qquad \begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \begin{pmatrix} 35.9 \\ 25.4 \\ 25.4 \end{pmatrix} \text{lb}$$

Problem 3-59

If each wire can sustain a maximum tension T_{max} before it fails, determine the greatest weight of the chandelier the wires will support in the position shown.

Given:

$T_{max} = 120\ \text{lb}$

$r = 1\ \text{ft}$

$h = 2.4\ \text{ft}$

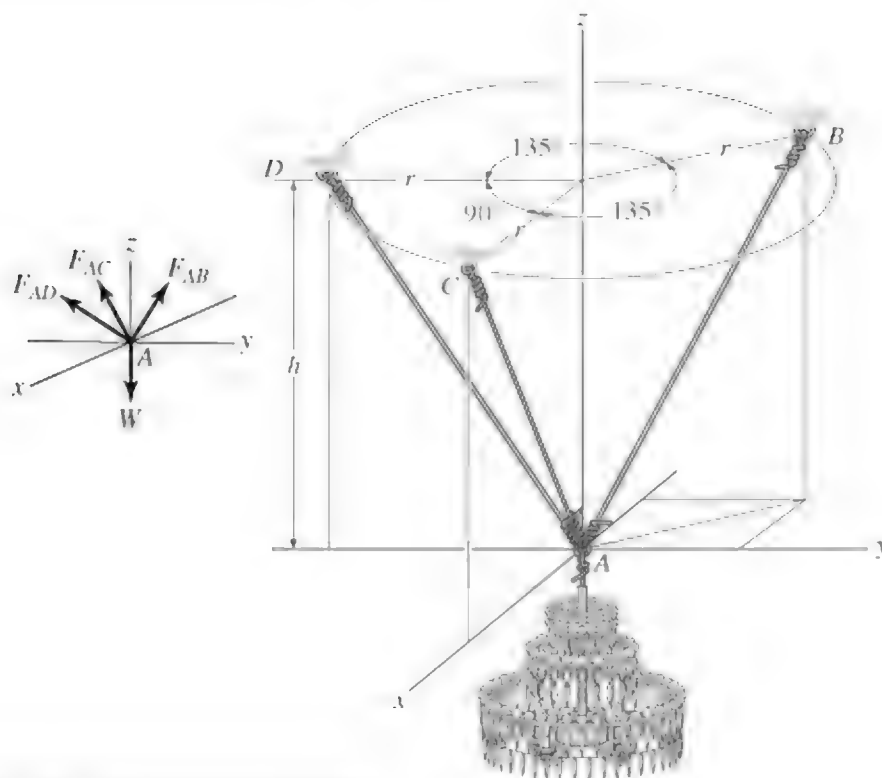
Solution:

The initial guesses:

$F_{AB} = T_{max}$

$F_{AC} = T_{max}$

$F_{AD} = T_{max}$

$W = T_{max}$

Case 1 Assume that cable AB has maximum tension

Given

$\Sigma\ F_x = 0;\qquad \frac{r}{\sqrt{r^2+h^2}}F_{AC} - \frac{r\cos(45\text{deg})}{\sqrt{r^2+h^2}}F_{AB} = 0$

$\Sigma\ F_y = 0;\qquad \frac{-r}{\sqrt{r^2+h^2}}F_{AD} + \frac{r\cos(45\text{deg})}{\sqrt{r^2+h^2}}F_{AB} = 0$

$\Sigma\ F_z = 0;$ $$\frac{h}{\sqrt{r^2+h^2}}F_{AC} + \frac{h}{\sqrt{r^2+h^2}}F_{AD} + \frac{h}{\sqrt{r^2+h^2}}F_{AB} - W = 0$$

$$\begin{pmatrix} W_1 \\ F_{AC1} \\ F_{AD1} \end{pmatrix} = \text{Find}(W, F_{AC}, F_{AD}) \qquad \begin{pmatrix} W_1 \\ F_{AC1} \\ F_{AD1} \end{pmatrix} = \begin{pmatrix} 267.4 \\ 84.9 \\ 84.9 \end{pmatrix} \text{lb}$$

Case 2 Assume that cable AC has maximum tension

Given

$\Sigma\ F_x = 0;$ $$\frac{r}{\sqrt{r^2+h^2}}F_{AC} - \frac{r\cos(45\ \text{deg})}{\sqrt{r^2+h^2}}F_{AB} = 0$$

$\Sigma\ F_y = 0;$ $$\frac{-r}{\sqrt{r^2+h^2}}F_{AD} + \frac{r\cos(45\ \text{deg})}{\sqrt{r^2+h^2}}F_{AB} = 0$$

$\Sigma\ F_z = 0;$ $$\frac{h}{\sqrt{r^2+h^2}}F_{AC} + \frac{h}{\sqrt{r^2+h^2}}F_{AD} + \frac{h}{\sqrt{r^2+h^2}}F_{AB} - W = 0$$

$$\begin{pmatrix} W_2 \\ F_{AB2} \\ F_{AD2} \end{pmatrix} = \text{Find}(W, F_{AB}, F_{AD}) \qquad \begin{pmatrix} W_2 \\ F_{AB2} \\ F_{AD2} \end{pmatrix} = \begin{pmatrix} 378.2 \\ 169.7 \\ 120 \end{pmatrix} \text{lb}$$

Case 3 Assume that cable AD has maximum tension

Given

$\Sigma\ F_x = 0;$ $$\frac{r}{\sqrt{r^2+h^2}}F_{AC} - \frac{r\cos(45\text{deg})}{\sqrt{r^2+h^2}}F_{AB} = 0$$

$\Sigma\ F_y = 0;$ $$\frac{-r}{\sqrt{r^2+h^2}}F_{AD} + \frac{r\cos(45\text{deg})}{\sqrt{r^2+h^2}}F_{AB} = 0$$

$\Sigma\ F_z = 0;$ $$\frac{h}{\sqrt{r^2+h^2}}F_{AC} + \frac{h}{\sqrt{r^2+h^2}}F_{AD} + \frac{h}{\sqrt{r^2+h^2}}F_{AB} - W = 0$$

$$\begin{pmatrix} W_3 \\ F_{AB3} \\ F_{AC3} \end{pmatrix} = \text{Find}(W, F_{AB}, F_{AC}) \qquad \begin{pmatrix} W_3 \\ F_{AB3} \\ F_{AC3} \end{pmatrix} = \begin{pmatrix} 378.2 \\ 169.7 \\ 120 \end{pmatrix} \text{lb}$$

$$W = \min(W_1, W_2, W_3) \qquad W = 267.42 \text{ lb}$$

Problem 3-60

Determine the force in each cable used to lift the surge arrester of mass M at constant velocity.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

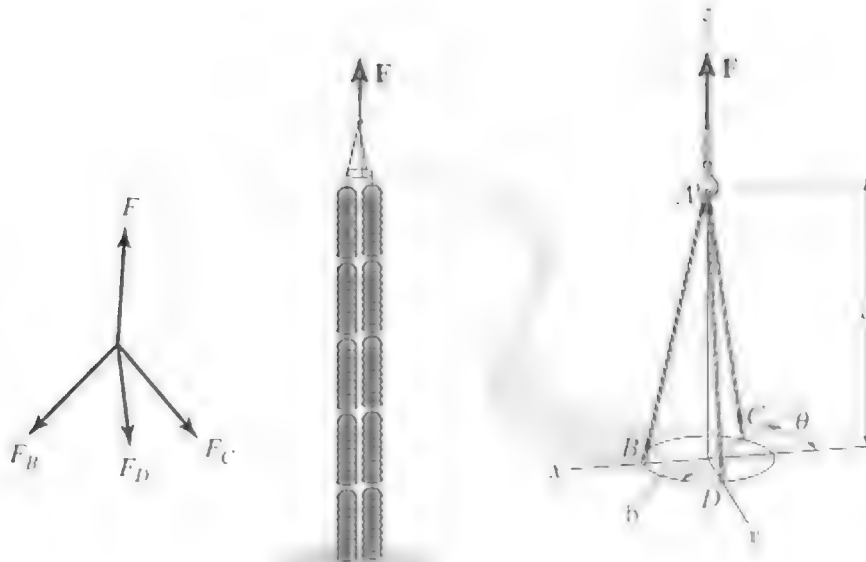
$$\text{Mg} = 10^3 \text{ kg}$$

Given:

$$M = 9.50 \text{ Mg}$$

$$a = 2 \text{ m}$$

$$b = 0.5 \text{ m}$$

$$\theta = 45 \text{ deg}$$

Solution:

Initial guesses: $F_B = 50$ kN $\quad F_C = 30$ kN $\quad F_D = 10$ kN

Given

$$\Sigma F_x = 0; \qquad F_B \frac{b}{\sqrt{b^2 + a^2}} - F_C \frac{b\cos(\theta)}{\sqrt{b^2 + a^2}} = 0$$

$$\Sigma F_y = 0; \qquad F_C \frac{-b\sin(\theta)}{\sqrt{b^2 + a^2}} + F_D \frac{b}{\sqrt{b^2 + a^2}} = 0$$

$$\Sigma F_z = 0; \qquad M g - F_B \frac{a}{\sqrt{b^2 + a^2}} - F_C \frac{a}{\sqrt{b^2 + a^2}} - F_D \frac{a}{\sqrt{b^2 + a^2}} = 0$$

$$\begin{pmatrix} F_B \\ F_C \\ F_D \end{pmatrix} = \text{Find}(F_B, F_C, F_D) \qquad \begin{pmatrix} F_B \\ F_C \\ F_D \end{pmatrix} = \begin{pmatrix} 28.13 \\ 39.78 \\ 28.13 \end{pmatrix} \text{kN}$$

Problem 3-61

The cylinder of weight W is supported by three chains as shown. Determine the force in each chain for equilibrium.

Given:

$W = 800$ lb

$r = 1$ ft

$d = 1$ ft

Solution:

The initial guesses:

$F_{AB} = 1$ lb

$F_{AC} = 1$ lb

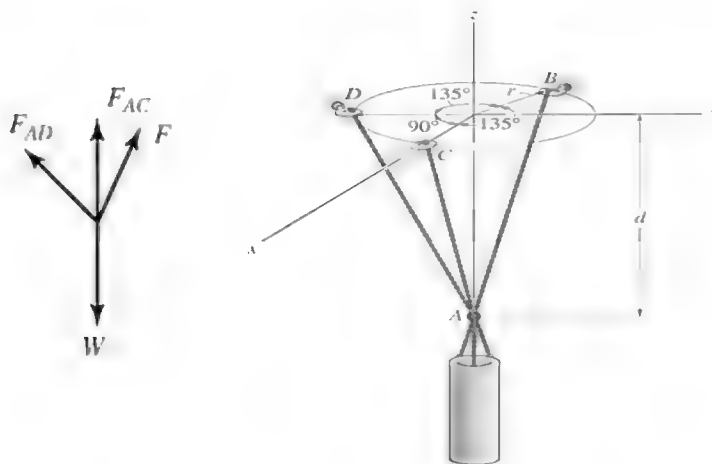
$F_{AD} = 1$ lb

Given

$\Sigma F_x = 0;$ $$\frac{r}{\sqrt{r^2 + d^2}} F_{AC} - \frac{r\cos(45\ \text{deg})}{\sqrt{r^2 + d^2}} F_{AB} = 0$$

$\Sigma F_y = 0;$ $$\frac{-r}{\sqrt{r^2 + d^2}} F_{AD} + \frac{r\sin(45\ \text{deg})}{\sqrt{r^2 + d^2}} F_{AB} = 0$$

$\Sigma F_z = 0;$ $$\frac{d}{\sqrt{r^2 + d^2}} F_{AD} + \frac{d}{\sqrt{r^2 + d^2}} F_{AC} + \frac{d}{\sqrt{r^2 + d^2}} F_{AB} - W = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \text{Find}(F_{AB}, F_{AC}, F_{AD}) \qquad \begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \begin{pmatrix} 469 \\ 331 \\ 331 \end{pmatrix} \text{lb}$$

Problem 3-62

The triangular frame *ABC* can be adjusted vertically between the three equal-length cords. If it remains in a horizontal plane, determine the required distance *s* so that the tension in each of the cords, *OA*, *OB*, and *OC*, equals **F**. The lamp has a mass *M*.

Given:

$F = 20\ \text{N}$

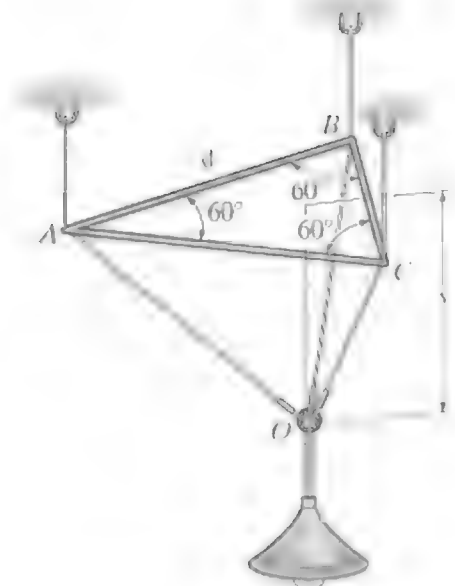
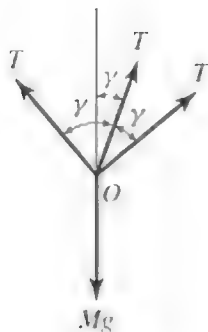
$M = 5\ \text{kg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$d = 0.5\ \text{m}$

Solution:

$\Sigma F_z = 0;\qquad 3F\cos(\gamma) = M g$

$\gamma = \text{acos}\left(\frac{M g}{3F}\right) \qquad \gamma = 35.16\,\text{deg}$

Geometry $\qquad \frac{2d\cos(30\ \text{deg})}{3} = s\tan(\gamma)$

$s = \frac{2d\cos(30\ \text{deg})}{3\tan(\gamma)} \qquad s = 410\,\text{mm}$

Problem 3-63

Determine the force in each cable needed to support the platform of weight *W*.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$W = 3500 \text{ lb}$ $d = 4 \text{ ft}$

$a = 2 \text{ ft}$ $e = 3 \text{ ft}$

$b = 4 \text{ ft}$ $f = 3 \text{ ft}$

$c = 4 \text{ ft}$ $g = 10 \text{ ft}$

Solution:

The initial guesses:

$F_{AB} = 1 \text{ lb}$ $F_{AC} = 1 \text{ lb}$ $F_{AD} = 1 \text{ lb}$

Given

$$\frac{-b}{\sqrt{g^2 + (e-a)^2 + b^2}} F_{AD} + \frac{c-d}{\sqrt{(c-d)^2 + e^2 + g^2}} F_{AC} + \frac{c}{\sqrt{c^2 + f^2 + g^2}} F_{AB} = 0$$

$$\frac{e-a}{\sqrt{g^2 + (e-a)^2 + b^2}} F_{AD} + \frac{e}{\sqrt{e^2 + (c-d)^2 + g^2}} F_{AC} - \frac{f}{\sqrt{c^2 + f^2 + g^2}} F_{AB} = 0$$

$$\frac{-g}{\sqrt{g^2 + (e-a)^2 + b^2}} F_{AD} - \frac{g}{\sqrt{e^2 + (c-d)^2 + g^2}} F_{AC} - \frac{g}{\sqrt{g^2 + f^2 + c^2}} F_{AB} + W = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \text{Find}(F_{AB}, F_{AC}, F_{AD}) \qquad \begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \begin{pmatrix} 1.467 \\ 0.914 \\ 1.42 \end{pmatrix} \text{kip}$$

Problem 3-64

A flowerpot of mass M is supported at A by the three cords. Determine the force acting in each cord for equilibrium.

Given:

$M = 25 \text{ kg}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

$\theta_1 = 30 \text{ deg}$

$\theta_2 = 30 \text{ deg}$

$\theta_3 = 60 \text{ deg}$

$\theta_4 = 45 \text{ deg}$

Solution:

Initial guesses:

$F_{AB} = 1 \text{ N}$

$F_{AD} = 1 \text{ N}$

$F_{AC} = 1 \text{ N}$

Given

$\Sigma F_x = 0; \quad F_{AD}\sin(\theta_1) - F_{AC}\sin(\theta_2) = 0$

$\Sigma F_y = 0; \quad -F_{AD}\cos(\theta_1)\sin(\theta_3) - F_{AC}\cos(\theta_2)\sin(\theta_3) + F_{AB}\sin(\theta_4) = 0$

$\Sigma F_z = 0; \quad F_{AD}\cos(\theta_1)\cos(\theta_3) + F_{AC}\cos(\theta_2)\cos(\theta_3) + F_{AB}\cos(\theta_4) - M\,g = 0$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \text{Find}(F_{AB}, F_{AC}, F_{AD}) \qquad \begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \begin{pmatrix} 219.89 \\ 103.65 \\ 103.65 \end{pmatrix} \text{N}$$

Problem 3-65

If each cord can sustain a maximum tension of **T** before it fails, determine the greatest weight of the flowerpot the cords can support.

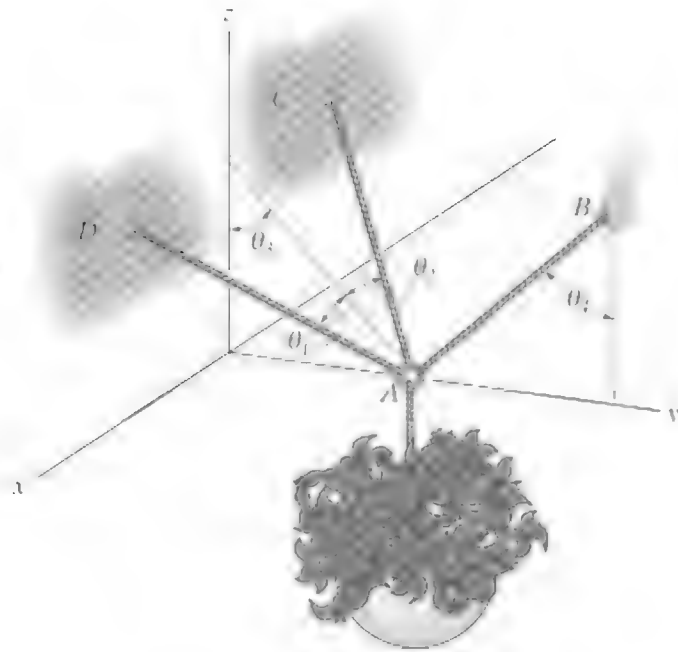
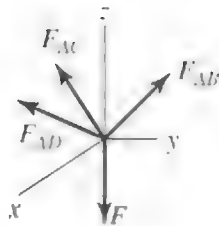
Given:

$T = 50\ \text{N}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 30\ \text{deg}$

$\theta_3 = 60\ \text{deg}$

$\theta_4 = 45\ \text{deg}$

Solution:

Initial guesses:

$F_{AB} = T$

$F_{AD} = T$

$F_{AC} = T$

$W = T$

Case 1 Assume that *AB* reaches maximum tension

Given

$$\Sigma F_x = 0;\ F_{AD}\sin(\theta_1) - F_{AC}\sin(\theta_2) = 0$$

$$\Sigma F_y = 0;\ -F_{AD}\cos(\theta_1)\sin(\theta_3) - F_{AC}\cos(\theta_2)\sin(\theta_3) + F_{AB}\sin(\theta_4) = 0$$

$$\Sigma F_z = 0;\ F_{AD}\cos(\theta_1)\cos(\theta_3) + F_{AC}\cos(\theta_2)\cos(\theta_3) + F_{AB}\cos(\theta_4) - W = 0$$

$$\begin{pmatrix} W_1 \\ F_{AC1} \\ F_{AD1} \end{pmatrix} = \text{Find}(W, F_{AC}, F_{AD}) \qquad \begin{pmatrix} W_1 \\ F_{AC1} \\ F_{AD1} \end{pmatrix} = \begin{pmatrix} 55.77 \\ 23.57 \\ 23.57 \end{pmatrix} \text{N}$$

Case 2 Assume that *AC* reaches maximum tension

Given

$$\Sigma F_x = 0;\ F_{AD}\sin(\theta_1) - F_{AC}\sin(\theta_2) = 0$$

$$\Sigma F_y = 0;\ -F_{AD}\cos(\theta_1)\sin(\theta_3) - F_{AC}\cos(\theta_2)\sin(\theta_3) + F_{AB}\sin(\theta_4) = 0$$

$$\Sigma F_z = 0;\ F_{AD}\cos(\theta_1)\cos(\theta_3) + F_{AC}\cos(\theta_2)\cos(\theta_3) + F_{AB}\cos(\theta_4) - W = 0$$

$$\begin{pmatrix} W_2 \\ F_{AB2} \\ F_{AD2} \end{pmatrix} = \text{Find}(W, F_{AB}, F_{AD}) \qquad \begin{pmatrix} W_2 \\ F_{AB2} \\ F_{AD2} \end{pmatrix} = \begin{pmatrix} 118.30 \\ 106.07 \\ 50.00 \end{pmatrix} \text{N}$$

Case 3 Assume that *AD* reaches maximum tension

Given

$$\Sigma F_x = 0;\ F_{AD}\sin(\theta_1) - F_{AC}\sin(\theta_2) = 0$$

$$\Sigma F_y = 0;\ -F_{AD}\cos(\theta_1)\sin(\theta_3) - F_{AC}\cos(\theta_2)\sin(\theta_3) + F_{AB}\sin(\theta_4) = 0$$

$$\Sigma F_z = 0;\ F_{AD}\cos(\theta_1)\cos(\theta_3) + F_{AC}\cos(\theta_2)\cos(\theta_3) + F_{AB}\cos(\theta_4) - W = 0$$

$$\begin{pmatrix} W_3 \\ F_{AB3} \\ F_{AC3} \end{pmatrix} = \text{Find}(W, F_{AB}, F_{AC}) \qquad \begin{pmatrix} W_3 \\ F_{AB3} \\ F_{AC3} \end{pmatrix} = \begin{pmatrix} 118.30 \\ 106.07 \\ 50.00 \end{pmatrix} \text{N}$$

$$W = \min(W_1, W_2, W_3) \qquad W = 55.77\ \text{N}$$

Problem 3-66

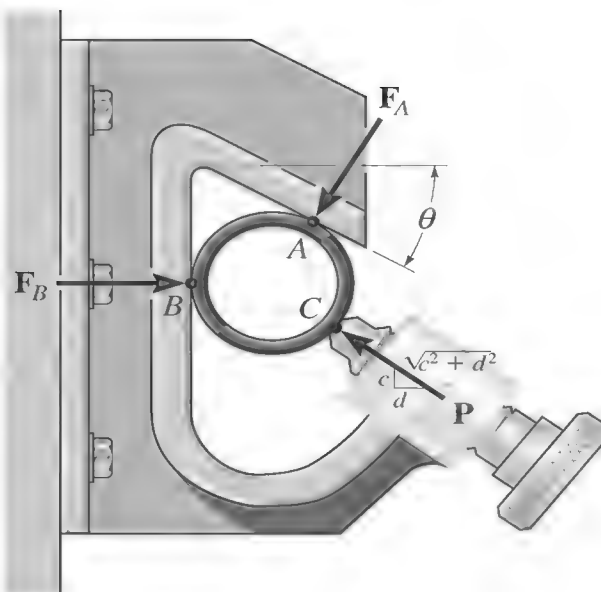
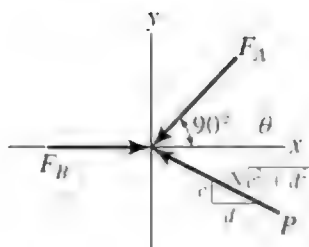
The pipe is held in place by the vice. If the bolt exerts force P on the pipe in the direction shown, determine the forces F_A and F_B that the smooth contacts at A and B exerton the pipe.

Given:

$P = 50\ \text{lb}$

$\theta = 30\ \text{deg}$

$c = 3$

$d = 4$

Solution:

Initial Guesses $F_A = 1\ \text{lb}$ $F_B = 1\ \text{lb}$

Given

$\xrightarrow{+} \Sigma F_x = 0;$ $$F_B - F_A \sin(\theta) - P\left(\frac{d}{\sqrt{c^2 + d^2}}\right) = 0$$

$+\uparrow \Sigma F_y = 0;$ $$-F_A \cos(\theta) + P\left(\frac{c}{\sqrt{c^2 + d^2}}\right) = 0$$

$$\begin{pmatrix} F_A \\ F_B \end{pmatrix} = \text{Find}(F_A, F_B) \qquad \begin{pmatrix} F_A \\ F_B \end{pmatrix} = \begin{pmatrix} 34.6 \\ 57.3 \end{pmatrix} \text{lb}$$

Problem 3-67

When y is zero, the springs sustain force F_0. Determine the magnitude of the applied vertical forces **F** and -**F** required to pull point A away from point B a distance y_1. The ends of cords CAD and CBD are attached to rings at C and D.

Given:

$F_0 = 60\ \text{lb}$

$k = 40\ \dfrac{\text{lb}}{\text{ft}}$

$d = 2\ \text{ft}$

$y_1 = 2\ \text{ft}$

Solution:

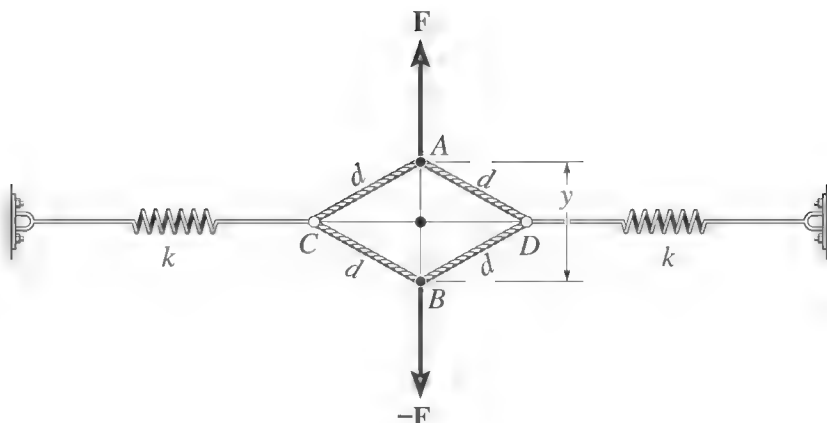
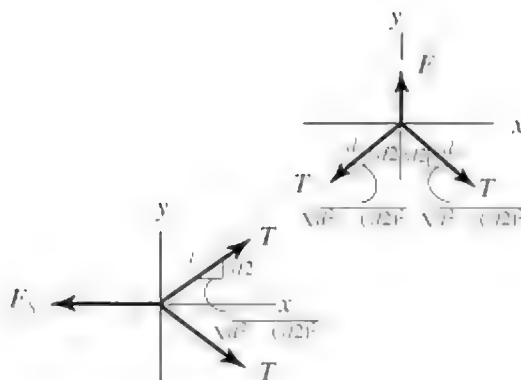
Initial spring stretch:

$$s_1 = \frac{F_0}{k} \qquad s_1 = 1.50\ \text{ft}$$

Initial guesses:

$F_s = 1\ \text{lb} \qquad T = 1\ \text{lb} \qquad F = 1\ \text{lb}$

Given

$$F - \frac{y_1}{2d}T = 0 \qquad \frac{\sqrt{d^2 - \left(\frac{y_1}{2}\right)^2}}{d}T - F_s = 0 \qquad F_s = k\left[d - \sqrt{d^2 - \left(\frac{y_1}{2}\right)^2} + s_1\right]$$

$$\begin{pmatrix} F_s \\ T \\ F \end{pmatrix} = \text{Find}(F_s, T, F) \qquad \begin{pmatrix} F_s \\ T \end{pmatrix} = \begin{pmatrix} 70.72 \\ 81.66 \end{pmatrix}\text{lb} \qquad F = 40.83\ \text{lb}$$

Problem 3-68

When y is zero, the springs are each stretched a distance δ. Determine the distance y if a force F is applied to points A and B as shown. The ends of cords CAD and CBD are attached to

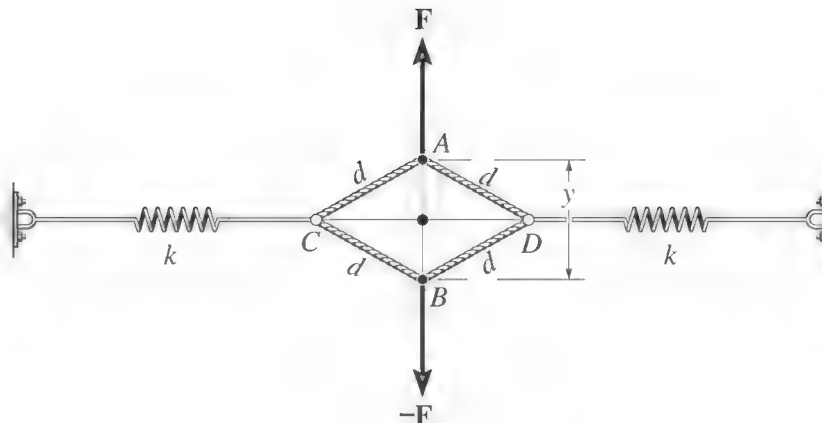
rings at C and D.

Given:

$\delta = 1.5\ \text{ft}$

$k = 40\ \frac{\text{lb}}{\text{ft}}$

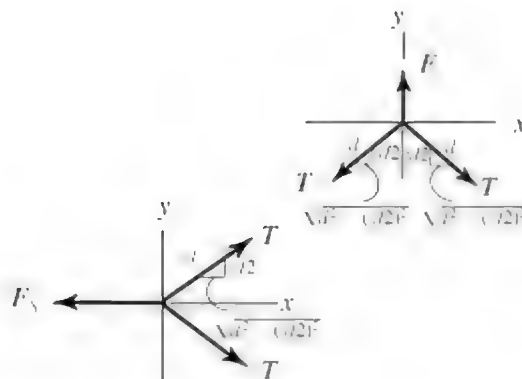
$d = 2\ \text{ft}$

$F = 60\ \text{lb}$

Solution:

Initial guesses:

$F_s = 1\ \text{lb} \qquad T = 1\ \text{lb} \qquad y = 1\ \text{ft}$

Given

$$F - \frac{y}{2d}T = 0$$

$$\frac{\sqrt{d^2 - \left(\frac{y}{2}\right)^2}}{d}T - F_s = 0 \qquad F_s = k\left[d - \sqrt{d^2 - \left(\frac{y}{2}\right)^2} + \delta\right]$$

$$\begin{pmatrix} F_s \\ T \\ y \end{pmatrix} = \text{Find}(F_s, T, y) \qquad \begin{pmatrix} F_s \\ T \end{pmatrix} = \begin{pmatrix} 76.92 \\ 97.55 \end{pmatrix}\text{lb} \qquad y = 2.46\,\text{ft}$$

Problem 3-69

Cord AB of length a is attached to the end B of a spring having an unstretched length b. The other end of the spring is attached to a roller C so that the spring remains horizontal as it stretches. If a weight W is suspended from B, determine the angle θ of cord AB for equilibrium.

Given:

$a = 5\ \text{ft}$

$b = 5\ \text{ft}$

$k = 10\ \frac{\text{lb}}{\text{ft}}$

$W = 10\ \text{lb}$

Solution:

Initial Guesses

$F_{BA} = 1\ \text{lb}$

$F_{sp} = 1\ \text{lb}$

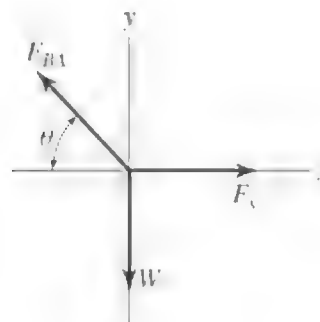
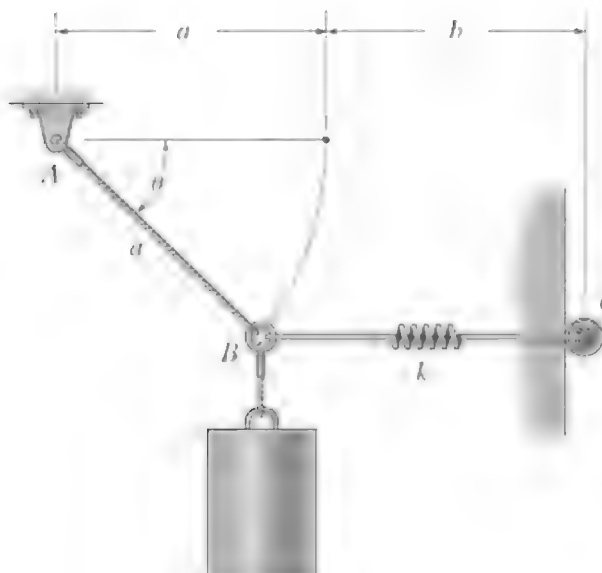
$\theta = 30\ \text{deg}$

Given

$$F_{sp} - F_{BA}\cos(\theta) = 0$$

$$F_{BA}\sin(\theta) - W = 0$$

$$F_{sp} = k(a - a\cos(\theta))$$

$$\begin{pmatrix} F_{sp} \\ F_{BA} \\ \theta \end{pmatrix} = \text{Find}(F_{sp}, F_{BA}, \theta) \qquad \begin{pmatrix} F_{sp} \\ F_{BA} \end{pmatrix} = \begin{pmatrix} 11.82 \\ 15.49 \end{pmatrix} \text{lb} \qquad \theta = 40.22\ \text{deg}$$

Problem 3-70

The uniform crate of mass M is suspended by using a cord of length l that is attached to the sides of the crate and passes over the small pulley at O. If the cord can be attached at either points A and B, or C and D, determine which attachment produces the least amount of tension in the cord and specify the cord tension in this case.

Given:

$M = 50$ kg

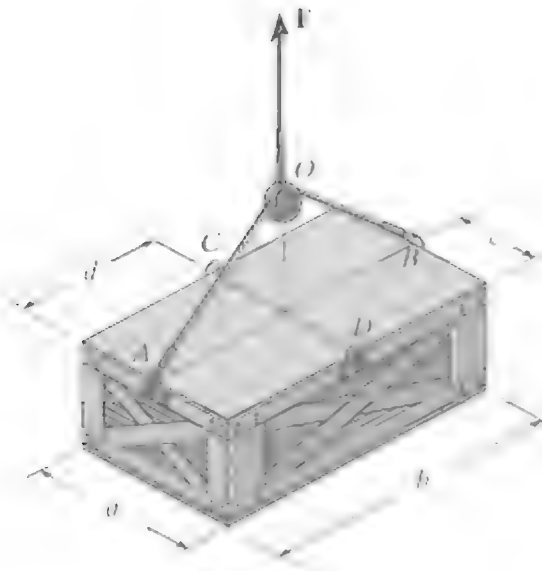
$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

$a = 0.6$ m

$b = 1.5$ m

$l = 2$ m

$c = \dfrac{a}{2}$

$d = \dfrac{b}{2}$

Solution:

Case 1 Attached at A and B Guess $T = 1$ N

Given $$M g - \left[\frac{\sqrt{\left(\frac{l}{2}\right)^2 - d^2}}{\frac{l}{2}}\right] 2T = 0 \qquad T_1 = \text{Find}(T) \qquad T_1 = 370.78 \text{ N}$$

Case 2 Attached at C and D Guess $T = 1$ N

Given $$M g - \left[\frac{\sqrt{\left(\frac{l}{2}\right)^2 - c^2}}{\frac{l}{2}}\right] 2T = 0 \qquad T_2 = \text{Find}(T) \qquad T_2 = 257.09 \text{ N}$$

Choose the arrangement that gives the smallest tension. $T = \min(T_1, T_2)$ $T = 257.09$ N

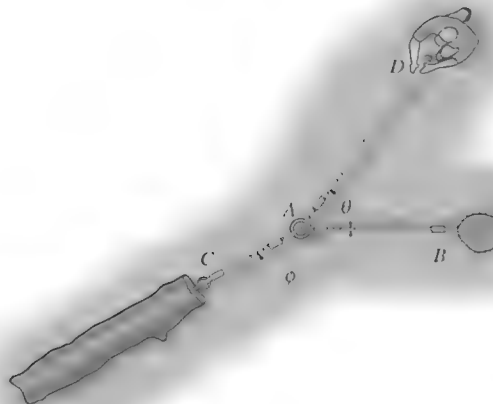
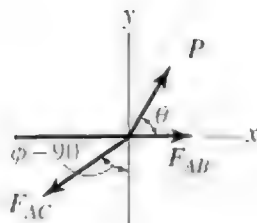
Problem 3-71

The man attempts to pull the log at C by using the three ropes. Determines the direction θ in which he should pull on his rope with a force P, so that he exerts a maximum

force on the log. What is the force on the log for this case ? Also, determine the direction in which he should pull in order to maximize the force in the rope attached to B.What is this maximum force?

Given:

$$P = 80\ \text{lb}$$

$$\phi = 150\ \text{deg}$$

Solution:

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{AB} + P\cos(\theta) - F_{AC}\sin(\phi - 90\ \text{deg}) = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad P\sin(\theta) - F_{AC}\cos(\phi - 90\ \text{deg}) = 0$

$$F_{AC} = \frac{P\sin(\theta)}{\cos(\phi - 90\text{deg})}$$

In order to maximize FAC we choose $\sin(\theta) = 1.$

Thus $\quad \theta = 90\ \text{deg} \quad F_{AC} = \dfrac{P\sin(\theta)}{\cos(\phi - 90\text{deg})} \quad F_{AC} = 160.00\ \text{lb}$

Now let's find the force in the rope AB.

$$F_{AB} = -P\cos(\theta) + F_{AC}\sin(\phi - 90\ \text{deg})$$

$$F_{AB} = -P\cos(\theta) + \frac{P\sin(\theta)\sin(\phi - 90\ \text{deg})}{\cos(\phi - 90\ \text{deg})}$$

$$F_{AB} = P\frac{\sin(\theta)\sin(\phi - 90\ \text{deg}) - \cos(\theta)\cos(\phi - 90\ \text{deg})}{\cos(\phi - 90\ \text{deg})} = -P\frac{\cos(\theta + \phi - 90\text{deg})}{\cos(\phi - 90\ \text{deg})}$$

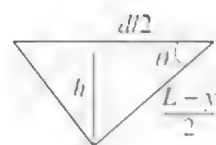
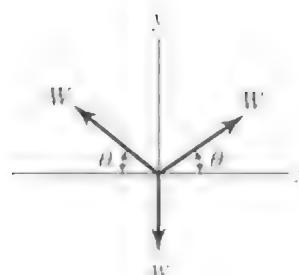
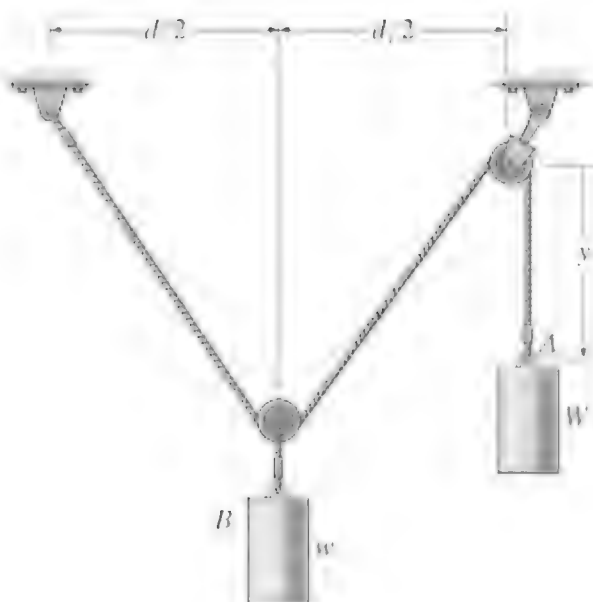
In order to maximize the force we set $\cos(\theta + \phi - 90\ \text{deg}) = -1$

$$\theta + \phi - 90\ \text{deg} = 180\ \text{deg}$$

$$\theta = 270\ \text{deg} - \phi \qquad \theta = 120.00\ \text{deg}$$

$$F_{AB} = -P\frac{\cos(\theta + \phi - 90\ \text{deg})}{\cos(\phi - 90\ \text{deg})} \qquad F_{AB} = 160.00\ \text{lb}$$

Problem 3-72

The "scale" consists of a known weight W which is suspended at A from a cord of total length L. Determine the weight w at B if A is at a distance y for equilibrium. Neglect the sizes and weights of the pulleys.

Solution:

$+\uparrow\ \Sigma F_y = 0; \quad 2W\sin(\theta) - w = 0$

Geometry $\quad h = \sqrt{\left(\frac{L-y}{2}\right)^2 - \left(\frac{d}{2}\right)^2} = \frac{1}{2}\sqrt{(L-y)^2 - d^2}$

$$w = 2W\left[\frac{\frac{1}{2}\sqrt{(L-y)^2 + d^2}}{\frac{L-y}{2}}\right] \qquad w = \frac{2W}{L-y}\sqrt{(L-y)^2 - d^2}$$

Problem 3-73

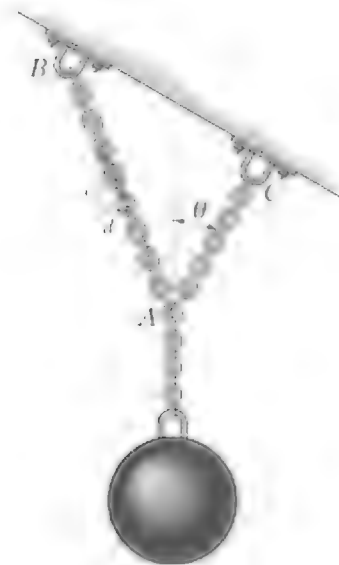
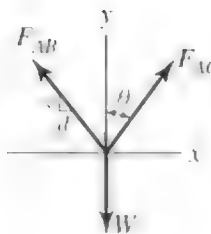
Determine the maximum weight W that can be supported in the position shown if each cable AC and AB can support a maximum tension of **F** before it fails.

Given:

$\theta = 30$ deg

$F = 600$ lb

$c = 12$

$d = 5$

Solution:

Initial Guesses $F_{AB} = F$ $F_{AC} = F$ $W = F$

Case 1 Assume that cable AC reaches maximum tension

Given $$F_{AC}\sin(\theta) - \frac{d}{\sqrt{c^2 + d^2}}F_{AB} = 0$$

$$F_{AC}\cos(\theta) + \frac{c}{\sqrt{c^2 + d^2}}F_{AB} - W = 0$$

$$\begin{pmatrix} W_1 \\ F_{AB1} \end{pmatrix} = \text{Find}(W, F_{AB}) \qquad \begin{pmatrix} W_1 \\ F_{AB1} \end{pmatrix} = \begin{pmatrix} 1239.62 \\ 780.00 \end{pmatrix} \text{lb}$$

Case 2 Assume that cable AB reaches maximum tension

Given $$F_{AC}\sin(\theta) - \frac{d}{\sqrt{c^2 + d^2}}F_{AB} = 0$$

$$F_{AC}\cos(\theta) + \frac{c}{\sqrt{c^2 + d^2}}F_{AB} - W = 0$$

$$\begin{pmatrix} W_2 \\ F_{AC2} \end{pmatrix} = \text{Find}(W, F_{AC}) \qquad \begin{pmatrix} W_2 \\ F_{AC2} \end{pmatrix} = \begin{pmatrix} 953.55 \\ 461.54 \end{pmatrix} \text{lb}$$

$$W = \min(W_1, W_2) \qquad W = 953.6 \text{ lb}$$

Problem 3-74

If the spring on rope OB has been stretched a distance δ. and fixed in place as shown, determine the tension developed in each of the other three ropes in order to hold the weight W in equilibrium. Rope OD lies in the x-y plane.

Given:

$a = 2$ ft

$b = 4$ ft

$c = 3$ ft

$d = 4$ ft

$e = 4$ ft

$f = 4$ ft

$x_B = -2$ ft

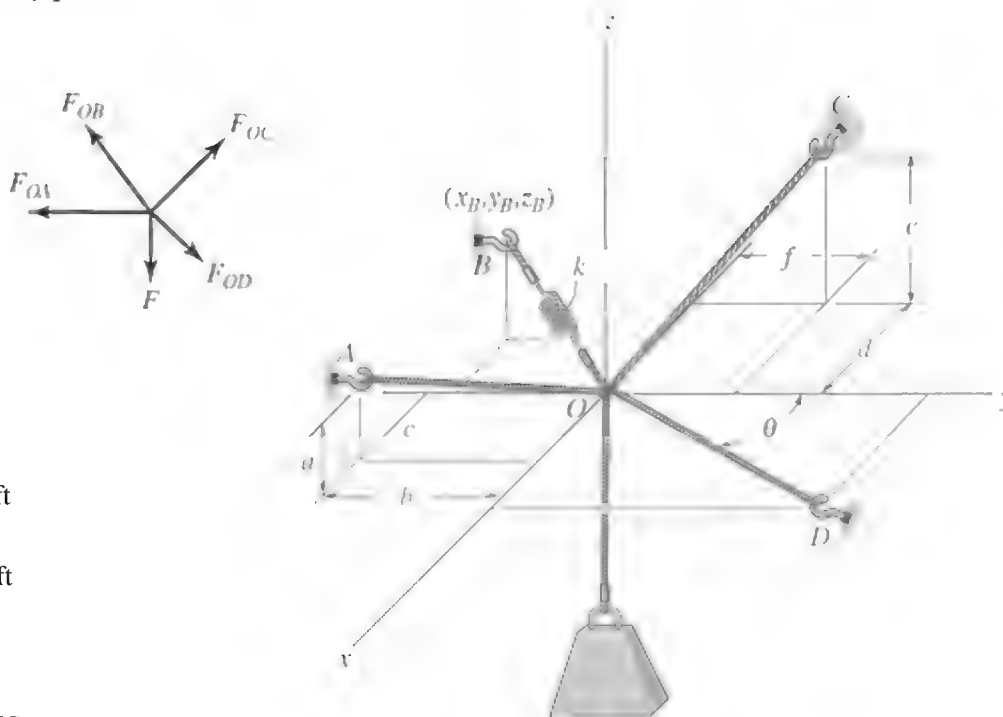
$y_B = -3$ ft

$z_B = 3$ ft

$\theta = 30$ deg

$k = 20 \frac{\text{lb}}{\text{in}}$

$\delta = 2$ in

$W = 225$ lb

Solution:

Initial Guesses $F_{OA} = 10 \text{ lb}$ $F_{OC} = 10 \text{ lb}$ $F_{OD} = 10 \text{ lb}$

Given

$$\frac{c}{\sqrt{a^2+b^2+c^2}}F_{OA} + k\delta\frac{x_B}{\sqrt{x_B^2+y_B^2+z_B^2}} + \frac{-d}{\sqrt{d^2+e^2+f^2}}F_{OC} + F_{OD}\sin(\theta) = 0$$

$$\frac{-b}{\sqrt{a^2+b^2+c^2}}F_{OA} + k\delta\frac{y_B}{\sqrt{x_B^2+y_B^2+z_B^2}} + \frac{f}{\sqrt{d^2+e^2+f^2}}F_{OC} + F_{OD}\cos(\theta) = 0$$

$$\frac{a}{\sqrt{a^2+b^2+c^2}}F_{OA} + k\delta\frac{z_B}{\sqrt{x_B^2+y_B^2+z_B^2}} + \frac{e}{\sqrt{d^2+e^2+f^2}}F_{OC} - W = 0$$

$$\begin{pmatrix} F_{OA} \\ F_{OC} \\ F_{OD} \end{pmatrix} = \text{Find}(F_{OA}, F_{OC}, F_{OD}) \qquad \begin{pmatrix} F_{OA} \\ F_{OC} \\ F_{OD} \end{pmatrix} = \begin{pmatrix} 201.6 \\ 215.7 \\ 58.6 \end{pmatrix} \text{lb}$$

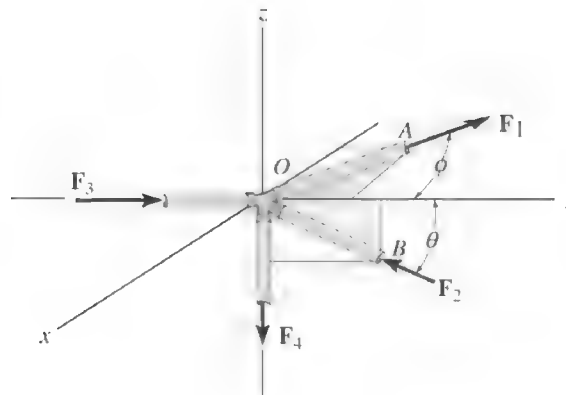
Problem 3-75

The joint of a space frame is subjected to four member forces. Member *OA* lies in the *x* - *y* plane and member *OB* lies in the *y* - *z* plane. Determine the forces acting in each of the members required for equilibrium of the joint.

Given:

$F_4 = 200 \text{ lb}$

$\theta = 40 \text{ deg}$

$\phi = 45 \text{ deg}$

Solution:

The initial guesses: $F_1 = 200 \text{ lb}$ $F_2 = 200 \text{ lb}$ $F_3 = 200 \text{ lb}$

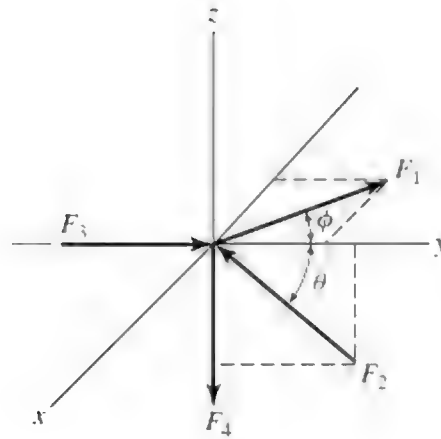
Given

$\Sigma\ F_y = 0;\quad F_3 + F_1\cos(\phi) - F_2\cos(\theta) = 0$

$\Sigma\ F_x = 0;\quad -F_1\sin(\phi) = 0$

$\Sigma\ F_z = 0;\quad F_2\sin(\theta) - F_4 = 0$

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \text{Find}(F_1, F_2, F_3)$$

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 311.1 \\ 238.4 \end{pmatrix} \text{lb}$$

Problem 4-1

If **A**, **B**, and **D** are given vectors, prove the distributive law for the vector cross product, i.e., $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.

Solution:

Consider the three vectors; with **A** vertical.

Note triangle *obd* is perpendicular to **A**.

$$od = |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}|(|\mathbf{B} + \mathbf{D}|)\sin(\theta_3)$$

$$ob = |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}|\ |\mathbf{B}|\ \sin(\theta_1)$$

$$bd = |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}|\ |\mathbf{B}|\ \sin(\theta_2)$$

Also, these three cross products all lie in the plane *obd* since they are all perpendicular to A. As noted the magnitude of each cross product is proportional to the length of each side of the triangle.

The three vector cross - products also form a closed triangle *o'b'd'* which is similar to triangle *obd*. Thus from the figure,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{D} \qquad \text{(QED)}$$

Note also,

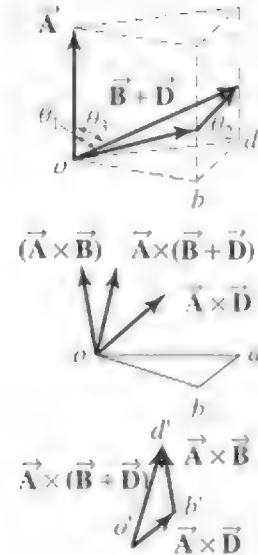
$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{K}$$

$$\mathbf{D} = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \left|\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{pmatrix}\right|$$

$$= [A_y(B_z + D_z) - A_z(B_y + D_y)]\mathbf{i} - [A_x(B_z + D_z) - A_z(B_x + D_x)]\mathbf{j} + [A_x(B_y + D_y) - A_y(B_x - D_x)]\mathbf{k}$$

$$= [(A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}] \ldots$$
$$+ [(A_yD_z - A_zD_y)\mathbf{i} - (A_xD_z - A_zD_x)\mathbf{j} + (A_xD_y - A_yD_x)\mathbf{k}]$$

$$= \left|\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix}\right| + \left|\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{pmatrix}\right|$$

$$= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \text{(QED)}$$

Problem 4-2

Prove the triple scalar product identity $\mathbf{A}\cdot(\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B})\cdot \mathbf{C}$.

Solution:

As shown in the figure

$$Area = B(C\sin(\theta)) = |\mathbf{B} \times \mathbf{C}|$$

Thus,

Volume of parallelopiped is $|B \times C|\ |h|$

But,

$$|h| = |\mathbf{A}\cdot\mathbf{u}_{B\times C}| = \left|\mathbf{A}\cdot\left(\frac{\mathbf{B} \times \mathbf{C}}{|\mathbf{B} \times \mathbf{C}|}\right)\right|$$

Thus,

$$Volume = |\mathbf{A}\cdot(\mathbf{B} \times \mathbf{C})|$$

Since $|\mathbf{A} \times \mathbf{B}\cdot\mathbf{C}|$ represents this same volume then

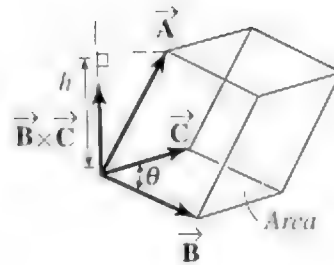
$$\mathbf{A}\cdot(\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B})\cdot\mathbf{C} \qquad \text{(QED)}$$

Also,

$$LHS = \mathbf{A}\cdot(\mathbf{B} \times \mathbf{C})$$

$$= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\left|\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{pmatrix}\right|$$

$$= A_x(B_yC_z - B_zC_y) - A_y(B_xC_z - B_zC_x) + A_z(B_xC_y - B_yC_x)$$

$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

$$RHS = (\mathbf{A} \times \mathbf{B})\cdot\mathbf{C}$$

$$= \left| \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix} \right| \left(C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} \right)$$

$$= C_x\left(A_y B_z - A_z B_y\right) - C_y\left(A_x B_z - A_z B_x\right) + C_z\left(A_x B_y - A_y B_x\right)$$

$$= A_x B_y C_z - A_x B_z C_y - A_y B_x C_z + A_y B_z C_x + A_z B_x C_y - A_z B_y C_x$$

Thus, $LHS = RHS$

$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C}$ (QED)

Problem 4-3

Given the three nonzero vectors **A**, **B**, and **C**, show that if $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = 0$, the three vectors *must* lie in the same plane.

Solution:

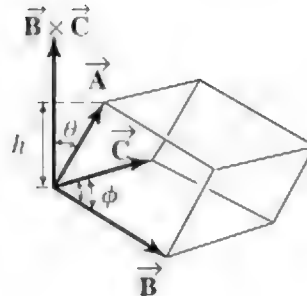
Consider,

$$\left|\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})\right| = |\mathbf{A}|\ |\mathbf{B}\times\mathbf{C}|\cos(\theta)$$

$$= \left(|\mathbf{A}|\cos(\theta)\right)|\mathbf{B}\times\mathbf{C}|$$

$$= |h|\ |\mathbf{B}\times\mathbf{C}|$$

$$= BC\ |h|\sin(\phi)$$

$=$ volume of parallelepiped.

If $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = 0$, then the volume equals zero, so that **A**, **B**, and **C** are coplanar.

Problem 4-4

Determine the magnitude and directional sense of the resultant moment of the forces at A and B about point O.

Given:

$F_1 = 40$ lb

$F_2 = 60$ lb

$\theta_1 = 30$ deg

$\theta_2 = 45$ deg

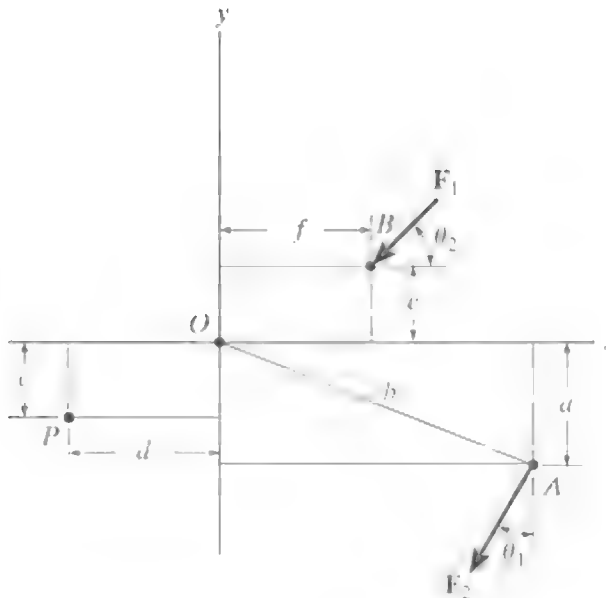
$a = 5$ in

$b = 13$ in

$c = 3$ in

$d = 6$ in

$e = 3$ in

$f = 6$ in

Solution:

$$M_{RO} = \Sigma M_O; \quad M_{RO} = F_1 \cos(\theta_2)e - F_1 \sin(\theta_2)f - F_2 \cos(\theta_1)\sqrt{b^2 - a^2} - F_2 \sin(\theta_1)a$$

$$M_{RO} = -858 \text{ lb}\cdot\text{in} \qquad |M_{RO}| = 858 \text{ lb}\cdot\text{in}$$

Problem 4-5

Determine the magnitude and directional sense of the resultant moment of the forces at A and B about point P.

Units Used:

kip = 1000 lb

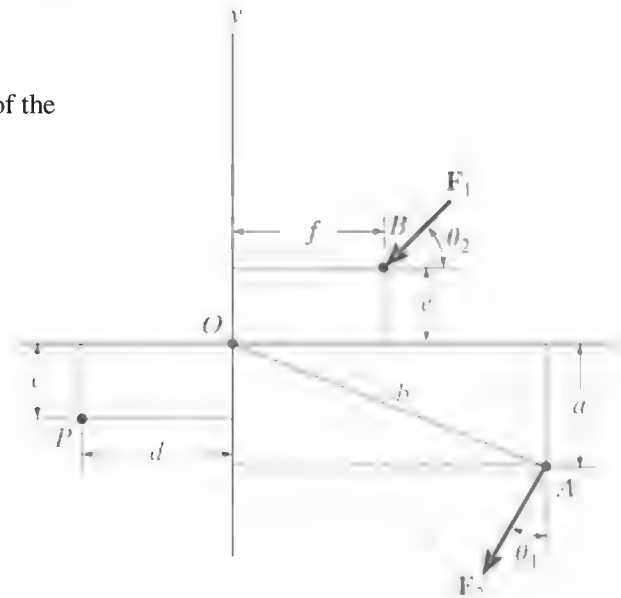
Given:

$F_1 = 40$ lb $\quad b = 13$ in

$F_2 = 60$ lb $\quad c = 3$ in

$\theta_1 = 30$ deg $\quad d = 6$ in

$\theta_2 = 45$ deg $\quad e = 3$ in

$a = 5$ in $\quad f = 6$ in

Solution:

$$M_{RP} = \Sigma M_P; \quad M_{RP} = F_1 \cos(\theta_2)(e + c) - F_1 \sin(\theta_2)(d + f) - F_2 \cos(\theta_1)\left(\sqrt{b^2 - a^2} + d\right) \ldots$$
$$+ -F_2 \sin(\theta_1)(a - c)$$

$$M_{RP} = -1165\,\text{lb}\cdot\text{in} \qquad |M_{RP}| = 1.17\,\text{kip}\cdot\text{in}$$

Problem 4-6

Determine the magnitude of the force **F** that should be applied at the end of the lever such that this force creates a clockwise moment *M* about point *O*.

Given:

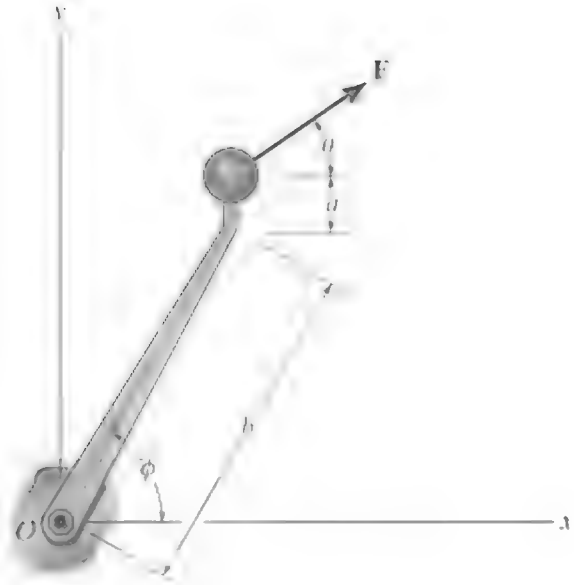
$M = 15\ \text{N m}$

$\phi = 60\ \text{deg}$

$\theta = 30\ \text{deg}$

$a = 50\ \text{mm}$

$b = 300\ \text{mm}$

Solution:

$$\curvearrowleft+ \quad M = F\cos(\theta)(a + b\sin(\phi)) - F\sin(\theta)(b\cos(\phi))$$

$$F = \frac{M}{\cos(\theta)(a + b\sin(\phi)) - \sin(\theta)(b\cos(\phi))} \qquad F = 77.6\ \text{N}$$

Problem 4-7

Determine the angle θ ($0 <= \theta <= 90$ deg) so that the force **F** develops a clockwise moment *M* about point *O*.

Given:

$F = 100\ \text{N} \qquad \phi = 60\ \text{deg}$

$M = 20\ \text{N}\cdot\text{m}$ $a = 50\ \text{mm}$

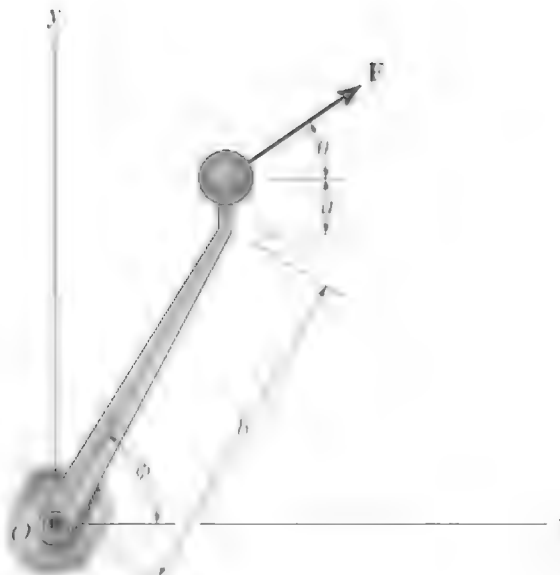
$\theta = 30\ \text{deg}$ $b = 300\ \text{mm}$

Solution:

Initial Guess $\theta = 30\ \text{deg}$

Given

$$M = F\cos(\theta)(a + b\sin(\phi)) - F\sin(\theta)(b\cos(\phi))$$

$\theta = \text{Find}(\theta)$ $\theta = 28.6\ \text{deg}$

Problem 4-8

Determine the magnitude and directional sense of the moment of the forces about point O.

Units Used:

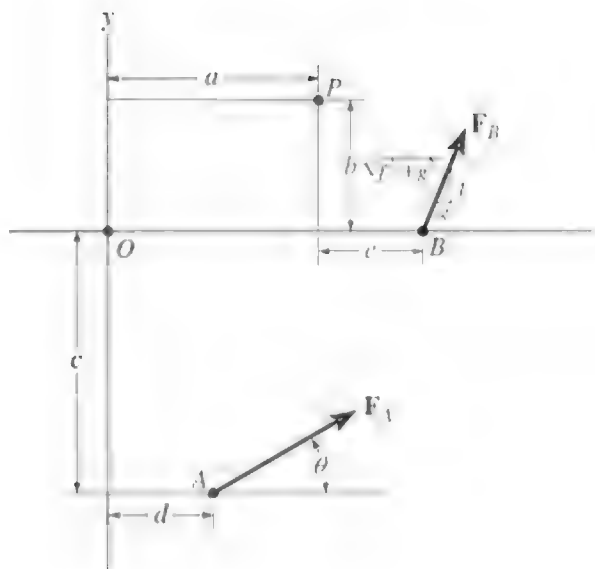
$\text{kN} = 10^3\ \text{N}$

Given:

$F_B = 260\ \text{N}$ $e = 2\ \text{m}$

$a = 4\ \text{m}$ $f = 12$

$b = 3\ \text{m}$ $g = 5$

$c = 5\ \text{m}$ $\theta = 30\ \text{deg}$

$d = 2\ \text{m}$ $F_A = 400\ \text{N}$

Solution:

$$M_O = F_A\sin(\theta)d + F_A\cos(\theta)c + F_B\frac{f}{\sqrt{f^2 + g^2}}(a + e)$$

$M_O = 3.57\ \text{kN}\cdot\text{m}$ (positive means counterclockwise)

Problem 4-9

Determine the magnitude and directional sense of the moment of the forces about point P.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

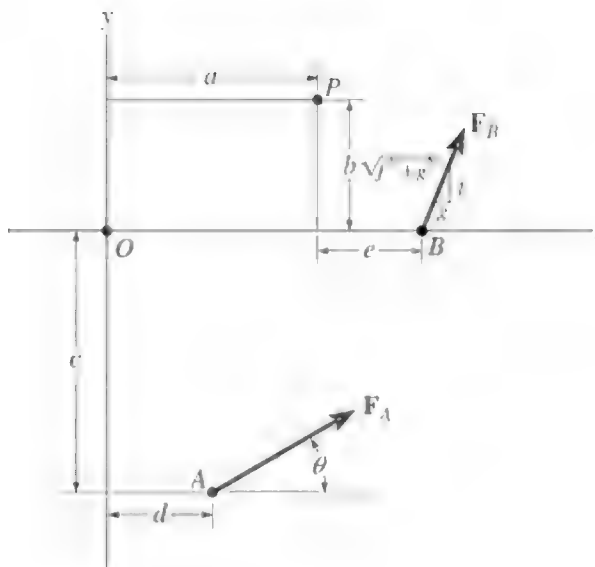
$F_B = 260\ \text{N}$ $\quad e = 2\ \text{m}$

$a = 4\ \text{m}$ $\quad f = 12$

$b = 3\ \text{m}$ $\quad g = 5$

$c = 5\ \text{m}$ $\quad \theta = 30\ \text{deg}$

$d = 2\ \text{m}$

$F_A = 400\ \text{N}$

Solution:

$$M_P = F_B \frac{g}{\sqrt{f^2+g^2}} b + F_B \frac{f}{\sqrt{f^2+g^2}} e - F_A \sin(\theta)(a-d) + F_A \cos(\theta)(b+c)$$

$M_P = 3.15\ \text{kN}\cdot\text{m}$ (positive means counterclockwise)

Problem 4-10

A force **F** is applied to the wrench. Determine the moment of this force about point O. Solve the problem using both a scalar analysis and a vector analysis.

Given:

$F = 40\ \text{N}$

$\theta = 20\ \text{deg}$

$a = 30\ \text{mm}$

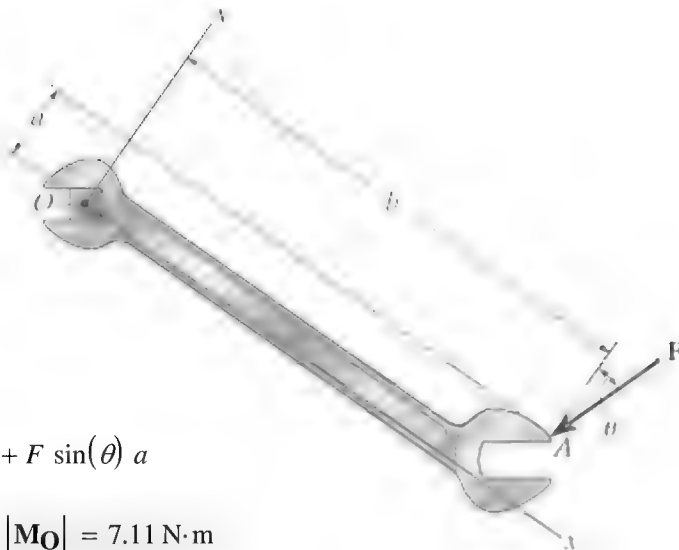
$b = 200\ \text{mm}$

Scalar Solution

$$+\ M_O = -F\cos(\theta)\,b + F\sin(\theta)\,a$$

$M_O = -7.11\ \text{N}\cdot\text{m}$ $\quad |M_O| = 7.11\ \text{N}\cdot\text{m}$

Vector Solution

$$M_O = \begin{pmatrix} b \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} -F\sin(\theta) \\ -F\cos(\theta) \\ 0 \end{pmatrix} \qquad M_O = \begin{pmatrix} 0 \\ 0 \\ -7.11 \end{pmatrix}\text{N}\cdot\text{m} \qquad |M_O| = 7.107\ \text{N}\cdot\text{m}$$

Problem 4-11

Determine the magnitude and directional sense of the resultant moment of the forces about point O.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

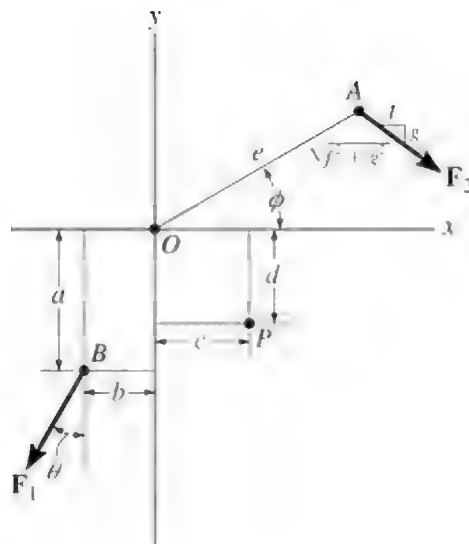
$F_1 = 300\ \text{lb}$ $\quad e = 10\ \text{ft}$

$F_2 = 250\ \text{lb}$ $\quad f = 4$

$a = 6\ \text{ft}$ $\quad g = 3$

$b = 3\ \text{ft}$ $\quad \theta = 30\ \text{deg}$

$c = 4\ \text{ft}$ $\quad \phi = 30\ \text{deg}$

$d = 4\ \text{ft}$

Solution:

$$M_O = F_2 \frac{f}{\sqrt{f^2 + g^2}} e \sin(\phi) + F_2 \frac{g}{\sqrt{f^2 + g^2}} e \cos(\phi) + F_1 \sin(\theta) a - F_1 \cos(\theta) b$$

$M_O = 2.42 \text{ kip}\cdot\text{ft}$ positive means clockwise

Problem 4-12

To correct a birth defect, the tibia of the leg is straightened using three wires that are attached through holes made in the bone and then to an external brace that is worn by the patient. Determine the moment of each wire force about joint A.

Given:

$F_1 = 4 \text{ N}$ $d = 0.15 \text{ m}$

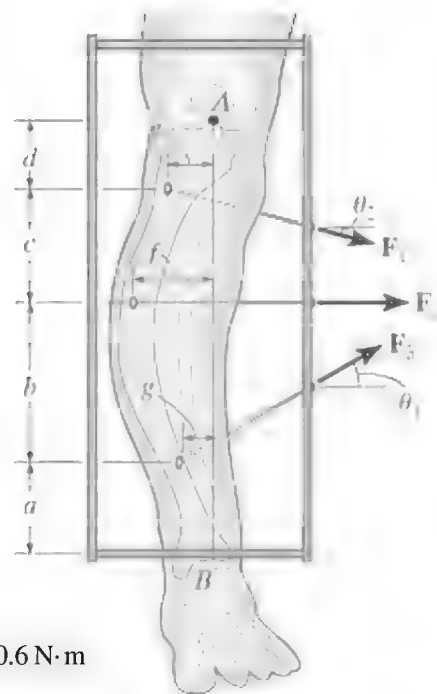
$F_2 = 8 \text{ N}$ $e = 20 \text{ mm}$

$F_3 = 6 \text{ N}$ $f = 35 \text{ mm}$

$a = 0.2 \text{ m}$ $g = 15 \text{ mm}$

$b = 0.35 \text{ m}$ $\theta_1 = 30 \text{ deg}$

$c = 0.25 \text{ m}$ $\theta_2 = 15 \text{ deg}$

Solution: Positive means counterclockwise

$M_{A1} = F_1 \cos(\theta_2) d + F_1 \sin(\theta_2) e$ $M_{A1} = 0.6 \text{ N}\cdot\text{m}$

$M_{A2} = F_2(c + d)$ $M_{A2} = 3.2 \text{ N}\cdot\text{m}$

$M_{A3} = F_3 \cos(\theta_1)(b + c + d) - F_3 \sin(\theta_1) g$ $M_{A3} = 3.852 \text{ N}\cdot\text{m}$

Problem 4-13

To correct a birth defect, the tibia of the leg is straightened using three wires that are attached through holes made in the bone and then to an external brace that is worn by the patient. Determine the moment of each wire force about joint B.

Given:

$F_1 = 4 \text{ N}$ $d = 0.15 \text{ m}$

$F_2 = 8\ \text{N}$ $e = 20\ \text{mm}$

$F_3 = 6\ \text{N}$ $f = 35\ \text{mm}$

$a = 0.2\ \text{m}$ $g = 15\ \text{mm}$

$b = 0.35\ \text{m}$ $\theta_1 = 30\ \text{deg}$

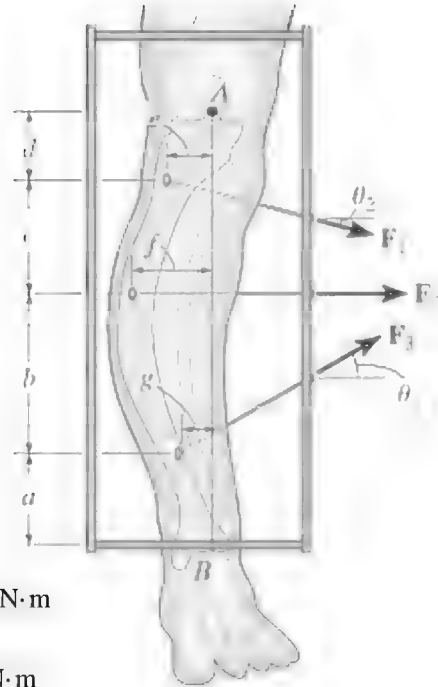
$c = 0.25\ \text{m}$ $\theta_2 = 15\ \text{deg}$

Solution:

Positive means clockwise

$$M_{B1} = F_1 \cos(\theta_2)(a + b + c) - F_1 \sin(\theta_2)e \qquad M_{B1} = 3.07\ \text{N}\cdot\text{m}$$

$$M_{B2} = F_2(a + b) \qquad M_{B2} = 4.4\ \text{N}\cdot\text{m}$$

$$M_{B3} = F_3 \cos(\theta_1)a + F_3 \sin(\theta_1)g \qquad M_{B3} = 1.084\ \text{N}\cdot\text{m}$$

Problem 4-14

Determine the moment of each force about the bolt located at A.

Given:

$F_B = 40\ \text{lb}$ $a = 2.5\ \text{ft}$ $\alpha = 20\ \text{deg}$ $\gamma = 30\ \text{deg}$

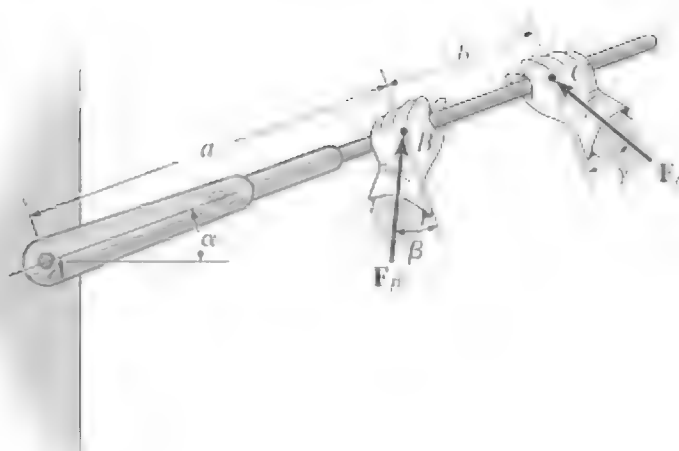
$F_C = 50\ \text{lb}$ $b = 0.75\ \text{ft}$ $\beta = 25\ \text{deg}$

Solution:

$$M_B = F_B \cos(\beta)a$$

$$M_B = 90.6 \text{ lb}\cdot\text{ft}$$

$$M_C = F_C \cos(\gamma)(a+b)$$

$$M_C = 141 \text{ lb}\cdot\text{ft}$$

Problem 4-15

Determine the resultant moment about the bolt located at A.

Given:

$F_B = 30$ lb

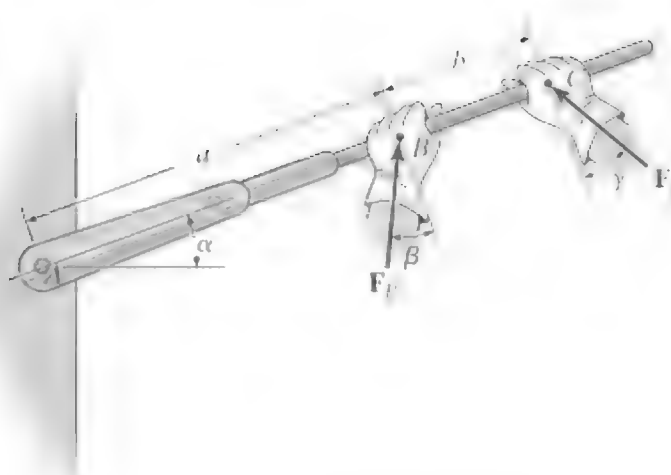
$F_C = 45$ lb

$a = 2.5$ ft

$b = 0.75$ ft

$\alpha = 20$ deg

$\beta = 25$ deg

$\gamma = 30$ deg

Solution:

$$M_A = F_B \cos(\beta)a + F_C \cos(\gamma)(a+b) \qquad M_A = 195 \text{ lb}\cdot\text{ft}$$

Problem 4-16

The elbow joint is flexed using the biceps brachii muscle, which remains essentially vertical as the arm moves in the vertical plane. If this muscle is located a distance a from the pivot point A on the humerus, determine the variation of the moment capacity about A if the constant force developed by the muscle is F. Plot these results of M vs. θ for $-60 \le \theta \le 80$.

Units Used:

$kN = 10^3 \ N$

Given:

$a = 16 \ mm$

$F = 2.30 \ kN$

$\theta = (-60 .. 80)$

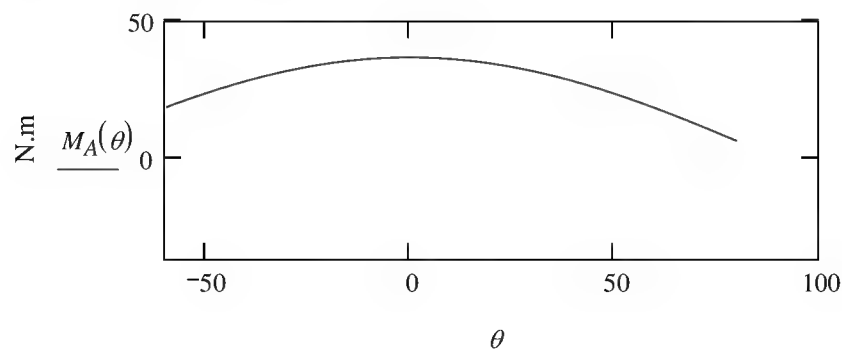
Solution:

$M_A(\theta) = F \ (a) \cos(\theta \ deg)$

50

N.m $\frac{M_A(\theta)}{}$ 0

−50 0 50 100

θ

Problem 4-17

The Snorkel Co.produces the articulating boom platform that can support weight W. If the boom is in the position shown, determine the moment of this force about points A, B, and C.

Units Used:

$kip = 10^3 lb$

Given:

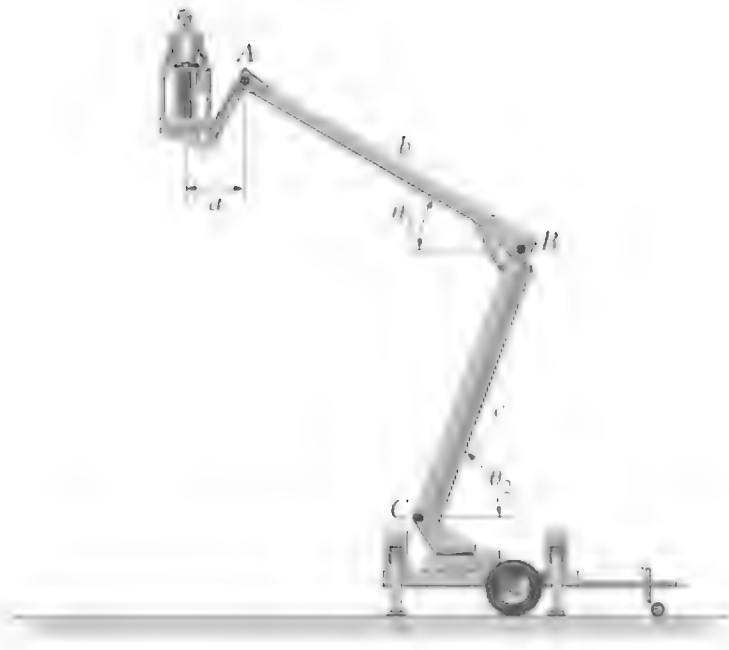
$a = 3\ \text{ft}$

$b = 16\ \text{ft}$

$c = 15\ \text{ft}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 70\ \text{deg}$

$W = 550\ \text{lb}$

Solution:

$M_A = W a$ $\quad M_A = 1.65\ \text{kip}\cdot\text{ft}$

$M_B = W\left(a + b\cos\left(\theta_1\right)\right)$ $\quad M_B = 9.27\ \text{kip}\cdot\text{ft}$

$M_C = W\left(a + b\cos\left(\theta_1\right) - c\cos\left(\theta_2\right)\right)$ $\quad M_C = 6.45\ \text{kip}\cdot\text{ft}$

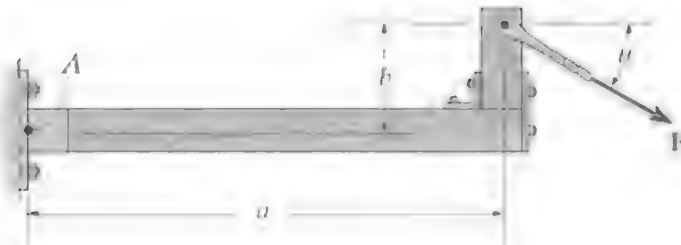
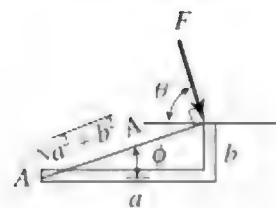
Problem 4-18

Determine the direction θ ($0° \le \theta \le 180°$) of the force **F** so that it produces **(a)** the maximum moment about point A and **(b)** the minimum moment about point A. Compute the moment in each case.

Given:

$F = 40\ \text{lb}$

$a = 8\ \text{ft}$

$b = 2\ \text{ft}$

Solution: The maximum occurs when the force is perpendicular to the line between A and the point of application of the force. The minimum occurs when the force is parallel to this line.

(a) $M_{Amax} = F\sqrt{a^2 + b^2}$ $\quad M_{Amax} = 329.848\ \text{lb}\cdot\text{ft}$

$\phi_a = \text{atan}\left(\frac{b}{a}\right)$ $\quad \phi_a = 14.04\ \text{deg}$

$\theta_a = 90\ \text{deg} - \phi_a$ $\quad \theta_a = 76.0\ \text{deg}$

(b) $M_{Amin} = 0\ \text{lb}\cdot\text{ft}$ $\quad M_{Amin} = 0\ \text{lb}\cdot\text{ft}$

$\phi_b = \text{atan}\left(\frac{b}{a}\right)$ $\quad \phi_b = 14.04\ \text{deg}$

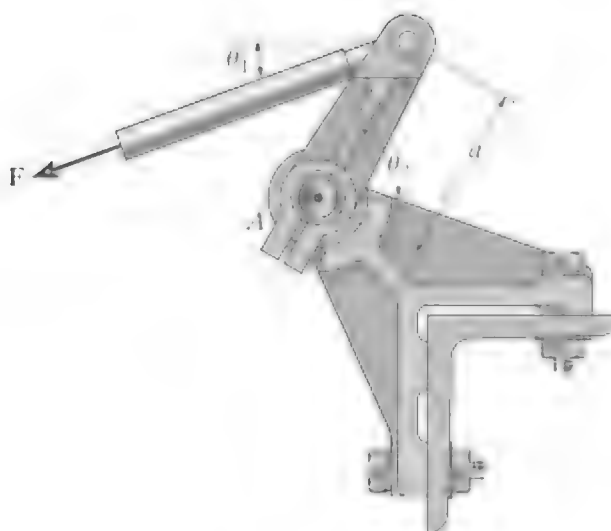
$\theta_b = 180\ \text{deg} - \phi_b$ $\quad \theta_b = 166\ \text{deg}$

Problem 4-19

The rod on the power control mechanism for a business jet is subjected to force **F**. Determine the moment of this force about the bearing at *A*.

Given:

$F = 80\ \text{N}$ $\quad \theta_1 = 20\ \text{deg}$

$a = 150\ \text{mm}$ $\quad \theta_2 = 60\ \text{deg}$

Solution:

$$M_A = F\cos(\theta_1)(a)\sin(\theta_2) - F\sin(\theta_1)(a)\cos(\theta_2) \qquad M_A = 7.71\ \text{N}\cdot\text{m}$$

Problem 4-20

The boom has length *L*, weight W_b, and mass center at *G*. If the maximum moment that can be developed by the motor at *A* is *M*, determine the maximum load *W*, having a mass center at *G'*, that can be lifted.

Given:

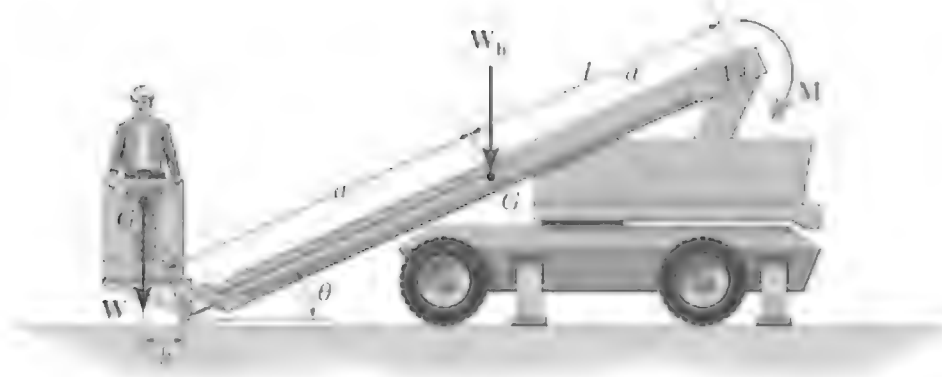
$L = 30 \text{ ft}$

$W_b = 800 \text{ lb}$

$a = 14 \text{ ft}$

$b = 2 \text{ ft}$

$\theta = 30 \text{ deg}$

$M = 20 \times 10^3 \text{ lb}\cdot\text{ft}$

Solution:

$$M = W_b (L - a)\cos(\theta) + W(L\cos(\theta) + b)$$

$$W = \frac{M - W_b (L - a)\cos(\theta)}{L\cos(\theta) + b} \qquad W = 319\,\text{lb}$$

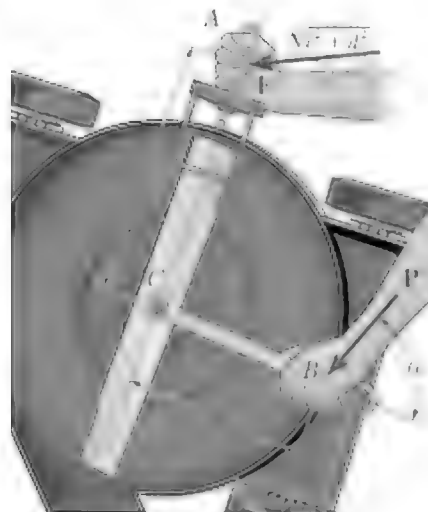
Problem 4-21

The tool at A is used to hold a power lawnmower blade stationary while the nut is being loosened with the wrench. If a force P is applied to the wrench at B in the direction shown, determine the moment it creates about the nut at C. What is the magnitude of force **F** at A so that it creates the opposite moment about C ?

Given:

$P = 50 \text{ N}$ $\quad b = 300 \text{ mm}$

$\theta = 60 \text{ deg}$ $\quad c = 5$

$a = 400 \text{ mm}$ $\quad d = 12$

Solution:

(a) $M_A = P\sin(\theta)b$

$M_A = 13.0\,\text{N}\cdot\text{m}$

(b) $M_A - F\dfrac{d}{\sqrt{c^2 + d^2}}a = 0$

$$F = M_A \left(\frac{\sqrt{c^2 + d^2}}{d\, a} \right)$$

$F = 35.2\ \text{N}$

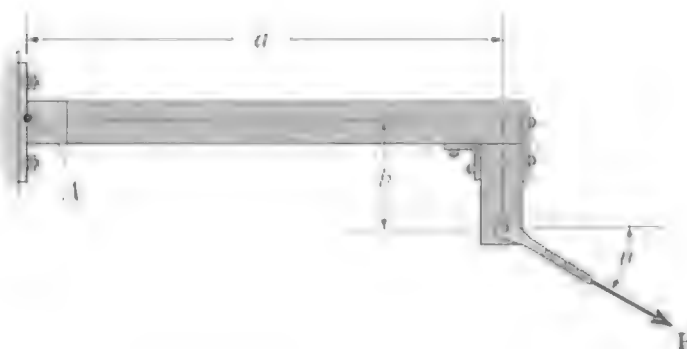
Problem 4-22

Determine the clockwise direction θ $(0\,\text{deg} \le \theta \le 180\,\text{deg})$ of the force **F** so that it produces (a) the maximum moment about point A and (b) no moment about point A. Compute the moment in each case.

Given:

$F = 80\ \text{lb}$

$a = 4\ \text{ft}$

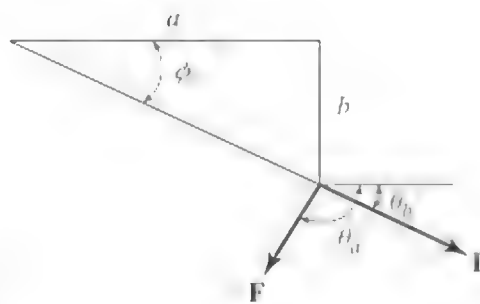
$b = 1\ \text{ft}$

Solution:

(a) $M_{Amax} = F\sqrt{a^2 + b^2}$ $M_{Amax} = 330\,\text{lb}\cdot\text{ft}$

$\phi = \text{atan}\left(\frac{b}{a}\right)$ $\phi = 14.0\,\text{deg}$

$\theta_a = 90\ \text{deg} + \phi$ $\theta_a = 104\,\text{deg}$

(b) $M_{Amin} = 0$

$\theta_b = \text{atan}\left(\frac{b}{a}\right)$ $\theta_b = 14.04\,\text{deg}$

Problem 4-23

The Y-type structure is used to support the high voltage transmission cables. If the supporting cables each exert a force F on the structure at B, determine the moment of each force about point A. Also, by the principle of transmissibility, locate the forces at points C and D and determine the moments.

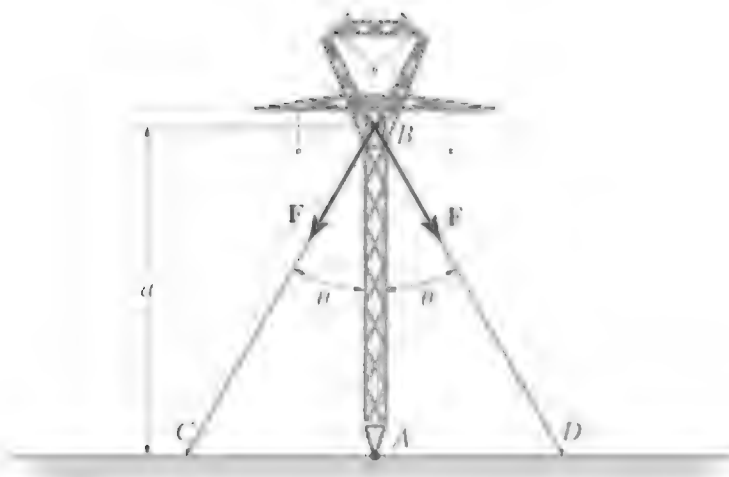
Units Used:

$kip = 1000\ lb$

Given:

$F = 275\ lb$

$a = 85\ ft$

$\theta = 30\ deg$

Solution:

$M_{A1} = F\ \sin(\theta)\ a \qquad M_{A1} = 11.7\,kip \cdot ft$

$M_{A2} = F \sin(\theta) a \qquad M_{A2} = 11.7\,kip \cdot ft$

Also $\quad b = (a)\tan(\theta)$

$M_{A1} = F\cos(\theta) b \qquad M_{A1} = 11.7\,kip \cdot ft$

$M_{A2} = F\cos(\theta) b \qquad M_{A2} = 11.7\,kip \cdot ft$

Problem 4-24

The force **F** acts on the end of the pipe at B. Determine (a) the moment of this force about point A, and (b) the magnitude and direction of a horizantal force, applied at C, which produces the same moment.

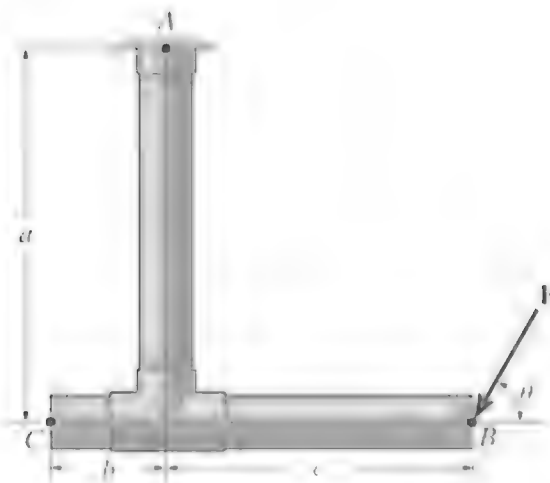
Given:

$F = 70\ N$

$a = 0.9\ m$

$b = 0.3\ m$

$c = 0.7\ m$

$\theta = 60\ deg$

Solution:

(a) $\quad M_A = F\ \sin(\theta)\ c + F\ \cos(\theta)\ a \qquad M_A = 73.9\,N \cdot m$

(b) $F_C\,(a) = M_A \qquad F_C = \frac{M_A}{a}$ $\qquad F_C = 82.2\ \text{N}$

Problem 4-25

The force F acts on the end of the pipe at B. Determine the angles θ ($0° \le \theta \le 180°$) of the force that will produce maximum and minimum moments about point A. What are the magnitudes of these moments?

Given:

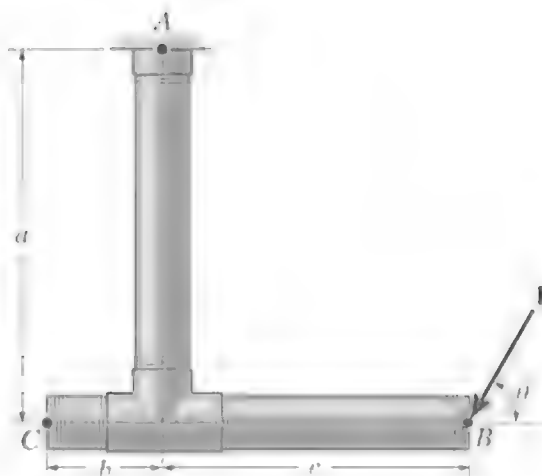
$F = 70\ \text{N}$

$a = 0.9\ \text{m}$

$b = 0.3\ \text{m}$

$c = 0.7\ \text{m}$

Solution:

$$M_A = F\sin(\theta)c + F\cos(\theta)a$$

For maximum moment $\frac{d}{d\theta}M_A = cF\cos(\theta) - aF\sin(\theta) = 0$

$$\theta_{max} = \operatorname{atan}\left(\frac{c}{a}\right) \qquad \theta_{max} = 37.9\ \text{deg}$$

$$M_{Amax} = F\sin(\theta_{max})c + F\cos(\theta_{max})a \qquad M_{Amax} = 79.812\ \text{N}\cdot\text{m}$$

For minimum moment $M_A = F\sin(\theta)c + F\cos(\theta)a = 0$

$$\theta_{min} = 180\ \text{deg} + \operatorname{atan}\left(\frac{-a}{c}\right) \qquad \theta_{min} = 128\ \text{deg}$$

$$M_{Amin} = F\,c\,\sin(\theta_{min}) + F\,(a)\,\cos(\theta_{min}) \qquad M_{Amin} = 0\ \text{N}\cdot\text{m}$$

Problem 4-26

The towline exerts force P at the end of the crane boom of length L. Determine the placement

x of the hook at A so that this force creates a maximum moment about point O. What is this moment?

Unit Used:

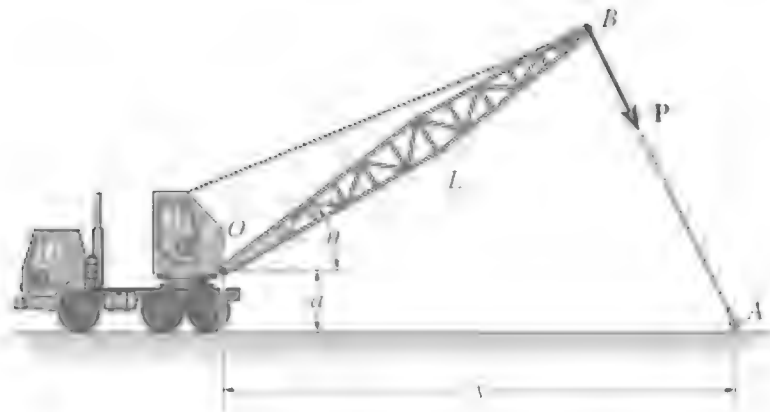
$$kN = 10^3\ N$$

Given:

$$P = 4\ kN$$

$$L = 20\ m$$

$$\theta = 30\ deg$$

$$a = 1.5\ m$$

Solution:

Maximum moment, $OB \perp BA$

Guesses $x = 1\ m$ $d = 1\ m$ (Length of the cable from B to A)

Given $L\cos(\theta) + d\sin(\theta) = x$

$$a + L\sin(\theta) = d\cos(\theta)$$

$$\begin{pmatrix} x \\ d \end{pmatrix} = \text{Find}(x, d) \qquad x = 23.96\ m$$

$$M_{max} = P\ L \qquad M_{max} = 80\,kN\cdot m$$

Problem 4-27

The towline exerts force P at the end of the crane boom of length L. Determine the position θ of the boom so that this force creates a maximum moment about point O. What is this moment?

Units Used:

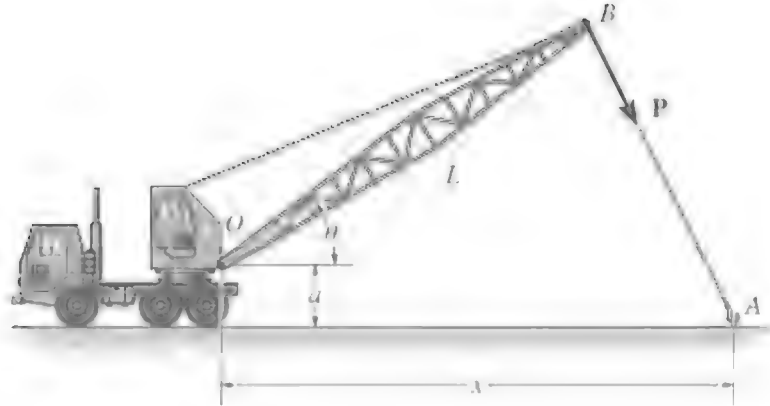
$$kN = 10^3\ N$$

Given:

$P = 4\ \text{kN}$

$x = 25\ \text{m}$

$L = 20\ \text{m}$

$a = 1.5\ \text{m}$

Solution:

Maximum moment, $OB \perp BA$

Guesses $\theta = 30\ \text{deg}$ $d = 1\ \text{m}$ (length of cable from B to A)

Given $L\cos(\theta) + d\sin(\theta) = x$

$a + L\sin(\theta) = d\cos(\theta)$

$\begin{pmatrix} \theta \\ d \end{pmatrix} = \text{Find}(\theta, d)$ $\theta = 33.573\ \text{deg}$

$M_{max} = P\,L$ $M_{max} = 80\ \text{kN}\cdot\text{m}$

Problem 4-28

Determine the resultant moment of the forces about point A. Solve the problem first by considering each force as a whole, and then by using the principle of moments.

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 250\ N \qquad a = 2\ m$

$F_2 = 300\ N \qquad b = 3\ m$

$F_3 = 500\ N \qquad c = 4\ m$

$\theta_1 = 60\ deg \qquad d = 3$

$\theta_2 = 30\ deg \qquad e = 4$

Solution Using Whole Forces:

Geometry $\quad \alpha = \operatorname{atan}\left(\frac{d}{e}\right) \qquad L = \left(a + b - \frac{d}{e}c\right)\frac{e}{\sqrt{e^2 + d^2}}$

$$M_A = -F_1\left[(a)\cos(\theta_2)\right] - F_2(a+b)\sin(\theta_1) - F_3 L \qquad M_A = -2.532\ \text{kN}\cdot\text{m}$$

Solution Using Principle of Moments:

$$M_A = -F_1\cos(\theta_2)a - F_2\sin(\theta_1)(a+b) + F_3\frac{d}{\sqrt{d^2+e^2}}c - F_3\frac{e}{\sqrt{d^2+e^2}}(a+b)$$

$$M_A = -2.532 \times 10^3\ \text{N}\cdot\text{m}$$

Problem 4-29

If the resultant moment about point A is M clockwise, determine the magnitude of $\mathbf{F}_3$.

Units Used:

$kN = 10^3\ N$

Given:

$M = 4.8\ kN\cdot m$ $a = 2\ m$

$F_1 = 300\ N$ $b = 3\ m$

$F_2 = 400\ N$ $c = 4\ m$

$\theta_1 = 60\ deg$ $d = 3$

$\theta_2 = 30\ deg$ $e = 4$

Solution:

Initial Guess $F_3 = 1\ N$

Given

$$-M = -F_1 \cos(\theta_2) a - F_2 \sin(\theta_1)(a+b) + F_3 \left(\frac{d}{\sqrt{d^2+e^2}}\right) c - F_3 \left(\frac{e}{\sqrt{d^2+e^2}}\right)(a+b)$$

$F_3 = \text{Find}(F_3)$ $F_3 = 1.593\ kN$

Problem 4-30

The flat-belt tensioner is manufactured by the Daton Co. and is used with V-belt drives on poultry and livestock fans. If the tension in the belt is **F**, when the pulley is not turning, determine the moment of each of these forces about the pin at A.

Given:

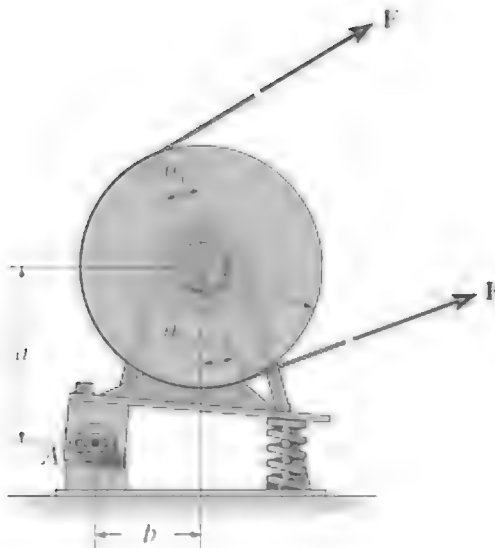
$F = 52$ lb

$a = 8$ in

$b = 5$ in

$c = 6$ in

$\theta_1 = 30$ deg

$\theta_2 = 20$ deg

Solution:

$$M_{A1} = F\cos(\theta_1)(a + c\cos(\theta_1)) - F\sin(\theta_1)(b - c\sin(\theta_1))$$

$$M_{A1} = 542\,\text{lb}\cdot\text{in}$$

$$M_{A2} = F\cos(\theta_2)(a - c\cos(\theta_2)) - F(\sin(\theta_2))(b + c\sin(\theta_2))$$

$$M_{A2} = -10.01\,\text{lb}\cdot\text{in}$$

Problem 4-31

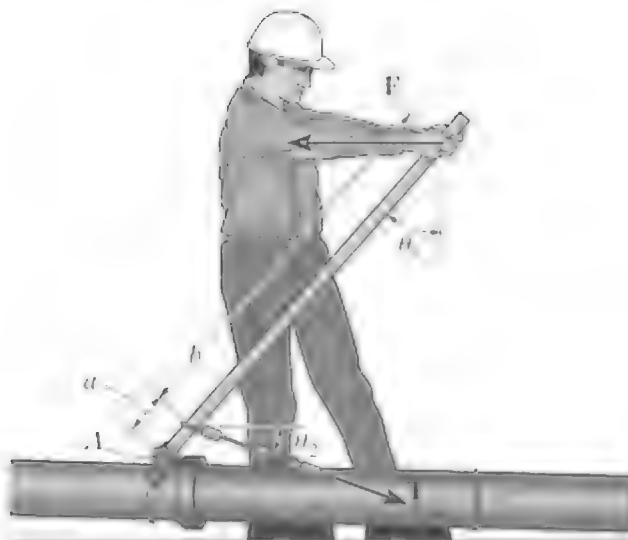
The worker is using the bar to pull two pipes together in order to complete the connection. If he applies a horizontal force **F** to the handle of the lever, determine the moment of this force about the end A. What would be the tension T in the cable needed to cause the opposite moment about point A.

Given:

$F = 80$ lb $\quad \theta_1 = 40$ deg $\quad \theta_2 = 20$ deg $\quad a = 0.5$ ft $\quad b = 4.5$ ft

Solution:

$$M_A = F(a+b)\cos(\theta_1)$$

$$M_A = 306\ \text{lb}\cdot\text{ft}$$

Require $\quad M_A = T\cos(\theta_2)(a)\cos(\theta_1) + T\sin(\theta_2)(a)\sin(\theta_1)$

$$T = \frac{M_A}{(a)(\cos(\theta_2)\cos(\theta_1) + \sin(\theta_2)\sin(\theta_1))} \qquad T = 652\ \text{lb}$$

Problem 4-32

If it takes a force **F** to pull the nail out, determine the smallest vertical force **P** that must be applied to the handle of the crowbar. *Hint:* This requires the moment of **F** about point A to be equal to the moment of **P** about A. Why?

Given:

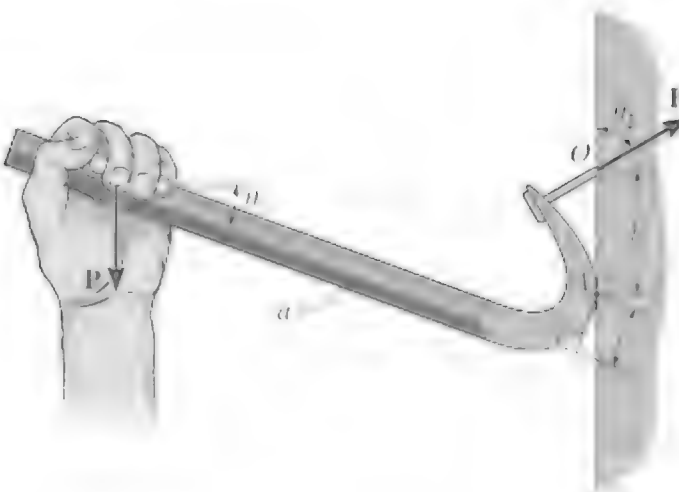
$F = 125$ lb

$a = 14$ in

$b = 3$ in

$c = 1.5$ in

$\theta_1 = 20$ deg

$\theta_2 = 60$ deg

Solution:

$$\curvearrowleft + \quad M_F = F\sin(\theta_2)(b) \qquad M_F = 325\,\text{lb}\cdot\text{in}$$

$$\curvearrowleft + \quad P\left[(a)\cos(\theta_1) + (c)\sin(\theta_1)\right] = M_F \qquad P = \frac{M_F}{(a)\,\cos(\theta_1) + (c)\,\sin(\theta_1)} \qquad P = 23.8\,\text{lb}$$

Problem 4-33

The pipe wrench is activated by pulling on the cable segment with a horizontal force **F**. Determine the moment M_A produced by the wrench on the pipe at θ. Neglect the size of the pulley.

Given:

$F = 500$ N

$a = 0.2$ m

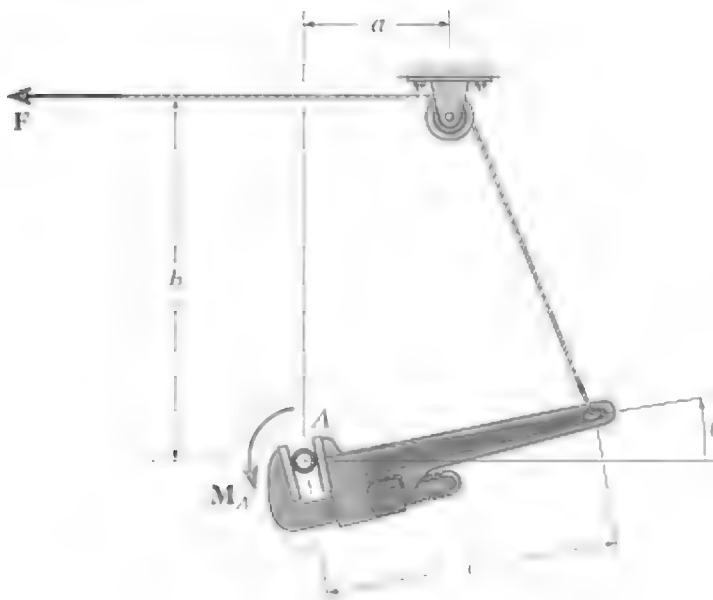
$b = 0.5$ m

$c = 0.4$ m

$\theta = 20$ deg

Solution:

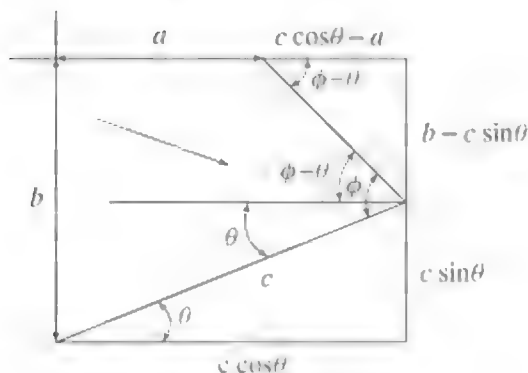
Initial Guesses

$\phi = 20$ deg

$M_A = 1$ N·m

Given

$$\frac{b - c\sin(\theta)}{c\cos(\theta) - a} = \tan(\phi - \theta)$$

$$M_A = F c \sin(\phi)$$

$$\begin{pmatrix} \phi \\ M_A \end{pmatrix} = \text{Find}(\phi, M_A) \qquad \phi = 84.161\text{ deg} \qquad M_A = 199\,\text{N}\cdot\text{m}$$

Problem 4-34

Determine the moment of the force at A about point O. Express the result as a Cartesian vector.

Given:

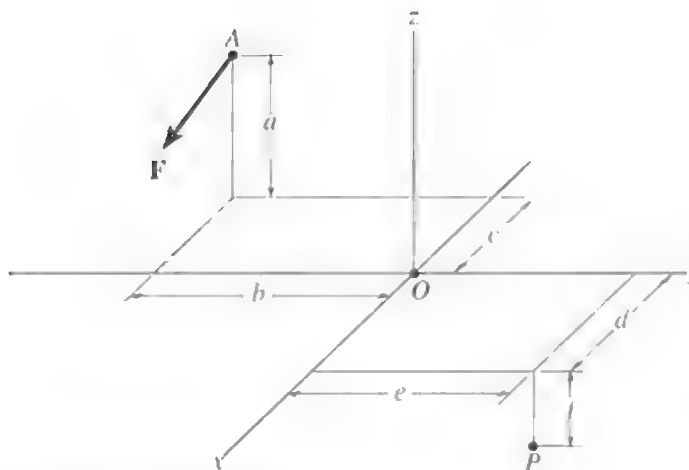
$$\mathbf{F} = \begin{pmatrix} 60 \\ -30 \\ -20 \end{pmatrix} \text{N}$$

$a = 4$ m $d = 4$ m

$b = 7$ m $e = 6$ m

$c = 3$ m $f = 2$ m

Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -c \\ -b \\ a \end{pmatrix} \qquad \mathbf{M_O} = \mathbf{r_{OA}} \times \mathbf{F} \qquad \mathbf{M_O} = \begin{pmatrix} 260 \\ 180 \\ 510 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-35

Determine the moment of the force at A about point P. Express the result as a Cartesian vector.

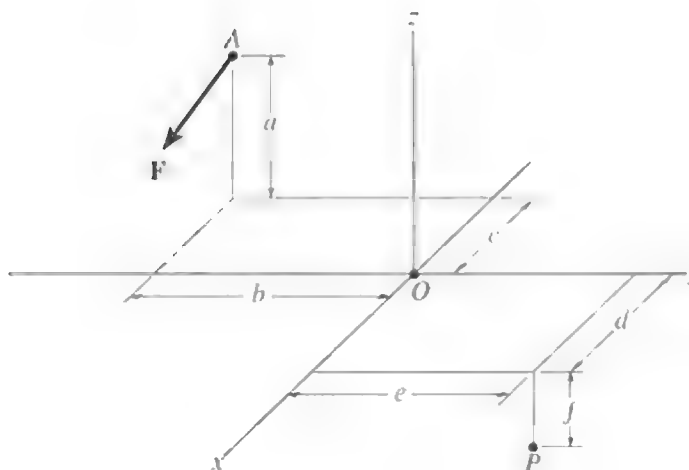
Given:

$a = 4$ m $b = 7$ m $c = 3$ m $d = 4$ m $e = 6$ m $f = 2$ m

$$\mathbf{F} = \begin{pmatrix} 60 \\ -30 \\ -20 \end{pmatrix} \text{N}$$

Solution:

$$\mathbf{r_{PA}} = \begin{pmatrix} -c-d \\ -b-e \\ a+f \end{pmatrix} \qquad \mathbf{M_P} = \mathbf{r_{PA}} \times \mathbf{F}$$

$$\mathbf{M_P} = \begin{pmatrix} 440 \\ 220 \\ 990 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-36

Determine the moment of the force **F** at A about point O. Express the result as a cartesian vector.

Units Used:

$kN = 10^3\ N$

Given:

$F = 13\ kN$

$a = 6\ m$

$b = 2.5\ m$

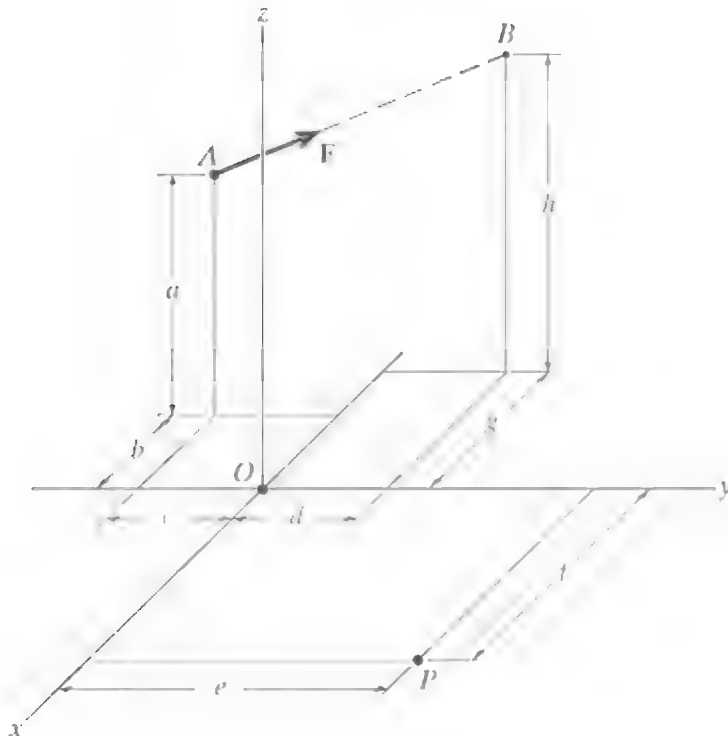
$c = 3\ m$

$d = 3\ m$

$e = 8\ m$

$f = 6\ m$

$g = 4\ m$

$h = 8\ m$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} b-g \\ c+d \\ h-a \end{pmatrix} \qquad \mathbf{r_{OA}} = \begin{pmatrix} -b \\ -c \\ a \end{pmatrix} \qquad \mathbf{F_1} = F \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|}$$

$$\mathbf{M_O} = \mathbf{r_{OA}} \times \mathbf{F_1} \qquad \mathbf{M_O} = \begin{pmatrix} -84 \\ -8 \\ -39 \end{pmatrix} \mathrm{kN \cdot m}$$

Problem 4-37

Determine the moment of the force **F** at A about point P. Express the result as a Cartesian vector.

Units Used: $kN = 10^3 N$

Given:

$F = 13\ \text{kN}$

$a = 6\ \text{m}$

$b = 2.5\ \text{m}$

$c = 3\ \text{m}$

$d = 3\ \text{m}$

$e = 8\ \text{m}$

$f = 6\ \text{m}$

$g = 4\ \text{m}$

$h = 8\ \text{m}$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} b - g \\ c + d \\ h - a \end{pmatrix} \qquad \mathbf{r_{PA}} = \begin{pmatrix} -b - f \\ -c - e \\ a \end{pmatrix} \qquad \mathbf{F_1} = F \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|}$$

$$\mathbf{M_O} = \mathbf{r_{PA}} \times \mathbf{F_1} \qquad \mathbf{M_O} = \begin{pmatrix} -116 \\ 16 \\ -135 \end{pmatrix} \text{kN} \cdot \text{m}$$

Problem 4-38

The curved rod lies in the *x-y* plane and has radius *r*. If a force **F** acts at its end as shown, determine the moment of this force about point *O*.

Given:

$r = 3\ \text{m}$ $a = 1\ \text{m}$ $\theta = 45\ \text{deg}$

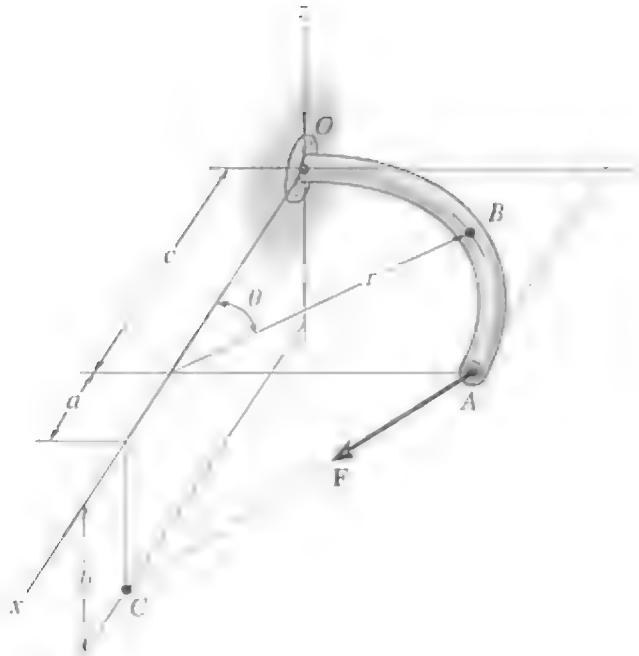
$F = 80\ \text{N}$ $b = 2\ \text{m}$

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} a \\ -r \\ -b \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} \text{m}$$

$$\mathbf{F_v} = F\frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad \mathbf{F_v} = \begin{pmatrix} 21.381 \\ -64.143 \\ -42.762 \end{pmatrix} \text{N}$$

$$\mathbf{r_{OA}} = \begin{pmatrix} r \\ r \\ 0 \end{pmatrix} \qquad \mathbf{r_{OA}} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \text{m}$$

$$\mathbf{M_O} = \mathbf{r_{OA}} \times \mathbf{F_v} \qquad \mathbf{M_O} = \begin{pmatrix} -128.285 \\ 128.285 \\ -256.571 \end{pmatrix} \text{N} \cdot \text{m}$$

Problem 4-39

The curved rod lies in the x-y plane and has a radius r. If a force **F** acts at its end as shown, determine the moment of this force about point B.

Given:

$F = 80 \text{ N}$ $\quad c = 3 \text{ m}$

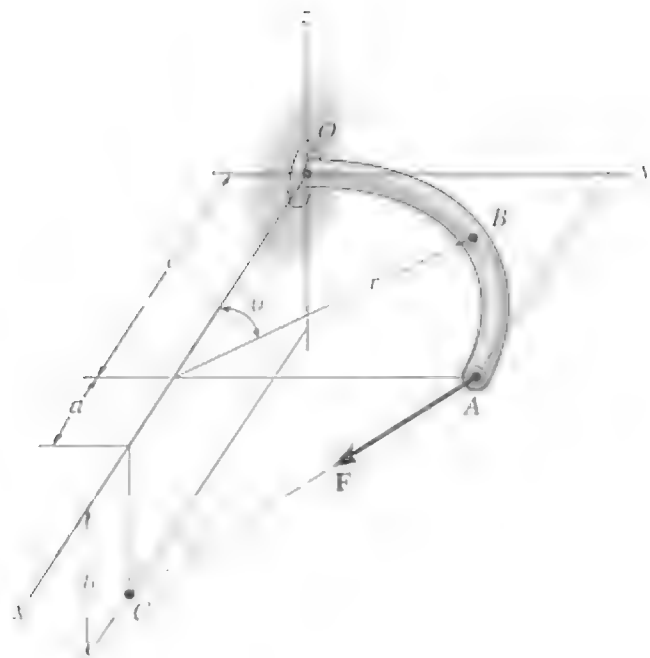
$a = 1 \text{ m}$ $\quad r = 3 \text{ m}$

$b = 2\ \text{m} \qquad \theta = 45\ \text{deg}$

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} a \\ -c \\ -b \end{pmatrix}$$

$$\mathbf{F_v} = F\frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad \mathbf{r_{BA}} = \begin{pmatrix} r\cos(\theta) \\ r - r\sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{M_B} = \mathbf{r_{BA}} \times \mathbf{F_v} \qquad \mathbf{M_B} = \begin{pmatrix} -37.6 \\ 90.7 \\ -154.9 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-40

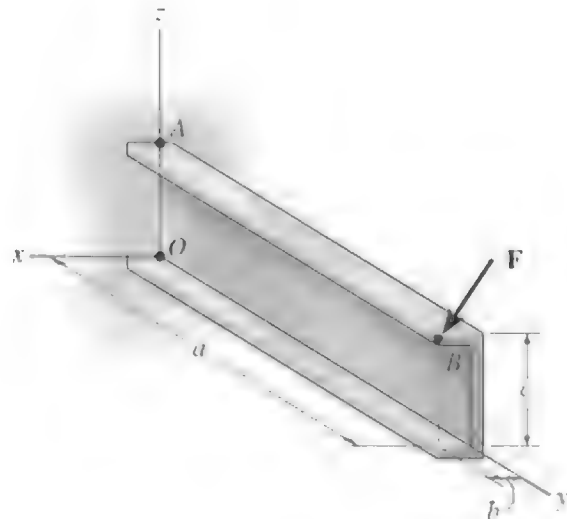
The force **F** acts at the end of the beam. Determine the moment of the force about point A.

Given:

$$\mathbf{F} = \begin{pmatrix} 600 \\ 300 \\ -600 \end{pmatrix} \text{N}$$

$a = 1.2\ \text{m}$

$b = 0.2\ \text{m}$

$c = 0.4\ \text{m}$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} b \\ a \\ 0 \end{pmatrix} \qquad \mathbf{M_A} = \mathbf{r_{AB}} \times \mathbf{F} \qquad \mathbf{M_A} = \begin{pmatrix} -720 \\ 120 \\ -660 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-41

The pole supports a traffic light of weight W. Using Cartesian vectors, determine the moment of the weight of the traffic light about the base of the pole at A.

Given:

$W = 22\ \text{lb} \qquad a = 12\ \text{ft} \qquad \theta = 30\ \text{deg}$

Solution:

$$\mathbf{r} = \begin{bmatrix} (a)\sin(\theta) \\ (a)\cos(\theta) \\ 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ -W \end{pmatrix}$$

$$\mathbf{M_A} = \mathbf{r} \times \mathbf{F}$$

$$\mathbf{M_A} = \begin{pmatrix} -229 \\ 132 \\ 0 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 4-42

The man pulls on the rope with a force **F**. Determine the moment that this force exerts about the base of the pole at O. Solve the problem two ways, i.e., by using a position vector from O to A, then O to B.

Given:

$F = 20\ \text{N}$

$a = 3\ \text{m}$

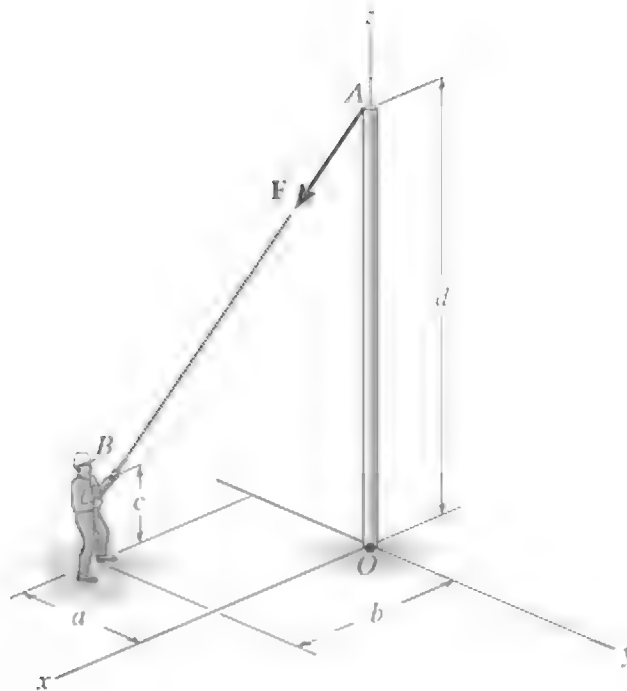
$b = 4 \text{ m}$

$c = 1.5 \text{ m}$

$d = 10.5 \text{ m}$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} b \\ -a \\ c-d \end{pmatrix} \qquad \mathbf{r_{OA}} = \begin{pmatrix} 0 \\ 0 \\ d \end{pmatrix}$$

$$\mathbf{r_{OB}} = \begin{pmatrix} b \\ -a \\ c \end{pmatrix} \qquad \mathbf{F_V} = F\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|}$$

$$\mathbf{M_{O1}} = \mathbf{r_{OA}} \times \mathbf{F_V} \qquad \mathbf{M_{O1}} = \begin{pmatrix} 61.2 \\ 81.6 \\ 0 \end{pmatrix} \text{N·m}$$

$$\mathbf{M_{O2}} = \mathbf{r_{OB}} \times \mathbf{F_V} \qquad \mathbf{M_{O2}} = \begin{pmatrix} 61.2 \\ 81.6 \\ -0 \end{pmatrix} \text{N·m}$$

Problem 4-43

Determine the smallest force F that must be applied along the rope in order to cause the curved rod, which has radius r, to fail at the support C. This requires a moment to be developed at C of magnitude M.

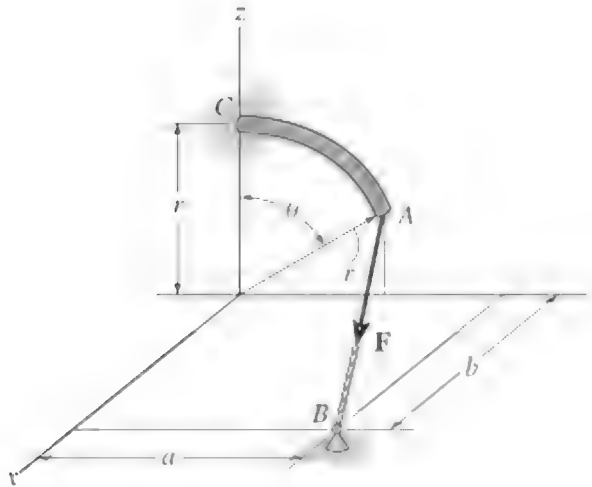
Given:

$r = 5\ \text{ft}$

$M = 80\ \text{lb}\cdot\text{ft}$

$\theta = 60\ \text{deg}$

$a = 7\ \text{ft}$

$b = 6\ \text{ft}$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} b \\ a - r\sin(\theta) \\ -r\cos(\theta) \end{pmatrix} \qquad \mathbf{u_{AB}} = \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{r_{CB}} = \begin{pmatrix} b \\ a \\ -r \end{pmatrix}$$

Guess $F = 1\ \text{lb}$

Given $|\mathbf{r_{CB}} \times (F\ \mathbf{u_{AB}})| = M \qquad F = \text{Find}(F) \qquad F = 18.6\,\text{lb}$

Problem 4-44

The pipe assembly is subjected to the force **F**. Determine the moment of this force about point A.

Given:

$F = 80\ \text{N}$

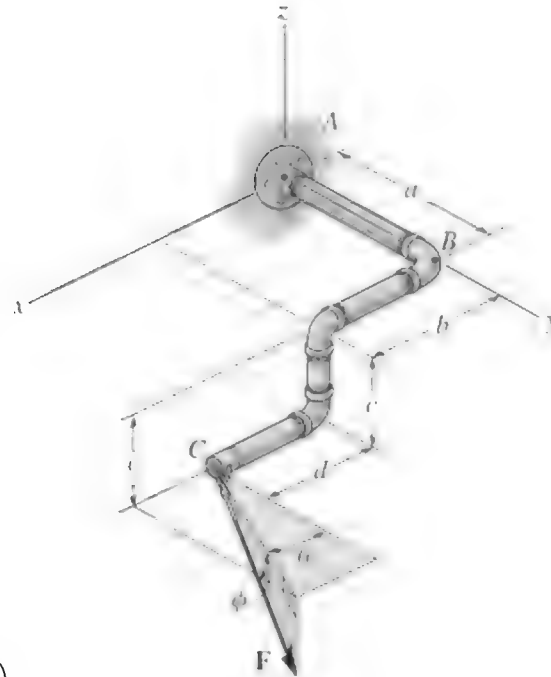
$a = 400\ \text{mm}$

$b = 300\ \text{mm}$

$c = 200\ \text{mm}$

$d = 250\ \text{mm}$

$\theta = 40\ \text{deg}$

$\phi = 30\ \text{deg}$

Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} b+d \\ a \\ -c \end{pmatrix} \qquad \mathbf{F_v} = F\begin{pmatrix} \cos(\phi)\sin(\theta) \\ \cos(\phi)\cos(\theta) \\ -\sin(\phi) \end{pmatrix}$$

$$\mathbf{M_A} = \mathbf{r_{AC}} \times \mathbf{F_v} \qquad \mathbf{M_A} = \begin{pmatrix} -5.385 \\ 13.093 \\ 11.377 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-45

The pipe assembly is subjected to the force **F**. Determine the moment of this force about point *B*.

Given:

$F = 80\ \text{N}$

$a = 400\ \text{mm}$

$b = 300\ \text{mm}$

$c = 200\ \text{mm}$

$d = 250\ \text{mm}$

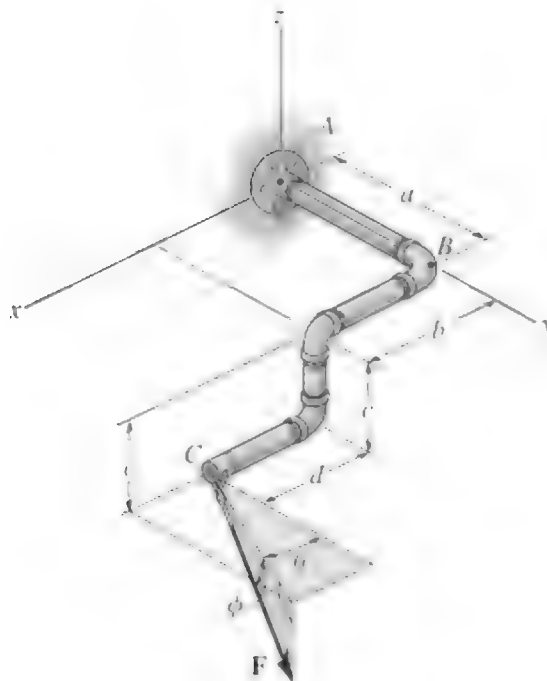
$\theta = 40\ \text{deg}$

$\phi = 30\ \text{deg}$

Solution:

$$\mathbf{r_{BC}} = \begin{pmatrix} b+d \\ 0 \\ -c \end{pmatrix} \qquad \mathbf{r_{BC}} = \begin{pmatrix} 550 \\ 0 \\ -200 \end{pmatrix} \text{mm}$$

$$\mathbf{F_v} = F\begin{pmatrix} \cos(\phi)\sin(\theta) \\ \cos(\phi)\cos(\theta) \\ -\sin(\phi) \end{pmatrix} \qquad \mathbf{F_v} = \begin{pmatrix} 44.534 \\ 53.073 \\ -40 \end{pmatrix} \text{N}$$

$$\mathbf{M_B} = \mathbf{r_{BC}} \times \mathbf{F_v} \qquad \mathbf{M_B} = \begin{pmatrix} 10.615 \\ 13.093 \\ 29.19 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-46

The *x*-ray machine is used for medical diagnosis. If the camera and housing at *C* have mass *M* and a mass center at *G*, determine the moment of its weight about point *O* when it is in the position shown.

Units Used:

$\text{kN} = 10^3\ \text{N}$

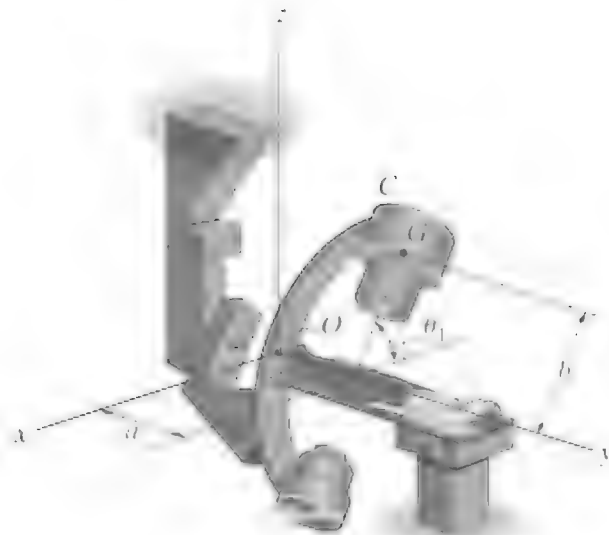
Given:

$M = 150\ \text{kg}$

$a = 1.2\ \text{m}$

$b = 1.5\ \text{m}$

$\theta = 60\ \text{deg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$\mathbf{M_O} = \begin{bmatrix} -(b)\cos(\theta) \\ a \\ (b)\sin(\theta) \end{bmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -Mg \end{pmatrix} \qquad \mathbf{M_O} = \begin{pmatrix} -1.77 \\ -1.1 \\ 0 \end{pmatrix} \text{kN} \cdot \text{m}$$

Problem 4-47

Using Cartesian vector analysis, determine the resultant moment of the three forces about the base of the column at A.

Units Used:

$\text{kN} = 10^3\ \text{N}$

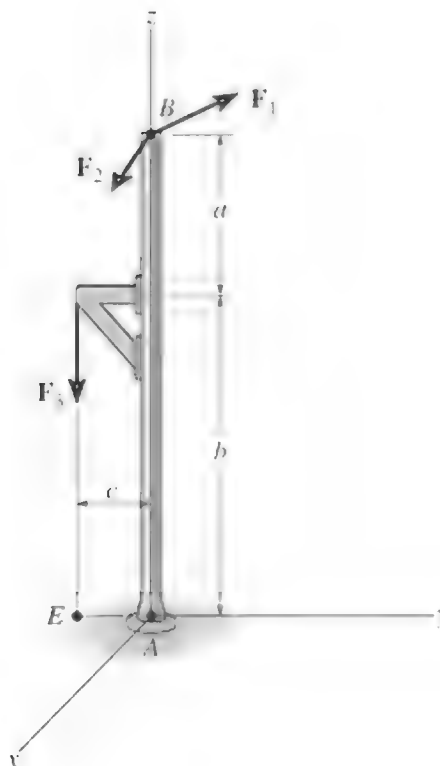
Given:

$$\mathbf{F_1} = \begin{pmatrix} 400 \\ 300 \\ 120 \end{pmatrix} \text{N}$$

$$\mathbf{F_2} = \begin{pmatrix} 100 \\ -100 \\ -60 \end{pmatrix} \text{N}$$

$$\mathbf{F_3} = \begin{pmatrix} 0 \\ 0 \\ -500 \end{pmatrix} \text{N}$$

$a = 4\ \text{m}$

$b = 8\ \text{m}$

$c = 1\ \text{m}$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} 0 \\ 0 \\ a+b \end{pmatrix} \qquad \mathbf{r_{A3}} = \begin{pmatrix} 0 \\ -c \\ b \end{pmatrix}$$

The individual moments

$$\mathbf{M_{A1}} = \mathbf{r_{AB}} \times \mathbf{F_1} \qquad \mathbf{M_{A2}} = \mathbf{r_{AB}} \times \mathbf{F_2} \qquad \mathbf{M_{A3}} = \mathbf{r_{A3}} \times \mathbf{F_3}$$

$$\mathbf{M_{A1}} = \begin{pmatrix} -3.6 \\ 4.8 \\ 0 \end{pmatrix} \text{kN}\cdot\text{m} \qquad \mathbf{M_{A2}} = \begin{pmatrix} 1.2 \\ 1.2 \\ 0 \end{pmatrix} \text{kN}\cdot\text{m} \qquad \mathbf{M_{A3}} = \begin{pmatrix} 0.5 \\ 0 \\ 0 \end{pmatrix} \text{kN}\cdot\text{m}$$

The total moment

$$\mathbf{M_A} = \mathbf{M_{A1}} + \mathbf{M_{A2}} + \mathbf{M_{A3}} \qquad \mathbf{M_A} = \begin{pmatrix} -1.9 \\ 6 \\ 0 \end{pmatrix} \text{kN}\cdot\text{m}$$

Problem 4-48

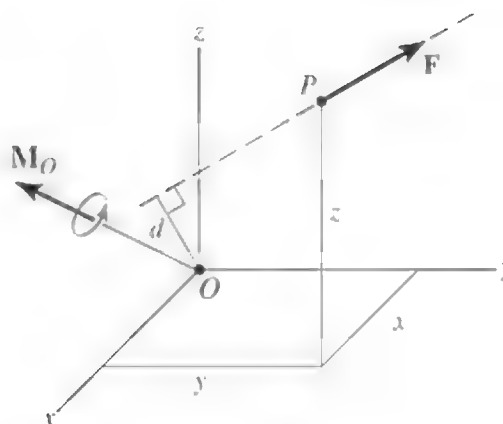
A force $\mathbf{F}$ produces a moment $\mathbf{M}_O$ about the origin of coordinates, point O. If the force acts at a point having the given x coordinate, determine the y and z coordinates.

Units Used: $\text{kN} = 10^3\ \text{N}$

Given:

$$\mathbf{F} = \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} \text{kN}$$

$$\mathbf{M_O} = \begin{pmatrix} 4 \\ 5 \\ -14 \end{pmatrix} \text{kN}\cdot\text{m}$$

$x = 1\ \text{m}$

Solution:

The initial guesses:

$y = 1\ \text{m} \qquad z = 1\ \text{m}$

Given

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \mathbf{F} = \mathbf{M_O} \qquad \begin{pmatrix} y \\ z \end{pmatrix} = \text{Find}(y, z) \qquad \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ m}$$

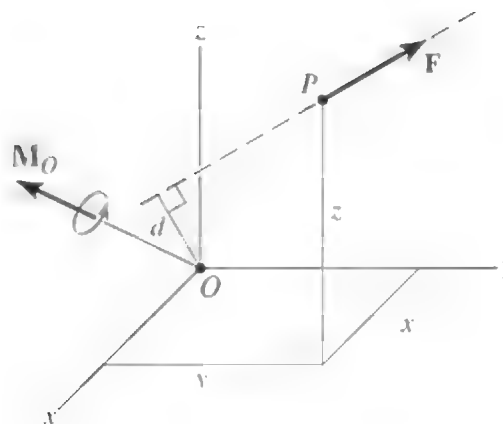
Problem 4-49

The force **F** creates a moment about point O of $\mathbf{M}_O$. If the force passes through a point having the given x coordinate, determine the y and z coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance d from point O to the line of action of **F**.

Given:

$$\mathbf{F} = \begin{pmatrix} 6 \\ 8 \\ 10 \end{pmatrix} \text{ N}$$

$$\mathbf{M_O} = \begin{pmatrix} -14 \\ 8 \\ 2 \end{pmatrix} \text{ N}\cdot\text{m}$$

$x = 1$ m

Solution:

The initial guesses: $y = 1$ m $\qquad z = 1$ m

Given

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \mathbf{F} = \mathbf{M_O} \qquad \begin{pmatrix} y \\ z \end{pmatrix} = \text{Find}(y, z) \qquad \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ m}$$

$$d = \frac{|\mathbf{M_O}|}{|\mathbf{F}|} \qquad d = 1.149 \text{ m}$$

Problem 4-50

The force **F** produces a moment $\mathbf{M}_O$ about the origin of coordinates, point O. If the force acts at a point having the given x-coordinate, determine the y and z coordinates.

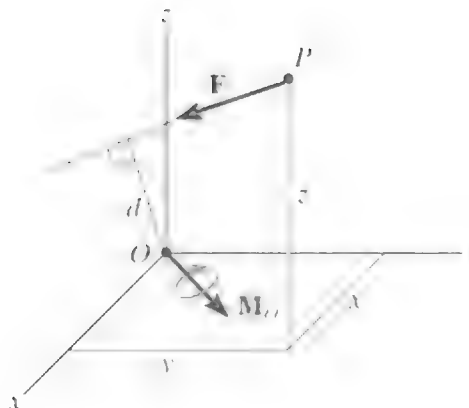
Units Used:

$kN = 10^3\ N$

Given:

$x = 1\ m$

$$\mathbf{F} = \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} kN$$

$$\mathbf{M_O} = \begin{pmatrix} 4 \\ 5 \\ -14 \end{pmatrix} kN\cdot m$$

Solution:

Initial Guesses: $y = 1\ m$ $z = 1\ m$

Given $\mathbf{M_O} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \mathbf{F}$ $\begin{pmatrix} y \\ z \end{pmatrix} = \text{Find}(y, z)$ $\begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} m$

Problem 4-51

Determine the moment of the force **F** about the *Oa* axis. Express the result as a Cartesian vector.

Given:

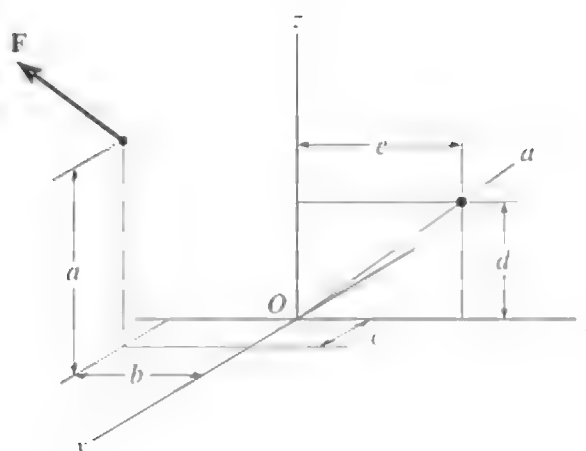
$$\mathbf{F} = \begin{pmatrix} 50 \\ -20 \\ 20 \end{pmatrix} N$$

$a = 6\ m$

$b = 2\ m$

$c = 1\ m$

$d = 3\ m$

$e = 4\ m$

Solution:

$$\mathbf{r_{OF}} = \begin{pmatrix} c \\ -b \\ a \end{pmatrix} \qquad \mathbf{r_{Oa}} = \begin{pmatrix} 0 \\ e \\ d \end{pmatrix} \qquad \mathbf{u_{Oa}} = \frac{\mathbf{r_{Oa}}}{|\mathbf{r_{Oa}}|}$$

$$\mathbf{M_{Oa}} = \left[\left(\mathbf{r_{OF}} \times \mathbf{F}\right)\cdot \mathbf{u_{Oa}}\right]\mathbf{u_{Oa}} \qquad \mathbf{M_{Oa}} = \begin{pmatrix} 0 \\ 217.6 \\ 163.2 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-52

Determine the moment of the force **F** about the *aa* axis. Express the result as a Cartesian vector.

Given:

$F = 600$ lb

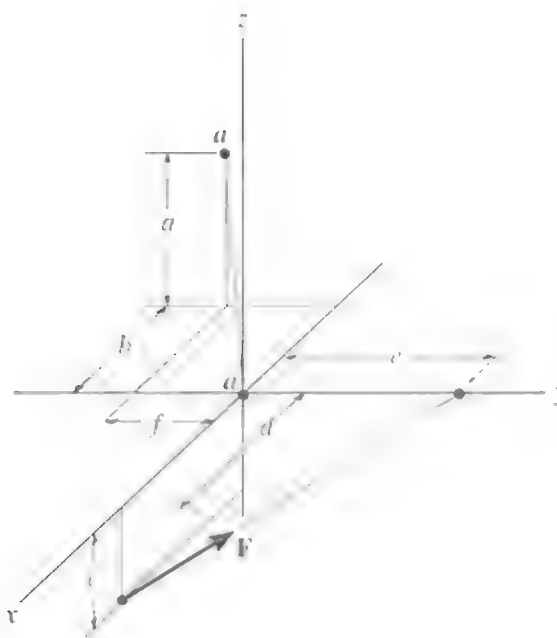
$a = 6$ ft

$b = 3$ ft

$c = 2$ ft

$d = 4$ ft

$e = 4$ ft

$f = 2$ ft

Solution:

$$\mathbf{F_v} = \frac{F}{\sqrt{c^2 + d^2 + e^2}}\begin{pmatrix} -d \\ e \\ c \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} d \\ 0 \\ -c \end{pmatrix} \qquad \mathbf{u_{aa}} = \frac{1}{\sqrt{a^2 + b^2 + f^2}}\begin{pmatrix} -b \\ -f \\ a \end{pmatrix}$$

$$\mathbf{M_{aa}} = \left[\left(\mathbf{r} \times \mathbf{F_v}\right)\cdot \mathbf{u_{aa}}\right]\mathbf{u_{aa}} \qquad \mathbf{M_{aa}} = \begin{pmatrix} -441 \\ -294 \\ 882 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 4-53

Determine the resultant moment of the two forces about the *Oa* axis. Express the result as a Cartesian vector.

Given:

$F_1 = 80\ \text{lb}$

$F_2 = 50\ \text{lb}$

$\alpha = 120\ \text{deg}$

$\beta = 60\ \text{deg}$

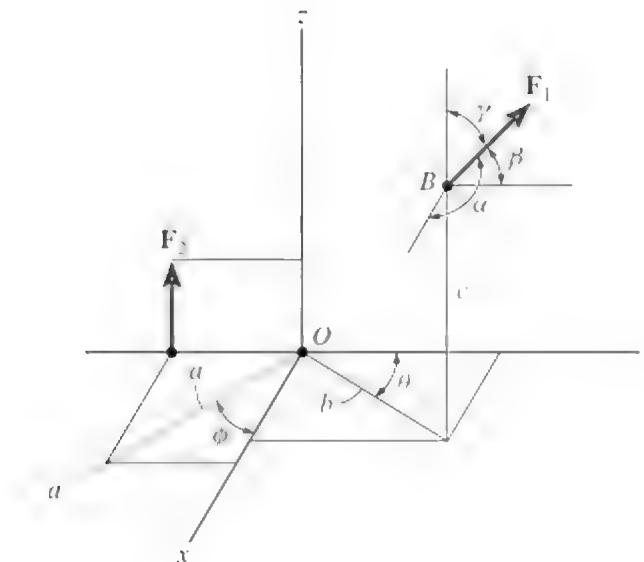
$\gamma = 45\ \text{deg}$

$a = 5\ \text{ft}$

$b = 4\ \text{ft}$

$c = 6\ \text{ft}$

$\theta = 30\ \text{deg}$

$\phi = 30\ \text{deg}$

Solution:

$$\mathbf{F_{1v}} = F_1\begin{pmatrix}\cos(\alpha)\\ \cos(\beta)\\ \cos(\gamma)\end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix}0\\ 0\\ F_2\end{pmatrix}$$

$$\mathbf{r_1} = \begin{bmatrix}(b)\sin(\theta)\\ (b)\cos(\theta)\\ c\end{bmatrix} \qquad \mathbf{r_2} = \begin{bmatrix}0\\ -(a)\sin(\phi)\\ 0\end{bmatrix}$$

$$\mathbf{u_{aa}} = \begin{pmatrix}\cos(\phi)\\ -\sin(\phi)\\ 0\end{pmatrix} \qquad \mathbf{M_{aa}} = \left[\left(\mathbf{r_1}\times\mathbf{F_{1v}} + \mathbf{r_2}\times\mathbf{F_{2v}}\right)\mathbf{u_{aa}}\right]\mathbf{u_{aa}}$$

$$\mathbf{M_{aa}} = \begin{pmatrix}26.132\\ -15.087\\ 0\end{pmatrix}\text{lb}\cdot\text{ft}$$

Problem 4-54

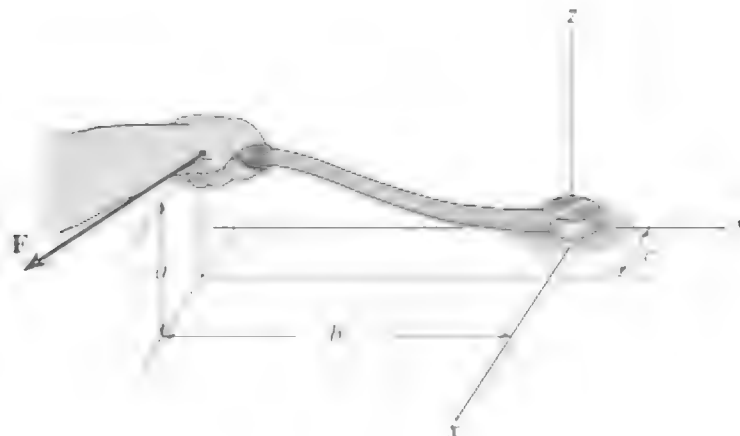
The force **F** is applied to the handle of the box wrench. Determine the component of the moment of this force about the z axis which is effective in loosening the bolt.

Given:

$a = 3$ in

$b = 8$ in

$c = 2$ in

$$\mathbf{F} = \begin{pmatrix} 8 \\ -1 \\ 1 \end{pmatrix} \text{ lb}$$

Solution:

$$\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} c \\ -b \\ a \end{pmatrix} \qquad M_z = (\mathbf{r} \times \mathbf{F}) \cdot \mathbf{k} \qquad M_z = 62 \text{ lb} \cdot \text{in}$$

Problem 4-55

The force **F** acts on the gear in the direction shown. Determine the moment of this force about the y axis.

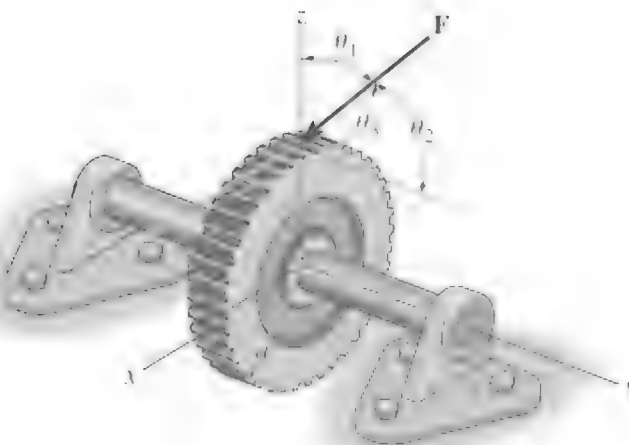
Given:

$F = 50$ lb

$a = 3$ in

$\theta_1 = 60$ deg

$\theta_2 = 45$ deg

$\theta_3 = 120$ deg

Solution:

$$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} \quad \mathbf{F_v} = F \begin{pmatrix} -\cos(\theta_3) \\ -\cos(\theta_2) \\ -\cos(\theta_1) \end{pmatrix} \quad M_y = (\mathbf{r} \times \mathbf{F_v}) \cdot \mathbf{j} \qquad M_y = 75\,\text{lb}\cdot\text{in}$$

Problem 4-56

The RollerBall skate is an in-line tandem skate that uses two large spherical wheels on each skate, rather than traditional wafer-shape wheels. During skating the two forces acting on the wheel of one skate consist of a normal force $\mathbf{F}_2$ and a friction force $\mathbf{F}_1$. Determine the moment of both of these forces about the axle AB of the wheel.

Given:

$\theta = 30$ deg

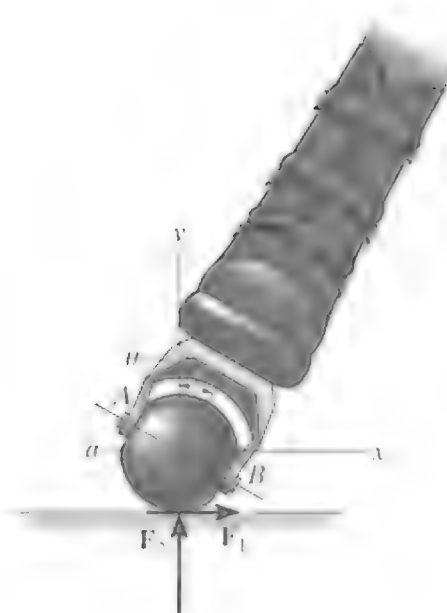
$F_1 = 13$ lb

$F_2 = 78$ lb

$a = 1.25$ in

Solution:

$$\mathbf{F} = \begin{pmatrix} F_1 \\ F_2 \\ 0 \end{pmatrix} \quad \mathbf{r} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}$$

$$\mathbf{ab} = \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix} \qquad M_{ab} = (\mathbf{r} \times \mathbf{F}) \cdot \mathbf{ab} \qquad M_{ab} = 0\,\text{lb}\cdot\text{in}$$

Problem 4-57

The cutting tool on the lathe exerts a force $\mathbf{F}$ on the shaft in the direction shown. Determine the moment of this force about the y axis of the shaft.

Units Used:

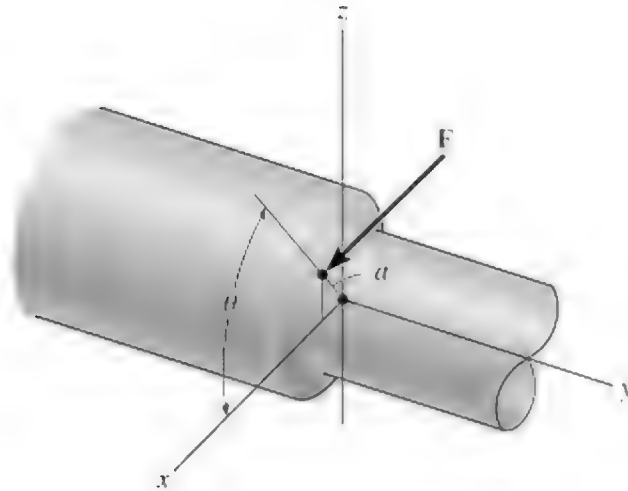
$kN = 10^3 \ N$

Given:

$$\mathbf{F} = \begin{pmatrix} 6 \\ -4 \\ -7 \end{pmatrix} kN$$

$a = 30 \ mm$

$\theta = 40 \ deg$

Solution:

$$\mathbf{r} = a\begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix} \qquad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad M_y = (\mathbf{r} \times \mathbf{F}) \cdot \mathbf{j} \qquad M_y = 0.277 \ kN \cdot m$$

Problem 4-58

The hood of the automobile is supported by the strut *AB*, which exerts a force **F** on the hood. Determine the moment of this force about the hinged axis *y*.

Given:

$F = 24 \ lb \qquad a = 2 \ ft \qquad b = 4 \ ft \qquad c = 2 \ ft \qquad d = 4 \ ft$

Solution:

$$\mathbf{r_A} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -b+c \\ a \\ d \end{pmatrix} \qquad \mathbf{F_v} = F\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{F_v} = \begin{pmatrix} -9.798 \\ 9.798 \\ 19.596 \end{pmatrix} \text{lb}$$

$$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad M_y = (\mathbf{r_A} \times \mathbf{F_v}) \cdot \mathbf{j} \qquad M_y = -78.384 \text{ lb}\cdot\text{ft}$$

Problem 4-59

The lug nut on the wheel of the automobile is to be removed using the wrench and applying the vertical force **F** at *A*. Determine if this force is adequate, provided a torque *M* about the *x* axis is initially required to turn the nut. If the force **F** can be applied at *A* in any other direction, will it be possible to turn the nut?

Given:

$F = 30\ \text{N}$

$M = 14\ \text{N}\cdot\text{m}$

$a = 0.25\ \text{m}$

$b = 0.3\ \text{m}$

$c = 0.5\ \text{m}$

$d = 0.1\ \text{m}$

Solution:

$$M_x = F\sqrt{c^2 - b^2}$$

$$M_x = 12\,\text{N}\cdot\text{m}$$

$M_x < M$ No

For M_{xmax}, apply force perpendicular to the handle and the x-axis.

$$M_{xmax} = F\,c$$

$$M_{xmax} = 15\,\text{N}\cdot\text{m}$$

$M_{xmax} > M$ Yes

Problem 4-60

The lug nut on the wheel of the automobile is to be removed using the wrench and applying the vertical force **F**. Assume that the cheater pipe *AB* is slipped over the handle of the wrench and the **F** force can be applied at any point and in any direction on the assembly. Determine if this force is adequate, provided a torque *M* about the *x* axis is initially required to turn the nut.

Given:

$F_1 = 30\ \text{N}$ $M = 14\ \text{N}\cdot\text{m}$ $a = 0.25\ \text{m}$ $b = 0.3\ \text{m}$ $c = 0.5\ \text{m}$ $d = 0.1\ \text{m}$

Solution:

$$M_x = F_1 \frac{a+c}{c}\sqrt{c^2 - b^2}$$

$$M_x = 18\,\text{N}\cdot\text{m}$$

$M_x > M$ Yes

M_{xmax} occurs when force is applied perpendicular to both the handle and the x-axis.

$$M_{xmax} = F_1(a + c)$$

$$M_{xmax} = 22.5\,\text{N}\cdot\text{m}$$

$M_{xmax} > M$ Yes

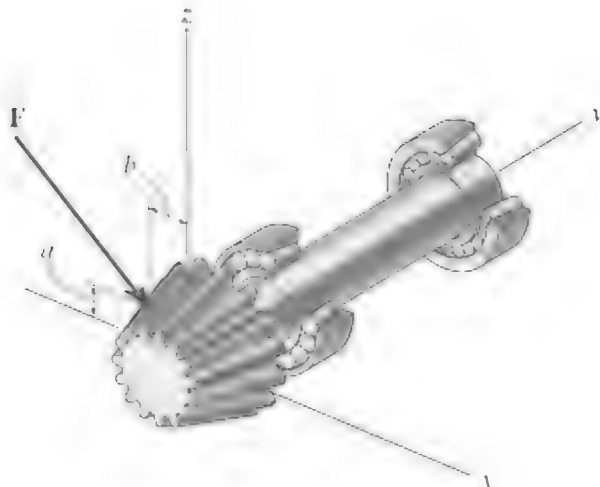
Problem 4-61

The bevel gear is subjected to the force **F** which is caused from contact with another gear. Determine the moment of this force about the y axis of the gear shaft.

Given:

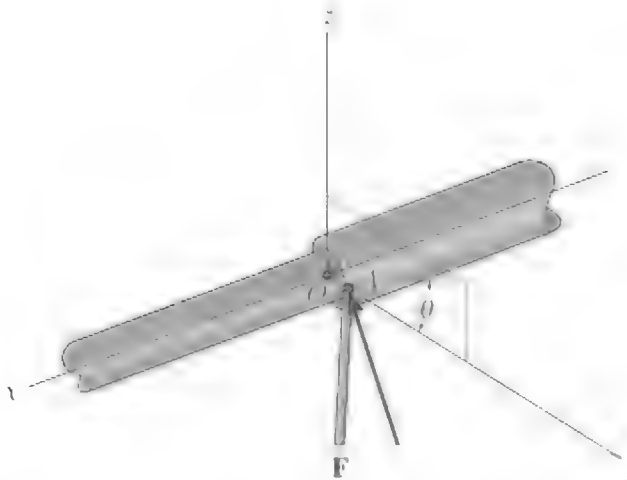
$a = 30$ mm

$b = 40$ mm

$$\mathbf{F} = \begin{pmatrix} 20 \\ 8 \\ -15 \end{pmatrix} \text{N}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} -b \\ 0 \\ a \end{pmatrix} \qquad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad M_y = (\mathbf{r} \times \mathbf{F}) \cdot \mathbf{j} \qquad M_y = 0 \, \text{N} \cdot \text{m}$$

Problem 4-62

The wooden shaft is held in a lathe. The cutting tool exerts force **F** on the shaft in the direction shown. Determine the moment of this force about the x axis of the shaft. Express the result as a Cartesian vector. The distance OA is a.

Given:

$a = 25$ mm

$\theta = 30$ deg

$$\mathbf{F} = \begin{pmatrix} -5 \\ -3 \\ 8 \end{pmatrix} \text{N}$$

Solution:

$$\mathbf{r} = \begin{bmatrix} 0 \\ (a)\cos(\theta) \\ (a)\sin(\theta) \end{bmatrix} \qquad \mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{M_x} = [(\mathbf{r} \times \mathbf{F}) \cdot \mathbf{i}]\mathbf{i} \qquad \mathbf{M_x} = \begin{pmatrix} 0.211 \\ 0 \\ 0 \end{pmatrix} \text{N} \cdot \text{m}$$

Problem 4-63

Determine the magnitude of the moment of the force **F** about the base line CA of the tripod.

Given:

$$\mathbf{F} = \begin{pmatrix} 50 \\ -20 \\ -80 \end{pmatrix} \text{N}$$

$a = 4 \text{ m}$

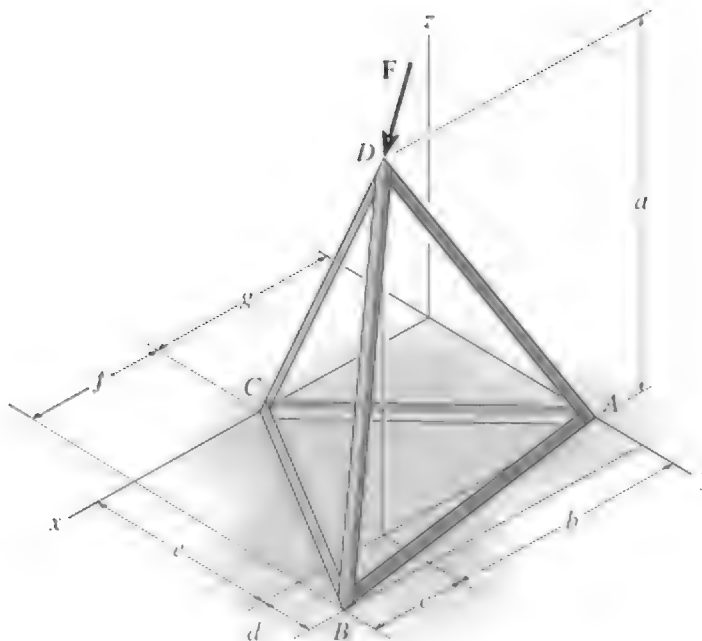
$b = 2.5 \text{ m}$

$c = 1 \text{ m}$

$d = 0.5 \text{ m}$

$e = 2 \text{ m}$

$f = 1.5 \text{ m}$

$g = 2 \text{ m}$

Solution:

$$\mathbf{r_{CA}} = \begin{pmatrix} -g \\ e \\ 0 \end{pmatrix} \quad \mathbf{u_{CA}} = \frac{\mathbf{r_{CA}}}{|\mathbf{r_{CA}}|} \quad \mathbf{r_{CD}} = \begin{pmatrix} b-g \\ e \\ a \end{pmatrix} \quad M_{CA} = (\mathbf{r_{CD}} \times \mathbf{F}) \cdot \mathbf{u_{CA}} \qquad M_{CA} = 226 \,\text{N}\cdot\text{m}$$

Problem 4-64

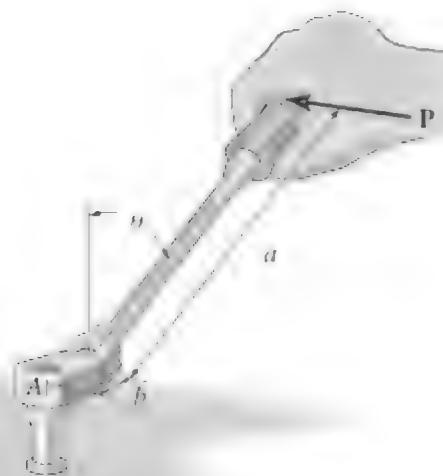
The flex-headed ratchet wrench is subjected to force **P**, applied perpendicular to the handle as shown. Determine the moment or torque this imparts along the vertical axis of the bolt at A.

Given:

$P = 16 \text{ lb} \quad a = 10 \text{ in}$

$\theta = 60 \text{ deg} \quad b = 0.75 \text{ in}$

Solution:

$$M = P\,[b + (a)\sin(\theta)] \qquad M = 150.564 \,\text{lb}\cdot\text{in}$$

Problem 4-65

If a torque or moment M is required to loosen the bolt at A, determine the force $\mathbf{P}$ that must be applied perpendicular to the handle of the flex-headed ratchet wrench.

Given:

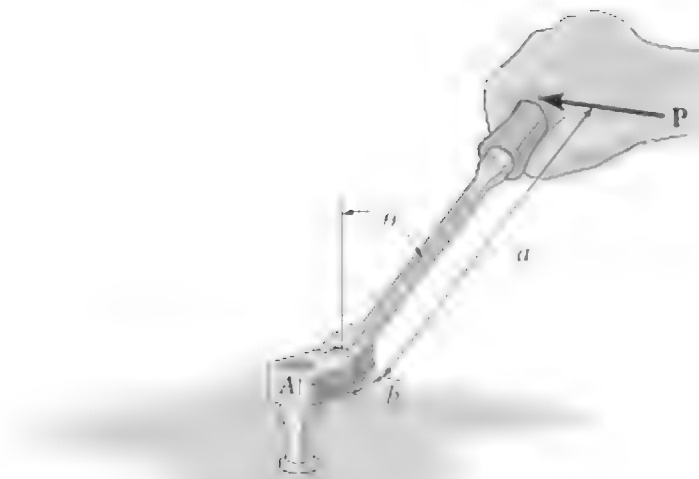
$M = 80 \text{ lb}\cdot\text{in}$

$\theta = 60 \text{ deg}$

$a = 10 \text{ in}$

$b = 0.75 \text{ in}$

Solution:

$$M = P\left[b + (a)\sin(\theta)\right] \qquad P = \frac{M}{b + (a)\sin(\theta)} \qquad P = 8.50 \text{ lb}$$

Problem 4-66

The A-frame is being hoisted into an upright position by the vertical force $\mathbf{F}$. Determine the moment of this force about the y axis when the frame is in the position shown.

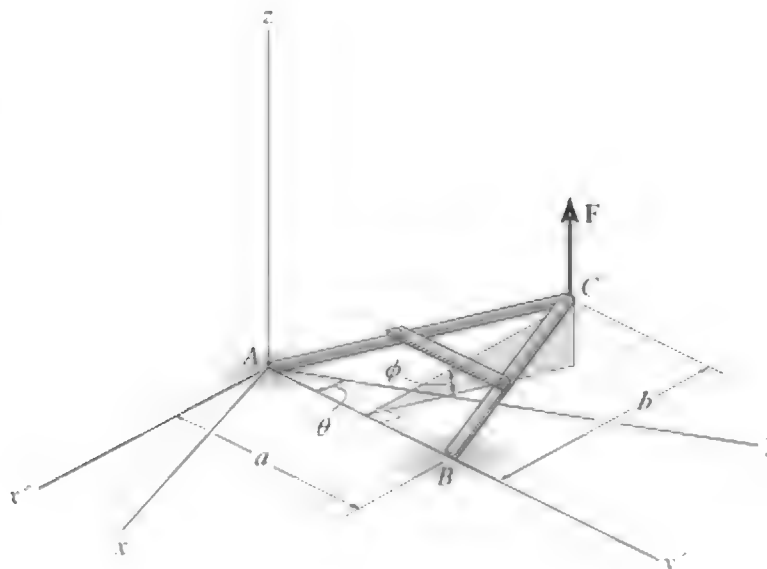
Given:

$F = 80 \text{ lb}$

$a = 6 \text{ ft}$

$b = 6 \text{ ft}$

$\theta = 30 \text{ deg}$

$\phi = 15 \text{ deg}$

Solution:

Using the primed coordinates we have

$$\mathbf{j} = \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{F_v} = F\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -b\cos(\phi) \\ \dfrac{a}{2} \\ b\sin(\phi) \end{pmatrix}$$

$$M_y = (\mathbf{r_{AC}} \times \mathbf{F_v})\cdot\mathbf{j} \qquad M_y = 281.528\ \text{lb}\cdot\text{ft}$$

Problem 4-67

Determine the moment of each force acting on the handle of the wrench about the a axis.

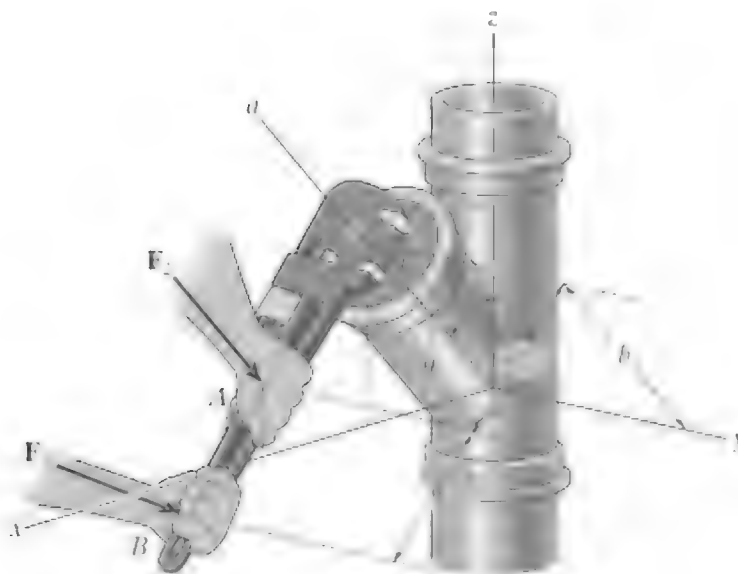
Given:

$$\mathbf{F_1} = \begin{pmatrix} -2 \\ 4 \\ -8 \end{pmatrix}\ \text{lb} \qquad \mathbf{F_2} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}\ \text{lb}$$

$b = 6$ in

$c = 4$ in

$d = 3.5$ in

$\theta = 45$ deg

Solution:

$$\mathbf{u_a} = \begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix}$$

$$\mathbf{r_1} = b\begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix} + (c+d)\begin{pmatrix} \sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix} \qquad \mathbf{r_2} = b\begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix} + c\begin{pmatrix} \sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix}$$

$$M_{1a} = (\mathbf{r_1} \times \mathbf{F_1})\cdot\mathbf{u_a} \qquad M_{1a} = 30\ \text{lb}\cdot\text{in}$$

$M_{2a} = (\mathbf{r_2} \times \mathbf{F_2}) \cdot \mathbf{u_a}$ $\qquad M_{2a} = 8\,\text{lb}\cdot\text{in}$

Problem 4-68

Determine the moment of each force acting on the handle of the wrench about the z axis.

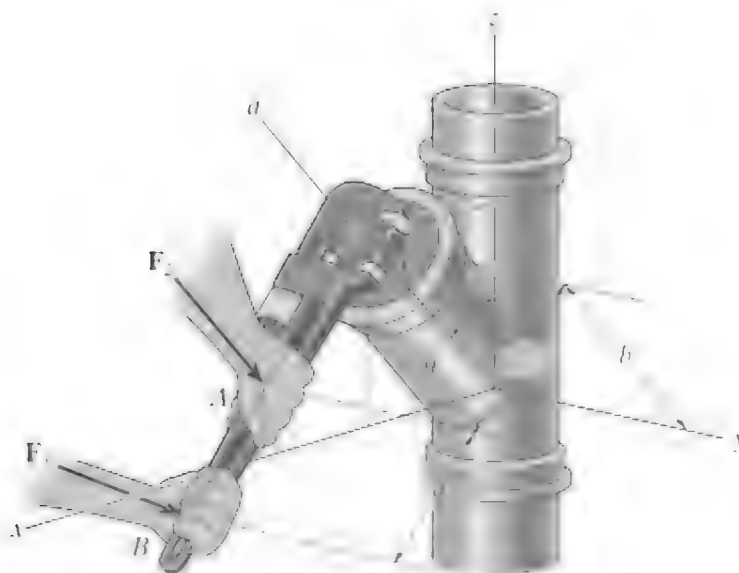
Given:

$$\mathbf{F_1} = \begin{pmatrix} -2 \\ 4 \\ -8 \end{pmatrix} \text{lb} \qquad \mathbf{F_2} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \text{lb}$$

$b = 6$ in

$c = 4$ in

$d = 3.5$ in

$\theta = 45$ deg

Solution:

$$\mathbf{r_1} = b\begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix} + (c+d)\begin{pmatrix} \sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix} \qquad \mathbf{r_2} = b\begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix} + c\begin{pmatrix} \sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$M_{1z} = (\mathbf{r_1} \times \mathbf{F_1}) \cdot \mathbf{k}$ $\qquad M_{1z} = 38.2\,\text{lb}\cdot\text{in}$

$M_{2z} = (\mathbf{r_2} \times \mathbf{F_2}) \cdot \mathbf{k}$ $\qquad M_{2z} = 14.1\,\text{lb}\cdot\text{in}$

Problem 4-69

Determine the magnitude and sense of the couple moment.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F = 5\ \text{kN}$

$\theta = 30\ \text{deg}$

$a = 0.5\ \text{m}$

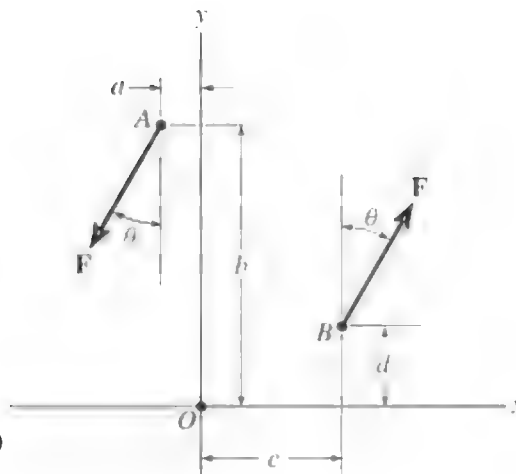
$b = 4\ \text{m}$

$c = 2\ \text{m}$

$d = 1\ \text{m}$

Solution:

$$M_C = F\cos(\theta)(a+c) + F\sin(\theta)(b-d)$$

$$M_C = 18.325\ \text{kN}\cdot\text{m}$$

Problem 4-70

Determine the magnitude and sense of the couple moment. Each force has a magnitude F.

Given:

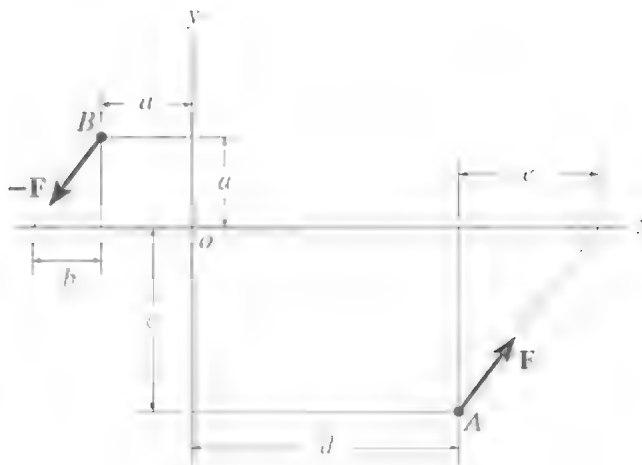
$F = 65\ \text{lb}$

$a = 2\ \text{ft}$

$b = 1.5\ \text{ft}$

$c = 4\ \text{ft}$

$d = 6\ \text{ft}$

$e = 3\ \text{ft}$

Solution:

$$M_c = \Sigma M_B; \qquad M_C = \left[F\left(\frac{c}{\sqrt{c^2+e^2}}\right)(d+a)\right] + \left[F\left(\frac{e}{\sqrt{c^2+e^2}}\right)(c+a)\right]$$

$$M_C = 650\ \text{lb}\cdot\text{ft} \quad \text{(Counterclockwise)}$$

Problem 4-71

Determine the magnitude and sense of the couple moment.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$F = 150 \text{ lb}$

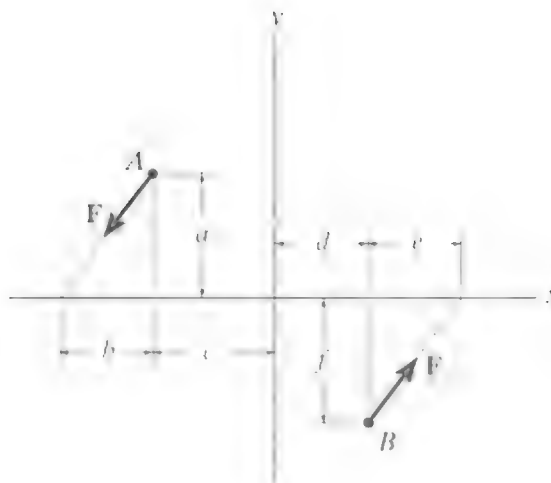
$a = 8 \text{ ft}$

$b = 6 \text{ ft}$

$c = 8 \text{ ft}$

$d = 6 \text{ ft}$

$e = 6 \text{ ft}$

$f = 8 \text{ ft}$

Solution:

$M_C = \Sigma M_A;$ $$M_C = F\frac{d}{\sqrt{d^2+f^2}}(a+f) + F\frac{f}{\sqrt{d^2+f^2}}(c+d)$$

$M_C = 3120 \text{ lb}\cdot\text{ft}$ $M_C = 3.120 \text{ kip}\cdot\text{ft}$

Problem 4-72

If the couple moment has magnitude M, determine the magnitude F of the couple forces.

Given:

$M = 300 \text{ lb}\cdot\text{ft}$

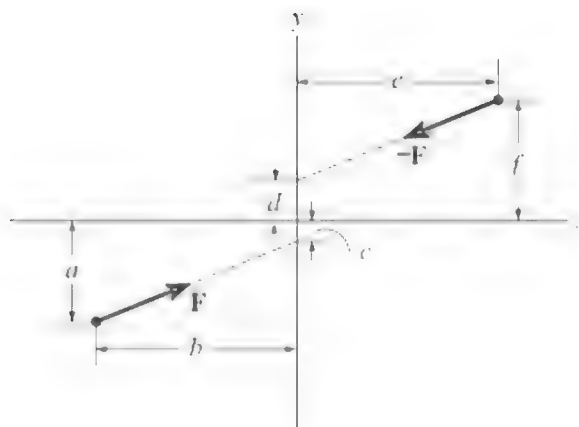
$a = 6 \text{ ft}$

$b = 12 \text{ ft}$

$c = 1 \text{ ft}$

$d = 2 \text{ ft}$

$e = 12 \text{ ft}$

$f = 7 \text{ ft}$

Solution:

$$M = F\left[\frac{e(f+a)}{\sqrt{(f-d)^2+e^2}} - \frac{(f-d)(b+e)}{\sqrt{(f-d)^2+e^2}}\right]$$

$$F = \frac{M}{\dfrac{e(f+a)}{\sqrt{(f-d)^2+e^2}} - \dfrac{(f-d)(b+e)}{\sqrt{(f-d)^2+e^2}}} \qquad F = 108\,\text{lb}$$

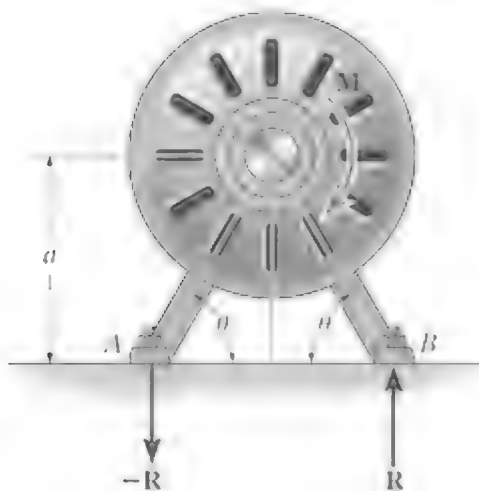
Problem 4-73

A clockwise couple M is resisted by the shaft of the electric motor. Determine the magnitude of the reactive forces $-\mathbf{R}$ and $\mathbf{R}$ which act at supports A and B so that the resultant of the two couples is zero.

Given:

$a = 150$ mm

$\theta = 60$ deg

$M = 5$ N·m

Solution:

$$M_C = -M + \frac{2Ra}{\tan(\theta)} = 0 \qquad R = \frac{M}{2}\,\frac{\tan(\theta)}{a} \qquad R = 28.9\,\text{N}$$

Problem 4-74

The resultant couple moment created by the two couples acting on the disk is $\mathbf{M_R}$. Determine the magnitude of force $\mathbf{T}$.

Units Used:

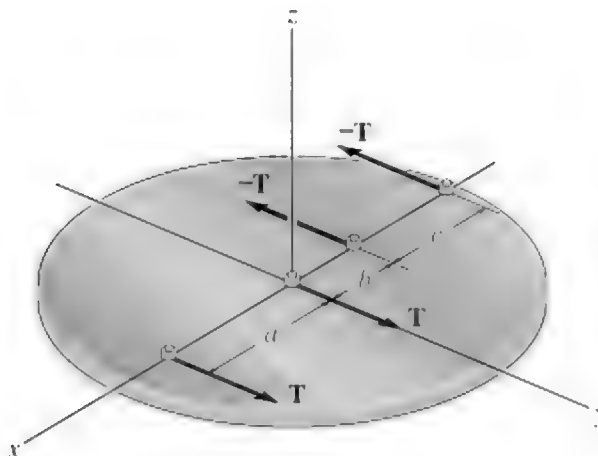
$kip = 10^3\ lb$

Given:

$$\mathbf{M_R} = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} kip\cdot in$$

$a = 4\ in$

$b = 2\ in$

$c = 3\ in$

Solution:

Initial Guess $T = 1\ kip$

Given
$$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ T \\ 0 \end{pmatrix} + \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -T \\ 0 \end{pmatrix} + \begin{pmatrix} -b-c \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -T \\ 0 \end{pmatrix} = \mathbf{M_R}$$

$T = \text{Find}(T)$ $T = 0.909\,kip$

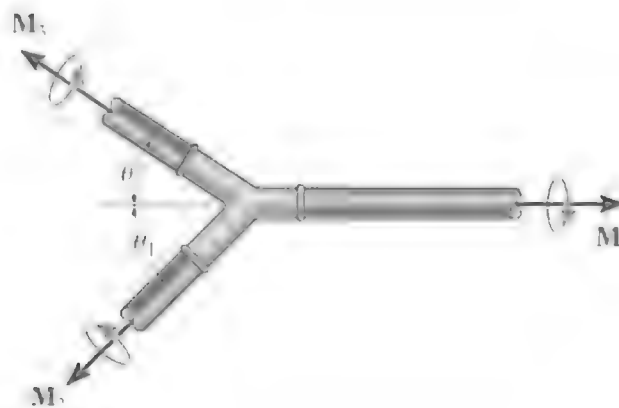
Problem 4-75

Three couple moments act on the pipe assembly. Determine the magnitude of M_3 and the bend angle θ so that the resultant couple moment is zero.

Given:

$\theta_1 = 45\ deg$

$M_1 = 900\ N\cdot m$

$M_2 = 500\ N\cdot m$

Solution:

Initial guesses: $\theta = 10\ \text{deg}$ $M_3 = 10\ \text{N}\cdot\text{m}$

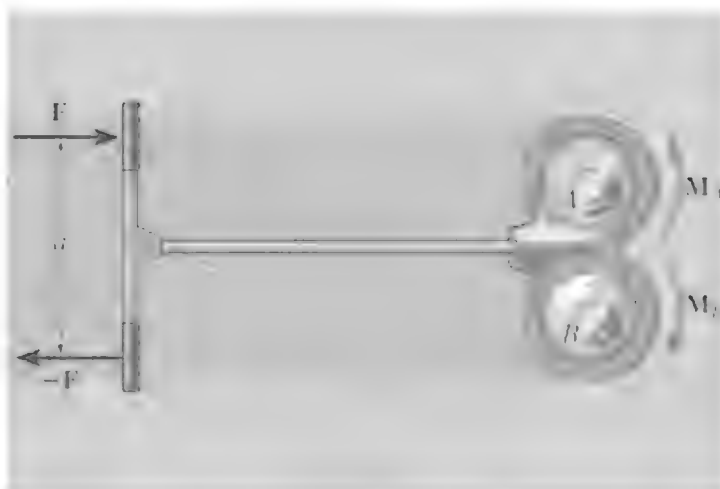
Given

$\xrightarrow{+} \Sigma M_x = 0;$ $M_1 - M_3\cos(\theta) - M_2\cos(\theta_1) = 0$

$+\uparrow \Sigma M_y = 0;$ $M_3\sin(\theta) - M_2\sin(\theta_1) = 0$

$$\begin{pmatrix} \theta \\ M_3 \end{pmatrix} = \text{Find}(\theta, M_3) \qquad \theta = 32.9\ \text{deg} \qquad M_3 = 651\ \text{N}\cdot\text{m}$$

Problem 4-76

The floor causes couple moments M_A and M_B on the brushes of the polishing machine. Determine the magnitude of the couple forces that must be developed by the operator on the handles so that the resultant couple moment on the polisher is zero. What is the magnitude of these forces if the brush at B suddenly stops so that $M_B = 0$?

Given:

$a = 0.3\ \text{m}$

$M_A = 40\ \text{N}\cdot\text{m}$

$M_B = 30\ \text{N}\cdot\text{m}$

Solution:

$M_A - M_B - F_1 a = 0$ $\qquad F_1 = \dfrac{M_A - M_B}{a}$ $\qquad F_1 = 33.3\ \text{N}$

$M_A - F_2 a = 0$ $\qquad F_2 = \dfrac{M_A}{a}$ $\qquad F_2 = 133\ \text{N}$

Problem 4-77

The ends of the triangular plate are subjected to three couples. Determine the magnitude of the force **F** so that the resultant couple moment is M clockwise.

Given:

$F_1 = 600 \text{ N}$

$F_2 = 250 \text{ N}$

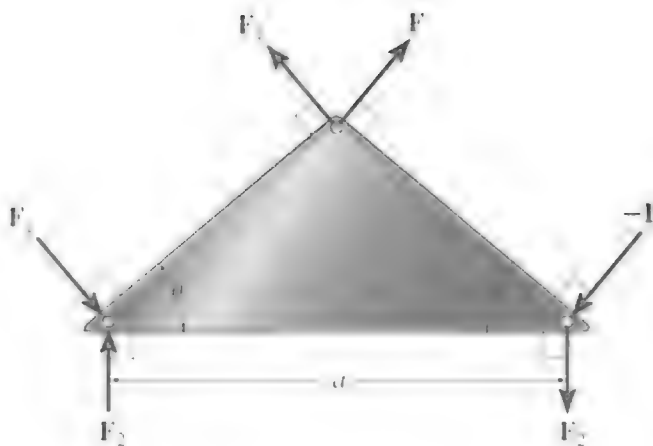
$a = 1 \text{ m}$

$\theta = 40 \text{ deg}$

$M = 400 \text{ N}\cdot\text{m}$

Solution:

Initial Guess $F = 1 \text{ N}$

Given $F_1\left(\frac{a}{2\cos(\theta)}\right) - F_2 a - F\left(\frac{a}{2\cos(\theta)}\right) = -M$ $\quad F = \text{Find}(F) \quad F = 830 \text{ N}$

Problem 4-78

Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is M counterclockwise. Where on the beam does the resultant couple moment act?

Given:

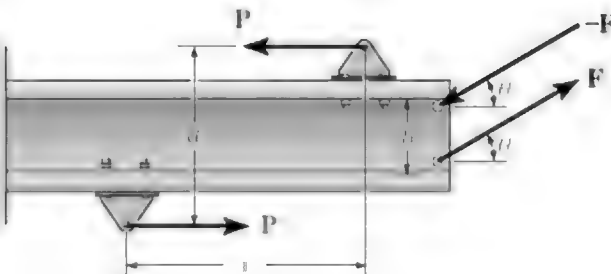
$M = 450 \text{ lb}\cdot\text{ft}$

$P = 200 \text{ lb}$

$a = 1.5 \text{ ft}$

$b = 1.25 \text{ ft}$

$c = 2 \text{ ft}$

$\theta = 30 \text{ deg}$

Solution:

$M_R = \Sigma M \qquad M = F\,b\cos(\theta) + P\,a \qquad F = \frac{M - P\,a}{b\cos(\theta)} \qquad F = 139\,\text{lb}$

The resultant couple moment is a free vector. It can act at any point on the beam.

Problem 4-79

Express the moment of the couple acting on the pipe assembly in Cartesian vector form. Solve the problem **(a)** using Eq. 4-13, and **(b)** summing the moment of each force about point *O*.

Given:

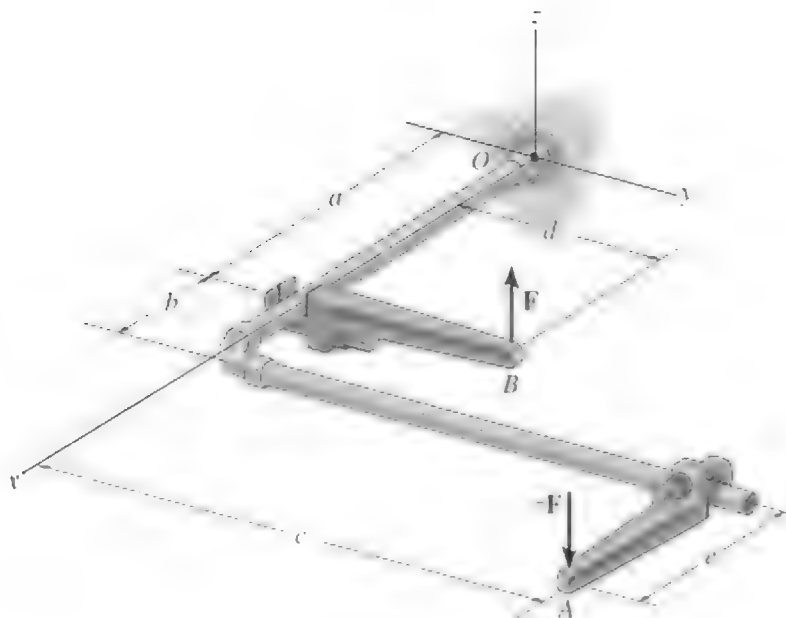
$$\mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ 25 \end{pmatrix} \text{N}$$

$a = 300$ mm

$b = 150$ mm

$c = 400$ mm

$d = 200$ mm

$e = 200$ mm

Solution:

(a) $$\mathbf{r_{AB}} = \begin{pmatrix} -e-b \\ -c+d \\ 0 \end{pmatrix} \qquad \mathbf{M} = \mathbf{r_{AB}} \times \mathbf{F} \qquad \mathbf{M} = \begin{pmatrix} -5 \\ 8.75 \\ 0 \end{pmatrix} \text{N}\cdot\text{m}$$

(b) $$\mathbf{r_{OB}} = \begin{pmatrix} a \\ d \\ 0 \end{pmatrix} \qquad \mathbf{r_{OA}} = \begin{pmatrix} a+b+e \\ c \\ 0 \end{pmatrix}$$

$$\mathbf{M} = \mathbf{r_{OB}} \times \mathbf{F} + \mathbf{r_{OA}} \times (-\mathbf{F}) \qquad \mathbf{M} = \begin{pmatrix} -5 \\ 8.75 \\ 0 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-80

If the couple moment acting on the pipe has magnitude M, determine the magnitude F of the vertical force applied to each wrench.

Given:

$M = 400\ \text{N}\cdot\text{m}$

$a = 300\ \text{mm}$

$b = 150\ \text{mm}$

$c = 400\ \text{mm}$

$d = 200\ \text{mm}$

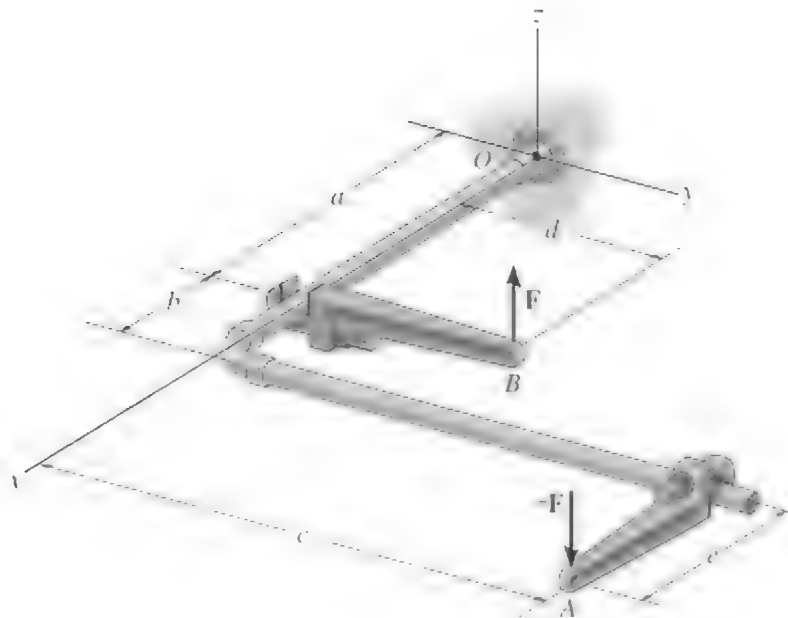
$e = 200\ \text{mm}$

Solution:

$$\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{r_{AB}} = \begin{pmatrix} -e-b \\ -c+d \\ 0 \end{pmatrix}$$

Guesss $F = 1\ \text{N}$

Given $\left|\mathbf{r_{AB}} \times (F\mathbf{k})\right| = M \qquad F = \text{Find}(F) \qquad F = 992.278\ \text{N}$

Problem 4-81

Determine the resultant couple moment acting on the beam. Solve the problem two ways: (a) sum moments about point O; and (b) sum moments about point A.

Units Used:

$\text{kN} = 10^3\ \text{N}$

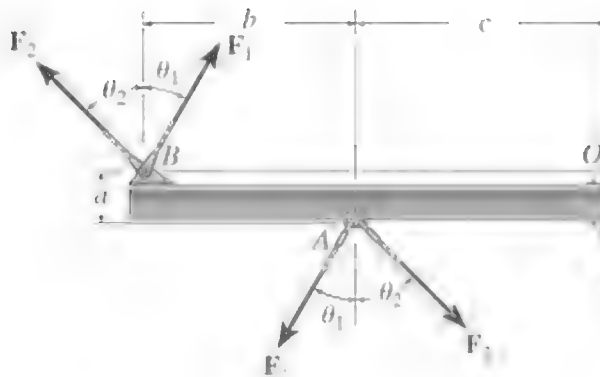
Given:

$F_1 = 2$ kN $\theta_1 = 30$ deg

$F_2 = 8$ kN $\theta_2 = 45$ deg

$a = 0.3$ m

$b = 1.5$ m

$c = 1.8$ m

Solution:

(a) $M_R = \Sigma M_O;$

$$M_{Ra} = \left(F_1 \cos(\theta_1) + F_2 \cos(\theta_2)\right)c + \left(F_2 \cos(\theta_2) - F_1 \sin(\theta_1)\right)a \; ... \\ + -\left(F_2 \cos(\theta_2) + F_1 \cos(\theta_1)\right)(b + c)$$

$$M_{Ra} = -9.69 \text{ kN}\cdot\text{m}$$

(b) $M_R = \Sigma M_A;$

$$M_{Rb} = \left(F_2 \sin(\theta_2) - F_1 \sin(\theta_1)\right)a - \left(F_2 \cos(\theta_2) + F_1 \cos(\theta_1)\right)b$$

$$M_{Rb} = -9.69 \text{ kN}\cdot\text{m}$$

Problem 4-82

Two couples act on the beam as shown. Determine the magnitude of **F** so that the resultant couple moment is M counterclockwise. Where on the beam does the resultant couple act?

Given:

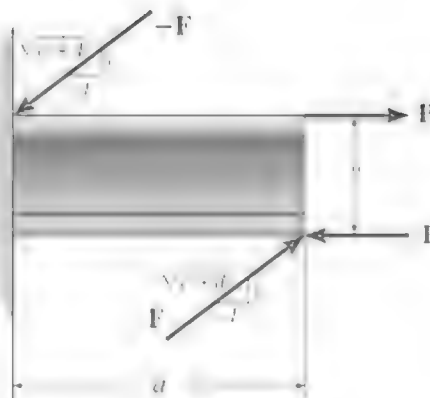
$M = 300$ lb·ft

$a = 4$ ft

$b = 1.5$ ft

$P = 200$ lb

$c = 3$

$d = 4$

Solution:

$$M = \frac{c}{\sqrt{c^2 + d^2}} F a + \frac{d}{\sqrt{c^2 + d^2}} F b - P b$$

$$F = \sqrt{c^2 + d^2} \left(\frac{M + P b}{c a + d b} \right)$$

$F = 167$ lb Resultant couple can act anywhere.

Problem 4-83

Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance d between the couple forces $\mathbf{F_1}$.

Given:

$F_1 = 80$ lb

$F_2 = 50$ lb

$a = 1$ ft

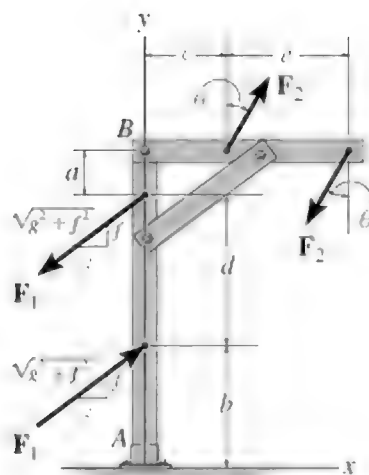
$b = 3$ ft

$c = 2$ ft

$e = 3$ ft

$f = 3$

$g = 4$

$\theta = 30$ deg

Solution:

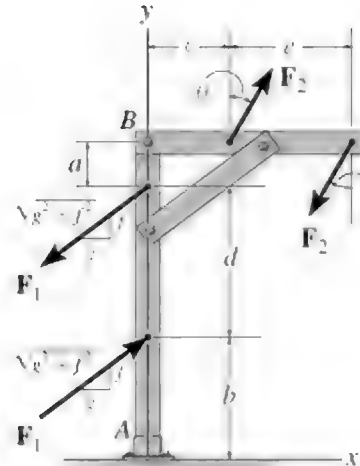
$$\left[-F_2 \cos(\theta) e + \left(\frac{g}{\sqrt{g^2 + f^2}} \right) F_1 d \right] = 0 \qquad d = \frac{F_2}{F_1} \cos(\theta)\, e \left(\frac{\sqrt{g^2 + f^2}}{g} \right) \qquad d = 2.03 \text{ ft}$$

Problem 4-84

Two couples act on the frame. Determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4-13) and (b) summing the moments of all the force components about point A.

Given:

$F_1 = 80$ lb $\quad c = 2$ ft $\quad g = 4$

$F_2 = 50$ lb $\quad d = 4$ ft $\quad \theta = 30$ deg

$a = 1$ ft $\quad e = 3$ ft

$b = 3$ ft $\quad f = 3$

Solution:

(a) $\mathbf{M} = \Sigma(\mathbf{r} \times \mathbf{F})$

$$\mathbf{M} = \begin{pmatrix} e \\ 0 \\ 0 \end{pmatrix} \times \left[F_2 \begin{pmatrix} -\sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix} \times \left[\frac{F_1}{\sqrt{f^2 + g^2}} \begin{pmatrix} -g \\ -f \\ 0 \end{pmatrix} \right] \qquad \mathbf{M} = \begin{pmatrix} 0 \\ 0 \\ 126.096 \end{pmatrix} \text{lb}\cdot\text{ft}$$

(b) Summing the moments of all force components aboout point A.

$$M_1 = \left(\frac{-g}{\sqrt{f^2 + g^2}} \right) F_1 b + \left(\frac{g}{\sqrt{f^2 + g^2}} \right) F_1 (b + d)$$

$$M_2 = F_2 \cos(\theta) c - F_2 \sin(\theta)(a + b + d) - F_2 \cos(\theta)(c + e) + F_2 \sin(\theta)(a + b + d)$$

$$M = M_1 + M_2 \qquad M = 126.096 \text{ lb}\cdot\text{ft}$$

Problem 4-85

Two couples act on the frame. Determine the resultant couple moment. Compute the result by resolving each force into x and y components and **(a)** finding the moment of each couple (Eq. 4 -13) and **(b)** summing the moments of all the force components about point B.

Given:

$F_1 = 80$ lb $d = 4$ft

$F_2 = 50$ lb $e = 3$ ft

$a = 1$ ft $f = 3$ ft

$b = 3$ ft $g = 4$ ft

$c = 2$ ft $\theta = 30$ deg

Solution:

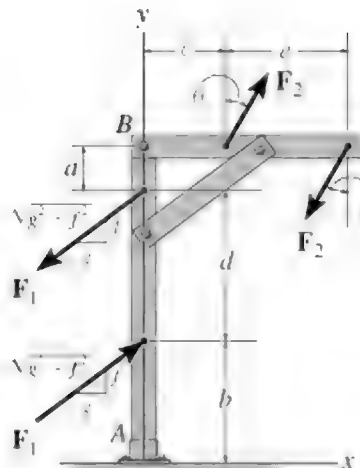
(a) $\mathbf{M} = \Sigma(\mathbf{r} \times \mathbf{F})$

$$\mathbf{M} = \begin{pmatrix} e \\ 0 \\ 0 \end{pmatrix} \times \left[F_2 \begin{pmatrix} -\sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix} \times \left[\frac{F_1}{\sqrt{f^2 + g^2}} \begin{pmatrix} -g \\ -f \\ 0 \end{pmatrix} \right] \qquad \mathbf{M} = \begin{pmatrix} 0 \\ 0 \\ 126.096 \end{pmatrix} \text{lb} \cdot \text{ft}$$

(b) Summing the moments of all force components about point B.

$$M_1 = \left(\frac{g}{\sqrt{f^2 + g^2}} \right) F_1 (a + d) - \left(\frac{g}{\sqrt{f^2 + g^2}} \right) F_1 a$$

$$M_2 = F_2 \cos(\theta) c - F_2 \cos(\theta)(c + e)$$

$$M = M_1 + M_2 \qquad M = 126.096 \text{ lb} \cdot \text{ft}$$

Problem 4-86

Determine the couple moment. Express the result as a Cartesian vector.

Given:

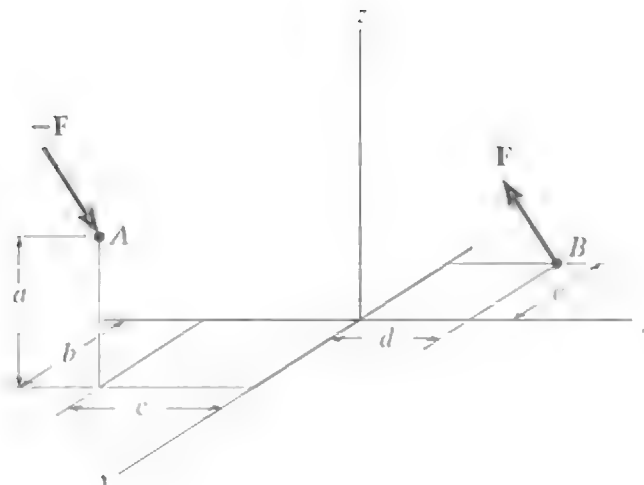
$$\mathbf{F} = \begin{pmatrix} 8 \\ -4 \\ 10 \end{pmatrix} \text{N}$$

$a = 5$ m

$b = 3$ m

$c = 4$ m

$d = 2$ m

$e = 3$ m

Solution:

$$\mathbf{r} = \begin{pmatrix} -b-e \\ c+d \\ -a \end{pmatrix} \qquad \mathbf{M} = \mathbf{r} \times \mathbf{F} \qquad \mathbf{M} = \begin{pmatrix} 40 \\ 20 \\ -24 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-87

Determine the couple moment. Express the result as a Cartesian vector.

Given:

$F = 80$ N

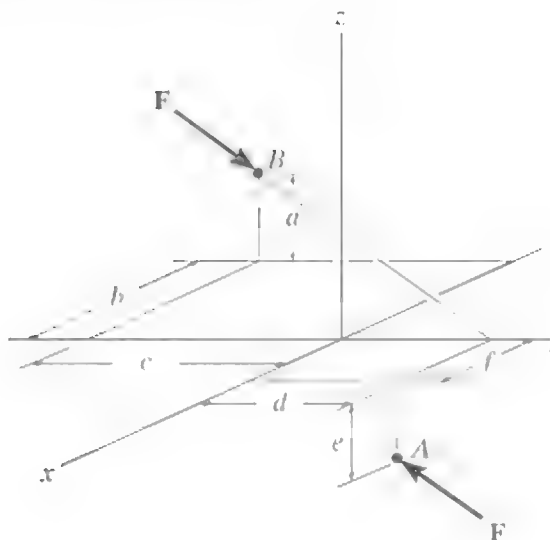
$a = 6$ m

$b = 10$ m

$c = 10$ m

$d = 5$ m

$e = 4$ m

$f = 4$ m

Solution:

$$\mathbf{u} = \frac{1}{\sqrt{a^2 + b^2 + (c+d)^2}}\begin{pmatrix} b \\ c+d \\ -a \end{pmatrix} \qquad \mathbf{F_V} = F\mathbf{u} \qquad \mathbf{r} = \begin{pmatrix} -f-b \\ -d-c \\ e+a \end{pmatrix}$$

$$\mathbf{M} = \mathbf{r} \times \mathbf{F_V} \qquad \mathbf{M} = \begin{pmatrix} -252.6 \\ 67.4 \\ -252.6 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-88

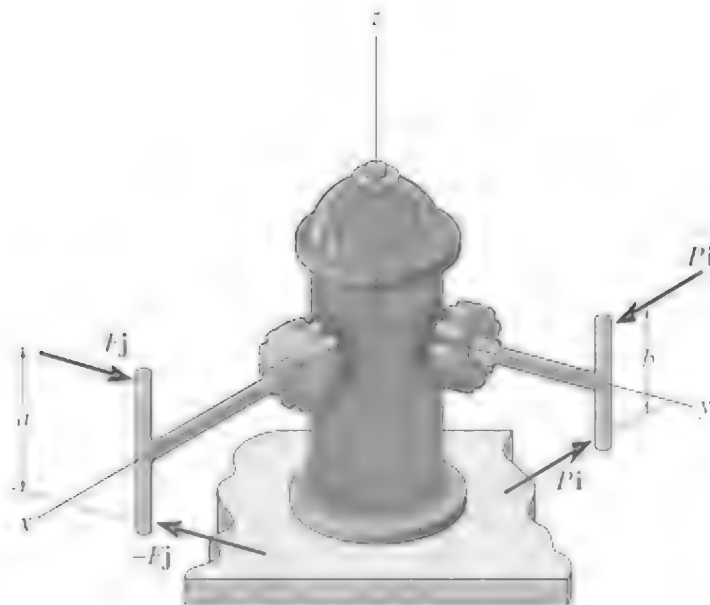
If the resultant couple of the two couples acting on the fire hydrant is $\mathbf{M_R} = \{-15\mathbf{i} + 30\mathbf{j}\}$ N·m, determine the force magnitude P.

Given:

$a = 0.2$ m

$b = 0.150$ m

$$\mathbf{M} = \begin{pmatrix} -15 \\ 30 \\ 0 \end{pmatrix} \text{N}\cdot\text{m}$$

$F = 75$ N

Solution:

Initial guess $\quad P = 1$ N

Given

$$\mathbf{M} = \begin{pmatrix} -Fa \\ Pb \\ 0 \end{pmatrix} \qquad P = \text{Find}(P) \qquad P = 200 \text{ N}$$

Problem 4-89

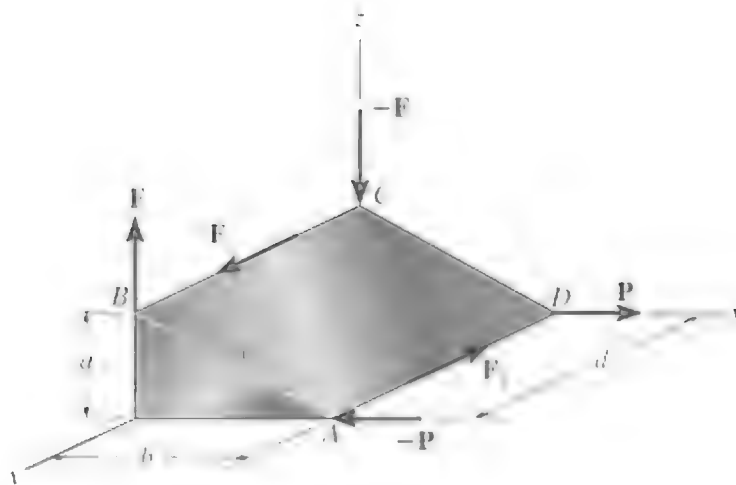
If the resultant couple of the three couples acting on the triangular block is to be zero, determine the magnitude of forces **F** and **P**.

Given:

$F_1 = 150$ N

$a = 300$ mm

$b = 400$ mm

$d = 600$ mm

Solution:

Initial guesses: $F = 1$ N $\quad P = 1$ N

Given $$\begin{pmatrix} d \\ 0 \\ a \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ F \end{pmatrix} + \begin{pmatrix} d \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -P \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} -F_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} \times \begin{pmatrix} F_1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} F \\ P \end{pmatrix} = \text{Find}(F, P) \qquad \begin{pmatrix} F \\ P \end{pmatrix} = \begin{pmatrix} 75 \\ 100 \end{pmatrix} \text{N}$$

Problem 4-90

Determine the couple moment that acts on the assembly. Express the result as a Cartesian vector. Member *BA* lies in the *x-y* plane.

Given:

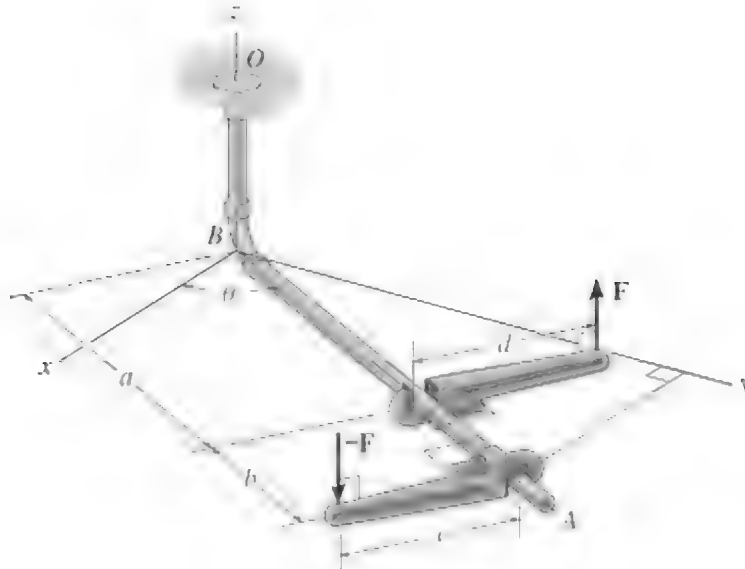
$$\mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ 100 \end{pmatrix} \text{N}$$

$a = 300$ mm

$b = 150$ mm

$c = 200$ mm

$d = 200$ mm

$\theta = 60$ deg

Solution:

$$\mathbf{r} = \begin{bmatrix} -(c+d)\sin(\theta) - b\cos(\theta) \\ -b\sin(\theta) + (c+d)\cos(\theta) \\ 0 \end{bmatrix} \qquad \mathbf{M} = \mathbf{r} \times \mathbf{F} \qquad \mathbf{M} = \begin{pmatrix} 7.01 \\ 42.14 \\ 0.00 \end{pmatrix} \text{N}\cdot\text{m}$$

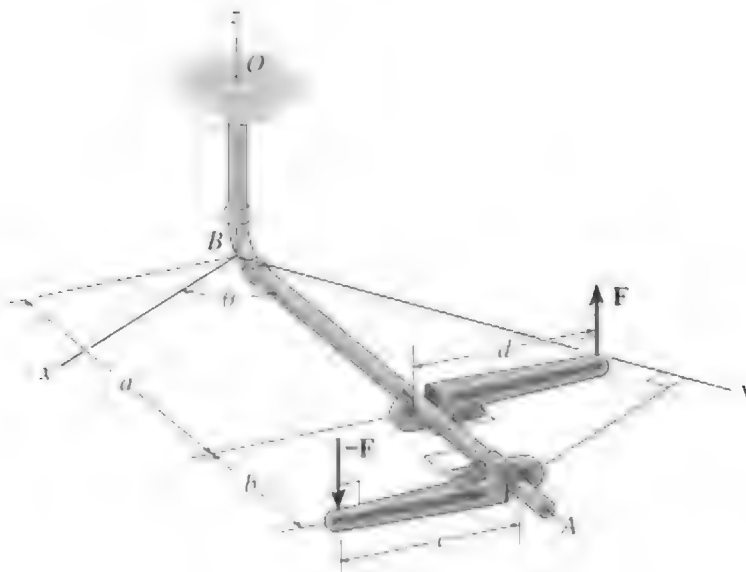
Problem 4-91

If the magnitude of the resultant couple moment is M, determine the magnitude F of the forces applied to the wrenches.

Given:

$M = 15$ N·m $\quad c = 200$ mm

$a = 300$ mm $\quad d = 200$ mm

$b = 150$ mm $\quad \theta = 60$ deg

Solution:

$$\mathbf{r} = \begin{bmatrix} -(c+d)\sin(\theta) - b\cos(\theta) \\ -b\sin(\theta) + (c+d)\cos(\theta) \\ 0 \end{bmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guess $F = 1\ \text{N}$

Given $|\mathbf{r} \times (F\mathbf{k})| = M$ $\quad F = \text{Find}(F) \quad F = 35.112\ \text{N}$

Problem 4-92

The gears are subjected to the couple moments shown. Determine the magnitude and coordinate direction angles of the resultant couple moment.

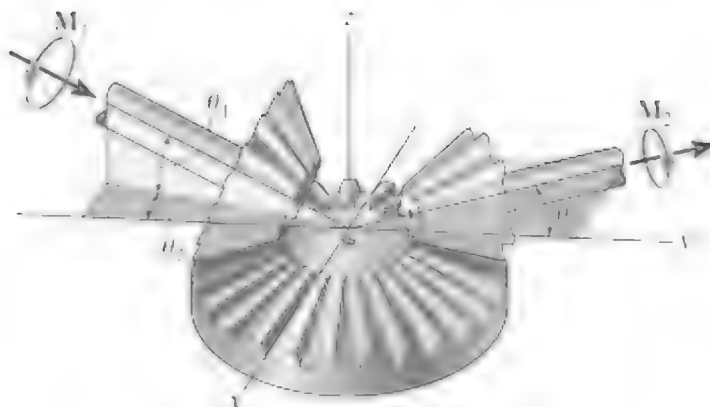
Given:

$M_1 = 40\ \text{lb}\cdot\text{ft}$

$M_2 = 30\ \text{lb}\cdot\text{ft}$

$\theta_1 = 20\ \text{deg}$

$\theta_2 = 15\ \text{deg}$

$\theta_3 = 30\ \text{deg}$

Solution:

$$\mathbf{M_1} = \begin{pmatrix} M_1\cos(\theta_1)\sin(\theta_2) \\ M_1\cos(\theta_1)\cos(\theta_2) \\ -M_1\sin(\theta_1) \end{pmatrix} \qquad \mathbf{M_2} = \begin{pmatrix} -M_2\sin(\theta_3) \\ M_2\cos(\theta_3) \\ 0 \end{pmatrix}$$

$$\mathbf{M_R} = \mathbf{M_1} + \mathbf{M_2} \qquad |\mathbf{M_R}| = 64.0\ \text{lb}\cdot\text{ft}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{M_R}}{|\mathbf{M_R}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 94.7 \\ 13.2 \\ 102.3 \end{pmatrix} \text{deg}$$

Problem 4-93

Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?

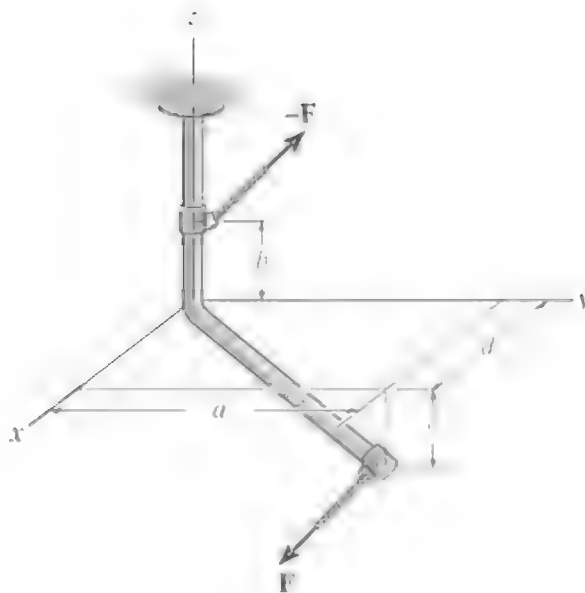
Given:

$$\mathbf{F} = \begin{pmatrix} 14 \\ -8 \\ -6 \end{pmatrix} \text{N}$$

$a = 1.5 \text{ m}$

$b = 0.5 \text{ m}$

$c = 0.5 \text{ m}$

$d = 0.8 \text{ m}$

Solution:

$$\mathbf{M} = \begin{pmatrix} d \\ a \\ -c \end{pmatrix} \times \mathbf{F} + \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times (-\mathbf{F}) \qquad \mathbf{M} = \begin{pmatrix} -17 \\ -9.2 \\ -27.4 \end{pmatrix} \text{N}\cdot\text{m} \qquad |\mathbf{M}| = 33.532 \,\text{N}\cdot\text{m}$$

Problem 4-94

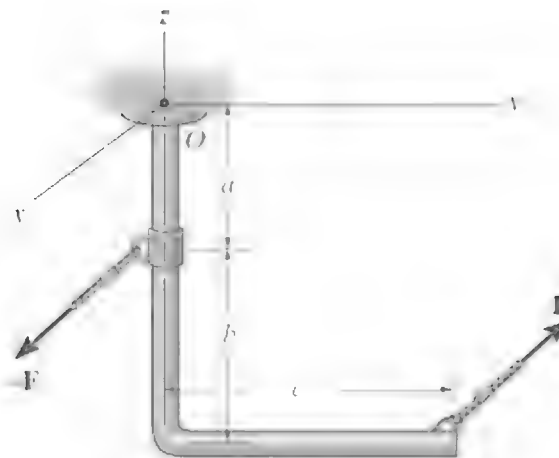
Express the moment of the couple acting on the pipe assembly in Cartesian vector form. Solve the problem (a) using Eq. 4-13, and (b) summing the moment of each force about point *O*.

Given:

$a = 0.3\ \text{m}$

$b = 0.4\ \text{m}$

$c = 0.6\ \text{m}$

$$\mathbf{F} = \begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix} \text{N}$$

Solution:

(a) $$\mathbf{M} = \begin{pmatrix} 0 \\ c \\ -b \end{pmatrix} \times \mathbf{F} \qquad \mathbf{M} = \begin{pmatrix} 2.6 \\ 2.4 \\ 3.6 \end{pmatrix} \text{N} \cdot \text{m}$$

(b) $$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times (-\mathbf{F}) + \begin{pmatrix} 0 \\ c \\ -a-b \end{pmatrix} \times \mathbf{F} \qquad \mathbf{M} = \begin{pmatrix} 2.6 \\ 2.4 \\ 3.6 \end{pmatrix} \text{N} \cdot \text{m}$$

Problem 4-95

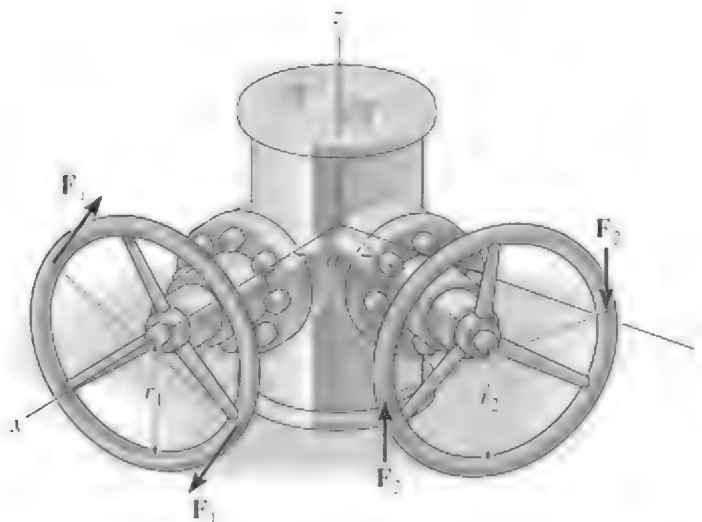
A couple acts on each of the handles of the minidual valve. Determine the magnitude and coordinate direction angles of the resultant couple moment.

Given:

$F_1 = 35\ \text{N} \quad \theta = 60\ \text{deg}$

$F_2 = 25\ \text{N}$

$r_1 = 175\ \text{mm}$

$r_2 = 175\ \text{mm}$

Solution:

$$\mathbf{M} = \begin{pmatrix} -F_1 2\ r_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -F_2 2 r_2 \cos(\theta) \\ -F_2 2\ r_2 \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} -16.63 \\ -7.58 \\ 0 \end{pmatrix} \text{N} \cdot \text{m} \qquad |\mathbf{M}| = 18.3\ \text{N} \cdot \text{m}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{M}}{|\mathbf{M}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 155.496 \\ 114.504 \\ 90 \end{pmatrix} \text{deg}$$

Problem 4-96

Express the moment of the couple acting on the pipe in Cartesian vector form. What is the magnitude of the couple moment?

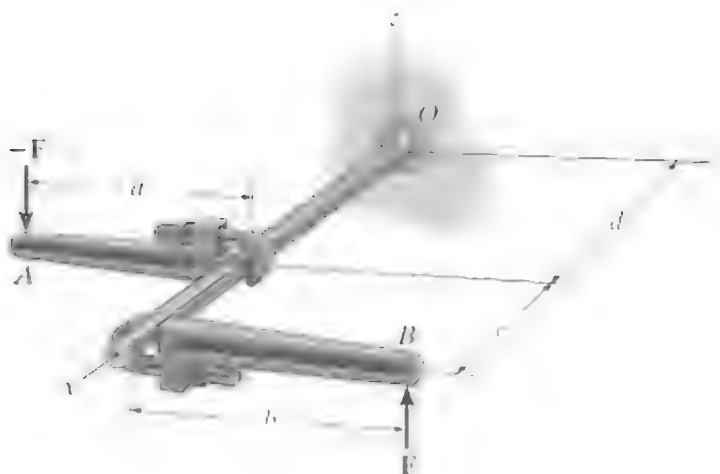
Given:

$F = 125 \text{ N}$

$a = 150 \text{ mm}$

$b = 150 \text{ mm}$

$c = 200 \text{ mm}$

$d = 600 \text{ mm}$

Solution:

$$\mathbf{M} = \begin{pmatrix} c \\ a+b \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ F \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} 37.5 \\ -25 \\ 0 \end{pmatrix} \text{N}\cdot\text{m} \qquad |\mathbf{M}| = 45.1 \text{ N}\cdot\text{m}$$

Problem 4-97

If the couple moment acting on the pipe has a magnitude M, determine the magnitude F of the forces applied to the wrenches.

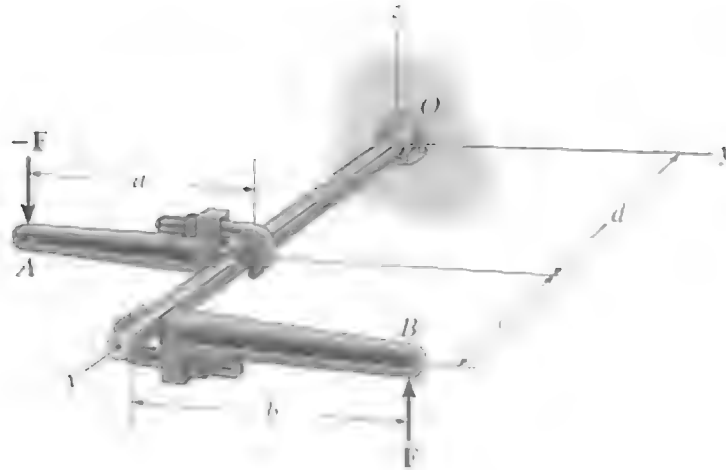
Given:

$M = 300\ \text{N}\cdot\text{m}$

$a = 150\ \text{mm}$

$b = 150\ \text{mm}$

$c = 200\ \text{mm}$

$d = 600\ \text{mm}$

Solution:

Initial guess: $F = 1\ \text{N}$

Given $\left|\begin{pmatrix} c \\ a+b \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ F \end{pmatrix}\right| = M$ $F = \text{Find}(F)$ $F = 832.1\ \text{N}$

Problem 4-98

Replace the force at A by an equivalent force and couple moment at point O.

Given:

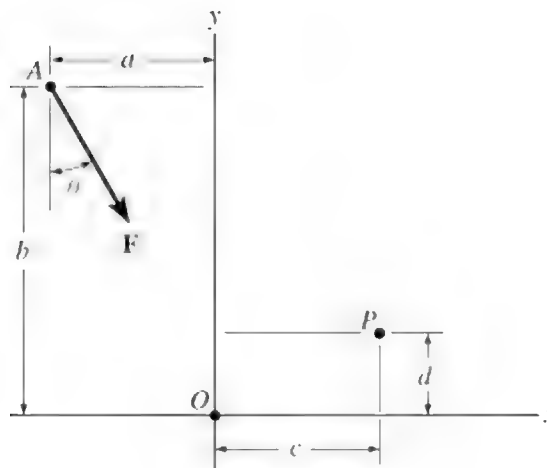
$F = 375\ \text{N}$

$a = 2\ \text{m}$

$b = 4\ \text{m}$

$c = 2\ \text{m}$

$d = 1\ \text{m}$

$\theta = 30\ \text{deg}$

Solution:

$$\mathbf{F_v} = F\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{F_v} = \begin{pmatrix} 187.5 \\ -324.76 \\ 0 \end{pmatrix} \text{N}$$

$$\mathbf{M_O} = \begin{pmatrix} -a \\ b \\ 0 \end{pmatrix} \times \mathbf{F_v} \qquad \mathbf{M_O} = \begin{pmatrix} 0 \\ 0 \\ -100.481 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-99

Replace the force at A by an equivalent force and couple moment at point P.

Given:

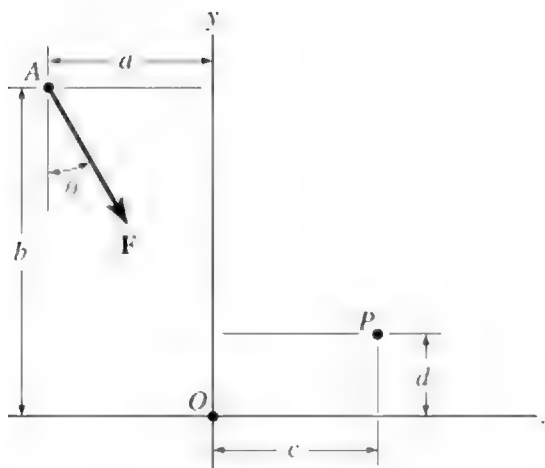
$F = 375$ N

$a = 2$ m

$b = 4$ m

$c = 2$ m

$d = 1$ m

$\theta = 30$ deg

Solution:

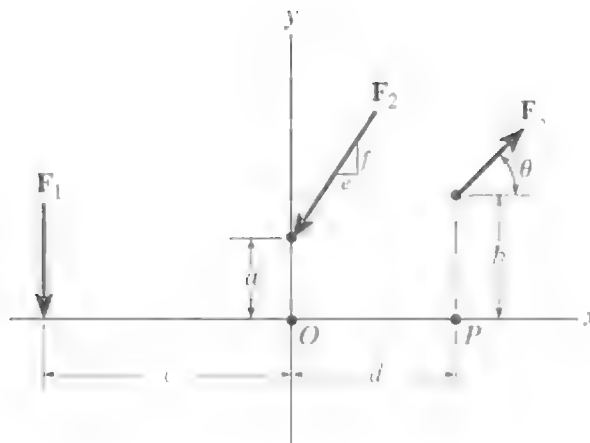
$$\mathbf{F_v} = F\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{F_v} = \begin{pmatrix} 187.5 \\ -324.76 \\ 0 \end{pmatrix} \text{N}$$

$$\mathbf{M_P} = \begin{pmatrix} -a-c \\ b-d \\ 0 \end{pmatrix} \times \mathbf{F_v} \qquad \mathbf{M_P} = \begin{pmatrix} 0 \\ 0 \\ 736.538 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-100

Replace the force system by an equivalent resultant force and couple moment at point O.

Given:

$F_1 = 60\ \text{lb}$ $a = 2\ \text{ft}$

$F_2 = 85\ \text{lb}$ $b = 3\ \text{ft}$

$F_3 = 25\ \text{lb}$ $c = 6\ \text{ft}$

$\theta = 45\ \text{deg}$ $d = 4\ \text{ft}$

$e = 3$

$f = 4$

Solution:

$$\mathbf{F} = \begin{pmatrix} 0 \\ -F_1 \\ 0 \end{pmatrix} + \frac{F_2}{\sqrt{e^2 + f^2}}\begin{pmatrix} -e \\ -f \\ 0 \end{pmatrix} + F_3\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} -33.322 \\ -110.322 \\ 0 \end{pmatrix} \text{lb} \qquad |\mathbf{F}| = 115.245\ \text{lb}$$

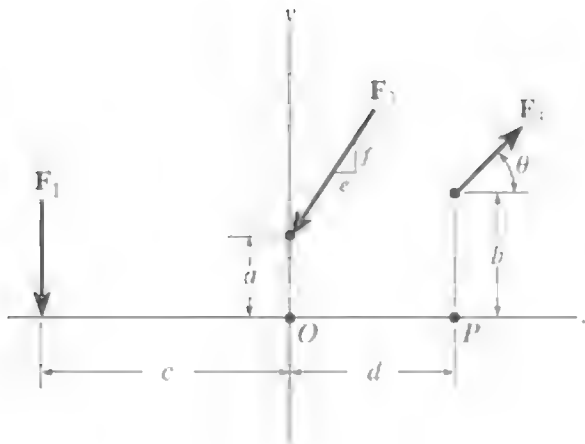
$$\mathbf{M_O} = \begin{pmatrix} -c \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -F_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \left[\frac{F_2}{\sqrt{e^2 + f^2}}\begin{pmatrix} -e \\ -f \\ 0 \end{pmatrix}\right] + \begin{pmatrix} d \\ b \\ 0 \end{pmatrix} \times \left[F_3\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{M_O} = \begin{pmatrix} 0 \\ 0 \\ 480 \end{pmatrix} \text{lb}\cdot\text{ft} \qquad |\mathbf{M_O}| = 480\ \text{lb}\cdot\text{ft}$$

Problem 4-101

Replace the force system by an equivalent resultant force and couple moment at point P.

Given:

$F_1 = 60\ \text{lb}$ $\quad a = 2\ \text{ft}$

$F_2 = 85\ \text{lb}$ $\quad b = 3\ \text{ft}$

$F_3 = 25\ \text{lb}$ $\quad c = 6\ \text{ft}$

$\theta = 45\ \text{deg}$ $\quad d = 4\ \text{ft}$

$e = 3$ $\quad f = 4$

Solution:

$$\mathbf{F} = \begin{pmatrix} 0 \\ -F_1 \\ 0 \end{pmatrix} + \frac{F_2}{\sqrt{e^2+f^2}}\begin{pmatrix} -e \\ -f \\ 0 \end{pmatrix} + F_3\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} -33.322 \\ -110.322 \\ 0 \end{pmatrix} \text{lb} \qquad |\mathbf{F}| = 115.245\ \text{lb}$$

$$\mathbf{M_P} = \begin{pmatrix} -c-d \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -F_1 \\ 0 \end{pmatrix} + \begin{pmatrix} -d \\ a \\ 0 \end{pmatrix} \times \left[\frac{F_2}{\sqrt{e^2+f^2}}\begin{pmatrix} -e \\ -f \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \times \left[F_3\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{M_P} = \begin{pmatrix} 0 \\ 0 \\ 921 \end{pmatrix} \text{lb}\cdot\text{ft} \qquad |\mathbf{M_P}| = 921\ \text{lb}\cdot\text{ft}$$

Problem 4-102

Replace the force system by an equivalent force and couple moment at point O.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$F_1 = 430\ \text{lb}$ $\quad F_2 = 260\ \text{lb}$

$a = 2\ \text{ft}$ $\quad e = 5\ \text{ft}$

$b = 8$ ft $\quad f = 12$

$c = 3$ ft $\quad g = 5$

$d = a \quad \theta = 60$ deg

Solution:

$$\mathbf{F_R} = F_1\begin{pmatrix}-\sin(\theta)\\-\cos(\theta)\\0\end{pmatrix} + \frac{F_2}{\sqrt{g^2+f^2}}\begin{pmatrix}g\\f\\0\end{pmatrix} \qquad \mathbf{F_R} = \begin{pmatrix}-272\\25\\0\end{pmatrix}\text{lb} \qquad |\mathbf{F_R}| = 274\,\text{lb}$$

$$\mathbf{M_O} = \begin{pmatrix}-d\\b\\0\end{pmatrix} \times \left[F_1\begin{pmatrix}-\sin(\theta)\\-\cos(\theta)\\0\end{pmatrix}\right] + \begin{pmatrix}e\\0\\0\end{pmatrix} \times \left[\frac{F_2}{\sqrt{g^2+f^2}}\begin{pmatrix}g\\f\\0\end{pmatrix}\right] \qquad \mathbf{M_O} = \begin{pmatrix}0\\0\\4.609\end{pmatrix}\text{kip}\cdot\text{ft}$$

Problem 4-103

Replace the force system by an equivalent force and couple moment at point P.

Units Used:

$\text{kip} = 10^3$ lb

Given:

$F_1 = 430$ lb $\quad F_2 = 260$ lb

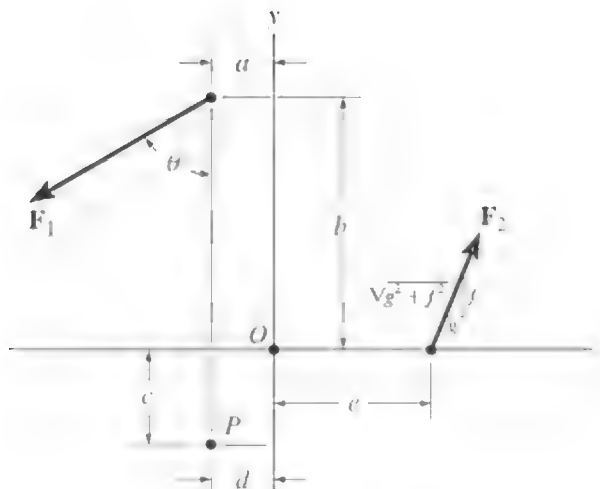
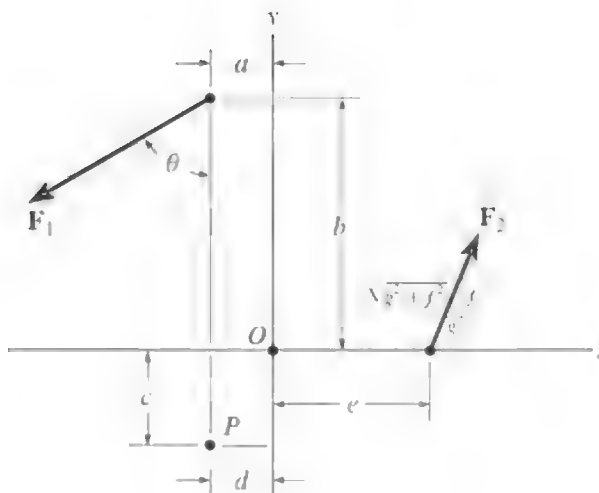
$a = 2$ ft $\quad e = 5$ ft

$b = 8$ ft $\quad f = 12$

$c = 3$ ft $\quad g = 5$

$d = a \quad \theta = 60$ deg

Solution:

$$\mathbf{F_R} = F_1\begin{pmatrix}-\sin(\theta)\\-\cos(\theta)\\0\end{pmatrix} + \frac{F_2}{\sqrt{g^2+f^2}}\begin{pmatrix}g\\f\\0\end{pmatrix} \qquad \mathbf{F_R} = \begin{pmatrix}-272\\25\\0\end{pmatrix}\text{lb} \qquad |\mathbf{F_R}| = 274\,\text{lb}$$

$$\mathbf{M_P} = \begin{pmatrix} 0 \\ b+c \\ 0 \end{pmatrix} \times \left[F_1 \begin{pmatrix} -\sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} d+e \\ c \\ 0 \end{pmatrix} \times \left[\frac{F_2}{\sqrt{g^2+f^2}} \begin{pmatrix} g \\ f \\ 0 \end{pmatrix} \right] \qquad \mathbf{M_P} = \begin{pmatrix} 0 \\ 0 \\ 5.476 \end{pmatrix} \text{kip}\cdot\text{ft}$$

Problem 4-104

Replace the loading system acting on the post by an equivalent resultant force and couple moment at point *O*.

Given:

$F_1 = 30$ lb $\quad a = 1$ ft $\quad d = 3$

$F_2 = 40$ lb $\quad b = 3$ ft $\quad e = 4$

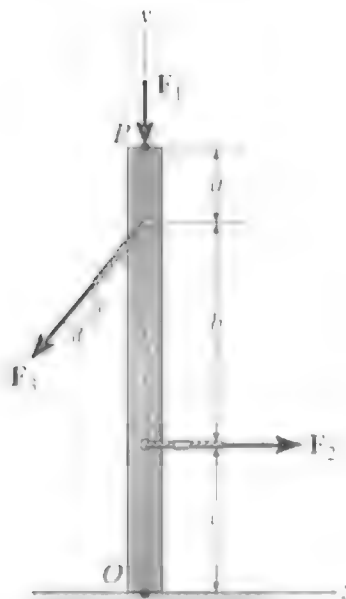
$F_3 = 60$ lb $\quad c = 2$ ft

Solution:

$$\mathbf{F_R} = F_1 \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + F_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{F_3}{\sqrt{d^2+e^2}} \begin{pmatrix} -d \\ -e \\ 0 \end{pmatrix}$$

$$\mathbf{F_R} = \begin{pmatrix} 4 \\ -78 \\ 0 \end{pmatrix} \text{lb} \qquad \left|\mathbf{F_R}\right| = 78.1 \text{ lb}$$

$$\mathbf{M_O} = \begin{pmatrix} 0 \\ a+b+c \\ 0 \end{pmatrix} \times \left[F_1 \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ c \\ 0 \end{pmatrix} \times \left[F_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ b+c \\ 0 \end{pmatrix} \times \left[\frac{F_3}{\sqrt{d^2+e^2}} \begin{pmatrix} -d \\ -e \\ 0 \end{pmatrix} \right]$$

$$\mathbf{M_O} = \begin{pmatrix} 0 \\ 0 \\ 100 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 4-105

Replace the loading system acting on the post by an equivalent resultant force and couple moment at point *P*.

Given:

$F_1 = 30\ \text{lb}$

$F_2 = 40\ \text{lb}$

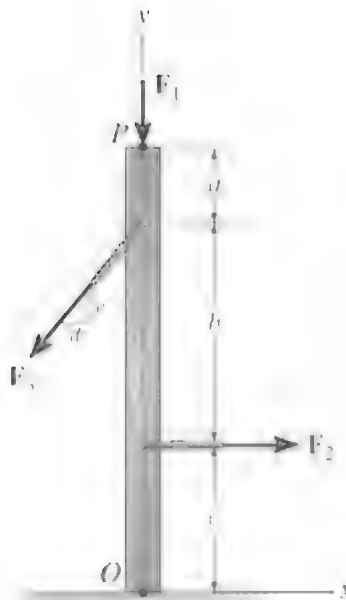
$F_3 = 60\ \text{lb}$

$a = 1\ \text{ft}$

$b = 3\ \text{ft}$

$c = 2\ \text{ft}$

$d = 3$

$e = 4$

Solution:

$$\mathbf{F_R} = F_1\begin{pmatrix}0\\-1\\0\end{pmatrix} + F_2\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{F_3}{\sqrt{d^2+e^2}}\begin{pmatrix}-d\\-e\\0\end{pmatrix}$$

$$\mathbf{F_R} = \begin{pmatrix}4\\-78\\0\end{pmatrix}\text{lb} \qquad \left|\mathbf{F_R}\right| = 78.1\ \text{lb}$$

$$\mathbf{M_P} = \left[\begin{pmatrix}0\\0\\0\end{pmatrix}\text{ft}\right] \times \left[F_1\begin{pmatrix}0\\-1\\0\end{pmatrix}\right] + \begin{pmatrix}0\\-a-b\\0\end{pmatrix} \times \left[F_2\begin{pmatrix}1\\0\\0\end{pmatrix}\right] + \begin{pmatrix}0\\-a\\0\end{pmatrix} \times \left[\frac{F_3}{\sqrt{d^2+e^2}}\begin{pmatrix}-d\\-e\\0\end{pmatrix}\right]$$

$$\mathbf{M_P} = \begin{pmatrix}0\\0\\124\end{pmatrix}\text{lb}\cdot\text{ft}$$

Problem 4-106

Replace the force and couple system by an equivalent force and couple moment at point O.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

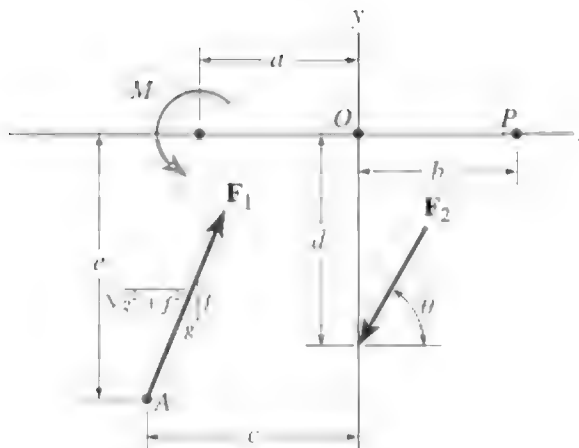
$M = 8\ \text{kN m}$ $\quad\theta = 60\ \text{deg}$

$a = 3\ \text{m}$ $\quad f = 12$

$b = 3\ \text{m}$ $\quad g = 5$

$c = 4\ \text{m}$ $\quad F_1 = 6\ \text{kN}$

$d = 4\text{m}$ $\quad F_2 = 4\ \text{kN}$

$e = 5\ \text{m}$

Solution:

$$\mathbf{F_R} = \frac{F_1}{\sqrt{f^2 + g^2}}\begin{pmatrix} g \\ f \\ 0 \end{pmatrix} + F_2\begin{pmatrix} -\cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{F_R} = \begin{pmatrix} 0.308 \\ 2.074 \\ 0 \end{pmatrix}\text{kN} \qquad |\mathbf{F_R}| = 2.097\ \text{kN}$$

$$\mathbf{M_O} = \begin{pmatrix} 0 \\ 0 \\ M \end{pmatrix} + \begin{pmatrix} -c \\ -e \\ 0 \end{pmatrix} \times \left[\frac{F_1}{\sqrt{f^2 + g^2}}\begin{pmatrix} g \\ f \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} \times \left[F_2\begin{pmatrix} -\cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{M_O} = \begin{pmatrix} 0 \\ 0 \\ -10.615 \end{pmatrix}\text{kN}\cdot\text{m}$$

Problem 4-107

Replace the force and couple system by an equivalent force and couple moment at point P.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

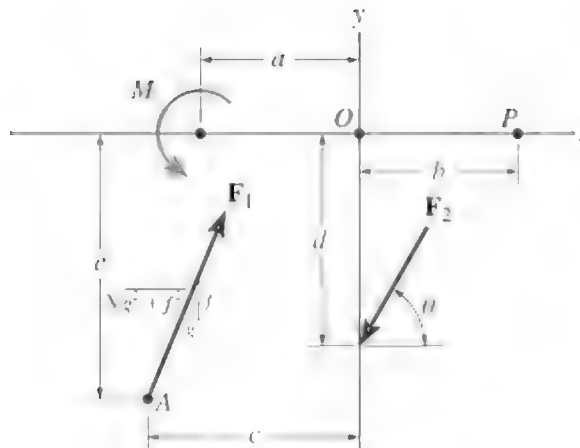
$M = 8\ \text{kN·m}$ $\theta = 60\ \text{deg}$

$a = 3\ \text{m}$ $f = 12$

$b = 3\ \text{m}$ $g = 5$

$c = 4\ \text{m}$ $F_1 = 6\ \text{kN}$

$d = 4\ \text{m}$ $F_2 = 4\ \text{kN}$

$e = 5\ \text{m}$

Solution:

$$\mathbf{F_R} = \frac{F_1}{\sqrt{f^2+g^2}}\begin{pmatrix} g \\ f \\ 0 \end{pmatrix} + F_2\begin{pmatrix} -\cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{F_R} = \begin{pmatrix} 0.308 \\ 2.074 \\ 0 \end{pmatrix}\text{kN} \qquad |\mathbf{F_R}| = 2.097\ \text{kN}$$

$$\mathbf{M_P} = \begin{pmatrix} 0 \\ 0 \\ M \end{pmatrix} + \begin{pmatrix} -c-b \\ -e \\ 0 \end{pmatrix} \times \left[\frac{F_1}{\sqrt{f^2+g^2}}\begin{pmatrix} g \\ f \\ 0 \end{pmatrix}\right] + \begin{pmatrix} -b \\ -d \\ 0 \end{pmatrix} \times \left[F_2\begin{pmatrix} -\cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{M_P} = \begin{pmatrix} 0 \\ 0 \\ -16.838 \end{pmatrix}\text{kN·m}$$

Problem 4-108

Replace the force system by a single force resultant and specify its point of application, measured along the *x* axis from point *O*.

Given:

$F_1 = 125\ \text{lb}$

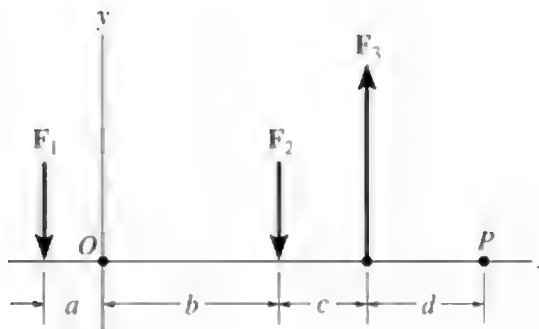
$F_2 = 350\ \text{lb}$

$F_3 = 850\ \text{lb}$

$a = 2\ \text{ft}$

$b = 6\ \text{ft}$

$c = 3\ \text{ft}$

$d = 4\ \text{ft}$

Solution:

$F_{Ry} = F_3 - F_2 - F_1$ $\qquad F_{Ry} = 375\ \text{lb}$

$F_{Ry}x = F_3(b + c) - F_2(b) + F_1(a)$

$$x = \frac{F_3(b + c) - F_2(b) + F_1\ a}{F_{Ry}} \qquad x = 15.5\ \text{ft}$$

Problem 4-109

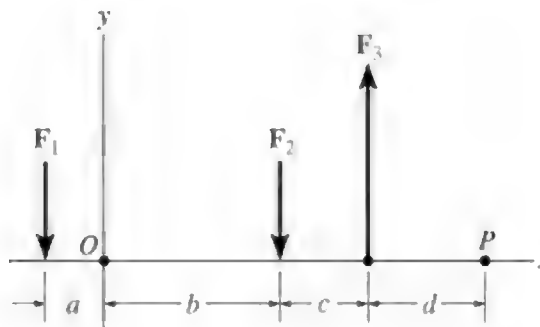
Replace the force system by a single force resultant and specify its point of application, measured along the x axis from point P.

Given:

$F_1 = 125\ \text{lb}$ $\quad a = 2\ \text{ft}$

$F_2 = 350\ \text{lb}$ $\quad b = 6\ \text{ft}$

$F_3 = 850\ \text{lb}$ $\quad c = 3\ \text{ft}$

$d = 4\ \text{ft}$

Solution:

$F_{Ry} = F_3 - F_2 - F_1$ $\qquad F_{Ry} = 375\ \text{lb}$

$F_{Ry}x = F_2(d + c) - F_3(d) + F_1(a + b + c + d)$

$$x = \frac{F_2\ d + F_2\ c - F_3\ d + F_1\ a + F_1\ b + F_1\ c + F_1\ d}{F_{Ry}}$$

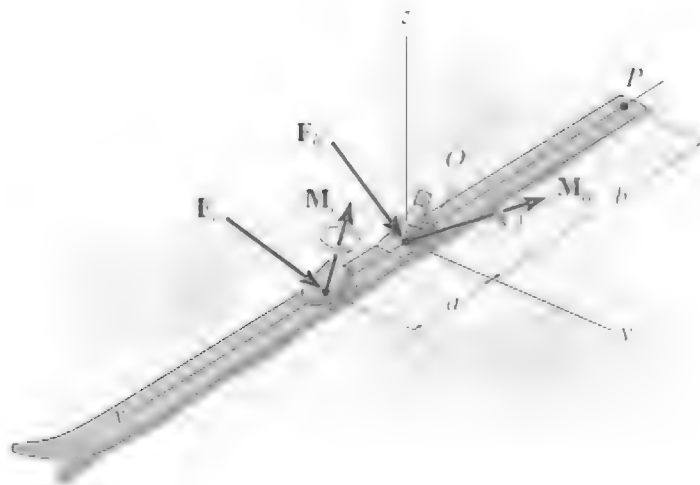
$x = 2.47\ \text{ft}$ (to the right of P)

Problem 4-110

The forces and couple moments which are exerted on the toe and heel plates of a snow ski are $\mathbf{F}_t$, $\mathbf{M}_t$, and $\mathbf{F}_h$, $\mathbf{M}_h$, respectively. Replace this system by an equivalent force and couple moment acting at point O. Express the results in Cartesian vector form.

Given:

$a = 120\text{ mm}$

$b = 800\text{ mm}$

Solution:

$$\mathbf{F_t} = \begin{pmatrix} -50 \\ 80 \\ -158 \end{pmatrix}\text{N} \quad \mathbf{F_h} = \begin{pmatrix} -20 \\ 60 \\ -250 \end{pmatrix}\text{N} \quad \mathbf{M_t} = \begin{pmatrix} -6 \\ 4 \\ 2 \end{pmatrix}\text{N}\cdot\text{m} \quad \mathbf{M_h} = \begin{pmatrix} -20 \\ 8 \\ 3 \end{pmatrix}\text{N}\cdot\text{m}$$

$$\mathbf{F_R} = \mathbf{F_t} + \mathbf{F_h}$$

$$\mathbf{F_R} = \begin{pmatrix} -70 \\ 140 \\ -408 \end{pmatrix}\text{N}$$

$$\mathbf{r_{0Ft}} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{M_{RP}} = (\mathbf{r_{0Ft}} \times \mathbf{F_t}) + \mathbf{M_t} + \mathbf{M_h}$$

$$\mathbf{M_{RP}} = \begin{pmatrix} -26 \\ 31 \\ 14.6 \end{pmatrix}\text{N}\cdot\text{m}$$

Problem 4-111

The forces and couple moments which are exerted on the toe and heel plates of a snow ski are $\mathbf{F}_t$, $\mathbf{M}_t$, and $\mathbf{F}_h$, $\mathbf{M}_h$, respectively. Replace this system by an equivalent force and couple moment

acting at point P. Express the results in Cartesian vector form.

Given:

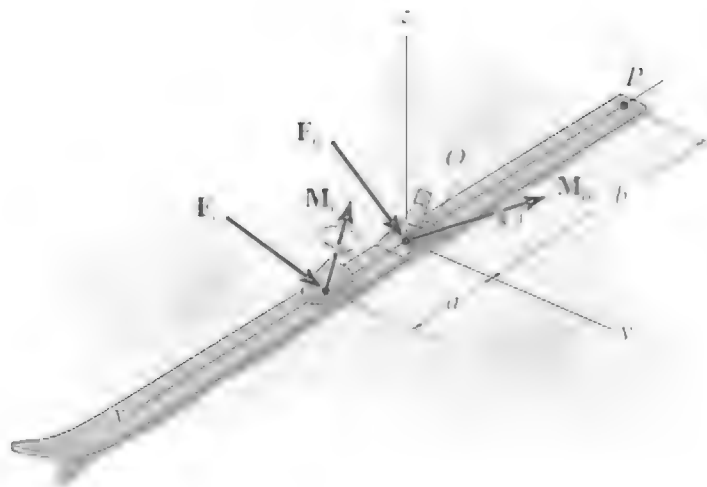
$a = 120\ \text{mm}$

$b = 800\ \text{mm}$

$\mathbf{F_t} = \begin{pmatrix} -50 \\ 80 \\ -158 \end{pmatrix} \text{N}$

$\mathbf{M_t} = \begin{pmatrix} -6 \\ 4 \\ 2 \end{pmatrix} \text{N}\cdot\text{m}$

$\mathbf{F_h} = \begin{pmatrix} -20 \\ 60 \\ -250 \end{pmatrix} \text{N}$

$\mathbf{M_h} = \begin{pmatrix} -20 \\ 8 \\ 3 \end{pmatrix} \text{N}\cdot\text{m}$

Solution:

$\mathbf{F_R} = \mathbf{F_t} + \mathbf{F_h}$ $\qquad \mathbf{F_R} = \begin{pmatrix} -70 \\ 140 \\ -408 \end{pmatrix} \text{N}$

$\mathbf{M_P} = \mathbf{M_t} + \mathbf{M_h} + \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \times \mathbf{F_h} + \begin{pmatrix} a+b \\ 0 \\ 0 \end{pmatrix} \times \mathbf{F_t}$ $\qquad \mathbf{M_P} = \begin{pmatrix} -26 \\ 357.4 \\ 126.6 \end{pmatrix} \text{N}\cdot\text{m}$

Problem 4-112

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end B.

Given:

$F_1 = 500\ \text{lb}$

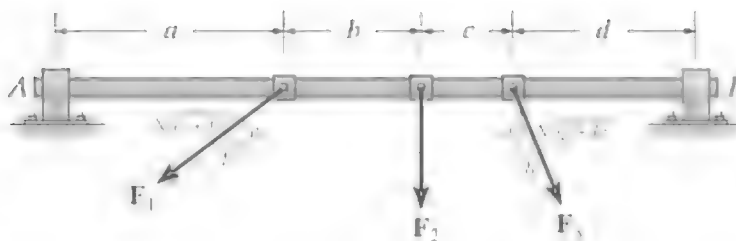
$F_2 = 200\ \text{lb}$

$F_3 = 260\ \text{lb}$

$a = 5\ \text{ft}$ $e = 3$

$b = 3\ \text{ft}$ $f = 4$

$c = 2\ \text{ft}$ $g = 12$

$d = 4\ \text{ft}$ $h = 5$

Solution:

$$\mathbf{F_R} = \frac{F_1}{\sqrt{e^2+f^2}}\begin{pmatrix}-f\\-e\\0\end{pmatrix} + F_2\begin{pmatrix}0\\-1\\0\end{pmatrix} + \frac{F_3}{\sqrt{g^2+h^2}}\begin{pmatrix}h\\-g\\0\end{pmatrix} \qquad \mathbf{F_R} = \begin{pmatrix}-300\\-740\\0\end{pmatrix}\text{lb} \qquad |\mathbf{F_R}| = 798\ \text{lb}$$

Initial guess: $x = 1\ \text{ft}$

Given

$$\begin{pmatrix}a\\0\\0\end{pmatrix}\times\left[\frac{F_1}{\sqrt{e^2+f^2}}\begin{pmatrix}-f\\-e\\0\end{pmatrix}\right] + \begin{pmatrix}a+b\\0\\0\end{pmatrix}\times\left[F_2\begin{pmatrix}0\\-1\\0\end{pmatrix}\right] + \begin{pmatrix}a+b+c\\0\\0\end{pmatrix}\times\left[\frac{F_3}{\sqrt{g^2+h^2}}\begin{pmatrix}h\\-g\\0\end{pmatrix}\right] = \begin{pmatrix}-x\\0\\0\end{pmatrix}\times\mathbf{F_R}$$

$x = \text{Find}(x)$ $x = -7.432\ \text{ft}$

Problem 4-113

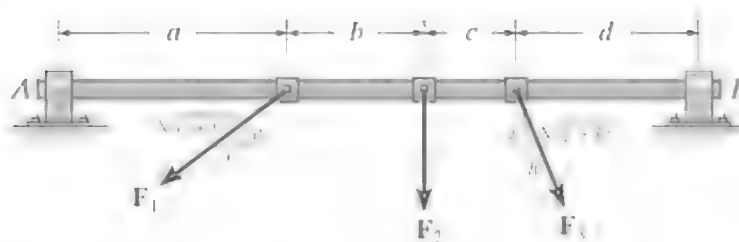
Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end B.

Given:

$F_1 = 500\ \text{lb}$

$F_2 = 200\ \text{lb}$

$F_3 = 260\ \text{lb}$

$a = 5\ \text{ft}$ $e = 3$

$b = 3\text{ ft}$ $\quad f = 4$

$c = 2\text{ ft}$ $\quad g = 12$

$d = 4\text{ ft}$ $\quad h = 5$

Solution:

$$\mathbf{F_R} = \frac{F_1}{\sqrt{e^2+f^2}}\begin{pmatrix}-f\\-e\\0\end{pmatrix} + F_2\begin{pmatrix}0\\-1\\0\end{pmatrix} + \frac{F_3}{\sqrt{g^2+h^2}}\begin{pmatrix}h\\-g\\0\end{pmatrix} \qquad \mathbf{F_R} = \begin{pmatrix}-300\\-740\\0\end{pmatrix}\text{lb} \qquad |\mathbf{F_R}| = 798\text{ lb}$$

Initial guess: $\quad x = 1\text{ft}$

Given

$$\begin{pmatrix}-b-c-d\\0\\0\end{pmatrix} \times \left[\frac{F_1}{\sqrt{e^2+f^2}}\begin{pmatrix}-f\\-e\\0\end{pmatrix}\right] + \begin{pmatrix}-c-d\\0\\0\end{pmatrix} \times \left[F_2\begin{pmatrix}0\\-1\\0\end{pmatrix}\right] + \begin{pmatrix}-d\\0\\0\end{pmatrix} \times \left[\frac{F_3}{\sqrt{g^2+h^2}}\begin{pmatrix}h\\-g\\0\end{pmatrix}\right] = \begin{pmatrix}-x\\0\\0\end{pmatrix} \times \mathbf{F}$$

$x = \text{Find}(x)$ $\quad x = 6.568\text{ ft}$ $\quad$ measured to the left of B

Problem 4-114

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.

Given:

$F_1 = 300\text{ lb}$ $\quad M = 600\text{ lb}\cdot\text{ft}$

$a = 3\text{ ft}$

$F_2 = 200\text{ lb}$

$b = 4\text{ ft}$

$F_3 = 400\text{ lb}$

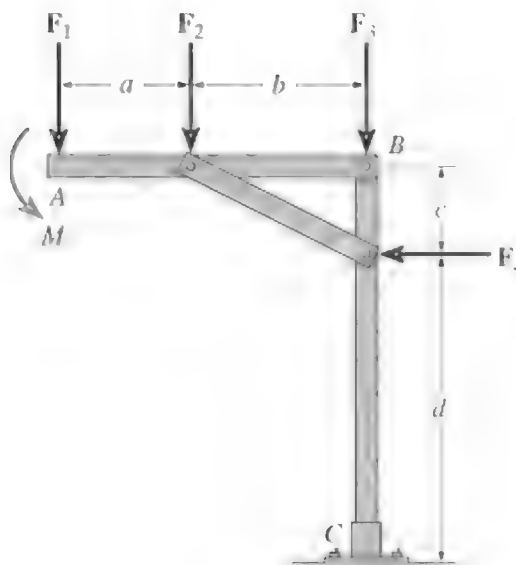
$c = 2\text{ ft}$

$F_4 = 200\text{ lb}$

$d = 7\text{ ft}$

Solution:

$F_{Rx} = -F_4$ $\qquad F_{Rx} = -200\text{ lb}$

$F_{Ry} = -F_1 - F_2 - F_3$ $\qquad F_{Ry} = -900\text{ lb}$

$F = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $F = 922\,\text{lb}$ $\theta = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right)$ $\theta = 77.5\,\text{deg}$

$F_{Ry}x = -F_2\,a - F_3(a+b) - F_4\,c + M$

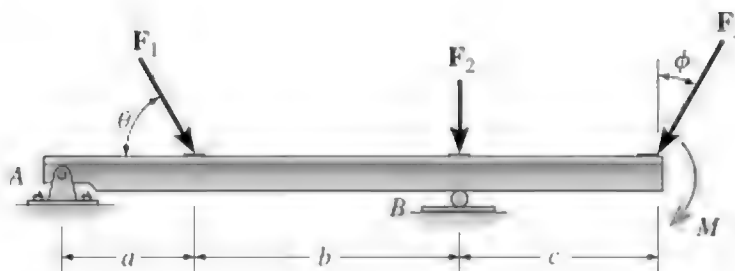
$$x = -\frac{F_2(a) + F_3(a+b) + F_4\,c - M}{F_{Ry}}$$ $x = 3.556\,\text{ft}$

Problem 4-115

Replace the loading on the frame by a single resultant force. Specify where the force acts, measured from end A.

Given:

$F_1 = 450\ \text{N}$ $a = 2\ \text{m}$

$F_2 = 300\ \text{N}$ $b = 4\ \text{m}$

$F_3 = 700\ \text{N}$ $c = 3\ \text{m}$

$\theta = 60\ \text{deg}$ $M = 1500\ \text{N}\cdot\text{m}$

$\phi = 30\ \text{deg}$

Solution:

$F_{Rx} = F_1\cos(\theta) - F_3\sin(\phi)$ $F_{Rx} = -125\ \text{N}$

$F_{Ry} = -F_1\sin(\theta) - F_3\cos(\phi) - F_2$ $F_{Ry} = -1.296 \times 10^3\ \text{N}$

$F = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $F = 1.302 \times 10^3\ \text{N}$

$\theta_1 = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right)$ $\theta_1 = 84.5\,\text{deg}$

$F_{Ry}(x) = -F_1\sin(\theta)a - F_2(a+b) - F_3\cos(\phi)(a+b+c) - M$

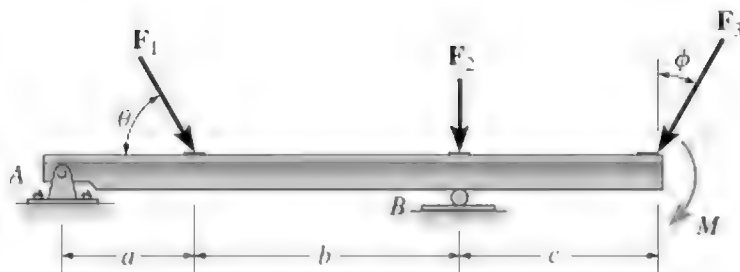
$$x = \frac{-F_1\sin(\theta)a - F_2(a+b) - F_3\cos(\phi)(a+b+c) - M}{F_{Ry}}$$ $x = 7.36\ \text{m}$

Problem 4-116

Replace the loading on the frame by a single resultant force. Specify where the force acts, measured from end B.

Given:

$F_1 = 450\ \text{N}$ $a = 2\ \text{m}$

$F_2 = 300\ \text{N}$ $b = 4\ \text{m}$

$F_3 = 700\ \text{N}$ $c = 3\ \text{m}$

$\theta = 60\ \text{deg}$ $M = 1500\ \text{N·m}$

$\phi = 30\text{deg}$

Solution:

$F_{Rx} = F_1 \cos(\theta) - F_3 \sin(\phi)$ $F_{Rx} = -125\ \text{N}$

$F_{Ry} = -F_1 \sin(\theta) - F_3 \cos(\phi) - F_2$ $F_{Ry} = -1.296 \times 10^3\ \text{N}$

$F = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $F = 1.302 \times 10^3\ \text{N}$

$\theta_1 = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right)$ $\theta_1 = 84.5\ \text{deg}$

$F_{Ry}x = F_1 \sin(\theta)b - F_3 \cos(\phi)c - M$

$x = \frac{F_1 \sin(\theta)b - F_3 \cos(\phi)c - M}{F_{Ry}}$ $x = 1.36\ \text{m}$ (to the right)

Problem 4-117

Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point O.

Given:

$F_1 = 200\ \text{N}$

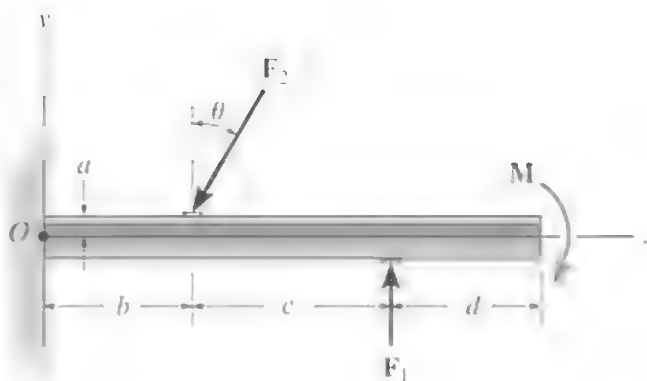
$F_2 = 450\ \text{N}$

$M = 200\ \text{N·m}$

$a = 0.2\ \text{m}$

$b = 1.5\ \text{m}$

$c = 2\ \text{m}$

$d = 1.5\ \text{m}$

$\theta = 30 \text{ deg}$

Solution:

$$\mathbf{F_R} = F_1\begin{pmatrix}0\\1\\0\end{pmatrix} + F_2\begin{pmatrix}-\sin(\theta)\\-\cos(\theta)\\0\end{pmatrix}$$

$$\mathbf{F_R} = \begin{pmatrix}-225\\-190\\0\end{pmatrix} \text{N} \qquad |\mathbf{F_R}| = 294 \text{ N}$$

$$\mathbf{M_O} = \begin{pmatrix}b+c\\a\\0\end{pmatrix} \times \left[F_1\begin{pmatrix}0\\1\\0\end{pmatrix}\right] + \begin{pmatrix}b\\a\\0\end{pmatrix} \times \left[F_2\begin{pmatrix}-\sin(\theta)\\-\cos(\theta)\\0\end{pmatrix}\right] + M\begin{pmatrix}0\\0\\-1\end{pmatrix}$$

$$\mathbf{M_O} = \begin{pmatrix}0\\0\\-39.6\end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-118

Determine the magnitude and direction θ of force **F** and its placement d on the beam so that the loading system is equivalent to a resultant force $\mathbf{F}_R$ acting vertically downward at point A and a clockwise couple moment M.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

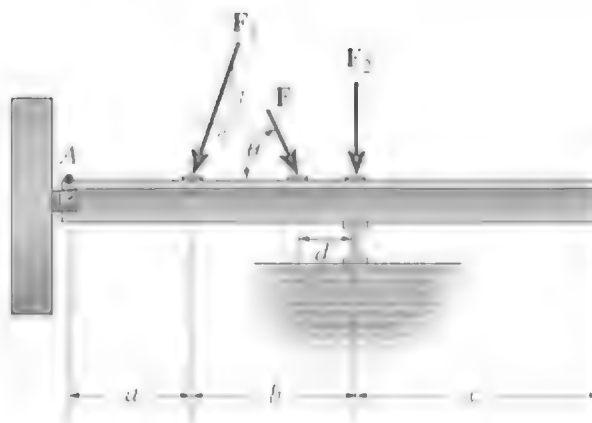
$F_1 = 5 \text{ kN} \qquad a = 3 \text{ m}$

$F_2 = 3 \text{ kN} \qquad b = 4 \text{ m}$

$F_R = 12 \text{ kN} \qquad c = 6 \text{ m}$

$M = 50 \text{ kN}\cdot\text{m} \qquad e = 7 \qquad f = 24$

Solution:

Initial guesses: $F = 1 \text{ kN} \qquad \theta = 30 \text{ deg} \qquad d = 2 \text{ m}$

Given $\left(\frac{-e}{\sqrt{e^2+f^2}}\right)F_1 + F\cos(\theta) = 0$

$$\left(\frac{-f}{\sqrt{e^2+f^2}}\right)F_1 - F\sin(\theta) - F_2 = -F_R$$

$$\left(\frac{f}{\sqrt{e^2+f^2}}\right)F_1 a + F\sin(\theta)(a+b-d) + F_2(a+b) = M$$

$$\begin{pmatrix} F \\ \theta \\ d \end{pmatrix} = \text{Find}(F, \theta, d) \qquad F = 4.427\ \text{kN} \qquad \theta = 71.565\ \text{deg} \qquad d = 3.524\ \text{m}$$

Problem 4-119

Determine the magnitude and direction θ of force **F** and its placement d on the beam so that the loading system is equivalent to a resultant force $\mathbf{F}_R$ acting vertically downward at point A and a clockwise couple moment M.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

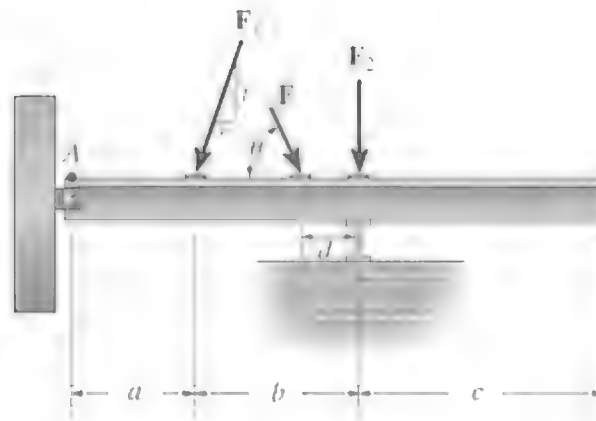
Given:

$F_1 = 5\ \text{kN}$ $\quad a = 3\ \text{m}$

$F_2 = 3\ \text{kN}$ $\quad b = 4\ \text{m}$

$F_R = 10\ \text{kN}$ $\quad c = 6\ \text{m}$

$M = 45\ \text{kN}\cdot\text{m}$ $\quad e = 7$

$f = 24$

Solution:

Initial guesses: $\quad F = 1\ \text{kN} \qquad \theta = 30\ \text{deg} \qquad d = 1\ \text{m}$

Given $\left(\frac{-e}{\sqrt{e^2+f^2}}\right)F_1 + F\cos(\theta) = 0$

$$\left(\frac{-f}{\sqrt{e^2+f^2}}\right)F_1 - F\sin(\theta) - F_2 = -F_R$$

$$\left(\frac{f}{\sqrt{e^2+f^2}}\right)F_1\,a + F\sin(\theta)(a+b-d) + F_2(a+b) = M$$

$$\begin{pmatrix} F \\ \theta \\ d \end{pmatrix} = \text{Find}(F, \theta, d) \qquad F = 2.608\ \text{kN} \qquad \theta = 57.529\ \text{deg} \qquad d = 2.636\ \text{m}$$

Problem 4-120

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.

Given:

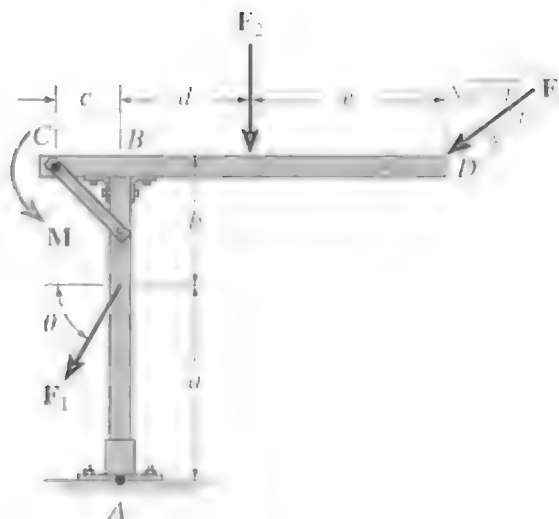
$F_1 = 500\ \text{N}$ $\quad a = 3\ \text{m}$

$F_2 = 300\ \text{N}$ $\quad b = 2\ \text{m}$

$c = 1\ \text{m}$

$F_3 = 250\ \text{N}$ $\quad d = 2\ \text{m}$

$M = 400\ \text{N}\cdot\text{m}$ $\quad e = 3\ \text{m}$

$\theta = 60\ \text{deg}$ $\quad f = 3$

$g = 4$

Solution:

$$F_{Rx} = -F_3\left(\frac{g}{\sqrt{g^2+f^2}}\right) - F_1(\cos(\theta)) \qquad F_{Rx} = -450\ \text{N}$$

$$F_{Ry} = -F_2 - F_3\left(\frac{f}{\sqrt{f^2+g^2}}\right) - F_1\sin(\theta) \qquad F_{Ry} = -883.0127\ \text{N}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F_R = 991.066\ \text{N}$$

$$\theta_1 = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \theta_1 = 62.996\ \text{deg}$$

$$-F_{Rx}(y) = M + F_1\cos(\theta)a + F_3\frac{g}{\sqrt{g^2+f^2}}(b+a) - F_2(d) - F_3\left(\frac{f}{\sqrt{g^2+f^2}}\right)(d+e)$$

$$y = \frac{M + F_1\cos(\theta)a + F_3\frac{g}{\sqrt{g^2+f^2}}(b+a) - F_2(d) - F_3\left(\frac{f}{\sqrt{g^2+f^2}}\right)(d+e)}{-F_{Rx}}$$

$$y = 1.78 \text{ m}$$

Problem 4-121

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member *CD*, measured from end *C*.

Given:

$F_1 = 500$ N	$a = 3$ m
$F_2 = 300$ N	$b = 2$ m
$F_3 = 250$ N	$c = 1$ m
$M = 400$ N·m	$d = 2$ m
$\theta = 60$ deg	$e = 3$ m
	$f = 3$
	$g = 4$

Solution:

$$F_{Rx} = -F_3\left(\frac{g}{\sqrt{g^2+f^2}}\right) - F_1(\cos(\theta)) \qquad F_{Rx} = -450 \text{ N}$$

$$F_{Ry} = -F_2 - F_3\left(\frac{f}{\sqrt{f^2+g^2}}\right) - F_1\sin(\theta) \qquad F_{Ry} = -883.0127 \text{ N}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F_R = 991.066 \text{ N}$$

$$\theta_1 = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \theta_1 = 62.996\,\text{deg}$$

$$F_{Ry}(x) = M - F_2(d+c) - F_3\left(\frac{f}{\sqrt{g^2+f^2}}\right)(c+d+e) - F_1(b)\cos(\theta) - F_1 c\sin(\theta)$$

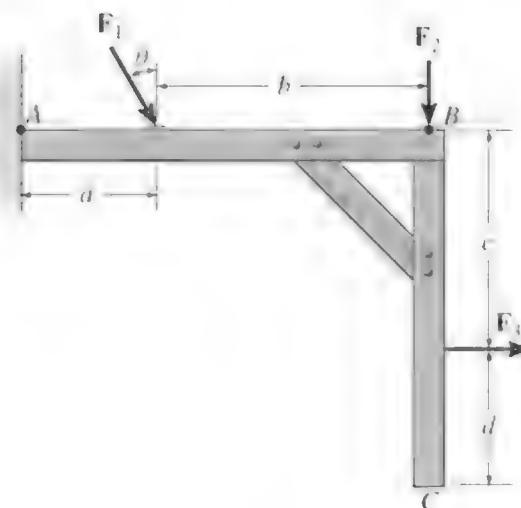
$$x = \frac{M - F_2(d+c) - F_3\left(\frac{f}{\sqrt{g^2+f^2}}\right)(c+d+e) - F_1(b)\cos(\theta) - F_1 c\sin(\theta)}{F_{Ry}}$$

$$x = 2.64\text{ m}$$

Problem 4-122

Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *AB*, measured from point *A*.

Given:

$F_1 = 35$ lb $\quad a = 2$ ft

$F_2 = 20$ lb $\quad b = 4$ ft

$F_3 = 25$ lb $\quad c = 3$ ft

$\theta = 30$ deg $\quad d = 2$ ft

Solution:

$F_{Rx} = F_1\sin(\theta) + F_3 \qquad F_{Rx} = 42.5\text{ lb}$

$F_{Ry} = -F_1\cos(\theta) - F_2 \qquad F_{Ry} = -50.311\text{ lb}$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F_R = 65.9\text{ lb}$

$\theta_1 = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \theta_1 = -49.8\text{ deg}$

$F_{Ry}x = -F_1\cos(\theta)a - F_2(a+b) + F_3(c)$

$$x = \frac{-F_1\cos(\theta)a - F_2(a+b) + F_3(c)}{F_{Ry}}$$ $x = 2.099$ ft

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Problem 4-123

Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *BC*, measured from point *B*.

Given:

$F_1 = 35$ lb

$F_2 = 20$ lb

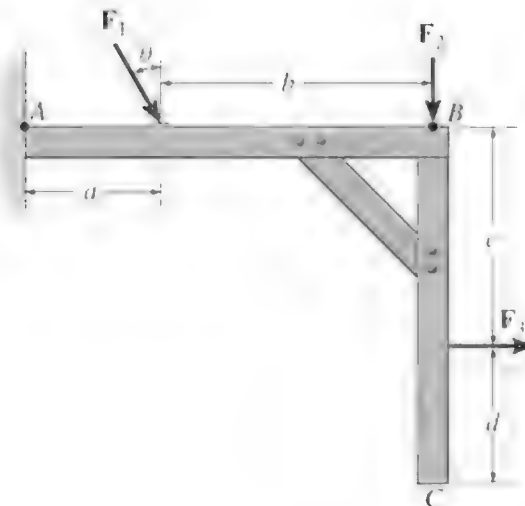
$F_3 = 25$ lb

$\theta = 30$ deg

$a = 2$ ft

$b = 4$ ft

$c = 3$ ft

$d = 2$ ft

Solution:

$F_{Rx} = F_1\sin(\theta) + F_3$ $F_{Rx} = 42.5$ lb

$F_{Ry} = -F_1\cos(\theta) - F_2$ $F_{Ry} = -50.311$ lb

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $F_R = 65.9$ lb

$\theta_1 = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right)$ $\theta_1 = -49.8$ deg

$F_{Rx}\,y = F_1\cos(\theta)b + F_3(c)$

$$y = \frac{F_1 \cos(\theta) b + F_3(c)}{F_{Rx}}$$ $y = 4.617\ \text{ft}$ (Below point B)

Problem 4-124

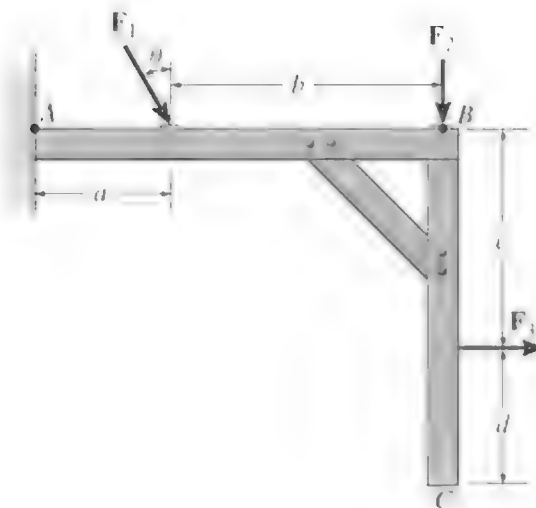
Replace the force system acting on the frame by an equivalent resultant force and couple moment acting at point A.

Given:

$F_1 = 35$ lb $a = 2$ ft

$F_2 = 20$ lb $b = 4$ ft

$F_3 = 25$ lb $c = 3$ ft

$\theta = 30$ deg $d = 2$ ft

Solution:

$F_{Rx} = F_1 \sin(\theta) + F_3$ $F_{Rx} = 42.5\ \text{lb}$

$F_{Ry} = F_1 \cos(\theta) + F_2$ $F_{Ry} = 50.311\ \text{lb}$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $F_R = 65.9\ \text{lb}$

$\theta_1 = \operatorname{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right)$ $\theta_1 = 49.8\ \text{deg}$

$M_{RA} = -F_1 \cos(\theta) a - F_2(a + b) + F_3(c)$ $M_{RA} = -106\ \text{lb}\cdot\text{ft}$

Problem 4-125

Replace the force and couple-moment system by an equivalent resultant force and couple moment at point O. Express the results in Cartesian vector form.

Units Used:

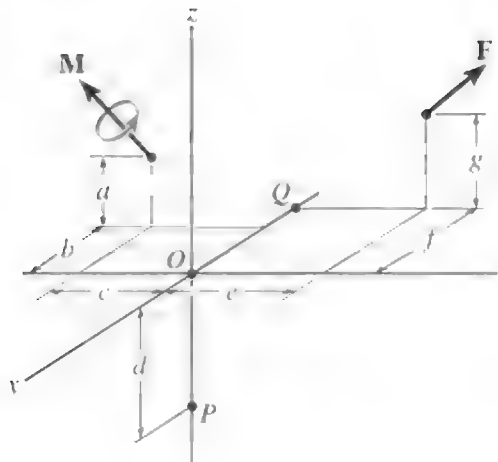
$$kN = 10^3\ N$$

Given:

$$\mathbf{F} = \begin{pmatrix} 8 \\ 6 \\ 8 \end{pmatrix} kN \qquad a = 3\ m \qquad e = 5\ m$$

$$b = 3\ m \qquad f = 6\ m$$

$$\mathbf{M} = \begin{pmatrix} -20 \\ -70 \\ 20 \end{pmatrix} kN{\cdot}m \qquad c = 4\ m \qquad g = 5\ m$$

$$d = 6\ m$$

Solution:

$$\mathbf{F_R} = \mathbf{F} \qquad \mathbf{M_R} = \mathbf{M} + \begin{pmatrix} -f \\ e \\ g \end{pmatrix} \times \mathbf{F} \qquad \mathbf{F_R} = \begin{pmatrix} 8 \\ 6 \\ 8 \end{pmatrix} kN \qquad \mathbf{M_R} = \begin{pmatrix} -10 \\ 18 \\ -56 \end{pmatrix} kN{\cdot}m$$

Problem 4-126

Replace the force and couple-moment system by an equivalent resultant force and couple moment at point P. Express the results in Cartesian vector form.

Units Used:

$$kN = 10^3\ N$$

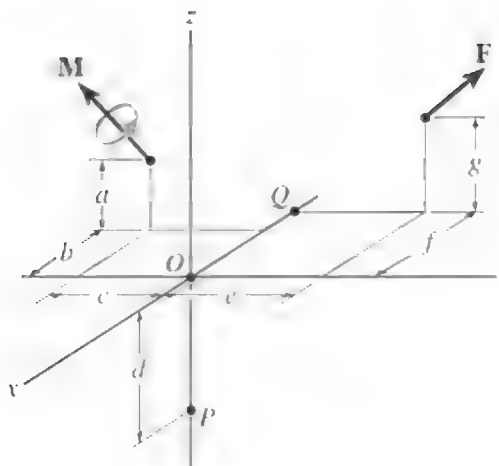
Given:

$$\mathbf{F} = \begin{pmatrix} 8 \\ 6 \\ 8 \end{pmatrix} kN$$

$$\mathbf{M} = \begin{pmatrix} -20 \\ -70 \\ 20 \end{pmatrix} kN{\cdot}m$$

$a = 3\ m$

$b = 3\ m \qquad e = 5\ m$

$c = 4\ m \qquad f = 6\ m$

$d = 6\ \text{m}$ $g = 5\ \text{m}$

Solution:

$$\mathbf{F_R} = \mathbf{F} \qquad \mathbf{M_R} = \mathbf{M} + \begin{pmatrix} -f \\ e \\ d+g \end{pmatrix} \times \mathbf{F} \qquad \mathbf{F_R} = \begin{pmatrix} 8 \\ 6 \\ 8 \end{pmatrix} \text{kN} \qquad \mathbf{M_R} = \begin{pmatrix} -46 \\ 66 \\ -56 \end{pmatrix} \text{kN}\cdot\text{m}$$

Problem 4-127

Replace the force and couple-moment system by an equivalent resultant force and couple moment at point Q. Express the results in Cartesian vector form.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

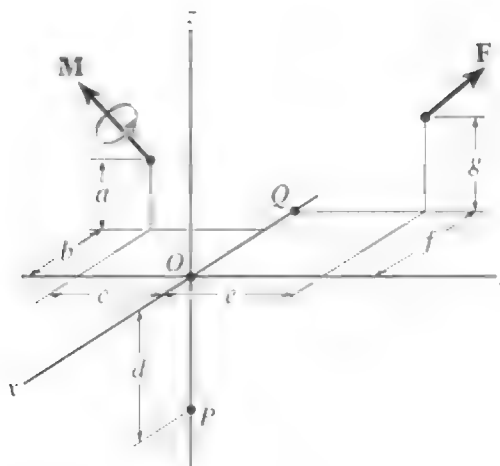
$$\mathbf{F} = \begin{pmatrix} 8 \\ 6 \\ 8 \end{pmatrix} \text{kN}$$

$$\mathbf{M} = \begin{pmatrix} -20 \\ -70 \\ 20 \end{pmatrix} \text{kN}\cdot\text{m}$$

$a = 3\ \text{m}$

$b = 3\ \text{m}$ $e = 5\ \text{m}$

$c = 4\ \text{m}$ $f = 6\ \text{m}$

$d = 6\ \text{m}$ $g = 5\ \text{m}$

Solution:

$$\mathbf{F_R} = \mathbf{F} \qquad \mathbf{M_R} = \mathbf{M} + \begin{pmatrix} 0 \\ e \\ g \end{pmatrix} \times \mathbf{F} \qquad \mathbf{F_R} = \begin{pmatrix} 8 \\ 6 \\ 8 \end{pmatrix} \text{kN} \qquad \mathbf{M_R} = \begin{pmatrix} -10 \\ -30 \\ -20 \end{pmatrix} \text{kN}\cdot\text{m}$$

Problem 4-128

The belt passing over the pulley is subjected to forces $\mathbf{F}_1$ and $\mathbf{F}_2$. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in

Cartesian vector form.

Given:

$F_1 = 40\ \text{N}$ $\quad r = 80\ \text{mm}$

$F_2 = 40\ \text{N}$ $\quad a = 300\ \text{mm}$

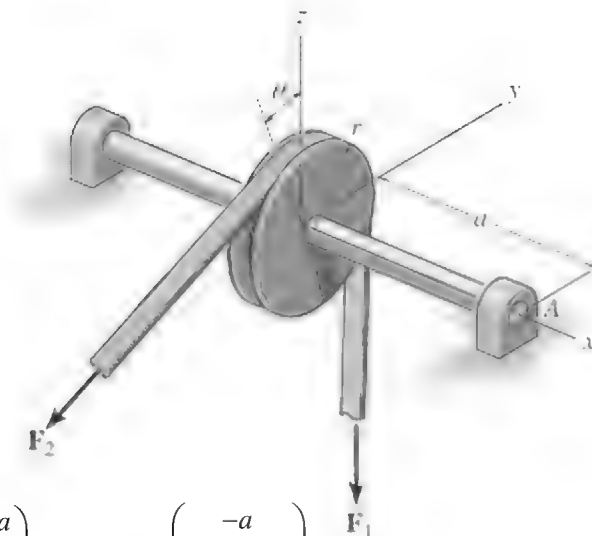
$\theta = 0\ \text{deg}$

Solution:

$$\mathbf{F_{1v}} = F_1\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\mathbf{F_{2v}} = F_2\begin{pmatrix} 0 \\ -\cos(\theta) \\ -\sin(\theta) \end{pmatrix} \qquad \mathbf{r_1} = \begin{pmatrix} -a \\ r \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \begin{pmatrix} -a \\ -r\sin(\theta) \\ r\cos(\theta) \end{pmatrix}$$

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}} \qquad \mathbf{M_A} = \mathbf{r_1} \times \mathbf{F_{1v}} + \mathbf{r_2} \times \mathbf{F_{2v}}$$

$$\mathbf{F_R} = \begin{pmatrix} 0 \\ -40 \\ -40 \end{pmatrix} \text{N} \qquad \mathbf{M_A} = \begin{pmatrix} 0 \\ -12 \\ 12 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-129

The belt passing over the pulley is subjected to forces $\mathbf{F_1}$ and $\mathbf{F_2}$. $\mathbf{F_1}$ acts in the $-\mathbf{k}$ direction.

Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form.

Given:

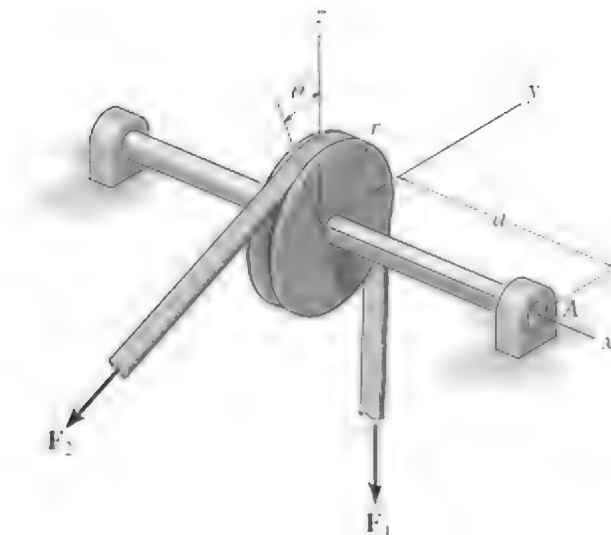
$F_1 = 40\ \text{N}$

$F_2 = 40\ \text{N}$

$\theta = 0\ \text{deg}$

$r = 80\ \text{mm}$

$a = 300\ \text{mm}$

$\theta = 45\ \text{deg}$

Solution:

$$\mathbf{F_{1v}} = F_1\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{F_{2v}} = F_2\begin{pmatrix} 0 \\ -\cos(\theta) \\ -\sin(\theta) \end{pmatrix} \qquad \mathbf{r_1} = \begin{pmatrix} -a \\ r \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \begin{pmatrix} -a \\ -r\sin(\theta) \\ r\cos(\theta) \end{pmatrix}$$

$$\mathbf{F_R} = \mathbf{F_{1v}} + \mathbf{F_{2v}} \qquad \mathbf{M_A} = \mathbf{r_1} \times \mathbf{F_{1v}} + \mathbf{r_2} \times \mathbf{F_{2v}}$$

$$\mathbf{F_R} = \begin{pmatrix} 0 \\ -28.28 \\ -68.28 \end{pmatrix} \text{N} \qquad \mathbf{M_A} = \begin{pmatrix} 0 \\ -20.49 \\ 8.49 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-130

Replace this system by an equivalent resultant force and couple moment acting at O. Express the results in Cartesian vector form.

Given:

$F_1 = 50\ \text{N}$

$F_2 = 80\ \text{N}$

$F_3 = 180\ \text{N}$

$a = 1.25\ \text{m}$

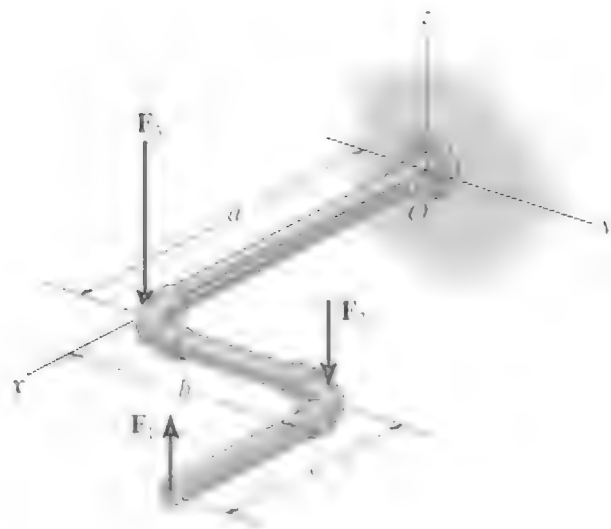
$b = 0.5\ \text{m}$

$c = 0.75\ \text{m}$

Solution:

$$\mathbf{F_R} = \begin{pmatrix} 0 \\ 0 \\ F_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -F_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -F_3 \end{pmatrix}$$

$$\mathbf{F_R} = \begin{pmatrix} 0 \\ 0 \\ -210 \end{pmatrix} \text{N}$$

$$\mathbf{M_O} = \begin{pmatrix} a+c \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ F_1 \end{pmatrix} + \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -F_2 \end{pmatrix} + \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -F_3 \end{pmatrix} \qquad \mathbf{M_O} = \begin{pmatrix} -15 \\ 225 \\ 0 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-131

Handle forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to the electric drill. Replace this system by an equivalent resultant force and couple moment acting at point O. Express the results in Cartesian vector form.

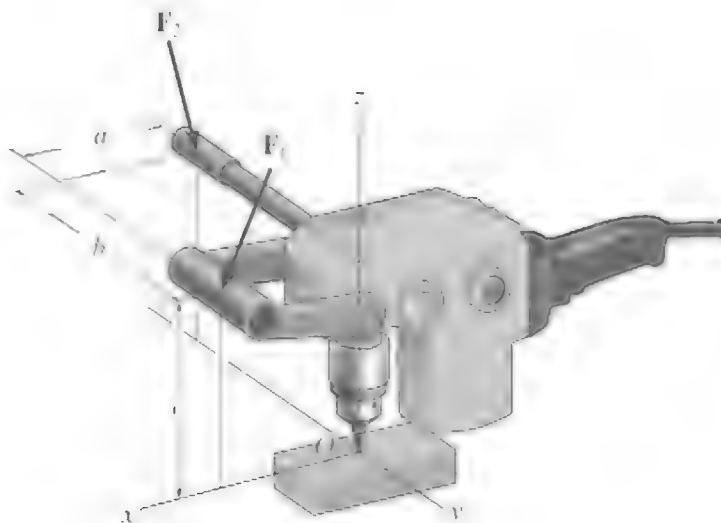
Given:

$a = 0.15\ \text{m}$

$b = 0.25\ \text{m}$

$c = 0.3\ \text{m}$

$$\mathbf{F_1} = \begin{pmatrix} 6 \\ -3 \\ -10 \end{pmatrix} \text{N}$$

$$\mathbf{F_2} = \begin{pmatrix} 0 \\ 2 \\ -4 \end{pmatrix} \text{N}$$

Solution:

$\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ $\qquad$ $\mathbf{F_R} = \begin{pmatrix} 6 \\ -1 \\ -14 \end{pmatrix}$ N

$\mathbf{M_O} = \begin{pmatrix} a \\ 0 \\ c \end{pmatrix} \times \mathbf{F_1} + \begin{pmatrix} 0 \\ -b \\ c \end{pmatrix} \times \mathbf{F_2}$ $\qquad$ $\mathbf{M_O} = \begin{pmatrix} 1.3 \\ 3.3 \\ -0.45 \end{pmatrix}$ N·m

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Problem 4-132

A biomechanical model of the lumbar region of the human trunk is shown. The forces acting in the four muscle groups consist of $\mathbf{F_R}$ for the rectus, $\mathbf{F_O}$ for the oblique, $\mathbf{F_L}$ for the lumbar latissimus dorsi, and $\mathbf{F_E}$ for the erector spinae. These loadings are symmetric with respect to the *y* - *z* plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point *O*. Express the results in Cartesian vector form.

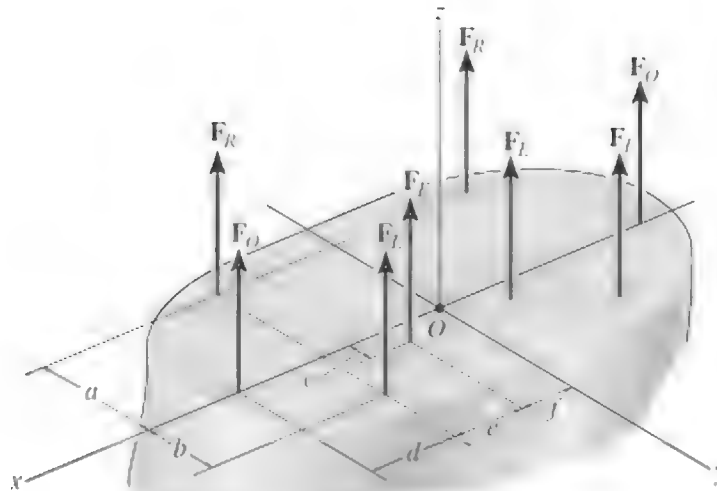
Given:

$F_R = 35$ N $\qquad a = 75$ mm

$F_O = 45$ N $\qquad b = 45$ mm

$F_L = 23$ N $\qquad c = 15$ mm

$F_E = 32$ N $\qquad d = 50$ mm

$e = 40$ mm $\qquad f = 30$ mm

Solution:

$F_{Res} = \Sigma F_i;$ $\qquad F_{Res} = 2(F_R + F_O + F_L + F_E)$ $\qquad F_{Res} = 270$ N

$M_{ROx} = \Sigma M_{Ox};$ $\qquad M_{RO} = -2F_R a + 2F_E c + 2F_L b$ $\qquad M_{RO} = -2.22$ N·m

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Problem 4-133

The building slab is subjected to four parallel column loadings.Determine the equivalent resultant force and specify its location (*x*, *y*) on the slab.

Units Used:

$kN = 10^3 \ N$

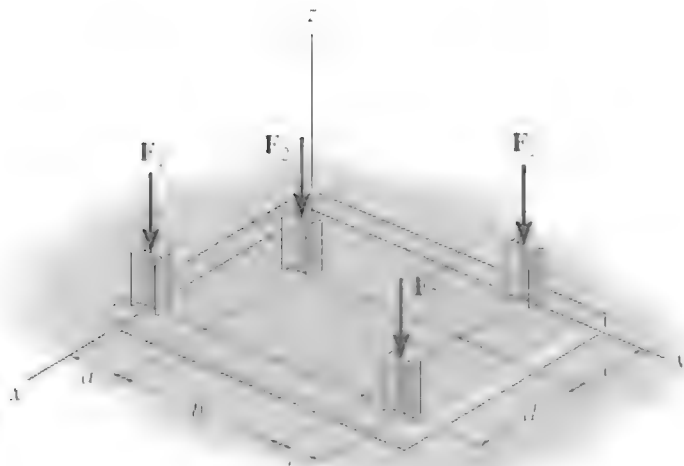
Given:

$F_1 = 30 \ kN \quad a = 3 \ m$

$F_2 = 40 \ kN \quad b = 8 \ m$

$F_3 = 20 \ kN \quad c = 2 \ m$

$F_4 = 50 \ kN \quad d = 6 \ m$

$e = 4 \ m$

Solution:

$+\uparrow F_R = \Sigma F_x; \qquad F_R = F_1 + F_2 + F_3 + F_4$

$$F_R = 140 \, kN$$

$M_{Rx} = \Sigma M_x; \qquad -F_R(y) = -(F_4)(a) - [(F_1)(a+b)] - [(F_2)(a+b+c)]$

$$y = \frac{F_4\ a + F_1\ a + F_1\ b + F_2\ a + F_2\ b + F_2\ c}{F_R}$$

$$y = 7.14 \ m$$

$M_{Ry} = \Sigma M_y; \qquad (F_R)x = (F_4)(e) + (F_3)(d+e) + (F_2)(b+c)$

$$x = \frac{F_4\ e + F_3\ d + F_3\ e + F_2\ b + F_2\ c}{F_R}$$

$$x = 5.71 \ m$$

Problem 4-134

The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab.

Units Used:

$kN = 10^3 \ N$

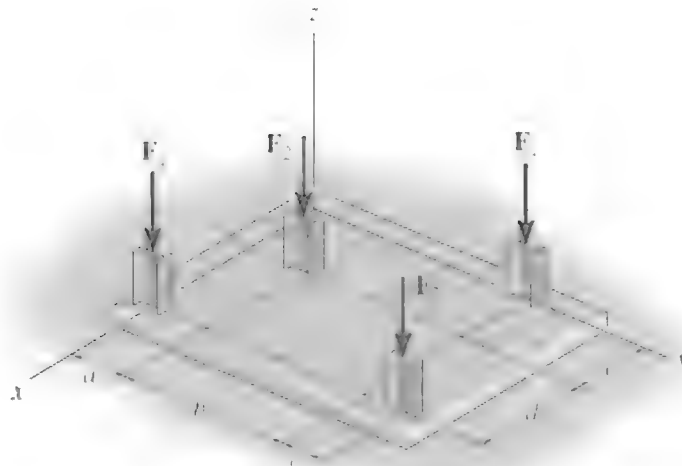
Given:

$F_1 = 20 \ kN$ $a = 3 \ m$

$F_2 = 50 \ kN$ $b = 8 \ m$

$F_3 = 20 \ kN$ $c = 2 \ m$

$F_4 = 50 \ kN$ $d = 6 \ m$

$e = 4 \ m$

Solution:

$$F_R = F_1 + F_2 + F_3 + F_4 \qquad F_R = 140 \, kN$$

$$F_R x = F_2 e + F_1 (d + e) + F_2 (d + e)$$

$$x = \frac{2 F_2 e + F_1 d + F_1 e + F_2 d}{F_R} \qquad x = 6.43 \, m$$

$$-F_R y = -F_2 a - F_3 (a + b) - F_2 (a + b + c)$$

$$y = \frac{2 F_2 a + F_3 a + F_3 b + F_2 b + F_2 c}{F_R} \qquad y = 7.29 \ m$$

Problem 4-135

The pipe assembly is subjected to the action of a wrench at B and a couple at A. Determine the magnitude F of the couple forces so that the system can be simplified to a wrench acting at point C.

Given:

$a = 0.6 \ m$

$b = 0.8\text{ m}$

$c = 0.25\text{ m}$

$d = 0.7\text{ m}$

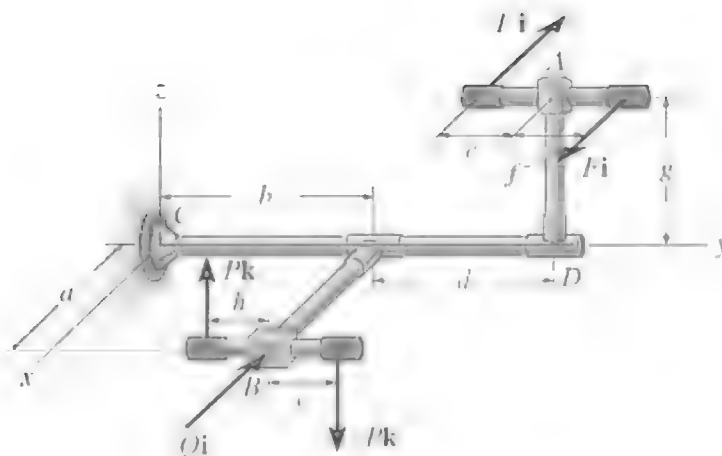
$e = 0.3\text{ m}$

$f = 0.3\text{ m}$

$g = 0.5\text{ m}$

$h = 0.25\text{ m}$

$P = 60\text{ N}$

$Q = 40\text{ N}$

Solution:

Initial Guess $F = 1\text{ N}$ $M_C = 1\text{ N}\cdot\text{m}$

Given

$$\begin{pmatrix} -M_C \\ 0 \\ 0 \end{pmatrix} = \begin{bmatrix} -P(c+h) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -F(e+f) \end{bmatrix} + \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} -Q \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} F \\ M_C \end{pmatrix} = \text{Find}(F, M_C) \qquad M_C = 30\text{N}\cdot\text{m} \qquad F = 53.3\text{ N}$$

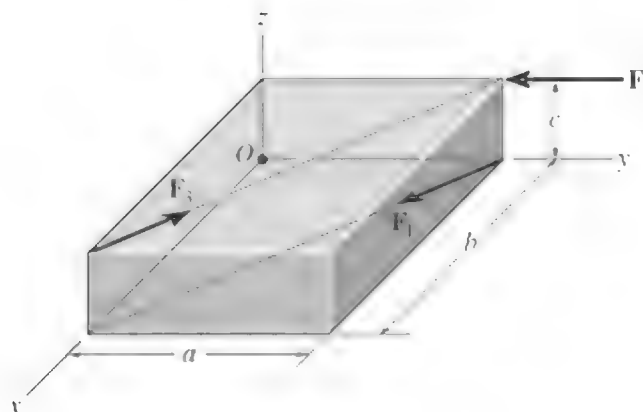
Problem 4-136

The three forces acting on the block each have a magnitude $F_1 = F_2 = F_3$. Replace this system by a wrench and specify the point where the wrench intersects the z axis, measured from point O.

Given:

$F_1 = 10\text{ lb}$ $a = 6\text{ ft}$

$F_2 = F_1$ $b = 6\text{ ft}$

$F_3 = F_1$ $c = 2\text{ ft}$

Solution:

The vectors

$$\mathbf{F_{1v}} = \frac{F_1}{\sqrt{b^2 + a^2}}\begin{pmatrix} b \\ -a \\ 0 \end{pmatrix} \qquad \mathbf{F_{2v}} = \begin{pmatrix} 0 \\ -F_2 \\ 0 \end{pmatrix} \qquad \mathbf{F_{3v}} = \frac{F_3}{\sqrt{b^2 + a^2}}\begin{pmatrix} -b \\ a \\ 0 \end{pmatrix}$$

Place the wrench in the x - z plane.

Guesses $\quad x = 1\text{ft} \quad z = 1\text{ft} \quad M = 1\ \text{lb}\cdot\text{ft} \quad R_x = 1\ \text{lb} \quad R_y = 1\ \text{lb} \quad R_z = 1\ \text{lb}$

Given $$\begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \mathbf{F_{1v}} + \mathbf{F_{2v}} + \mathbf{F_{3v}}$$

$$\begin{pmatrix} x \\ 0 \\ z \end{pmatrix} \times \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} + \frac{M}{\sqrt{R_x^2 + R_y^2 + R_z^2}}\begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \begin{pmatrix} 0 \\ a \\ c \end{pmatrix} \times \mathbf{F_{2v}} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \mathbf{F_{1v}} + \begin{pmatrix} b \\ 0 \\ c \end{pmatrix} \times \mathbf{F_{3v}}$$

$$\begin{pmatrix} x \\ z \\ M \\ R_x \\ R_y \\ R_z \end{pmatrix} = \text{Find}(x, z, M, R_x, R_y, R_z) \qquad \mathbf{M_v} = \frac{M}{\sqrt{R_x^2 + R_y^2 + R_z^2}}\begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}$$

$$\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0.586 \end{pmatrix}\text{ft} \qquad \mathbf{M_v} = \begin{pmatrix} 0 \\ -14.142 \\ 0 \end{pmatrix}\text{lb}\cdot\text{ft} \qquad \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \begin{pmatrix} 0 \\ -10 \\ 0 \end{pmatrix}\text{lb}$$

Problem 4-137

Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.

Units Used:

$$\text{kN} = 10^3\text{N}$$

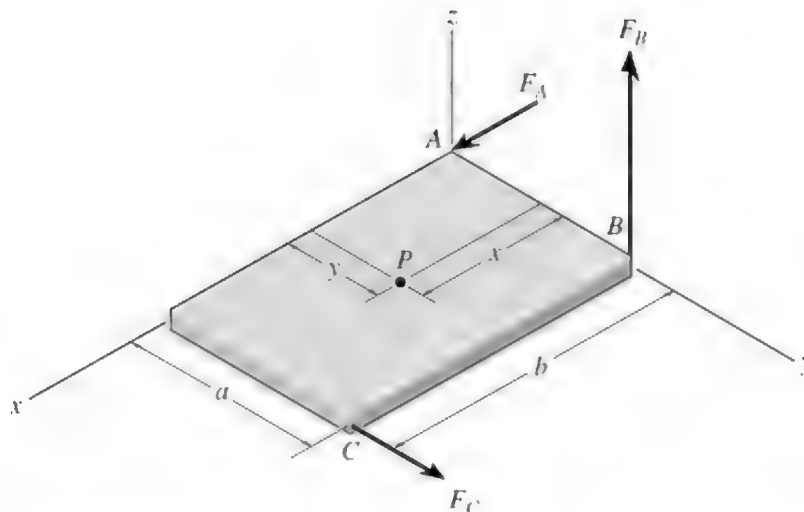
Given:

$F_A = 500$ N

$F_B = 800$ N

$F_C = 300$ N

$a = 4$ m

$b = 6$ m

Solution:

$$\mathbf{F_R} = \begin{pmatrix} F_A \\ F_C \\ F_B \end{pmatrix} \qquad |\mathbf{F_R}| = 0.9899\,\text{kN}$$

Guesses $x = 1$ m $y = 1$ m $M = 100$ N·m

Given

$$M\frac{\mathbf{F_R}}{|\mathbf{F_R}|} + \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times \mathbf{F_R} = \begin{pmatrix} b \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ F_C \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ F_B \end{pmatrix}$$

$$\begin{pmatrix} M \\ x \\ y \end{pmatrix} = \text{Find}(M, x, y) \qquad M = 3.07\,\text{kN}\cdot\text{m} \qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.163 \\ 2.061 \end{pmatrix}\,\text{m}$$

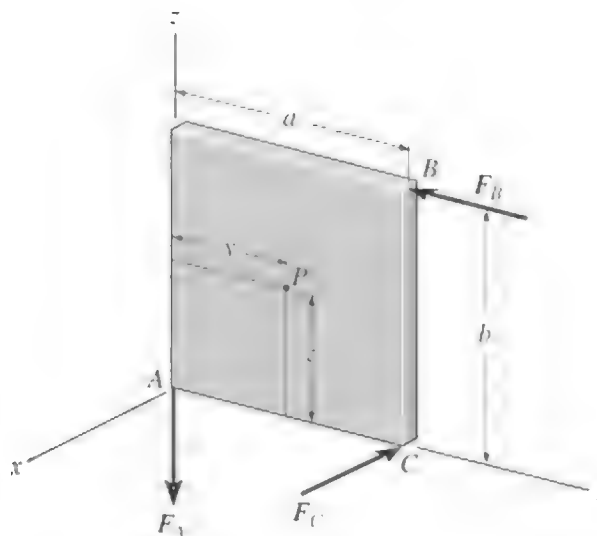
Problem 4-138

Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(y, z)$ where its line of action intersects the plate.

Given:

$F_A = 80$ lb $a = 12$ ft

$F_B = 60$ lb $b = 12$ ft

$F_C = 40$ lb

Solution:

$$\mathbf{F_R} = \begin{pmatrix} -F_C \\ -F_B \\ -F_A \end{pmatrix} \qquad |\mathbf{F_R}| = 108\,\text{lb}$$

Guesses $y = 1$ ft $\quad z = 1$ ft $\quad M = 1$ lb·ft

Given
$$M\frac{\mathbf{F_R}}{|\mathbf{F_R}|} + \begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times \mathbf{F_R} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} -F_C \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ b \end{pmatrix} \times \begin{pmatrix} 0 \\ -F_B \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} M \\ y \\ z \end{pmatrix} = \text{Find}(M, y, z) \qquad M = -624\,\text{lb}\cdot\text{ft} \qquad \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 0.414 \\ 8.69 \end{pmatrix}\text{ft}$$

Problem 4-139

The loading on the bookshelf is distributed as shown. Determine the magnitude of the equivalent resultant location, measured from point O.

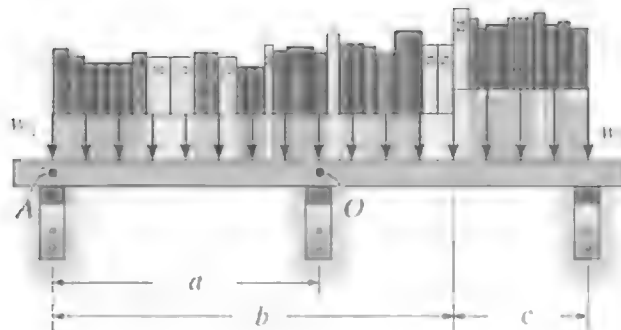
Given:

$w_1 = 2\frac{\text{lb}}{\text{ft}}$

$w_2 = 3.5\frac{\text{lb}}{\text{ft}}$

$a = 2.75$ ft

$b = 4$ ft

$c = 1.5$ ft

Solution: Guesses $R = 1$ lb $\quad d = 1$ft

Given $\quad w_1 b + w_2 c = R$

$$w_1 b\left(a - \frac{b}{2}\right) - w_2 c\left(\frac{c}{2} + b - a\right) = -dR$$

$$\begin{pmatrix} R \\ d \end{pmatrix} = \text{Find}(R, d) \qquad R = 13.25\,\text{lb} \qquad d = 0.34\,\text{ft}$$

Problem 4-140

Replace the loading by an equivalent resultant force and couple moment acting at point A.

Units Used:

$kN = 10^3\ N$

Given:

$w_1 = 600\ \frac{N}{m}$

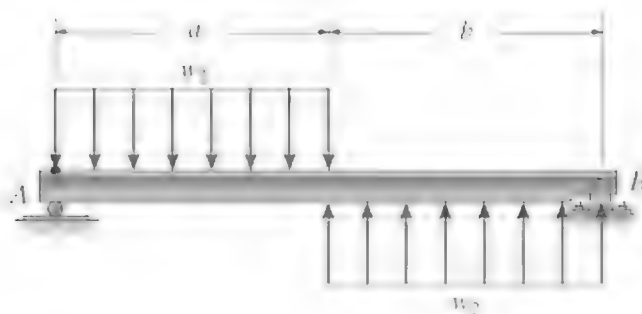
$w_2 = 600\ \frac{N}{m}$

$a = 2.5\ m$

$b = 2.5\ m$

Solution:

$F_R = w_1 a - w_2 b$ $\qquad F_R = 0\ N$

$M_{RA} = w_1 a\left(\frac{a+b}{2}\right)$ $\qquad M_{RA} = 3.75\ kN \cdot m$

Problem 4-141

Replace the loading by an equivalent force and couple moment acting at point O.

Units Used:

$kN = 10^3\ N$

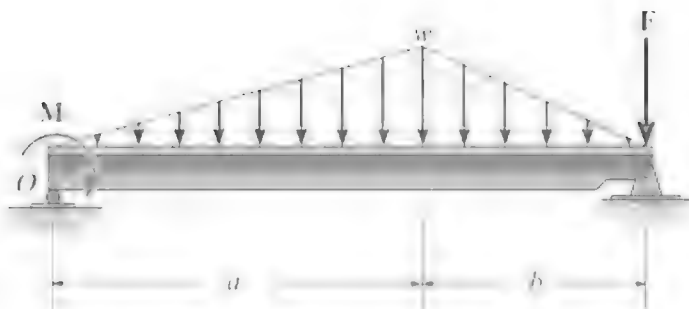
Given:

$w = 6\ \frac{kN}{m}$

$F = 15\ kN$

$M = 500\ kN \cdot m$

$a = 7.5\ m$

$b = 4.5\ m$

Solution:

$F_R = \frac{1}{2}w(a+b) + F \qquad F_R = 51.0\,\text{kN}$

$M_R = -M - \left(\frac{1}{2}wa\right)\left(\frac{2}{3}a\right) - \left(\frac{1}{2}wb\right)\left(a + \frac{b}{3}\right) - F(a+b) \qquad M_R = -914\,\text{kN}\cdot\text{m}$

Problem 4-142

Replace the loading by a single resultant force, and specify the location of the force on the beam measured from point O.

Units Used:

$\text{kN} = 10^3\ \text{N}$

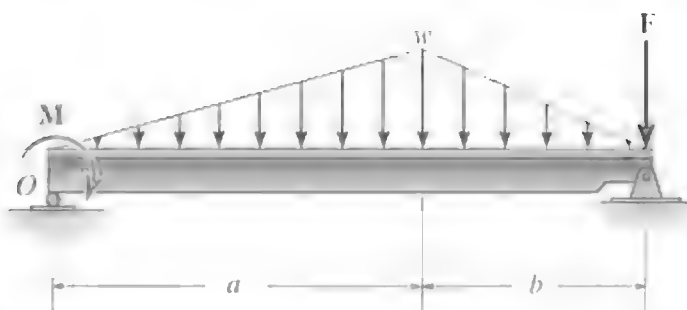
Given:

$w = 6\ \frac{\text{kN}}{\text{m}}$

$F = 15\ \text{kN}$

$M = 500\ \text{kN}\cdot\text{m}$

$a = 7.5\ \text{m}$

$b = 4.5\ \text{m}$

Solution:

Initial Guesses: $F_R = 1\ \text{kN} \qquad d = 1\ \text{m}$

Given

$F_R = \frac{1}{2}w(a+b) + F$

$-F_R d = -M - \left(\frac{1}{2}\ wa\right)\left(\frac{2}{3}a\right) - \left(\frac{1}{2}w\ b\right)\left(a + \frac{b}{3}\right) - F(a+b)$

$\begin{pmatrix} F_R \\ d \end{pmatrix} = \text{Find}(F_R, d) \qquad F_R = 51\,\text{kN} \qquad d = 17.922\ \text{m}$

Problem 4-143

The column is used to support the floor which exerts a force P on the top of the column. The effect of soil pressure along its side is distributed as shown. Replace this loading by an

equivalent resultant force and specify where it acts along the column, measured from its base *A*.

Units Used: $\text{kip} = 10^3 \text{ lb}$

Given:

$P = 3000 \text{ lb}$

$w_1 = 80 \frac{\text{lb}}{\text{ft}}$

$w_2 = 200 \frac{\text{lb}}{\text{ft}}$

$h = 9 \text{ ft}$

Solution:

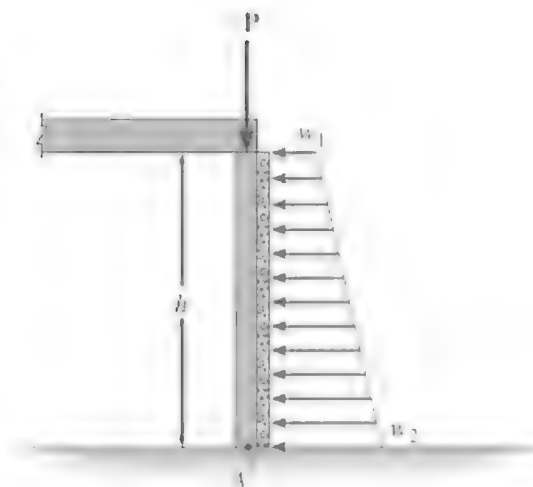
$$F_{Rx} = w_1 h + \frac{1}{2}(w_2 - w_1)h$$

$F_{Rx} = 1260 \text{ lb}$ $F_{Ry} = P$

$F_R = \sqrt{F_{Rx}^2 + P^2}$ $F_R = 3.25 \text{ kip}$

$\theta = \text{atan}\left(\frac{P}{F_{Rx}}\right)$ $\theta = 67.2 \text{ deg}$

$$F_{Rx} y = \frac{1}{2}(w_2 - w_1)h\frac{h}{3} + w_1 h\frac{h}{2}$$

$y = \frac{1}{6} h^2 \frac{w_2 + 2 w_1}{F_{Rx}}$ $y = 3.86 \text{ ft}$

Problem 4-144

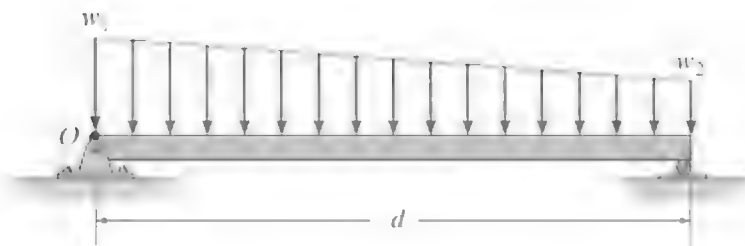
Replace the loading by an equivalent force and couple moment at point *O*.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$w_1 = 15 \frac{\text{kN}}{\text{m}}$

$w_2 = 5 \frac{\text{kN}}{\text{m}}$

$d = 9\ \text{m}$

Solution:

$F_R = \frac{1}{2}(w_1 + w_2)d$ $F_R = 90\,\text{kN}$

$M_{RO} = w_2 d\frac{d}{2} + \frac{1}{2}(w_1 - w_2)d\frac{d}{3}$ $M_{RO} = 338\,\text{kN}\cdot\text{m}$

Problem 4-145

Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at C.

Units Used:

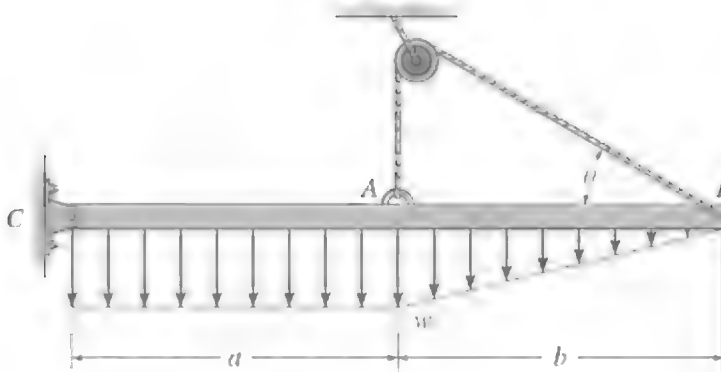
$\text{kip} = 10^3\ \text{lb}$

Given:

$w = 800\ \frac{\text{lb}}{\text{ft}}$

$a = 15\ \text{ft}$

$b = 15\ \text{ft}$

$\theta = 30\ \text{deg}$

Solution:

$F_R = wa + \frac{wb}{2}$ $F_R = 18\,\text{kip}$

$F_R x = wa\frac{a}{2} + \frac{wb}{2}\left(a + \frac{b}{3}\right)$

$x = \frac{wa\frac{a}{2} + \frac{wb}{2}\left(a + \frac{b}{3}\right)}{F_R}$ $x = 11.7\,\text{ft}$

Problem 4-146

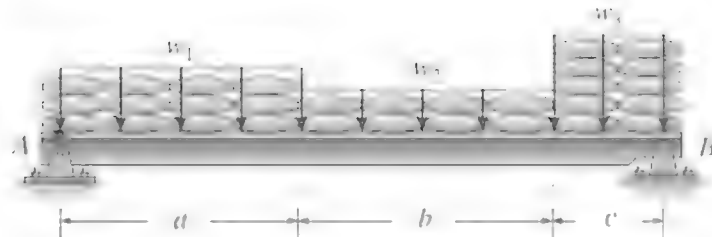
The beam supports the distributed load caused by the sandbags. Determine the resultant force on the beam and specify its location measured from point A.

Units Used: $\text{kN} = 10^3\ \text{N}$

Given:

$w_1 = 1.5\ \frac{\text{kN}}{\text{m}}$ $a = 3\ \text{m}$

$w_2 = 1\ \frac{\text{kN}}{\text{m}}$ $b = 3\ \text{m}$

$w_3 = 2.5\ \frac{\text{kN}}{\text{m}}$ $c = 1.5\ \text{m}$

Solution:

$$F_R = w_1 a + w_2 b + w_3 c \qquad F_R = 11.25\ \text{kN}$$

$$M_A = w_1 a \frac{a}{2} + w_2 b\left(a + \frac{b}{2}\right) + w_3 c\left(a + b + \frac{c}{2}\right)$$

$$M_A = 45.563\ \text{kN}\cdot\text{m} \qquad d = \frac{M_A}{F_R} \qquad d = 4.05\ \text{m}$$

Problem 4-147

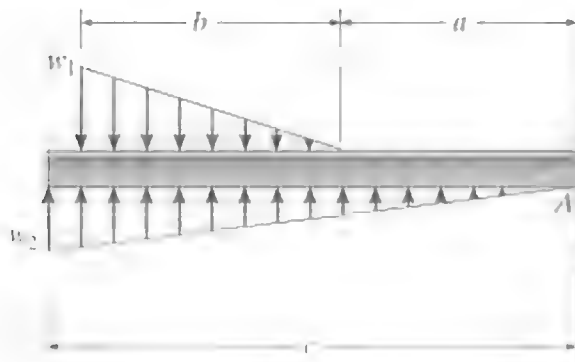
Determine the length b of the triangular load and its position a on the beam such that the equivalent resultant force is zero and the resultant couple moment is M clockwise.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$w_1 = 4\ \frac{\text{kN}}{\text{m}}$ $w_2 = 2.5\ \frac{\text{kN}}{\text{m}}$

$M = 8\ \text{kN}\cdot\text{m}$ $c = 9\ \text{m}$

Solution:

Initial Guesses: $a = 1\ \text{m}$ $b = 1\ \text{m}$

Given $\frac{-1}{2} w_1 b + \frac{1}{2} w_2 c = 0$

$$\frac{1}{2}w_1 b\left(a+\frac{2b}{3}\right)-\frac{1}{2}w_2 c\frac{2c}{3}=-M$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \text{Find}(a, b) \qquad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1.539 \\ 5.625 \end{pmatrix} \text{ m}$$

Problem 4-148

Replace the distributed loading by an equivalent resultant force and specify its location, measured from point A.

Units Used:

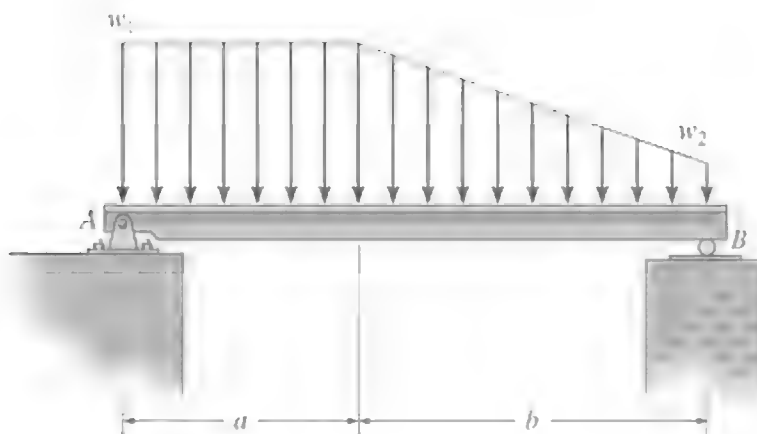
$\text{kN} = 10^3\text{N}$

Given:

$w_1 = 800\frac{\text{N}}{\text{m}}$

$w_2 = 200\frac{\text{N}}{\text{m}}$

$a = 2 \text{ m}$

$b = 3 \text{ m}$

Solution:

$$F_R = w_2 b + w_1 a + \frac{1}{2}(w_1 - w_2)b \qquad F_R = 3.10\,\text{kN}$$

$$xF_R = w_1 a\frac{a}{2} + \frac{1}{2}(w_1 - w_2)b\left(a+\frac{b}{3}\right) + w_2 b\left(a+\frac{b}{2}\right)$$

$$x = \frac{w_1 a\frac{a}{2} + \frac{1}{2}(w_1 - w_2)b\left(a+\frac{b}{3}\right) + w_2 b\left(a+\frac{b}{2}\right)}{F_R} \qquad x = 2.06 \text{ m}$$

Problem 4-149

The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.

Units Used:

$kip = 10^3 lb$

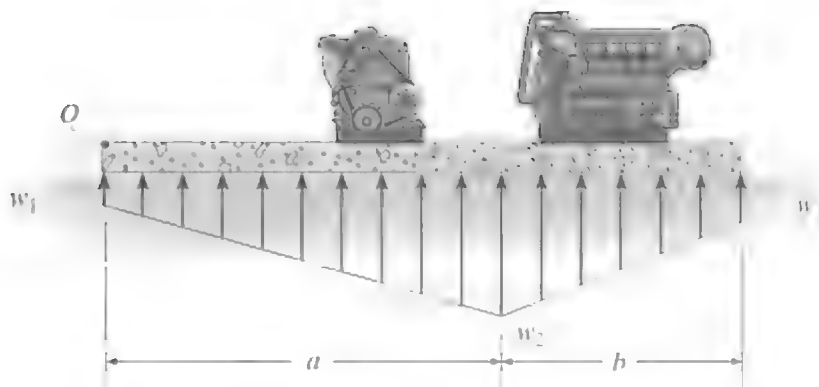
Given:

$w_1 = 50 \frac{lb}{ft}$

$w_2 = 300 \frac{lb}{ft}$

$w_3 = 100 \frac{lb}{ft}$

$a = 12\ ft$

$b = 9\ ft$

Solution:

$$F_R = w_1 a + \frac{1}{2}(w_2 - w_1)a + \frac{1}{2}(w_2 - w_3)b + w_3 b \qquad F_R = 3.9\ kip$$

$$F_R\, d = w_1 a \frac{a}{2} + \frac{1}{2}(w_2 - w_1)a\frac{2a}{3} + \frac{1}{2}(w_2 - w_3)b\left(a + \frac{b}{3}\right) + w_3 b\left(a + \frac{b}{2}\right)$$

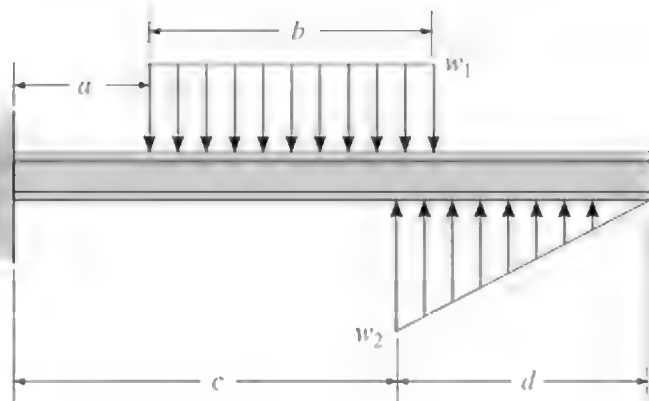
$$d = \frac{3\, w_3\, b\, a + 2\, w_3\, b^2 + w_1\, a^2 + 2\, a^2\, w_2 + 3\, b\, w_2\, a + w_2\, b^2}{6F_R} \qquad d = 11.3\ ft$$

Problem 4-150

The beam is subjected to the distributed loading. Determine the length b of the uniform load and its position a on the beam such that the resultant force and couple moment acting on the beam are zero.

Given:

$w_1 = 40 \frac{lb}{ft}$ $\quad c = 10ft$

$w_2 = 60 \frac{lb}{ft}$ $\quad d = 6\ ft$

Solution:

Initial Guesses: $a = 1\ ft$ $\quad b = 1ft$

Given

$$\frac{1}{2}w_2 d - w_1 b = 0 \qquad \frac{1}{2}w_2 d\left(c + \frac{d}{3}\right) - w_1 b\left(a + \frac{b}{2}\right) = 0$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \text{Find}(a, b) \qquad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 9.75 \\ 4.5 \end{pmatrix} \text{ft}$$

Problem 4-151

Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point B.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

$$w_1 = 800 \frac{\text{lb}}{\text{ft}}$$

$$w_2 = 500 \frac{\text{lb}}{\text{ft}}$$

$$a = 12 \text{ ft}$$

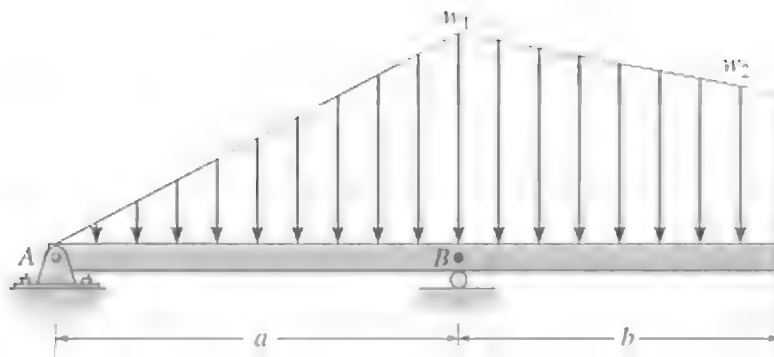
$$b = 9 \text{ ft}$$

Solution:

$$F_R = \frac{1}{2}a w_1 + \frac{1}{2}(w_1 - w_2)b + w_2 b \qquad F_R = 10.65 \text{ kip}$$

$$F_R x = -\frac{1}{2} a w_1 \frac{a}{3} + \frac{1}{2}(w_1 - w_2)b\frac{b}{3} + w_2 b \frac{b}{2}$$

$$x = \frac{-\frac{1}{2} a w_1 \frac{a}{3} + \frac{1}{2}(w_1 - w_2)b\frac{b}{3} + w_2 b \frac{b}{2}}{F_R} \qquad x = 0.479 \text{ ft}$$

(to the right of B)

Problem 4-152

Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects member AB, measured from A.

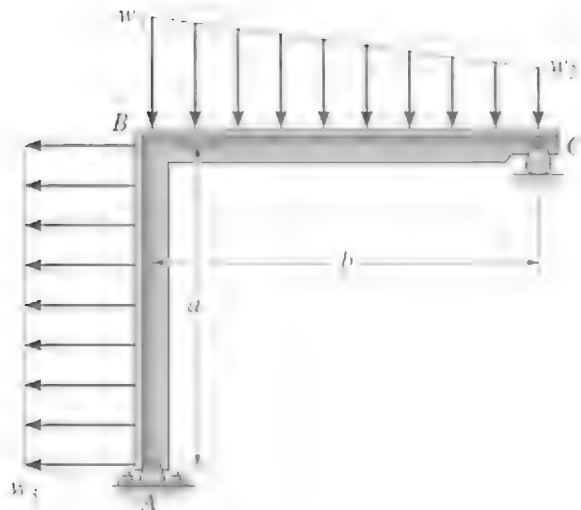
Given:

$$w_1 = 200\frac{\text{N}}{\text{m}}$$

$$w_2 = 100\frac{\text{N}}{\text{m}}$$

$$w_3 = 200\frac{\text{N}}{\text{m}}$$

$$a = 5\text{ m}$$

$$b = 6\text{ m}$$

Solution:

$$F_{Rx} = -w_3 a \qquad F_{Rx} = -1000\text{ N}$$

$$F_{Ry} = \frac{-1}{2}(w_1 + w_2)b \qquad F_{Ry} = -900\text{ N}$$

$$-yF_{Rx} = w_3 a\frac{a}{2} - w_2 b\frac{b}{2} - \frac{1}{2}(w_1 - w_2)b\frac{b}{3}$$

$$y = \frac{w_3 a\frac{a}{2} - w_2 b\frac{b}{2} - \frac{1}{2}(w_1 - w_2)b\frac{b}{3}}{-F_{Rx}} \qquad y = 0.1\text{ m}$$

Problem 4-153

Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects member *BC*, measured from *C*.

Units Used:

$$\text{kN} = 10^3\text{N}$$

Given:

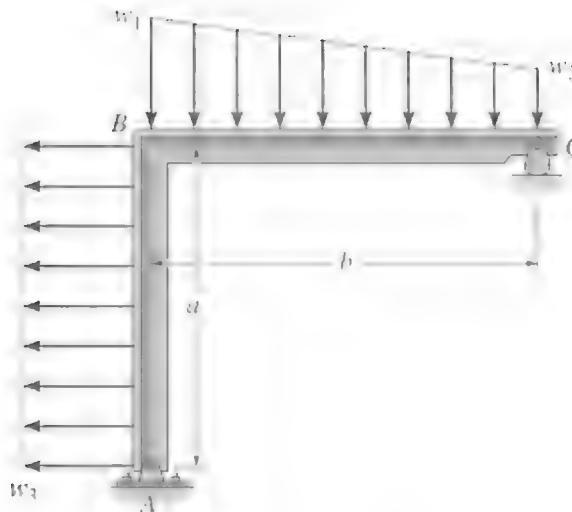
$w_1 = 200 \frac{N}{m}$

$w_2 = 100 \frac{N}{m}$

$w_3 = 200 \frac{N}{m}$

$a = 5\ m$

$b = 6\ m$

Solution:

$F_{Rx} = -w_3 a$ $\qquad F_{Rx} = -1000\ N$

$F_{Ry} = \frac{-1}{2}(w_1 + w_2)b$ $\qquad F_{Ry} = -900\ N$

$-xF_{Ry} = -w_3 a \frac{a}{2} + w_2 b \frac{b}{2} + \frac{1}{2}(w_1 - w_2)b \frac{2b}{3}$

$$x = \frac{-w_3 a \frac{a}{2} + w_2 b \frac{b}{2} + \frac{1}{2}(w_1 - w_2)b \frac{2b}{3}}{-F_{Ry}}$$ $\qquad x = 0.556\ m$

$\left|\begin{pmatrix} F_{Rx} \\ F_{Ry} \end{pmatrix}\right| = 1.345\ kN$

Problem 4-154

Replace the loading by an equivalent resultant force and couple moment acting at point O.

Units Used:

$kN = 10^3\ N$

Given:

$w_1 = 7.5 \frac{kN}{m}$

$$w_2 = 20\ \frac{\text{kN}}{\text{m}}$$

$$a = 3\ \text{m}$$

$$b = 3\ \text{m}$$

$$c = 4.5\ \text{m}$$

Solution:

$$F_R = \frac{1}{2}\left(w_2 - w_1\right)c + w_1\,c + w_1\,b + \frac{1}{2}w_1\,a$$

$$F_R = 95.6\,\text{kN}$$

$$M_{Ro} = -\frac{1}{2}\left(w_2 - w_1\right)c\,\frac{c}{3} - w_1\,c\,\frac{c}{2} - w_1\,b\left(c + \frac{b}{2}\right) - \frac{1}{2}w_1\,a\left(b + c + \frac{a}{3}\right) \qquad M_{Ro} = -349\,\text{kN}\cdot\text{m}$$

Problem 4-155

Determine the equivalent resultant force and couple moment at point O.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

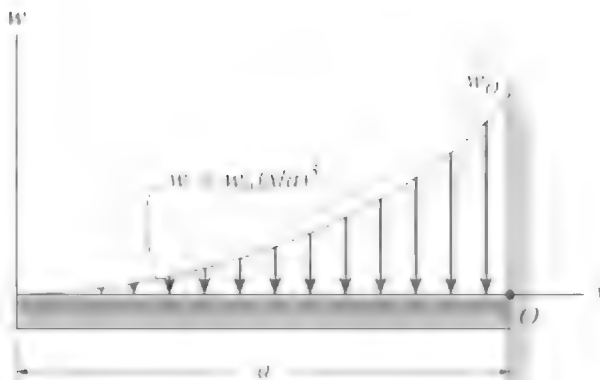
$$a = 3\ \text{m}$$

$$w_O = 3\ \frac{\text{kN}}{\text{m}}$$

$$w(x) = w_O\left(\frac{x}{a}\right)^2$$

Solution:

$$F_R = \int_0^a w(x)\ \text{d}x \qquad F_R = 3\,\text{kN}$$

$$M_O = \int_0^a w(x)(a - x)\ \text{d}x \qquad M_O = 2.25\,\text{kN}\cdot\text{m}$$

Problem 4-156

Wind has blown sand over a platform such that the intensity of the load can be approximated by the function $w = w_0\left(\frac{x}{d}\right)^3$. Simplify this distributed loading to an equivalent resultant force and specify the magnitude and location of the force, measured from A.

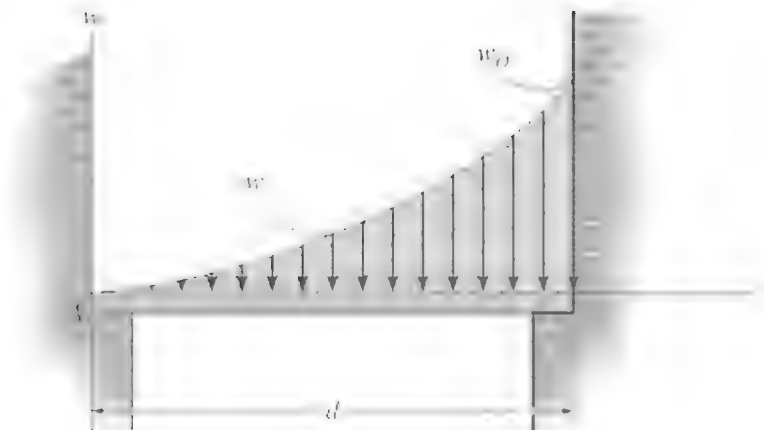
Units Used:

$$kN = 10^3 N$$

Given:

$$w_0 = 500\,\frac{N}{m}$$

$$d = 10\ m$$

$$w(x) = w_0\left(\frac{x}{d}\right)^3$$

Solution:

$$F_R = \int_0^d w(x)\,dx \qquad F_R = 1.25\,kN$$

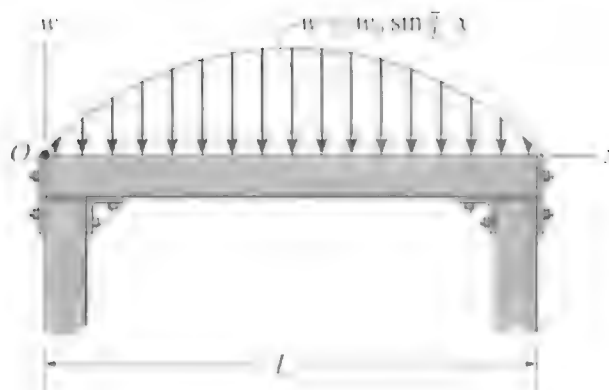
$$d = \frac{\int_0^d x\,w(x)\,dx}{F_R} \qquad d = 8\ m$$

Problem 4-157

Determine the equivalent resultant force and its location, measured from point O.

Solution:

$$F_R = \int_0^L w_0 \sin\left(\frac{\pi x}{L}\right) dx = \frac{2w_0 L}{\pi}$$

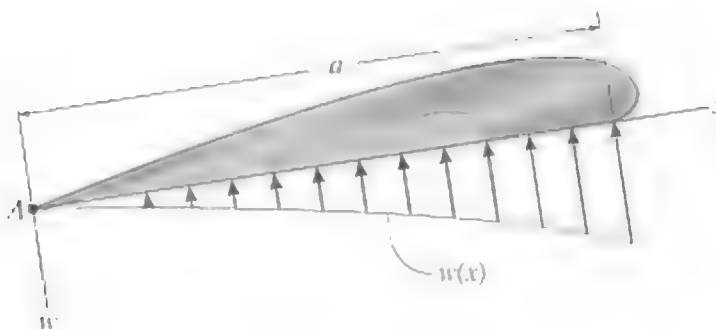
$$d = \frac{\int_0^L x w_0 \sin\left(\frac{\pi x}{L}\right) dx}{F_R} = \frac{L}{2}$$

Problem 4-158

Determine the equivalent resultant force acting on the bottom of the wing due to air pressure and specify where it acts, measured from point A.

Given:

$a = 3$ ft

$k = 86 \frac{\text{lb}}{\text{ft}^3}$

$w(x) = kx^2$

Solution:

$$F_R = \int_0^a w(x)\, dx \qquad F_R = 774\,\text{lb}$$

$$x = \frac{\int_0^a x w(x)\, dx}{F_R} \qquad x = 2.25\,\text{ft}$$

Problem 4-159

Currently eighty-five percent of all neck injuries are caused by rear-end car collisions. To

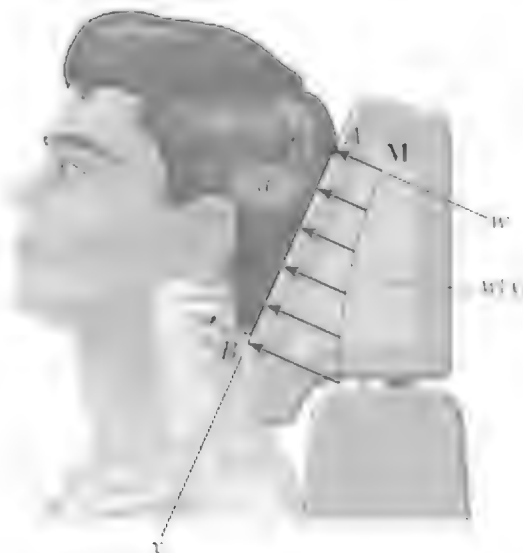
alleviate this problem, an automobile seat restraint has been developed that provides additional pressure contact with the cranium. During dynamic tests the distribution of load on the cranium has been plotted and shown to be parabolic. Determine the equivalent resultant force and its location, measured from point *A*.

Given:

$a = 0.5\ \text{ft}$

$w_0 = 12\ \frac{\text{lb}}{\text{ft}}$

$k = 24\ \frac{\text{lb}}{\text{ft}^3}$

$w(x) = w_0 + kx^2$

Solution:

$$F_R = \int_0^a w(x)\, dx \qquad F_R = 7\,\text{lb}$$

$$x = \frac{\int_0^a x w(x)\, dx}{F_R} \qquad x = 0.268\,\text{ft}$$

Problem 4-160

Determine the equivalent resultant force of the distributed loading and its location, measured from point A. Evaluate the integrals using Simpson's rule.

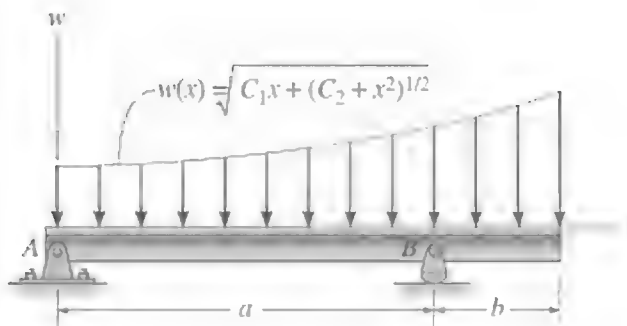
Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$c_1 = 5$

$c_2 = 16$

$a = 3$

$b = 1$

Solution:

$$F_R = \int_0^{a+b} \sqrt{c_1 x + \sqrt{c_2 + x^2}}\, dx \qquad F_R = 14.9$$

$$d = \frac{\int_0^{a+b} x\sqrt{c_1 x + \sqrt{c_2 + x^2}}\, dx}{F_R} \qquad d = 2.27$$

Problem 4-161

Determine the coordinate direction angles of F, which is applied to the end A of the pipe assembly, so that the moment of F about O is zero.

Given:

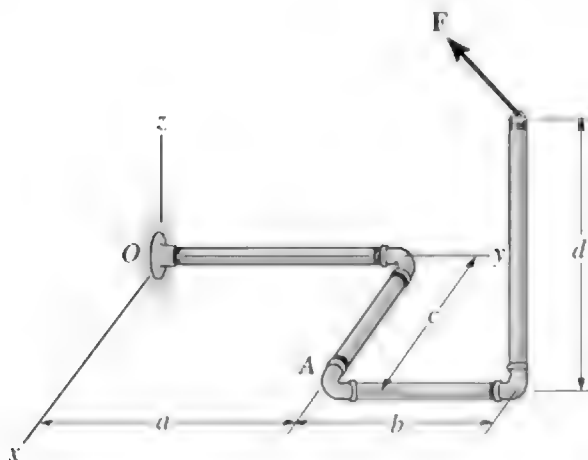
$F = 20$ lb

$a = 8$ in

$b = 6$ in

$c = 6$ in

$d = 10$ in

Solution:

Require $M_o = 0$. This happens when force F is directed either towards or away from point O.

$$\mathbf{r} = \begin{pmatrix} c \\ a+b \\ d \end{pmatrix} \qquad \mathbf{u} = \frac{\mathbf{r}}{|\mathbf{r}|} \qquad \mathbf{u} = \begin{pmatrix} 0.329 \\ 0.768 \\ 0.549 \end{pmatrix}$$

If the force points away from O, then

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}(\mathbf{u}) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 70.774 \\ 39.794 \\ 56.714 \end{pmatrix} \text{deg}$$

If the force points towards O, then

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}(-\mathbf{u}) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 109.226 \\ 140.206 \\ 123.286 \end{pmatrix} \text{deg}$$

Problem 4-162

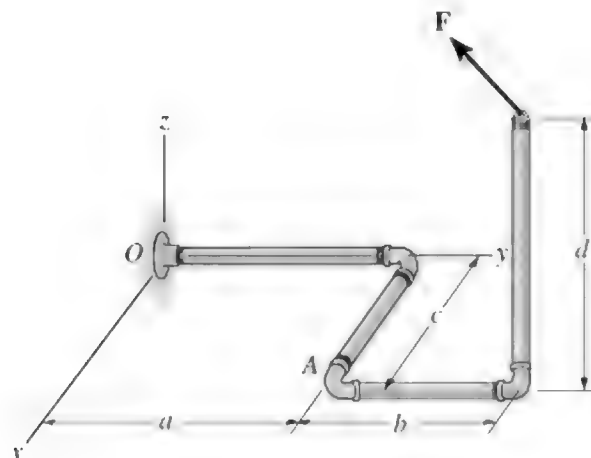
Determine the moment of the force *F* about point *O*. The force has coordinate direction angles α, β, γ. Express the result as a Cartesian vector.

Given:

$F = 20 \text{ lb}$ $\qquad a = 8 \text{ in}$

$\alpha = 60 \text{ deg}$ $\qquad b = 6 \text{ in}$

$\beta = 120 \text{ deg}$ $\qquad c = 6 \text{ in}$

$\gamma = 45 \text{ deg}$ $\qquad d = 10 \text{ in}$

Solution:

$$\mathbf{r} = \begin{pmatrix} c \\ a+b \\ d \end{pmatrix} \qquad \mathbf{F_v} = F\begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix} \qquad \mathbf{M} = \mathbf{r} \times \mathbf{F_v} \qquad \mathbf{M} = \begin{pmatrix} 297.99 \\ 15.147 \\ -200 \end{pmatrix} \text{lb}\cdot\text{in}$$

Problem 4-163

Replace the force at *A* by an equivalent resultant force and couple moment at point *P*. Express the results in Cartesian vector form.

Units Used:

$kN = 10^3\ N$

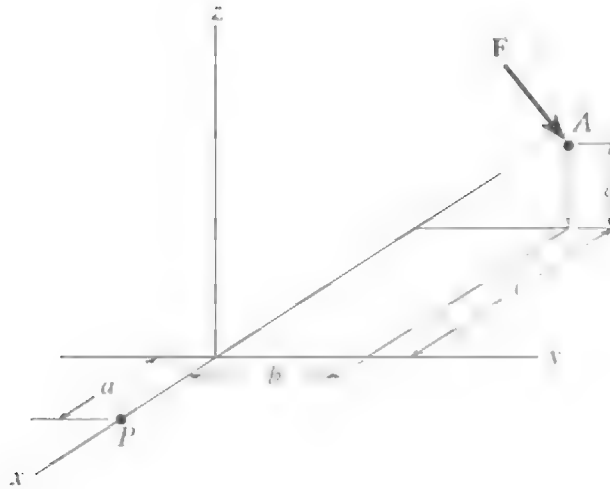
Given:

$a = 4\ m$

$b = 6\ m$

$c = 8\ m$

$d = 4\ m$

$$\mathbf{F} = \begin{pmatrix} -300 \\ 200 \\ -500 \end{pmatrix} N$$

Solution:

$$\mathbf{F_R} = \mathbf{F} \qquad \mathbf{F_R} = \begin{pmatrix} -300 \\ 200 \\ -500 \end{pmatrix} N$$

$$\mathbf{M_P} = \begin{pmatrix} -a-c \\ b \\ d \end{pmatrix} \times \mathbf{F} \qquad \mathbf{M_P} = \begin{pmatrix} -3.8 \\ -7.2 \\ -0.6 \end{pmatrix} kN\cdot m$$

Problem 4-164

Determine the moment of the force $\mathbf{F_C}$ about the door hinge at A. Express the result as a Cartesian vector.

Given:

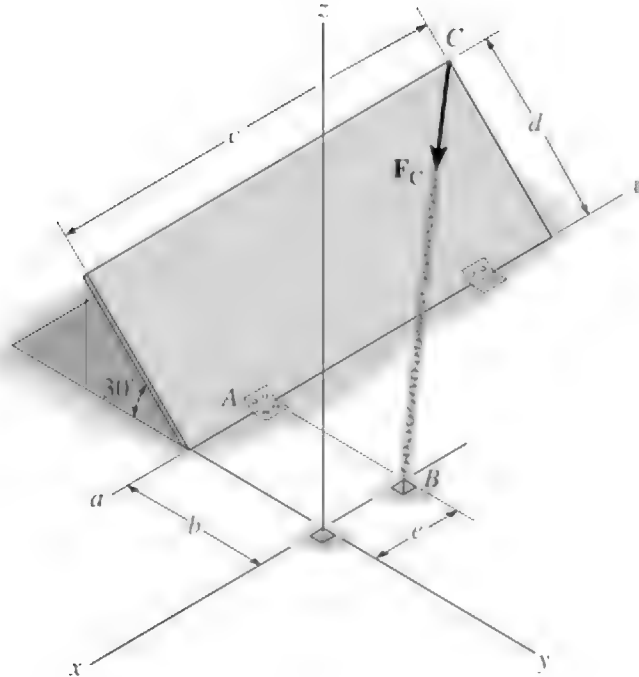
$F = 250\ \text{N}$

$b = 1\ \text{m}$

$c = 2.5\ \text{m}$

$d = 1.5\ \text{m}$

$e = 0.5\ \text{m}$

$\theta = 30\ \text{deg}$

Solution:

$$\mathbf{r_{CB}} = \begin{pmatrix} c - e \\ b + d\cos(\theta) \\ -d\sin(\theta) \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad \mathbf{F_v} = F\frac{\mathbf{r_{CB}}}{|\mathbf{r_{CB}}|}$$

$$\mathbf{M_A} = \mathbf{r_{AB}} \times \mathbf{F_v} \qquad \mathbf{M_A} = \begin{pmatrix} -59.7 \\ 0.0 \\ -159.3 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 4-165

Determine the magnitude of the moment of the force $\mathbf{F_C}$ about the hinged axis aa of the door.

Given:

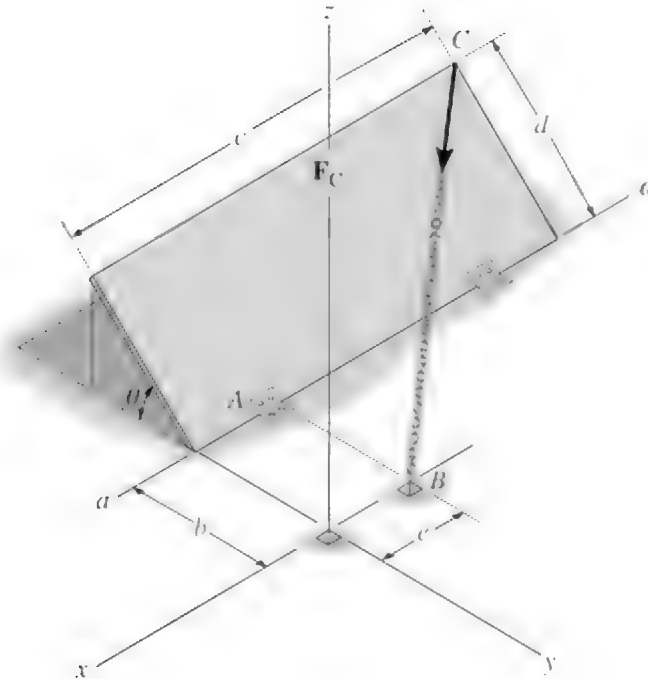
$F = 250\ \text{N}$

$b = 1\ \text{m}$

$c = 2.5\ \text{m}$

$d = 1.5\ \text{m}$

$e = 0.5\ \text{m}$

$\theta = 30\ \text{deg}$

Solution:

$$r_{CB} = \begin{pmatrix} c-e \\ b + d\cos(\theta) \\ -d\sin(\theta) \end{pmatrix} \qquad r_{AB} = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad F_v = F\frac{r_{CB}}{|r_{CB}|} \qquad u_a = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$M_{aa} = (r_{AB} \times F_v)\cdot u_a \qquad M_{aa} = -59.7\ \text{N}\cdot\text{m}$$

Problem 4-166

A force F_1 acts vertically downward on the Z-bracket. Determine the moment of this force about the bolt axis (z axis), which is directed at angle θ from the vertical.

Given:

$F_1 = 80\ \text{N}$

$a = 100\ \text{mm}$

$b = 300\ \text{mm}$

$c = 200\ \text{mm}$

$\theta = 15\ \text{deg}$

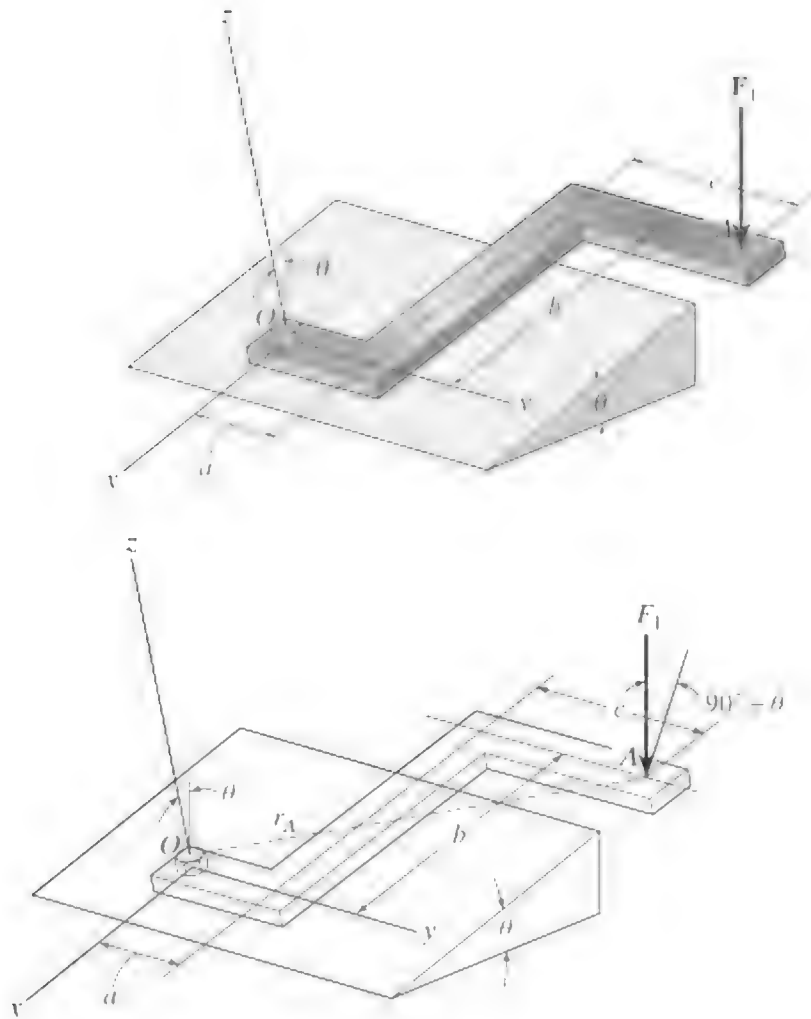
Solution:

$$\mathbf{r} = \begin{pmatrix} -b \\ a+c \\ 0 \end{pmatrix}$$

$$\mathbf{F} = F_1 \begin{pmatrix} \sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix}$$

$$\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$M_z = (\mathbf{r} \times \mathbf{F})\mathbf{k}$$

$$M_z = -6.212\ \text{N}\cdot\text{m}$$

Problem 4-167

Replace the force F having acting at point A by an equivalent force and couple moment at point C.

Units Used: $\text{kip} = 10^3\ \text{lb}$

Given:

$F = 50\ \text{lb}$

$a = 10\ \text{ft}$

$b = 20\ \text{ft}$

$c = 15\ \text{ft}$

$d = 10\ \text{ft}$

$e = 30\ \text{ft}$

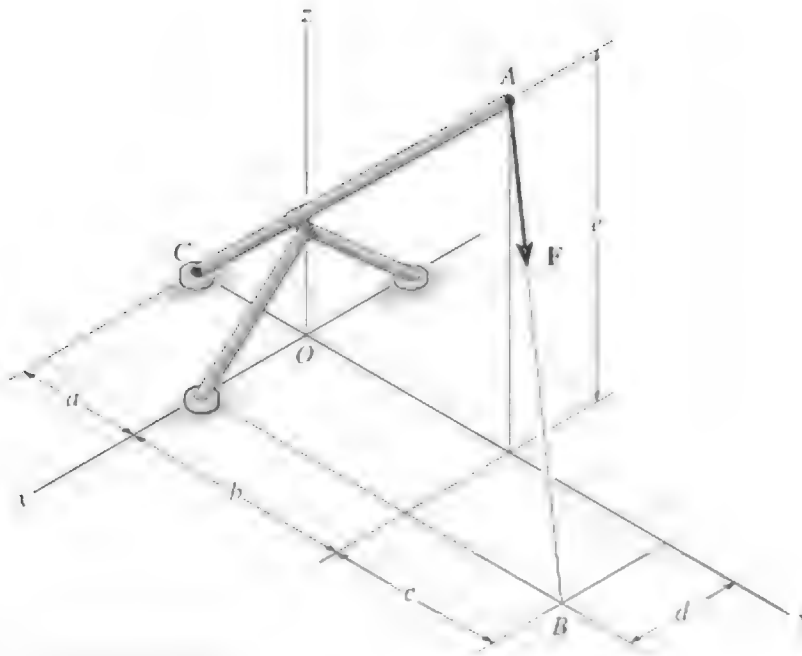
Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} d \\ c \\ -e \end{pmatrix}$$

$$\mathbf{F_v} = F\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|}$$

$$\mathbf{r_{CA}} = \begin{pmatrix} 0 \\ a+b \\ e \end{pmatrix}$$

$$\mathbf{F_R} = \mathbf{F_v} \qquad \mathbf{F_R} = \begin{pmatrix} 14.286 \\ 21.429 \\ -42.857 \end{pmatrix} \text{lb}$$

$$\mathbf{M_R} = \mathbf{r_{CA}} \times \mathbf{F_v} \qquad \mathbf{M_R} = \begin{pmatrix} -1.929 \\ 0.429 \\ -0.429 \end{pmatrix} \text{kip}\cdot\text{ft}$$

Problem 4-168

The horizontal force F acts on the handle of the wrench. What is the magnitude of the moment of this force about the z axis?

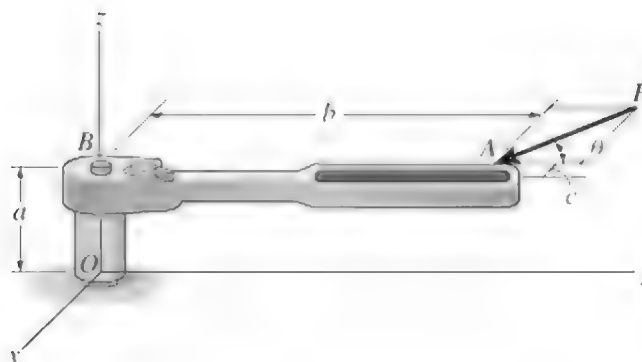
Given:

$F = 30\ \text{N}$

$a = 50\ \text{mm}$

$b = 200\ \text{mm}$

$c = 10\ \text{mm}$

$\theta = 45\ \text{deg}$

Solution:

$$\mathbf{F_v} = F\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{r_{OA}} = \begin{pmatrix} -c \\ b \\ a \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$M_z = (\mathbf{r_{OA}} \times \mathbf{F_v})\mathbf{k} \qquad M_z = -4.03\ \text{N}\cdot\text{m}$$

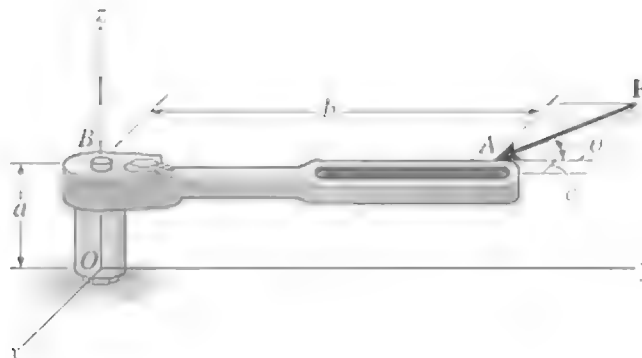
Problem 4-169

The horizontal force **F** acts on the handle of the wrench. Determine the moment of this force about point *O*. Specify the coordinate direction angles α, β, γ of the moment axis.

Given:

$F = 30$ N $\quad c = 10$ mm

$a = 50$ mm $\quad \theta = 45$ deg

$b = 200$ mm

Solution:

$$\mathbf{F_v} = F\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{r_{OA}} = \begin{pmatrix} -c \\ b \\ a \end{pmatrix}$$

$$\mathbf{M_O} = \mathbf{r_{OA}} \times \mathbf{F_v} \qquad \mathbf{M_O} = \begin{pmatrix} 1.06 \\ 1.06 \\ -4.03 \end{pmatrix} \text{N}\cdot\text{m}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}\left(\frac{\mathbf{M_O}}{|\mathbf{M_O}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 75.7 \\ 75.7 \\ 159.6 \end{pmatrix} \text{deg}$$

Problem 4-170

If the resultant couple moment of the three couples acting on the triangular block is to be zero, determine the magnitudes of forces **F** and **P.**

Given:

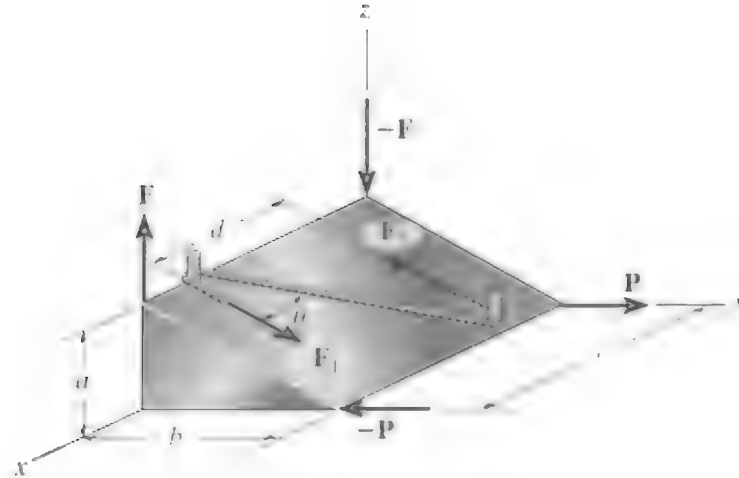
$F_1 = 10 \text{ lb}$

$a = 3 \text{ in}$

$b = 4 \text{ in}$

$c = 6 \text{ in}$

$d = 3 \text{ in}$

$\theta = 30 \text{ deg}$

Solution:

Initial Guesses: $F = 1 \text{ lb}$ $P = 1 \text{ lb}$

Given

$$\begin{pmatrix} 0 \\ -F c \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -P c \end{pmatrix} + \frac{F_1 d}{\sqrt{a^2 + b^2}} \begin{pmatrix} 0 \\ a \\ b \end{pmatrix} = 0 \qquad \begin{pmatrix} F \\ P \end{pmatrix} = \text{Find}(F, P) \qquad \begin{pmatrix} F \\ P \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{lb}$$

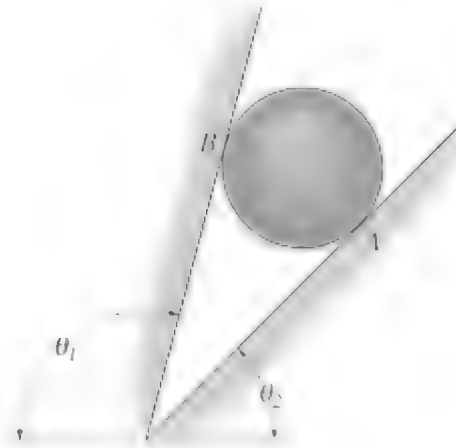
Problem 5-1

Draw the free-body diagram of the sphere of weight W resting between the smooth inclined planes. Explain the significance of each force on the diagram.

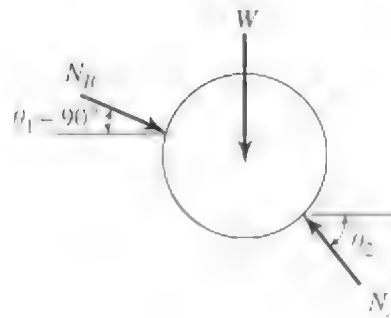
Given:

$W = 10$ lb

$\theta_1 = 105$ deg

$\theta_2 = 45$ deg

Solution:

N_A, N_B force of plane on sphere.

W force of gravity on sphere.

Problem 5-2

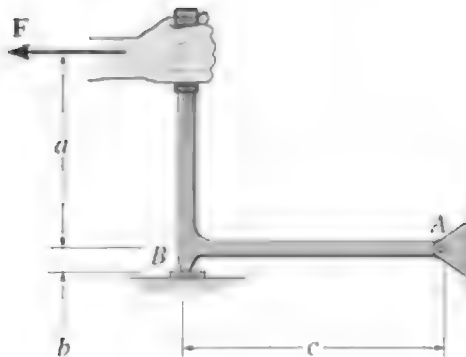
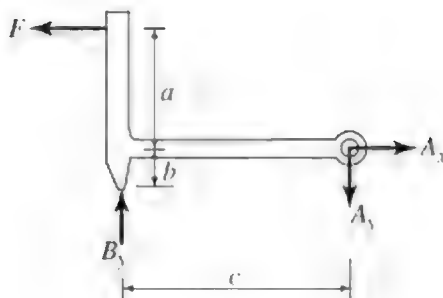
Draw the free-body diagram of the hand punch, which is pinned at A and bears down on the smooth surface at B.

Given:

$F = 8$ lb

$a = 1.5$ ft

$b = 0.2$ ft

$c = 2$ ft

Solution:

Problem 5-3

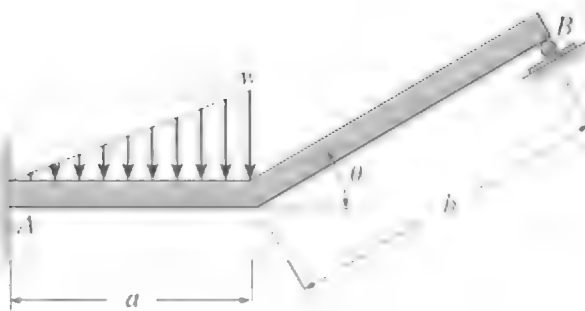
Draw the free-body diagram of the beam supported at A by a fixed support and at B by a roller. Explain the significance of each force on the diagram.

Given:

$$w = 40\ \frac{\text{lb}}{\text{ft}}$$

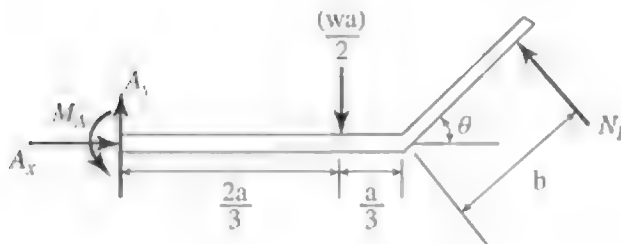
$$a = 3\ \text{ft}$$

$$b = 4\ \text{ft}$$

$$\theta = 30\ \text{deg}$$

Solution:

A_x, A_y, M_A effect of wall on beam.

N_B force of roller on beam.

$\frac{w a}{2}$ resultant force of distributed load on beam.

Problem 5-4

Draw the free-body diagram of the jib crane AB, which is pin-connected at A and supported by member (link) BC.

Units Used:

$kN = 10^3 \ N$

Given:

$F = 8 \ kN$

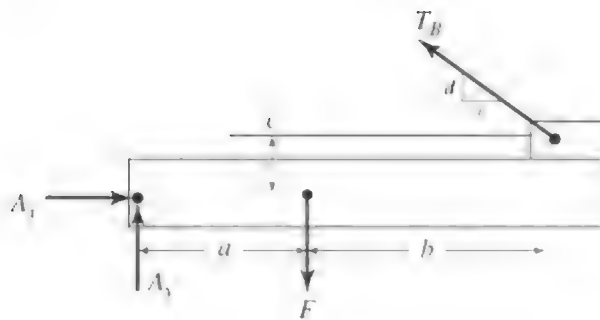
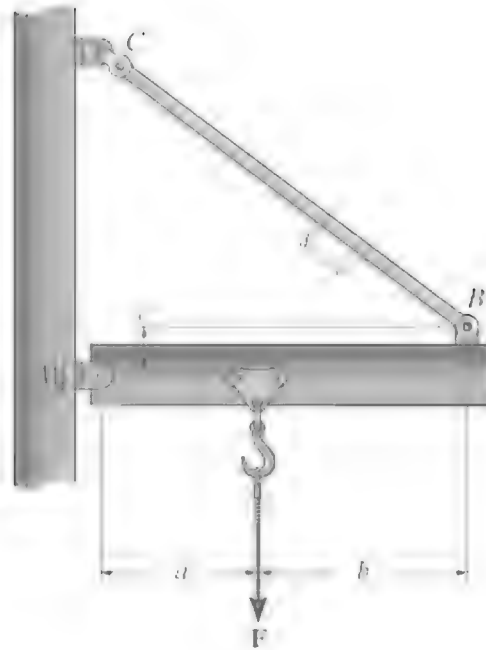
$a = 3 \ m$

$b = 4 \ m$

$c = 0.4 \ m$

$d = 3$

$e = 4$

Solution:

Problem 5-5

Draw the free-body diagram of the C-bracket supported at A, B, and C by rollers. Explain the significance of each forcce on the diagram.

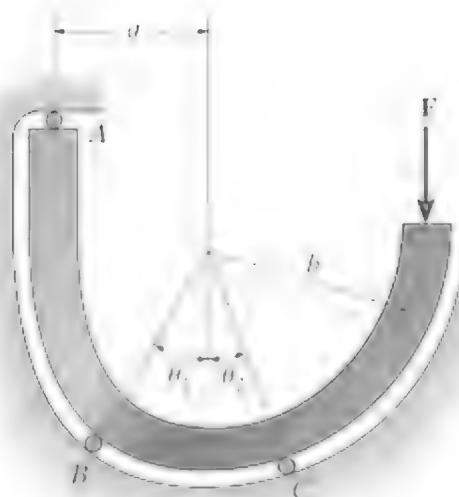
Given:

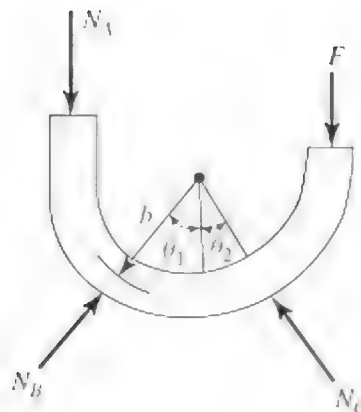
$a = 3 \ ft$

$b = 4 \ ft$

$\theta_1 = 30 \ deg$

$\theta_2 = 20 \ deg$

$F = 200 \ lb$

Solution:

N_A, N_B, N_C force of rollers on beam.

Problem 5-6

Draw the free-body diagram of the smooth rod of mass M which rests inside the glass. Explain the significance of each force on the diagram.

Given:

$M = 20$ gm

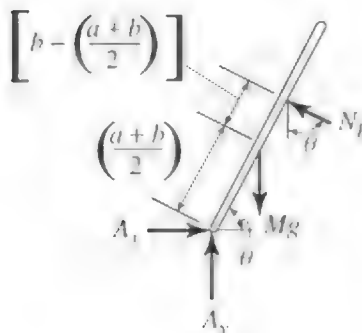
$a = 75$ mm

$b = 200$ mm

$\theta = 40$ deg

Solution:

A_x, A_y, N_B force of glass on rod.

$M(g)$ N force of gravity on rod.

Problem 5-7

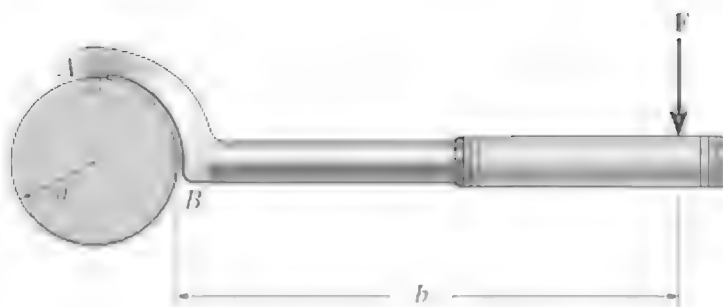
Draw the free-body diagram of the "spanner wrench" subjected to the force **F**. The support at A can be considered a pin, and the surface of contact at B is smooth. Explain the significance of

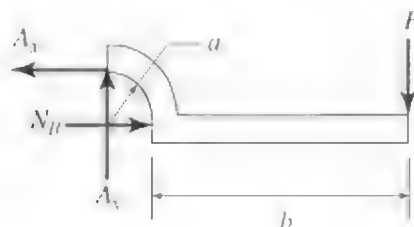
each force on the diagram.

Given:

$F = 20\ \text{lb}$

$a = 1\ \text{in}$

$b = 6\ \text{in}$

Solution:

A_x, A_y, N_B force of cylinder on wrench.

Problem 5-8

Draw the free-body diagram of the automobile, which is being towed at constant velocity up the incline using the cable at *C*. The automobile has a mass *M* and center of mass at *G*. The tires are free to roll. Explain the significance of each force on the diagram.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$

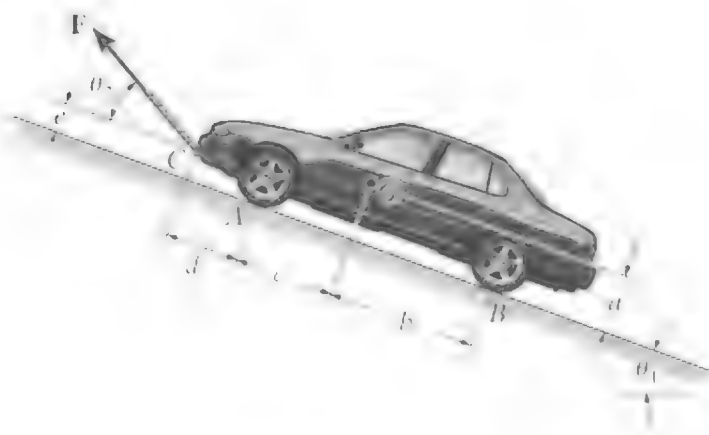
Given:

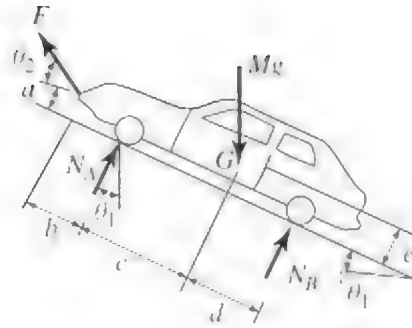
$M = 5\ \text{Mg}$ $\quad d = 1.50\ \text{m}$

$a = 0.3\ \text{m}$ $\quad e = 0.6\ \text{m}$

$b = 0.75\ \text{m}$ $\quad \theta_1 = 20\ \text{deg}$

$c = 1\ \text{m}$ $\quad \theta_2 = 30\ \text{deg}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

N_A, N_B force of road on car.
F force of cable on car.
Mg force of gravity on car.

Problem 5-9

Draw the free-body diagram of the uniform bar, which has mass M and center of mass at G. The supports A, B, and C are smooth.

Given:

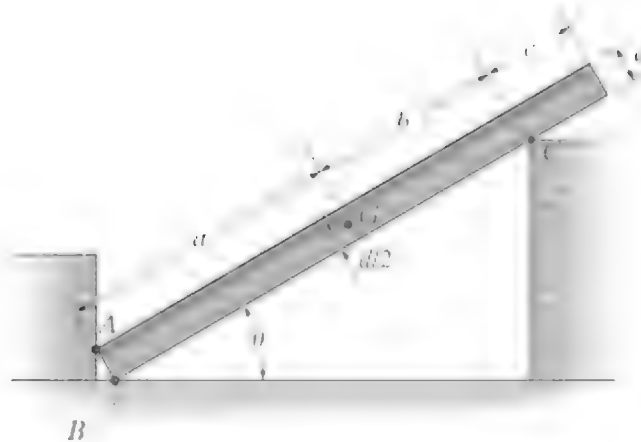
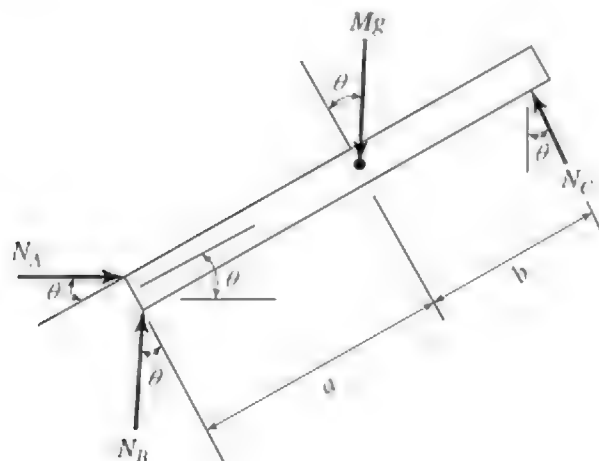
$M = 100 \text{ kg}$

$a = 1.75 \text{ m}$

$b = 1.25 \text{ m}$

$c = 0.5 \text{ m}$

$d = 0.2 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

Problem 5-10

Draw the free-body diagram of the beam, which is pin-connected at A and rocker-supported at B.

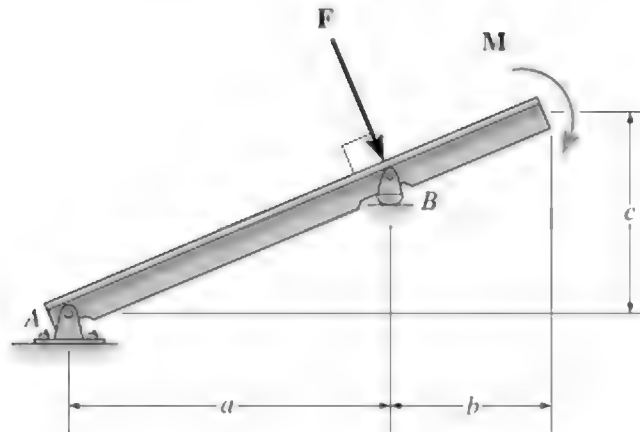
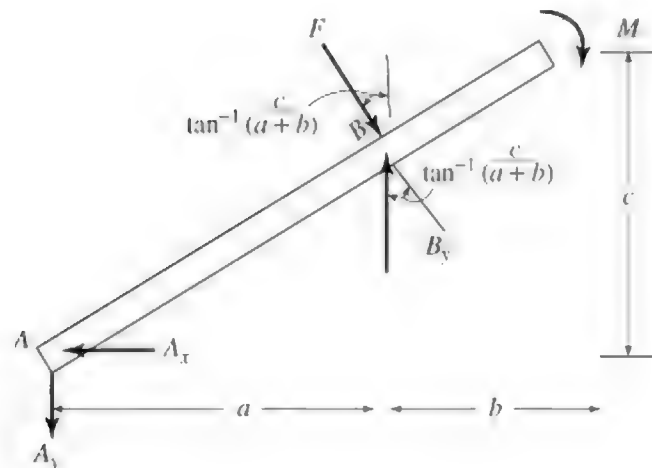
Given:

$F = 500\text{ N}$

$M = 800\text{ N}\cdot\text{m}$

$a = 8\text{ m}$

$b = 4\text{ m}$

$c = 5\text{ m}$

Solution:

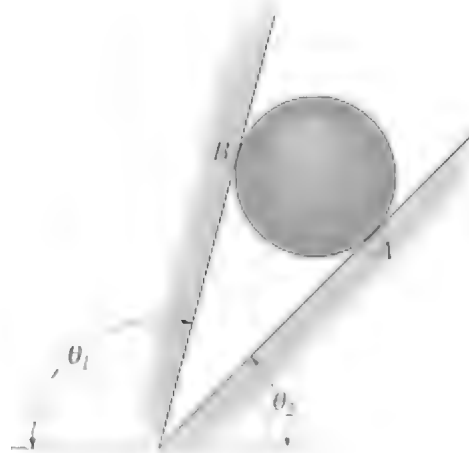
Problem 5-11

The sphere of weight W rests between the smooth inclined planes. Determine the reaactions at the supports.

Given:

$W = 10\text{ lb}$

$\theta_1 = 105\text{ deg}$

$\theta_2 = 45\text{ deg}$

Solution:

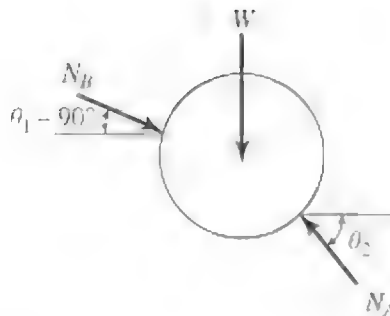
Initial guesses

$N_A = 1 \text{ lb} \qquad N_B = 1 \text{ lb}$

Given

$$N_B \cos(\theta_1 - 90 \text{ deg}) - N_A \cos(\theta_2) = 0$$

$$N_A \sin(\theta_2) - N_B \sin(\theta_1 - 90 \text{ deg}) - W = 0$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 19.3 \\ 14.1 \end{pmatrix} \text{lb}$$

Problem 5-12

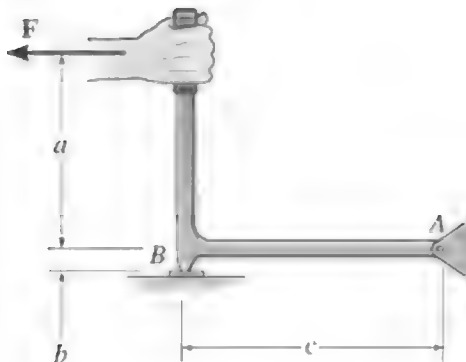
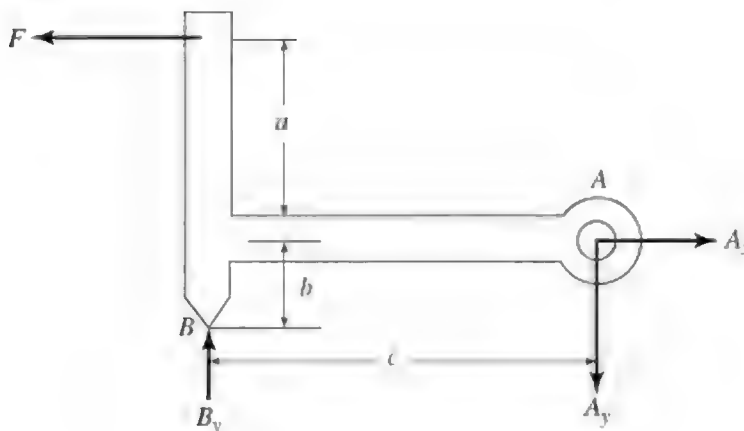
Determine the magnitude of the resultant force acting at pin A of the handpunch.

Given:

$F = 8 \text{ lb}$

$a = 1.5 \text{ ft}$

$b = 0.2 \text{ ft}$

$c = 2 \text{ ft}$

Solution:

$\Sigma F_x = 0; \qquad A_x - F = 0 \qquad A_x = F \qquad A_x = 8 \text{ lb}$

$\Sigma M = 0; \qquad F a - A_y c = 0 \qquad A_y = F \dfrac{a}{c} \qquad A_y = 6 \text{ lb}$

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} \qquad F_A = 10\,\text{lb}$$

Problem 5-13

The *C*-bracket is supported at *A*, *B*, and *C* by rollers. Determine the reactions at the supports.

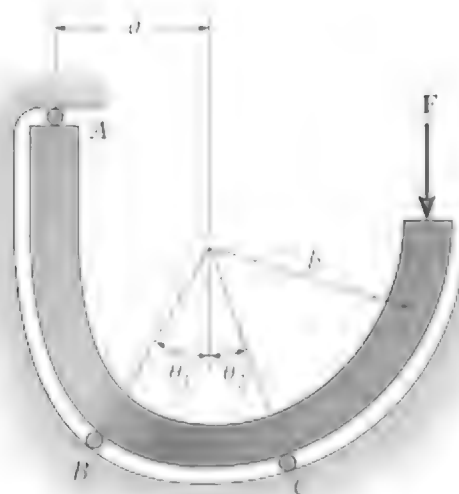
Given:

$a = 3\ \text{ft}$

$b = 4\ \text{ft}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 20\ \text{deg}$

$F = 200\ \text{lb}$

Solution:

Initial Guesses:

$N_A = 1\ \text{lb}$

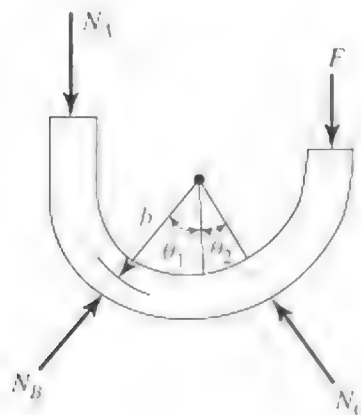
$N_B = 1\ \text{lb}$

$N_C = 1\ \text{lb}$

Given

$$N_A a - F b = 0$$

$$N_B \sin(\theta_1) - N_C \sin(\theta_2) = 0$$

$$N_B \cos(\theta_1) + N_C \cos(\theta_2) - N_A - F = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ N_C \end{pmatrix} = \text{Find}(N_A, N_B, N_C) \qquad \begin{pmatrix} N_A \\ N_B \\ N_C \end{pmatrix} = \begin{pmatrix} 266.7 \\ 208.4 \\ 304.6 \end{pmatrix} \text{lb}$$

Problem 5-14

The smooth rod of mass M rests inside the glass. Determine the reactions on the rod.

Given:

$M = 20 \text{ gm}$

$a = 75 \text{ mm}$

$b = 200 \text{ mm}$

$\theta = 40 \text{ deg}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

Solution:

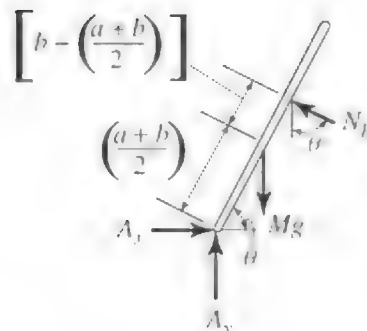
Initial Guesses:

$A_x = 1 \text{ N} \quad A_y = 1 \text{ N} \quad N_B = 1 \text{ N}$

Given

$$A_x - N_B \sin(\theta) = 0$$

$$A_y - M g + N_B \cos(\theta) = 0$$

$$-M g \frac{a+b}{2} \cos(\theta) + N_B b = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ N_B \end{pmatrix} = \text{Find}(A_x, A_y, N_B) \qquad \begin{pmatrix} A_x \\ A_y \\ N_B \end{pmatrix} = \begin{pmatrix} 0.066 \\ 0.117 \\ 0.103 \end{pmatrix} \text{N}$$

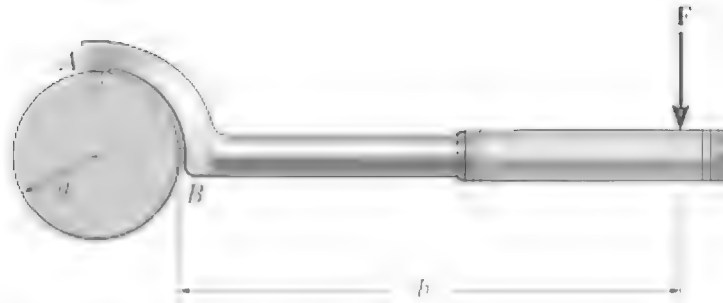
Problem 5-15

The "spanner wrench" is subjected to the force **F**. The support at A can be considered a pin, and the surface of contact at B is smooth. Determine the reactions on the spanner wrench.

Given:

$F = 20\ \text{lb}$

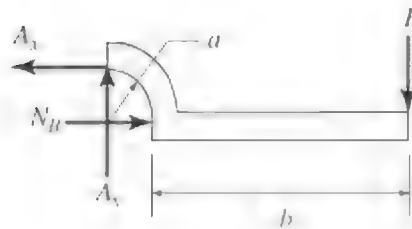
$a = 1\ \text{in}$

$b = 6\ \text{in}$

Solution:

Initial Guesses:

$A_x = 1\ \text{lb}$

$A_y = 1\ \text{lb}$

$N_B = 1\ \text{lb}$

Given

$$-A_x + N_B = 0 \qquad A_y - F = 0 \qquad -F\ (a + b) + A_x\ a = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ N_B \end{pmatrix} = \text{Find}(A_x, A_y, N_B) \qquad \begin{pmatrix} A_x \\ A_y \\ N_B \end{pmatrix} = \begin{pmatrix} 140 \\ 20 \\ 140 \end{pmatrix} \text{lb}$$

Problem 5-16

The automobile is being towed at constant velocity up the incline using the cable at C. The automobile has a mass M and center of mass at G. The tires are free to roll. Determine the reactions on both wheels at A and B and the tension in the cable at C.

Units Used:

$\text{Mg} = 10^3\ \text{kg} \quad \text{kN} = 10^3\ \text{N}$

Given:

$M = 5 \text{ Mg}$ $d = 1.50 \text{ m}$

$a = 0.3 \text{ m}$ $e = 0.6 \text{ m}$

$b = 0.75 \text{ m}$ $\theta_1 = 20 \text{ deg}$

$c = 1 \text{ m}$ $\theta_2 = 30 \text{ deg}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

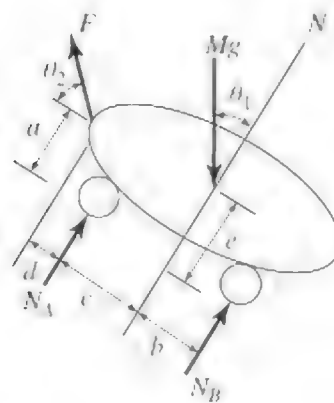
Solution:

Guesses $F = 1 \text{ kN}$ $N_A = 1 \text{ kN}$ $N_B = 1 \text{ kN}$

Given

$$N_A + N_B + F \sin(\theta_2) - M g \cos(\theta_1) = 0$$

$$-F \cos(\theta_2) + M g \sin(\theta_1) = 0$$

$$F \cos(\theta_2) a - F \sin(\theta_2) b - M g \cos(\theta_1) c - M g \sin(\theta_1) e + N_B(c + d) = 0$$

$$\begin{pmatrix} F \\ N_A \\ N_B \end{pmatrix} = \text{Find}(F, N_A, N_B) \qquad \begin{pmatrix} F \\ N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 19.37 \\ 13.05 \\ 23.36 \end{pmatrix} \text{kN}$$

Problem 5-17

The uniform bar has mass M and center of mass at G. The supports A, B, and C are smooth. Determine the reactions at the points of contact at A, B, and C.

Given:

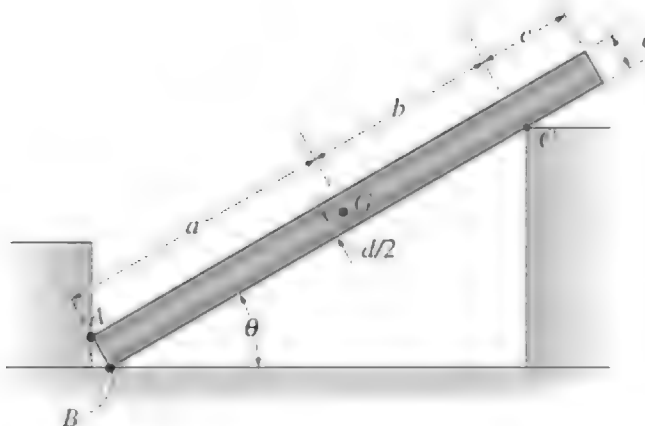
$M = 100 \text{ kg}$

$a = 1.75 \text{ m}$

$b = 1.25 \text{ m}$

$c = 0.5\ \text{m}$

$d = 0.2\ \text{m}$

$\theta = 30\ \text{deg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

The initial guesses:

$N_A = 20\ \text{N}$

$N_B = 30\ \text{N}$

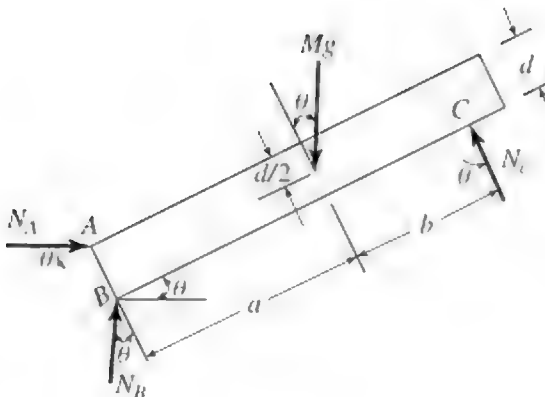
$N_C = 40\ \text{N}$

Given

$\Sigma M_A = 0; \quad -M\,g\cos(\theta)a - M\,g\sin(\theta)\,\frac{d}{2} + N_B\,\sin(\theta)d + N_C(a+b) = 0$

$+\uparrow\Sigma\ F_y = 0; \quad N_B - M\,g + N_C\,\cos(\theta) = 0$

$+\uparrow\Sigma\ F_y = 0; \quad N_A - N_C\,\sin(\theta) = 0$

$$\begin{pmatrix} N_C \\ N_B \\ N_A \end{pmatrix} = \text{Find}(N_C, N_B, N_A) \qquad \begin{pmatrix} N_C \\ N_B \\ N_A \end{pmatrix} = \begin{pmatrix} 493 \\ 554 \\ 247 \end{pmatrix} \text{N}$$

Problem 5-18

The beam is pin-connected at A and rocker-supported at B. Determine the reactions at the pin A and at the roller at B.

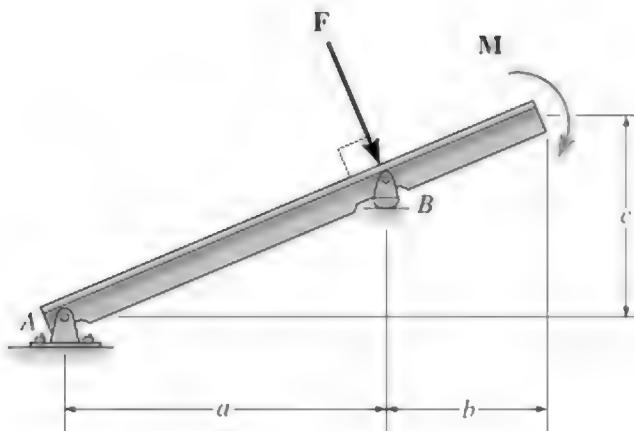
Given:

$F = 500\ \text{N}$

$M = 800\ \text{N}\cdot\text{m}$

$a = 8\ \text{m}$

$b = 4 \text{ m}$

$c = 5 \text{ m}$

Solution:

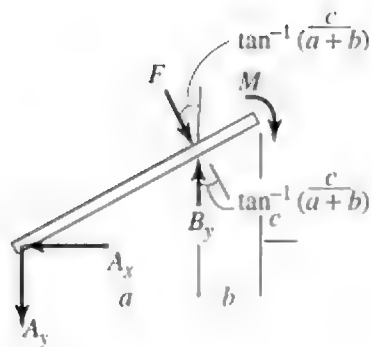
$\alpha = \operatorname{atan}\left(\frac{c}{a+b}\right)$

$\Sigma M_A = 0;$ $-F\frac{a}{\cos(\alpha)} - M + B_y a = 0$

$B_y = \frac{F\ a + M\ \cos(\alpha)}{\cos(\alpha)\ a}$

$B_y = 642 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $-A_x + F\ \sin(\alpha) = 0$ $A_x = F\ \sin(\alpha)$ $A_x = 192 \text{ N}$

$+\uparrow \Sigma F_y = 0;$ $-A_y - F\ \cos(\alpha) + B_y = 0$ $A_y = -F\ \cos(\alpha) + B_y$ $A_y = 180 \text{ N}$

Problem 5-19

Determine the magnitude of the reactions on the beam at A and B. Neglect the thickness of the beam.

Given:

$F_1 = 600 \text{ N}$

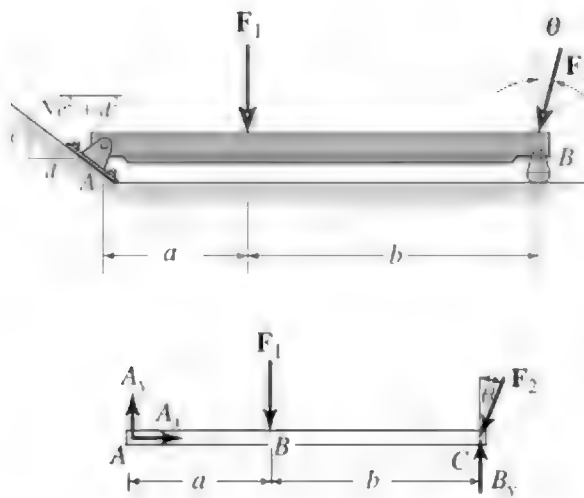
$F_2 = 400 \text{ N}$

$\theta = 15 \text{ deg}$

$a = 4 \text{ m}$

$b = 8 \text{ m}$

$c = 3$

$d = 4$

Solution:

$\Sigma M_A = 0;$ $B_y(a+b) - F_2\cos(\theta)(a+b) - F_1\ a = 0$

$$B_y = \frac{F_2 \cos(\theta)\ (a+b) + F_1\ a}{a+b} \qquad B_y = 586\ \text{N}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - F_2 \sin(\theta) = 0$

$$A_x = F_2 \sin(\theta) \qquad A_x = 104\ \text{N}$$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - F_2 \cos(\theta) + B_y - F_1 = 0$

$$A_y = F_2 \cos(\theta) - B_y + F_1 \qquad A_y = 400\ \text{N}$$

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} \qquad F_A = 413\ \text{N}$$

Problem 5-20

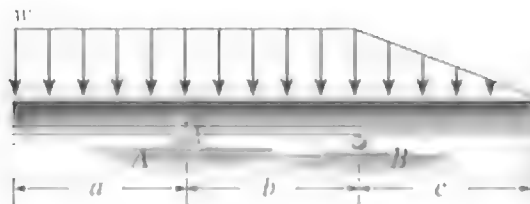
Determine the reactions at the supports.

Given:

$w = 250\ \frac{\text{lb}}{\text{ft}}$

$a = 6\ \text{ft}$

$b = 6\ \text{ft}$

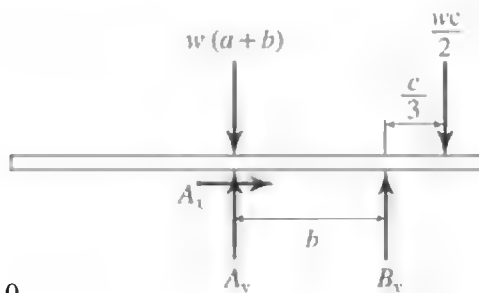
$c = 6\ \text{ft}$

Solution:

Guesses

$A_x = 1\ \text{lb} \qquad A_y = 1\ \text{lb} \qquad B_y = 1\ \text{lb}$

Given

$A_x = 0 \qquad A_y + B_y - w(a+b) - \frac{1}{2}\ wc = 0$

$$wa\left(\frac{a}{2}\right) - wb\left(\frac{b}{2}\right) - \frac{1}{2}wc\left(b + \frac{c}{3}\right) + B_y\ b = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_y) \qquad \begin{pmatrix} A_x \\ A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 0 \\ 2750 \\ 1000 \end{pmatrix} \text{lb}$$

Problem 5-21

When holding the stone of weight W in equilibrium, the humerus H, assumed to be smooth, exerts normal forces $\mathbf{F}_C$ and $\mathbf{F}_A$ on the radius C and ulna A as shown. Determine these forces and the force $\mathbf{F}_B$ that the biceps B exerts on the radius for equilibrium. The stone has a center of mass at G. Neglect the weight of the arm.

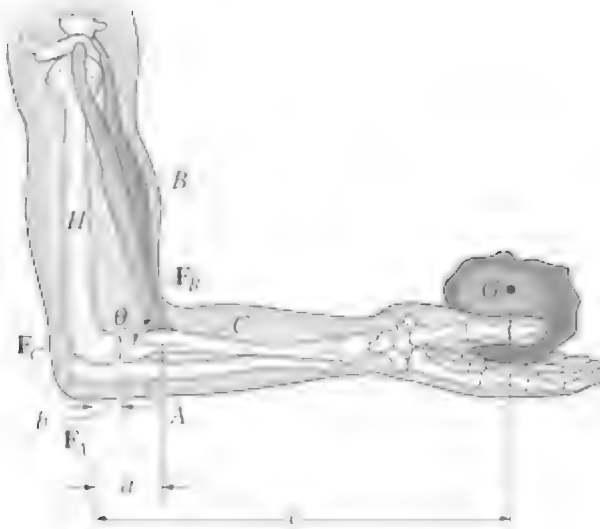
Given:

$W = 5$ lb

$\theta = 75$ deg

$a = 2$ in

$b = 0.8$ in

$c = 14$ in

Solution:

$\Sigma M_B = 0; \qquad -W(c-a) + F_A\, a = 0$

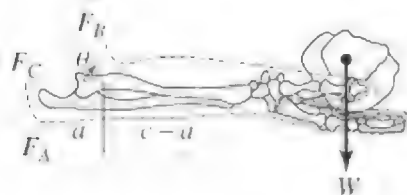
$$F_A = W\left(\frac{c-a}{a}\right)$$

$F_A = 30\,\text{lb}$

$+\uparrow \Sigma F_y = 0; \qquad F_B \sin(\theta) - W - F_A = 0$

$$F_B = \frac{W + F_A}{\sin(\theta)}$$

$F_B = 36.2\,\text{lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_C - F_B \cos(\theta) = 0$

$$F_C = F_B \cos(\theta)$$

$$F_C = 9.378 \text{ lb}$$

Problem 5-22

The uniform door has a weight W and a center of gravity at G. Determine the reactions at the hinges if the hinge at A supports only a horizontal reaction on the door, whereas the hinge at B exerts both horizontal and vertical reactions.

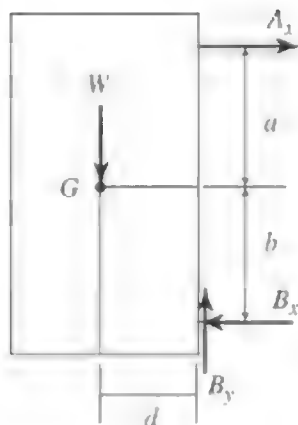
Given:

$W = 100$ lb

$a = 3$ ft

$b = 3$ ft

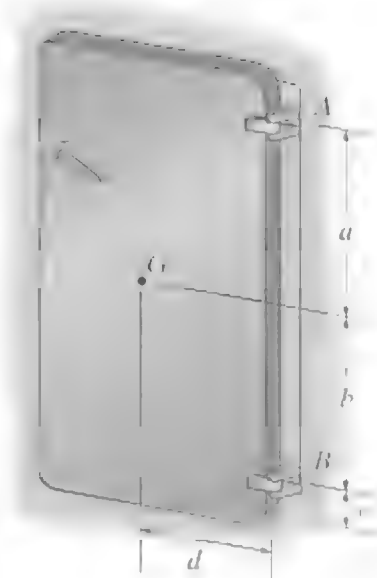
$c = 0.5$ ft

$d = 2$ ft

Solution:

$\Sigma M_B = 0;\quad W d - A_x (a + b) = 0$

$$A_x = W\left(\frac{d}{a+b}\right) \qquad A_x = 33.3 \text{ lb}$$

$\Sigma F_x = 0;\quad B_x = A_x \qquad B_x = 33.3 \text{ lb}$

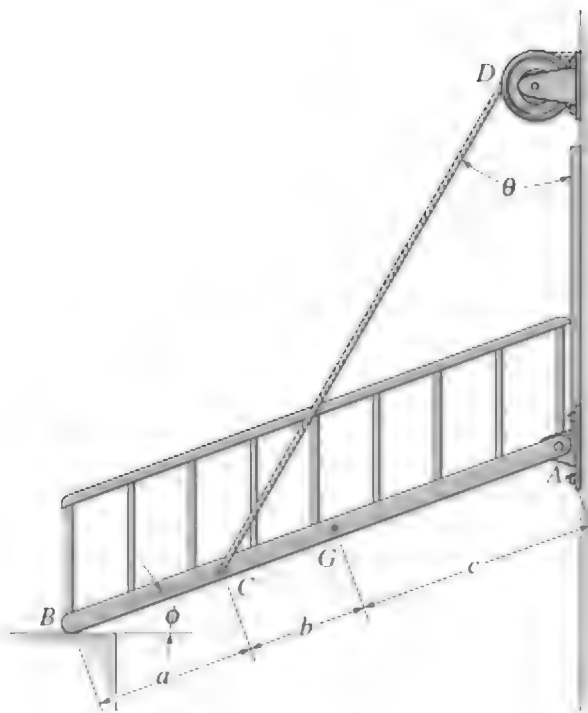
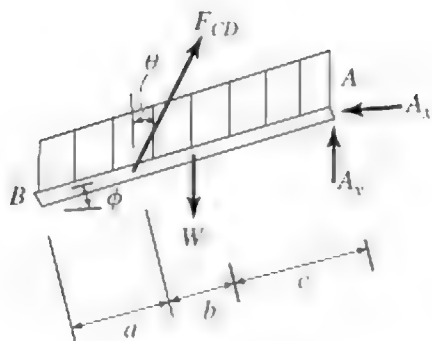
$\Sigma F_y = 0;\quad B_y = W \qquad B_y = 100 \text{ lb}$

Problem 5-23

The ramp of a ship has weight W and center of gravity at G. Determine the cable force in CD needed to just start lifting the ramp, (i.e., so the reaction at B becomes zero). Also, determine the horizontal and vertical components of force at the hinge (pin) at A.

Given:

$W = 200$ lb $\quad a = 4$ ft

$\theta = 30$ deg $\quad b = 3$ ft

$\phi = 20$ deg $\quad c = 6$ ft

Solution:

$\Sigma M_A = 0;$

$$-F_{CD}\cos(\theta)(b+c)\cos(\phi) + F_{CD}\sin(\theta)(b+c)\sin(\phi) + Wc\cos(\phi) = 0$$

$$F_{CD} = \frac{W\,c\,\cos(\phi)}{(b+c)(\cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi))} \qquad F_{CD} = 195\text{ lb}$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CD}\sin(\theta) - A_x = 0$

$$A_x = F_{CD}\sin(\theta) \qquad A_x = 97.5\text{ lb}$$

$+\uparrow \Sigma F_y = 0; \qquad A_y - W + F_{CD}\cos(\theta) = 0$

$$A_y = W - F_{CD}\cos(\theta) \qquad A_y = 31.2\text{ lb}$$

Problem 5-24

The drainpipe of mass M is held in the tines of the fork lift. Determine the normal forces at A and B as functions of the blade angle θ and plot the results of force (ordinate) versus θ (abscissa) for $0 \le \theta \le 90$ deg.

Units used:

$$Mg = 10^3 \text{ kg}$$

Given:

$$M = 1.4 \text{ Mg}$$

$$a = 0.4 \text{ m}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution:

$$\theta = 0..90$$

$$N_A(\theta) = \frac{M g \sin(\theta \text{ deg})}{10^3}$$

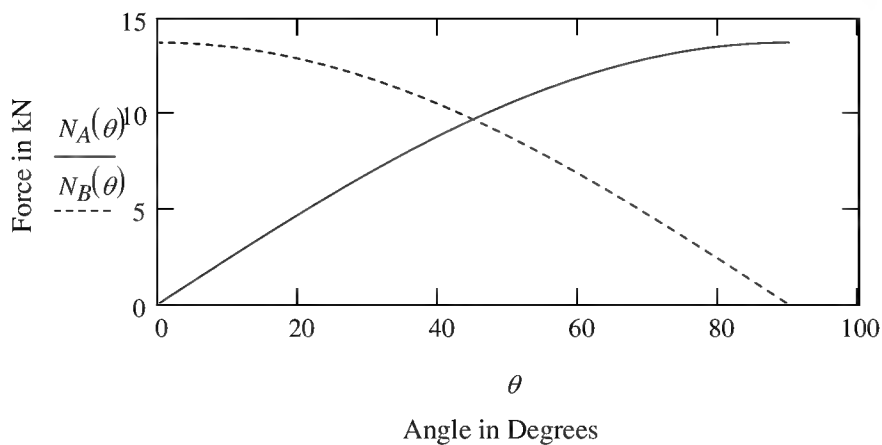
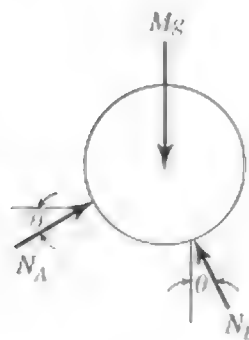
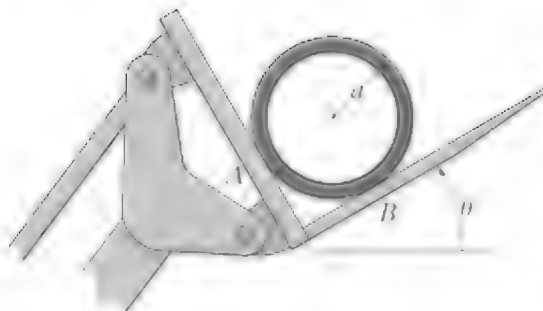
$$N_B(\theta) = \frac{M g \cos(\theta \text{ deg})}{10^3}$$

Force in kN

$N_A(\theta)$ ——

$N_B(\theta)$ - - - - -

θ

Angle in Degrees

Problem 5-25

While *slowly* walking, a man having a total mass M places all his weight on *one foot*. Assuming that the normal force $\mathbf{N}_C$ of the ground acts on his foot at C, determine the resultant vertical compressive force $\mathbf{F}_B$ which the tibia T exerts on the astragalus B, and the vertical tension $\mathbf{F}_A$ in the achilles tendon A at the instant shown.

Units Used:

$kN = 10^3\ N$

Given:

$M = 80\ kg$

$a = 15\ mm$

$b = 5\ mm$

$c = 20\ mm$

$d = 100\ mm$

Solution:

$N_C = M g$

$N_C = 785\ N$

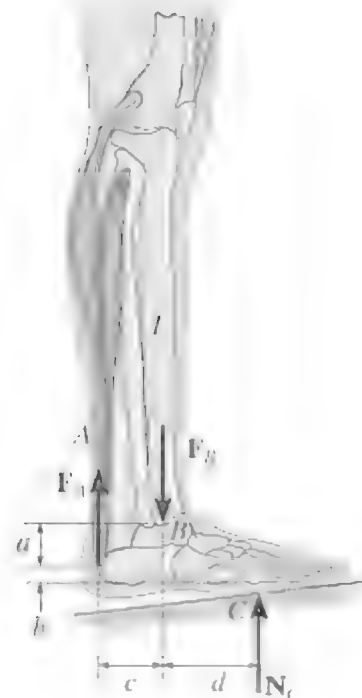
$\Sigma M_A = 0;$ $-F_B\, c + N_C\, (c + d) = 0$

$F_B = N_C \left(\frac{c + d}{c}\right)$

$F_B = 4.71\ kN$

$\Sigma F_y = 0;$ $F_A - F_B + N_C = 0$

$F_A = F_B - N_C$

$F_A = 3.92\ kN$

Problem 5-26

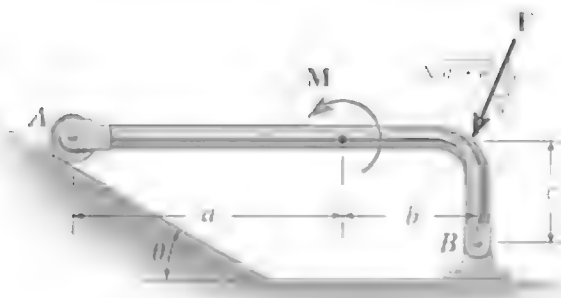
Determine the reactions at the roller A and pin B.

Given:

$M = 800 \text{ lb ft}$ $c = 3 \text{ ft}$

$F = 390 \text{ lb}$ $d = 5$

$a = 8 \text{ ft}$ $e = 12$

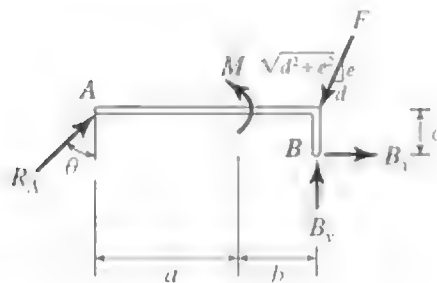
$b = 4 \text{ ft}$ $\theta = 30 \text{ deg}$

Solution:

Guesses $R_A = 1 \text{ lb}$ $B_x = 1 \text{ lb}$ $B_y = 1 \text{ lb}$

Given

$$R_A \sin(\theta) + B_x - \left(\frac{d}{\sqrt{e^2 + d^2}}\right)F = 0$$

$$R_A \cos(\theta) + B_y - \left(\frac{e}{\sqrt{e^2 + d^2}}\right)F = 0$$

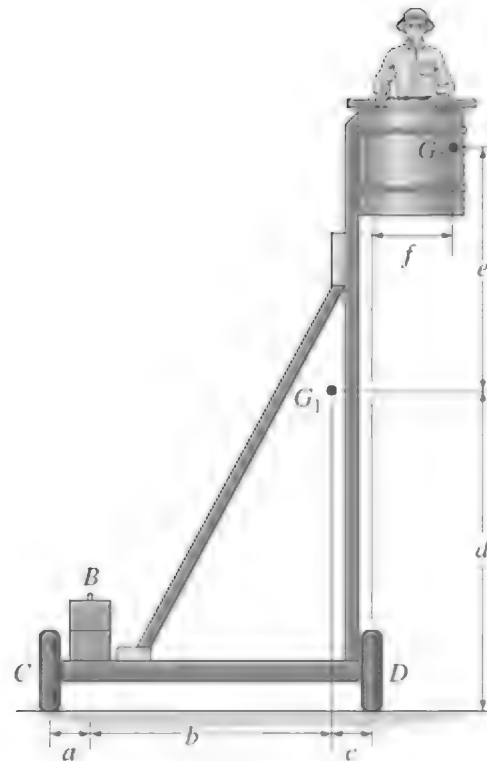
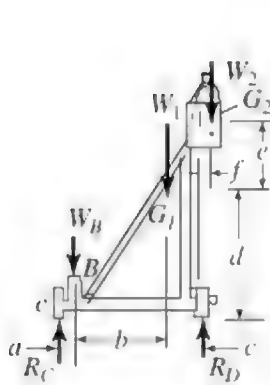
$$M - R_A \cos(\theta)(a + b) + B_x c = 0$$

$$\begin{pmatrix} R_A \\ B_x \\ B_y \end{pmatrix} = \text{Find}(R_A, B_x, B_y) \qquad R_A = 105.1 \text{ lb} \qquad \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 97.4 \\ 269 \end{pmatrix} \text{lb}$$

Problem 5-27

The platform assembly has weight W_1 and center of gravity at G_1. If it is intended to support a maximum load W_2 placed at point G_2, determine the smallest counterweight W that should be placed at B in order to prevent the platform from tipping over.

Given:

$W_1 = 250 \text{ lb}$ $a = 1 \text{ ft}$ $c = 1 \text{ ft}$ $e = 6 \text{ ft}$

$W_2 = 400 \text{ lb}$ $b = 6 \text{ ft}$ $d = 8 \text{ ft}$ $f = 2 \text{ ft}$

Solution:

When tipping occurs, $R_c = 0$

$\Sigma M_D = 0;\qquad -W_2\, f + W_1\, c + W_B\,(b + c) = 0$

$$W_B = \frac{W_2\, f - W_1\, c}{b + c}$$

$$W_B = 78.6\ \text{lb}$$

Problem 5-28

The articulated crane boom has a weight *W* and mass center at *G*. If it supports a load *L*, determine the force acting at the pin *A* and the compression in the hydraulic cylinder *BC* when the boom is in the position shown.

Units Used:

$$\text{kip} = 10^3\ \text{lb}$$

Given:

$$W = 125\ \text{lb}$$

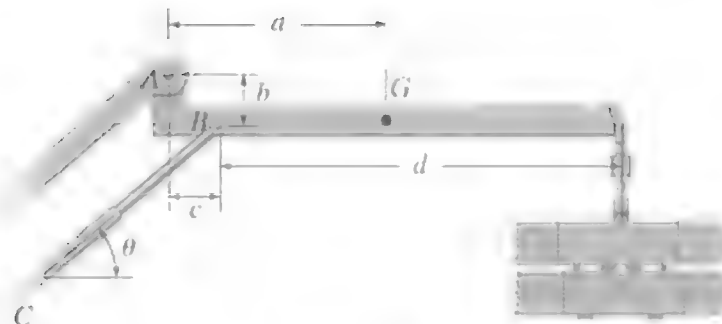
$L = 600 \text{ lb}$

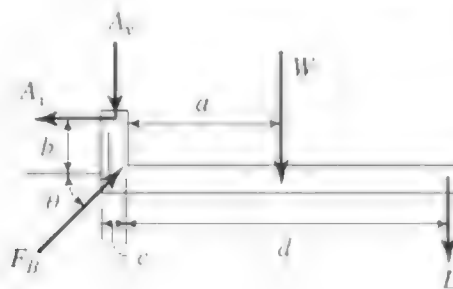
$a = 4 \text{ ft}$

$b = 1 \text{ ft}$

$c = 1 \text{ ft}$

$d = 8 \text{ ft}$

$\theta = 40 \text{ deg}$

Solution:

Guesses $A_x = 1 \text{ lb}$ $A_y = 1 \text{ lb}$ $F_B = 1 \text{ lb}$

Given $-A_x + F_B\cos(\theta) = 0$ $-A_y + F_B\sin(\theta) - W - L = 0$

$$F_B\cos(\theta)b + F_B\sin(\theta)c - Wa - L(d+c) = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ F_B \end{pmatrix} = \text{Find}(A_x, A_y, F_B) \qquad F_B = 4.19 \text{ kip} \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 3.208 \\ 1.967 \end{pmatrix} \text{kip}$$

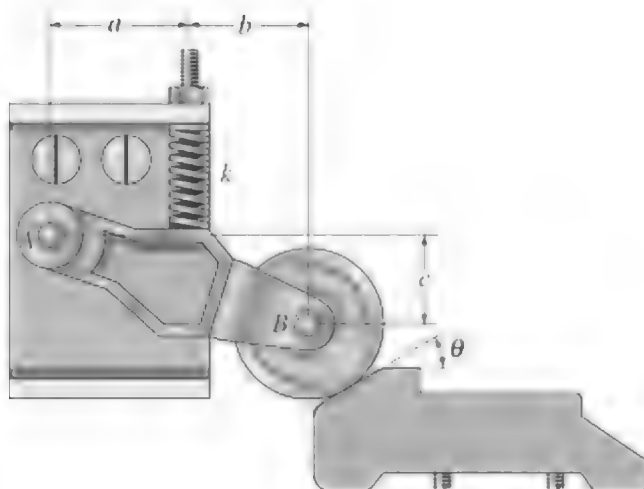
Problem 5-29

The device is used to hold an elevator door open. If the spring has stiffness k and it is compressed a distnace δ, determine the horizontal and vertical components of reaction at the pin A and the resultant force at the wheel bearing B.

Given:

$k = 40 \frac{\text{N}}{\text{m}}$ $b = 125 \text{ mm}$

$\delta = 0.2 \text{ m}$ $c = 100 \text{ mm}$

$a = 150 \text{ mm}$ $\theta = 30 \text{ deg}$

Solution:

$$F_s = k\delta$$

$\Sigma M_A = 0;$ $\quad -F_s\, a + F_B \cos(\theta)(a+b) - F_B \sin(\theta) c = 0$

$$F_B = F_s \frac{a}{\cos(\theta)\,(a+b) - \sin(\theta)\, c}$$

$$F_B = 6.378 \text{ N}$$

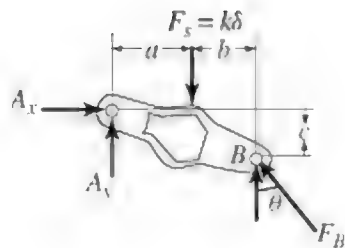
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - F_B \sin(\theta) = 0$

$$A_x = F_B \sin(\theta)$$

$$A_x = 3.189 \text{ N}$$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - F_s + F_B \cos(\theta) = 0$

$$A_y = F_s - F_B \cos(\theta)$$

$$A_y = 2.477 \text{ N}$$

Problem 5-30

Determine the reactions on the bent rod which is supported by a smooth surface at B and by a collar at A, which is fixed to the rod and is free to slide over the fixed inclined rod.

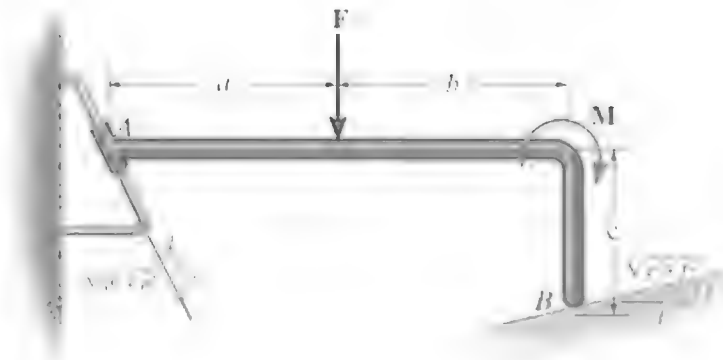
Given:

$F = 100$ lb

$M = 200$ lb ft

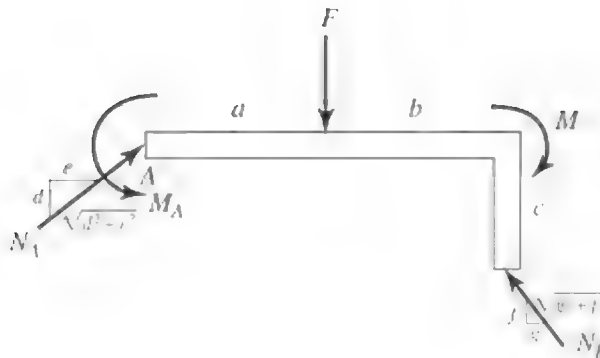
$a = 3$ ft

$b = 3$ ft

$c = 2$ft

$d = 3$

$e = 4$

$f = 12$

$g = 5$

Solution:

Initial Guesses:

$N_A = 20\ \text{lb}$ $N_B = 10\ \text{lb}$ $M_A = 30\ \text{lb ft}$

Given

$\Sigma M_A = 0;$ $$M_A - F\,a - M + N_B\left(\frac{f}{\sqrt{f^2+g^2}}\right)(a+b) - N_B\left(\frac{g}{\sqrt{f^2+g^2}}\right)c = 0$$

$\Sigma F_x = 0;$ $$N_A\left(\frac{e}{\sqrt{e^2+d^2}}\right) - N_B\left(\frac{g}{\sqrt{f^2+g^2}}\right) = 0$$

$\Sigma F_y = 0;$ $$N_A\left(\frac{d}{\sqrt{e^2+d^2}}\right) + N_B\left(\frac{f}{\sqrt{f^2+g^2}}\right) - F = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ M_A \end{pmatrix} = \text{Find}(N_A, N_B, M_A) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 39.7 \\ 82.5 \end{pmatrix}\text{lb} \qquad M_A = 106\,\text{lb}\cdot\text{ft}$$

Problem 5-31

The cantilevered jib crane is used to support the load F. If the trolley T can be placed anywhere in the range $x_1 \le x \le x_2$, determine the maximum magnitude of reaction at the supports A and B. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at B supports a force in the vertical direction, whereas the one at A does not.

Units Used:

$\text{kip} = 1000\ \text{lb}$

Given:

$F = 780\ \text{lb}$

$a = 4 \text{ ft}$

$b = 8 \text{ ft}$

$x_1 = 1.5 \text{ ft}$

$x_2 = 7.5 \text{ ft}$

Solution:

The maximum occurs when $x = x_2$

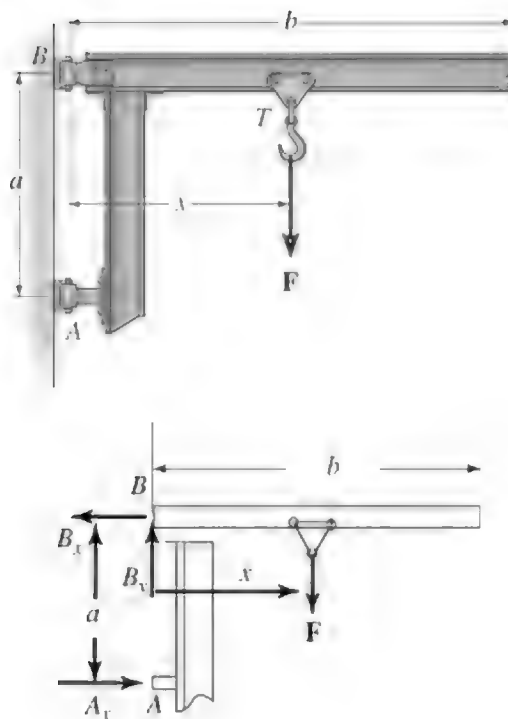
$\Sigma M_A = 0;\quad -F x_2 + B_x a = 0$

$$B_x = F \frac{x_2}{a}$$

$$B_x = 1.462 \times 10^3 \text{ lb}$$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - B_x = 0 \quad A_x = B_x \quad A_x = 1.462 \times 10^3 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad B_y - F = 0 \quad B_y = F \quad B_y = 780 \text{ lb}$

$$F_B = \sqrt{B_x^2 + B_y^2} \qquad F_B = 1.657 \text{ kip}$$

Problem 5-32

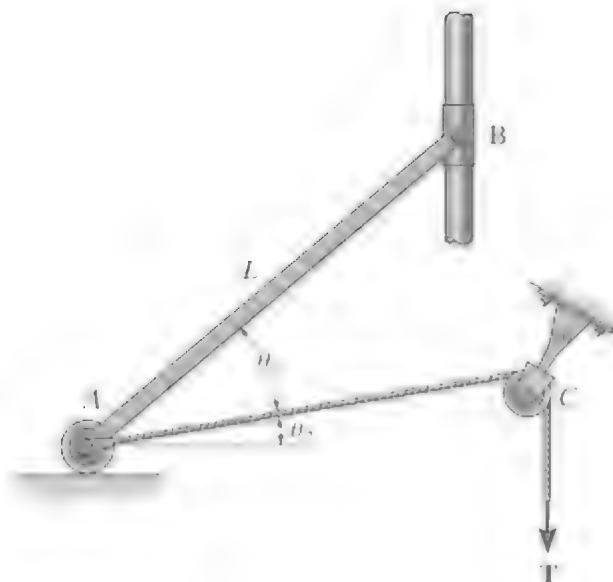
The uniform rod AB has weight W. Determine the force in the cable when the rod is in the position shown.

Given:

$W = 15 \text{ lb}$

$L = 5 \text{ ft}$

$\theta_1 = 30$ deg

$\theta_2 = 10$ deg

Solution:

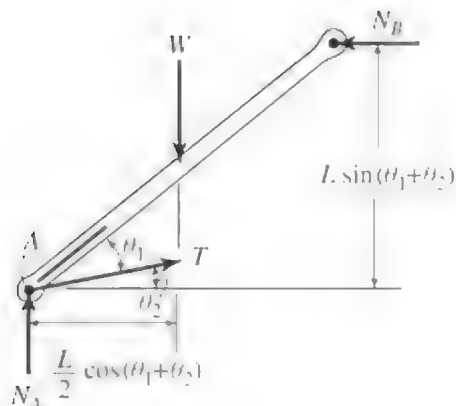
$\Sigma M_A = 0;\quad N_B\, L \sin(\theta_1 + \theta_2) - W\left(\frac{L}{2}\right)\cos(\theta_1 + \theta_2) = 0$

$$N_B = \frac{W\cos(\theta_1 + \theta_2)}{2\,\sin(\theta_1 + \theta_2)}$$

$N_B = 8.938$ lb

$\Sigma F_x = 0;\quad T\cos(\theta_2) - N_B$

$$T = \frac{N_B}{\cos(\theta_2)}$$

$T = 9.08$ lb

Problem 5-33

The power pole supports the three lines, each line exerting a vertical force on the pole due to its weight as shown. Determine the reactions at the fixed support *D*. If it is possible for wind or ice to snap the lines, determine which line(s) when removed create(s) a condition for the greatest moment reaction at *D*.

Units Used:

kip $= 10^3$ lb

Given:

$W_1 = 800$ lb

$W_2 = 450$ lb

$W_3 = 400$ lb

$a = 2$ft

$b = 4$ ft

$c = 3$ ft

Solution:

$\xrightarrow{+} \Sigma F_x = 0;$ $D_x = 0$

$+\uparrow \Sigma F_y = 0;$ $D_y - (W_1 + W_2 + W_3) = 0$

$D_y = W_1 + W_2 + W_3$ $D_y = 1.65$ kip

$\Sigma M_D = 0;$ $-W_2 b - W_3 c + W_1 a + M_D = 0$

$M_D = W_2 b + W_3 c - W_1 a$ $M_D = 1.4$ kip·ft

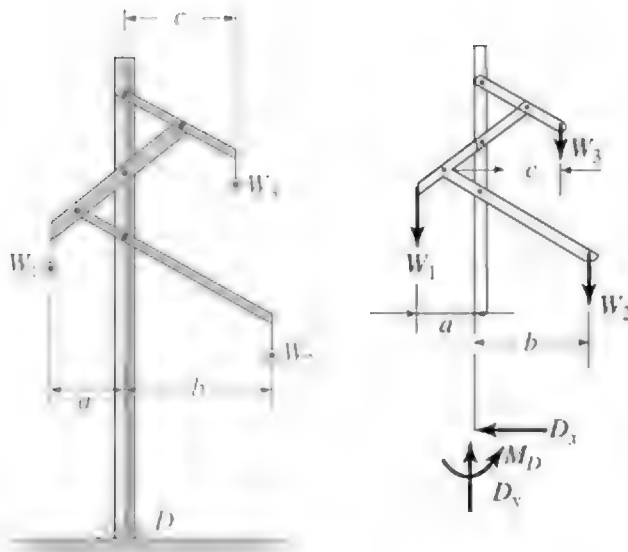
Examine all cases. For these numbers we require line 1 to snap.

$M_{Dmax} = W_2 b + W_3 c$ $M_{Dmax} = 3$ kip·ft

Problem 5-34

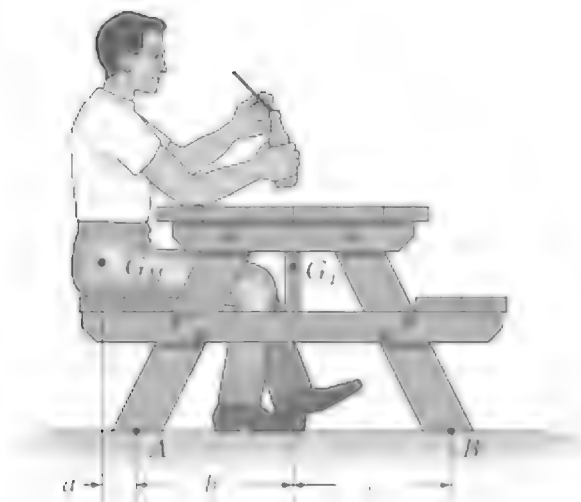
The picnic table has a weight W_T and a center of gravity at G_T. If a man weighing W_M has a center of gravity at G_M and sits down in the centered position shown, determine the vertical reaction at each of the two legs at B.Neglect the thickness of the legs. What can you conclude from the results?

Given:

$W_T = 50$ lb

$W_M = 225 \text{ lb}$

$a = 6 \text{ in}$

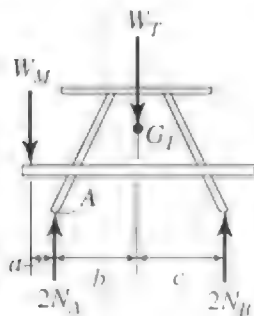
$b = 20 \text{ in}$

$c = 20 \text{ in}$

Solution:

$\Sigma M_A = 0;$ $\quad 2\,N_B(b+c) + W_M a - W_T\, b = 0$

$$N_B = \frac{W_T\, b - W_M a}{2\,(b+c)}$$

$$N_B = -4.37 \text{ lb}$$

Since N_B has a negative sign, the table will tip over.

Problem 5-35

If the wheelbarrow and its contents have a mass of M and center of mass at G, determine the magnitude of the resultant force which the man must exert on *each* of the two handles in order to hold the wheelbarrow in equilibrium.

Given:

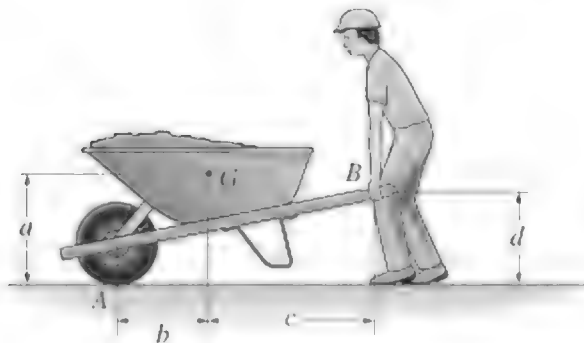
$M = 60 \text{ kg}$

$a = 0.6 \text{ m}$

$b = 0.5 \text{ m}$

$c = 0.9 \text{ m}$

$d = 0.5 \text{ m}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

Solution:

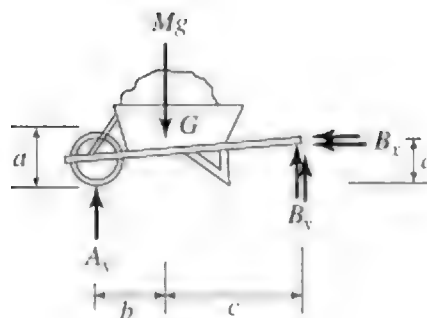
$\Sigma M_B = 0;$ $\quad -A_y\,(b+c) + M\,g\,c = 0$

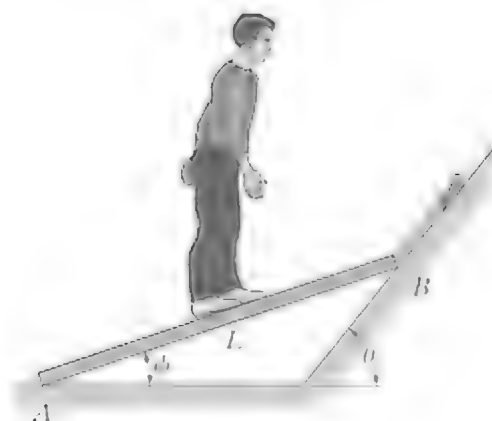
$$A_y = \frac{M\,g\,c}{b+c}$$

$$A_y = 378.386\text{ N}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad B_x = 0\text{N}$ $\qquad B_x = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - M\,g + 2\,B_y = 0$

$$B_y = \frac{M\,g - A_y}{2} \qquad B_y = 105.107\text{ N}$$

Problem 5-36

The man has weight W and stands at the center of the plank. If the planes at A and B are smooth, determine the tension in the cord in terms of W and θ.

Solution:

$\Sigma M_B = 0;$ $\quad W\,\frac{L}{2}\cos(\phi) - N_A\,L\cos(\phi) = 0$

$$N_A = \frac{W}{2}$$

$\Sigma F_x = 0;$ $\quad T\cos(\theta) - N_B\,\sin(\theta) = 0 \qquad (1)$

$\Sigma F_y = 0;$ $T\sin(\theta) + N_B\cos(\theta) + N_A - W = 0$ (2)

Solving Eqs. (1) and (2) yields:

$$T = \frac{W}{2}\sin(\theta)$$

$$N_B = \frac{W}{2}\cos(\theta)$$

Problem 5-37

When no force is applied to the brake pedal of the lightweight truck, the retainer spring AB keeps the pedal in contact with the smooth brake light switch at C. If the force on the switch is $\mathbf{F}$, determine the unstretched length of the spring if the stiffness of the spring is k.

Given:

$F = 3\ \text{N}$

$k = 80\ \dfrac{\text{N}}{\text{m}}$

$a = 100\ \text{mm}$

$b = 50\ \text{mm}$

$c = 40\ \text{mm}$

$d = 10\ \text{mm}$

$\theta = 30\ \text{deg}$

Solution:

$\Sigma M_D = 0;$ $F_s\, b - F\cos(\theta)c - F\sin(\theta)d = 0$

$$F_s = F\frac{\cos(\theta)\ c + \sin(\theta)\ d}{b} \qquad F_s = 2.378\ \text{N}$$

$$F_s = kx \qquad x = \frac{F_s}{k} \qquad x = 29.73\ \text{mm}$$

$$L_0 = a - x \qquad L_0 = 70.3\ \text{mm}$$

Problem 5-38

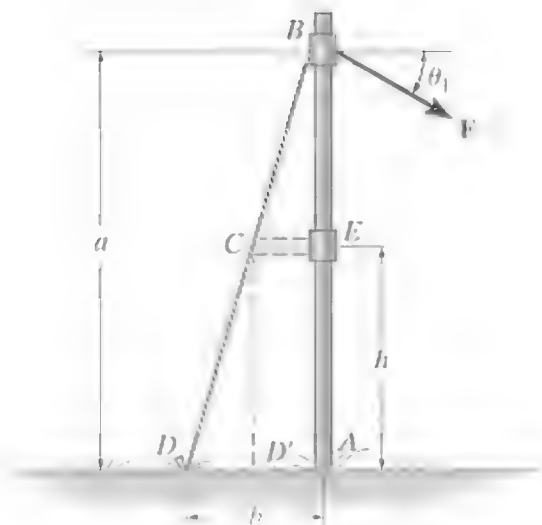
The telephone pole of negligible thickness is subjected to the force **F** directed as shown. It is supported by the cable *BCD* and can be assumed pinned at its base *A*. In order to provide clearance for a sidewalk right of way, where *D* is located, the strut *CE* is attached at *C*, as shown by the dashed lines (cable segment *CD* is removed). If the tension in *CD'* is to be twice the tension in *BCD*, determine the height *h* for placement of the strut *CE*.

Given:

$F = 80 \text{ lb}$

$\theta = 30 \text{ deg}$

$a = 30 \text{ ft}$

$b = 10 \text{ ft}$

Solution:

$+\Sigma M_A = 0;$ $$-F\cos(\theta)\,a + \left(\frac{b}{\sqrt{a^2+b^2}}\right)T_{BCD}\,a = 0$$

$$T_{BCD} = F\cos(\theta)\frac{\sqrt{a^2+b^2}}{b} \qquad T_{BCD} = 219.089 \text{ lb}$$

Require $T_{CD'} = 2\,T_{BCD}$ $\qquad T_{CD'} = 438.178 \text{ lb}$

$+\Sigma M_A = 0;$ $$T_{CD'}d - F\cos(\theta)a = 0$$

$$d = F\,a\left(\frac{\cos(\theta)}{T_{CD'}}\right) \qquad d = 4.7434 \text{ ft}$$

Geometry:

$$\frac{a-h}{d} = \frac{a}{b} \qquad h = a - a\left(\frac{d}{b}\right) \qquad h = 15.8 \text{ ft}$$

Problem 5-39

The worker uses the hand truck to move material down the ramp. If the truck and its contents are held in the position shown and have weight W with center of gravity at G, determine the resultant normal force of both wheels on the ground A and the magnitude of the force required at the grip B.

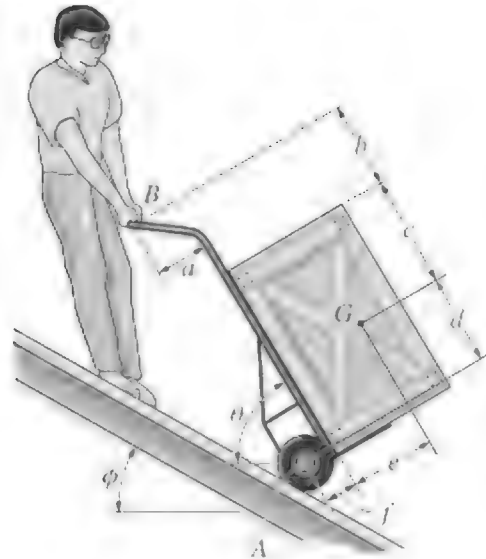
Given:

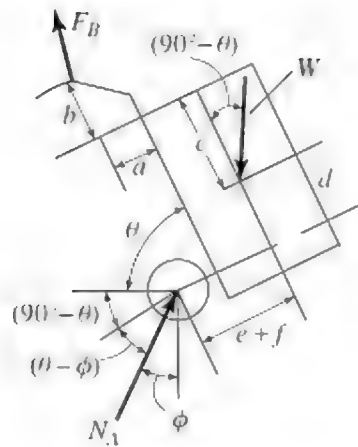
$W = 100$ lb $e = 1.5$ ft

$a = 1$ ft $f = 0.5$ ft

$b = 1.5$ ft $\theta = 60$ deg

$c = 2$ ft $\phi = 30$ deg

$d = 1.75$ ft

Solution:

$\Sigma M_B = 0;$

$$N_A \cos(\theta - \phi)(b + c + d) + N_A \sin(\theta - \phi)(a - f) - W\cos(\theta)(b + c) - W\sin(\theta)(e + a) = 0$$

$$N_A = \frac{W\cos(\theta)(b + c) + W\sin(\theta)(e + a)}{\cos(\theta - \phi)(b + c + d) + \sin(\theta - \phi)(a - f)}$$

$N_A = 81.621$ lb

$\Sigma F_x = 0;$ $-B_x + N_A \sin(\phi) = 0$ $B_x = N_A \sin(\phi)$ $B_x = 40.811$ lb

$\Sigma F_y = 0;$ $B_y + N_A (\cos(\phi) - W = 0)$ $B_y = W - N_A \cos(\phi)$ $B_y = 29.314$ lb

$$F_B = \sqrt{B_x^{\ 2} + B_y^{\ 2}}$$ $F_B = 50.2$ lb

$\Sigma F_y = 0;$ $T\sin(\theta) + N_B\cos(\theta) + N_A - W = 0$ (2)

Solving Eqs. (1) and (2) yields:

$$T = \frac{W}{2}\sin(\theta)$$

$$N_B = \frac{W}{2}\cos(\theta)$$

Problem 5-37

When no force is applied to the brake pedal of the lightweight truck, the retainer spring AB keeps the pedal in contact with the smooth brake light switch at C. If the force on the switch is $\mathbf{F}$, determine the unstretched length of the spring if the stiffness of the spring is k.

Given:

$F = 3\text{ N}$

$k = 80\ \frac{\text{N}}{\text{m}}$

$a = 100\text{ mm}$

$b = 50\text{ mm}$

$c = 40\text{ mm}$

$d = 10\text{ mm}$

$\theta = 30\text{ deg}$

Solution:

$\Sigma M_D = 0;$ $F_s\,b - F\cos(\theta)c - F\sin(\theta)d = 0$

$$F_s = F\,\frac{\cos(\theta)\ c + \sin(\theta)\ d}{b} \qquad F_s = 2.378\ \text{N}$$

$$F_s = kx \qquad x = \frac{F_s}{k} \qquad x = 29.73\ \text{mm}$$

$$L_0 = a - x \qquad L_0 = 70.3\ \text{mm}$$

The shelf supports the electric motor which has mass m_1 and mass center at G_m. The platform upon which it rests has mass m_2 and mass center at G_p. Assuming that a single bolt B holds the shelf up and the bracket bears against the smooth wall at A, determine this normal force at A and the horizontal and vertical components of reaction of the bolt B on the bracket.

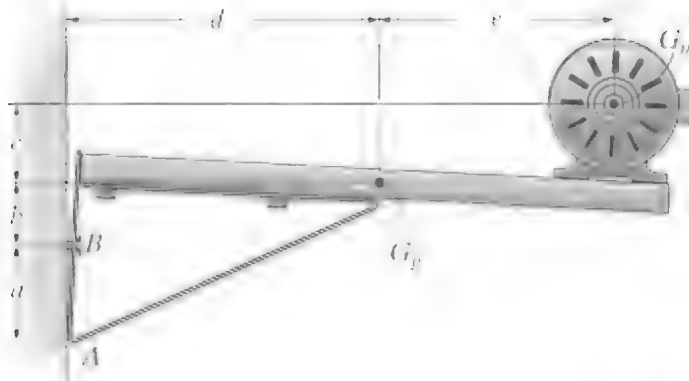
Given:

$m_1 = 15\ \text{kg}$ $c = 50\ \text{mm}$

$m_2 = 4\ \text{kg}$ $d = 200\ \text{mm}$

$a = 60\ \text{mm}$ $e = 150\ \text{mm}$

$b = 40\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

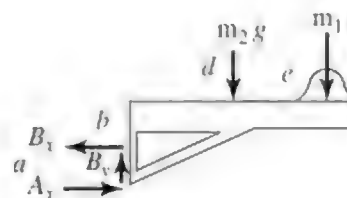
$\Sigma M_A = 0;$ $B_x a - m_2 g d - m_1 g(d+e) = 0$

$$B_x = g\,\frac{m_2\, d + m_1\,(d+e)}{a} \qquad B_x = 989\ \text{N}$$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;$ $A_x - B_x = 0$

$$A_x = B_x \qquad A_x = 989\ \text{N}$$

$+\uparrow \Sigma F_y = 0;$ $B_y - m_2\, g - m_1\, g = 0$

$$B_y = m_2\, g + m_1\, g \qquad B_y = 186\ \text{N}$$

Problem 5-42

A cantilever beam, having an extended length L, is subjected to a vertical force $\mathbf{F}$. Assuming that the wall resists this load with linearly varying distributed loads over the length a of the beam portion inside the wall, determine the intensities w_1 and w_2 for equilibrium.

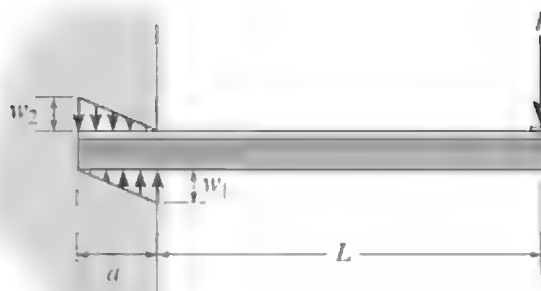
Units Used:

$kN = 10^3\ N$

Given:

$F = 500\ N$

$a = 0.15\ m$

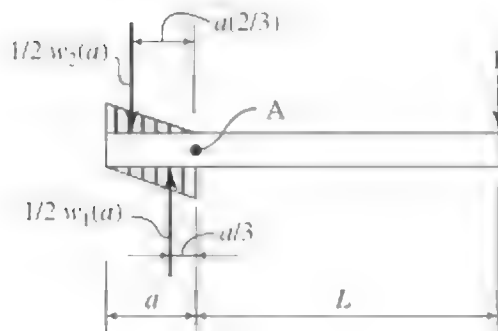
$L = 3\ m$

Solution:

The initial guesses

$w_1 = 1\ \frac{kN}{m}$ $\qquad w_2 = 1\ \frac{kN}{m}$

Given

$+\uparrow \Sigma F_y = 0;$ $\qquad \frac{1}{2} w_1\ a - \frac{1}{2} w_2\ a - F = 0$

$\Sigma M_A = 0;$ $\qquad -F\ L - \frac{1}{2} w_1\ a\left(\frac{a}{3}\right) + \frac{1}{2} w_2\ a\left(\frac{2\ a}{3}\right) = 0$

$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \text{Find}(w_1, w_2)$ $\qquad \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 413 \\ 407 \end{pmatrix} \frac{kN}{m}$

Problem 5-43

The upper portion of the crane boom consists of the jib *AB*, which is supported by the pin at *A*, the guy line *BC*, and the backstay *CD*, each cable being separately attached to the mast at *C*. If the load *F* is supported by the hoist line, which passes over the pulley at *B*, determine the magnitude of the resultant force the pin exerts on the jib at *A* for equilibrium, the tension in the guy line *BC*, and the tension *T* in the hoist line. Neglect the weight of the jib. The pulley at *B* has a radius of *r*.

Units Used:

$$kN = 10^3 \ N$$

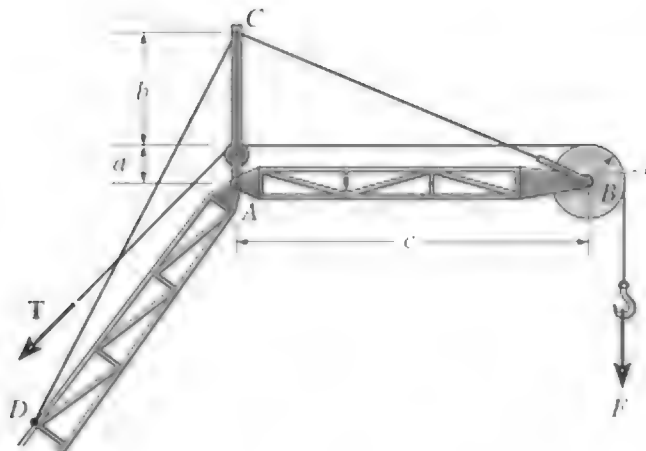
Given:

$F = 5 \ kN$

$r = 0.1 \ m$

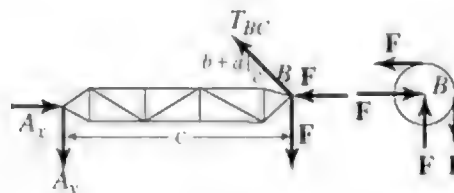
$a = r$

$b = 1.5 \ m$

$c = 5 \ m$

Solution:

From pulley, tension in the hoist line is

$\Sigma M_B = 0;$ $\quad T(a) - F(r) = 0$

$$T = F\frac{r}{a} \qquad T = 5\,\text{kN}$$

From the jib,

$\Sigma M_A = 0;$ $\quad -F(c) + T_{BC}\dfrac{b+a}{\sqrt{c^2 + (b+a)^2}}c = 0$

$$T_{BC} = F\frac{\sqrt{c^2 + (b+a)^2}}{b+a} \qquad T_{BC} = 16.406\,\text{kN}$$

$+\uparrow \Sigma F_y = 0;$ $\quad -A_y + T_{BC}\left[\dfrac{b+a}{\sqrt{c^2 + (b+a)^2}}\right] - F = 0$

$$A_y = T_{BC}\left[\frac{b+a}{\sqrt{c^2 + (b+a)^2}}\right] - F \qquad A_y = 0\,\text{kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - T_{BC}\left[\dfrac{c}{\sqrt{c^2 + (b+a)^2}}\right] - F = 0$

$$A_x = T_{BC}\frac{c}{\sqrt{c^2+(b+a)^2}} + F \qquad A_x = 20.6\,\text{kN}$$

$$F_A = \sqrt{A_x^2 + A_y^2} \qquad F_A = 20.6\,\text{kN}$$

Problem 5-44

The mobile crane has weight W_1 and center of gravity at G_1; the boom has weight W_2 and center of gravity at G_2. Determine the smallest angle of tilt θ of the boom, without causing the crane to overturn if the suspended load has weight W. Neglect the thickness of the tracks at A and B.

Given:

$W_1 = 120000$ lb

$W_2 = 30000$ lb

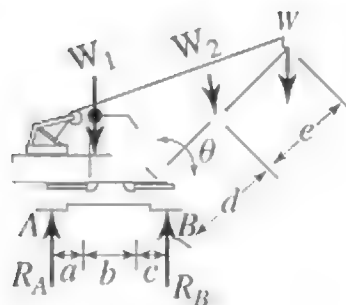
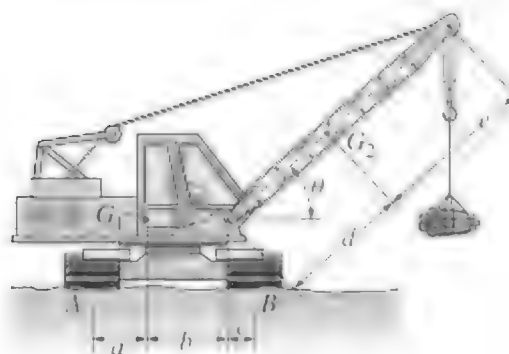
$W = 40000$ lb

$a = 4$ ft

$b = 6$ ft

$c = 3$ ft

$d = 12$ ft

$e = 15$ ft

Solution:

When tipping occurs, $R_A = 0$

$\Sigma M_B = 0;$ $\quad -W_2(d\cos(\theta) - c) - W[(d+e)\cos(\theta) - c] + W_1(b+c) = 0$

$$\theta = \text{acos}\left[\frac{W_2\,c + W\,c + W_1\,(b+c)}{W_2\,d + W\,(d+e)}\right]$$

$\theta = 26.4\,\text{deg}$

Problem 5-45

The mobile crane has weight W_1 and center of gravity at G_1; the boom has weight W_2 and center of gravity at G_2. If the suspended load has weight W determine the normal reactions at the tracks A and B. For the calculation, neglect the thickness of the tracks.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

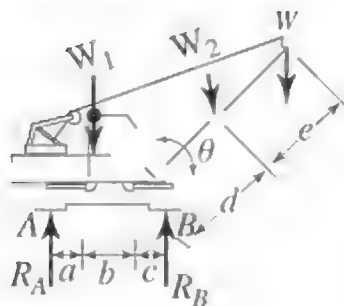
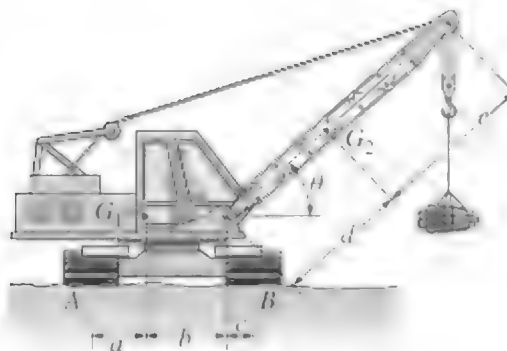
Given:

$W_1 = 120000$ lb $\quad a = 4$ ft

$W_2 = 30000$ lb $\quad b = 6$ ft

$W = 16000$ lb $\quad c = 3$ ft

$\theta = 30$ deg $\quad d = 12$ ft

$e = 15$ ft

Solution:

$\Sigma M_B = 0;$

$$-W_2(d\cos(\theta) - c) - W[(d+e)\cos(\theta) - c] - R_A(a+b+c) + W_1(b+c) = 0$$

$$R_A = \frac{-W_2\,(d\cos(\theta) - c) - W[(d+e)\cos(\theta) - c] + W_1\,(b+c)}{a+b+c} \qquad R_A = 40.9 \text{ kip}$$

$+\uparrow \Sigma F_y = 0; \qquad R_A + R_B - W_1 - W_2 - W = 0$

$$R_B = -R_A + W_1 + W_2 + W \qquad R_B = 125 \text{ kip}$$

Problem 5-46

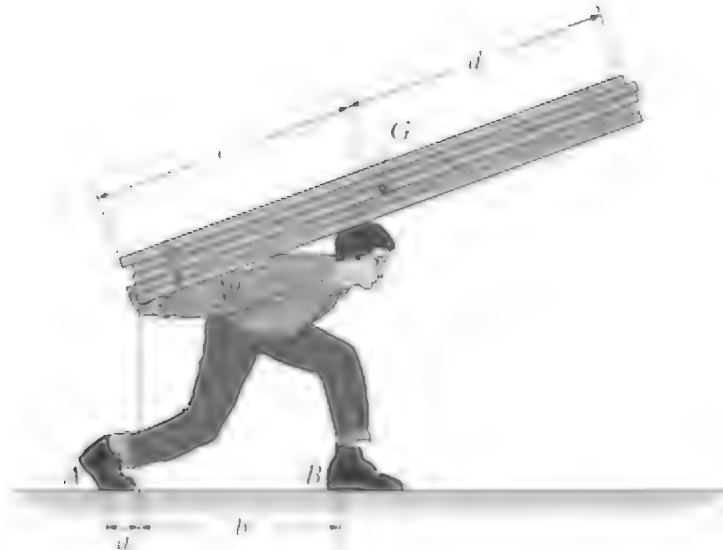
The man attempts to support the load of boards having a weight W and a center of gravity at G. If he is standing on a smooth floor, determine the smallest angle θ at which he can hold them up in the position shown. Neglect his weight.

Given:

$a = 0.5$ ft

$b = 3$ ft

$c = 4$ ft

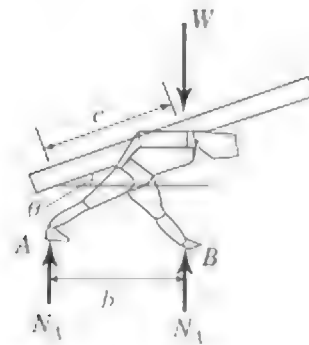
$d = 4$ ft

Solution:

$\Sigma M_B = 0;\quad -N_A(a+b) + W(b - c\cos(\theta)) = 0$

As θ becomes smaller, N_A goes to 0 so that,

$$\cos(\theta) = \frac{b}{c}$$

$$\theta = \operatorname{acos}\left(\frac{b}{c}\right)$$

$$\theta = 41.4\,\text{deg}$$

Problem 5-47

The motor has a weight W. Determine the force that each of the chains exerts on the supporting hooks at A, B, and C. Neglect the size of the hooks and the thickness of the beam.

Given:

$W = 850 \text{ lb}$

$a = 0.5 \text{ ft}$

$b = 1 \text{ ft}$

$c = 1.5 \text{ ft}$

$\theta_1 = 10 \text{ deg}$

$\theta_2 = 30 \text{ deg}$

$\theta_3 = 10 \text{ deg}$

Solution:

Guesses

$F_A = 1 \text{ lb} \qquad F_B = 1 \text{ lb} \qquad F_C = 1 \text{ lb}$

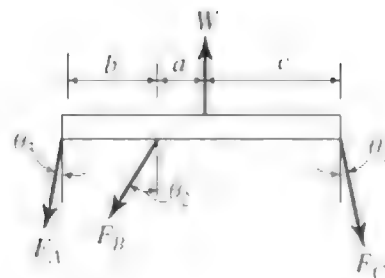
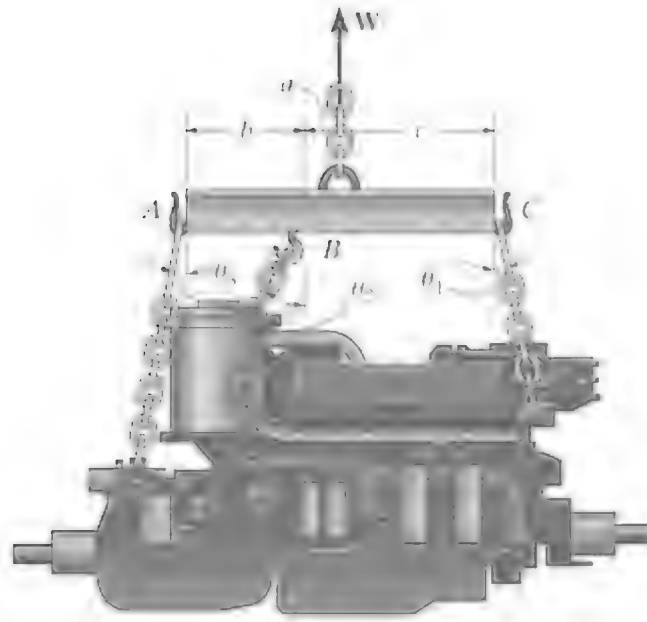
Given

$\Sigma M_B = 0; \qquad F_A \cos(\theta_3) b + W a - F_C \cos(\theta_1)(a + c) = 0$

$\Sigma F_x = 0; \qquad F_C \sin(\theta_1) - F_B \sin(\theta_2) - F_A \sin(\theta_3) = 0$

$\Sigma F_y = 0; \qquad W - F_A \cos(\theta_3) - F_B \cos(\theta_2) - F_C \cos(\theta_1) = 0$

$$\begin{pmatrix} F_A \\ F_B \\ F_C \end{pmatrix} = \text{Find}(F_A, F_B, F_C)$$

$$\begin{pmatrix} F_A \\ F_B \\ F_C \end{pmatrix} = \begin{pmatrix} 432 \\ -0 \\ 432 \end{pmatrix} \text{lb}$$

Problem 5-48

The boom supports the two vertical loads. Neglect the size of the collars at *D* and *B* and the thickness of the boom, and compute the horizontal and vertical components of force at the pin *A* and the force in cable *CB*.

Given:

$F_1 = 800\ \text{N}$

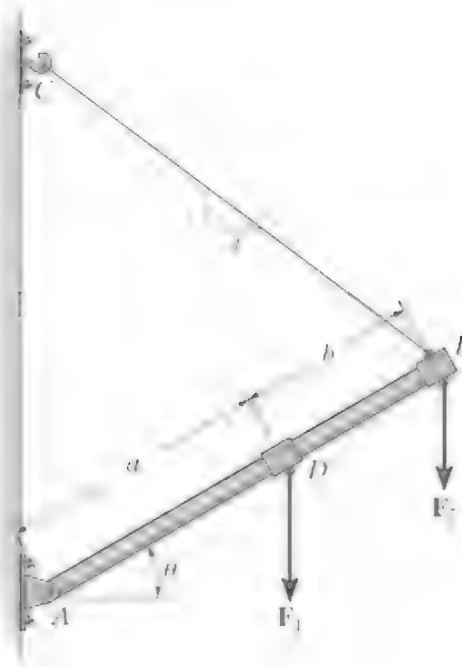
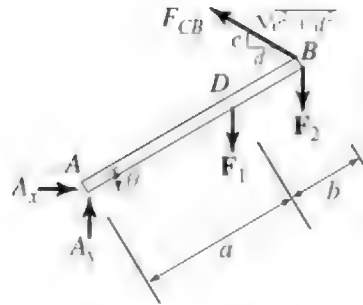
$F_2 = 350\ \text{N}$

$a = 1.5\ \text{m}$

$b = 1\ \text{m}$

$c = 3$

$d = 4$

$\theta = 30\ \text{deg}$

Solution:

$\Sigma M_A = 0;$

$$-F_1 a\cos(\theta) - F_2(a+b)\cos(\theta) + \frac{d}{\sqrt{c^2+d^2}}F_{CB}\ (a+b)\sin(\theta) + \frac{c}{\sqrt{c^2+d^2}}F_{CB}(a+b)\cos(\theta) = 0$$

$$F_{CB} = \frac{\left[F_1 a + F_2(a+b)\right]\cos(\theta)\sqrt{c^2+d^2}}{d\sin(\theta)(a+b) + c\cos(\theta)(a+b)} \qquad F_{CB} = 782\ \text{N}$$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;$

$$A_x - \frac{d}{\sqrt{c^2+d^2}}F_{CB} = 0$$

$$A_x = \frac{d}{\sqrt{c^2+d^2}}F_{CB} \qquad A_x = 625\ \text{N}$$

$+\uparrow \Sigma F_y = 0;$

$$A_y - F_1 - F_2 + \frac{c}{\sqrt{c^2+d^2}}F_{CB} = 0$$

$$A_y = F_1 + F_2 - \frac{c}{\sqrt{c^2+d^2}}F_{CB} \qquad A_y = 681\ \text{N}$$

Problem 5-49

The boom is intended to support two vertical loads $\mathbf{F}_1$ and $\mathbf{F}_2$. If the cable *CB* can sustain a

maximum load T_{max} before it fails, determine the critical loads if $F_1 = 2F_2$. Also, what is the magnitude of the maximum reaction at pin A?

Units Used:

$$kN = 10^3\ N$$

Given:

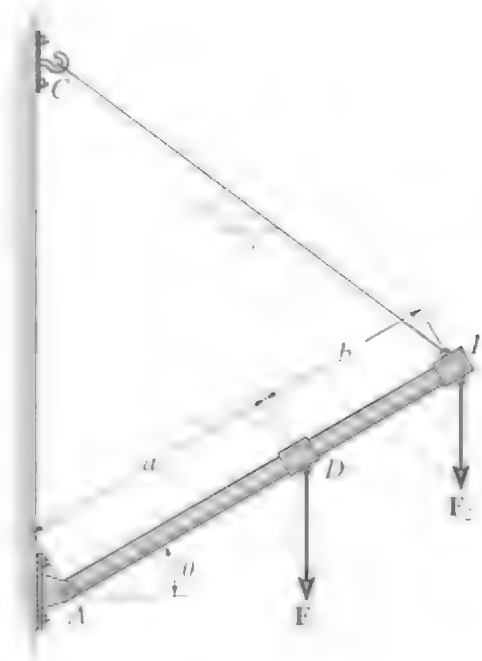
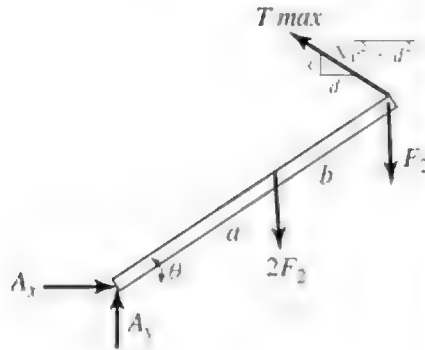
$$T_{max} = 1500\ N$$

$$a = 1.5\ m$$

$$b = 1\ m$$

$$c = 3$$

$$d = 4$$

$$\theta = 30deg$$

Solution:

$\Sigma M_A = 0; \qquad F_1 = 2\ F_2$

$$-2\ F_2\, a \cos(\theta) - F_2 (a+b)\cos(\theta) + \frac{d}{\sqrt{c^2+d^2}} T_{max}(a+b)\sin(\theta) + \frac{c}{\sqrt{c^2+d^2}} T_{max}(a+b)\cos(\theta) = 0$$

$$F_2 = \frac{(a+b)T_{max}(d\sin(\theta) + c\cos(\theta))}{\sqrt{c^2+d^2}\cos(\theta)(3\ a+b)} \qquad F_2 = 724\,N$$

$$F_1 = 2\ F_2 \qquad F_1 = 1.448\,kN$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - \frac{d}{\sqrt{c^2+d^2}} T_{max} = 0$

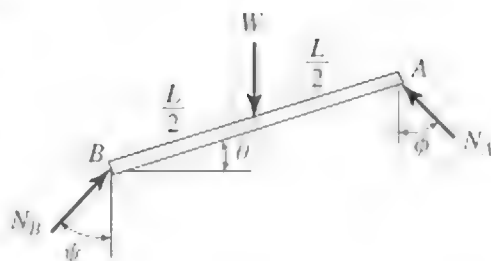
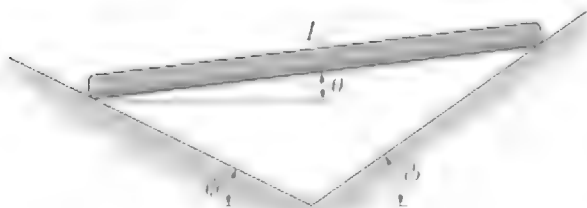
$$A_x = \frac{d}{\sqrt{c^2+d^2}} T_{max} \qquad A_x = 1.20\,kN$$

$+\uparrow \Sigma\ F_y = 0; \qquad A_y - F_2 - F_1 + \frac{c}{\sqrt{c^2+d^2}} T_{max} = 0$

$$A_y = F_2 + F_1 - \frac{c}{\sqrt{c^2 + d^2}} T_{max} \qquad A_y = 1.27\,\text{kN}$$

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} \qquad F_A = 1.749\,\text{kN}$$

Problem 5-50

The uniform rod of length L and weight W is supported on the smooth planes. Determine its position θ for equilibrium. Neglect the thickness of the rod.

Solution:

$\Sigma M_B = 0;$ $\quad -W\frac{L}{2}\cos(\theta) + N_A\cos(\phi - \theta)L = 0 \qquad N_A = \frac{W\cos(\theta)}{2\cos(\phi - \theta)}$

$\Sigma M_A = 0;$ $\quad W\frac{L}{2}\cos(\theta) - N_B\cos(\psi + \theta)L = 0 \qquad N_B = \frac{W\cos(\theta)}{2\cos(\psi + \theta)}$

$\Sigma F_x = 0;$ $\quad N_B\sin(\psi) - N_A\sin(\phi) = 0$

$$\frac{W\cos(\theta)}{2\cos(\psi + \theta)}\sin(\psi) - \frac{W\cos(\theta)}{2\cos(\phi - \theta)}\sin(\phi) = 0$$

$$\sin(\psi)\cos(\phi - \theta) - \sin(\phi)\cos(\psi + \theta) = 0$$

$$\sin(\psi)(\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)) - \sin(\phi)(\cos(\psi)\cos(\theta) - \sin(\psi)\sin(\theta)) = 0$$

$$2\sin(\psi)\sin(\phi)\sin(\theta) = (\sin(\phi)\cos(\psi) - \sin(\psi)\cos(\phi))\cos(\theta)$$

$$\tan(\theta) = \frac{\sin(\phi)\cos(\psi) - \sin(\psi)\cos(\phi)}{2\sin(\psi)\sin(\phi)} = \frac{\cot(\psi) - \cot(\phi)}{2}$$

$$\theta = \text{atan}\left(\frac{\cot(\psi) - \cot(\phi)}{2}\right)$$

Problem 5-51

The toggle switch consists of a cocking lever that is pinned to a fixed frame at A and held in place by the spring which has unstretched length δ. Determine the magnitude of the resultant force at A and the normal force on the peg at B when the lever is in the position shown.

Given:

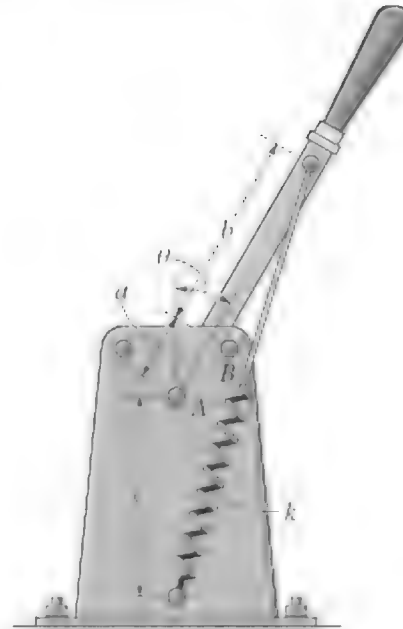
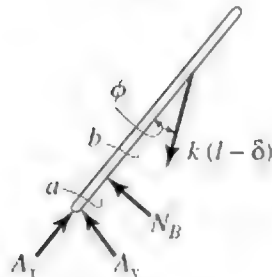
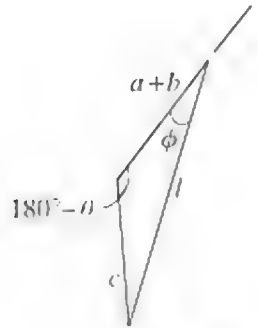
$\delta = 200 \text{ mm}$

$k = 5 \dfrac{\text{N}}{\text{m}}$

$a = 100 \text{ mm}$

$b = 300 \text{ mm}$

$c = 300 \text{ mm}$

$\theta = 30 \text{ deg}$

Solution:

Using the law of cosines and the law of sines

$$l = \sqrt{c^2 + (a+b)^2 - 2\,c(a+b)\cos(180 \text{ deg} - \theta)}$$

$$\frac{\sin(\phi)}{c} = \frac{\sin(180 \text{ deg} - \theta)}{l} \qquad \phi = \text{asin}\left(c\,\frac{\sin(180 \text{ deg} - \theta)}{l}\right) \qquad \phi = 12.808 \text{ deg}$$

$$F_s = k\,s = k(l - \delta) \qquad F_s = k(l - \delta) \qquad F_s = 2.3832 \text{ N}$$

$$\Sigma M_A = 0; \quad -F_s \sin(\phi)(a+b) + N_B\,a = 0 \qquad N_B = F_s \sin(\phi)\,\frac{a+b}{a} \qquad N_B = 2.11 \text{ N}$$

$$\Sigma F_x = 0; \quad A_x - F_s \cos(\phi) = 0 \qquad A_x = F_s \cos(\phi) \qquad A_x = 2.3239 \text{ N}$$

$$\Sigma F_y = 0; \quad A_y + N_B - F_s \sin(\phi) = 0 \qquad A_y = F_s \sin(\phi) - N_B \qquad A_y = -1.5850 \text{ N}$$

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} \qquad F_A = 2.813 \text{ N}$$

Problem 5-52

The rigid beam of negligible weight is supported horizontally by two springs and a pin. If the springs are uncompressed when the load is removed, determine the force in each spring when the load **P** is applied. Also, compute the vertical deflection of end C. Assume the spring stiffness k is large enough so that only small deflections occur. *Hint*: The beam rotates about A so the deflections in the springs can be related.

Solution:

$\Sigma M_A = 0;$

$$F_B L + F_C 2\ L - P\frac{3}{2}L = 0$$

$$F_B + 2\ F_C = 1.5\ P$$

$$\Delta_C = 2\ \Delta_B$$

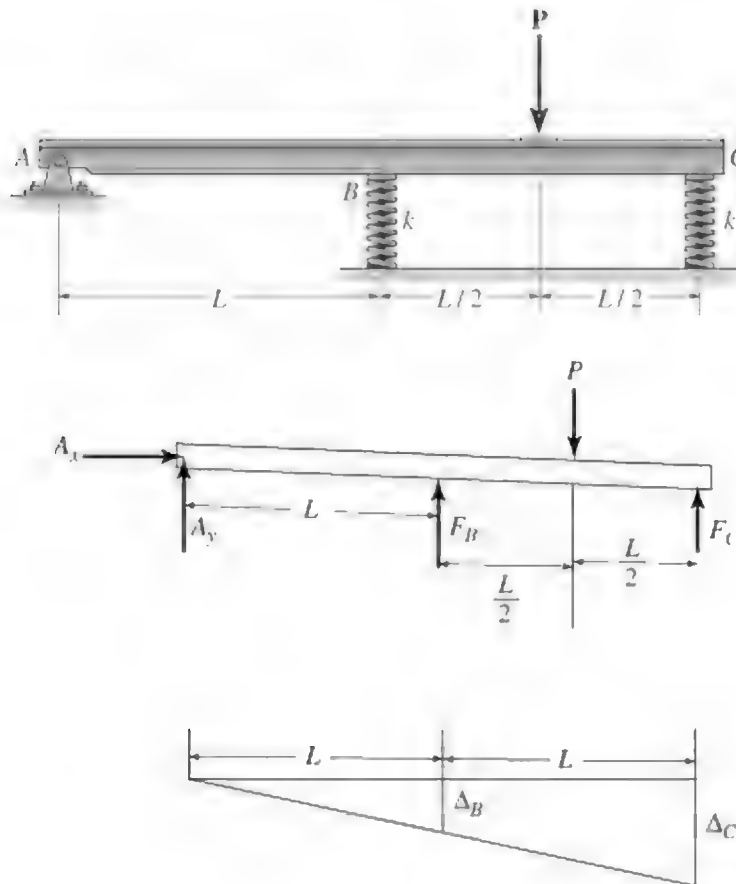
$$\frac{F_C}{k} = \frac{2\ F_B}{k}$$

$$F_C = 2F_B$$

$$5\ F_B = 1.5\ P$$

$$F_B = 0.3\ P$$

$$F_c = 0.6\ P$$

$$\Delta_C = \frac{0.6\ P}{k}$$

Problem 5-53

The rod supports a weight W and is pinned at its end A. If it is also subjected to a couple moment of M, determine the angle θ for equilibrium. The spring has an unstretched length δ and a stiffness k.

Given:

$W = 200 \text{ lb}$

$M = 100 \text{ lb ft}$

$\delta = 2 \text{ ft}$

$k = 50 \frac{\text{lb}}{\text{ft}}$

$a = 3 \text{ ft}$

$b = 3 \text{ ft}$

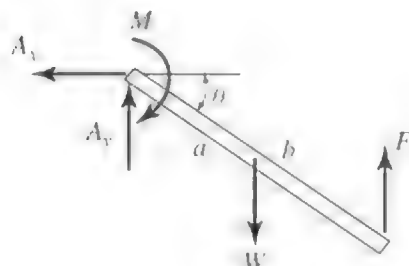
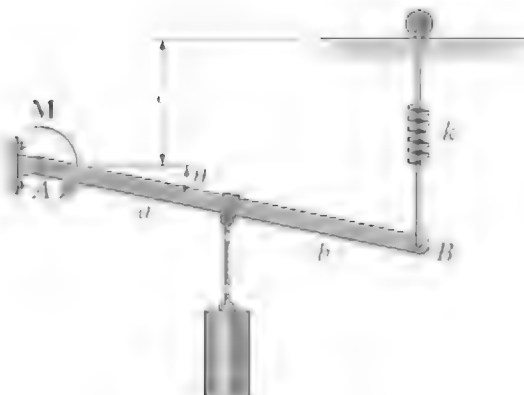
$c = 2 \text{ ft}$

Solution:

Initial Guess: $\theta = 10 \text{ deg}$

Given

$$k[(a+b)\sin(\theta) + c - \delta](a+b)\cos(\theta) - W a\cos(\theta) - M = 0$$

$\theta = \text{Find}(\theta)$ $\theta = 23.2 \text{ deg}$

Problem 5-54

The smooth pipe rests against the wall at the points of contact *A, B,* and *C*. Determine the reactions at these points needed to support the vertical force **F**. Neglect the pipe's thickness in the calculation.

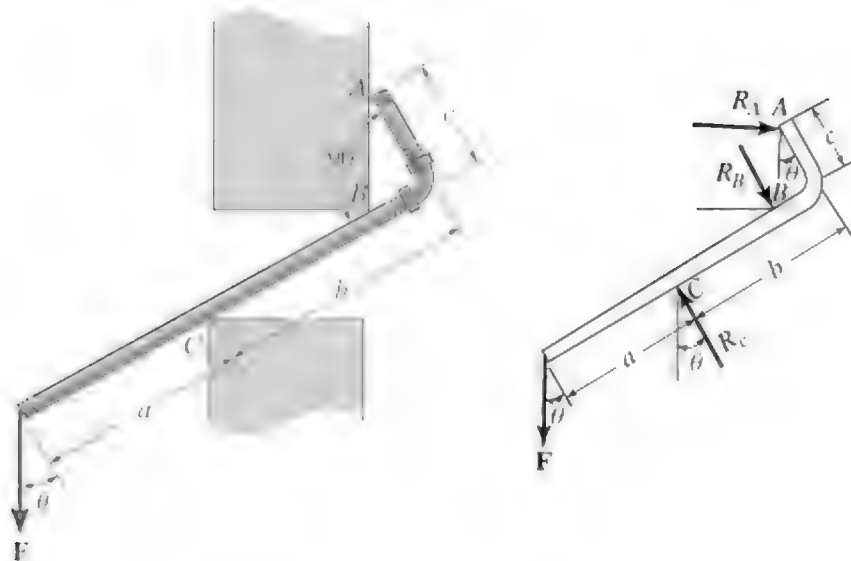
Given:

$F = 45 \text{ lb}$

$\theta = 30\ \text{deg}$

$a = 16\ \text{in}$

$b = 20\ \text{in}$

$c = 8\ \text{in}$

Solution:

Initial Guesses: $R_A = 1\ \text{lb}$ $R_B = 1\ \text{lb}$ $R_C = 1\ \text{lb}$

Given

$\Sigma M_A = 0;$ $F\cos(\theta)(a+b) - F\sin(\theta)c - R_C b + R_B c\tan(\theta) = 0$

$+\uparrow \Sigma F_y = 0;$ $R_C\cos(\theta) - R_B\cos(\theta) - F = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $R_A + R_B\sin(\theta) - R_C\sin(\theta) = 0$

$$\begin{pmatrix} R_A \\ R_B \\ R_C \end{pmatrix} = \text{Find}(R_A, R_B, R_C) \qquad \begin{pmatrix} R_A \\ R_B \\ R_C \end{pmatrix} = \begin{pmatrix} 25.981 \\ 11.945 \\ 63.907 \end{pmatrix} \text{lb}$$

Problem 5-55

The rigid metal strip of negligible weight is used as part of an electromagnetic switch. If the stiffness of the springs at A and B is k, and the strip is originally horizontal when the springs are unstretched, determine the smallest force needed to close the contact gap at C.

Units Used:

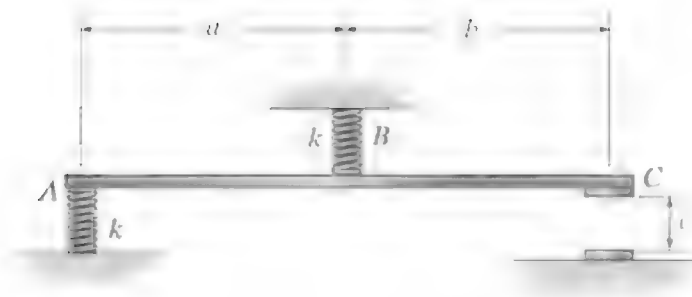
$\text{mN} = 10^{-3}\ \text{N}$

Given:

$a = 50 \text{ mm}$

$b = 50 \text{ mm}$

$c = 10 \text{ mm}$

$k = 5 \frac{\text{N}}{\text{m}}$

Solution:

Initial Guesses: $F = 0.5 \text{ N}$ $y_A = 1 \text{ mm}$ $y_B = 1 \text{ mm}$

Given

$$\frac{c - y_A}{a + b} = \frac{y_B - y_A}{a} \qquad k\,y_A + k\,y_B - F = 0 \qquad k\,y_B a - F(a + b) = 0$$

$$\begin{pmatrix} y_A \\ y_B \\ F \end{pmatrix} = \text{Find}(y_A, y_B, F) \qquad \begin{pmatrix} y_A \\ y_B \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \text{mm} \qquad F = 10\,\text{mN}$$

Problem 5-56

The rigid metal strip of negligible weight is used as part of an electromagnetic switch. Determine the maximum stiffness k of the springs at A and B so that the contact at C closes when the vertical force developed there is $\mathbf{F}$. Originally the strip is horizontal as shown.

Units Used:

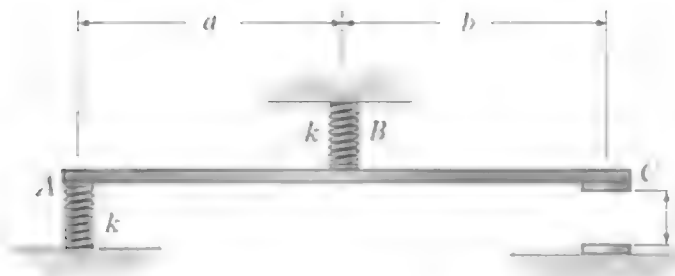
$\text{mN} = 10^{-3} \text{ N}$

Given:

$a = 50 \text{ mm}$

$b = 50 \text{ mm}$

$c = 10 \text{ mm}$

$F = 0.5 \text{ N}$

Solution:

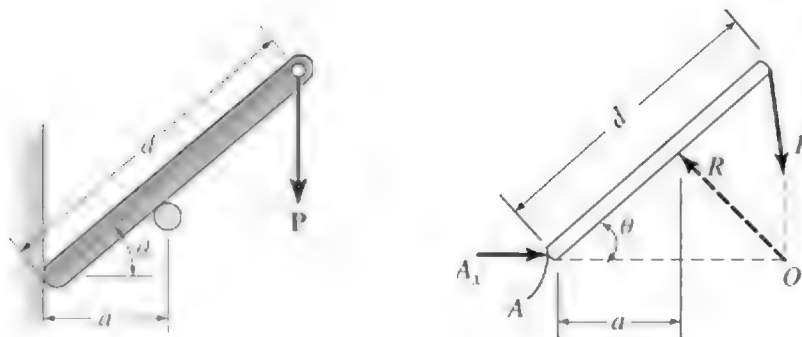
Initial Guesses: $k = 1\,\frac{\text{N}}{\text{m}}$ $\quad y_A = 1\ \text{mm}$ $\quad y_B = 1\ \text{mm}$

Given

$$\frac{c - y_A}{a + b} = \frac{y_B - y_A}{a} \qquad k\,y_A + k\,y_B - F = 0 \qquad k\,y_B a - F(a + b) = 0$$

$$\begin{pmatrix} y_A \\ y_B \\ k \end{pmatrix} = \text{Find}(y_A, y_B, k) \qquad \begin{pmatrix} y_A \\ y_B \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}\text{mm} \qquad k = 250\,\frac{\text{N}}{\text{m}}$$

Problem 5-57

Determine the distance d for placement of the load **P** for equilibrium of the smooth bar in the position θ as shown. Neglect the weight of the bar.

Solution:

$+\!\uparrow \Sigma F_y = 0;$ $\qquad R\cos(\theta) - P = 0$

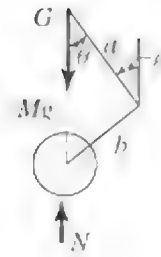
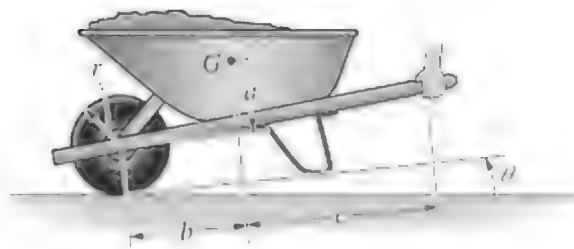
$\Sigma M_A = 0;$ $\qquad -P\,d\cos(\theta) + R\,\frac{a}{\cos(\theta)} = 0$

$$R\ d\cos(\theta)^2 = R\,\frac{a}{\cos(\theta)}$$

$$d = \frac{a}{\cos(\theta)^3}$$

Problem 5-58

The wheelbarrow and its contents have mass m and center of mass at G. Determine the greatest

angle of tilt θ without causing the wheelbarrow to tip over.

Solution:

Require point G to be over the wheel axle for tipping. Thus

$$b\cos(\theta) = a\sin(\theta)$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Problem 5-59

Determine the force **P** needed to pull the roller of mass M over the smooth step.

Given:

$M = 50$ kg

$a = 0.6$ m

$b = 0.1$ m

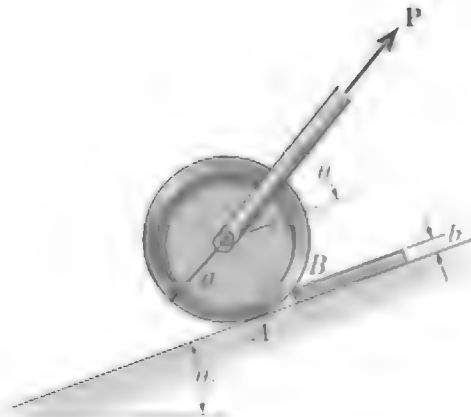
$\theta = 60$ deg

$\theta_1 = 20$ deg

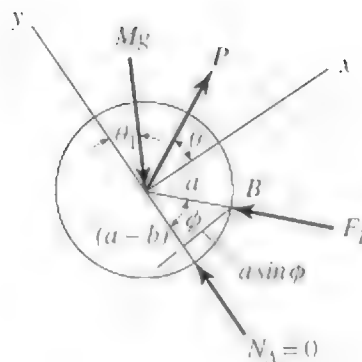
$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

$$\phi = \operatorname{acos}\left(\frac{a-b}{a}\right)$$

$$\phi = 33.56\,\text{deg}$$

$\Sigma M_B = 0,\quad M\,g\sin(\theta_1)(a-b) + M\,g\cos(\theta_1)a\sin(\phi)\ \ldots = 0$
$\quad + P\cos(\theta)(a-b) - P\sin(\theta)a\sin(\phi)$

$$P = Mg\left[\frac{\sin(\theta_1)\,(a-b) + \cos(\theta_1)\,a\,\sin(\phi)}{\cos(\theta)\,(a-b) + \sin(\theta)\,a\,\sin(\phi)}\right]$$

$$P = 441\ \text{N}$$

Problem 5-60

Determine the magnitude and direction θ of the minimum force **P** needed to pull the roller of mass M over the smooth step.

Given:

$a = 0.6\ \text{m}$

$b = 0.1\ \text{m}$

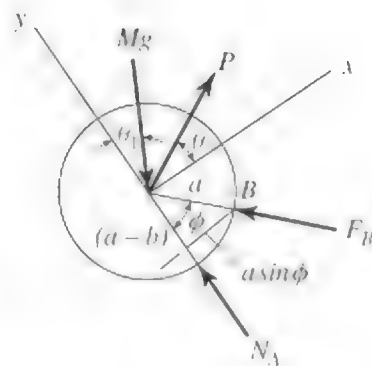
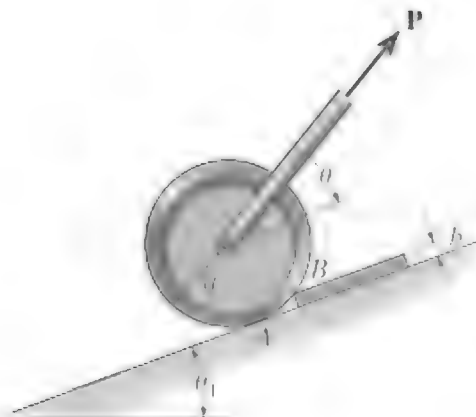
$\theta_1 = 20\ \text{deg}$

$M = 50\ \text{kg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

For P_{min}, N_A tends to 0 $\quad \phi = \operatorname{acos}\left(\frac{a-b}{a}\right)$

$$\phi = 33.56\,\text{deg}$$

$\Sigma M_B = 0\quad M\,g\sin(\theta_1)(a-b) + M\,g\cos(\theta_1)a\sin(\phi)\ \ldots = 0$
$\quad + [-P\cos(\theta)(a-b)] - P\sin(\theta)a\sin(\phi)$

$$P = \frac{M\ g\left[\sin(\theta_1)(a-b) + \cos(\theta_1)\, a \sin(\phi)\right]}{\cos(\theta)(a-b) + a\sin(\phi)\sin(\theta)}$$

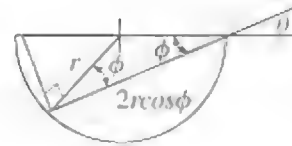
For P_{min}:

$$\frac{dP}{d\theta} = \frac{M\ g\left[\sin(\theta_1)(a-b) + \cos(\theta_1)\, a\sin(\phi)\right]}{\left[\cos(\theta)(a-b) + a\sin(\phi)\sin(\theta)\right]^2}\left[a\sin(\phi)\cos(\theta) - (a-b)\sin(\theta)\right] = 0$$

which gives, $\theta = \operatorname{atan}\left(\sin(\phi)\cdot\frac{a}{a-b}\right)$ $\theta = 33.6\ \text{deg}$

$$P = \frac{M\ g\left[\sin(\theta_1)(a-b) + \cos(\theta_1)\, a\sin(\phi)\right]}{\cos(\theta)(a-b) + a\sin(\phi)\sin(\theta)} \qquad P = 395\ \text{N}$$

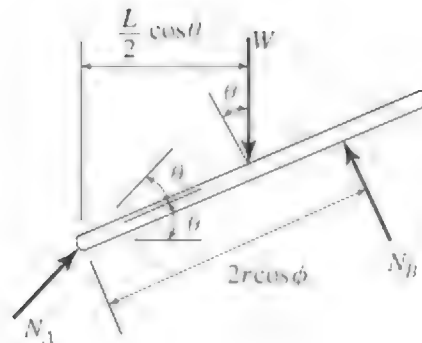
Problem 5-61

A uniform glass rod having a length L is placed in the smooth hemispherical bowl having a radius r. Determine the angle of inclination θ for equilibrium.

Solution:

By Observation $\phi = \theta$.

Equilibirium :

$$\Sigma M_A = 0;\quad N_B 2\ r\cos(\theta) - W\frac{L}{2}\cos(\theta) = 0$$

$$N_B = \frac{W\ L}{4\ r}$$

$$\Sigma F_x = 0;\quad N_A\cos(\theta) - W\sin(\theta) = 0 \qquad N_A = W\tan(\theta)$$

$$\Sigma F_y = 0; \quad W\tan(\theta)\sin(\theta) + \frac{W L}{4\, r} - W\cos(\theta) = 0$$

$$\sin(\theta)^2 - \cos(\theta)^2 = 1 - 2\cos(\theta)^2 = \frac{-L}{4\, r}\cos(\theta)$$

$$2\cos(\theta)^2 - \frac{L}{4\, r}\cos(\theta) - 1 = 0$$

$$\cos(\theta) = \frac{L + \sqrt{L^2 + 128\, r^2}}{16\, r} \qquad \theta = \mathrm{acos}\left(\frac{L + \sqrt{L^2 + 128\, r^2}}{16\, r}\right)$$

Problem 5-62

The disk has mass M and is supported on the smooth cylindrical surface by a spring having stiffness k and unstretched length l_0. The spring remains in the horizontal position since its end A is attached to the small roller guide which has negligible weight. Determine the angle θ to the nearest degree for equilibrium of the roller.

Given:

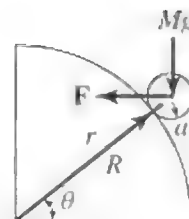
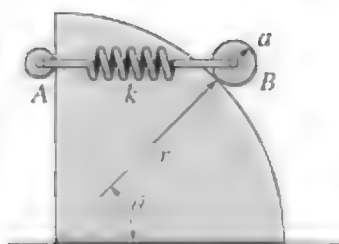
$M = 20\text{ kg}$

$k = 400\ \frac{\text{N}}{\text{m}}$

$l_0 = 1\text{ m}$

$r = 2\text{ m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$a = 0.2\text{ m}$

Guesses $F = 10\text{ N}$ $R = 10\text{ N}$ $\theta = 30\text{ deg}$

Solution: Given

$\overset{+}{\rightarrow}\ \Sigma F_y = 0; \quad R\sin(\theta) - M\, g = 0$

$+\uparrow \Sigma F_x = 0; \quad R\cos(\theta) - F = 0$

Spring $\quad F = k\left[(r + a)\cos(\theta) - l_0\right]$

$$\begin{pmatrix} F \\ R \\ \theta \end{pmatrix} = \text{Find}(F, R, \theta) \qquad \begin{pmatrix} F \\ R \end{pmatrix} = \begin{pmatrix} 163.633 \\ 255.481 \end{pmatrix} \text{N} \qquad \theta = 50.171 \text{ deg}$$

There is also another answer that we can find by choosing different starting guesses.

Guesses $F = 200 \text{ N}$ $R = 200 \text{ N}$ $\theta = 20 \text{ deg}$

Solution: Given

$\overset{+}{\rightarrow} \Sigma F_y = 0;$ $R\sin(\theta) - M\,g = 0$

$+\uparrow \Sigma F_x = 0;$ $R\cos(\theta) - F = 0$

Spring $F = k\left[(r+a)\cos(\theta) - l_0\right]$

$$\begin{pmatrix} F \\ R \\ \theta \end{pmatrix} = \text{Find}(F, R, \theta) \qquad \begin{pmatrix} F \\ R \end{pmatrix} = \begin{pmatrix} 383.372 \\ 430.66 \end{pmatrix} \text{N} \qquad \theta = 27.102 \text{ deg}$$

Problem 5-63

Determine the *x, y, z* components of reaction at the fixed wall *A*.The force $\mathbf{F}_2$ is parallel to the *z* axis and the force $\mathbf{F}_1$ is parallel to the *y* axis.

Given:

$a = 2 \text{ m}$ $d = 2 \text{ m}$

$b = 1 \text{ m}$ $F_1 = 200 \text{ N}$

$c = 2.5 \text{ m}$ $F_2 = 150 \text{ N}$

Solution:

$\Sigma F_x = 0;\quad A_x = 0$

$\Sigma F_x = 0;\quad A_y = -F_1$

$A_y = -200\ \text{N}$

$\Sigma F_x = 0;\quad A_z = F_2$

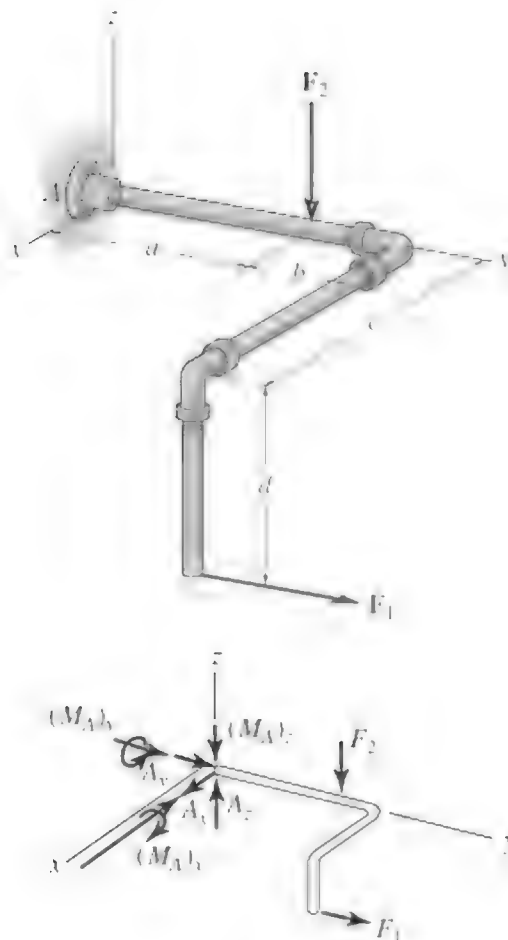
$A_z = 150\ \text{N}$

$\Sigma M_x = 0;\quad M_{Ax} = -F_2 a + F_1 d$

$M_{Ax} = 100\,\text{N}\cdot\text{m}$

$\Sigma M_y = 0;\quad M_{Ay} = 0$

$\Sigma M_z = 0;\quad M_{Az} = F_1 c$

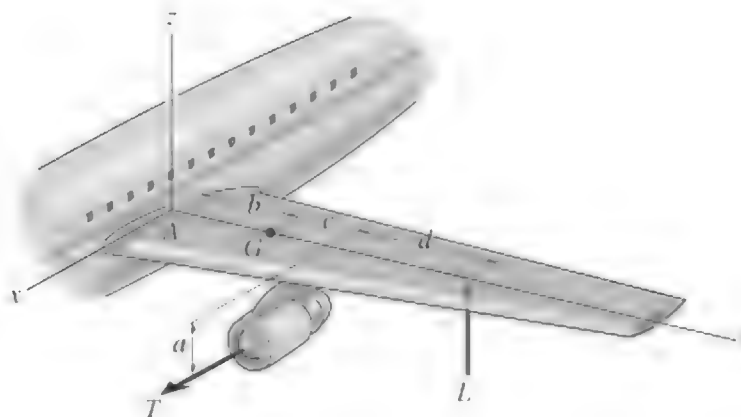
$M_{Az} = 500\,\text{N}\cdot\text{m}$

Problem 5-64

The wing of the jet aircraft is subjected to thrust T from its engine and the resultant lift force L. If the mass of the wing is M and the mass center is at G, determine the x, y, z components of reaction where the wing is fixed to the fuselage at A.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

$g = 9.81\,\dfrac{\text{m}}{\text{s}^2}$

Given:

$T = 8 \text{ kN}$

$L = 45 \text{ kN}$

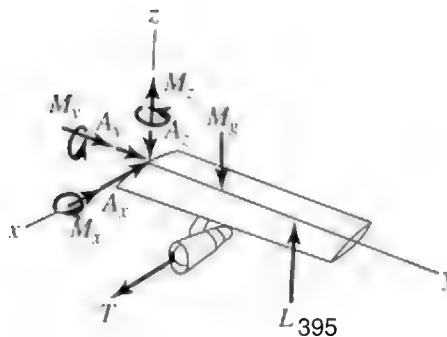
$M = 2.1 \text{ Mg}$

$a = 2.5 \text{ m}$

$b = 5 \text{ m}$

$c = 3 \text{ m}$

$d = 7 \text{ m}$

395

Solution:

$\Sigma F_x = 0;\quad -A_x + T = 0$

$A_x = T \qquad A_x = 8\,\text{kN}$

$\Sigma F_y = 0;\quad A_y = 0 \qquad A_y = 0$

$\Sigma F_z = 0;\quad -A_z - M\,g + L = 0$

$A_z = L - M\,g \qquad A_z = 24.4\,\text{kN}$

$\Sigma M_y = 0;\quad M_y - T(a) = 0$

$M_y = T\,a \qquad M_y = 20.0\,\text{kN}\cdot\text{m}$

$\Sigma M_x = 0;\quad L(b + c + d) - M\,g\,b - M_x = 0$

$M_x = L(b + c + d) - M\,g\,b \qquad M_x = 572\,\text{kN}\cdot\text{m}$

$\Sigma M_z = 0;\quad M_z - T(b + c) = 0$

$M_z = T(b + c) \qquad M_z = 64.0\,\text{kN}\cdot\text{m}$

Problem 5-65

The uniform concrete slab has weight W. Determine the tension in each of the three parallel supporting cables when the slab is held in the horizontal plane as shown.

Units Used:

$kip = 10^3\ lb$

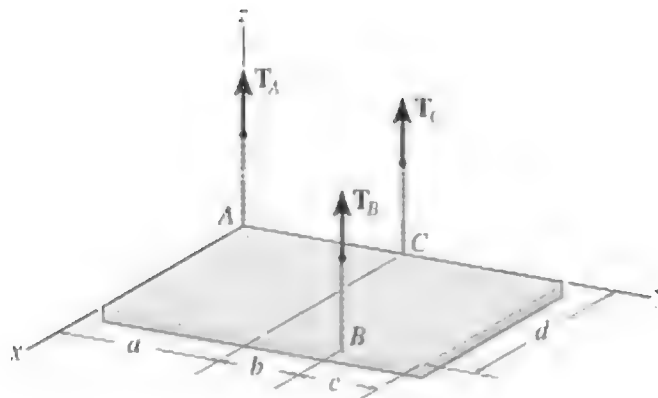
Given:

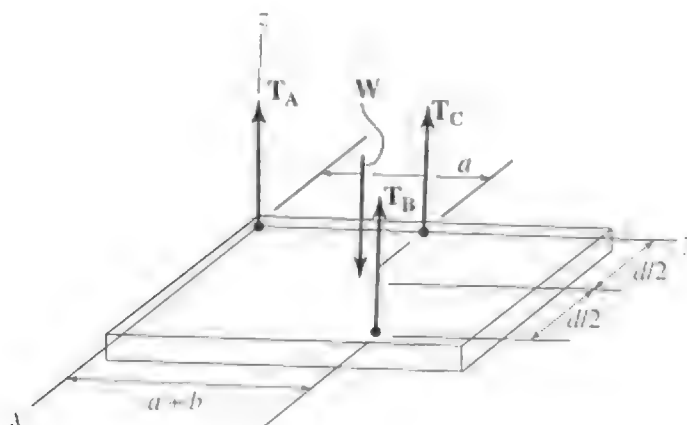
$W = 5500\ lb$

$a = 6\ ft$

$b = 3\ ft$

$c = 3\ ft$

$d = 6\ ft$

Solution:

Equations of Equilibrium : The cable tension T_B can be obtained directly by summing moments about the y axis.

$\Sigma M_y = 0;$ $W\frac{d}{2} - T_B d = 0$ $T_B = \frac{W}{2}$ $T_B = 2.75\ kip$

$\Sigma M_x = 0;$ $T_C a + T_B(a + b) - W\left(\frac{a + b + c}{2}\right) = 0$

$$T_C = \frac{1}{a}\left[W\frac{a + b + c}{2} - T_B(a + b)\right] \qquad T_C = 1.375\ kip$$

$\Sigma F_z = 0;$ $T_A + T_B + T_C - W = 0$ $T_A = -T_B - T_C + W$ $T_A = 1.375\ kip$

Problem 5-66

The air-conditioning unit is hoisted to the roof of a building using the three cables. If the tensions in the cables are $\mathbf{T_A}$, $\mathbf{T_B}$ and $\mathbf{T_C}$, determine the weight of the unit and the location (x, y) of its center of gravity G.

Given:

$T_A = 250\ \text{lb}$

$T_B = 300\ \text{lb}$

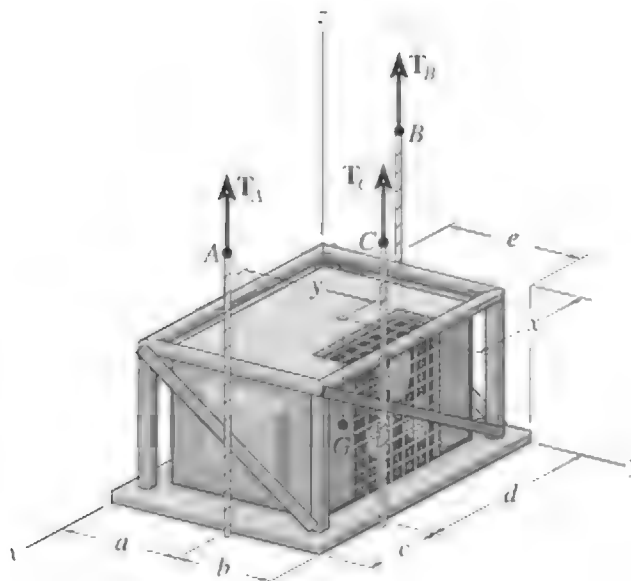
$T_C = 200\ \text{lb}$

$a = 5\ \text{ft}$

$b = 4\ \text{ft}$

$c = 3\ \text{ft}$

$d = 7\ \text{ft}$

$e = 6\ \text{ft}$

Solution:

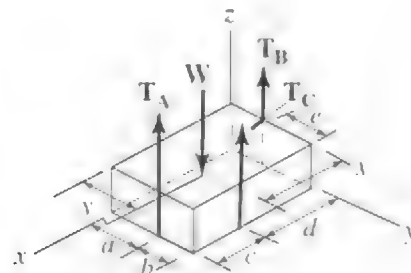
$\Sigma F_z = 0;\quad T_A + T_B + T_C - W = 0$

$$W = T_A + T_B + T_C \qquad W = 750\ \text{lb}$$

$\Sigma M_y = 0;\quad W\,x - T_A(c+d) - T_C d = 0$

$$x = \frac{T_A(c+d) + T_C d}{W} \qquad x = 5.2\ \text{ft}$$

$\Sigma M_x = 0;\quad T_A a + T_B(a+b-e) + T_C(a+b) - W\,y = 0$

$$y = \frac{T_A a + T_B(a+b-e) + T_C(a+b)}{W} \qquad y = 5.267\ \text{ft}$$

Problem 5-67

The platform truck supports the three loadings shown. Determine the normal reactions on each of its three wheels.

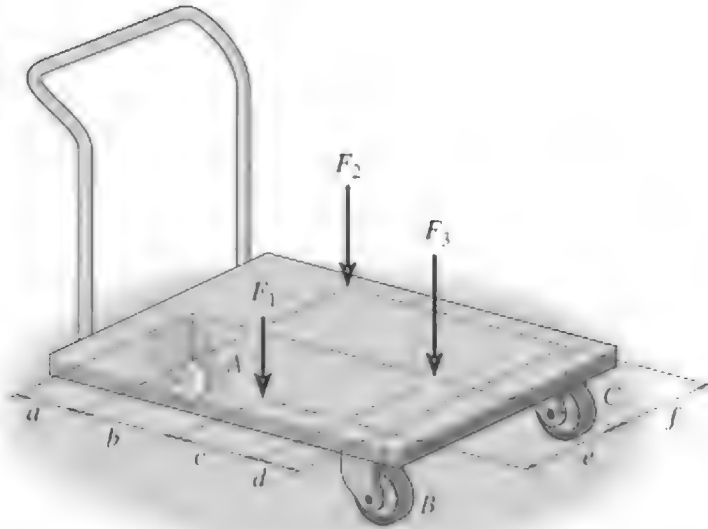
Given:

$F_1 = 380$ lb $b = 12$ in

$F_2 = 500$ lb $c = 10$ in

$d = 5$ in

$F_3 = 800$ lb $e = 12$ in

$a = 8$ in $f = 12$ in

Solution:

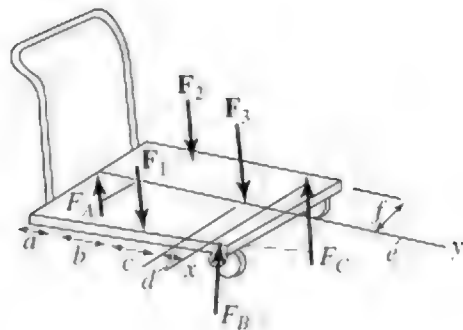
The initail guesses are $F_A = 1$ lb $F_B = 1$ lb $F_C = 1$ lb

Given

$\Sigma M_x = 0;$ $F_1(c+d) + F_2(b+c+d) + F_3 d - F_A(a+b+c+d) = 0$

$\Sigma M_y = 0;$ $F_1 e - F_B e - F_2 f + F_C f = 0$

$\Sigma F_y = 0;$ $F_B + F_C - F_2 + F_A - F_1 - F_3 = 0$

$$\begin{pmatrix} F_A \\ F_B \\ F_C \end{pmatrix} = \text{Find}(F_A, F_B, F_C) \qquad \begin{pmatrix} F_A \\ F_B \\ F_C \end{pmatrix} = \begin{pmatrix} 663 \\ 449 \\ 569 \end{pmatrix} \text{lb}$$

Problem 5-68

Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage *A* and wings *B* and *C* are located as shown. If these components have weights W_A, W_B and W_C, determine the normal reactions of the wheels *D, E*, and *F* on the ground.

Units Used:

$kip = 10^3\ lb$

Given:

$W_A = 45000\ lb$

$W_B = 8000\ lb$

$W_C = 6000\ lb$

$a = 8\ ft$ $e = 20\ ft$

$b = 6\ ft$ $f = 4\ ft$

$c = 8\ ft$ $g = 3\ ft$

$d = 6\ ft$

Solution:

Initial guesses:

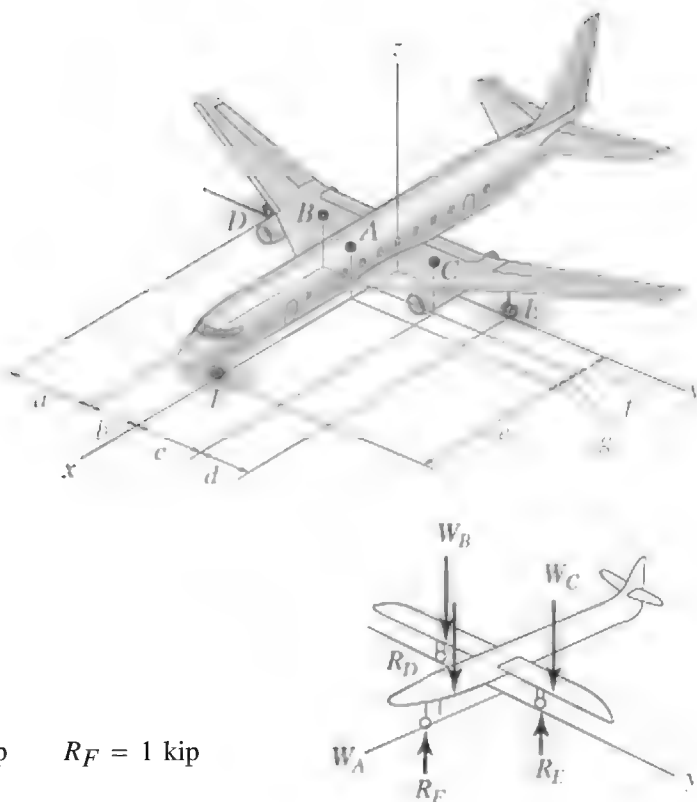
$R_D = 1\ kip$ $R_E = 1\ kip$ $R_F = 1\ kip$

Given

$\Sigma M_x = 0;\quad W_B b - R_D(a+b) - W_C c + R_E(c+d) = 0$

$\Sigma M_y = 0;\quad W_B f + W_A(g+f) + W_C f - R_F(e+g+f) = 0$

$\Sigma F_z = 0;\quad R_D + R_E + R_F - W_A - W_B - W_C = 0$

$$\begin{pmatrix} R_D \\ R_E \\ R_F \end{pmatrix} = \text{Find}(R_D, R_E, R_F) \qquad \begin{pmatrix} R_D \\ R_E \\ R_F \end{pmatrix} = \begin{pmatrix} 22.6 \\ 22.6 \\ 13.7 \end{pmatrix} kip$$

Problem 5-69

If the cable can be subjected to a maximum tension **T**, determine the maximum force **F** which may be applied to the plate. Compute the *x, y, z* components of reaction at the hinge *A* for this loading.

Given:

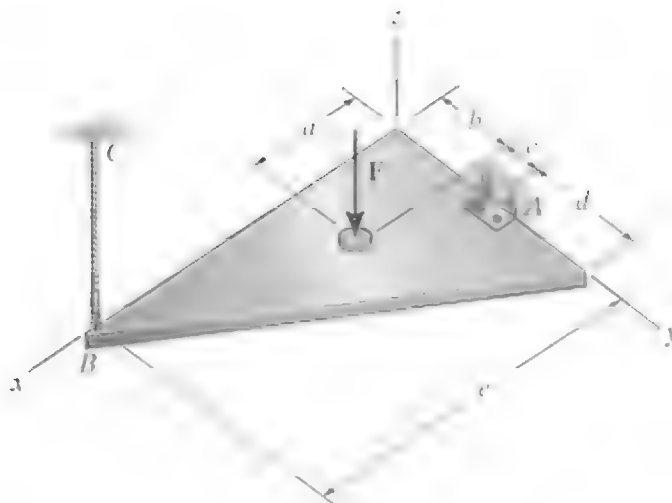
$a = 3\ \text{ft}$

$b = 2\ \text{ft}$

$c = 1\ \text{ft}$

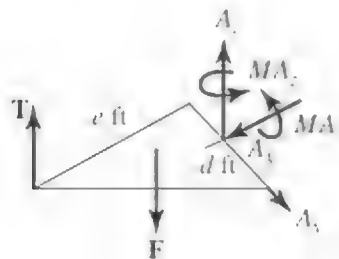
$d = 3\ \text{ft}$

$e = 9\ \text{ft}$

$T = 300\ \text{lb}$

Solution:

Initial guesses:

$F = 10\ \text{lb}$ $\quad M_{Ax} = 10\ \text{lb ft}$ $\quad M_{Az} = 10\ \text{lb ft}$

$A_x = 10\ \text{lb}$ $\quad A_y = 10\ \text{lb}$ $\quad A_z = 10\ \text{lb}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ T - F \end{pmatrix} = 0 \qquad \begin{pmatrix} M_{Ax} \\ 0 \\ M_{Az} \end{pmatrix} + \begin{pmatrix} a \\ -c \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} + \begin{pmatrix} e \\ -b-c \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} = 0$$

$$\begin{pmatrix} F \\ A_x \\ A_y \\ A_z \\ M_{Ax} \\ M_{Az} \end{pmatrix} = \text{Find}(F, A_x, A_y, A_z, M_{Ax}, M_{Az}) \qquad \begin{pmatrix} M_{Ax} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{lb ft}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 600 \end{pmatrix} \text{lb} \qquad F = 900\ \text{lb}$$

Problem 5-70

The boom *AB* is held in equilibrium by a ball-and-socket joint *A* and a pulley and cord system as shown. Determine the *x, y, z* components of reaction at *A* and the tension in cable *DEC*.

Given:

$$\mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ -1500 \end{pmatrix} \text{lb}$$

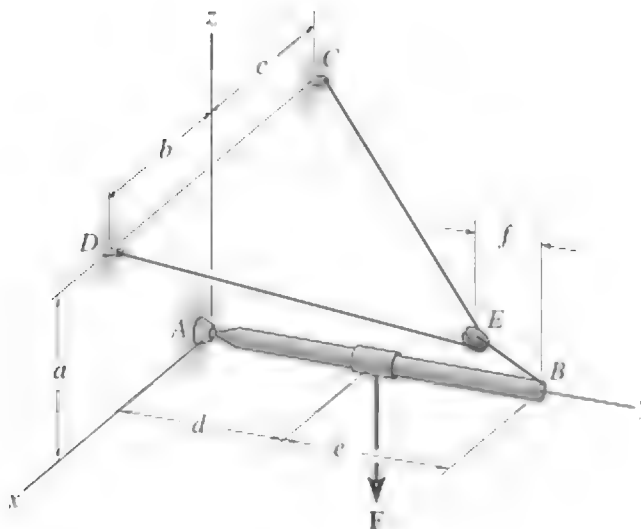
$a = 5 \text{ ft}$

$b = 4 \text{ ft}$

$c = b$

$d = 5 \text{ ft}$

$e = 5 \text{ ft}$

$f = 2 \text{ ft}$

Solution:

$$\alpha = \text{atan}\left(\frac{a}{d+e}\right)$$

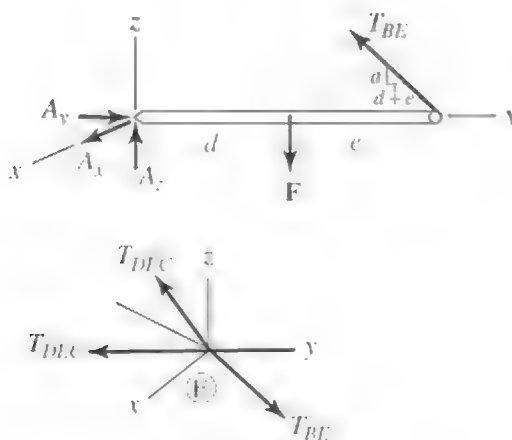
$$L = \sqrt{a^2 + (d+e)^2}$$

$$\beta = \text{atan}\left(\frac{b}{L - \frac{f L}{d+e}}\right)$$

Guesses $T_{BE} = 1 \text{ lb}$ $T_{DEC} = 1 \text{ lb}$

$A_x = 1 \text{ lb}$ $A_y = 1 \text{ lb}$ $A_z = 1 \text{ lb}$

Given $2\,T_{DEC}\cos(\beta) = T_{BE}$

$$\begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix} \times \mathbf{F} + \begin{pmatrix} 0 \\ d+e \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -T_{BE}\cos(\alpha) \\ T_{BE}\sin(\alpha) \end{pmatrix} = 0 \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \mathbf{F} + T_{BE}\begin{pmatrix} 0 \\ -\cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ T_{BE} \\ T_{DEC} \end{pmatrix} = \text{Find}(A_x, A_y, A_z, T_{BE}, T_{DEC}) \quad T_{DEC} = 919\,\text{lb} \quad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ 1.5 \times 10^3 \\ 750 \end{pmatrix} \text{lb}$$

Problem 5-71

The cable *CED* can sustain a maximum tension T_{max} before it fails. Determine the greatest vertical force *F* that can be applied to the boom. Also, what are the *x, y, z* components of reaction at the ball-and-socket joint *A*?

Given:

$T_{max} = 800\ \text{lb}$

$a = 5\ \text{ft}$

$b = 4\ \text{ft}$

$c = b$

$d = 5\ \text{ft}$

$e = 5\ \text{ft}$

$f = 2\ \text{ft}$

Solution:

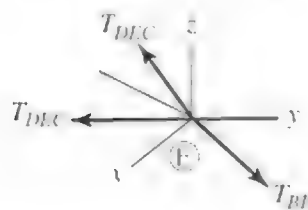
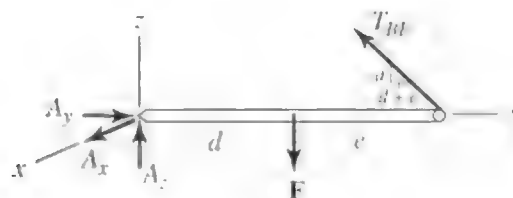
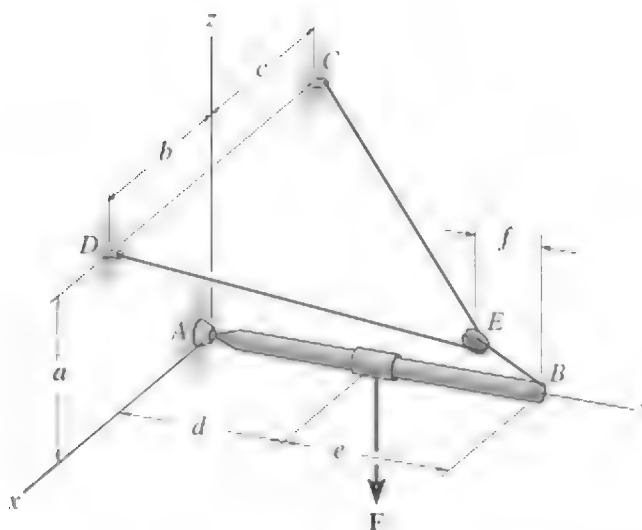
$$\alpha = \text{atan}\left(\frac{a}{d+e}\right)$$

$$L = \sqrt{a^2 + (d+e)^2}$$

$$\beta = \text{atan}\left(\frac{b}{L - \frac{f\,L}{d+e}}\right) \qquad T_{DEC} = T_{max}$$

Guesses $\quad T_{BE} = 1\ \text{lb} \quad F = 1\ \text{lb}$

$A_x = 1\ \text{lb} \quad A_y = 1\ \text{lb} \quad A_z = 1\ \text{lb}$

Given $\quad 2\,T_{DEC}\cos(\beta) = T_{BE}$

$$\begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} + \begin{pmatrix} 0 \\ d+e \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -T_{BE}\cos(\alpha) \\ T_{BE}\sin(\alpha) \end{pmatrix} = 0 \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} + T_{BE}\begin{pmatrix} 0 \\ -\cos(\alpha) \\ \sin(\alpha) \end{pmatrix} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ T_{BE} \\ F \end{pmatrix} = \text{Find}(A_x, A_y, A_z, T_{BE}, F) \qquad F = 1306\,\text{lb} \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ 1.306 \times 10^3 \\ 653.197 \end{pmatrix}\text{lb}$$

Problem 5-72

The uniform table has a weight W and is supported by the framework shown. Determine the smallest vertical force **P** that can be applied to its surface that will cause it to tip over. Where should this force be applied?

Given:

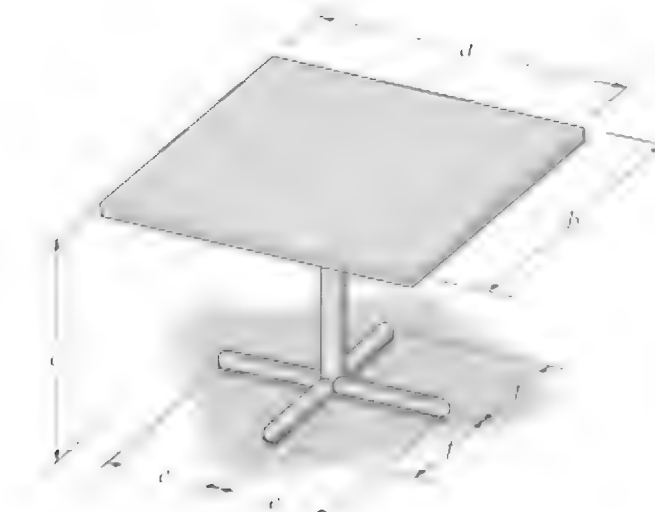
$W = 20$ lb

$a = 3.5$ ft

$b = 2.5$ ft

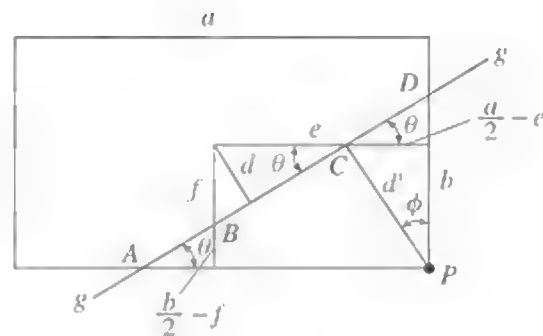
$c = 3$ ft

$e = 1.5$ ft

$f = 1$ ft

Solution:

$$\theta = \text{atan}\left(\frac{f}{e}\right) \qquad \theta = 33.69\,\text{deg}$$

$$d = e\sin(\theta) \qquad d = 0.832\,\text{ft}$$

$$\phi = \text{atan}\left(\frac{\frac{a}{2} - e}{\frac{b}{2}}\right) \qquad \phi = 11.31\,\text{deg}$$

$$d' = \sqrt{\left(\frac{a}{2} - e\right)^2 + \left(\frac{b}{2}\right)^2} \qquad d' = 1.275\text{ ft}$$

Tipping will occur about the g - g axis.
Require P to be applied at the corner of the table for P_{min}.

$$W\ d = P\ d' \sin(90\text{ deg} - \phi + \theta)$$

$$P = W \frac{d}{d' \sin(90\text{ deg} - \phi + \theta)} \qquad P = 14.1\text{ lb}$$

Problem 5-73

The windlass is subjected to load W. Determine the horizontal force P needed to hold the handle in the position shown, and the components of reaction at the ball-and-socket joint A and the smooth journal bearing B. The bearing at B is in proper alignment and exerts only force reactions perpendicular to the shaft on the windlass.

Given:

$W = 150$ lb

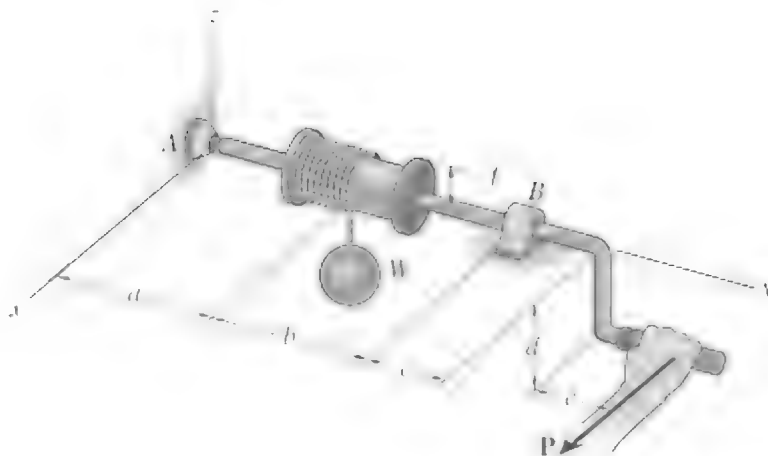
$a = 2$ ft

$b = 2$ ft

$c = 1$ ft

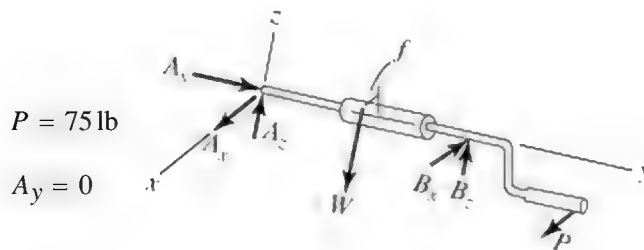
$d = 1$ ft

$e = 1$ ft

$f = 0.5$ ft

Solution:

$\Sigma M_y = 0;\quad W\ f - P\ d = 0$

$$P = \frac{W\ f}{d} \qquad P = 75\text{ lb}$$

$\Sigma F_y = 0;\quad A_y = 0\text{ lb} \qquad A_y = 0$

$\Sigma M_x = 0;\quad -W\ a + B_z(a + b) = 0$

$$B_z = \frac{W\ a}{a+b} \qquad B_z = 75\,\text{lb}$$

$\Sigma F_z = 0;\quad A_z + B_z - W = 0$

$$A_z = W - B_z \qquad A_z = 75\,\text{lb}$$

$\Sigma M_z = 0;\quad B_x(a+b) - (a+b+c+e)P = 0$

$$B_x = \frac{P(a+b+c+e)}{a+b} \qquad B_x = 112\,\text{lb}$$

$\Sigma F_x = 0;\quad A_x - B_x + P = 0$

$$A_x = B_x - P \qquad A_x = 37.5\,\text{lb}$$

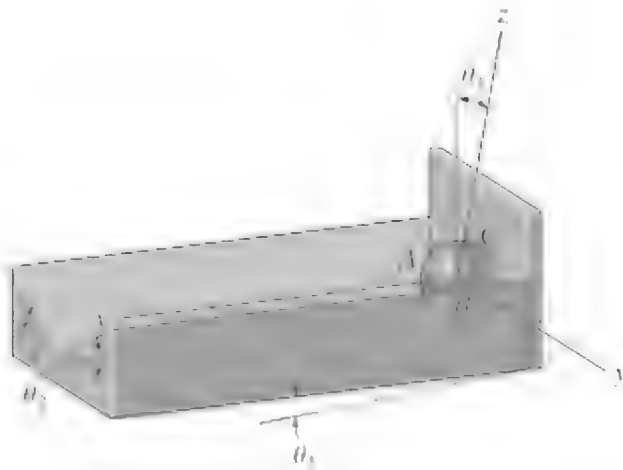
Problem 5-74

A ball of mass *M* rests between the grooves *A* and *B* of the incline and against a vertical wall at *C*. If all three surfaces of contact are smooth, determine the reactions of the surfaces on the ball. *Hint:* Use the *x, y, z* axes, with origin at the center of the ball, and the *z* axis inclined as shown.

Given:

$M = 2\ \text{kg}$

$\theta_1 = 10\ \text{deg}$

$\theta_2 = 45\ \text{deg}$

Solution:

$\Sigma F_x = 0;\quad F_C\cos(\theta_1) - M\ g\sin(\theta_1) = 0$

$$F_C = M\ g\cdot\tan(\theta_1)$$

$$F_C = 0.32\ \text{kg}\cdot\text{m}$$

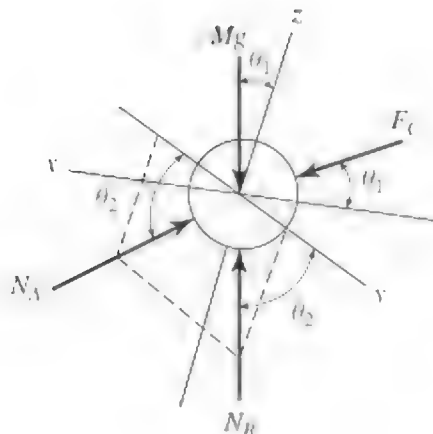
$\Sigma F_y = 0; \quad N_A \cos(\theta_2) - N_B \cos(\theta_2) = 0$

$N_A = N_B$

$\Sigma F_z = 0; \quad 2\ N_A \sin(\theta_2) - M\ g \cos(\theta_1) - F_c \sin(\theta_1) = 0$

$$N_A = \frac{1}{2} \cdot \frac{M\ g \cdot \cos(\theta_1) + F_c \cdot \sin(\theta_1)}{\sin(\theta_2)}$$

$N_A = 1.3\ \text{kg} \cdot \text{m}$

$N = N_A = N_B$

Problem 5-75

Member *AB* is supported by cable *BC* and at *A* by a *square* rod which fits loosely through the square hole at the end joint of the member as shown. Determine the components of reaction at *A* and the tension in the cable needed to hold the cylinder of weight *W* in equilibrium.

Units Used:

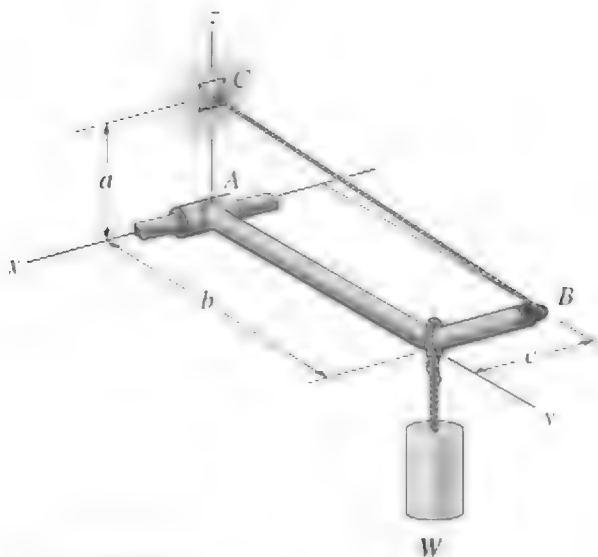
$\text{kip} = 10^3\ \text{lb}$

Given:

$W = 800\ \text{lb}$

$a = 2\ \text{ft}$

$b = 6\ \text{ft}$

$c = 3\ \text{ft}$

Solution:

$\Sigma F_x = 0 \quad F_{BC}\left(\frac{c}{\sqrt{c^2 + b^2 + a^2}}\right) = 0 \quad F_{BC} = 0\ \text{lb}$

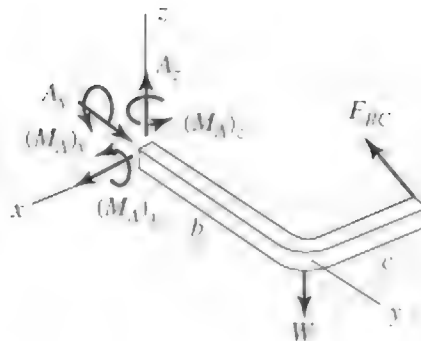
$\Sigma F_y = 0 \quad A_y = 0 \quad A_y = 0\text{lb} \quad A_y = 0\,\text{lb}$

$\Sigma F_z = 0 \quad A_z - W = 0 \quad A_z = W \quad A_z = 800\,\text{lb}$

$\Sigma M_x = 0 \quad M_{Ax} - W\,b = 0$

$M_{Ax} = W\,b \quad M_{Ax} = 4.80\,\text{kip}\cdot\text{ft}$

$\Sigma M_y = 0 \quad M_{Ay} = 0\ \text{lb ft} \quad M_{Ay} = 0\,\text{lb}\cdot\text{ft}$

$\Sigma M_z = 0 \quad M_{Az} = 0\ \text{lb ft} \quad M_{Az} = 0\,\text{lb}\cdot\text{ft}$

Problem 5-76

The pipe assembly supports the vertical loads shown. Determine the components of reaction at the ball-and-socket joint A and the tension in the supporting cables BC and BD.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F_1 = 3\ \text{kN} \quad d = 2\ \text{m}$

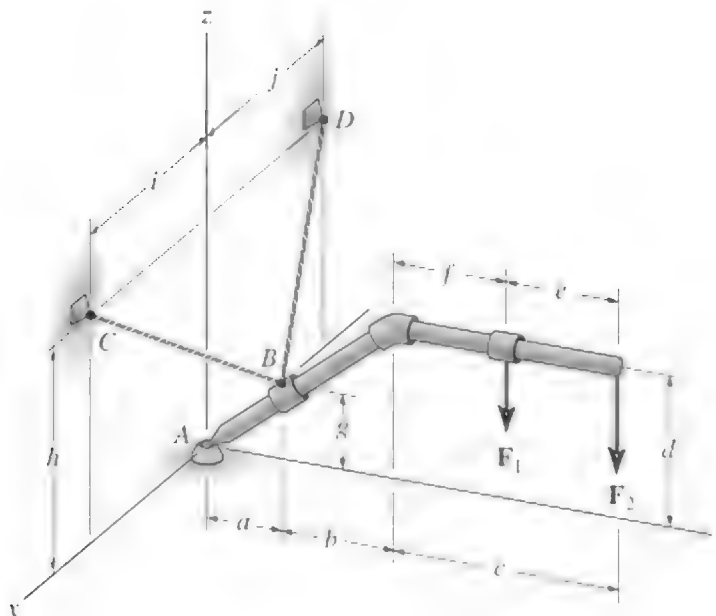
$F_2 = 4\ \text{kN} \quad e = 1.5\ \text{m}$

$a = 1\ \text{m} \quad g = 1\ \text{m}$

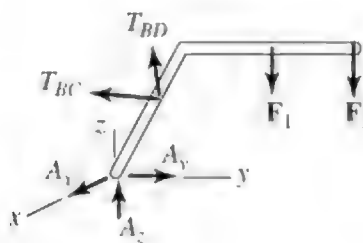
$b = 1.5\ \text{m} \quad h = 3\ \text{m}$

$c = 3\ \text{m} \quad i = 2\ \text{m}$

$f = c - e \quad j = 2\text{m}$

Solution:

The initial guesses are:

$T_{BD} = 1\ \text{kN} \quad T_{BC} = 1\ \text{kN}$

$A_x = 1\ \text{kN} \quad A_y = 1\ \text{kN} \quad A_z = 1\ \text{kN}$

The vectors

$$\mathbf{r_1} = \begin{pmatrix} 0 \\ a+b+f \\ d \end{pmatrix} \qquad \mathbf{r_2} = \begin{pmatrix} 0 \\ a+b+c \\ d \end{pmatrix}$$

$$\mathbf{r_{BC}} = \begin{pmatrix} i \\ -a \\ h-g \end{pmatrix} \qquad \mathbf{r_{BD}} = \begin{pmatrix} -j \\ -a \\ h-g \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} 0 \\ a \\ g \end{pmatrix}$$

$$\mathbf{u_{BC}} = \frac{\mathbf{r_{BC}}}{|\mathbf{r_{BC}}|} \qquad \mathbf{u_{BD}} = \frac{\mathbf{r_{BD}}}{|\mathbf{r_{BD}}|} \qquad i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Given $\quad A_x i + A_y j + A_z k - F_1 k - F_2 k + T_{BD}\mathbf{u_{BD}} + T_{BC}\mathbf{u_{BC}} = 0$

$$\mathbf{r_{AB}} \times \left(T_{BD}\mathbf{u_{BD}} + T_{BC}\mathbf{u_{BC}}\right) + \mathbf{r_1} \times \left(-F_1 k\right) + \mathbf{r_2} \times \left(-F_2 k\right) = 0$$

$$\begin{pmatrix} T_{BD} \\ T_{BC} \\ A_x \\ A_y \\ A_z \end{pmatrix} = \text{Find}\left(T_{BD}, T_{BC}, A_x, A_y, A_z\right) \qquad \begin{pmatrix} T_{BD} \\ T_{BC} \end{pmatrix} = \begin{pmatrix} 17 \\ 17 \end{pmatrix} \text{kN}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ 11.333 \\ -15.667 \end{pmatrix} \text{kN}$$

Problem 5-77

The hatch door has a weight W and center of gravity at G. If the force $\mathbf{F}$ applied to the handle at C has coordinate direction angles of α, β and γ, determine the magnitude of $\mathbf{F}$ needed to hold the door slightly open as shown. The hinges are in proper alignment and exert only force reactions on the door. Determine the components of these reactions if A exerts only x and z components of force and B exerts x, y, z force components.

Given:

$W = 80$ lb

$\alpha = 60$ deg

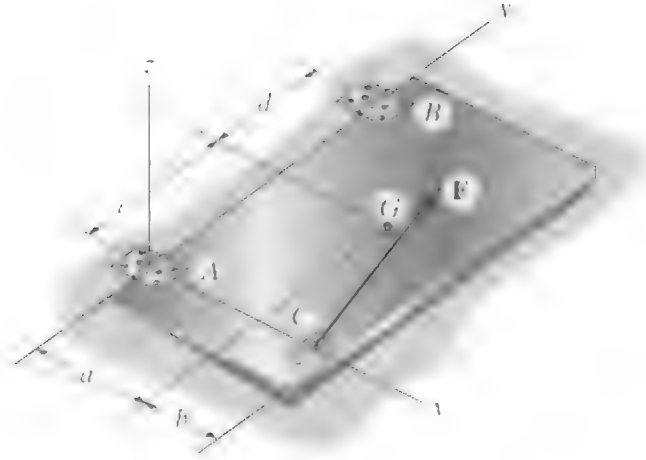
$\beta = 45 \text{ deg}$

$\gamma = 60 \text{ deg}$

$a = 3 \text{ ft}$

$b = 2 \text{ ft}$

$c = 4 \text{ ft}$

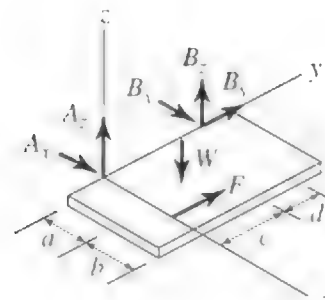
$d = 3 \text{ ft}$

Solution:

Initial Guesses:

$A_x = 1 \text{ lb}$ $\quad A_z = 1 \text{ lb}$ $\quad F = 1 \text{ lb}$

$B_x = 1 \text{ lb}$ $\quad B_y = 1 \text{ lb}$ $\quad B_z = 1 \text{ lb}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + F\begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -W \end{pmatrix} = 0$$

$$\begin{pmatrix} a+b \\ 0 \\ 0 \end{pmatrix} \times \left[F\begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix}\right] + \begin{pmatrix} a \\ c \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -W \end{pmatrix} + \begin{pmatrix} 0 \\ c+d \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = 0$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_y \\ B_z \\ F \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_y, B_z, F)$$

$$\begin{pmatrix} A_x \\ A_z \end{pmatrix} = \begin{pmatrix} -96.5 \\ -13.7 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 48.5 \\ -67.9 \\ 45.7 \end{pmatrix} \text{lb}$$

$F = 96 \text{ lb}$

Problem 5-78

The hatch door has a weight W and center of gravity at G. If the force $\mathbf{F}$ applied to the handle at C has coordinate direction angles α, β, γ determine the magnitude of $\mathbf{F}$ needed to hold the door slightly open as shown. If the hinge at A becomes loose from its attachment and is ineffective, what are the x, y, z components of reaction at hinge B?

Given:

$W = 80 \text{ lb}$

$\alpha = 60 \text{ deg}$

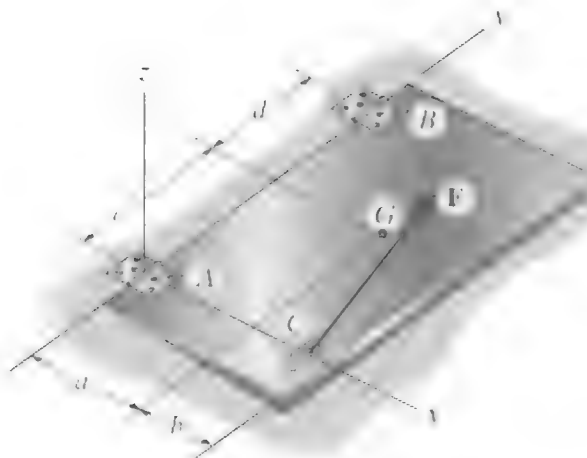
$\beta = 45 \text{ deg}$

$\gamma = 60 \text{ deg}$

$a = 3 \text{ ft}$

$b = 2 \text{ ft}$

$c = 4 \text{ ft}$

$d = 3 \text{ ft}$

Solution:

$\Sigma M_y = 0;$ $\quad F = W \dfrac{a}{\cos(\gamma)\ (a+b)}$ $\quad F = 96 \text{ lb}$

$\Sigma F_x = 0;$ $\quad B_x + F\cos(\alpha) = 0$

$B_x = -F\cos(\alpha)$ $\quad B_x = -48 \text{ lb}$

$\Sigma F_y = 0;$ $\quad B_y + F\cos(\beta) = 0$

$B_y = -F\cos(\beta)$ $\quad B_y = -67.9 \text{ lb}$

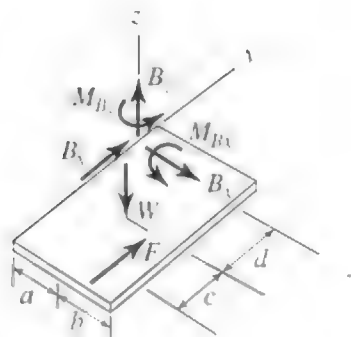
$\Sigma F_z = 0;$ $\quad B_z - W + F\cos(\gamma) = 0$

$B_z = W - F\cos(\gamma)$ $\quad B_z = 32 \text{ lb}$

$\Sigma M_x = 0;$ $\quad M_{Bx} + W d - F\cos(\gamma)(c+d) = 0$

$M_{Bx} = -W\,d + F\cos(\gamma)\ (c+d)$ $\quad M_{Bx} = 96 \text{ lb}\cdot\text{ft}$

$\Sigma M_z = 0;$ $\quad M_{Bz} + F\cos(\alpha)(c+d) + F\cos(\beta)(a+b) = 0$

$M_{Bz} = -F\cos(\alpha)\ (c+d) - F\cos(\beta)\ (a+b)$ $\quad M_{Bz} = -675 \text{ lb}\cdot\text{ft}$

Problem 5-79

The bent rod is supported at A, B, and C by smooth journal bearings. Compute the x, y, z components of reaction at the bearings if the rod is subjected to forces $\mathbf{F_1}$ and $\mathbf{F_2}$. $\mathbf{F_1}$ lies in the y-z plane. The bearings are in proper alignment and exert only force reactions on the rod.

Given:

$F_1 = 300\ \text{lb}$ $\quad d = 3\ \text{ft}$

$F_2 = 250\ \text{lb}$ $\quad e = 5\ \text{ft}$

$a = 1\ \text{ft}$ $\quad \alpha = 30\ \text{deg}$

$b = 4\ \text{ft}$ $\quad \beta = 45\ \text{deg}$

$c = 2\ \text{ft}$ $\quad \theta = 45\ \text{deg}$

Solution:

The initial guesses:

$A_x = 100\ \text{lb}$ $\quad A_y = 200\ \text{lb}$

$B_x = 300\ \text{lb}$ $\quad B_z = 400\ \text{lb}$

$C_y = 500\ \text{lb}$ $\quad C_z = 600\ \text{lb}$

Given

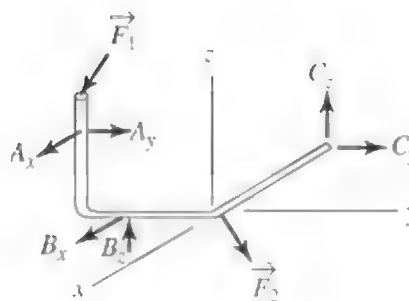
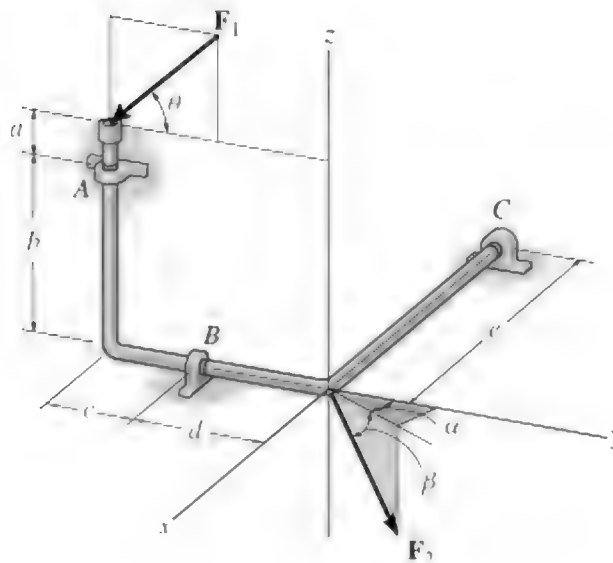
$$A_x + B_x + F_2\cos(\beta)\sin(\alpha) = 0$$

$$A_y + C_y - F_1\cos(\theta) + F_2\cos(\beta)\cos(\alpha) = 0$$

$$B_z + C_z - F_1\sin(\theta) - F_2\sin(\beta) = 0$$

$$F_1\cos(\theta)(a + b) + F_1\sin(\theta)(c + d) - B_z d - A_y b = 0$$

$$A_x b + C_z e = 0$$

$$A_x(c + d) + B_x d - C_y e = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_z \\ C_y \\ C_z \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_z, C_y, C_z)$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 632.883 \\ -141.081 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} B_x \\ B_z \end{pmatrix} = \begin{pmatrix} -721.271 \\ 895.215 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} C_y \\ C_z \end{pmatrix} = \begin{pmatrix} 200.12 \\ -506.306 \end{pmatrix} \text{lb}$$

Problem 5-80

The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of $\mathbf{F_2}$ which will cause the reaction $\mathbf{C}_y$ at the bearing C to be equal to zero. The bearings are in proper alignment and exert only force reactions on the rod.

Given:

$F_1 = 300$ lb $\quad d = 3$ ft

$C_y = 0$ lb $\quad e = 5$ ft

$a = 1$ ft $\quad \alpha = 30$ deg

$b = 4$ ft $\quad \beta = 45$ deg

$c = 2$ ft $\quad \theta = 45$ deg

Solution:

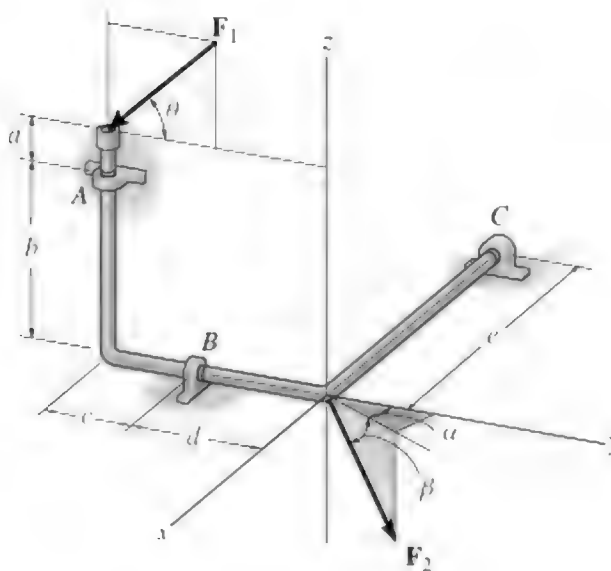
The initial guesses:

$A_x = 100$ lb $\quad A_y = 200$ lb

$B_x = 300$ lb $\quad B_z = 400$ lb

$F_2 = 500$ lb $\quad C_z = 600$ lb

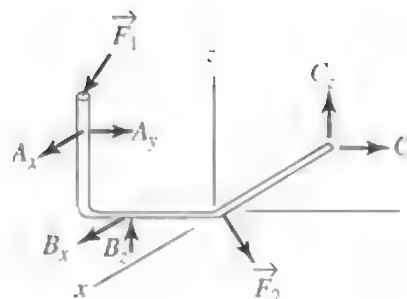
Given

$$A_x + B_x + F_2\cos(\beta)\sin(\alpha) = 0$$

$$A_y + C_y - F_1 \cos(\theta) + F_2 \cos(\beta)\cos(\alpha) = 0$$

$$B_z + C_z - F_1 \sin(\theta) - F_2 \sin(\beta) = 0$$

$$F_1 \cos(\theta)(a + b) + F_1 \sin(\theta)(c + d) - B_z d - A_y b = 0$$

$$A_x b + C_z e = 0$$

$$A_x(c + d) + B_x d - C_y e = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_z \\ C_z \\ F_2 \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_z, C_z, F_2) \qquad F_2 = 673.704 \text{ lb}$$

Problem 5-81

Determine the tension in cables BD and CD and the x, y, z components of reaction at the ball-and-socket joint at A.

Given:

$F = 300$ N

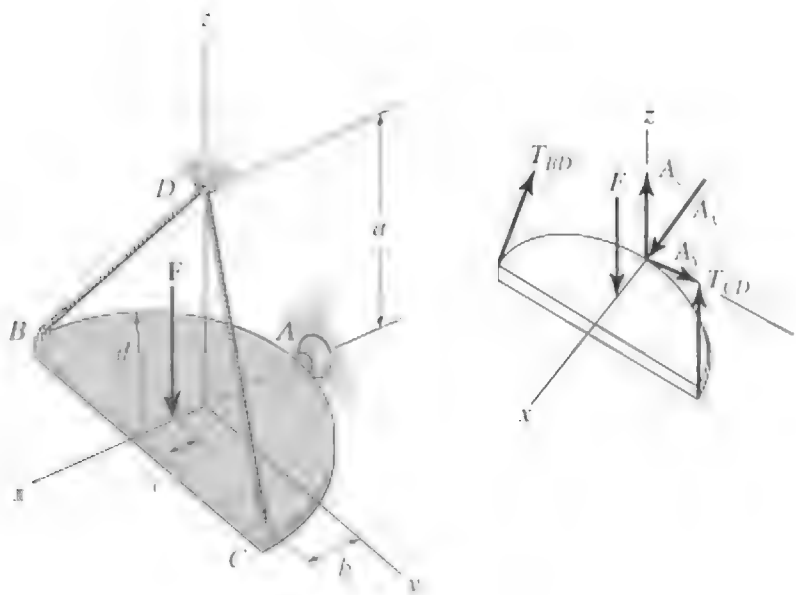
$a = 3$ m

$b = 1$ m

$c = 0.5$ m

$d = 1.5$ m

Solution:

$$r_{BD} = \begin{pmatrix} -b \\ d \\ a \end{pmatrix}$$

$$r_{CD} = \begin{pmatrix} -b \\ -d \\ a \end{pmatrix}$$

Initial Guesses: $T_{BD} = 1\ \text{N}$ $T_{CD} = 1\ \text{N}$ $A_x = 1\ \text{N}$ $A_y = 1\ \text{N}$ $A_z = 1\ \text{N}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + T_{BD}\frac{r_{BD}}{|r_{BD}|} + T_{CD}\frac{r_{CD}}{|r_{CD}|} + \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} = 0$$

$$\begin{pmatrix} d \\ -d \\ 0 \end{pmatrix} \times \left(T_{BD}\frac{r_{BD}}{|r_{BD}|}\right) + \begin{pmatrix} d \\ d \\ 0 \end{pmatrix} \times \left(T_{CD}\frac{r_{CD}}{|r_{CD}|}\right) + \begin{pmatrix} d-c \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} = 0$$

$$\begin{pmatrix} T_{BD} \\ T_{CD} \\ A_x \\ A_y \\ A_z \end{pmatrix} = \text{Find}(T_{BD}, T_{CD}, A_x, A_y, A_z) \qquad \begin{pmatrix} T_{BD} \\ T_{CD} \end{pmatrix} = \begin{pmatrix} 116.7 \\ 116.7 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 66.7 \\ 0 \\ 100 \end{pmatrix} \text{N}$$

Problem 5-82

Determine the tensions in the cables and the components of reaction acting on the smooth collar at A necessary to hold the sign of weight W in equilibrium. The center of gravity for the sign is at G.

Given:

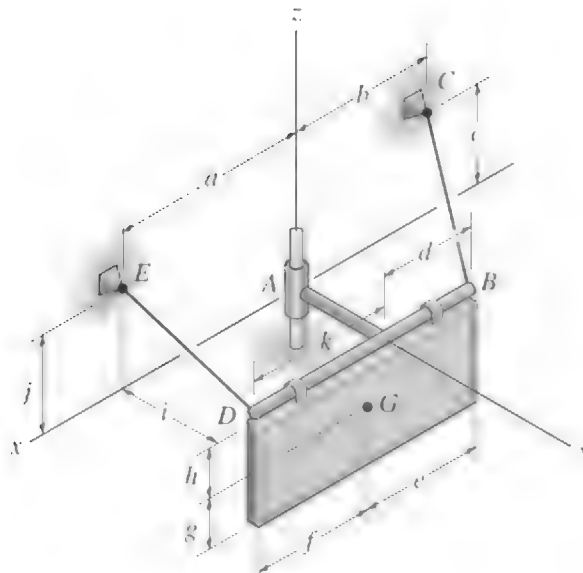
$W = 50\ \text{lb}$ $f = 2.5\ \text{ft}$

$a = 4\ \text{ft}$ $g = 1\ \text{ft}$

$b = 3\ \text{ft}$ $h = 1\ \text{ft}$

$c = 2\ \text{ft}$ $\quad i = 2\ \text{ft}$

$d = 2\ \text{ft}$ $\quad j = 2\ \text{ft}$

$e = 2.5\ \text{ft}$ $\quad k = 3\ \text{ft}$

Solution:

The initial guesses are:

$T_{BC} = 10\ \text{lb}$ $\quad A_x = 10\ \text{lb}$ $\quad M_{Ax} = 10\ \text{lb}\cdot\text{ft}$

$T_{DE} = 10\ \text{lb}$ $\quad A_y = 10\ \text{lb}$ $\quad M_{Ay} = 10\ \text{lb}\cdot\text{ft}$

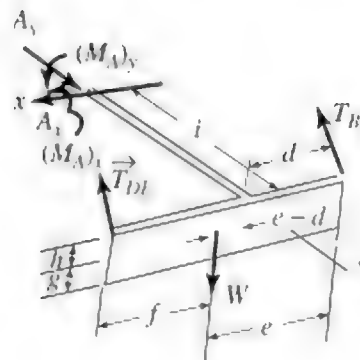
Given

$$(a-k)\frac{T_{DE}}{\sqrt{(a-k)^2+i^2+j^2}} + (-b+d)\frac{T_{BC}}{\sqrt{(d-b)^2+i^2+c^2}} + A_x = 0$$

$$-i\frac{T_{DE}}{\sqrt{(a-k)^2+i^2+j^2}} - i\frac{T_{BC}}{\sqrt{(d-b)^2+i^2+c^2}} + A_y = 0$$

$$j\frac{T_{DE}}{\sqrt{(a-k)^2+i^2+j^2}} + c\frac{T_{BC}}{\sqrt{(d-b)^2+i^2+c^2}} - W = 0$$

$$M_{Ax} + T_{DE}\,j\frac{i}{\sqrt{(a-k)^2+i^2+j^2}} + c\,T_{BC}\frac{i}{\sqrt{(d-b)^2+i^2+c^2}} - W i = 0$$

$$M_{Ay} - T_{DE}\,k\frac{j}{\sqrt{(a-k)^2+i^2+j^2}} + T_{BC}\,c\frac{d}{\sqrt{(d-b)^2+i^2+c^2}} + W(k-f) = 0$$

$$-T_{DE}a\frac{i}{\sqrt{(a-k)^2+i^2+j^2}}+T_{BC}b\frac{i}{\sqrt{(d-b)^2+i^2+c^2}}=0$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ T_{BC} \\ T_{DE} \\ A_x \\ A_y \end{pmatrix} = \text{Find}(M_{Ax}, M_{Ay}, T_{BC}, T_{DE}, A_x, A_y)$$

$$\begin{pmatrix} T_{BC} \\ T_{DE} \end{pmatrix} = \begin{pmatrix} 42.857 \\ 32.143 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 3.571 \\ 50 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \end{pmatrix} = \begin{pmatrix} 2.698 \times 10^{-13} \\ -17.857 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 5-83

The member is supported by a pin at A and a cable BC. If the load at D is W, determine the x, y, z components of reaction at these supports.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$W = 300\ \text{lb}$

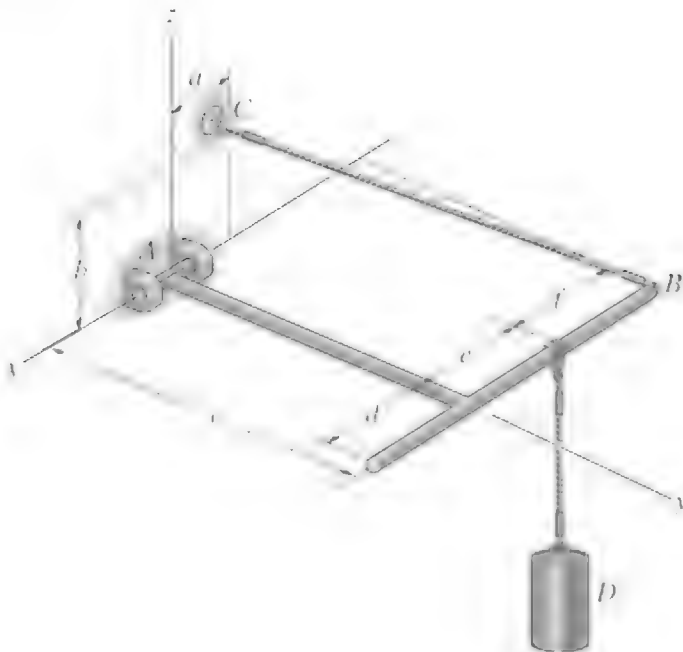
$a = 1\ \text{ft}$

$b = 2\ \text{ft}$

$c = 6\ \text{ft}$

$d = 2\ \text{ft}$

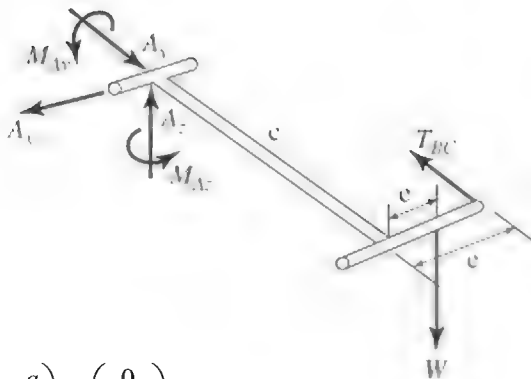
$e = 2\ \text{ft}$

$f = 2\ \text{ft}$

Solution:

Initial Guesses:

$T_{BC} = 1\text{ lb}$ $A_x = 1\text{ lb}$

$A_y = 1\text{ lb}$ $A_z = 1\text{ lb}$

$M_{Ay} = 1\text{ lb ft}$ $M_{Az} = 1\text{ lb ft}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \frac{T_{BC}}{\sqrt{b^2 + c^2 + (e+f-a)^2}}\begin{pmatrix} e+f-a \\ -c \\ b \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -W \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ M_{Ay} \\ M_{Az} \end{pmatrix} + \begin{pmatrix} -e \\ c \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -W \end{pmatrix} + \begin{pmatrix} -a \\ 0 \\ b \end{pmatrix} \times \left[\frac{T_{BC}}{\sqrt{b^2 + c^2 + (e+f-a)^2}}\begin{pmatrix} e+f-a \\ -c \\ b \end{pmatrix}\right] = 0$$

$$\begin{pmatrix} T_{BC} \\ A_x \\ A_y \\ A_z \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \text{Find}(T_{BC}, A_x, A_y, A_z, M_{Ay}, M_{Az})$$

$T_{BC} = 1.05\text{ kip}$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} -450 \\ 900 \\ 0 \end{pmatrix}\text{ lb}$$

$$\begin{pmatrix} M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} -600 \\ -900 \end{pmatrix}\text{ lb}\cdot\text{ft}$$

Problem 5-84

Determine the *x*, *y*, *z* components of reaction at the pin *A* and the tension in the cable *BC* necessary for equilibrium of the rod.

Given:

$F = 350$ lb $e = 12$ ft

$a = 4$ ft $f = 4$ ft

$b = 5$ ft $g = 10$ ft

$c = 4$ ft $h = 4$ ft

$d = 2$ ft $i = 10$ ft

Solution:

Initial Guesses:

$F_{BC} = 1$ lb

$A_y = 1$ lb

$M_{Ay} = 1$ lb·ft

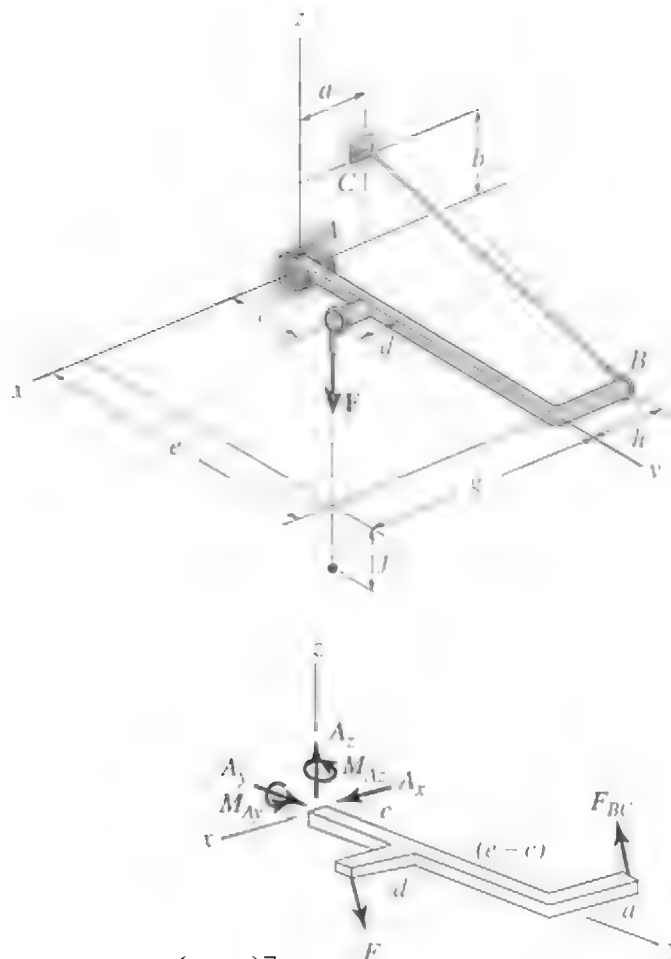
$A_x = 1$ lb

$A_z = 1$ lb

$M_{Az} = 1$ lb·ft

Given

$$\begin{pmatrix} 0 \\ M_{Ay} \\ M_{Az} \end{pmatrix} + \begin{pmatrix} d \\ c \\ 0 \end{pmatrix} \times \left[\frac{F}{\sqrt{(g-d)^2 + (e-c)^2 + f^2}} \begin{pmatrix} g-d \\ e-c \\ -f \end{pmatrix} \right] \ldots = 0$$
$$+ \begin{pmatrix} -a \\ 0 \\ b \end{pmatrix} \times \left[\frac{F_{BC}}{\sqrt{(a-h)^2 + e^2 + b^2}} \begin{pmatrix} a-h \\ -e \\ b \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \frac{F}{\sqrt{(g-d)^2 + (e-c)^2 + f^2}} \begin{pmatrix} g-d \\ e-c \\ -f \end{pmatrix} + \frac{F_{BC}}{\sqrt{(a-h)^2 + e^2 + b^2}} \begin{pmatrix} a-h \\ -e \\ b \end{pmatrix} = 0$$

$$\begin{pmatrix} F_{BC} \\ A_x \\ A_y \\ A_z \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \text{Find}(F_{BC}, A_x, A_y, A_z, M_{Ay}, M_{Az})$$

$F_{BC} = 101\ \text{lb}$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} -233.3 \\ -140 \\ 77.8 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} -388.9 \\ 93.3 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 5-85

Rod *AB* is supported by a ball-and-socket joint at *A* and a cable at *B*. Determine the *x*, *y*, *z* components of reaction at these supports if the rod is subjected to a vertical force **F** as shown.

Given:

$F = 50\ \text{lb}$

$a = 2\ \text{ft}$ $\quad c = 2\ \text{ft}$

$b = 4\ \text{ft}$ $\quad d = 2\ \text{ft}$

Solution:

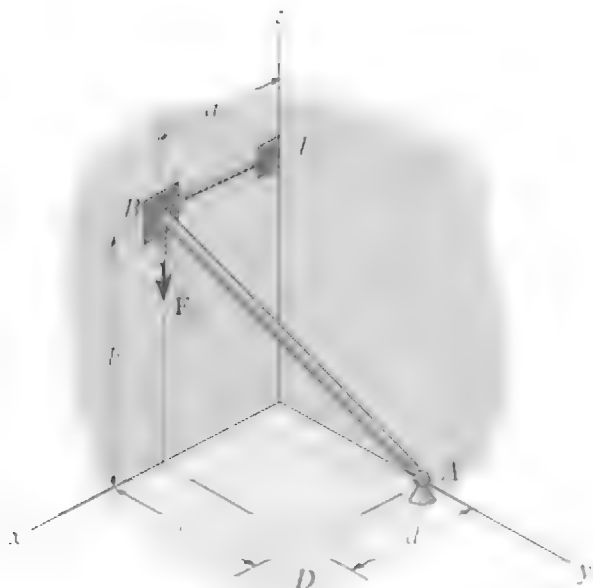
$T_B = 10\ \text{lb}$ $\quad A_x = 10\ \text{lb}$

$A_y = 10\ \text{lb}$ $\quad A_z = 10\ \text{lb}$

$B_y = 10\ \text{lb}$

Given

$\Sigma F_x = 0;$ $\quad -T_B + A_x = 0$

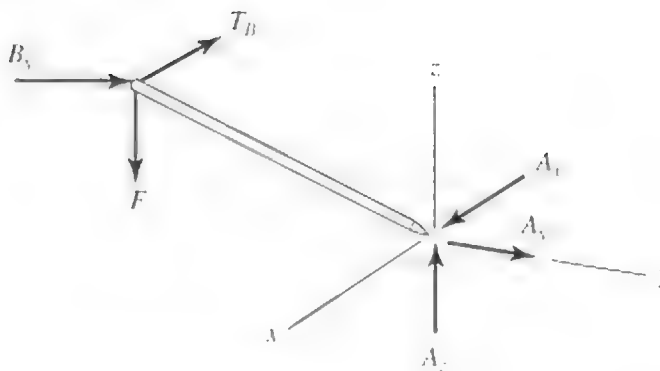
$\Sigma F_y = 0;$ $\quad A_y + B_y = 0$

$\Sigma F_z = 0; \quad -F + A_z = 0$

$\Sigma M_{Ax} = 0; \quad F(c) - B_y(b) = 0$

$\Sigma M_{Ay} = 0; \quad F(a) - T_B(b) = 0$

$\Sigma M_{Az} = 0; \quad B_y(a) - T_B(c) = 0$

Solving,

$$\begin{pmatrix} T_B \\ A_x \\ A_y \\ A_z \\ B_y \end{pmatrix} = \text{Find}(T_B, A_x, A_y, A_z, B_y) \qquad \begin{pmatrix} T_B \\ A_x \\ A_y \\ A_z \\ B_y \end{pmatrix} = \begin{pmatrix} 25 \\ 25 \\ -25 \\ 50 \\ 25 \end{pmatrix} \text{lb}$$

Problem 5-86

The member is supported by a square rod which fits loosely through a smooth square hole of the attached collar at *A* and by a roller at *B*. Determine the *x*, *y*, *z* components of reaction at these supports when the member is subjected to the loading shown.

Given:

$M = 50 \text{ lb}\cdot\text{ft}$

$$F = \begin{pmatrix} 20 \\ -40 \\ -30 \end{pmatrix} \text{lb}$$

$a = 2 \text{ ft}$

$b = 1 \text{ ft}$

$c = 2 \text{ ft}$

Solution:

Initial Guesses

$A_x = 1 \text{ lb} \qquad A_y = 1 \text{ lb}$

$M_{Ax} = 1\ \text{lb ft}$ $\quad M_{Ay} = 1\ \text{lb ft}$

$M_{Az} = 1\ \text{lb ft}$ $\quad B_z = 1\ \text{lb}$

Given

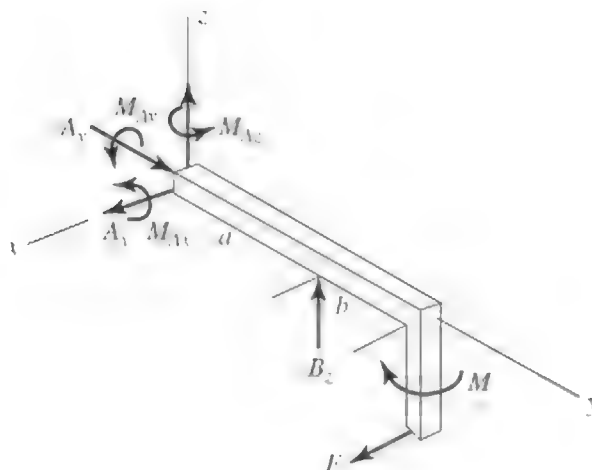
$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix} + F = 0$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix} + \left[\begin{pmatrix} 0 \\ a+b \\ -c \end{pmatrix} \times F + \begin{pmatrix} 0 \\ 0 \\ -M \end{pmatrix} \right] = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ M_{Ax} \\ M_{Ay} \\ M_{Az} \\ B_z \end{pmatrix} = \text{Find}(A_x, A_y, M_{Ax}, M_{Ay}, M_{Az}, B_z)$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} -20 \\ 40 \end{pmatrix} \text{lb}$$

$B_z = 30\,\text{lb}$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} 110 \\ 40 \\ 110 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 5-87

The platform has mass M and center of mass located at G. If it is lifted using the three cables, determine the force in each of these cables.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$ $\quad \text{kN} = 10^3\ \text{N}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Given:

$M = 3\ \text{Mg}$

$a = 4\ \text{m}$

$b = 3\ \text{m}$

$c = 3\ \text{m}$

$d = 4\ \text{m}$

$e = 2\ \text{m}$

Solution:

The initial guesses are:

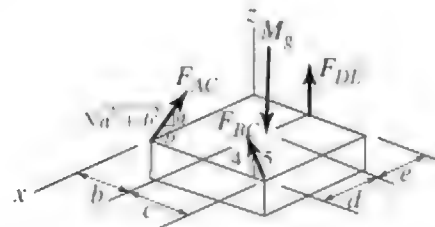
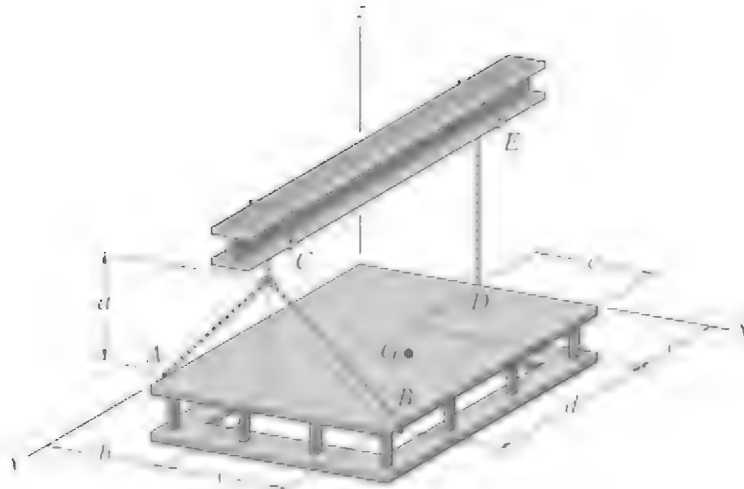
$F_{AC} = 10\ \text{N}$ $\quad F_{BC} = 10\ \text{N}$ $\quad F_{DE} = 10\ \text{N}$

Given

$$\frac{b\left(F_{AC}\right)}{\sqrt{a^2+b^2}} - \frac{c\left(F_{BC}\right)}{\sqrt{a^2+c^2}} = 0$$

$$M\ g\ e - \left(F_{AC}\right)a\frac{d+e}{\sqrt{a^2+b^2}} - F_{BC}\frac{a(d+e)}{\sqrt{a^2+c^2}} = 0$$

$$\frac{a}{\sqrt{a^2+c^2}}F_{BC}(b+c) - M\ g\ b + F_{DE}b = 0$$

$$\begin{pmatrix} F_{AC} \\ F_{BC} \\ F_{DE} \end{pmatrix} = \text{Find}\left(F_{AC}, F_{BC}, F_{DE}\right) \qquad \begin{pmatrix} F_{AC} \\ F_{BC} \\ F_{DE} \end{pmatrix} = \text{kN}$$

Problem 5-88

The platform has a mass of M and center of mass located at G. If it is lifted using the three cables, determine the force in each of the cables. Solve for each force by using a single moment equation of equilibrium.

Units Used:

$$\text{Mg} = 1000\ \text{kg}$$

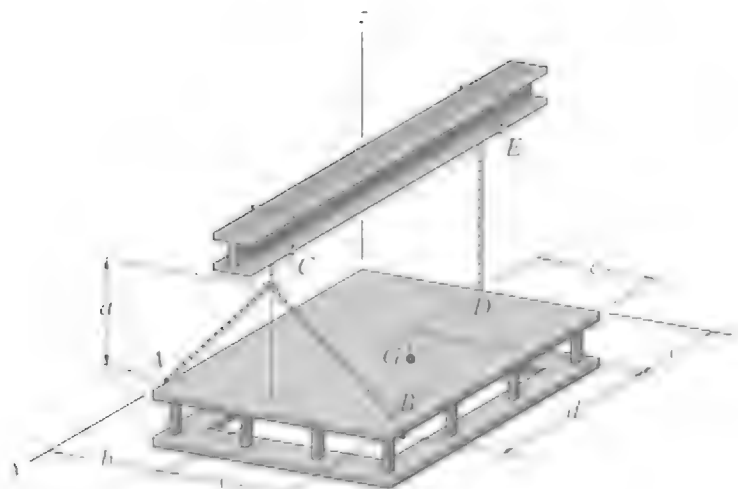
$$\text{kN} = 10^3\ \text{N}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

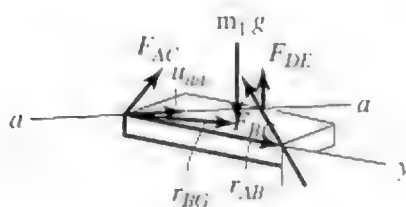
Given:

$M = 2\ \text{Mg}$ $\quad c = 3\ \text{m}$

$a = 4\ \text{m}$ $\quad d = 4\ \text{m}$

$b = 3\ \text{m}$ $\quad e = 2\ \text{m}$

Solution:

$$r_{BC} = \begin{pmatrix} 0 \\ -c \\ a \end{pmatrix} \qquad r_{AC} = \begin{pmatrix} 0 \\ b \\ a \end{pmatrix}$$

$$r_{AD} = \begin{pmatrix} -e-d \\ b \\ 0 \end{pmatrix} \qquad r_{BD} = \begin{pmatrix} -d-e \\ -c \\ 0 \end{pmatrix}$$

First find F_{DE}.

$$\Sigma M_y' = 0; \qquad F_{DE}(d+e) - M\ g\ d = 0 \qquad F_{DE} = \frac{M\ g\ d}{d+e} \qquad F_{DE} = 4.1\ \text{s}^2\ \text{kN}$$

Next find F_{BC}. Guess $F_{BC} = 1\ \text{kN}$

$$\text{Given} \quad \left[\begin{pmatrix} e \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -M\ g \end{pmatrix} + \begin{pmatrix} e+d \\ c \\ 0 \end{pmatrix} \times \left(F_{BC}\frac{r_{BC}}{|r_{BC}|}\right)\right] r_{AD} = 0 \qquad F_{BC} = \text{Find}(F_{BC})$$

$$F_{BC} = \ \text{kN}$$

Now find F_{AC}. Guess $F_{AC} = 1\ \text{kN}$

Given $\left[\begin{pmatrix} e \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -M\,g \end{pmatrix} + \begin{pmatrix} e+d \\ -b \\ 0 \end{pmatrix} \times \left(F_{AC}\frac{r_{AC}}{|r_{AC}|}\right)\right] r_{BD} = 0$ $\qquad F_{AC} = \text{Find}(F_{AC})$

$F_{AC} =$ kN

Problem 5-89

The cables exert the forces shown on the pole. Assuming the pole is supported by a ball-and-socket joint at its base, determine the components of reaction at A. The forces $\mathbf{F_1}$ and $\mathbf{F_2}$ lie in a horizontal plane.

Given:

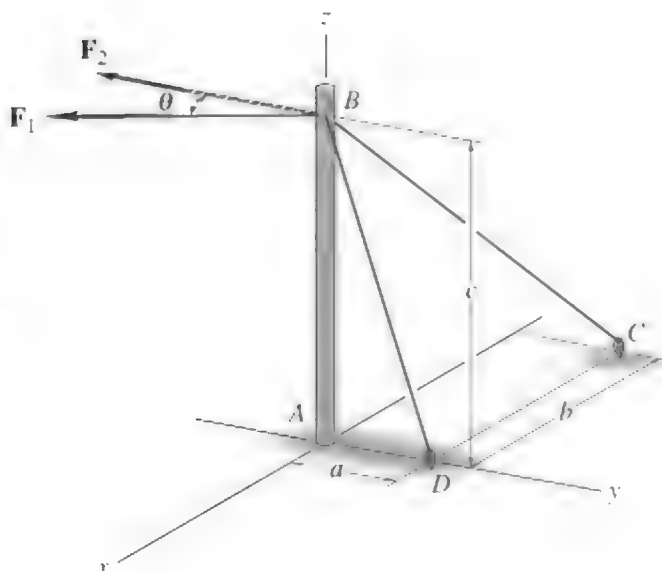
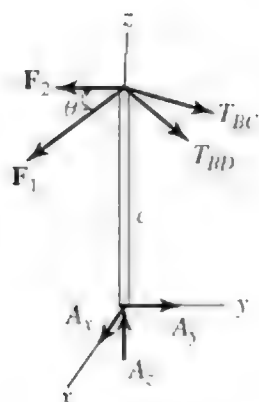
$F_1 = 140$ lb

$F_2 = 75$ lb

$\theta = 30$ deg

$a = 5$ ft

$b = 10$ ft

$c = 15$ ft

Solution:

The initial guesses are

$T_{BC} = 100$ lb $\quad T_{BD} = 100$ lb $\quad A_x = 100$ lb $\quad A_y = 100$ lb $\quad A_z = 100$ lb

Given

$$\left(F_1\cos(\theta) + F_2\right)c - T_{BC}\,a\left(\frac{c}{\sqrt{a^2+b^2+c^2}}\right) - T_{BD}\,a\left(\frac{c}{\sqrt{a^2+c^2}}\right) = 0$$

$$F_1\sin(\theta)c - b\,T_{BC}\left(\frac{c}{\sqrt{a^2+b^2+c^2}}\right) = 0$$

$$A_x + F_1 \sin(\theta) - b\left(\frac{T_{BC}}{\sqrt{a^2+b^2+c^2}}\right) = 0$$

$$A_y - F_1 \cos(\theta) - F_2 + T_{BD}\left(\frac{a}{\sqrt{a^2+c^2}}\right) + a\left(\frac{T_{BC}}{\sqrt{a^2+b^2+c^2}}\right) = 0$$

$$A_z - c\left(\frac{T_{BD}}{\sqrt{a^2+c^2}}\right) - c\left(\frac{T_{BC}}{\sqrt{a^2+b^2+c^2}}\right) = 0$$

$$\begin{pmatrix} T_{BC} \\ T_{BD} \\ A_x \\ A_y \\ A_z \end{pmatrix} = \text{Find}(T_{BC}, T_{BD}, A_x, A_y, A_z) \qquad \begin{pmatrix} T_{BC} \\ T_{BD} \end{pmatrix} = \begin{pmatrix} 131.0 \\ 509.9 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} -0.0 \\ 0.0 \\ 588.7 \end{pmatrix} \text{lb}$$

Problem 5-90

The silo has a weight *W*, a center of gravity at *G* and a radius *r*. Determine the vertical component of force that each of the three struts at *A*, *B*, and *C* exerts on the silo if it is subjected to a resultant wind loading of **F** which acts in the direction shown.

Given:

$W = 3500$ lb

$F = 250$ lb

$\theta_1 = 30$ deg

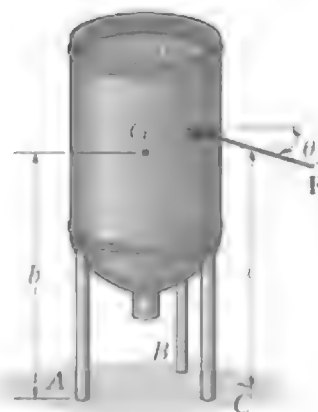
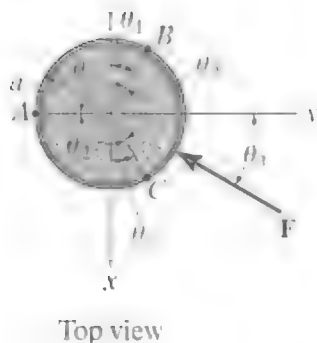
$\theta_2 = 120$ deg

$\theta_3 = 30$ deg

$r = 5$ ft

$b = 12$ ft

$c = 15$ ft

Top view

Solution:

Initial Guesses: $A_z = 1\ \text{lb}$ $B_z = 2\ \text{lb}$ $C_z = 31\ \text{lb}$

Given

$\Sigma M_y = 0;\ B_z r\cos(\theta_1) - C_z r\cos(\theta_1) - F\sin(\theta_3)c = 0$ [1]

$\Sigma M_x = 0;\ -B_z r\sin(\theta_1) - C_z r\sin(\theta_1) + A_z r - F\cos(\theta_3)c = 0$ [2]

$\Sigma F_z = 0;\ A_z + B_z + C_z = W$ [3]

Solving Eqs.[1], [2] and [3] yields:

$$\begin{pmatrix} A_z \\ B_z \\ C_z \end{pmatrix} = \text{Find}(A_z, B_z, C_z) \qquad \begin{pmatrix} A_z \\ B_z \\ C_z \end{pmatrix} = \begin{pmatrix} 1600 \\ 1167 \\ 734 \end{pmatrix} \text{lb}$$

Problem 5-91

The shaft assembly is supported by two smooth journal bearings A and B and a short link DC. If a couple moment is applied to the shaft as shown, determine the components of force reaction at the bearings and the force in the link. The link lies in a plane parallel to the y-z plan and the bearings are properly aligned on the shaft.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

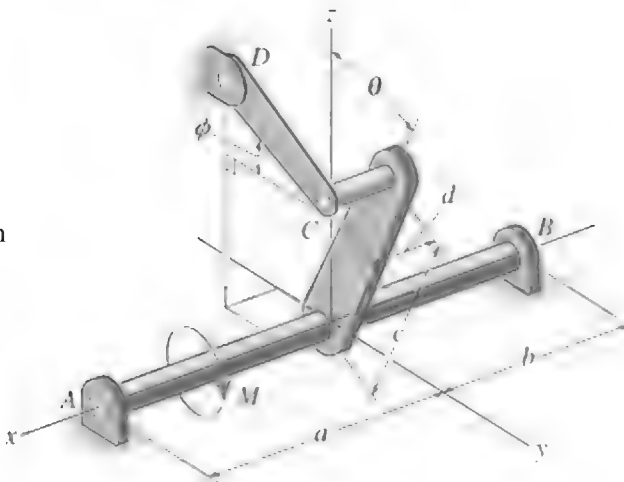
$M = 250\ \text{N}\cdot\text{m}$

$a = 400\ \text{mm}$

$b = 300\ \text{mm}$

$c = 250\ \text{mm}$

$d = 120\ \text{mm}$

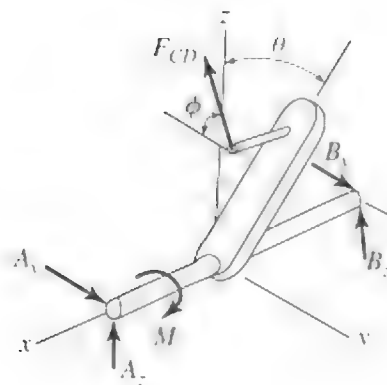
$\theta = 30\ \text{deg}$

$\phi = 20$ deg

Solution:

Initial Guesses:

$A_y = 1$ kN $\quad A_z = 1$ kN $\quad B_y = 1$ kN

$B_z = 1$ kN $\quad F_{CD} = 1$ kN

Given

$$\begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ -F_{CD}\cos(\phi) \\ F_{CD}\sin(\phi) \end{pmatrix} = 0$$

$$\begin{pmatrix} d-a \\ c\sin(\theta) \\ c\cos(\theta) \end{pmatrix} \times \begin{pmatrix} 0 \\ -F_{CD}\cos(\phi) \\ F_{CD}\sin(\phi) \end{pmatrix} + \begin{pmatrix} -a-b \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} -M \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} A_y \\ A_z \\ B_y \\ B_z \\ F_{CD} \end{pmatrix} = \text{Find}(A_y, A_z, B_y, B_z, F_{CD})$$

$$\begin{pmatrix} A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 573 \\ -208 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 382 \\ -139 \end{pmatrix} \text{N}$$

$$F_{CD} = 1.015 \text{ kN}$$

Problem 5-92

If neither the pin at A nor the roller at B can support a load no greater than F_{max}, determine the maximum intensity of the distributed load w, so that failure of a support does not occur.

Units Used:

$kN = 10^3 N$

Given:

$F_{max} = 6\ kN$

$a = 3\ m$

$b = 3\ m$

Solution:

The greatest reaction is at A. Require

$\Sigma M_B = 0;\quad -F_{max}(a+b) + w\,a\left(\frac{a}{2}+b\right) + \frac{1}{2}w\,b\frac{2}{3}b = 0$

$$w = \frac{F_{max}(a+b)}{a\left(\frac{a}{2}+b\right)+\frac{b^2}{3}} \qquad w = 2.18\ \frac{kN}{m}$$

Problem 5-93

If the maximum intensity of the distributed load acting on the beam is w, determine the reactions at the pin A and roller B.

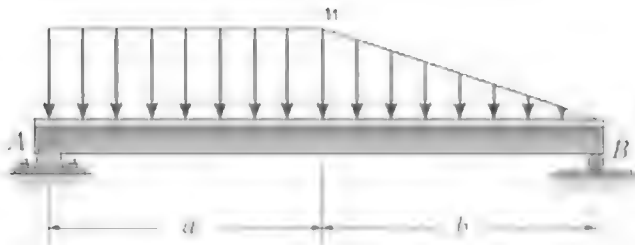
Units Used:

$kN = 10^3\ N$

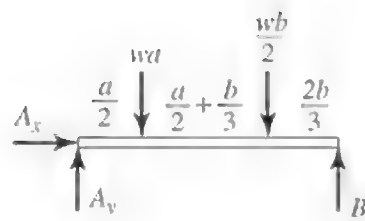
Given:

$F = 6\ kN$

$a = 3\ m$

$b = 3\ m$

$w = 4\ \frac{\text{kN}}{\text{m}}$

Solution:

$\Sigma F_x = 0;\qquad A_x = 0$

$\Sigma M_A = 0;\qquad -w\,a\frac{a}{2} - \frac{1}{2}\,w\,b\left(a + \frac{b}{3}\right) + B_y(a+b) = 0$

$$B_y = \frac{1}{6}\,w\,\frac{3a^2 + 3\,a\,b + b^2}{a+b} \qquad B_y = 7\,\text{kN}$$

$\Sigma F_y = 0;\qquad A_y + B_y - w\,a - \frac{1}{2}\,w\,b = 0$

$$A_y = -B_y + w\,a + \frac{1}{2}\,w\,b \qquad A_y = 11\,\text{kN}$$

Problem 5-94

Determine the normal reaction at the roller A and horizontal and vertical components at pin B for equilibrium of the member.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F_1 = 10\ \text{kN}$

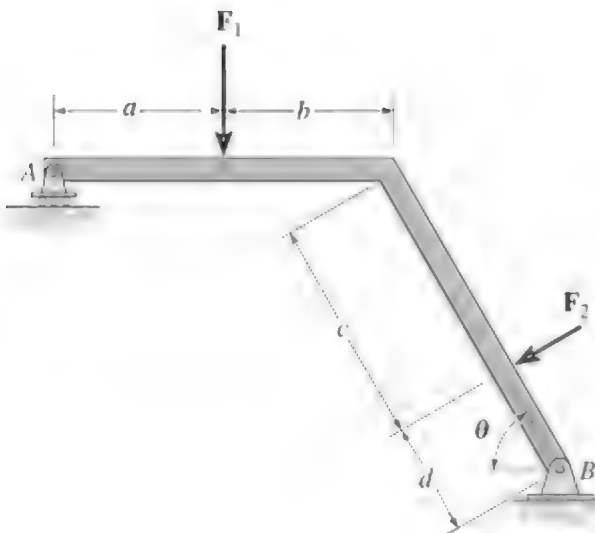
$F_2 = 6\ \text{kN}$

$a = 0.6\ \text{m}$

$b = 0.6\ \text{m}$

$c = 0.8\ \text{m}$

$d = 0.4\ \text{m}$

$\theta = 60\ \text{deg}$

Solution:

Initial Guesses:

$N_A = 1 \text{ kN}$

$B_x = 1 \text{ kN}$

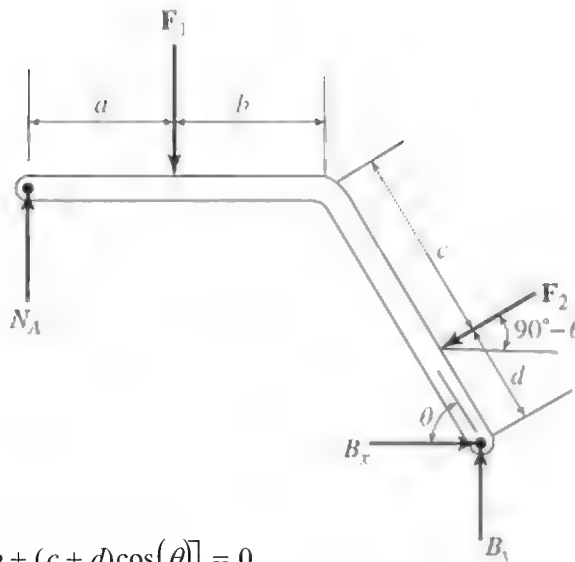
$B_y = 1 \text{ kN}$

Given

$B_x - F_2 \sin(\theta) = 0$

$B_y + N_A - F_1 - F_2 \cos(\theta) = 0$

$F_2 d + F_1[b + (c + d)\cos(\theta)] - N_A[a + b + (c + d)\cos(\theta)] = 0$

$$\begin{pmatrix} N_A \\ B_x \\ B_y \end{pmatrix} = \text{Find}(N_A, B_x, B_y) \qquad \begin{pmatrix} N_A \\ B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 8 \\ 5.196 \\ 5 \end{pmatrix} \text{kN}$$

Problem 5-95

The symmetrical shelf is subjected to uniform pressure P. Support is provided by a bolt (or pin) located at each end A and A' and by the symmetrical brace arms, which bear against the smooth wall on both sides at B and B'. Determine the force resisted by each bolt at the wall and the normal force at B for equilibrium.

Units Used:

$\text{kPa} = 10^3 \text{ Pa}$

Given:

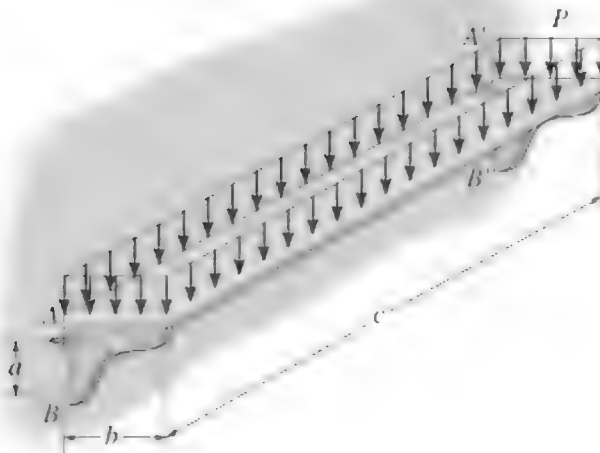
$P = 4 \text{ kPa}$

$a = 0.15 \text{ m}$

$b = 0.2 \text{ m}$

$c = 1.5 \text{ m}$

Solution:

$\Sigma M_A = 0;$

$$N_B a - P\left(b\frac{c}{2}\right)\frac{b}{2} = 0$$

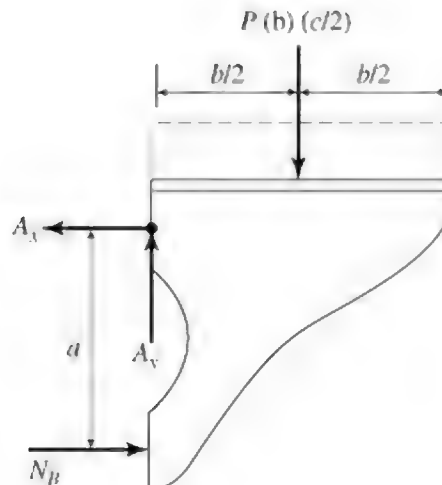
$$N_B = P\frac{b^2 c}{4\,a} \qquad N_B = 400\text{ N}$$

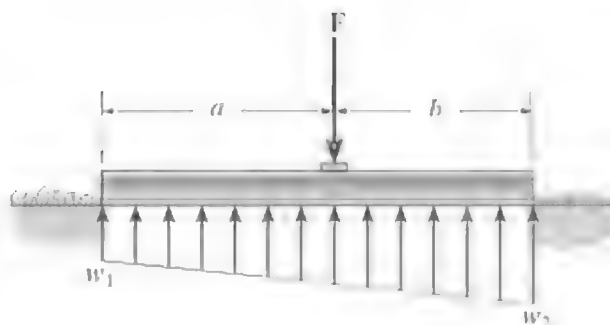
$\Sigma F_x = 0;$

$$A_x = N_B \qquad A_x = 400\text{ N}$$

$\Sigma F_y = 0;$

$$A_y = P\,b\frac{c}{2} \qquad A_y = 600\text{ N}$$

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} \qquad F_A = 721\text{ N}$$

Problem 5-96

A uniform beam having a weight W supports a vertical load **F**. If the ground pressure varies linearly as shown, determine the load intensities w_1 and w_2 measured in lb/ft, necessary for equilibrium.

Given:

$W = 200$ lb

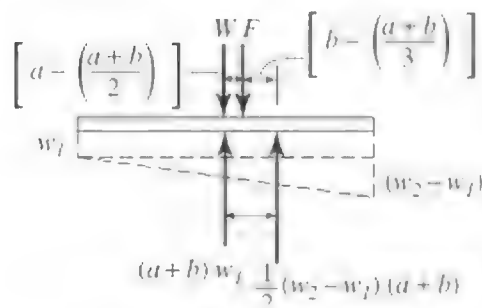
$F = 800$ lb

$a = 7$ ft

$b = 6$ ft

Solution:

Initial Guesses:

$$w_1 = 1\,\frac{\text{lb}}{\text{ft}} \qquad w_2 = 1\,\frac{\text{lb}}{\text{ft}}$$

Given

$$w_1(a+b) + \frac{1}{2}(w_2 - w_1)(a+b) - F - W = 0$$

$$w_1(a+b)\frac{a+b}{2} + \frac{1}{2}(w_2 - w_1)(a+b)\frac{2}{3}(a+b) - W\frac{a+b}{2} - F\,a = 0$$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \text{Find}(w_1, w_2) \qquad \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 62.7 \\ 91.1 \end{pmatrix}\frac{\text{lb}}{\text{ft}}$$

Problem 5-97

The uniform ladder rests along the wall of a building at A and on the roof at B. If the ladder has a weight W and the surfaces at A and B are assumed smooth, determine the angle θ for equilibrium.

Given:

$a = 18$ ft

$W = 25$ lb

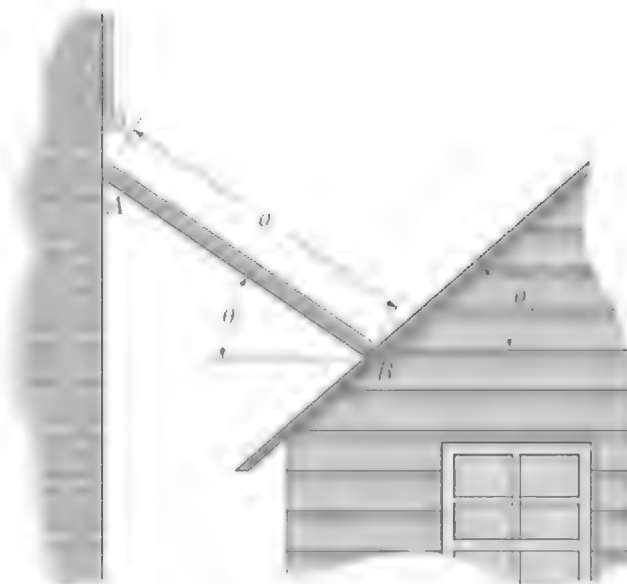
$\theta_1 = 40$ deg

Solution:

Initial guesses:

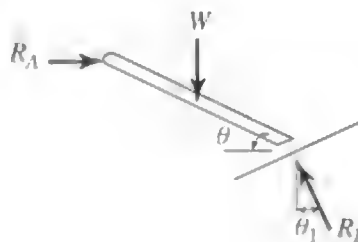
$R_A = 10$ lb

$R_B = 10$ lb

$\theta = 10$ deg

Given

$\Sigma M_B = 0;\quad -R_A\, a \sin(\theta) + W\frac{a}{2}\cos(\theta) = 0$

$\Sigma F_x = 0;\quad R_A - R_B \sin(\theta_1) = 0$

$\Sigma F_y = 0;\quad R_B \cos(\theta_1) - W = 0$

Solving,

$$\begin{pmatrix} R_B \\ R_A \\ \theta \end{pmatrix} = \mathrm{Find}(R_B, R_A, \theta) \qquad \begin{pmatrix} R_A \\ R_B \end{pmatrix} = \begin{pmatrix} 21 \\ 32.6 \end{pmatrix} \mathrm{lb} \qquad \theta = 30.8\,\mathrm{deg}$$

Problem 5-98

Determine the x, y, z components of reaction at the ball supports B and C and the ball-and-socket A (not shown) for the uniformly loaded plate.

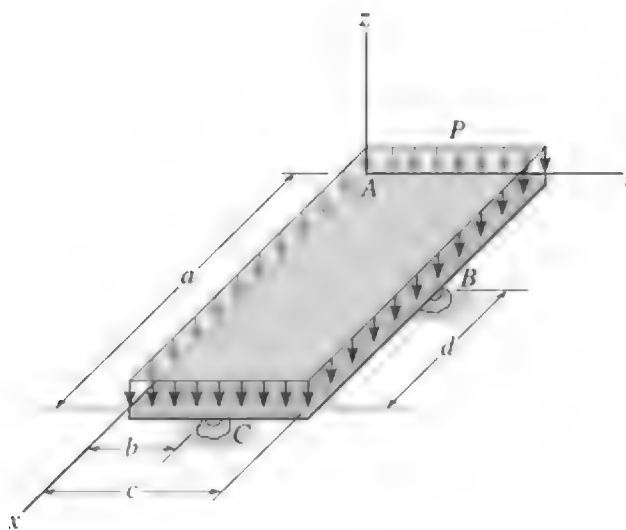
Given:

$P = 2\ \frac{\mathrm{lb}}{\mathrm{ft}^2}$

$a = 4$ ft

$b = 1$ ft

$c = 2$ ft

$d = 2$ ft

Solution:

The initial guesses are $A_x = 1$ lb $\quad A_y = 1$ lb $\quad A_z = 1$ lb $\quad B_z = 1$ lb $\quad C_z = 1$ lb

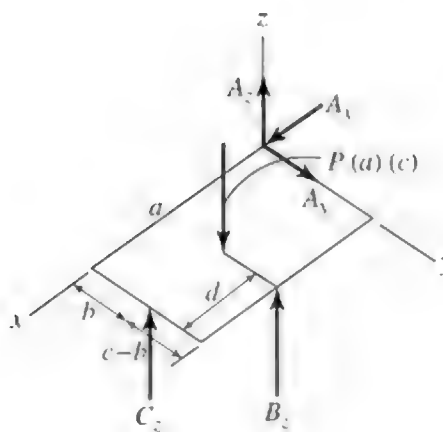
Given

$\Sigma F_x = 0;\quad A_x = 0$

$\Sigma F_y = 0;\quad A_y = 0$

$\Sigma F_z = 0;\quad A_z + B_z + C_z - P\,a\,c = 0$

$\Sigma M_x = 0;\quad c\,B_z - P\,a\,c\left(\frac{c}{2}\right) + C_z\,b = 0$

$\Sigma M_y = 0;\quad -B_z\,(a-d) + P\,a\,c\left(\frac{a}{2}\right) - C_z\,a = 0$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ B_z \\ C_z \end{pmatrix} = \text{Find}(A_x, A_y, A_z, B_z, C_z) \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5.333 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} B_z \\ C_z \end{pmatrix} = \begin{pmatrix} 5.333 \\ 5.333 \end{pmatrix} \text{lb}$$

Problem 5-99

A vertical force **F** acts on the crankshaft. Determine the horizontal equilibrium force **P** that must be applied to the handle and the x, y, z components of force at the smooth journal bearing A and the thrust bearing B. The bearings are properly aligned and exert the force reactions on the shaft.

Given:

$F = 80$ lb

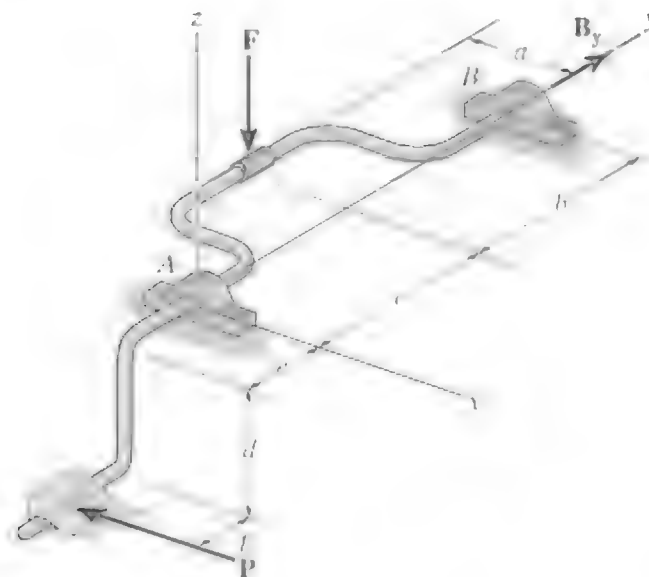
$a = 10$ in

$b = 14$ in

$c = 14$ in

$d = 8$ in

$e = 6$ in

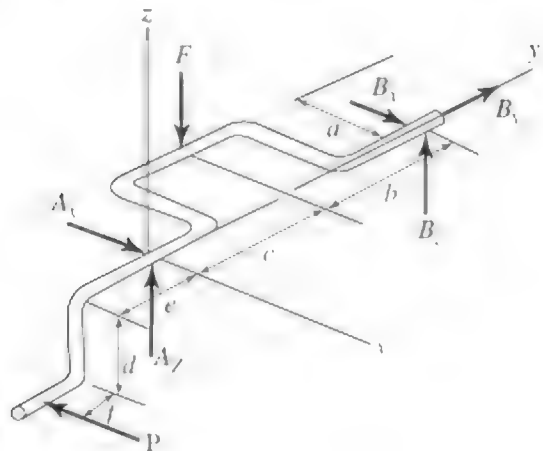
$f = 4$ in

Solution:

$\Sigma M_y = 0;$ $\quad P\,d - F\,a = 0$

$$P = F\left(\frac{a}{d}\right) \qquad P = 100\,\text{lb}$$

$\Sigma M_x = 0;$ $\quad B_z(b + c) - F\,c = 0$

$$B_z = F\left(\frac{c}{b+c}\right) \qquad B_z = 40\,\text{lb}$$

$\Sigma M_z = 0;$ $\quad -B_x(b + c) - P(e + f) = 0$

$$B_x = -P\left(\frac{e+f}{b+c}\right) \qquad B_x = -35.7\,\text{lb}$$

$\Sigma F_x = 0;\quad A_x + B_x - P = 0$

$$A_x = -B_x + P \qquad A_x = 135.7\,\text{lb}$$

$\Sigma F_y = 0;\qquad B_y = 0$

$\Sigma F_z = 0;\quad A_z + B_z - F = 0$

$$A_z = -B_z + F \qquad A_z = 40\,\text{lb}$$

Problem 5-100

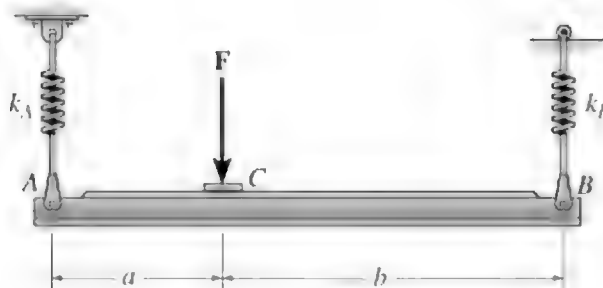
The horizontal beam is supported by springs at its ends. If the stiffness of the spring at A is k_A, determine the required stiffness of the spring at B so that if the beam is loaded with the force **F**, it remains in the horizontal position both before and after loading.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$k_A = 5\,\frac{\text{kN}}{\text{m}} \qquad a = 1\ \text{m}$$

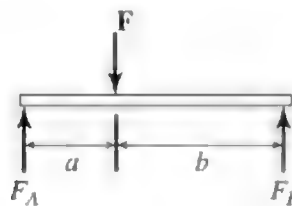
$$F = 800\ \text{N} \qquad b = 2\ \text{m}$$

Solution:

Equilibrium:

$\Sigma M_A = 0;\qquad F_B(a+b) - F\,a = 0$

$$F_B = F\left(\frac{a}{a+b}\right)$$

$$F_B = 266.667\ \text{N}$$

$\Sigma M_B = 0;\qquad F\,b - F_A(a+b) = 0$

$$F_A = F\left(\frac{b}{a+b}\right)$$

$$F_A = 533.333\ \text{N}$$

Spring force formula:

$$x_A = x_B \qquad \frac{F_A}{k_A} = \frac{F_B}{k_B} \qquad k_B = \frac{F_B}{F_A}k_A \qquad k_B = 2.5\,\frac{\text{kN}}{\text{m}}$$

Problem 6-1

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$$kN = 10^3 \ N$$

Given:

$$P_1 = 7 \ kN$$

$$P_2 = 7 \ kN$$

Solution:

$$\theta = 45 \ deg$$

Initial Guesses:

$F_{AB} = 1 \ kN$ $F_{AD} = 1 \ kN$ $F_{DB} = 1 \ kN$

$F_{DC} = 1 \ kN$ $F_{CB} = 1 \ kN$

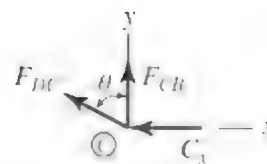
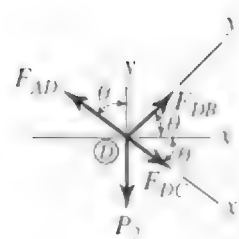
Given

Joint *A*: $$F_{AB} + F_{AD}\cos(\theta) = 0$$

$$-P_1 - F_{AD}\sin(\theta) = 0$$

Joint *D*: $$F_{DB}\cos(\theta) - F_{AD}\cos(\theta) + F_{DC}\cos(\theta) = 0$$

$$(F_{AD} + F_{DB} - F_{DC})\sin(\theta) - P_2 = 0$$

Joint *C*: $$F_{CB} + F_{DC}\sin(\theta) = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{DB} \\ F_{DC} \\ F_{CB} \end{pmatrix} = \text{Find}(F_{AB}, F_{AD}, F_{DB}, F_{DC}, F_{CB}) \qquad \begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{DB} \\ F_{DC} \\ F_{CB} \end{pmatrix} = \begin{pmatrix} 7 \\ -9.9 \\ 4.95 \\ -14.85 \\ 10.5 \end{pmatrix} kN$$

Positive means Tension, Negative means Compression

Problem 6-2

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$kN = 10^3\ N$

Given:

$P_1 = 8\ kN$

$P_2 = 10\ kN$

Solution:

$\theta = 45\ deg$

Initial Guesses:

$F_{AB} = 1\ kN$ $F_{AD} = 1\ kN$ $F_{DB} = 1\ kN$

$F_{DC} = 1\ kN$ $F_{CB} = 1\ kN$

Given

Joint A: $F_{AB} + F_{AD}\cos(\theta) = 0$

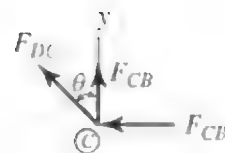
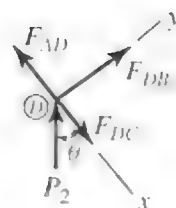
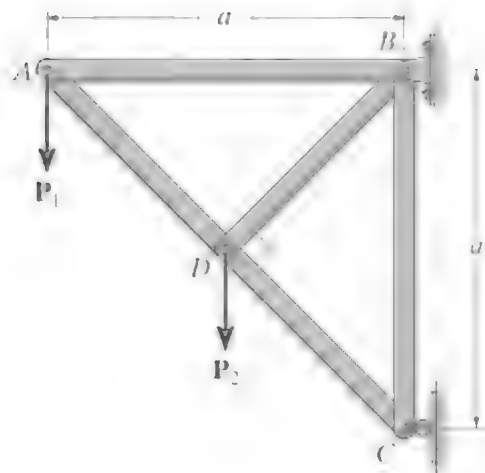
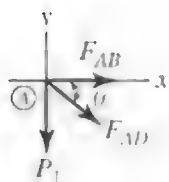
$-P_1 - F_{AD}\sin(\theta) = 0$

Joint D: $F_{DB}\cos(\theta) - F_{AD}\cos(\theta) + F_{DC}\cos(\theta) = 0$

$(F_{AD} + F_{DB} - F_{DC})\sin(\theta) - P_2 = 0$

Joint C: $F_{CB} + F_{DC}\sin(\theta) = 0$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{DB} \\ F_{DC} \\ F_{CB} \end{pmatrix} = \text{Find}(F_{AB}, F_{AD}, F_{DB}, F_{DC}, F_{CB}) \quad \begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{DB} \\ F_{DC} \\ F_{CB} \end{pmatrix} = \begin{pmatrix} 8 \\ -11.31 \\ 7.07 \\ -18.38 \\ 13 \end{pmatrix} kN$$

Positive means Tension, Negative means Compression

Problem 6-3

The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

$$P_1 = 600 \text{ lb}$$

$$P_2 = 400 \text{ lb}$$

$$a = 4 \text{ ft}$$

$$\theta = 45 \text{ deg}$$

Solution:

Initial Guesses

$F_{AB} = 1 \text{ lb}$ $\quad F_{AD} = 1 \text{ lb}$ $\quad F_{DC} = 1 \text{ lb}$

$F_{BC} = 1 \text{ lb}$ $\quad F_{BD} = 1 \text{ lb}$ $\quad F_{DE} = 1 \text{ lb}$

Given

Joint *A*: $\quad F_{AB} + F_{AD}\cos(\theta) = 0$

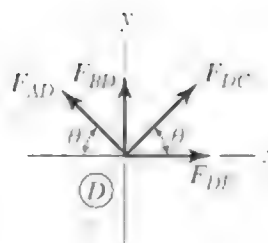
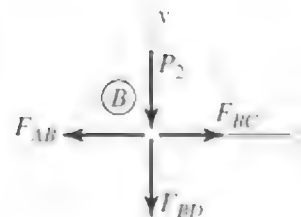
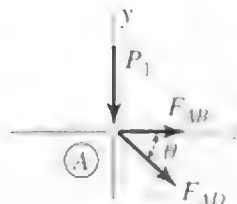
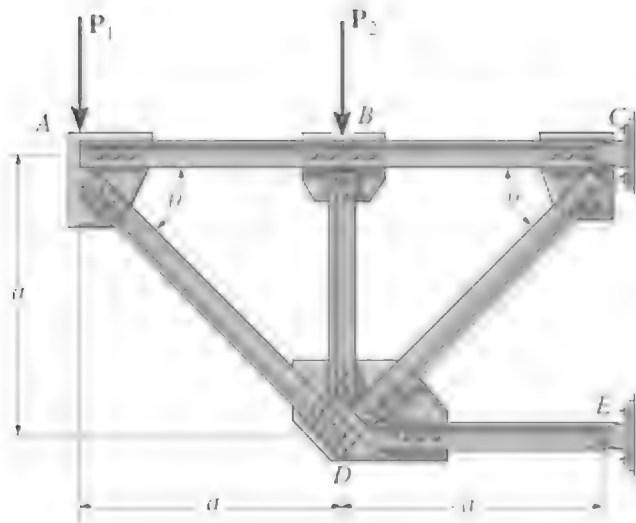
$$-P_1 - F_{AD}\sin(\theta) = 0$$

Joint *B*: $\quad F_{BC} - F_{AB} = 0$

$$-P_2 - F_{BD} = 0$$

Joint *D*: $\quad (F_{DC} - F_{AD})\cos(\theta) + F_{DE} = 0$

$$(F_{DC} + F_{AD})\sin(\theta) + F_{BD} = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{DC} \\ F_{DE} \end{pmatrix} = \text{Find}(F_{AB}, F_{AD}, F_{BC}, F_{BD}, F_{DC}, F_{DE})$$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{DC} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 600 \\ -849 \\ 600 \\ -400 \\ 1414 \\ -1600 \end{pmatrix} \text{lb}$$

Positive means Tension,
Negative means Compression

Problem 6-4

The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$P_1 = 800 \text{ lb}$

$P_2 = 0 \text{ lb}$

$a = 4 \text{ ft}$

$\theta = 45 \text{ deg}$

Solution:

Initial Guesses

$F_{AB} = 1 \text{ lb}$ $F_{AD} = 1 \text{ lb}$ $F_{DC} = 1 \text{ lb}$

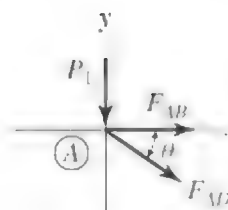
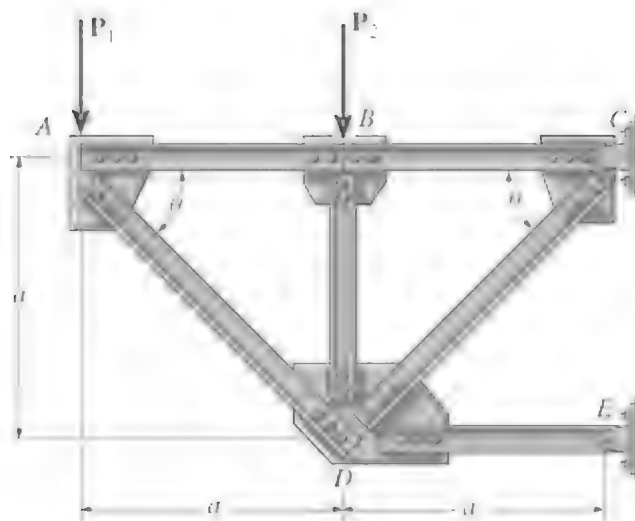
$F_{BC} = 1 \text{ lb}$ $F_{BD} = 1 \text{ lb}$ $F_{DE} = 1 \text{ lb}$

Given

Joint A: $F_{AB} + F_{AD}\cos(\theta) = 0$

$-P_1 - F_{AD}\sin(\theta) = 0$

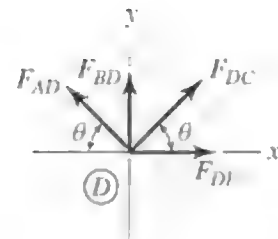
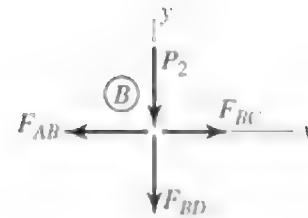
Joint B: $F_{BC} - F_{AB} = 0$

$-P_2 - F_{BD} = 0$

Joint D: $(F_{DC} - F_{AD})\cos(\theta) + F_{DE} = 0$

$$(F_{DC} + F_{AD})\sin(\theta) + F_{BD} = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{DC} \\ F_{DE} \end{pmatrix} = \text{Find}(F_{AB}, F_{AD}, F_{BC}, F_{BD}, F_{DC}, F_{DE})$$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{DC} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 800 \\ -1131 \\ 800 \\ 0 \\ 1131 \\ -1600 \end{pmatrix} \text{lb}$$

Positive means Tension,
Negative means Compression

Problem 6-5

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$P_1 = 20 \text{ kN}$$

$$P_2 = 10 \text{ kN}$$

$$a = 1.5 \text{ m}$$

$$e = 2 \text{ m}$$

Solution: $\theta = \text{atan}\left(\frac{e}{a}\right)$

Initial Guesses:

$F_{AB} = 1$ kN $F_{AG} = 1$ kN $F_{CF} = 1$ kN

$F_{BC} = 1$ kN $F_{BG} = 1$ kN $F_{DE} = 1$ kN

$F_{CG} = 1$ kN $F_{FG} = 1$ kN $F_{EF} = 1$ kN

$F_{CD} = 1$ kN $F_{DF} = 1$ kN

Given

Joint *B* $F_{BC} - F_{AB}\cos(\theta) = 0$

$-F_{BG} - F_{AB}\sin(\theta) = 0$

Joint *G* $F_{FG} + F_{CG}\cos(\theta) - F_{AG} = 0$

$F_{CG}\sin(\theta) + F_{BG} - P_1 = 0$

Joint *C* $-F_{BC} + F_{CD} + (F_{CF} - F_{CG})\cos(\theta) = 0$

$-(F_{CG} + F_{CF})\sin(\theta) = 0$

Joint *D* $-F_{CD} + F_{DE}\cos(\theta) = 0$

$-F_{DF} - F_{DE}\sin(\theta) = 0$

Joint *F* $F_{EF} - F_{FG} - F_{CF}\cos(\theta) = 0$

$F_{DF} + F_{CF}\sin(\theta) - P_2 = 0$

Joint *E* $-F_{DE}\cos(\theta) - F_{EF} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CG} \\ F_{CD} \\ F_{AG} \\ F_{BG} \\ F_{FG} \\ F_{DF} \\ F_{CF} \\ F_{DE} \\ F_{EF} \end{pmatrix} = \text{Find}(F_{AB}, F_{BC}, F_{CG}, F_{CD}, F_{AG}, F_{BG}, F_{FG}, F_{DF}, F_{CF}, F_{DE}, F_{EF})$$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CG} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} -21.88 \\ -13.13 \\ 3.13 \\ -9.37 \end{pmatrix} \text{kN} \quad \begin{pmatrix} F_{AG} \\ F_{BG} \\ F_{FG} \\ F_{DF} \end{pmatrix} = \begin{pmatrix} 13.13 \\ 17.5 \\ 11.25 \\ 12.5 \end{pmatrix} \text{kN} \quad \begin{pmatrix} F_{CF} \\ F_{DE} \\ F_{EF} \end{pmatrix} = \begin{pmatrix} -3.13 \\ -15.62 \\ 9.37 \end{pmatrix} \text{kN}$$

Positive means Tension, Negative means Compression

Problem 6-6

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$P_1 = 40 \text{ kN}$

$P_2 = 20 \text{ kN}$

$a = 1.5 \text{ m}$

$e = 2 \text{ m}$

Solution: $\theta = \operatorname{atan}\left(\frac{e}{a}\right)$

Initial Guesses:

$F_{AB} = 1\text{ kN}$ $F_{AG} = 1\text{ kN}$ $F_{CF} = 1\text{ kN}$

$F_{BC} = 1\text{ kN}$ $F_{BG} = 1\text{ kN}$ $F_{DE} = 1\text{ kN}$

$F_{CG} = 1\text{ kN}$ $F_{FG} = 1\text{ kN}$ $F_{EF} = 1\text{ kN}$

$F_{CD} = 1\text{ kN}$ $F_{DF} = 1\text{ kN}$

Given

Joint B $F_{BC} - F_{AB}\cos(\theta) = 0$

$-F_{BG} - F_{AB}\sin(\theta) = 0$

Joint G $F_{FG} + F_{CG}\cos(\theta) - F_{AG} = 0$

$F_{CG}\sin(\theta) + F_{BG} - P_1 = 0$

Joint C $-F_{BC} + F_{CD} + (F_{CF} - F_{CG})\cos(\theta) = 0$

$-(F_{CG} + F_{CF})\sin(\theta) = 0$

Joint D $-F_{CD} + F_{DE}\cos(\theta) = 0$

$-F_{DF} - F_{DE}\sin(\theta) = 0$

Joint F $F_{EF} - F_{FG} - F_{CF}\cos(\theta) = 0$

$F_{DF} + F_{CF}\sin(\theta) - P_2 = 0$

Joint E $-F_{DE}\cos(\theta) - F_{EF} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CG} \\ F_{CD} \\ F_{AG} \\ F_{BG} \\ F_{FG} \\ F_{DF} \\ F_{CF} \\ F_{DE} \\ F_{EF} \end{pmatrix} = \text{Find}\left(F_{AB}, F_{BC}, F_{CG}, F_{CD}, F_{AG}, F_{BG}, F_{FG}, F_{DF}, F_{CF}, F_{DE}, F_{EF}\right)$$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CG} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} -43.75 \\ -26.25 \\ 6.25 \\ -18.75 \end{pmatrix} \text{kN} \quad \begin{pmatrix} F_{AG} \\ F_{BG} \\ F_{FG} \\ F_{DF} \end{pmatrix} = \begin{pmatrix} 26.25 \\ 35 \\ 22.5 \\ 25 \end{pmatrix} \text{kN} \quad \begin{pmatrix} F_{CF} \\ F_{DE} \\ F_{EF} \end{pmatrix} = \begin{pmatrix} -6.25 \\ -31.25 \\ 18.75 \end{pmatrix} \text{kN}$$

Positive means Tension, Negative means Compression

Problem 6-7

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$F_1 = 3 \text{ kN}$$

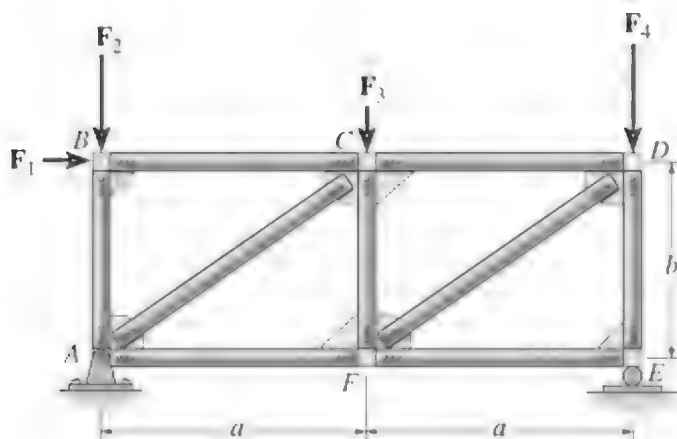
$$F_2 = 8 \text{ kN}$$

$$F_3 = 4 \text{ kN}$$

$$F_4 = 10 \text{ kN}$$

$a = 2\ \text{m}$

$b = 1.5\ \text{m}$

Solution: $\theta = \text{atan}\left(\frac{b}{a}\right)$

Initial Guesses

$F_{BA} = 1\ \text{kN}$ $F_{BC} = 1\ \text{kN}$ $F_{AC} = 1\ \text{kN}$

$F_{AF} = 1\ \text{kN}$ $F_{CD} = 1\ \text{kN}$ $F_{CF} = 1\ \text{kN}$

$F_{DF} = 1\ \text{kN}$ $F_{ED} = 1\ \text{kN}$ $F_{EF} = 1\ \text{kN}$

Given

Joint B $F_1 + F_{BC} = 0$

$-F_2 - F_{BA} = 0$

Joint C $F_{CD} - F_{BC} - F_{AC}\cos(\theta) = 0$

$-F_3 - F_{AC}\sin(\theta) - F_{CF} = 0$

Joint E $-F_{EF} = 0$

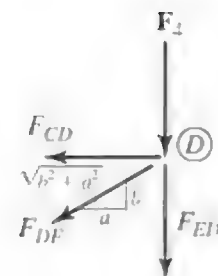
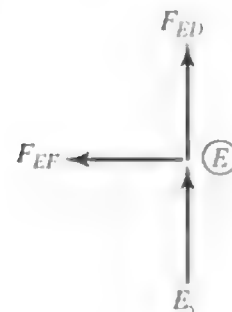
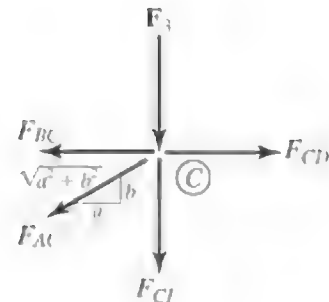
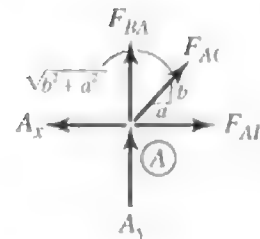
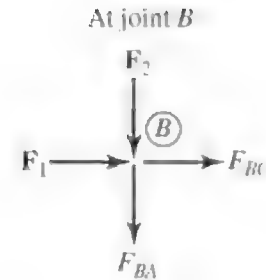
Joint D $-F_{CD} - F_{DF}\cos(\theta) = 0$

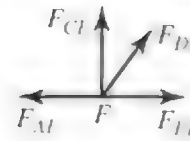
$-F_4 - F_{DF}\sin(\theta) - F_{ED} = 0$

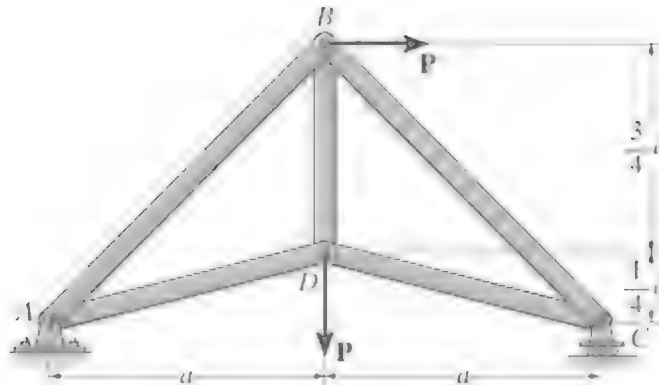
Joint F $-F_{AF} + F_{EF} + F_{DF}\cos(\theta) = 0$

$F_{CF} + F_{DF}\sin(\theta) = 0$

$$\begin{pmatrix} F_{BA} \\ F_{AF} \\ F_{DF} \\ F_{BC} \\ F_{CD} \\ F_{ED} \\ F_{AC} \\ F_{CF} \\ F_{EF} \end{pmatrix} = \text{Find}(F_{BA}, F_{AF}, F_{DF}, F_{BC}, F_{CD}, F_{ED}, F_{AC}, F_{CF}, F_{EF})$$

At joint B

$$\begin{pmatrix} F_{BA} \\ F_{AF} \\ F_{DF} \\ F_{BC} \\ F_{CD} \\ F_{ED} \\ F_{AC} \\ F_{CF} \\ F_{EF} \end{pmatrix} = \begin{pmatrix} -8 \\ 4.167 \\ 5.208 \\ -3 \\ -4.167 \\ -13.125 \\ -1.458 \\ -3.125 \\ 0 \end{pmatrix} \text{kN}$$

Positive means tension,
Negative means compression.

Problem 6-8

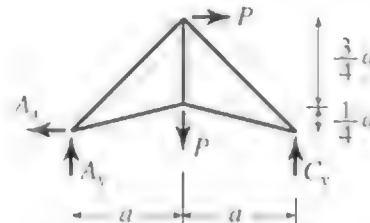
Determine the force in each member of the truss in terms of the external loading and state if the members are in tension or compression.

Solution:

$\Sigma M_A = 0;$ $\quad -P\ a + C_y 2a - P\,a = 0$

$$C_y = P$$

Joint C:

$\Sigma F_x = 0;$ $\quad \frac{1}{\sqrt{2}} F_{BC} - \frac{4}{\sqrt{17}} F_{CD} = 0$

$\Sigma F_y = 0;$ $\quad P + \frac{1}{\sqrt{17}} F_{CD} - \frac{1}{\sqrt{2}} F_{BC} = 0$

$$F_{BC} = \frac{4\sqrt{2}P}{3} = 1.886\ P\ \ (C)$$

$$F_{CD} = \frac{\sqrt{17}P}{3} = 1.374\ P\ \ (T)$$

Joint *B*:

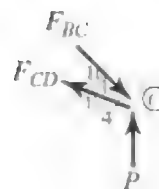
$\Sigma F_x = 0;\qquad P - \frac{1}{\sqrt{2}}F_{CD} + \frac{1}{\sqrt{2}}F_{AB} = 0$

$\Sigma F_y = 0;\qquad \frac{1}{\sqrt{2}}F_{CD} + \frac{1}{\sqrt{2}}F_{AB} - F_{BD} = 0$

$$F_{AB} = \frac{\sqrt{2}P}{3} = 0.471P\qquad (C)$$

$$F_{BD} = \frac{5P}{3} = 1.667P\qquad (T)$$

Joint *D*:

$\Sigma F_x = 0;\qquad F_{DA} = F_{CD} = 1.374P\qquad (T)$

Problem 6-9

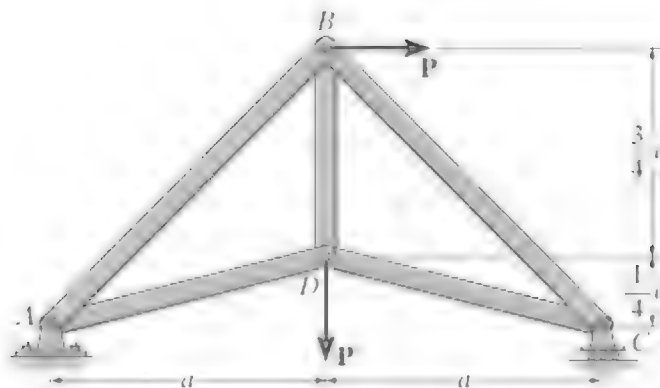
The maximum allowable tensile force in the members of the truss is T_{max}, and the maximum allowable compressive force is C_{max}. Determine the maximum magnitude P of the two loads that can be applied to the truss.

Given:

$T_{max} = 1500\ \text{lb}$

$C_{max} = 800\ \text{lb}$

Solution:

Set $\quad P = 1\ \text{lb}$

Initial Guesses

$F_{AB} = 1\ \text{lb}\qquad F_{AD} = 1\ \text{lb}\qquad F_{BD} = 1\ \text{lb}$

$F_{BC} = 1\ \text{lb}\qquad F_{CD} = 1\ \text{lb}$

Given

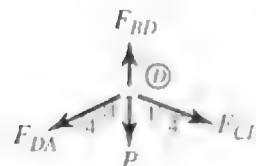
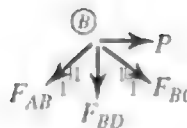
Joint B $\left(F_{BC} - F_{AB}\right)\frac{1}{\sqrt{2}} + P = 0$

$$-F_{BD} - \left(F_{AB} + F_{BC}\right)\frac{1}{\sqrt{2}} = 0$$

Joint D $\left(F_{CD} - F_{AD}\right)\frac{4}{\sqrt{17}} = 0$

$$F_{BD} - P - \left(F_{AD} + F_{CD}\right)\frac{1}{\sqrt{17}} = 0$$

Joint C $-F_{BC}\frac{1}{\sqrt{2}} - F_{CD}\frac{4}{\sqrt{17}} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{AD} \\ F_{CD} \\ F_{BD} \end{pmatrix} = \text{Find}\left(F_{AB}, F_{BC}, F_{AD}, F_{CD}, F_{BD}\right) \qquad \begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{AD} \\ F_{CD} \\ F_{BD} \end{pmatrix} = \begin{pmatrix} -0.471 \\ -1.886 \\ 1.374 \\ 1.374 \\ 1.667 \end{pmatrix} \text{lb}$$

Now find the critical load

$$P_1 = P\frac{T_{max}}{\max\left(F_{AB}, F_{BC}, F_{AD}, F_{CD}, F_{BD}\right)} \qquad P_1 = 900\,\text{lb}$$

$$P_2 = P\frac{C_{max}}{\left|\min\left(F_{AB}, F_{BC}, F_{AD}, F_{CD}, F_{BD}\right)\right|} \qquad P_2 = 424.264\,\text{lb}$$

$$P = \min\left(P_1, P_2\right) \qquad P = 424.3\,\text{lb}$$

Problem 6-10

Determine the force in each member of the truss and state if the members are in tension or compression.

Given:

$P_1 = 0$ lb

$P_2 = 1000 \text{ lb}$

$a = 10 \text{ ft}$

$b = 10 \text{ ft}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{b}{a}\right)$$

Initial Guesses:

$F_{AB} = 1 \text{ lb}$ $F_{AG} = 1 \text{ lb}$ $F_{BG} = 1 \text{ lb}$

$F_{BC} = 1 \text{ lb}$ $F_{DC} = 1 \text{ lb}$ $F_{DE} = 1 \text{ lb}$

$F_{EG} = 1 \text{ lb}$ $F_{EC} = 1 \text{ lb}$ $F_{CG} = 1 \text{ lb}$

Given

Joint B $\quad F_{BC} - F_{AB} = 0$

$F_{BG} - P_1 = 0$

Joint G $\quad (F_{CG} - F_{AG})\cos(\theta) + F_{EG} = 0$

$-(F_{CG} + F_{AG})\sin(\theta) - F_{BG} = 0$

Joint C $\quad F_{DC} - F_{BC} - F_{CG}\cos(\theta) = 0$

$F_{EC} + F_{CG}\sin(\theta) - P_2 = 0$

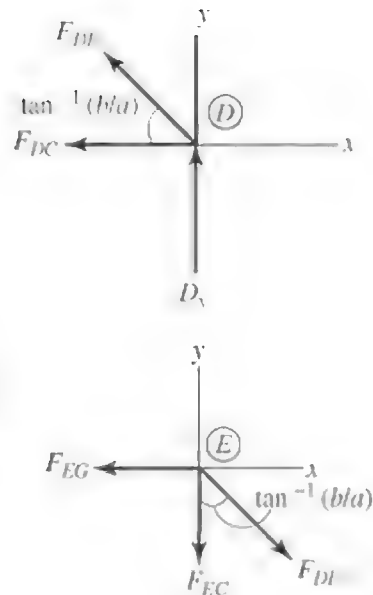
Joint E $\quad F_{DE}\cos(\theta) - F_{EG} = 0$

$-F_{EC} - F_{DE}\sin(\theta) = 0$

Joint D $\quad -F_{DE}\cos(\theta) - F_{DC} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{EG} \\ F_{AG} \\ F_{DC} \\ F_{EC} \\ F_{BG} \\ F_{DE} \\ F_{CG} \end{pmatrix} = \text{Find}(F_{AB}, F_{BC}, F_{EG}, F_{AG}, F_{DC}, F_{EC}, F_{BG}, F_{DE}, F_{CG})$$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{EG} \\ F_{AG} \\ F_{DC} \\ F_{EC} \\ F_{BG} \\ F_{DE} \\ F_{CG} \end{pmatrix} = \begin{pmatrix} 333 \\ 333 \\ -667 \\ -471 \\ 667 \\ 667 \\ 0 \\ -943 \\ 471 \end{pmatrix} \text{lb}$$

Positive means tension,
Negative means compression.

Problem 6-11

Determine the force in each member of the truss and state if the members are in tension or compression.

Given:

$P_1 = 500 \text{ lb}$

$P_2 = 1500 \text{ lb}$

$a = 10\ \text{ft}$

$b = 10\ \text{ft}$

Solution:

$$\theta = \text{atan}\left(\frac{b}{a}\right)$$

Initial Guesses:

$F_{AB} = 1\ \text{lb}$ $F_{AG} = 1\ \text{lb}$ $F_{BG} = 1\ \text{lb}$

$F_{BC} = 1\ \text{lb}$ $F_{DC} = 1\ \text{lb}$ $F_{DE} = 1\ \text{lb}$

$F_{EG} = 1\ \text{lb}$ $F_{EC} = 1\ \text{lb}$ $F_{CG} = 1\ \text{lb}$

Given

Joint *B* $F_{BC} - F_{AB} = 0$

$F_{BG} - P_1 = 0$

Joint *G* $(F_{CG} - F_{AG})\cos(\theta) + F_{EG} = 0$

$-(F_{CG} + F_{AG})\sin(\theta) - F_{BG} = 0$

Joint *C* $F_{DC} - F_{BC} - F_{CG}\cos(\theta) = 0$

$F_{EC} + F_{CG}\sin(\theta) - P_2 = 0$

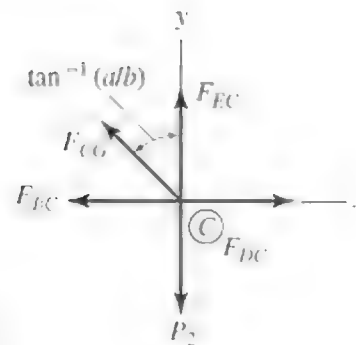
Joint *E* $F_{DE}\cos(\theta) - F_{EG} = 0$

$-F_{EC} - F_{DE}\sin(\theta) = 0$

Joint *D* $-F_{DE}\cos(\theta) - F_{DC} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{EG} \\ F_{AG} \\ F_{DC} \\ F_{EC} \\ F_{BG} \\ F_{DE} \\ F_{CG} \end{pmatrix} = \text{Find}(F_{AB}, F_{BC}, F_{EG}, F_{AG}, F_{DC}, F_{EC}, F_{BG}, F_{DE}, F_{CG})$$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{EG} \\ F_{AG} \\ F_{DC} \\ F_{EC} \\ F_{BG} \\ F_{DE} \\ F_{CG} \end{pmatrix} = \begin{pmatrix} 833 \\ 833 \\ -1167 \\ -1179 \\ 1167 \\ 1167 \\ 500 \\ -1650 \\ 471 \end{pmatrix} \text{lb}$$

Positive means tension,
Negative means compression.

Problem 6-12

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$P_1 = 10 \text{ kN}$$

$$P_2 = 15 \text{ kN}$$

$a = 2 \text{ m}$

$b = 4 \text{ m}$

$c = 4 \text{ m}$

Solution: $\alpha = \text{atan}\left(\frac{c}{a}\right)$ $\beta = \text{atan}\left(\frac{c}{b}\right)$

Initial Guesses:

$F_{AB} = 1 \text{ kN}$ $F_{AF} = 1 \text{ kN}$ $F_{GB} = 1 \text{ kN}$

$F_{BF} = 1 \text{ kN}$ $F_{FC} = 1 \text{ kN}$ $F_{FE} = 1 \text{ kN}$

$F_{BC} = 1 \text{ kN}$ $F_{EC} = 1 \text{ kN}$ $F_{CD} = 1 \text{ kN}$

$F_{ED} = 1 \text{ kN}$

Given

Joint B $-F_{GB} + F_{BC} - F_{AB}\cos(\alpha) = 0$

$-F_{AB}\sin(\alpha) - F_{BF} = 0$

Joint F $-F_{AF} + F_{FE} + F_{FC}\cos(\beta) = 0$

$F_{BF} + F_{FC}\sin(\beta) - P_1 = 0$

Joint C $-F_{BC} - F_{FC}\cos(\beta) + F_{CD}\cos(\alpha) = 0$

$-F_{FC}\sin(\beta) - F_{CD}\sin(\alpha) - F_{EC} = 0$

Joint E $-F_{FE} + F_{ED} = 0$

$F_{EC} - P_2 = 0$

Joint D $-F_{CD}\cos(\alpha) - F_{ED} = 0$

$F_{CD}\sin(\alpha) = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BF} \\ F_{BC} \\ F_{ED} \\ F_{AF} \\ F_{FC} \\ F_{EC} \\ F_{GB} \\ F_{FE} \\ F_{CD} \end{pmatrix} = \text{Find}(F_{AB}, F_{BF}, F_{BC}, F_{ED}, F_{AF}, F_{FC}, F_{EC}, F_{GB}, F_{FE}, F_{CD})$$

$$\begin{pmatrix} F_{AB} \\ F_{BF} \\ F_{BC} \\ F_{ED} \\ F_{AF} \end{pmatrix} = \begin{pmatrix} -27.951 \\ 25 \\ 15 \\ 0 \\ -15 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} F_{FC} \\ F_{EC} \\ F_{GB} \\ F_{FE} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} -21.213 \\ 15 \\ 27.5 \\ 0 \\ 0 \end{pmatrix} \text{kN}$$

Positive means Tension,
Negative means Compression

Problem 6-13

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$P_1 = 0\ \text{kN}$

$P_2 = 20\ \text{kN}$

$a = 2\ \text{m}$

$b = 4\ \text{m}$

$c = 4\ \text{m}$

Solution: $\alpha = \operatorname{atan}\left(\frac{c}{a}\right)$ $\beta = \operatorname{atan}\left(\frac{c}{b}\right)$

Initial Guesses:

$F_{AB} = 1\text{ kN}$ $F_{AF} = 1\text{ kN}$ $F_{GB} = 1\text{ kN}$

$F_{BF} = 1\text{ kN}$ $F_{FC} = 1\text{ kN}$ $F_{FE} = 1\text{ kN}$

$F_{BC} = 1\text{ kN}$ $F_{EC} = 1\text{ kN}$ $F_{CD} = 1\text{ kN}$

$F_{ED} = 1\text{ kN}$

Given

Joint B $-F_{GB} + F_{BC} - F_{AB}\cos(\alpha) = 0$

$-F_{AB}\sin(\alpha) - F_{BF} = 0$

Joint F $-F_{AF} + F_{FE} + F_{FC}\cos(\beta) = 0$

$F_{BF} + F_{FC}\sin(\beta) - P_1 = 0$

Joint C $-F_{BC} - F_{FC}\cos(\beta) + F_{CD}\cos(\alpha) = 0$

$-F_{FC}\sin(\beta) - F_{CD}\sin(\alpha) - F_{EC} = 0$

Joint E $-F_{FE} + F_{ED} = 0$

$F_{EC} - P_2 = 0$

Joint D $-F_{CD}\cos(\alpha) - F_{ED} = 0$

$F_{CD}\sin(\alpha) = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BF} \\ F_{BC} \\ F_{ED} \\ F_{AF} \\ F_{FC} \\ F_{EC} \\ F_{GB} \\ F_{FE} \\ F_{CD} \end{pmatrix} = \operatorname{Find}(F_{AB}, F_{BF}, F_{BC}, F_{ED}, F_{AF}, F_{FC}, F_{EC}, F_{GB}, F_{FE}, F_{CD})$$

$$\begin{pmatrix} F_{AB} \\ F_{BF} \\ F_{BC} \\ F_{ED} \\ F_{AF} \end{pmatrix} = \begin{pmatrix} -22.361 \\ 20 \\ 20 \\ 0 \\ -20 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} F_{FC} \\ F_{EC} \\ F_{GB} \\ F_{FE} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} -28.284 \\ 20 \\ 30 \\ 0 \\ 0 \end{pmatrix} \text{kN}$$

Positive means Tension,
Negative means Compression

Problem 6-14

Determine the force in each member of the truss and state if the members are in tension or compression.

Given:

$P_1 = 100$ lb

$P_2 = 200$ lb

$P_3 = 300$ lb

$a = 10$ ft

$b = 10$ ft

$\theta = 30$ deg

Solution: $\phi = \text{atan}\left(\frac{b}{a}\right)$

Initial Guesses:

$F_{AB} = 1$ lb $\quad F_{AF} = 1$ lb $\quad F_{BC} = 1$ lb

$F_{BF} = 1$ lb $\quad F_{FC} = 1$ lb $\quad F_{FE} = 1$ lb

$F_{ED} = 1$ lb $\quad F_{EC} = 1$ lb $\quad F_{CD} = 1$ lb

Given

Joint B $F_{BC} - F_{AB}\cos(\phi) = 0$

$-F_{BF} - F_{AB}\sin(\phi) = 0$

Joint F $-F_{AF} + F_{FE} + F_{FC}\cos(\phi) = 0$

$-P_2 + F_{BF} + F_{FC}\sin(\phi) = 0$

Joint C $-F_{BC} + F_{CD}\cos(\phi) - F_{FC}\cos(\phi) = 0$

$-F_{EC} - F_{CD}\sin(\phi) - F_{FC}\sin(\phi) = 0$

Joint E $-F_{FE} + F_{ED} = 0$

$F_{EC} - P_3 = 0$

Joint D $-F_{ED}\cos(\theta) - F_{CD}\cos(\phi + \theta) = 0$

$$\begin{pmatrix} F_{AB} \\ F_{AF} \\ F_{BC} \\ F_{BF} \\ F_{FC} \\ F_{FE} \\ F_{ED} \\ F_{EC} \\ F_{CD} \end{pmatrix} = \text{Find}(F_{AB}, F_{AF}, F_{BC}, F_{BF}, F_{FC}, F_{FE}, F_{ED}, F_{EC}, F_{CD})$$

$$\begin{pmatrix} F_{AB} \\ F_{AF} \\ F_{BC} \\ F_{BF} \\ F_{FC} \\ F_{FE} \\ F_{ED} \\ F_{EC} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} -330.0 \\ 79.4 \\ -233.3 \\ 233.3 \\ -47.1 \\ 112.7 \\ 112.7 \\ 300.0 \\ -377.1 \end{pmatrix} \text{lb}$$

Positive means Tension,
Negative means Compression

Problem 6-15

Determine the force in each member of the truss and state if the members are in tension or compression.

Given:

$P_1 = 400 \text{ lb}$

$P_2 = 400 \text{ lb}$

$P_3 = 0 \text{ lb}$

$a = 10 \text{ ft}$

$b = 10 \text{ ft}$

$\theta = 30 \text{ deg}$

Solution: $\phi = \text{atan}\left(\frac{b}{a}\right)$

Initial Guesses:

$F_{AB} = 1 \text{ lb}$ $F_{AF} = 1 \text{ lb}$ $F_{BC} = 1 \text{ lb}$

$F_{BF} = 1 \text{ lb}$ $F_{FC} = 1 \text{ lb}$ $F_{FE} = 1 \text{lb}$

$F_{ED} = 1 \text{ lb}$ $F_{EC} = 1 \text{ lb}$ $F_{CD} = 1 \text{ lb}$

Given

Joint B $\quad F_{BC} - F_{AB}\cos(\phi) = 0$

$$-F_{BF} - F_{AB}\sin(\phi) = 0$$

Joint F $\quad -F_{AF} + F_{FE} + F_{FC}\cos(\phi) = 0$

$$-P_2 + F_{BF} + F_{FC}\sin(\phi) = 0$$

Joint C $\quad -F_{BC} + F_{CD}\cos(\phi) - F_{FC}\cos(\phi) = 0$

$$-F_{EC} - F_{CD}\sin(\phi) - F_{FC}\sin(\phi) = 0$$

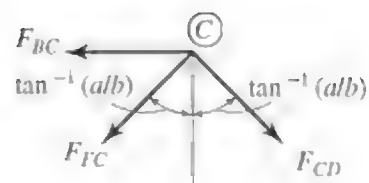
Joint E $\quad -F_{FE} + F_{ED} = 0$

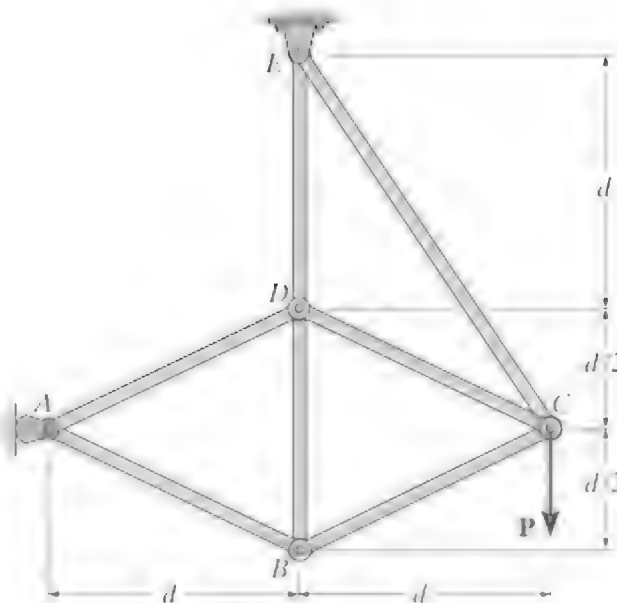
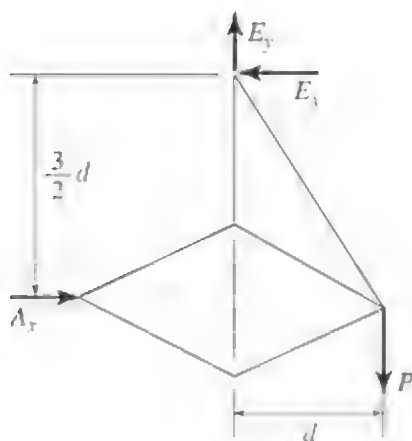
$$F_{EC} - P_3 = 0$$

Joint D $\quad -F_{ED}\cos(\theta) - F_{CD}\cos(\phi + \theta) = 0$

$$\begin{pmatrix} F_{AB} \\ F_{AF} \\ F_{BC} \\ F_{BF} \\ F_{FC} \\ F_{FE} \\ F_{ED} \\ F_{EC} \\ F_{CD} \end{pmatrix} = \text{Find}(F_{AB}, F_{AF}, F_{BC}, F_{BF}, F_{FC}, F_{FE}, F_{ED}, F_{EC}, F_{CD})$$

$$\begin{pmatrix} F_{AB} \\ F_{AF} \\ F_{BC} \\ F_{BF} \\ F_{FC} \\ F_{FE} \\ F_{ED} \\ F_{EC} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} -377.1 \\ 189.7 \\ -266.7 \\ 266.7 \\ 188.6 \\ 56.4 \\ 56.4 \\ 0.0 \\ -188.6 \end{pmatrix} \text{lb}$$

Positive means Tension,
Negative means Compression

Problem 6-16

Determine the force in each member of the truss in terms of the load P and state if the members are in tension or compression.

Solution:

Support reactions:

$\Sigma M_E = 0;\quad A_x \frac{3}{2} d - P\,d = 0 \qquad A_x = \frac{2P}{3}$

$\Sigma F_x = 0;\quad A_x - E_x = 0 \qquad E_x = \frac{2P}{3}$

$\Sigma F_y = 0;\quad E_y - P = 0 \qquad E_y = P$

Joint E:

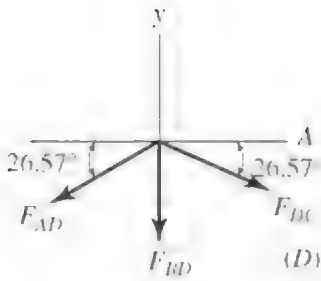
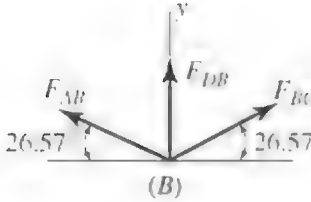
$\Sigma F_x = 0;\quad F_{EC}\frac{2}{\sqrt{13}} - E_x = 0 \qquad F_{EC} = \sqrt{\frac{13}{3}}P = 1.20P \quad \text{(T)}$

$\Sigma F_y = 0;\quad P - F_{ED} - F_{EC}\frac{3}{\sqrt{13}} = 0 \qquad F_{ED} = 0$

Joint A:

$\Sigma F_y = 0;\quad F_{AB}\frac{1}{\sqrt{5}} - F_{AD}\frac{1}{\sqrt{5}} = 0 \qquad F_{AB} = F_{AD}$

$\Sigma F_x = 0;\quad A_x - 2F_{AB}\frac{2}{\sqrt{5}} = 0 \qquad F_{AB} = F_{AD} = \frac{\sqrt{5}}{6}P = 0.373P \quad \text{(C)}$

Joint D:

$\Sigma F_x = 0;\quad F_{AD}\frac{2}{\sqrt{5}} - F_{DC}\frac{2}{\sqrt{5}} = 0 \qquad F_{DC} = \frac{\sqrt{5}}{6}P = 0.373P \quad \text{(C)}$

$\Sigma F_y = 0;\quad 2F_{AD}\frac{1}{\sqrt{5}} - F_{DB} = 0 \qquad F_{DB} = \frac{P}{3} \quad \text{(T)}$

Joint B:

$\Sigma F_x = 0;\quad F_{AB}\frac{1}{\sqrt{5}} - F_{BC}\frac{1}{\sqrt{5}} = 0 \qquad F_{BC} = \sqrt{\frac{5}{6}}P = 0.373P \quad \text{(C)}$

Problem 6-17

The maximum allowable tensile force in the members of the truss is T_{max} and the maximum allowable compressive force is C_{max}. Determine the maximum magnitude of the load P that can be applied to the truss.

Units Used:

$kN = 10^3\ N$

Given:

$T_{max} = 5\ kN$

$C_{max} = 3\ kN$

$d = 2\ m$

Solution:

Set $P = 1\ kN$

Initial Guesses:

$F_{AD} = 1\ kN$ $F_{AB} = 1\ kN$

$F_{BC} = 1\ kN$ $F_{BD} = 1\ kN$

$F_{CD} = 1\ kN$ $F_{CE} = 1\ kN$ $F_{DE} = 1\ kN$

Given

Joint A $$F_{AD}\frac{1}{\sqrt{5}} - F_{AB}\frac{1}{\sqrt{5}} = 0$$

Joint B $$F_{BC}\frac{2}{\sqrt{5}} - F_{AB}\frac{2}{\sqrt{5}} = 0$$

$$\left(F_{BC} + F_{AB}\right)\frac{1}{\sqrt{5}} + F_{BD} = 0$$

Joint D $$\left(F_{CD} - F_{AD}\right)\frac{2}{\sqrt{5}} = 0$$

$$F_{DE} - F_{BD} - \left(F_{AD} + F_{CD}\right)\frac{1}{\sqrt{5}} = 0$$

Joint C $$-\left(F_{CD} + F_{BC}\right)\frac{2}{\sqrt{5}} - F_{CE}\frac{2}{\sqrt{13}} = 0$$

$$\left(F_{CD} - F_{BC}\right)\frac{1}{\sqrt{5}} + F_{CE}\frac{3}{\sqrt{13}} - P = 0$$

$$\begin{pmatrix} F_{AD} \\ F_{AB} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ F_{CE} \\ F_{DE} \end{pmatrix} = \text{Find}(F_{AD}, F_{AB}, F_{BC}, F_{BD}, F_{CD}, F_{CE}, F_{DE})$$

$$\begin{pmatrix} F_{AD} \\ F_{AB} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ F_{CE} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} -0.373 \\ -0.373 \\ -0.373 \\ 0.333 \\ -0.373 \\ 1.202 \\ 0 \end{pmatrix} \text{kN}$$

Now Scale the answer

$$P_1 = P \frac{T_{max}}{\max(F_{AD}, F_{AB}, F_{BC}, F_{BD}, F_{CD}, F_{CE}, F_{DE})}$$

$$P_2 = P \frac{C_{max}}{|\min(F_{AD}, F_{AB}, F_{BC}, F_{BD}, F_{CD}, F_{CE}, F_{DE})|}$$

$$P = \min(P_1, P_2) \qquad P = 4.16\,\text{kN}$$

Problem 6-18

Determine the force in each member of the truss and state if the members are in tension or compression. *Hint:* The horizontal force component at A must be zero. Why?

Units Used:

$kip = 10^3\ lb$

Given:

$F_1 = 600\ lb$

$F_2 = 800\ lb$

$a = 4\ ft$

$b = 3\ ft$

$\theta = 60\ deg$

Solution:

Initial Guesses

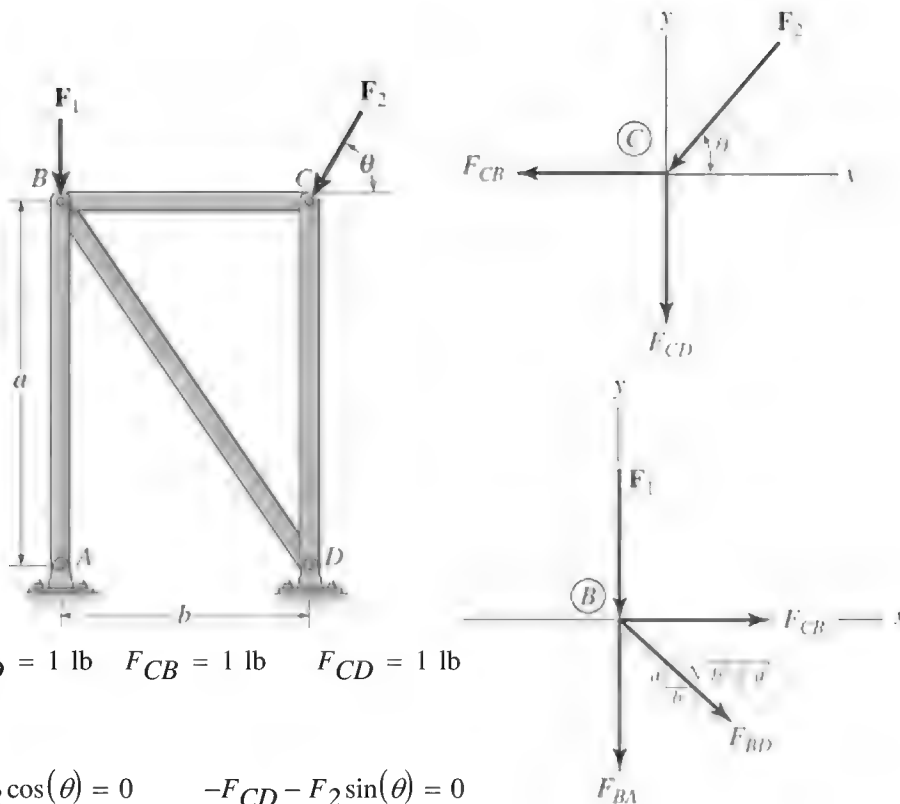
$F_{BA} = 1\ lb \quad F_{BD} = 1\ lb \quad F_{CB} = 1\ lb \quad F_{CD} = 1\ lb$

Given

Joint C $\quad -F_{CB} - F_2\cos(\theta) = 0 \qquad -F_{CD} - F_2\sin(\theta) = 0$

Joint B $\quad F_{CB} + F_{BD}\dfrac{b}{\sqrt{a^2+b^2}} = 0 \qquad -F_{BA} - F_{BD}\dfrac{a}{\sqrt{a^2+b^2}} - F_1 = 0$

$$\begin{pmatrix} F_{BA} \\ F_{BD} \\ F_{CB} \\ F_{CD} \end{pmatrix} = \text{Find}(F_{BA}, F_{BD}, F_{CB}, F_{CD}) \qquad \begin{pmatrix} F_{BA} \\ F_{BD} \\ F_{CB} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} -1.133 \times 10^3 \\ 666.667 \\ -400 \\ -692.82 \end{pmatrix} lb$$

Positive means Tension
Negative means Compression

Problem 6-19

Determine the force in each member of the truss and state if the members are in tension or compression. *Hint:* The resultant force at the pin E acts along member ED. Why?

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 3\ \text{kN}$

$F_2 = 2\ \text{kN}$

$a = 3\ \text{m}$

$b = 4\ \text{m}$

Solution:

Initial Guesses:

$F_{CB} = 1\ \text{kN} \quad F_{CD} = 1\ \text{kN} \quad F_{BA} = 1\ \text{kN}$

$F_{BD} = 1\ \text{kN} \quad F_{DA} = 1\ \text{kN} \quad F_{DE} = 1\ \text{kN}$

Given

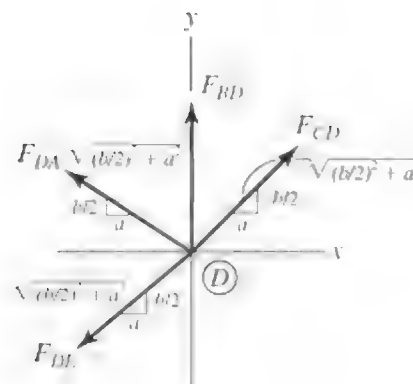
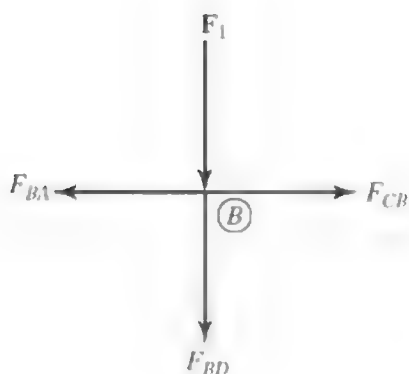
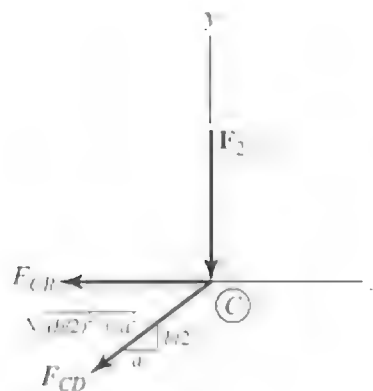
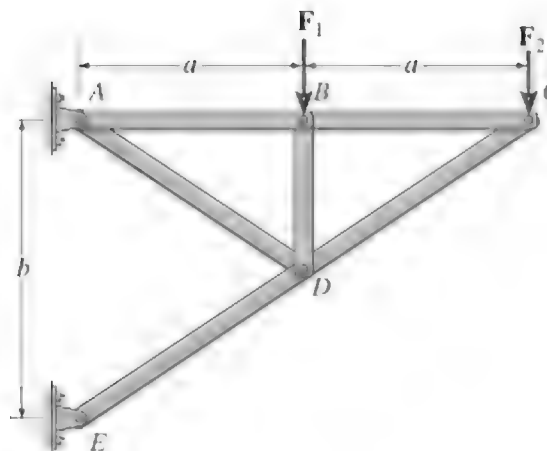
Joint C $\quad -F_{CB} - F_{CD}\dfrac{2\ a}{\sqrt{(2\ a)^2 + b^2}} = 0$

$$-F_2 - F_{CD}\frac{b}{\sqrt{(2\ a)^2 + b^2}} = 0$$

Joint B $\quad -F_{BA} + F_{CB} = 0$

$$-F_1 - F_{BD} = 0$$

Joint D $\quad \left(F_{CD} - F_{DA} - F_{DE}\right)\dfrac{2\ a}{\sqrt{(2\ a)^2 + b^2}} = 0$

$$F_{BD} + \left(F_{CD} + F_{DA} - F_{DE}\right)\frac{b}{\sqrt{(2\ a)^2 + b^2}} = 0$$

$$\begin{pmatrix} F_{CB} \\ F_{CD} \\ F_{BA} \\ F_{BD} \\ F_{DA} \\ F_{DE} \end{pmatrix} = \text{Find}(F_{CB}, F_{CD}, F_{BA}, F_{BD}, F_{DA}, F_{DE})$$

$$\begin{pmatrix} F_{CB} \\ F_{CD} \\ F_{BA} \\ F_{BD} \\ F_{DA} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 3 \\ -3.606 \\ 3 \\ -3 \\ 2.704 \\ -6.31 \end{pmatrix} \text{kN}$$

Positive means Tension,
Negative means Compression

Problem 6-20

Each member of the truss is uniform and has a mass density ρ. Determine the approximate force in each member due to the weight of the truss. State if the members are in tension or compression. Solve the problem by *assuming* the weight of each member can be represented as a vertical force, half of which is applied at each end of the member.

Given:

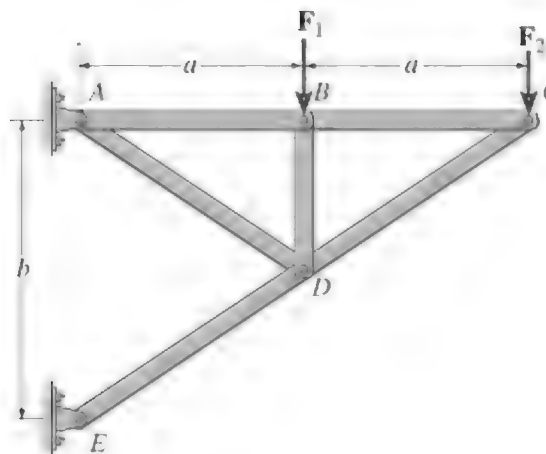
$\rho = 8 \frac{\text{kg}}{\text{m}}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

$F_1 = 0 \text{ N}$

$F_2 = 0 \text{ N}$

$a = 3 \text{ m}$

$b = 4 \text{ m}$

Solution:

Initial Guesses:

$F_{CB} = 1\ \text{N}$ $F_{CD} = 1\ \text{N}$ $F_{BA} = 1\ \text{N}$

$F_{BD} = 1\ \text{N}$ $F_{DA} = 1\ \text{N}$ $F_{DE} = 1\ \text{N}$

Given

Joint C $-F_{CB} - F_{CD}\dfrac{2\ a}{\sqrt{(2\ a)^2 + b^2}} = 0$

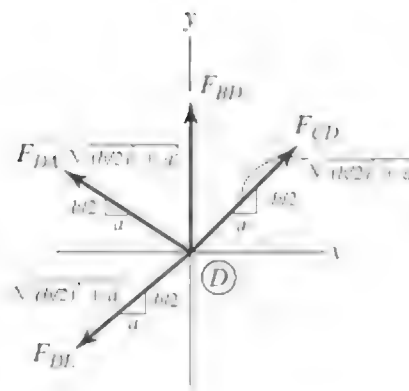
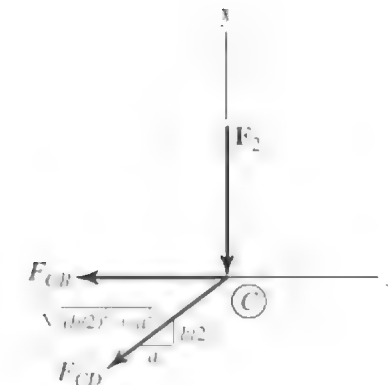
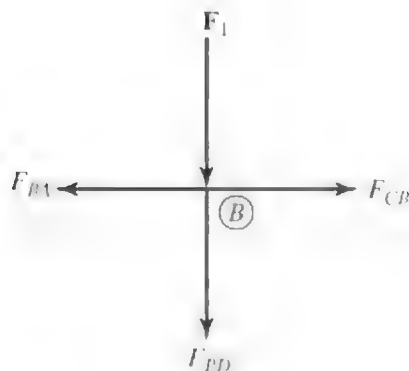
$$-F_2 - F_{CD}\frac{b}{\sqrt{(2\ a)^2 + b^2}} - \rho g\left[\frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{4}\right)^2}\right] = 0$$

Joint B $-F_{BA} + F_{CB} = 0$

$$-F_1 - F_{BD} - \rho g\left(a + \frac{b}{4}\right) = 0$$

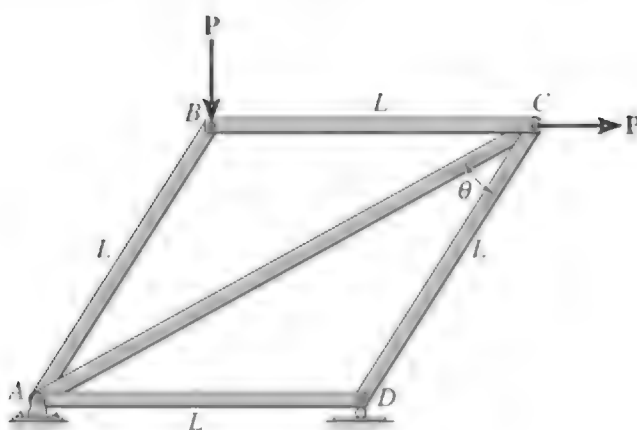
Joint D $F_{BD} + \left(F_{CD} + F_{DA} - F_{DE}\right)\dfrac{b}{\sqrt{(2\ a)^2 + b^2}} - \rho g\left[\dfrac{b}{4} + 3\sqrt{\left(\dfrac{a}{2}\right)^2 + \left(\dfrac{b}{4}\right)^2}\right] = 0$

$$\left(F_{CD} - F_{DA} - F_{DE}\right)\frac{2\ a}{\sqrt{(2\ a)^2 + b^2}} = 0$$

$$\begin{pmatrix} F_{CB} \\ F_{CD} \\ F_{BA} \\ F_{BD} \\ F_{DA} \\ F_{DE} \end{pmatrix} = \text{Find}\left(F_{CB}, F_{CD}, F_{BA}, F_{BD}, F_{DA}, F_{DE}\right)$$

$$\begin{pmatrix} F_{CB} \\ F_{CD} \\ F_{BA} \\ F_{BD} \\ F_{DA} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 389 \\ -467 \\ 389 \\ -314 \\ 736 \\ -1204 \end{pmatrix} \text{N}$$

Positive means Tension,
Negative means Compression

Problem 6-21

Determine the force in each member of the truss in terms of the external loading and state if the members are in tension or compression.

Solution:

Joint B:

$+\uparrow\Sigma F_y = 0;\quad F_{BA}\sin(2\theta) - P = 0$

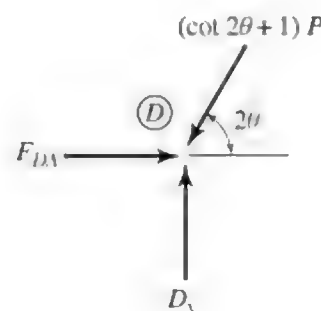
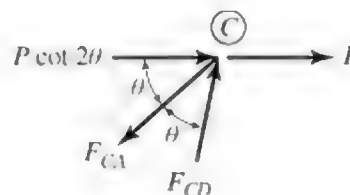
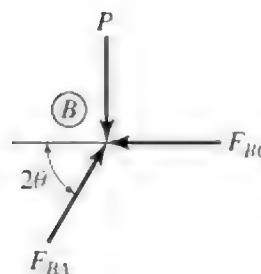
$F_{BA} = P\csc(2\theta)\quad$ (C)

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{BA}\cos(2\theta) - F_{BC} = 0$

$F_{BC} = P\cot(2\theta)\quad$ (C)

Joint C:

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad P\cot(2\theta) + P + F_{CD}\cos(2\theta) - F_{CA}\cos(\theta) = 0$

$+\uparrow\Sigma F_y = 0;\quad F_{CD}\sin(2\theta) - F_{CA}\sin(\theta) = 0$

$$F_{CA} = \frac{\cot(2\ \theta) + 1}{\cos(\theta) - \sin(\theta)\cot(2\ \theta)}P$$

$$F_{CA} = (\cot(\theta)\csc(\theta) - \sin(\theta) + 2\cos(\theta))P \qquad \text{(T)}$$

$$F_{CD} = (\cot(2\ \theta) + 1)\ P \qquad \text{(C)}$$

Joint D:

$\overset{+}{\rightarrow}\ \Sigma\ F_x = 0;\quad F_{DA} - [\cot(2\ \theta) + 1]\ [\cos(2\ \theta)]\ P = 0$

$$F_{DA} = [\cot(2\ \theta) + 1]\ [\cos(2\ \theta)]P \qquad \text{(C)}$$

Problem 6-22

The maximum allowable tensile force in the members of the truss is T_{max}, and the maximum allowable compressive force is C_{max}. Determine the maximum magnitude P of the two loads that can be applied to the truss.

Units Used:

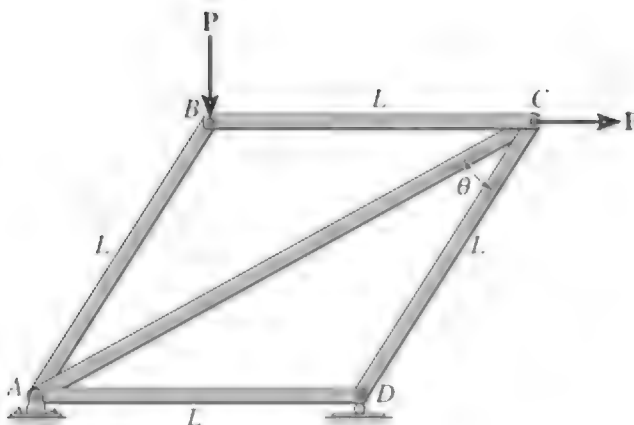
$kN = 10^3\ N$

Given:

$T_{max} = 2\ kN$

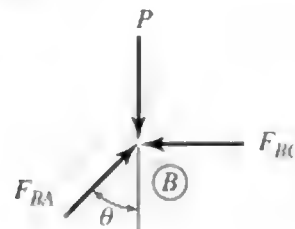
$C_{max} = 1.2\ kN$

$L = 2\ m$

$\theta = 30\ deg$

Solution:

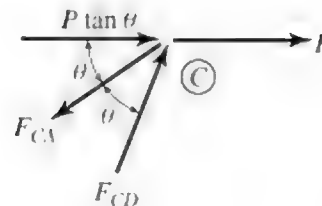
Initial guesses (assume all bars are in tension). Use a unit load for P and then scale the answer later.

$F_{BA} = 1\ kN \qquad F_{BC} = 1\ kN \qquad F_{CA} = 1\ kN$

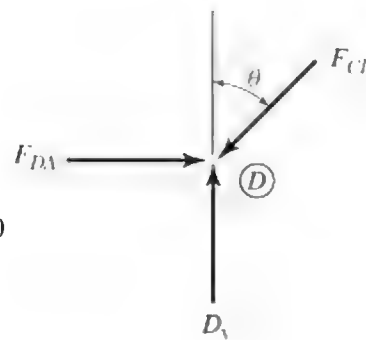
$F_{CD} = 1\ kN \qquad F_{DA} = 1\ kN \qquad P = 1\ kN$

Given

Joint B

$+\uparrow \Sigma F_y = 0; \quad -F_{BA}\sin(2\ \theta) - P = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{BA}\cos(2\ \theta) + F_{BC} = 0$

Joint C

$+\uparrow \Sigma F_y = 0; \quad -F_{CA}\sin(\theta) - F_{CD}\sin(2\ \theta) = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$

$-F_{BC} + P - F_{CD}\cos(2\ \theta) - F_{CA}\cos(\theta) = 0$

Joint D

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{DA} + F_{CD}\cos(2\ \theta) = 0$

$$\begin{pmatrix} F_{BA} \\ F_{BC} \\ F_{CA} \\ F_{CD} \\ F_{DA} \end{pmatrix} = \text{Find}(F_{BA}, F_{BC}, F_{CA}, F_{CD}, F_{DA}) \qquad \text{ans} = \begin{pmatrix} F_{BA} \\ F_{BC} \\ F_{CA} \\ F_{CD} \\ F_{DA} \end{pmatrix} \qquad \text{ans} = \begin{pmatrix} -1.155 \\ -0.577 \\ 2.732 \\ -1.577 \\ -0.789 \end{pmatrix} \text{kN}$$

Now find the biggest tension and the biggest compression.

$T = \max(\text{ans}) \qquad T = 2.732\,\text{kN}$

$C = \min(\text{ans}) \qquad C = -1.577\,\text{kN}$

Decide which is more important and scale the answer

$$P = \min\left[\begin{pmatrix} \dfrac{T_{max}}{T} \\ \dfrac{-C_{max}}{C} \end{pmatrix} P\right] \qquad P = 732.051\,\text{N}$$

Problem 6-23

The *Fink truss* supports the loads shown. Determine the force in each member and state if the members are in tension or compression. Approximate each joint as a pin.

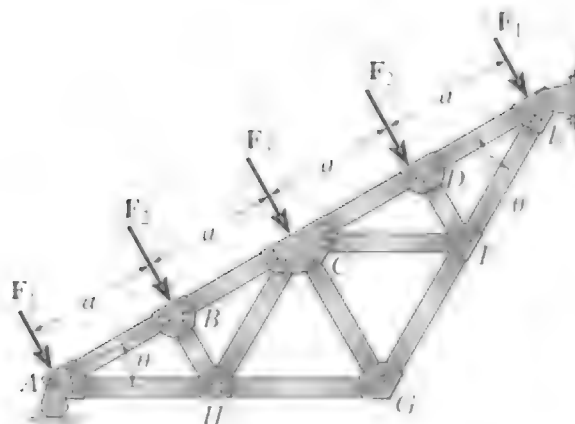
Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$F_1 = 500\ \text{lb}$ $a = 2.5\ \text{ft}$

$F_2 = 1\ \text{kip}$ $\theta = 30\ \text{deg}$

$F_3 = 1\ \text{kip}$

Solution:

Entire truss:

$\Sigma F_x = 0;$ $E_x = (F_1 + F_2 + F_3 + F_2 + F_1)\sin(\theta)$ $E_x = 2000\ \text{lb}$

$\Sigma M_E = 0;$ $-A_y 4a\cos(\theta) + F_1 4a + F_2 3a + F_3 2a + F_2 a = 0$

$$A_y = \frac{2\ F_1 + 2\ F_2 + F_3}{2\ \cos(\theta)}$$ $A_y = 2309.4\ \text{lb}$

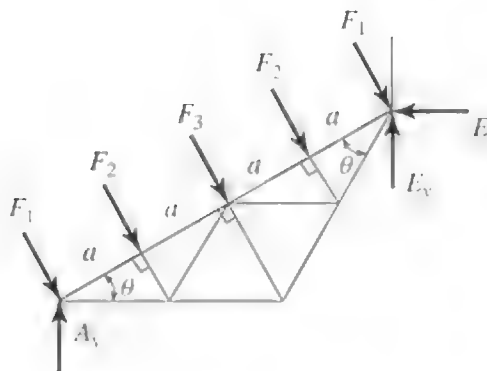
$\Sigma F_y = 0;$ $E_y = -A_y + 2\ \cos(\theta)\ F_1 + 2\ \cos(\theta)\ F_2 + \cos(\theta)\ F_3$ $E_y = 1154.7\ \text{lb}$

Joint A:

$\Sigma F_y = 0;$ $$F_{AB} = \frac{-\cos(\theta)\ F_1 + A_y}{\sin(\theta)}$$

$F_{AB} = 3.75\ \text{kip}$ (C)

$\Sigma F_x = 0;$ $F_{AH} = -\sin(\theta)\ F_1 + F_{AB}\cos(\theta)$

$F_{AH} = 3\ \text{kip}$ (T)

Joint B:

$\Sigma F_x = 0;$ $F_{BC} = F_{AB}$ $F_{BC} = 3.75\,\text{kip}$ (C)

$\Sigma F_y = 0;$ $F_{BH} = F_2$ $F_{BH} = 1\,\text{kip}$ (C)

Joint H:

$\Sigma F_y = 0;$ $F_{HC} = F_2$ $F_{HC} = 1\,\text{kip}$ (T)

$\Sigma F_x = 0;$ $F_{GH} = -F_2\cos(-90\text{ deg} + \theta) - F_{HC}\cos(-90\text{ deg} + \theta) + F_{AH}$

$F_{GH} = 2\,\text{kip}$ (T)

Joint E:

$$\Sigma F_y = 0; \quad F_{EF} = \frac{-(F_1 - E_x\sin(\theta) - E_y\cos(\theta))}{\sin(\theta)}$$

$F_{EF} = 3\,\text{kip}$ (T)

$\Sigma F_x = 0;$ $F_{ED} = -E_y\sin(\theta) + E_x\cos(\theta) + F_{EF}\cos(\theta)$

$F_{ED} = 3.75\,\text{kip}$ (C)

Joint D:

$\Sigma F_x = 0;$ $F_{DC} = F_{ED}$

$F_{DC} = 3.75\,\text{kip}$ (C)

$\Sigma F_y = 0;$ $F_{DF} = F_2$

$F_{DF} = 1\,\text{kip}$ (C)

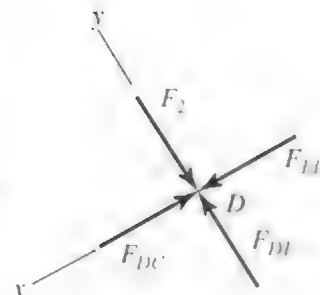
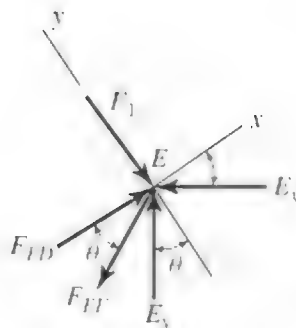
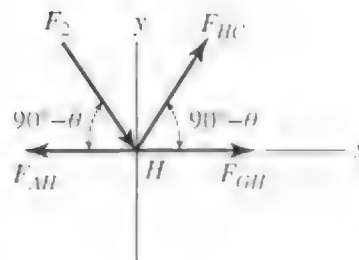
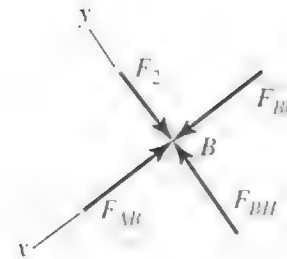
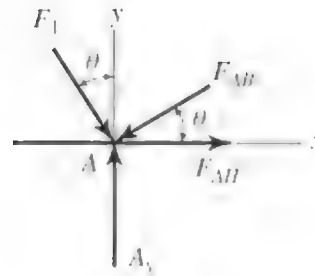
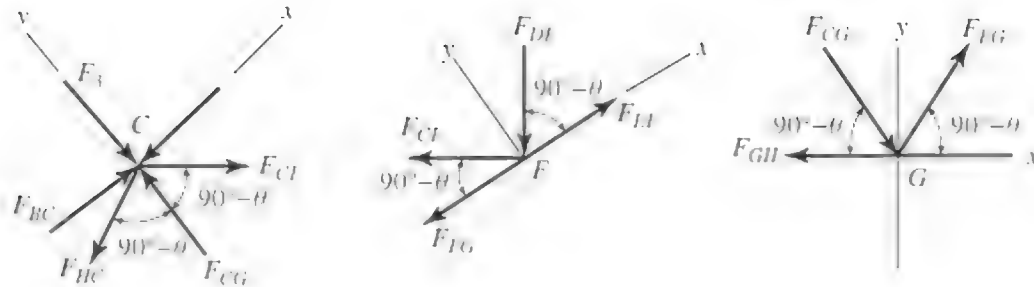
Joint C:

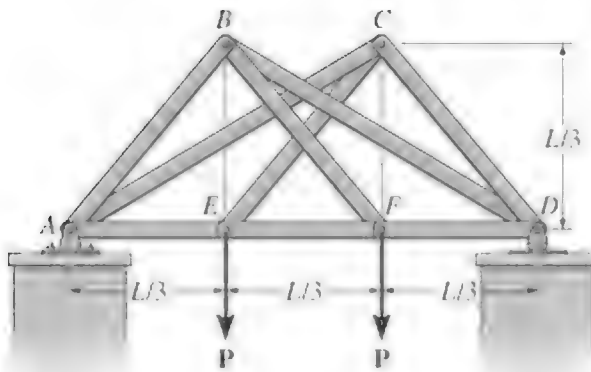
$\Sigma F_x = 0;$ $F_{CF} = F_{HC}$

$F_{CF} = 1\,\text{kip}$ (T)

$\Sigma F_y = 0;$ $F_{CG} = F_3 + F_{HC}\cos(90\text{ deg} - \theta)(2)$ $F_{CG} = 2\,\text{kip}$ (C)

Joint F:

$\Sigma F_x = 0;$ $F_{FG} = F_{EF} - F_{CF}\cos(90\text{ deg} - \theta)(2)$ $F_{FG} = 2\,\text{kip}$ (T)

Problem 6-24

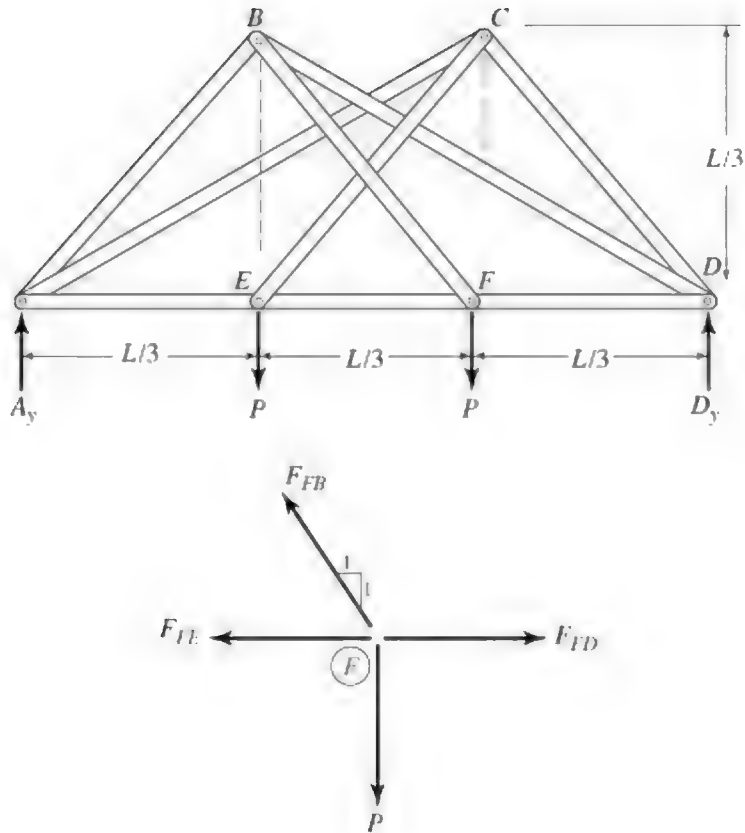
Determine the force in each member of the double scissors truss in terms of the load P and state if the members are in tension or compression.

Solution:

$\Sigma M_A = 0;$ $$P\frac{L}{3} + P\frac{2\,L}{3} - D_y\,L = 0$$

$+\uparrow\ \Sigma F_y = 0;$ $$A_y + D_y - 2\,P = 0$$

Joint F:

$+\uparrow\ \Sigma F_y = 0;$ $$F_{FB}\frac{1}{\sqrt{2}} - P = 0$$

$\xrightarrow{+}\ \Sigma F_x = 0;$ $$F_{FD} - F_{FE} - F_{FB}\frac{1}{\sqrt{2}} = 0$$

Joint E:

$+\uparrow \Sigma F_y = 0; \quad F_{EC}\frac{1}{\sqrt{2}} - P = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{EF} - F_{EA} + F_{EC}\frac{1}{\sqrt{2}} = 0$

Joint B:

$+\uparrow \Sigma F_y = 0; \quad F_{BA}\frac{1}{\sqrt{2}} + F_{BD}\frac{1}{\sqrt{5}} - F_{FB}\frac{1}{\sqrt{2}} = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BA}\frac{1}{\sqrt{2}} + F_{FB}\frac{1}{\sqrt{2}} - F_{BD}\left(\frac{2}{\sqrt{5}}\right) = 0$

Joint C:

$+\uparrow \Sigma F_y = 0; \quad F_{CA}\frac{1}{\sqrt{5}} + F_{CD}\frac{1}{\sqrt{2}} - F_{EC}\frac{1}{\sqrt{2}} = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CA}\frac{2}{\sqrt{5}} - F_{EC}\frac{1}{\sqrt{2}} - F_{CD}\frac{1}{\sqrt{2}} = 0$

Joint A:

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AE} - F_{BA}\frac{1}{\sqrt{2}} - F_{CA}\frac{2}{\sqrt{5}} = 0$

Solving we find

$F_{EF} = 0.667\,P(\mathrm{T})$

$F_{FD} = 1.67\,P(\mathrm{T})$

$F_{AB} = 0.471\,P(\mathrm{C})$

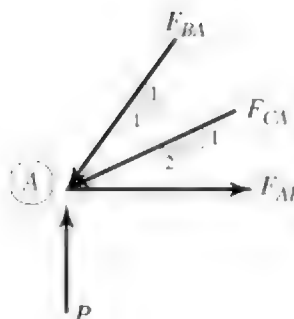
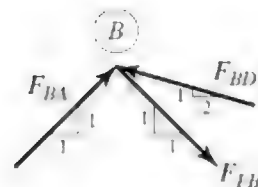
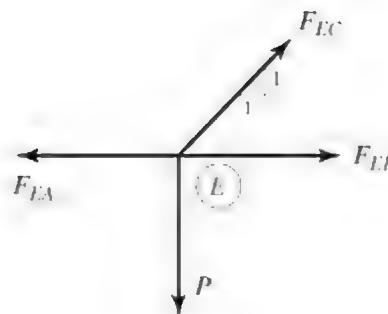
$F_{AE} = 1.67\,P(\mathrm{T})$

$F_{AC} = 1.49\,P(\mathrm{C})$

$F_{BF} = 1.41\,P(\mathrm{T})$

$F_{BD} = 1.49\,P(\mathrm{C})$

$F_{EC} = 1.41\,P(\mathrm{T})$

$F_{CD} = 0.471\,P(\mathrm{C})$

Problem 6-25

Determine the force in each member of the truss and state if the members are in tension or compression. *Hint*: The vertical component of force at C must equal zero. Why?

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 6\ kN$

$F_2 = 8 \text{ kN}$

$a = 1.5 \text{ m}$

$b = 2 \text{ m}$

$c = 2 \text{ m}$

Solution:

Initial Guesses:

$F_{AB} = 1 \text{ kN} \quad F_{AE} = 1 \text{ kN} \quad F_{EB} = 1 \text{ kN}$

$F_{BC} = 1 \text{ kN} \quad F_{BD} = 1 \text{ kN} \quad F_{ED} = 1 \text{ kN}$

Given

Joint *A* $\quad F_{AB}\frac{a}{\sqrt{a^2+c^2}} + F_{AE} = 0$

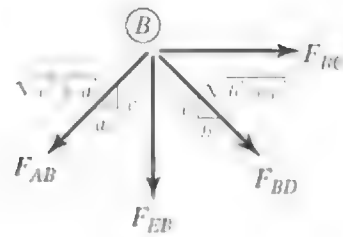
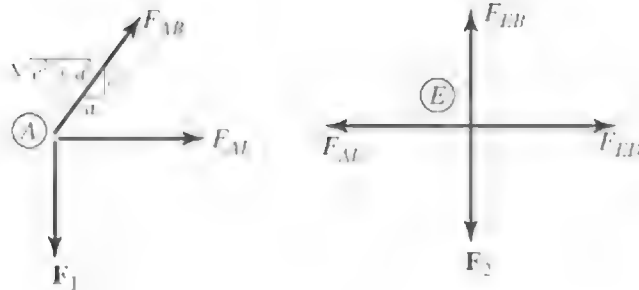
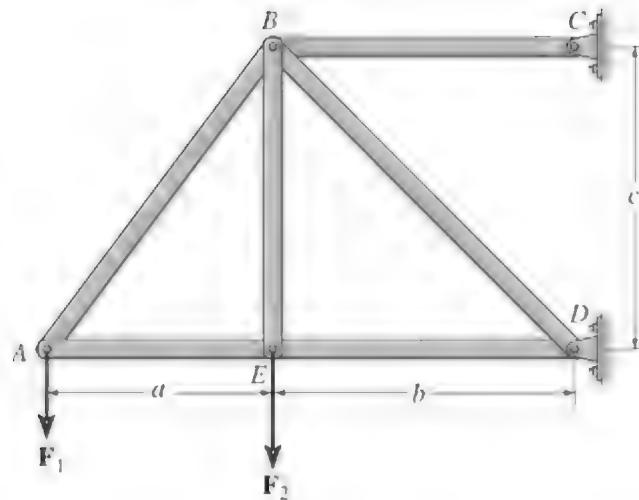
$$F_{AB}\frac{c}{\sqrt{a^2+c^2}} - F_1 = 0$$

Joint *E* $\quad F_{ED} - F_{AE} = 0$

$$F_{EB} - F_2 = 0$$

Joint *B* $\quad F_{BC} + F_{BD}\frac{b}{\sqrt{b^2+c^2}} - F_{AB}\frac{a}{\sqrt{a^2+c^2}} = 0$

$$-F_{EB} - F_{BD}\frac{c}{\sqrt{b^2+c^2}} - F_{AB}\frac{c}{\sqrt{a^2+c^2}} = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AE} \\ F_{EB} \\ F_{BC} \\ F_{BD} \\ F_{ED} \end{pmatrix} = \text{Find}\left(F_{AB}, F_{AE}, F_{EB}, F_{BC}, F_{BD}, F_{ED}\right)$$

$$\begin{pmatrix} F_{AB} \\ F_{AE} \\ F_{EB} \\ F_{BC} \\ F_{BD} \\ F_{ED} \end{pmatrix} = \begin{pmatrix} 7.5 \\ -4.5 \\ 8 \\ 18.5 \\ -19.799 \\ -4.5 \end{pmatrix} \text{kN}$$

Positive means Tension,
Negative means Compresson.

Problem 6-26

Each member of the truss is uniform and has a mass density ρ. Remove the external loads F_1 and F_2 and determine the approximate force in each member due to the weight of the truss. State if the members are in tension or compression. Solve the problem by *assuming* the weight of each member can be represented as a vertical force, half of which is applied at each end of the member.

Given:

$F_1 = 0$

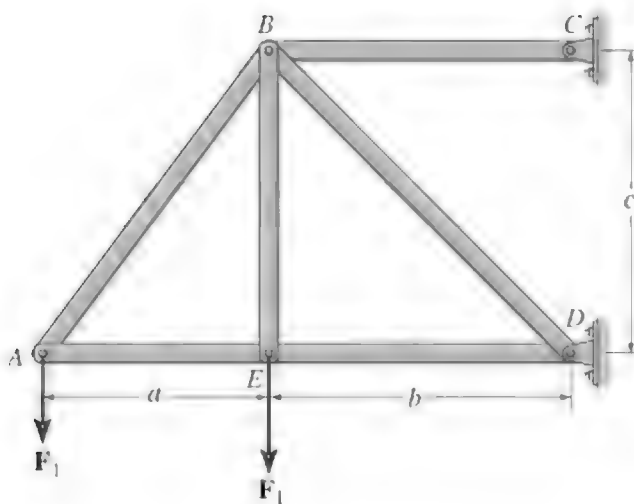
$F_2 = 0$

$\rho = 8 \frac{\text{kg}}{\text{m}}$

$a = 1.5 \text{ m}$

$b = 2 \text{ m}$

$c = 2 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

Find the weights of each bar.

$W_{AB} = \rho g \sqrt{a^2 + c^2}$ $\quad W_{BC} = \rho g b$ $\quad W_{BE} = \rho g c$

$W_{AE} = \rho g a$ $\quad W_{BD} = \rho g \sqrt{b^2 + c^2}$ $\quad W_{DE} = \rho g b$

Guesses $F_{AB} = 1\text{ N}$ $F_{AE} = 1\text{ N}$ $F_{BE} = 1\text{ N}$

$F_{BC} = 1\text{ N}$ $F_{BD} = 1\text{ N}$ $F_{DE} = 1\text{ N}$

Given

Joint A $$F_{AE} + \frac{a}{\sqrt{a^2+c^2}}F_{AB} = 0$$

$$\frac{c}{\sqrt{a^2+c^2}}F_{AB} - \frac{W_{AB}+W_{AE}}{2} = 0$$

Joint E $$F_{DE} - F_{AE} = 0$$

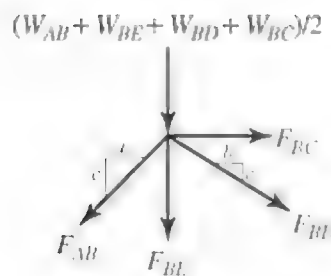
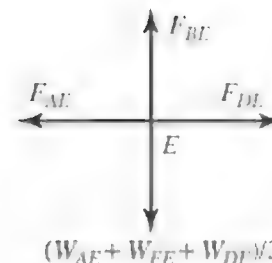
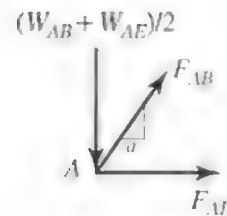
$$F_{BE} - \frac{W_{AE}+W_{BE}+W_{DE}}{2} = 0$$

Joint B $$F_{BC} + \frac{b}{\sqrt{b^2+c^2}}F_{BD} - \frac{a}{\sqrt{a^2+c^2}}F_{AB} = 0$$

$$\frac{-c}{\sqrt{a^2+c^2}}F_{AB} - F_{BE} - \frac{c}{\sqrt{b^2+c^2}}F_{BD} - \frac{W_{AB}+W_{BE}+W_{BD}+W_{BC}}{2} = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AE} \\ F_{BC} \\ F_{BD} \\ F_{BE} \\ F_{DE} \end{pmatrix} = \text{Find}(F_{AB}, F_{AE}, F_{BC}, F_{BD}, F_{BE}, F_{DE})$$

$$\begin{pmatrix} F_{AB} \\ F_{AE} \\ F_{BC} \\ F_{BD} \\ F_{BE} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 196 \\ -118 \\ 857 \\ -1045 \\ 216 \\ -118 \end{pmatrix}\text{ N}$$

Positive means tension,
Negative means Compression.

Problem 6-27

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$kN = 10^3\ N$

Given:

$P_1 = 4\ kN$

$P_2 = 0\ kN$

$a = 2\ m$

$\theta = 15\ deg$

Solution:

Take advantage of the symetry.

Initial Guesses:

$F_{BD} = 1\ kN \quad F_{CD} = 1\ kN \quad F_{AB} = 1\ kN$

$F_{CA} = 1\ kN \quad F_{BC} = 1\ kN$

Given

Joint D $\quad \dfrac{-P_1}{2} - F_{BD}\sin(2\ \theta) - F_{CD}\sin(3\ \theta) = 0$

Joint B $\quad -P_2\cos(2\ \theta) - F_{BC} = 0$

$F_{BD} - F_{AB} - P_2\sin(2\ \theta) = 0$

Joint C $\quad F_{CD}\cos(\theta) - F_{CA}\cos(\theta) = 0$

$(F_{CD} + F_{CA})\sin(\theta) + F_{BC} = 0$

$$\begin{pmatrix} F_{BD} \\ F_{CD} \\ F_{AB} \\ F_{CA} \\ F_{BC} \end{pmatrix} = \text{Find}(F_{BD}, F_{CD}, F_{AB}, F_{CA}, F_{BC}) \qquad \begin{pmatrix} F_{FD} \\ F_{ED} \\ F_{GF} \\ F_{EG} \\ F_{FE} \end{pmatrix} = \begin{pmatrix} F_{BD} \\ F_{CD} \\ F_{AB} \\ F_{CA} \\ F_{BC} \end{pmatrix}$$

$$\begin{pmatrix} F_{BD} \\ F_{CD} \\ F_{AB} \\ F_{CA} \\ F_{BC} \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ -4 \\ 0 \\ 0 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} F_{FD} \\ F_{ED} \\ F_{GF} \\ F_{EG} \\ F_{FE} \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ -4 \\ 0 \\ 0 \end{pmatrix} \text{kN}$$

Positvive means Tension,
Negative means Compression

Problem 6-28

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

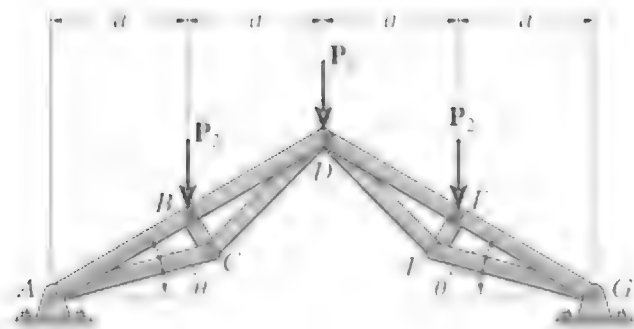
$\text{kN} = 10^3 \text{ N}$

Given:

$P_1 = 2 \text{ kN}$

$P_2 = 4 \text{ kN}$

$a = 2 \text{ m}$

$\theta = 15 \text{ deg}$

Solution:

Take advantage of the symmetry.

Initial Guesses:

$F_{BD} = 1 \text{ kN} \quad F_{CD} = 1 \text{ kN} \quad F_{AB} = 1 \text{ kN}$

$F_{CA} = 1 \text{ kN} \quad F_{BC} = 1 \text{ kN}$

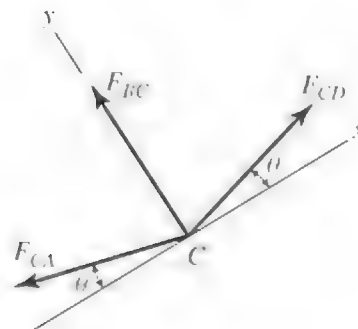
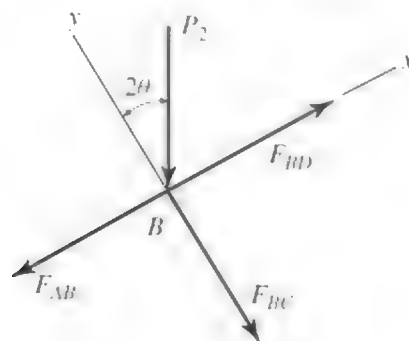
Given

Joint D $\quad \dfrac{-P_1}{2} - F_{BD}\sin(2\ \theta) - F_{CD}\sin(3\ \theta) = 0$

Joint B $\quad -P_2\cos(2\ \theta) - F_{BC} = 0$

$F_{BD} - F_{AB} - P_2\sin(2\theta) = 0$

Joint C $\quad F_{CD}\cos(\theta) - F_{CA}\cos(\theta) = 0$

$(F_{CD} + F_{CA})\sin(\theta) + F_{BC} = 0$

$$\begin{pmatrix} F_{BD} \\ F_{CD} \\ F_{AB} \\ F_{CA} \\ F_{BC} \end{pmatrix} = \text{Find}(F_{BD}, F_{CD}, F_{AB}, F_{CA}, F_{BC}) \qquad \begin{pmatrix} F_{FD} \\ F_{ED} \\ F_{GF} \\ F_{EG} \\ F_{FE} \end{pmatrix} = \begin{pmatrix} F_{BD} \\ F_{CD} \\ F_{AB} \\ F_{CA} \\ F_{BC} \end{pmatrix}$$

$$\begin{pmatrix} F_{BD} \\ F_{CD} \\ F_{AB} \\ F_{CA} \\ F_{BC} \end{pmatrix} = \begin{pmatrix} -11.46 \\ 6.69 \\ -13.46 \\ 6.69 \\ -3.46 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} F_{FD} \\ F_{ED} \\ F_{GF} \\ F_{EG} \\ F_{FE} \end{pmatrix} = \begin{pmatrix} -11.46 \\ 6.69 \\ -13.46 \\ 6.69 \\ -3.46 \end{pmatrix} \text{kN}$$

Positvive means Tension,
Negative means Compression

Problem 6-29

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$F_1 = 2 \text{ kip}$

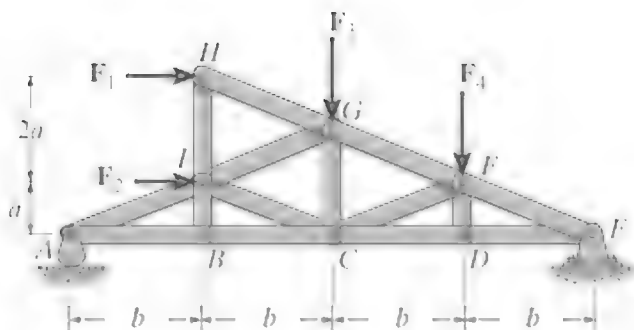
$F_2 = 1.5 \text{ kip}$

$F_3 = 3 \text{ kip}$

$F_4 = 3 \text{ kip}$

$a = 4 \text{ ft}$

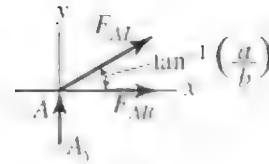
$b = 10 \text{ ft}$

Solution: $\theta = \text{atan}\left(\frac{a}{b}\right)$

Initial Guesses

$F_{AB} = 1\ \text{lb}$ $F_{BC} = 1\ \text{lb}$ $F_{CD} = 1\ \text{lb}$ $F_{DE} = 1\ \text{lb}$

$F_{AI} = 1\ \text{lb}$ $F_{BI} = 1\ \text{lb}$ $F_{CI} = 1\ \text{lb}$ $F_{CG} = 1\ \text{lb}$

$F_{CF} = 1\ \text{lb}$ $F_{DF} = 1\ \text{lb}$ $F_{EF} = 1\ \text{lb}$ $F_{HI} = 1\ \text{lb}$

$F_{GI} = 1\ \text{lb}$ $F_{GH} = 1\ \text{lb}$ $F_{FG} = 1\ \text{lb}$

Given

Joint A $F_{AI}\cos(\theta) + F_{AB} = 0$

Joint B $F_{BC} - F_{AB} = 0$

$F_{BI} = 0$

Joint C $F_{CD} - F_{BC} + (F_{CF} - F_{CI})\cos(\theta) = 0$

$F_{CG} + (F_{CF} + F_{CI})\sin(\theta) = 0$

Joint D $F_{DE} - F_{CD} = 0$

$F_{DF} = 0$

Joint I $F_2 + (F_{GI} + F_{CI} - F_{AI})\cos(\theta) = 0$

$F_{HI} - F_{BI} + (F_{GI} - F_{AI} - F_{CI})\sin(\theta) = 0$

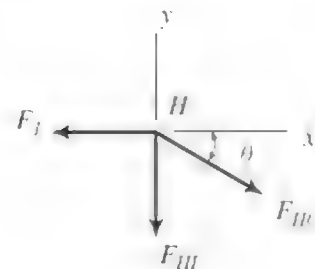
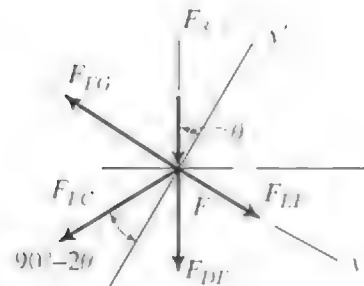
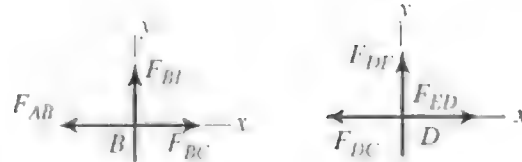
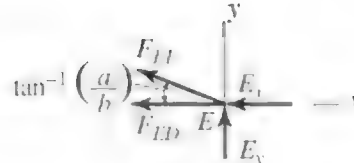
Joint H $F_{GH}\cos(\theta) + F_1 = 0$

$-F_{GH}\sin(\theta) - F_{HI} = 0$

Joint G $(F_{FG} - F_{GH} - F_{GI})\cos(\theta) = 0$

$-F_3 - F_{CG} + (F_{GH} - F_{FG} - F_{GI})\sin(\theta) = 0$

Joint F $(F_{EF} - F_{FG} - F_{CF})\cos(\theta) = 0$

$(F_{FG} - F_{CF} - F_{EF})\sin(\theta) - F_4 - F_{DF} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \\ F_{DE} \\ F_{AI} \\ F_{BI} \\ F_{CI} \\ F_{CG} \\ F_{CF} \\ F_{DF} \\ F_{EF} \\ F_{HI} \\ F_{GI} \\ F_{GH} \\ F_{FG} \end{pmatrix} = \text{Find}\left(F_{AB}, F_{BC}, F_{CD}, F_{DE}, F_{AI}, F_{BI}, F_{CI}, F_{CG}, F_{CF}, F_{DF}, F_{EF}, F_{HI}, F_{GI}, F_{GH}, F\right.$$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \\ F_{DE} \\ F_{AI} \end{pmatrix} = \begin{pmatrix} 3.75 \\ 3.75 \\ 7.75 \\ 7.75 \\ -4.04 \end{pmatrix} \text{kip} \qquad \begin{pmatrix} F_{BI} \\ F_{CI} \\ F_{CG} \\ F_{CF} \\ F_{DF} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.27 \\ 1.4 \\ -4.04 \\ 0 \end{pmatrix} \text{kip} \qquad \begin{pmatrix} F_{EF} \\ F_{HI} \\ F_{GI} \\ F_{GH} \\ F_{FG} \end{pmatrix} = \begin{pmatrix} -12.12 \\ 0.8 \\ -5.92 \\ -2.15 \\ -8.08 \end{pmatrix} \text{kip}$$

Positive means Tension, Negative means Compression

Problem 6-30

The *Howe bridge truss* is subjected to the loading shown. Determine the force in members *DE*, *EH*, and *HG*, and state if the members are in tension or compression.

Units Used:

$$kN = 10^3 \text{ N}$$

Given:

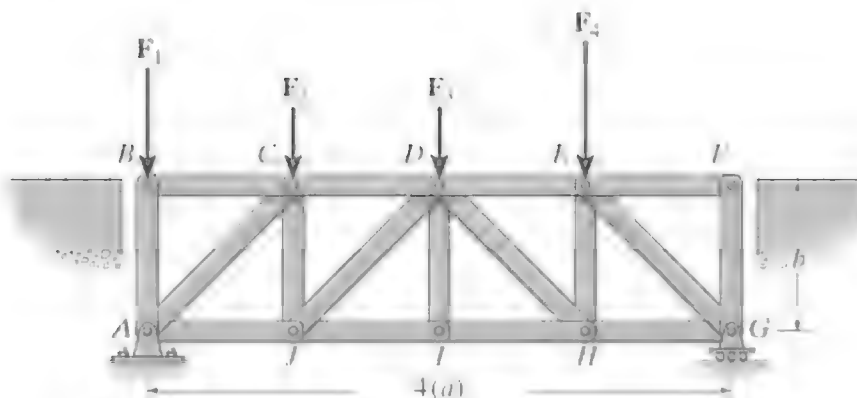
$F_1 = 30 \text{ kN}$

$F_2 = 20 \text{ kN}$

$F_3 = 20 \text{ kN}$

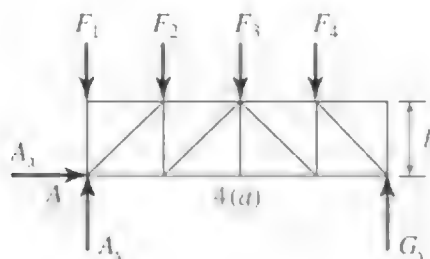
$F_4 = 40 \text{ kN}$

$a = 4 \text{ m}$

$b = 4 \text{ m}$

Solution:

$-F_2\,a - F_3(2a) - F_4(3a) + G_y(4a) = 0$

$$G_y = \frac{F_2 + 2F_3 + 3F_4}{4}$$

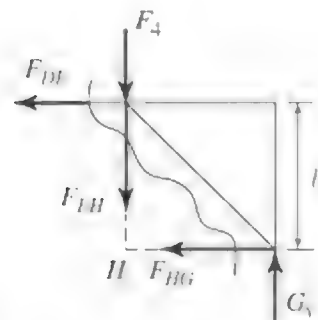
$G_y = 45 \text{ kN}$

Guesses $F_{DE} = 1 \text{ kN}$ $F_{EH} = 1 \text{ kN}$ $F_{HG} = 1 \text{ kN}$

Given

$-F_{DE} - F_{HG} = 0$ $G_y - F_4 - F_{EH} = 0$

$F_{DE}\,b + G_y\,a = 0$

$$\begin{pmatrix} F_{DE} \\ F_{EH} \\ F_{HG} \end{pmatrix} = \text{Find}(F_{DE}, F_{EH}, F_{HG}) \qquad \begin{pmatrix} F_{DE} \\ F_{EH} \\ F_{HG} \end{pmatrix} = \begin{pmatrix} -45 \\ 5 \\ 45 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-31

The *Pratt bridge truss* is subjected to the loading shown. Determine the force in members *LD*, *LK, CD*, and *KD*, and state if the members are in tension or compression.

Units Used:

$\text{kN} = 10^3 \text{ N}$

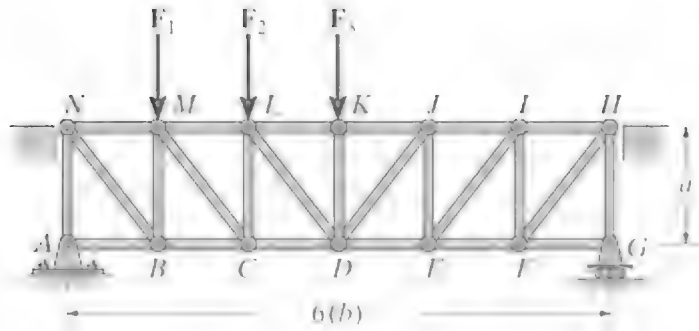
Given:

$F_1 = 50\text{ kN}$

$F_2 = 50\text{ kN}$

$F_3 = 50\text{ kN}$

$a = 4\text{ m}$

$b = 3\text{ m}$

Solution:

$A_x = 0$

$$A_y = \frac{3F_3 + 4F_2 + 5F_1}{6}$$

Guesses

$F_{LD} = 1\text{ kN}$ $\quad F_{LK} = 1\text{ kN}$

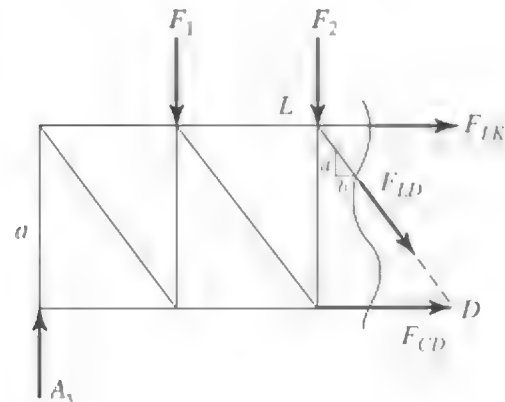
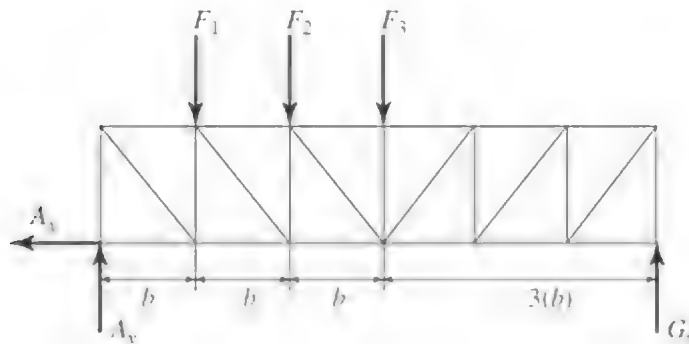
$F_{CD} = 1\text{ kN}$ $\quad F_{KD} = 1\text{ kN}$

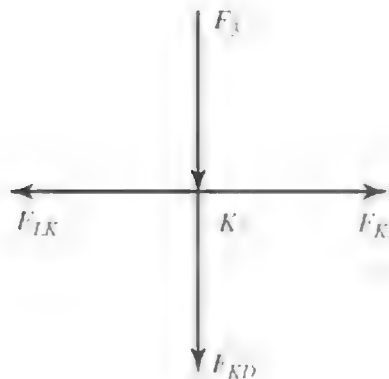
Given

$$F_2\, b + F_1(2b) - A_y(3b) - F_{LK}\, a = 0$$

$$F_{CD}a + F_1 b - A_y(2b) = 0$$

$$A_y - F_1 - F_2 - \left(\frac{a}{\sqrt{a^2 + b^2}}\right)F_{LD} = 0$$

$$-F_3 - F_{KD} = 0$$

$$\begin{pmatrix} F_{LD} \\ F_{LK} \\ F_{CD} \\ F_{KD} \end{pmatrix} = \text{Find}(F_{LD}, F_{LK}, F_{CD}, F_{KD}) \qquad \begin{pmatrix} F_{LD} \\ F_{LK} \\ F_{CD} \\ F_{KD} \end{pmatrix} = \begin{pmatrix} 0 \\ -112.5 \\ 112.5 \\ -50 \end{pmatrix}\text{kN}$$

Positive (T)
Negative (C)

Problem 6-32

The *Pratt bridge truss* is subjected to the loading shown. Determine the force in members *JI*, *JE*, and *DE*, and state if the members are in tension or compression.

Units Used:

$$kN = 10^3\ N$$

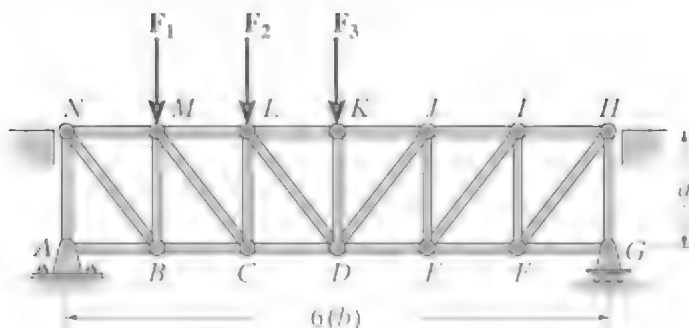
Given:

$$F_1 = 50\ kN$$

$$F_2 = 50\ kN$$

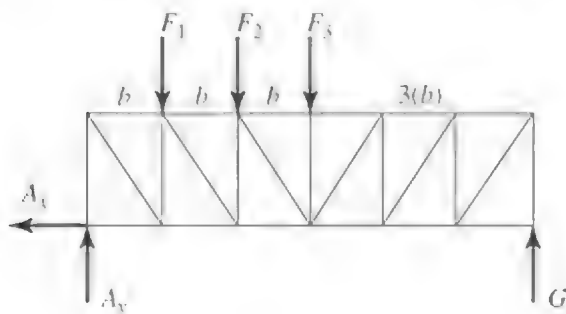
$$F_3 = 50\ kN$$

$$a = 4\ m$$

$$b = 3\ m$$

Solution:

Initial Guesses

$G_y = 1\ kN$ $\quad F_{JI} = 1\ kN$

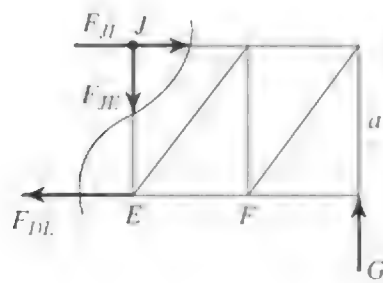
$F_{JE} = 1\ kN$ $\quad F_{DE} = 1\ kN$

Given

Entire Truss

$$-F_1 b - F_2(2b) - F_3(3b) + G_y(6b) = 0$$

Section

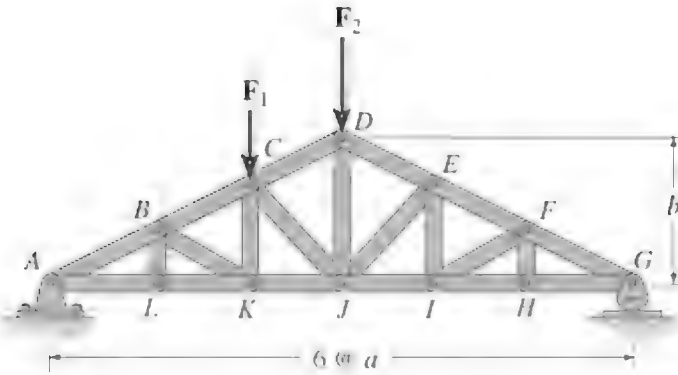
$$-F_{DE} - F_{JI} = 0 \qquad F_{JE} + G_y = 0$$

$$G_y(2b) - F_{DE}a = 0$$

$$\begin{pmatrix} G_y \\ F_{JI} \\ F_{JE} \\ F_{DE} \end{pmatrix} = \text{Find}(G_y, F_{JI}, F_{JE}, F_{DE}) \quad G_y = 50\,\text{kN}$$

$$\begin{pmatrix} F_{JI} \\ F_{JE} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} -75 \\ -50 \\ 75 \end{pmatrix} \text{kN}$$

Positive means Tension,
Negative means Compression

Problem 6-33

The roof truss supports the vertical loading shown. Determine the force in members *BC*, *CK*, and *KJ* and state if these members are in tension or compression.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$F_1 = 4\ \text{kN}$$

$$F_2 = 8\ \text{kN}$$

$$a = 2\ \text{m}$$

$$b = 3\ \text{m}$$

Solution:

Initial Guesses

$A_x = 1 \text{ kN}$ $A_y = 1 \text{ kN}$

$F_{BC} = 1 \text{ kN}$ $F_{CK} = 1 \text{ kN}$

$F_{KJ} = 1 \text{ kN}$

Given

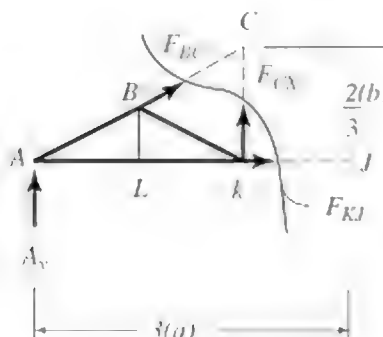
$A_x = 0$

$F_2(3a) + F_1(4a) - A_y(6a) = 0$

$$F_{KJ}\left(\frac{2b}{3}\right) + A_x\left(\frac{2b}{3}\right) - A_y(2a) = 0$$

$$F_{KJ} + A_x + \left(\frac{3a}{\sqrt{b^2 + 9a^2}}\right)F_{BC} = 0$$

$$F_{CK} + A_y + \left(\frac{b}{\sqrt{b^2 + 9a^2}}\right)F_{BC} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{KJ} \\ F_{CK} \\ F_{BC} \end{pmatrix} = \text{Find}(A_x, A_y, F_{KJ}, F_{CK}, F_{BC})$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{KJ} \\ F_{CK} \\ F_{BC} \end{pmatrix} = \begin{pmatrix} 0 \\ 6.667 \\ 13.333 \\ 0 \\ -14.907 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-34

Determine the force in members *CD*, *CJ*, *KJ*, and *DJ* of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

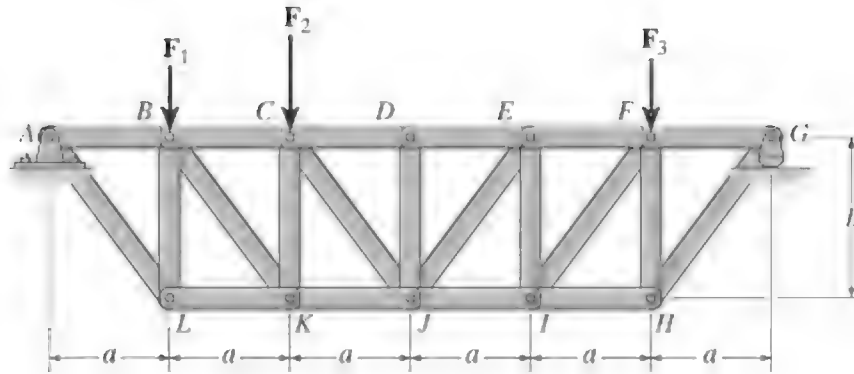
Given:

$F_1 = 4000 \text{ lb}$

$F_2 = 8000 \text{ lb}$

$F_3 = 5000 \text{ lb}$

$a = 9 \text{ ft}$

$b = 12 \text{ ft}$

Solution

Initial Guesses:

$A_y = 1 \text{ kip}$ $F_{CJ} = 1 \text{ kip}$ $F_{DJ} = 1 \text{ kip}$ $F_{CD} = 1 \text{ kip}$ $F_{KJ} = 1 \text{ kip}$

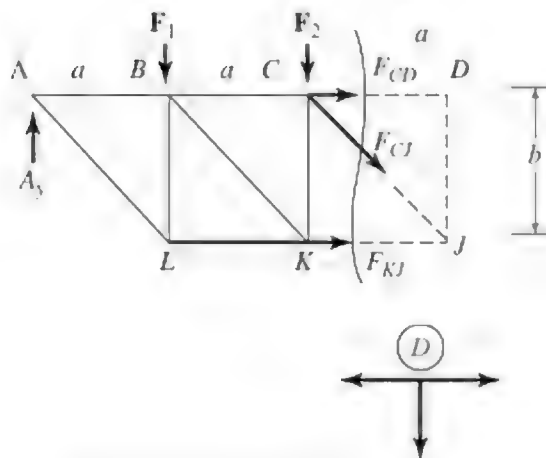
Given

$$F_3\,a + F_2(4a) + F_1(5a) - A_y(6a) = 0$$

$$-A_y(2a) + F_1\,a + F_{KJ}\,b = 0$$

$$F_{CD} + F_{KJ} + \left(\frac{a}{\sqrt{a^2+b^2}}\right)F_{CJ} = 0$$

$$A_y - F_1 - F_2 - \left(\frac{b}{\sqrt{a^2+b^2}}\right)F_{CJ} = 0$$

$$-F_{DJ} = 0$$

$$\begin{pmatrix} A_y \\ F_{KJ} \\ F_{CJ} \\ F_{DJ} \\ F_{CD} \end{pmatrix} = \text{Find}(A_y, F_{KJ}, F_{CJ}, F_{DJ}, F_{CD})$$

$$\begin{pmatrix} A_y \\ F_{KJ} \\ F_{CJ} \\ F_{DJ} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} 9.5 \\ 11.25 \\ -3.125 \\ 0 \\ -9.375 \end{pmatrix} \text{kip}$$

Positive (T)
Negative (C)

Problem 6-35

Determine the force in members *EI* and *JI* of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

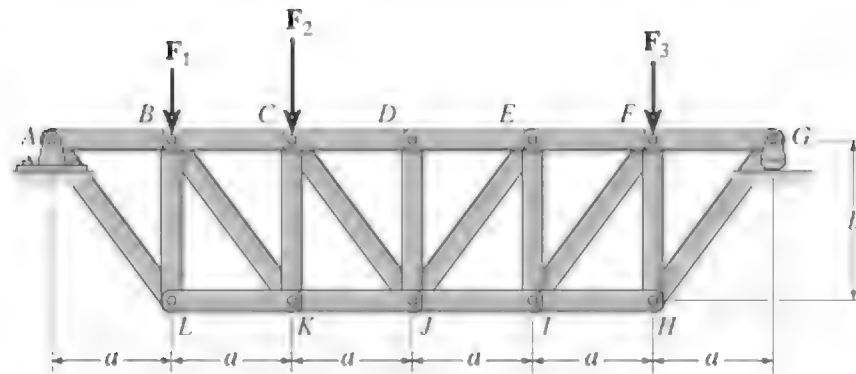
Given:

$F_1 = 4000\ \text{lb}$

$F_2 = 8000\ \text{lb}$

$F_3 = 5000\ \text{lb}$

$a = 9\ \text{ft}$

$b = 12\ \text{ft}$

Solution:

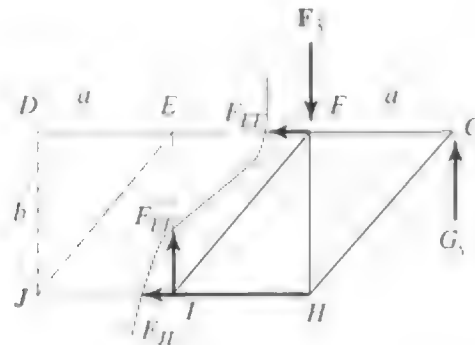
Initial Guesses:

$G_y = 1\ \text{kip} \quad F_{EI} = 1\ \text{kip} \quad F_{JI} = 1\ \text{kip}$

Given

$$-F_1 a - F_2 2a - F_3 5a + G_y 6a = 0$$

$$G_y 2a - F_3 a - F_{JI} b = 0$$

$$F_{EI} - F_3 + G_y = 0$$

$$\begin{pmatrix} G_y \\ F_{JI} \\ F_{EI} \end{pmatrix} = \text{Find}(G_y, F_{JI}, F_{EI}) \qquad \begin{pmatrix} G_y \\ F_{JI} \\ F_{EI} \end{pmatrix} = \begin{pmatrix} 7.5 \\ 7.5 \\ -2.5 \end{pmatrix} \text{kip}$$

Positive (T)
Negative (C)

Problem 6-36

Determine the force in members *BE*, *EF*, and *CB*, and state if the members are in tension or compression.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F_1 = 5 \text{ kN}$ $\quad F_4 = 10 \text{ kN}$

$F_2 = 10 \text{ kN}$ $\quad a = 4 \text{ m}$

$F_3 = 5 \text{ kN}$ $\quad b = 4 \text{ m}$

Solution: $\theta = \operatorname{atan}\left(\frac{a}{b}\right)$

Inital Guesses

$F_{CB} = 1 \text{ kN}$ $\quad F_{BE} = 1 \text{ kN}$ $\quad F_{EF} = 1 \text{ kN}$

Given

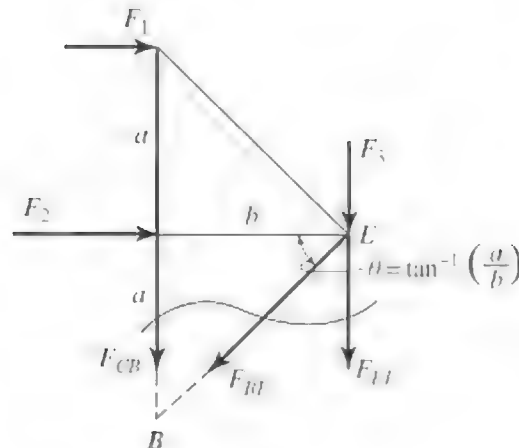
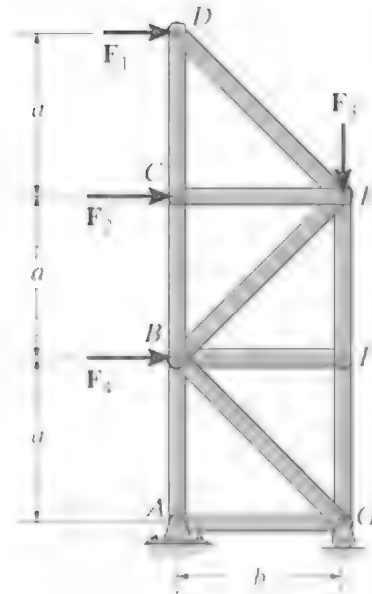
$$F_1 + F_2 - F_{BE}\cos(\theta) = 0$$

$$-F_{CB} - F_{EF} - F_{BE}\sin(\theta) - F_3 = 0$$

$$-F_1\, a + F_{CB}\, b = 0$$

$$\begin{pmatrix} F_{CB} \\ F_{BE} \\ F_{EF} \end{pmatrix} = \operatorname{Find}(F_{CB}, F_{BE}, F_{EF})$$

$$\begin{pmatrix} F_{CB} \\ F_{BE} \\ F_{EF} \end{pmatrix} = \begin{pmatrix} 5 \\ 21.2 \\ -25 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-37

Determine the force in members BF, BG, and AB, and state if the members are in tension or compression.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$F_1 = 5 \text{ kN}$ $\quad F_4 = 10 \text{ kN}$

$F_2 = 10\ \text{kN}$ $a = 4\ \text{m}$

$F_3 = 5\ \text{kN}$ $b = 4\ \text{m}$

Solution: $\theta = \text{atan}\left(\frac{a}{b}\right)$

Inital Guesses

$F_{AB} = 1\ \text{kN}$ $F_{BG} = 1\ \text{kN}$ $F_{BF} = 1\ \text{kN}$

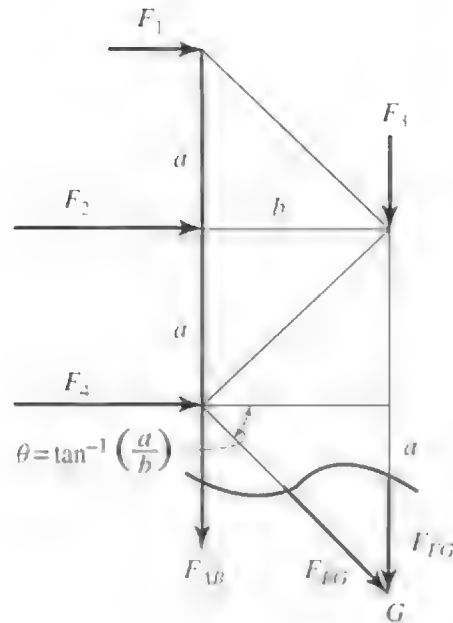
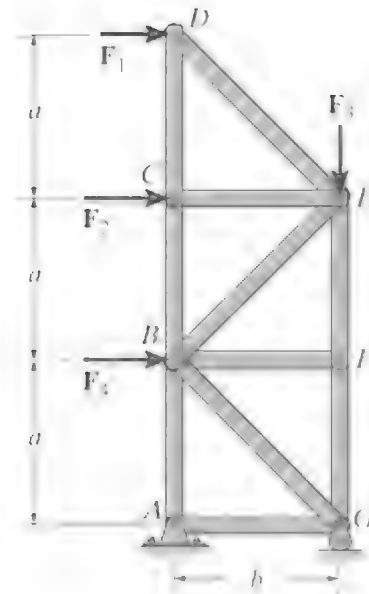
Given

$$F_1 + F_2 + F_4 + F_{BG}\cos(\theta) = 0$$

$$-F_1 3a - F_2 2a - F_4 a + F_{AB} b = 0$$

$$-F_{BF} = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{BG} \\ F_{BF} \end{pmatrix} = \text{Find}(F_{AB}, F_{BG}, F_{BF})$$

$$\begin{pmatrix} F_{AB} \\ F_{BG} \\ F_{BF} \end{pmatrix} = \begin{pmatrix} 45 \\ -35.4 \\ 0 \end{pmatrix} \text{kN}$$ Positive (T) Negative (C)

Problem 6-38

Determine the force developed in members *GB* and *GF* of the bridge truss and state if these members are in tension or compression.

Given:

$F_1 = 600\ \text{lb}$

$F_2 = 800 \text{ lb}$

$a = 10 \text{ ft}$

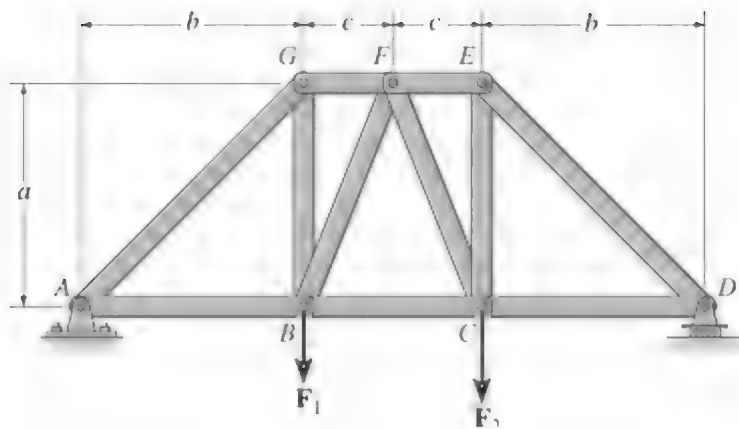
$b = 10 \text{ ft}$

$c = 4 \text{ ft}$

Solution:

Initial Guesses

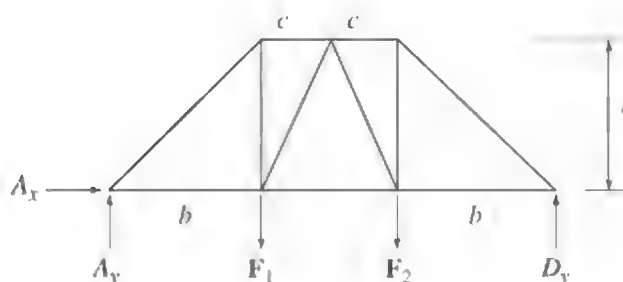
$A_x = 1 \text{ lb}$ $\quad A_y = 1 \text{ lb}$

$F_{GB} = 1 \text{ lb}$ $\quad F_{GF} = 1 \text{ lb}$

Given

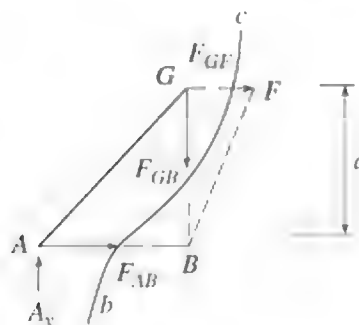
$$F_2 b + F_1(b + 2c) - A_y 2(b + c) = 0$$

$$A_x = 0$$

$$A_y - F_{GB} = 0$$

$$-A_y b - F_{GF} a = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{GB} \\ F_{GF} \end{pmatrix} = \text{Find}(A_x, A_y, F_{GB}, F_{GF})$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{GB} \\ F_{GF} \end{pmatrix} = \begin{pmatrix} 0 \\ 671.429 \\ 671.429 \\ -671.429 \end{pmatrix} \text{lb}$$

Positive (T)
Negative (C)

Problem 6-39

Determine the force members *BC*, *FC*, and *FE*, and state if the members are in tension or compression.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$F_1 = 6 \text{ kN}$

$F_2 = 6 \text{ kN}$

$a = 3 \text{ m}$

$b = 3 \text{ m}$

Solution: $\theta = \operatorname{atan}\left(\frac{a}{b}\right)$

Initial Guesses

$D_y = 1 \text{ kN}$ $F_{BC} = 1 \text{ kN}$

$F_{FC} = 1 \text{ kN}$ $F_{FE} = 1 \text{ kN}$

Given

$$-F_1 b - F_2(2b) + D_y(3b) = 0$$

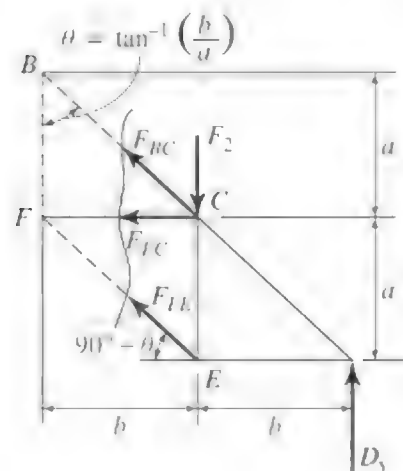
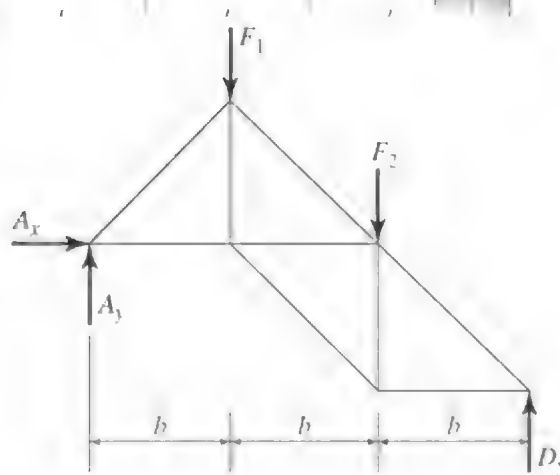
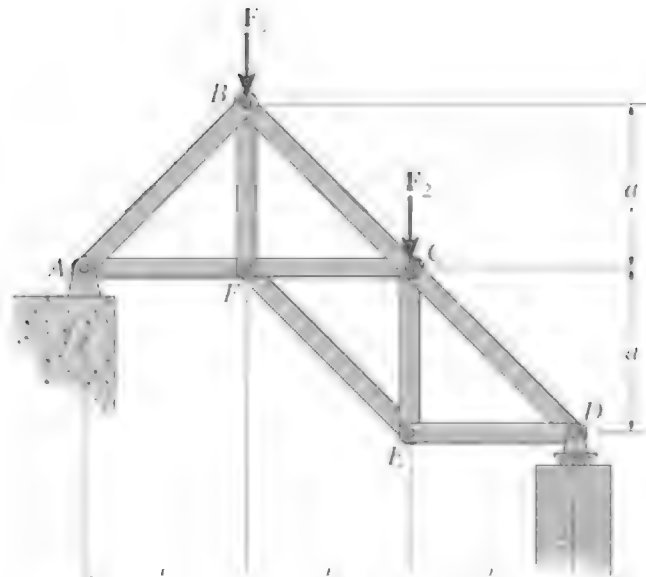
$$D_y b - F_{FE}\cos(\theta)a = 0$$

$$-F_{FC} - (F_{BC} + F_{FE})\cos(\theta) = 0$$

$$-F_2 + D_y + (F_{FE} + F_{BC})\sin(\theta) = 0$$

$$\begin{pmatrix} D_y \\ F_{BC} \\ F_{FC} \\ F_{FE} \end{pmatrix} = \operatorname{Find}(D_y, F_{BC}, F_{FC}, F_{FE})$$

$$\begin{pmatrix} D_y \\ F_{BC} \\ F_{FC} \\ F_{FE} \end{pmatrix} = \begin{pmatrix} 6 \\ -8.49 \\ 0 \\ 8.49 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-40

Determine the force in members *IC* and *CG* of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 6\ kN$

$F_2 = 6\ kN$

$a = 1.5\ m$

$b = 2\ m$

Solution:

By inspection of joints *B*, *D*, *H* and *I*.

AB, *BC*, *CD*, *DE*, *HI*, and *GI* are all zero-force members.

Guesses $A_y = 1\ kN$ $F_{IC} = 1\ kN$ $F_{CG} = 1\ kN$ $F_{CJ} = 1\ kN$

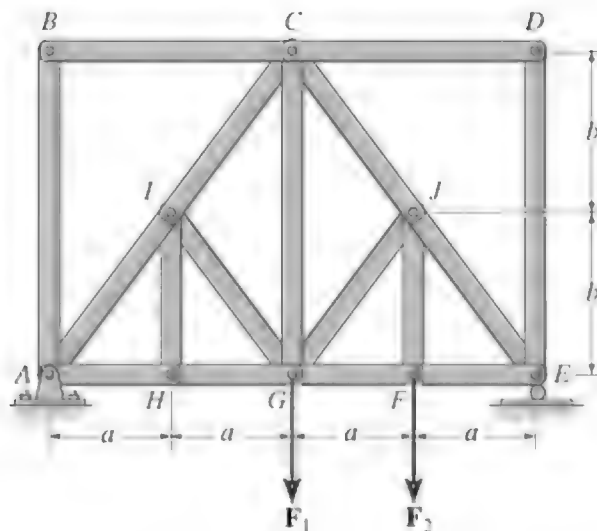
Given $-A_y(4a) + F_1(2a) + F_2 a = 0$

$$-A_y(2a) - \frac{b}{\sqrt{a^2+b^2}} F_{IC}\, a - \frac{a}{\sqrt{a^2+b^2}} F_{IC}\, b = 0$$

$$\frac{-a}{\sqrt{a^2+b^2}} F_{IC} + \frac{a}{\sqrt{a^2+b^2}} F_{CJ} = 0$$

$$\frac{-b}{\sqrt{a^2+b^2}} F_{IC} - \frac{b}{\sqrt{a^2+b^2}} F_{CJ} - F_{CG} = 0$$

$$\begin{pmatrix} A_y \\ F_{IC} \\ F_{CG} \\ F_{CJ} \end{pmatrix} = \text{Find}(A_y, F_{IC}, F_{CG}, F_{CJ}) \qquad \begin{pmatrix} A_y \\ F_{IC} \\ F_{CG} \\ F_{CJ} \end{pmatrix} = \begin{pmatrix} 4.5 \\ -5.625 \\ 9 \\ -5.625 \end{pmatrix} kN$$

Positive (T)
Negative (C)

Problem 6-41

Determine the force in members *JE* and *GF* of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 6\ kN$

$F_2 = 6\ kN$

$a = 1.5\ m$

$b = 2\ m$

Solution:

By inspection of joints *B*, *D*, *H* and *I*.

AB, *BC*, *CD*, *DE*, *HI*, and *GI* are all zero-force members.

Guesses $E_y = 1\ kN$ $F_{JE} = 1\ kN$ $F_{GF} = 1\ kN$

Given $-F_1(2a) - F_2(3a) + E_y(4a) = 0$

$$E_y + \frac{b}{\sqrt{a^2 + b^2}} F_{JE} = 0$$

$$\frac{-a}{\sqrt{a^2 + b^2}} F_{JE} - F_{GF} = 0$$

$$\begin{pmatrix} E_y \\ F_{JE} \\ F_{GF} \end{pmatrix} = \text{Find}(E_y, F_{JE}, F_{GF}) \qquad \begin{pmatrix} E_y \\ F_{JE} \\ F_{GF} \end{pmatrix} = \begin{pmatrix} 7.5 \\ -9.375 \\ 5.625 \end{pmatrix} kN$$

Positive (T)
Negative (C)

Problem 6-42

Determine the force in members *BC*, *HC*, and *HG*. After the truss is sectioned use a single equation of equilibrium for the calculation of each force. State if these members are in tension or compression.

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 2 \text{ kN}$ $F_4 = 5 \text{ kN}$ $a = 5 \text{ m}$

$F_2 = 4 \text{ kN}$ $F_5 = 3 \text{ kN}$ $b = 2 \text{ m}$

$F_3 = 4 \text{ kN}$ $c = 3 \text{ m}$

Solution:

Guesses

$A_x = 1 \text{ kN}$ $A_y = 1 \text{ kN}$

$F_{BC} = 1 \text{ kN}$ $F_{HC} = 1 \text{ kN}$

$F_{HG} = 1 \text{ kN}$ $d = 1 \text{ m}$

Given

$$\frac{c}{a+d} = \frac{b}{a} \qquad -A_x = 0$$

$$(F_1 - A_y)(4a) + F_2(3a) + F_3(2a) + F_4(a) = 0$$

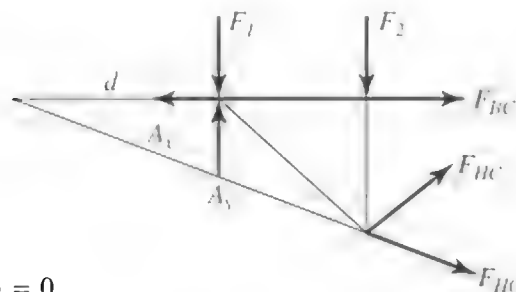
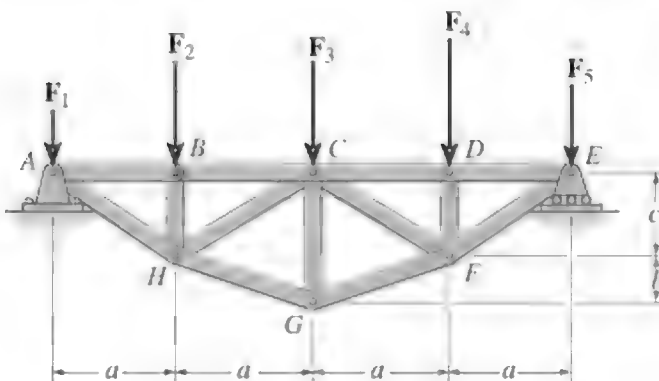
$$(F_1 - A_y)(a) + A_x(c) - F_{BC}(c) = 0$$

$$(F_1 - A_y)(2a) + F_2(a) + \frac{a}{\sqrt{a^2+b^2}} F_{HG}(c) + \frac{b}{\sqrt{a^2+b^2}} F_{HG}(a) = 0$$

$$(A_y - F_1)(d) - F_2(a+d) + \frac{c}{\sqrt{a^2+c^2}} F_{HC}(a+d) + \frac{a}{\sqrt{a^2+c^2}} F_{HC}(c) = 0$$

$$\begin{pmatrix} A_y \\ A_x \\ F_{BC} \\ F_{HC} \\ F_{HG} \\ d \end{pmatrix} = \text{Find}(A_y, A_x, F_{BC}, F_{HC}, F_{HG}, d) \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 8.25 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} F_{BC} \\ F_{HC} \\ F_{HG} \end{pmatrix} = \begin{pmatrix} -10.417 \\ 2.235 \\ 9.155 \end{pmatrix} \text{kN}$$

$d = 2.5 \text{ m}$

Positive (T)
Negative (C)

Problem 6-43

Determine the force in members *CD*, *CF*, and *CG* and state if these members are in tension or

compression.

Units Used:

$$kN = 10^3\ N$$

Given:

$F_1 = 2\ kN$ $F_4 = 5\ kN$ $a = 5\ m$

$F_2 = 4\ kN$ $F_5 = 3\ kN$ $b = 2\ m$

$F_3 = 4\ kN$ $c = 3\ m$

Solution:

Guesses

$E_y = 1\ kN$ $F_{CD} = 1\ kN$

$F_{CF} = 1\ kN$ $F_{CG} = 1\ kN$

$F_{FG} = 1\ kN$ $F_{GH} = 1\ kN$

Given

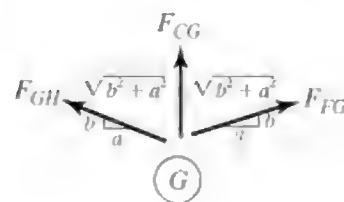
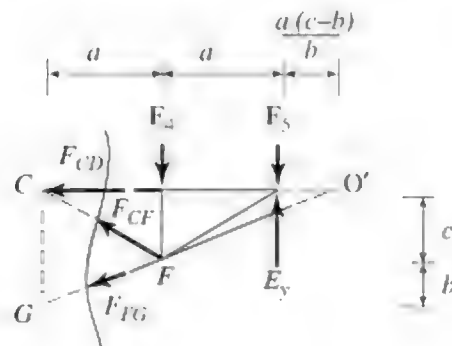
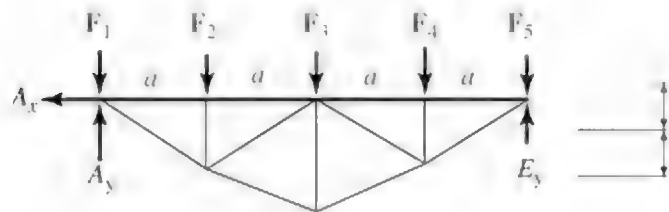
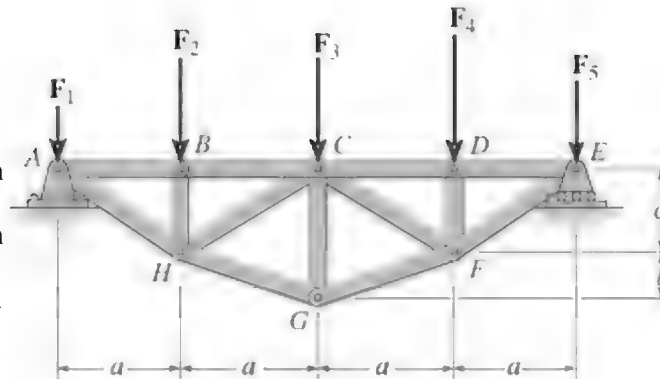
$$-F_2(a) - F_3(2a) - F_4(3a) + (E_y - F_5)(4a) = 0$$

$$F_{CD}(c) + (E_y - F_5)(a) = 0$$

$$-F_4(a) - (F_5 - E_y)(2a) - \frac{a}{\sqrt{a^2 + b^2}}F_{FG}(b + c) = 0$$

$$\frac{a}{\sqrt{a^2 + b^2}}F_{FG} - \frac{a}{\sqrt{a^2 + b^2}}F_{GH} = 0$$

$$\frac{b}{\sqrt{a^2 + b^2}}(F_{FG} + F_{GH}) + F_{CG} = 0$$

$$(F_5 - E_y)\frac{a(c - b)}{b} + F_4\left[a + \frac{a(c - b)}{b}\right] - \frac{c}{\sqrt{a^2 + c^2}}F_{CF}\left[2\ a + \frac{a(c - b)}{b}\right] = 0$$

$$\begin{pmatrix} E_y \\ F_{CD} \\ F_{CF} \\ F_{CG} \\ F_{FG} \\ F_{GH} \end{pmatrix} = \text{Find}(E_y, F_{CD}, F_{CF}, F_{CG}, F_{FG}, F_{GH}) \qquad \begin{pmatrix} E_y \\ F_{CD} \\ F_{CF} \\ F_{CG} \\ F_{FG} \\ F_{GH} \end{pmatrix} = \begin{pmatrix} 9.75 \\ -11.25 \\ 3.207 \\ -6.8 \\ 9.155 \\ 9.155 \end{pmatrix} \text{kN} \quad \begin{matrix} \text{Positive (T)} \\ \text{Negative (C)} \end{matrix}$$

Problem 6-44

Determine the force in members *OE*, *LE*, and *LK* of the *Baltimore truss* and state if the members are in tension or compression.

Units Used:

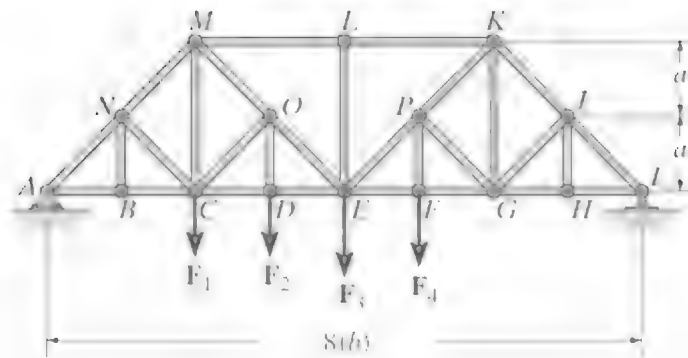
$kN = 10^3 \text{ N}$

Given:

$F_1 = 2 \text{ kN} \quad a = 2 \text{ m}$

$F_2 = 2 \text{ kN} \quad b = 2 \text{ m}$

$F_3 = 5 \text{ kN}$

$F_4 = 3 \text{ kN}$

Solution:

$A_x = 0 \text{ kN}$

Initial Guesses

$A_y = 1 \text{ kN} \quad F_{OE} = 1 \text{ kN}$

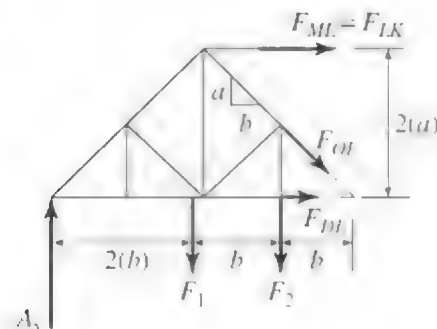
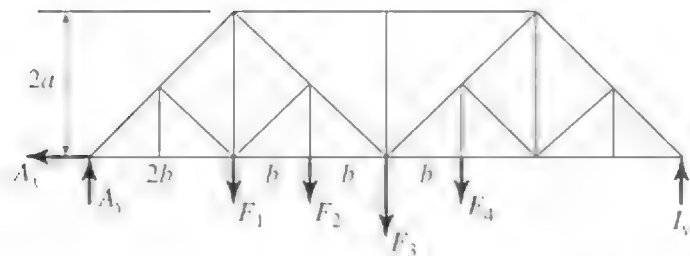
$F_{DE} = 1 \text{ kN} \quad F_{LK} = 1 \text{ kN}$

$F_{LE} = 1 \text{ kN}$

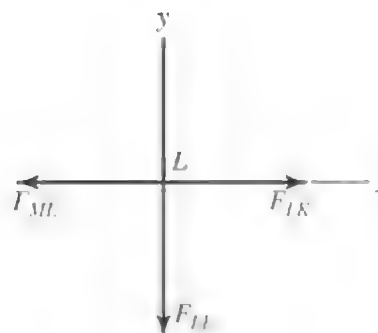
Given

$F_{LE} = 0$

$F_4(3b) + F_3(4b) + F_2(5b) + F_1(6b) - A_y(8b) = 0$

$$F_{LK} + F_{DE} + F_{OE}\frac{b}{\sqrt{a^2 + b^2}} = 0$$

$$A_y - F_1 - F_2 - F_{OE}\frac{a}{\sqrt{a^2 + b^2}} = 0$$

$$-F_{LK}(2a) + F_2(b) + F_1(2b) - A_y(4b) = 0$$

$$\begin{pmatrix} A_y \\ F_{OE} \\ F_{DE} \\ F_{LK} \\ F_{LE} \end{pmatrix} = \text{Find}(A_y, F_{OE}, F_{DE}, F_{LK}, F_{LE})$$

$$\begin{pmatrix} A_y \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 6.375 \\ 7.375 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} F_{OE} \\ F_{LE} \\ F_{LK} \end{pmatrix} = \begin{pmatrix} 3.36 \\ 0 \\ -9.75 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-45

Determine the force in member *GJ* of the truss and state if this member is in tension or compression.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

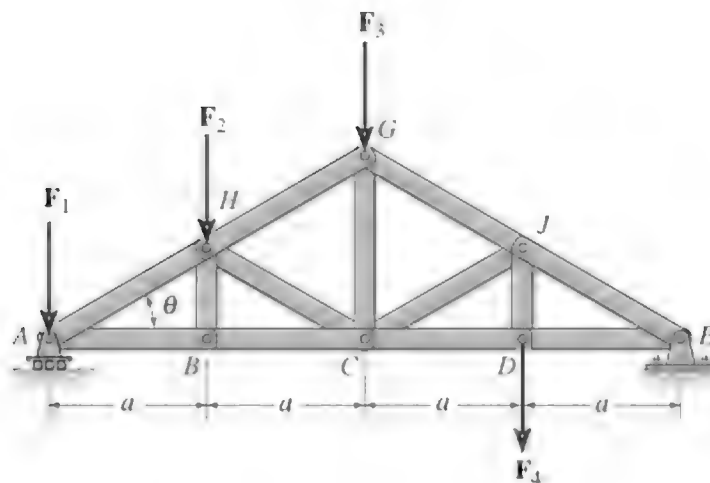
$F_1 = 1000 \text{ lb}$

$F_2 = 1000 \text{ lb}$

$F_3 = 1000 \text{ lb}$

$F_4 = 1000 \text{ lb}$

$a = 10 \text{ ft}$

$\theta = 30 \text{ deg}$

Solution:

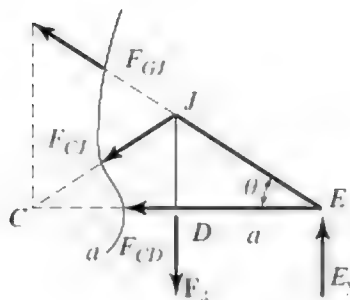
Guess $E_y = 1\ \text{lb}$ $F_{GJ} = 1\ \text{lb}$

Given

$$-F_2(a) - F_3(2a) - F_4(3a) + E_y(4a) = 0$$

$$-F_4(a) + E_y(2a) + F_{GJ}\sin(\theta)(2a) = 0$$

$$\begin{pmatrix} E_y \\ F_{GJ} \end{pmatrix} = \text{Find}(E_y, F_{GJ}) \qquad \begin{pmatrix} E_y \\ F_{GJ} \end{pmatrix} = \begin{pmatrix} 1.5 \\ -2 \end{pmatrix} \text{kip}$$

Positive (T)
Negative (C)

Problem 6-46

Determine the force in member *GC* of the truss and state if this member is in tension or compression.

Units Used:

$$\text{kip} = 10^3\ \text{lb}$$

Given:

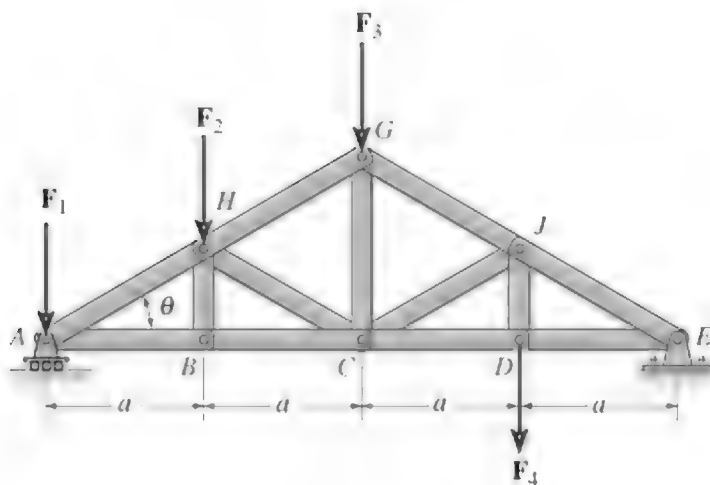
$F_1 = 1000\ \text{lb}$

$F_2 = 1000\ \text{lb}$

$F_3 = 1000\ \text{lb}$

$F_4 = 1000\ \text{lb}$

$a = 10\ \text{ft}$

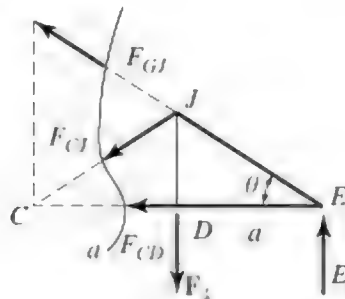
$\theta = 30\ \text{deg}$

Solution:

Guess $E_y = 1\ \text{lb}$ $F_{GJ} = 1\ \text{lb}$

$F_{HG} = 1\ \text{lb}$ $F_{GC} = 1\ \text{lb}$

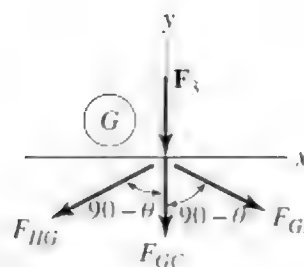
Given

$$-F_2(a) - F_3(2a) - F_4(3a) + E_y(4a) = 0$$

$$-F_4(a) + E_y(2a) + F_{GJ}\sin(\theta)(2a) = 0$$

$$-F_{HG}\cos(\theta) + F_{GJ}\cos(\theta) = 0$$

$$-F_3 - F_{GC} - (F_{HG} + F_{GJ})\sin(\theta) = 0$$

$$\begin{pmatrix} E_y \\ F_{GJ} \\ F_{GC} \\ F_{HG} \end{pmatrix} = \text{Find}(E_y, F_{GJ}, F_{GC}, F_{HG}) \qquad \begin{pmatrix} E_y \\ F_{GJ} \\ F_{GC} \\ F_{HG} \end{pmatrix} = \begin{pmatrix} 1.5 \\ -2 \\ 1 \\ -2 \end{pmatrix} \text{kip}$$

Positive (T)
Negative (C)

Problem 6-47

Determine the force in members *KJ*, *JN*, and *CD*, and state if the members are in tension or compression. Also indicate all zero-force members.

Units Used:

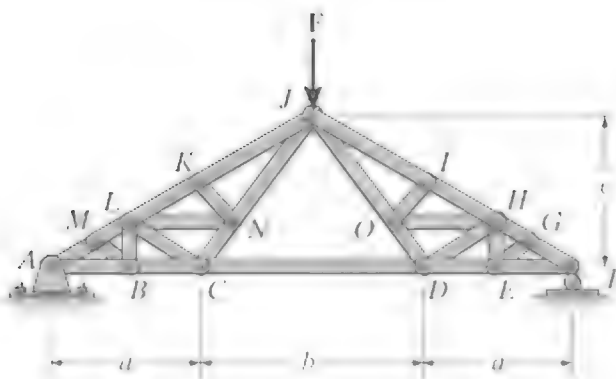
$\text{kip} = 10^3 \text{ lb}$

Given:

$F = 3 \text{ kip}$

$a = 20 \text{ ft}$

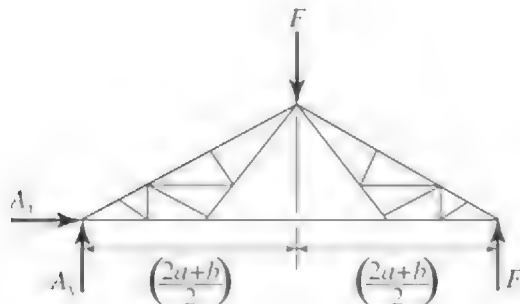
$b = 30 \text{ ft}$

$c = 20 \text{ ft}$

Solution: $A_x = 0$

$$\theta = \text{atan}\left(\frac{2c}{2a+b}\right) \qquad \phi = \text{atan}\left(\frac{2c}{b}\right)$$

Initial Guesses:

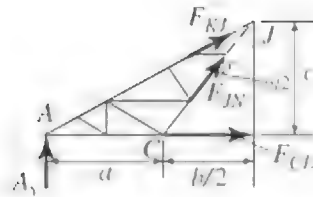
$A_y = 1 \text{ lb}$ $\quad F_{CD} = 1 \text{ lb}$

$F_{KJ} = 1 \text{ lb}$ $\quad F_{JN} = 1 \text{ lb}$

Given

$$F\left(a + \frac{b}{2}\right) - A_y(2a + b) = 0$$

$$F_{CD}c - A_y\left(a + \frac{b}{2}\right) = 0$$

$$F_{CD} + F_{JN}\cos(\phi) + F_{KJ}\cos(\theta) = 0$$

$$A_y + F_{JN}\sin(\phi) + F_{KJ}\sin(\theta) = 0$$

$$\begin{pmatrix} A_y \\ F_{CD} \\ F_{JN} \\ F_{KJ} \end{pmatrix} = \text{Find}(A_y, F_{CD}, F_{JN}, F_{KJ}) \qquad A_y = 1.5\ \text{kip} \qquad \begin{pmatrix} F_{CD} \\ F_{JN} \\ F_{KJ} \end{pmatrix} = \begin{pmatrix} 2.625 \\ 0 \\ -3.023 \end{pmatrix}\text{kip}$$

Positive (T), Negative (C)

Problem 6-48

Determine the force in members *BG*, *HG*, and *BC* of the truss and state if the members are in tension or compression.

Units Used:

$kN = 10^3\ N$

Given:

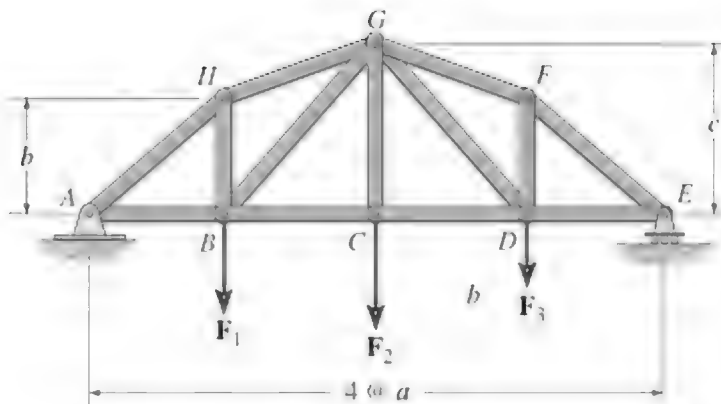
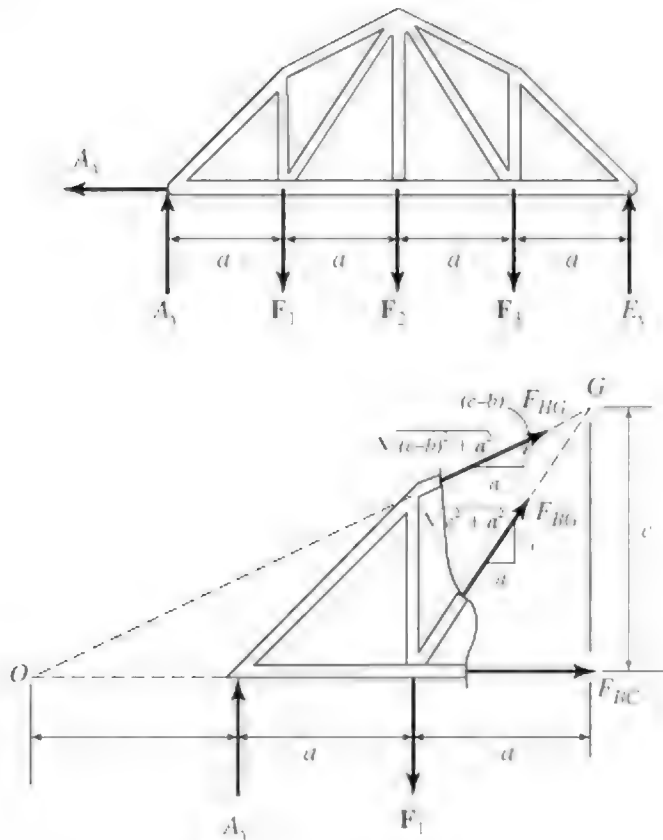
$F_1 = 6\ kN$

$F_2 = 7\ kN$

$F_3 = 4\ kN$

$a = 3\ m$

$b = 3\ m$

$c = 4.5\ m$

Solution:

Initial Guesses $F_{BG} = 1\ \text{kN}$

$A_x = 1\ \text{kN}$ $F_{HG} = 1\ \text{kN}$

$A_y = 1\ \text{kN}$ $F_{BC} = 1\ \text{kN}$

Given

$$-A_x = 0$$

$$-A_y(a) - \left[\frac{a}{\sqrt{(c-b)^2 + a^2}}\right] F_{HG}(b) = 0$$

$$F_3(a) + F_2(2a) + F_1(3a) - A_y(4a) = 0$$

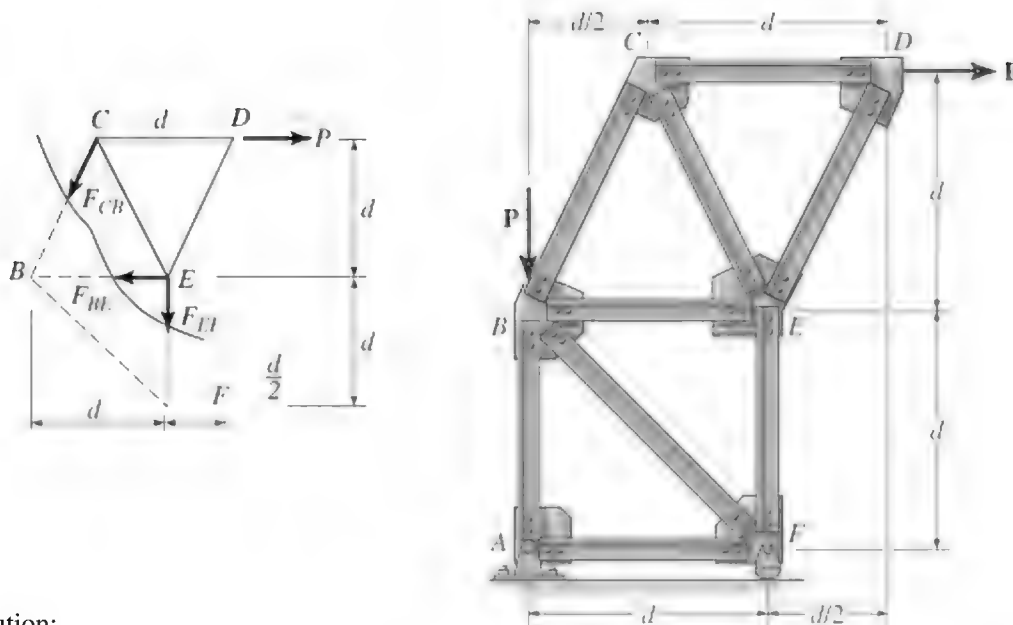
$$F_{BC} + \left[\frac{a}{\sqrt{(c-b)^2 + a^2}}\right] F_{HG} + \left(\frac{a}{\sqrt{a^2 + c^2}}\right) F_{BG} - A_x = 0$$

$$A_y - F_1 + \left[\frac{c-b}{\sqrt{(c-b)^2 + a^2}}\right] F_{HG} + \left(\frac{c}{\sqrt{a^2 + c^2}}\right) F_{BG} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{HG} \\ F_{BG} \\ F_{BC} \end{pmatrix} = \text{Find}\left(A_x, A_y, F_{HG}, F_{BG}, F_{BC}\right) \qquad \begin{pmatrix} A_x \\ A_y \\ F_{HG} \\ F_{BG} \\ F_{BC} \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \\ -10.062 \\ 1.803 \\ 8 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-49

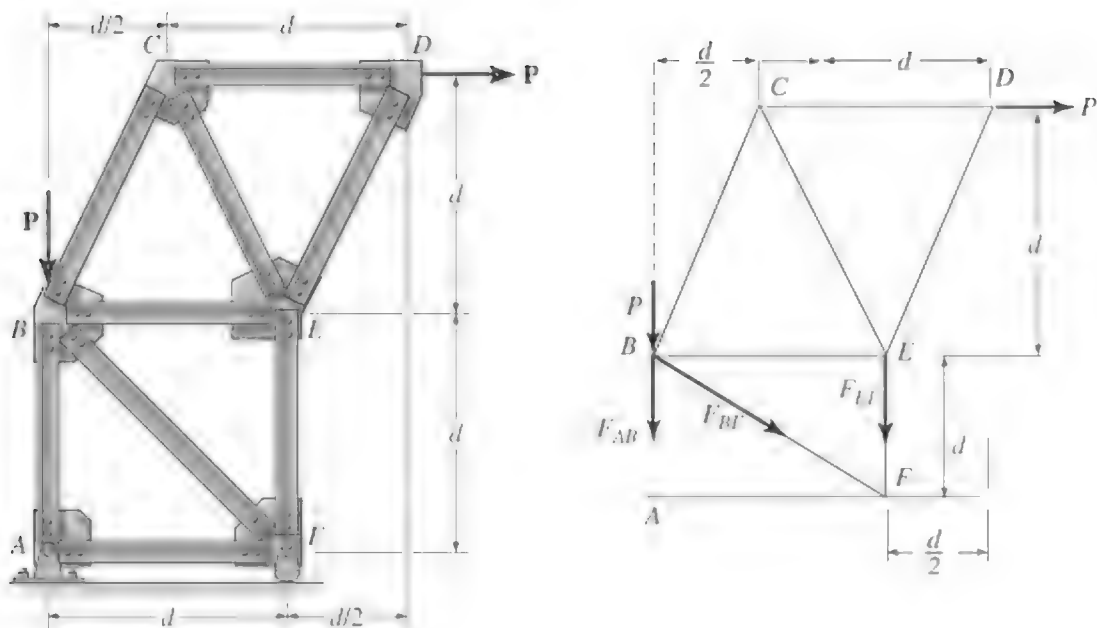
The skewed truss carries the load shown. Determine the force in members *CB*, *BE*, and *EF* and state if these members are in tension or compression. Assume that all joints are pinned.

Solution:

$\Sigma M_B = 0;\quad -P\,d - F_{EF}\,d = 0 \qquad F_{EF} = -P \qquad F_{EF} = P \quad \text{(C)}$

$\Sigma M_E = 0;\quad -P\,d + \frac{2}{\sqrt{5}}F_{CB}\,d = 0 \qquad F_{CB} = \frac{\sqrt{5}}{2}P \qquad F_{CB} = 1.12P \quad \text{(T)}$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;\quad P - \frac{1}{\sqrt{5}}F_{CB} - F_{BE} = 0 \qquad P_{BE} = \frac{P}{2} \qquad F_{BE} = 0.5P \quad \text{(T)}$

Problem 6-50

The skewed truss carries the load shown. Determine the force in members *AB*, *BF*, and *EF* and state if these members are in tension or compression. Assume that all joints are pinned.

Solution:

$\Sigma M_F = 0;$ $-P\,2d + P\,d + F_{AB}\,d = 0$ $F_{AB} = P$ $F_{AB} = P$ (T)

$\Sigma M_B = 0;$ $-P\,d - F_{EF}\,d = 0$ $F_{EF} = -P$ $F_{EF} = P$ (C)

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;$ $P + F_{BF}\frac{1}{\sqrt{2}} = 0$ $F_{BE} = -\sqrt{2}P$ $F_{BF} = 1.41P$ (C)

Problem 6-51

Determine the force developed in members BC and CH of the roof truss and state if the members are in tension or compression.

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 1.5\ kN$

$F_2 = 2\ kN$

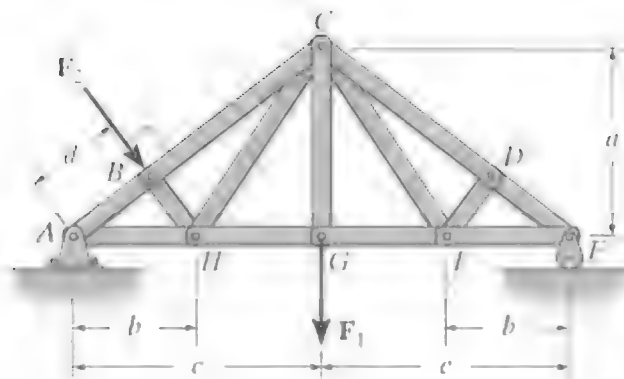
$a = 1.5\ \text{m}$

$b = 1\ \text{m}$

$c = 2\ \text{m}$

$d = 0.8\ \text{m}$

Solution:

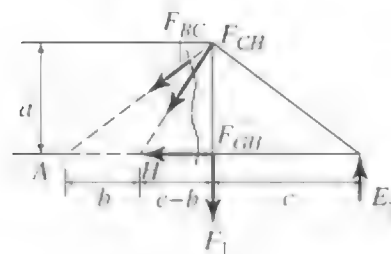
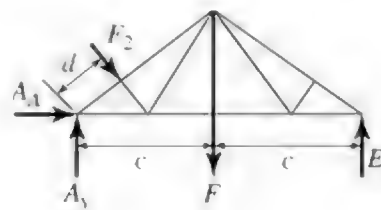
$\theta = \operatorname{atan}\left(\frac{a}{c}\right) \qquad \phi = \operatorname{atan}\left(\frac{a}{c-b}\right)$

Initial Guesses:

$E_y = 1\ \text{kN} \qquad F_{BC} = 1\ \text{kN} \qquad F_{CH} = 1\ \text{kN}$

Given

$$-F_2(d) - F_1(c) + E_y(2c) = 0$$

$$F_{BC}\sin(\theta)(c) + F_{CH}\sin(\phi)(c-b) + E_y(c) = 0$$

$$-F_{BC}\sin(\theta) - F_{CH}\sin(\phi) - F_1 + E_y = 0$$

$$\begin{pmatrix} E_y \\ F_{BC} \\ F_{CH} \end{pmatrix} = \text{Find}(E_y, F_{BC}, F_{CH}) \qquad E_y = 1.15\,\text{kN} \qquad \begin{pmatrix} F_{BC} \\ F_{CH} \end{pmatrix} = \begin{pmatrix} -3.25 \\ 1.923 \end{pmatrix}\text{kN}$$

Positive (T)
Negative (C)

Problem 6-52

Determine the force in members *CD* and *GF* of the truss and state if the members are in tension or compression. Also indicate all zero-force members.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F_1 = 1.5\ \text{kN}$

$F_2 = 2\ \text{kN}$

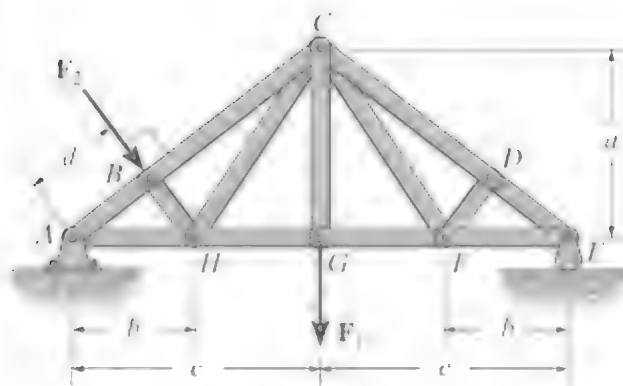
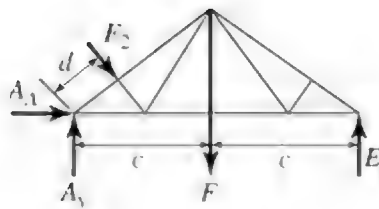
$a = 1.5\ \text{m}$

$b = 1\ \text{m}$

$c = 2\ \text{m}$

$d = 0.8\ \text{m}$

Solution:

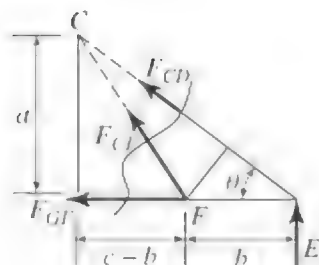
$\theta = \text{atan}\left(\frac{a}{c}\right) \qquad \phi = \text{atan}\left(\frac{a}{c-b}\right)$

Initial Guesses:

$E_y = 1\ \text{kN} \qquad F_{CD} = 1\ \text{kN} \qquad F_{GF} = 1\ \text{kN}$

Given

$-F_2(d) - F_1(c) + E_y(2c) = 0$

$E_y(b) + F_{CD}\sin(\theta)(b) = 0$

$E_y(c) - F_{GF}(a) = 0$

$\begin{pmatrix} E_y \\ F_{CD} \\ F_{GF} \end{pmatrix} = \text{Find}(E_y, F_{CD}, F_{GF}) \qquad E_y = 1.15\,\text{kN} \qquad \begin{pmatrix} F_{CD} \\ F_{GF} \end{pmatrix} = \begin{pmatrix} -1.917 \\ 1.533 \end{pmatrix}\text{kN}$ Positive (T) Negative (C)

DF and *CF* are zero force members.

Problem 6-53

Determine the force in members *DE*, *DL*, and *ML* of the roof truss and state if the members are in tension or compression.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F_1 = 6\ \text{kN}$

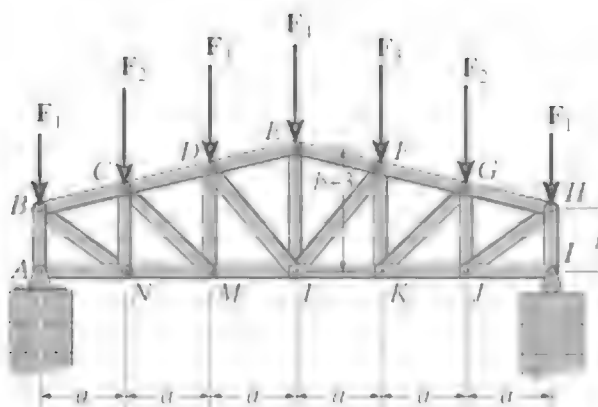
$F_2 = 12\ \text{kN}$

$F_3 = 12\ \text{kN}$

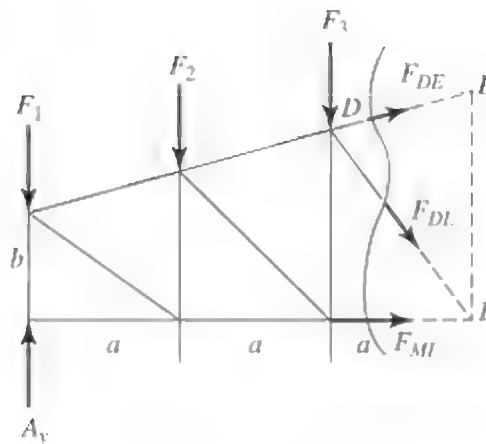
$F_4 = 12 \text{ kN}$

$a = 4 \text{ m}$

$b = 3 \text{ m}$

$c = 6 \text{ m}$

Solution:

$$\theta = \text{atan}\left(\frac{c-b}{3a}\right)$$

$$\phi = \text{atan}\left[\frac{b + \frac{2}{3}(c-b)}{a}\right]$$

Initial Guesses:

$A_y = 1 \text{ kN}$ $F_{ML} = 1 \text{ kN}$ $F_{DL} = 1 \text{ kN}$ $F_{DE} = 1 \text{ kN}$

Given

$$F_2(a) + F_3(2a) + F_4(3a) + F_3(4a) + F_2(5a) + F_1(6a) - A_y(6a) = 0$$

$$F_1(2a) + F_2(a) - A_y(2a) + F_{ML}\left[b + \frac{2}{3}(c-b)\right] = 0$$

$$A_y - F_1 - F_2 - F_3 + F_{DE}\sin(\theta) - F_{DL}\sin(\phi) = 0$$

$$F_{ML} + F_{DL}\cos(\phi) + F_{DE}\cos(\theta) = 0$$

$$\begin{pmatrix} A_y \\ F_{ML} \\ F_{DE} \\ F_{DL} \end{pmatrix} = \text{Find}(A_y, F_{ML}, F_{DE}, F_{DL}) \qquad A_y = 36\text{kN} \qquad \begin{pmatrix} F_{ML} \\ F_{DE} \\ F_{DL} \end{pmatrix} = \begin{pmatrix} 38.4 \\ -37.1 \\ -3.8 \end{pmatrix} \text{kN}$$

Positive (T), Negative (C)

Problem 6-54

Determine the force in members *EF* and *EL* of the roof truss and state if the members are in

tension or compression.

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 6\ kN$

$F_2 = 12\ kN$

$F_3 = 12\ kN$

$F_4 = 12\ kN$

$a = 4\ m$

$b = 3\ m$

$c = 6\ m$

Solution:

$$\theta = \operatorname{atan}\left(\frac{c-b}{3a}\right)$$

Initial Guesses:

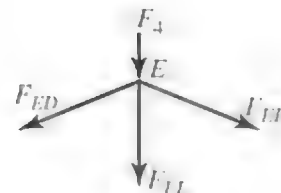
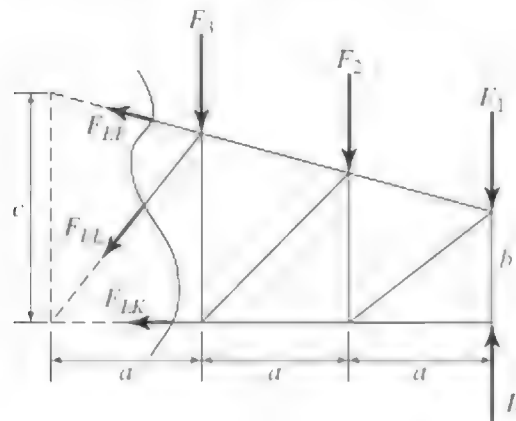
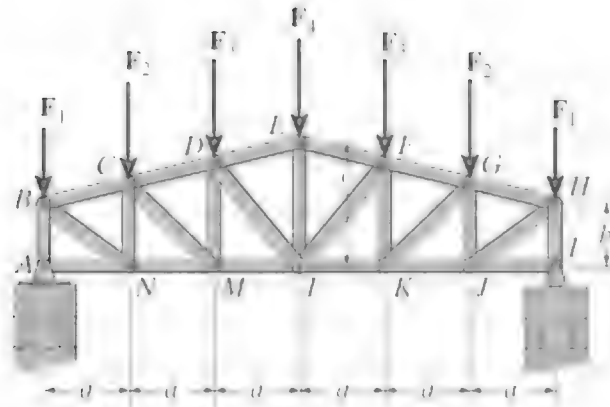
$I_y = 1\ kN \qquad F_{EF} = 1\ kN$

$F_{EL} = 1\ kN$

Given

$$-F_2(a) - F_3(2a) - F_4(3a) - F_3(4a) - F_2(5a) - F_1(6a) + I_y(6a) = 0$$

$$-F_3(a) - F_2(2a) - F_1(3a) + I_y(3a) + F_{EF}\cos(\theta)(c) = 0$$

$$-F_4 - F_{EL} - 2F_{EF}\sin(\theta) = 0$$

$$\begin{pmatrix} I_y \\ F_{EF} \\ F_{EL} \end{pmatrix} = \text{Find}(I_y, F_{EF}, F_{EL}) \qquad I_y = 36\,kN \qquad \begin{pmatrix} F_{EF} \\ F_{EL} \end{pmatrix} = \begin{pmatrix} -37.108 \\ 6 \end{pmatrix} kN$$

Positive (T)
Negative (C)

Problem 6-55

Two space trusses are used to equally support the uniform sign of mass M. Determine the force developed in members AB, AC, and BC of truss $ABCD$ and state if the members are in tension or compression. Horizontal short links support the truss at joints B and D and there is a ball-and-

socket joint at C.

Given:

$M = 50\ \text{kg}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$a = 0.25\ \text{m}$

$b = 0.5\ \text{m}$

$c = 2\ \text{m}$

Solution: $\quad h = \sqrt{b^2 - a^2}$

$$\mathbf{AB} = \begin{pmatrix} -a \\ -c \\ h \end{pmatrix} \quad \mathbf{AD} = \begin{pmatrix} a \\ -c \\ h \end{pmatrix} \quad \mathbf{AC} = \begin{pmatrix} 0 \\ -c \\ 0 \end{pmatrix}$$

$$\mathbf{BD} = \begin{pmatrix} 2a \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{BC} = \begin{pmatrix} a \\ 0 \\ -h \end{pmatrix}$$

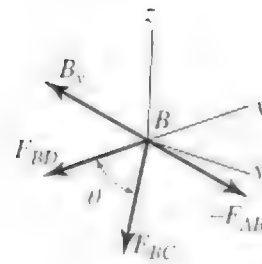
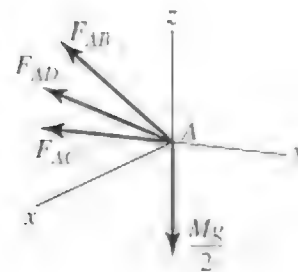
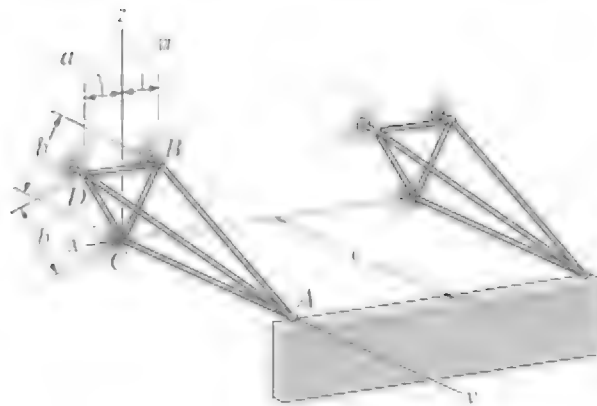
Guesses

$F_{AB} = 1\ \text{N}$ $\quad F_{AD} = 1\ \text{N}$ $\quad F_{AC} = 1\ \text{N}$

$F_{BC} = 1\ \text{N}$ $\quad F_{BD} = 1\ \text{N}$ $\quad B_y = 1\ \text{N}$

Given

$$F_{AB}\frac{\mathbf{AB}}{|\mathbf{AB}|} + F_{AD}\frac{\mathbf{AD}}{|\mathbf{AD}|} + F_{AC}\frac{\mathbf{AC}}{|\mathbf{AC}|} + \begin{pmatrix} 0 \\ 0 \\ \frac{-Mg}{2} \end{pmatrix} = 0$$

$$F_{AB}\frac{-\mathbf{AB}}{|\mathbf{AB}|} + F_{BD}\frac{\mathbf{BD}}{|\mathbf{BD}|} + F_{BC}\frac{\mathbf{BC}}{|\mathbf{BC}|} + \begin{pmatrix} 0 \\ -B_y \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AD} \\ F_{AC} \\ F_{BC} \\ F_{BD} \\ B_y \end{pmatrix} = \text{Find}(F_{AB}, F_{AD}, F_{AC}, F_{BC}, F_{BD}, B_y)$$

$$\begin{pmatrix} B_y \\ F_{AD} \\ F_{BD} \end{pmatrix} = \begin{pmatrix} 566 \\ 584 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{BC} \end{pmatrix} = \begin{pmatrix} 584 \\ -1133 \\ -142 \end{pmatrix} \text{N}$$

Positive (T), Negative (C)

Problem 6-56

Determine the force in each member of the space truss and state if the members are in tension or compression.The truss is supported by short links at *B*, *C*, and *D*.

Given:

$F = 600$ N

$a = 3$ m

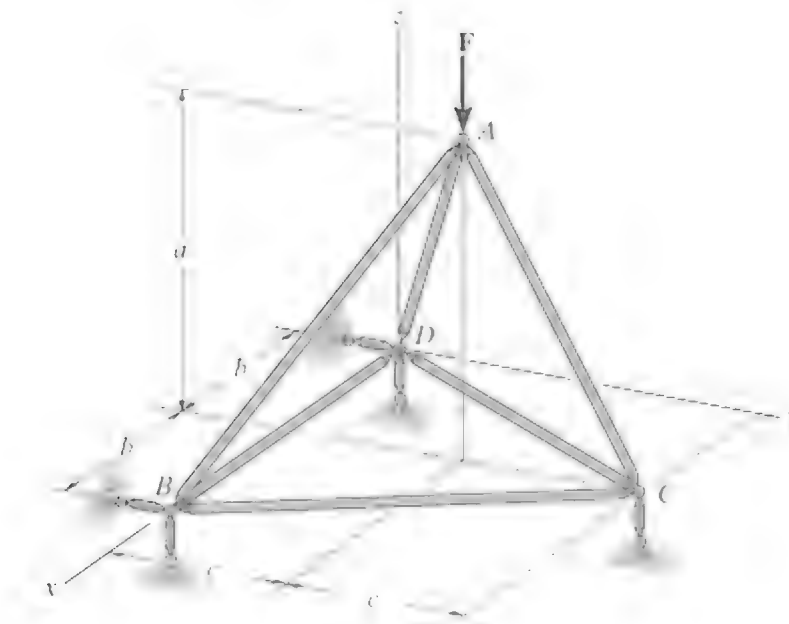
$b = 1$ m

$c = 1.5$ m

Solution:

$$\mathbf{AB} = \begin{pmatrix} b \\ -c \\ -a \end{pmatrix}$$

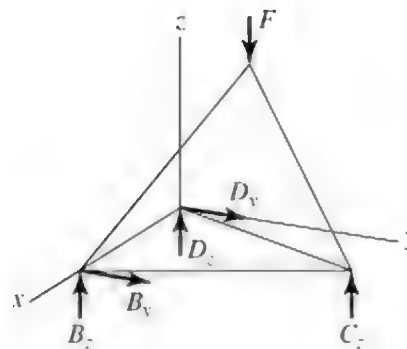
$$\mathbf{AC} = \begin{pmatrix} 0 \\ c \\ -a \end{pmatrix}$$

$$\mathbf{AD} = \begin{pmatrix} -b \\ -c \\ -a \end{pmatrix} \qquad \mathbf{CD} = \begin{pmatrix} -b \\ -2c \\ 0 \end{pmatrix}$$

$$\mathbf{CB} = \begin{pmatrix} b \\ -2c \\ 0 \end{pmatrix} \qquad \mathbf{BD} = \begin{pmatrix} -2b \\ 0 \\ 0 \end{pmatrix}$$

Guesses

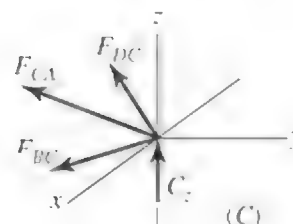
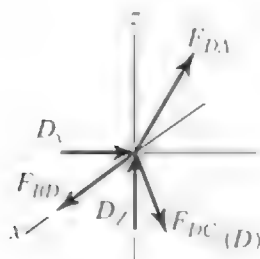
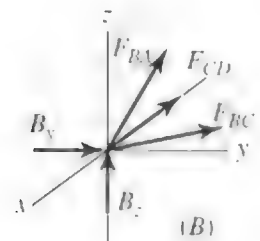
$F_{BA} = 1\text{ N}$ $\quad F_{BC} = 1\text{ N}$ $\quad F_{CA} = 1\text{ N}$

$F_{DA} = 1\text{ N}$ $\quad F_{BD} = 1\text{ N}$ $\quad F_{DC} = 1\text{ N}$

$B_y = 1\text{ N}$ $\quad B_z = 1\text{ N}$ $\quad C_z = 1\text{ N}$

Given

$$F_{BA}\frac{\mathbf{AB}}{|\mathbf{AB}|} + F_{CA}\frac{\mathbf{AC}}{|\mathbf{AC}|} + F_{DA}\frac{\mathbf{AD}}{|\mathbf{AD}|} + \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} = 0$$

$$F_{CA}\frac{-\mathbf{AC}}{|\mathbf{AC}|} + F_{DC}\frac{\mathbf{CD}}{|\mathbf{CD}|} + F_{BC}\frac{\mathbf{CB}}{|\mathbf{CB}|} + \begin{pmatrix} 0 \\ 0 \\ C_z \end{pmatrix} = 0$$

$$F_{BC}\frac{-\mathbf{CB}}{|\mathbf{CB}|} + F_{BD}\frac{\mathbf{BD}}{|\mathbf{BD}|} + F_{BA}\frac{-\mathbf{AB}}{|\mathbf{AB}|} + \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} = 0$$

$$\begin{pmatrix} F_{BA} \\ F_{BC} \\ F_{CA} \\ F_{DA} \\ F_{BD} \\ F_{DC} \\ B_y \\ B_z \\ C_z \end{pmatrix} = \text{Find}(F_{BA}, F_{BC}, F_{CA}, F_{DA}, F_{BD}, F_{DC}, B_y, B_z, C_z)$$

$$\begin{pmatrix} B_y \\ B_z \\ C_z \end{pmatrix} = \begin{pmatrix} 1.421 \times 10^{-14} \\ 150 \\ 300 \end{pmatrix} \text{N} \quad \begin{pmatrix} F_{BA} \\ F_{BC} \\ F_{CA} \\ F_{DA} \\ F_{BD} \\ F_{DC} \end{pmatrix} = \begin{pmatrix} -175 \\ 79.1 \\ -335.4 \\ -175 \\ 25 \\ 79.1 \end{pmatrix} \text{N} \quad \begin{array}{l}\text{Positive (T),} \\ \text{Negative (C)}\end{array}$$

Problem 6-57

Determine the force in each member of the space truss and state if the members are in tension or compression.The truss is supported by short links at A, B, and C.

Given:

$a = 4$ ft

$b = 2$ ft

$c = 3$ ft

$d = 2$ ft

$e = 8$ ft

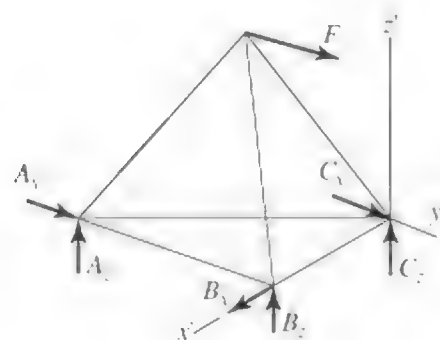
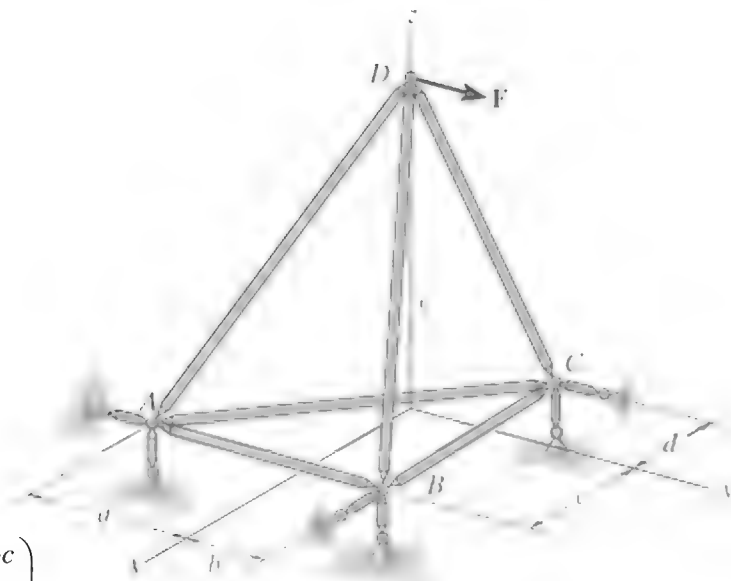
$$\mathbf{F} = \begin{pmatrix} 0 \\ 500 \\ 0 \end{pmatrix} \text{lb}$$

Solution:

$$\mathbf{AD} = \begin{pmatrix} -c \\ a \\ e \end{pmatrix} \qquad \mathbf{BD} = \begin{pmatrix} -c \\ -b \\ e \end{pmatrix}$$

$$\mathbf{CD} = \begin{pmatrix} d \\ -b \\ e \end{pmatrix} \qquad \mathbf{AB} = \begin{pmatrix} 0 \\ a+b \\ 0 \end{pmatrix}$$

$$\mathbf{AC} = \begin{pmatrix} -c-d \\ a+b \\ 0 \end{pmatrix} \qquad \mathbf{BC} = \begin{pmatrix} -c-d \\ 0 \\ 0 \end{pmatrix}$$

Guesses

$F_{BA} = 1\text{ lb}$ $F_{BC} = 1\text{ lb}$ $F_{BD} = 1\text{ lb}$

$F_{AD} = 1\text{ lb}$ $F_{AC} = 1\text{ lb}$ $F_{CD} = 1\text{ lb}$

$A_y = 1\text{ lb}$ $A_z = 1\text{ lb}$ $B_x = 1\text{ lb}$

$B_z = 1\text{ lb}$ $C_y = 1\text{ lb}$ $C_z = 1\text{ lb}$

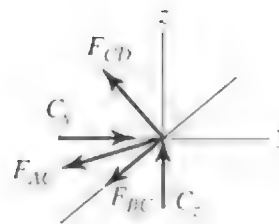
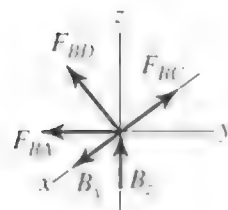
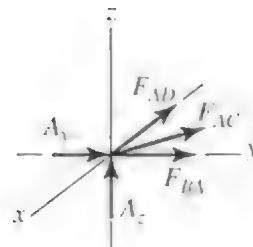
Given

$$\mathbf{F} + F_{AD}\frac{-\mathbf{AD}}{|\mathbf{AD}|} + F_{BD}\frac{-\mathbf{BD}}{|\mathbf{BD}|} + F_{CD}\frac{-\mathbf{CD}}{|\mathbf{CD}|} = \mathbf{0}$$

$$F_{AD}\frac{\mathbf{AD}}{|\mathbf{AD}|} + F_{BA}\frac{\mathbf{AB}}{|\mathbf{AB}|} + F_{AC}\frac{\mathbf{AC}}{|\mathbf{AC}|} + \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} = \mathbf{0}$$

$$F_{BA}\frac{-\mathbf{AB}}{|\mathbf{AB}|} + F_{BC}\frac{\mathbf{BC}}{|\mathbf{BC}|} + F_{BD}\frac{\mathbf{BD}}{|\mathbf{BD}|} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} = \mathbf{0}$$

$$F_{CD}\frac{\mathbf{CD}}{|\mathbf{CD}|} + F_{AC}\frac{-\mathbf{AC}}{|\mathbf{AC}|} + F_{BC}\frac{-\mathbf{BC}}{|\mathbf{BC}|} + \begin{pmatrix} 0 \\ C_y \\ C_z \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} F_{BA} \\ F_{BC} \\ F_{BD} \\ F_{AD} \\ F_{AC} \\ F_{CD} \\ A_y \\ A_z \\ B_x \\ B_z \\ C_y \\ C_z \end{pmatrix} = \text{Find}(F_{BA}, F_{BC}, F_{BD}, F_{AD}, F_{AC}, F_{CD}, A_y, A_z, B_x, B_z, C_y, C_z)$$

$$\begin{pmatrix} A_y \\ A_z \\ B_x \\ B_z \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} -200 \\ -667 \\ 0 \\ 667 \\ -300 \\ 0 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} F_{BA} \\ F_{BC} \\ F_{BD} \\ F_{AD} \\ F_{AC} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} 167 \\ 250 \\ -731 \\ 786 \\ -391 \\ 0 \end{pmatrix} \text{lb}$$

Positive (T)
Negative (C)

Problem 6-58

The space truss is supported by a ball-and-socket joint at D and short links at C and E. Determine the force in each member and state if the members are in tension or compression.

Given:

$$\mathbf{F_1} = \begin{pmatrix} 0 \\ 0 \\ -500 \end{pmatrix} \text{lb}$$

$$\mathbf{F_2} = \begin{pmatrix} 0 \\ 400 \\ 0 \end{pmatrix} \text{lb}$$

$a = 4$ ft

$b = 3$ ft

$c = 3$ ft

Solution:

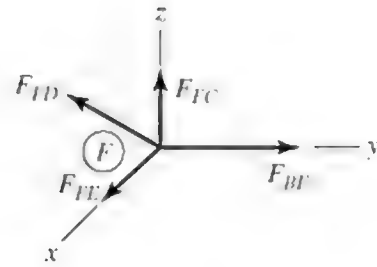
Find the external reactions

Guesses

$E_y = 1$ lb $\quad C_y = 1$ lb $\quad C_z = 1$ lb

$D_x = 1$ lb $\quad D_y = 1$ lb $\quad D_z = 1$ lb

Given

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} + \begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ C_y \\ C_z \end{pmatrix} + \mathbf{F_1} + \mathbf{F_2} = 0$$

$$\begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \mathbf{F_1} + \begin{pmatrix} -b \\ a \\ 0 \end{pmatrix} \times \mathbf{F_2} + \begin{pmatrix} 0 \\ 0 \\ c \end{pmatrix} \times \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} + \begin{pmatrix} -b \\ 0 \\ c \end{pmatrix} \times \begin{pmatrix} 0 \\ C_y \\ C_z \end{pmatrix} = 0$$

$$\begin{pmatrix} E_y \\ C_y \\ C_z \\ D_x \\ D_y \\ D_z \end{pmatrix} = \text{Find}(E_y, C_y, C_z, D_x, D_y, D_z) \qquad \begin{pmatrix} E_y \\ C_y \\ C_z \\ D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} 266.667 \\ -400 \\ 0 \\ 0 \\ -266.667 \\ 500 \end{pmatrix} \text{lb}$$

Now find the force in each member.

$$\mathbf{AB} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{AC} = \begin{pmatrix} -b \\ -a \\ c \end{pmatrix} \quad \mathbf{AD} = \begin{pmatrix} 0 \\ -a \\ c \end{pmatrix} \quad \mathbf{AE} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}$$

$$\mathbf{BC} = \begin{pmatrix} 0 \\ -a \\ c \end{pmatrix} \quad \mathbf{BE} = \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix} \quad \mathbf{BF} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \quad \mathbf{CD} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{CF} = \begin{pmatrix} 0 \\ 0 \\ -c \end{pmatrix} \quad \mathbf{DE} = \begin{pmatrix} 0 \\ 0 \\ -c \end{pmatrix} \quad \mathbf{DF} = \begin{pmatrix} -b \\ 0 \\ -c \end{pmatrix} \quad \mathbf{EF} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix}$$

Guesses

$F_{AB} = 1$ lb $\quad F_{AC} = 1$ lb $\quad F_{AD} = 1$ lb $\quad F_{AE} = 1$ lb

$F_{BC} = 1$ lb $\quad F_{BE} = 1$ lb $\quad F_{BF} = 1$ lb $\quad F_{CD} = 1$ lb

$F_{CF} = 1\ \text{lb} \quad F_{DE} = 1\text{lb} \qquad F_{DF} = 1\ \text{lb} \qquad F_{EF} = 1\ \text{lb}$

Given

$$\mathbf{F_1} + F_{AB}\frac{\mathbf{AB}}{|\mathbf{AB}|} + F_{AC}\frac{\mathbf{AC}}{|\mathbf{AC}|} + F_{AD}\frac{\mathbf{AD}}{|\mathbf{AD}|} + F_{AE}\frac{\mathbf{AE}}{|\mathbf{AE}|} = \mathbf{0}$$

$$\mathbf{F_2} + F_{BC}\frac{\mathbf{BC}}{|\mathbf{BC}|} + F_{BF}\frac{\mathbf{BF}}{|\mathbf{BF}|} + F_{BE}\frac{\mathbf{BE}}{|\mathbf{BE}|} + F_{AB}\frac{-\mathbf{AB}}{|\mathbf{AB}|} = \mathbf{0}$$

$$\begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} + F_{AE}\frac{-\mathbf{AE}}{|\mathbf{AE}|} + F_{BE}\frac{-\mathbf{BE}}{|\mathbf{BE}|} + F_{EF}\frac{\mathbf{EF}}{|\mathbf{EF}|} + F_{DE}\frac{-\mathbf{DE}}{|\mathbf{DE}|} = \mathbf{0}$$

$$F_{BF}\frac{-\mathbf{BF}}{|\mathbf{BF}|} + F_{CF}\frac{-\mathbf{CF}}{|\mathbf{CF}|} + F_{DF}\frac{-\mathbf{DF}}{|\mathbf{DF}|} + F_{EF}\frac{-\mathbf{EF}}{|\mathbf{EF}|} = \mathbf{0}$$

$$\begin{pmatrix} 0 \\ C_y \\ C_z \end{pmatrix} + F_{BC}\frac{-\mathbf{BC}}{|\mathbf{BC}|} + F_{AC}\frac{-\mathbf{AC}}{|\mathbf{AC}|} + F_{CD}\frac{\mathbf{CD}}{|\mathbf{CD}|} + F_{CF}\frac{\mathbf{CF}}{|\mathbf{CF}|} = \mathbf{0}$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \\ F_{AE} \\ F_{BC} \\ F_{BE} \\ F_{BF} \\ F_{CD} \\ F_{CF} \\ F_{DE} \\ F_{DF} \\ F_{EF} \end{pmatrix} = \text{Find}(F_{AB}, F_{AC}, F_{AD}, F_{AE}, F_{BC}, F_{BE}, F_{BF}, F_{CD}, F_{CF}, F_{DE}, F_{DF}, F_{EF})$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \\ F_{AE} \\ F_{BC} \\ F_{BE} \end{pmatrix} = \begin{pmatrix} -300 \\ 583.095 \\ 333.333 \\ -666.667 \\ 0 \\ 500 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} F_{BF} \\ F_{CD} \\ F_{CF} \\ F_{DE} \\ F_{DF} \\ F_{EF} \end{pmatrix} = \begin{pmatrix} 0 \\ -300 \\ -300 \\ 0 \\ 424.264 \\ -300 \end{pmatrix} \text{lb} \qquad \begin{array}{l} \text{Positive (T)} \\ \text{Negative (C)} \end{array}$$

Problem 6-59

The space truss is supported by a ball-and-socket joint at *D* and short links at *C* and *E*. Determine the force in each member and state if the members are in tension or compression.

Given:

$$\mathbf{F_1} = \begin{pmatrix} 200 \\ 300 \\ -500 \end{pmatrix} \text{lb}$$

$$\mathbf{F_2} = \begin{pmatrix} 0 \\ 400 \\ 0 \end{pmatrix} \text{lb}$$

$a = 4$ ft

$b = 3$ ft

$c = 3$ ft

Solution:

Find the external reactions

Guesses

$E_y = 1$ lb $\quad C_y = 1$ lb $\quad C_z = 1$ lb

$D_x = 1$ lb $\quad D_y = 1$ lb $\quad D_z = 1$ lb

Given

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} + \begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ C_y \\ C_z \end{pmatrix} + \mathbf{F_1} + \mathbf{F_2} = \mathbf{0}$$

$$\begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \mathbf{F_1} + \begin{pmatrix} -b \\ a \\ 0 \end{pmatrix} \times \mathbf{F_2} + \begin{pmatrix} 0 \\ 0 \\ c \end{pmatrix} \times \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} + \begin{pmatrix} -b \\ 0 \\ c \end{pmatrix} \times \begin{pmatrix} 0 \\ C_y \\ C_z \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} E_y \\ C_y \\ C_z \\ D_x \\ D_y \\ D_z \end{pmatrix} = \text{Find}(E_y, C_y, C_z, D_x, D_y, D_z) \qquad \begin{pmatrix} E_y \\ C_y \\ C_z \\ D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} -33.333 \\ -666.667 \\ 200 \\ -200 \\ -1.253 \times 10^{-13} \\ 300 \end{pmatrix} \text{lb}$$

Now find the force in each member.

$$\mathbf{AB} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{AC} = \begin{pmatrix} -b \\ -a \\ c \end{pmatrix} \quad \mathbf{AD} = \begin{pmatrix} 0 \\ -a \\ c \end{pmatrix} \quad \mathbf{AE} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}$$

$$\mathbf{BC} = \begin{pmatrix} 0 \\ -a \\ c \end{pmatrix} \quad \mathbf{BE} = \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix} \quad \mathbf{BF} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \quad \mathbf{CD} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{CF} = \begin{pmatrix} 0 \\ 0 \\ -c \end{pmatrix} \quad \mathbf{DE} = \begin{pmatrix} 0 \\ 0 \\ -c \end{pmatrix} \quad \mathbf{DF} = \begin{pmatrix} -b \\ 0 \\ -c \end{pmatrix} \quad \mathbf{EF} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix}$$

Guesses

$F_{AB} = 1\text{ lb} \quad F_{AC} = 1\text{ lb} \quad F_{AD} = 1\text{ lb} \quad F_{AE} = 1\text{ lb}$

$F_{BC} = 1\text{ lb} \quad F_{BE} = 1\text{ lb} \quad F_{BF} = 1\text{ lb} \quad F_{CD} = 1\text{ lb}$

$F_{CF} = 1\text{ lb} \quad F_{DE} = 1\text{lb} \quad F_{DF} = 1\text{ lb} \quad F_{EF} = 1\text{ lb}$

Given

$$\mathbf{F_1} + F_{AB}\frac{\mathbf{AB}}{|\mathbf{AB}|} + F_{AC}\frac{\mathbf{AC}}{|\mathbf{AC}|} + F_{AD}\frac{\mathbf{AD}}{|\mathbf{AD}|} + F_{AE}\frac{\mathbf{AE}}{|\mathbf{AE}|} = \mathbf{0}$$

$$\mathbf{F_2} + F_{BC}\frac{\mathbf{BC}}{|\mathbf{BC}|} + F_{BF}\frac{\mathbf{BF}}{|\mathbf{BF}|} + F_{BE}\frac{\mathbf{BE}}{|\mathbf{BE}|} + F_{AB}\frac{-\mathbf{AB}}{|\mathbf{AB}|} = \mathbf{0}$$

$$\begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} + F_{AE}\frac{-\mathbf{AE}}{|\mathbf{AE}|} + F_{BE}\frac{-\mathbf{BE}}{|\mathbf{BE}|} + F_{EF}\frac{\mathbf{EF}}{|\mathbf{EF}|} + F_{DE}\frac{-\mathbf{DE}}{|\mathbf{DE}|} = \mathbf{0}$$

$$F_{BF}\frac{-\mathbf{BF}}{|\mathbf{BF}|} + F_{CF}\frac{-\mathbf{CF}}{|\mathbf{CF}|} + F_{DF}\frac{-\mathbf{DF}}{|\mathbf{DF}|} + F_{EF}\frac{-\mathbf{EF}}{|\mathbf{EF}|} = \mathbf{0}$$

$$\begin{pmatrix} 0 \\ C_y \\ C_z \end{pmatrix} + F_{BC}\frac{-\mathbf{BC}}{|\mathbf{BC}|} + F_{AC}\frac{-\mathbf{AC}}{|\mathbf{AC}|} + F_{CD}\frac{\mathbf{CD}}{|\mathbf{CD}|} + F_{CF}\frac{\mathbf{CF}}{|\mathbf{CF}|} = \mathbf{0}$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \\ F_{AE} \\ F_{BC} \\ F_{BE} \\ F_{BF} \\ F_{CD} \\ F_{CF} \\ F_{DE} \\ F_{DF} \\ F_{EF} \end{pmatrix} = \text{Find}(F_{AB}, F_{AC}, F_{AD}, F_{AE}, F_{BC}, F_{BE}, F_{BF}, F_{CD}, F_{CF}, F_{DE}, F_{DF}, F_{EF})$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \\ F_{AE} \\ F_{BC} \\ F_{BE} \end{pmatrix} = \begin{pmatrix} -300 \\ 971.825 \\ 1.121 \times 10^{-11} \\ -366.667 \\ 0 \\ 500 \end{pmatrix} \quad \begin{pmatrix} F_{BF} \\ F_{CD} \\ F_{CF} \\ F_{DE} \\ F_{DF} \\ F_{EF} \end{pmatrix} = \begin{pmatrix} 0 \\ -500 \\ -300 \\ 0 \\ 424.264 \\ -300 \end{pmatrix} \text{lb}$$

Positive (T)
Negative (C)

Problem 6-60

Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by a ball-and-socket joints at A, B, and E. *Hint:* The support reaction at E acts along member EC. Why?

Given:

$$\mathbf{F} = \begin{pmatrix} -200 \\ 400 \\ 0 \end{pmatrix} \text{N}$$

$a = 2$ m

$b = 1.5$ m

$c = 5$ m

$d = 1$ m

$e = 2$ m

Solution:

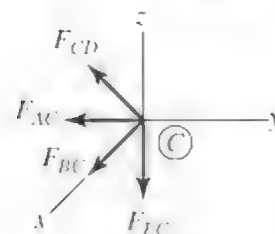
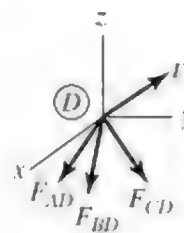
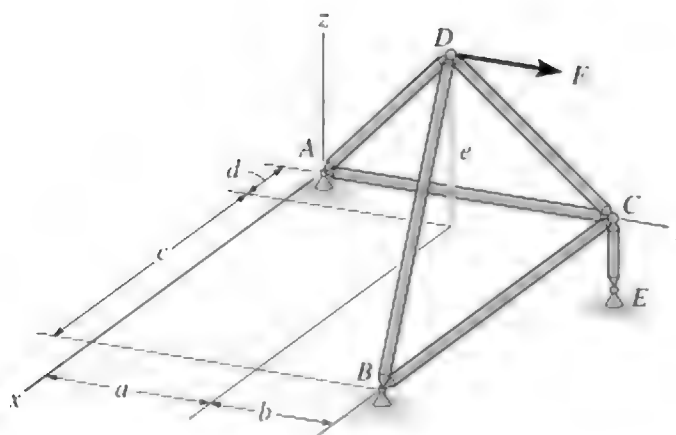
$$\mathbf{AC} = \begin{pmatrix} 0 \\ a+b \\ 0 \end{pmatrix} \qquad \mathbf{AD} = \begin{pmatrix} d \\ a \\ e \end{pmatrix}$$

$$\mathbf{BC} = \begin{pmatrix} -c-d \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{BD} = \begin{pmatrix} -c \\ -b \\ e \end{pmatrix} \qquad \mathbf{CD} = \begin{pmatrix} d \\ -b \\ e \end{pmatrix}$$

Guesses $F_{AC} = 1$ N $F_{AD} = 1$ N $F_{BC} = 1$ N

$F_{CD} = 1$ N $F_{EC} = 1$ N $F_{BD} = 1$ N

Given

$$\mathbf{F} + F_{AD}\frac{-\mathbf{AD}}{|\mathbf{AD}|} + F_{BD}\frac{-\mathbf{BD}}{|\mathbf{BD}|} + F_{CD}\frac{-\mathbf{CD}}{|\mathbf{CD}|} = \mathbf{0}$$

$$F_{CD}\frac{\mathbf{CD}}{|\mathbf{CD}|} + F_{BC}\frac{-\mathbf{BC}}{|\mathbf{BC}|} + F_{AC}\frac{-\mathbf{AC}}{|\mathbf{AC}|} + \begin{pmatrix} 0 \\ 0 \\ -F_{EC} \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} F_{AC} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ F_{EC} \end{pmatrix} = \text{Find}(F_{AC}, F_{AD}, F_{BC}, F_{BD}, F_{CD}, F_{EC}) \qquad \begin{pmatrix} F_{AC} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ F_{EC} \end{pmatrix} = \begin{pmatrix} 221 \\ 343 \\ 148 \\ 186 \\ -397 \\ -295 \end{pmatrix} \text{N}$$

Positive (T)
Negative (C)

Problem 6-61

Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at C, D, E, and G.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$F = 3$ kN

$a = 2$ m

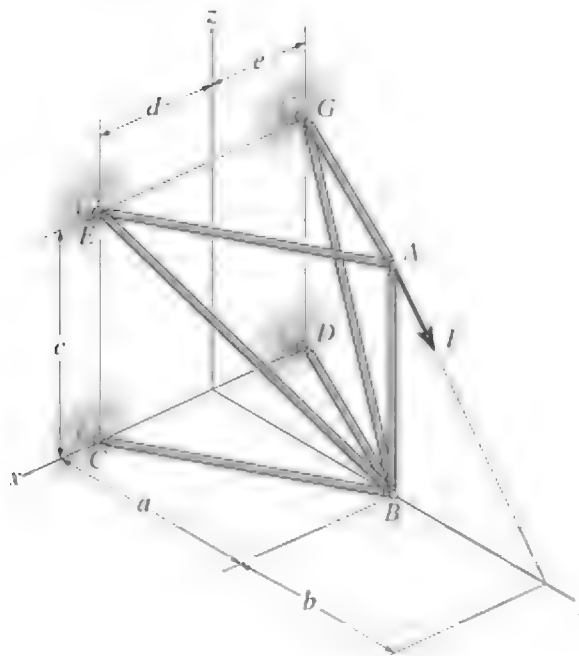
$b = 1.5$ m

$c = 2$ m

$d = 1$ m

$e = 1$ m

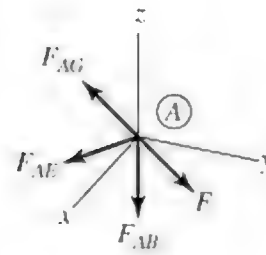
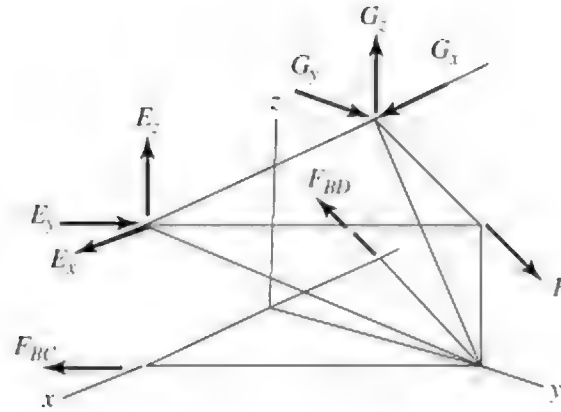
Solution:

$$\mathbf{F_v} = \frac{F}{\sqrt{b^2 + c^2}}\begin{pmatrix} 0 \\ b \\ -c \end{pmatrix}$$

$$\mathbf{u_{AG}} = \frac{1}{\sqrt{a^2 + e^2}} \begin{pmatrix} -e \\ -a \\ 0 \end{pmatrix}$$

$$\mathbf{u_{AE}} = \frac{1}{\sqrt{a^2 + d^2}} \begin{pmatrix} d \\ -a \\ 0 \end{pmatrix}$$

$$\mathbf{u_{AB}} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{u_{BC}} = \mathbf{u_{AE}} \qquad \mathbf{u_{BD}} = \mathbf{u_{AG}}$$

$$\mathbf{u_{BE}} = \frac{1}{\sqrt{a^2 + c^2 + d^2}} \begin{pmatrix} d \\ -a \\ c \end{pmatrix}$$

$$\mathbf{u_{BG}} = \frac{1}{\sqrt{a^2 + e^2 + c^2}} \begin{pmatrix} -e \\ -a \\ c \end{pmatrix}$$

Guesses

$F_{AB} = 1\ \text{kN}$ $\quad F_{AE} = 1\ \text{kN}$ $\quad F_{AG} = 1\ \text{kN}$

$F_{BC} = 1\ \text{kN}$ $\quad F_{BD} = 1\ \text{kN}$

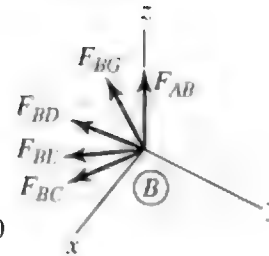
$F_{BE} = 1\ \text{kN}$ $\quad F_{BG} = 1\ \text{kN}$

Given

$$\frac{-c}{\sqrt{c^2 + b^2}} F(a) - \frac{a}{\sqrt{a^2 + d^2}} F_{BC}(c) - \frac{a}{\sqrt{a^2 + e^2}} F_{BD}(c) = 0$$

$$\frac{e}{\sqrt{a^2 + e^2}} F_{BD}(a) - \frac{d}{\sqrt{a^2 + d^2}} F_{BC}(a) = 0$$

$$\mathbf{F_v} + F_{AE}\mathbf{u_{AE}} + F_{AG}\mathbf{u_{AG}} + F_{AB}\mathbf{u_{AB}} = \mathbf{0}$$

$$-F_{AB}\mathbf{u_{AB}} + F_{BG}\mathbf{u_{BG}} + F_{BE}\mathbf{u_{BE}} + F_{BC}\mathbf{u_{BC}} + F_{BD}\mathbf{u_{BD}} = \mathbf{0}$$

$$\begin{pmatrix} F_{AB} \\ F_{AE} \\ F_{AG} \\ F_{BC} \\ F_{BD} \\ F_{BE} \\ F_{BG} \end{pmatrix} = \text{Find}(F_{AB}, F_{AE}, F_{AG}, F_{BC}, F_{BD}, F_{BE}, F_{BG}) \qquad \begin{pmatrix} F_{AB} \\ F_{AE} \\ F_{AG} \\ F_{BC} \\ F_{BD} \\ F_{BE} \\ F_{BG} \end{pmatrix} = \begin{pmatrix} -2.4 \\ 1.006 \\ 1.006 \\ -1.342 \\ -1.342 \\ 1.8 \\ 1.8 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-62

Determine the force in members *BD*, *AD*, and *AF* of the space truss and state if the members are in tension or compression. The truss is supported by short links at *A*, *B*, *D*, and *F*.

Given:

$$\mathbf{F} = \begin{pmatrix} 0 \\ 250 \\ -250 \end{pmatrix} \text{lb}$$

$a = 6$ ft

$b = 6$ ft

$\theta = 60$ deg

Solution:

Find the external reactions

$h = b \sin(\theta)$

Guesses

$A_x = 1$ lb $\quad B_y = 1$ lb $\quad B_z = 1$ lb

$D_y = 1$ lb $\quad F_y = 1$ lb $\quad F_z = 1$ lb

Given

$$\mathbf{F} + \begin{pmatrix} A_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ D_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ F_y \\ F_z \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} 0.5b \\ a \\ h \end{pmatrix} \times \mathbf{F} + \begin{pmatrix} b \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} 0.5b \\ 0 \\ h \end{pmatrix} \times \begin{pmatrix} 0 \\ D_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ F_y \\ F_z \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} A_x \\ B_y \\ B_z \\ D_y \\ F_y \\ F_z \end{pmatrix} = \text{Find}(A_x, B_y, B_z, D_y, F_y, F_z) \qquad \begin{pmatrix} A_x \\ B_y \\ B_z \\ D_y \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} 0 \\ 72 \\ 125 \\ -394 \\ 72 \\ 125 \end{pmatrix} \text{lb}$$

Now find the forces in the members

$$\mathbf{AB} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \quad \mathbf{AC} = \begin{pmatrix} -b \\ -a \\ 0 \end{pmatrix} \quad \mathbf{AD} = \begin{pmatrix} -0.5b \\ -a \\ h \end{pmatrix}$$

$$\mathbf{AE} = \begin{pmatrix} -0.5b \\ 0 \\ h \end{pmatrix} \quad \mathbf{AF} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{BC} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{BD} = \begin{pmatrix} -0.5b \\ 0 \\ h \end{pmatrix} \quad \mathbf{CD} = \begin{pmatrix} 0.5b \\ 0 \\ h \end{pmatrix} \quad \mathbf{CF} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix}$$

$$\mathbf{DE} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \quad \mathbf{DF} = \begin{pmatrix} -0.5b \\ a \\ -h \end{pmatrix} \quad \mathbf{EF} = \begin{pmatrix} -0.5b \\ 0 \\ -h \end{pmatrix}$$

Guesses

$F_{AB} = 1 \text{ lb}$ $F_{AC} = 1 \text{ lb}$ $F_{AD} = 1 \text{ lb}$

$F_{AE} = 1 \text{ lb}$ $F_{AF} = 1 \text{ lb}$ $F_{BC} = 1 \text{ lb}$

$F_{BD} = 1 \text{ lb}$ $F_{CD} = 1 \text{ lb}$ $F_{CF} = 1 \text{ lb}$

$F_{DE} = 1\ \text{lb} \quad F_{DF} = 1\ \text{lb} \quad F_{EF} = 1\ \text{lb}$

Given

$$\mathbf{F} + F_{DE}\frac{-\mathbf{DE}}{|\mathbf{DE}|} + F_{AE}\frac{-\mathbf{AE}}{|\mathbf{AE}|} + F_{EF}\frac{\mathbf{EF}}{|\mathbf{EF}|} = \mathbf{0}$$

$$F_{CF}\frac{\mathbf{CF}}{|\mathbf{CF}|} + F_{CD}\frac{\mathbf{CD}}{|\mathbf{CD}|} + F_{BC}\frac{-\mathbf{BC}}{|\mathbf{BC}|} + F_{AC}\frac{-\mathbf{AC}}{|\mathbf{AC}|} = \mathbf{0}$$

$$F_{DE}\frac{\mathbf{DE}}{|\mathbf{DE}|} + F_{DF}\frac{\mathbf{DF}}{|\mathbf{DF}|} + F_{AD}\frac{-\mathbf{AD}}{|\mathbf{AD}|} + F_{BD}\frac{-\mathbf{BD}}{|\mathbf{BD}|} + F_{CD}\frac{-\mathbf{CD}}{|\mathbf{CD}|} + \begin{pmatrix} 0 \\ D_y \\ 0 \end{pmatrix} = \mathbf{0}$$

$$F_{AB}\frac{-\mathbf{AB}}{|\mathbf{AB}|} + F_{BC}\frac{\mathbf{BC}}{|\mathbf{BC}|} + F_{BD}\frac{\mathbf{BD}}{|\mathbf{BD}|} + \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} = \mathbf{0}$$

$$F_{AB}\frac{\mathbf{AB}}{|\mathbf{AB}|} + F_{AC}\frac{\mathbf{AC}}{|\mathbf{AC}|} + F_{AF}\frac{\mathbf{AF}}{|\mathbf{AF}|} + F_{AD}\frac{\mathbf{AD}}{|\mathbf{AD}|} + F_{AE}\frac{\mathbf{AE}}{|\mathbf{AE}|} + \begin{pmatrix} A_x \\ 0 \\ 0 \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \\ F_{AE} \\ F_{AF} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ F_{CF} \\ F_{DE} \\ F_{DF} \\ F_{EF} \end{pmatrix} = \text{Find}\left(F_{AB}, F_{AC}, F_{AD}, F_{AE}, F_{AF}, F_{BC}, F_{BD}, F_{CD}, F_{CF}, F_{DE}, F_{DF}, F_{EF}\right)$$

$$\begin{pmatrix} F_{BD} \\ F_{AD} \\ F_{AF} \end{pmatrix} = \begin{pmatrix} -144.3 \\ 204.1 \\ 72.2 \end{pmatrix} \text{lb}$$

Positive (T)
Negative (C)

Problem 6-63

Determine the force in members *CF*, *EF*, and *DF* of the space truss and state if the members are in tension or compression. The truss is supported by short links at *A*, *B*, *D*, and *F*.

Given:

$$\mathbf{F} = \begin{pmatrix} 0 \\ 250 \\ -250 \end{pmatrix} \text{lb}$$

$a = 6$ ft

$b = 6$ ft

$\theta = 60$ deg

Solution:

Find the external reactions

$h = b\sin(\theta)$

Guesses

$A_x = 1$ lb $\quad B_y = 1$ lb $\quad B_z = 1$ lb

$D_y = 1$ lb $\quad F_y = 1$ lb $\quad F_z = 1$ lb

Given

$$\mathbf{F} + \begin{pmatrix} A_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ D_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ F_y \\ F_z \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} 0.5b \\ a \\ h \end{pmatrix} \times \mathbf{F} + \begin{pmatrix} b \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} 0.5b \\ 0 \\ h \end{pmatrix} \times \begin{pmatrix} 0 \\ D_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ F_y \\ F_z \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} A_x \\ B_y \\ B_z \\ D_y \\ F_y \\ F_z \end{pmatrix} = \text{Find}(A_x, B_y, B_z, D_y, F_y, F_z) \qquad \begin{pmatrix} A_x \\ B_y \\ B_z \\ D_y \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} 0 \\ 72 \\ 125 \\ -394 \\ 72 \\ 125 \end{pmatrix} \text{lb}$$

Now find the forces in the members

$$\mathbf{AB} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \qquad \mathbf{AC} = \begin{pmatrix} -b \\ -a \\ 0 \end{pmatrix} \qquad \mathbf{AD} = \begin{pmatrix} -0.5b \\ -a \\ h \end{pmatrix}$$

$$\mathbf{AE} = \begin{pmatrix} -0.5b \\ 0 \\ h \end{pmatrix} \qquad \mathbf{AF} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{BC} = \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{BD} = \begin{pmatrix} -0.5b \\ 0 \\ h \end{pmatrix} \qquad \mathbf{CD} = \begin{pmatrix} 0.5b \\ 0 \\ h \end{pmatrix} \qquad \mathbf{CF} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix}$$

$$\mathbf{DE} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \qquad \mathbf{DF} = \begin{pmatrix} -0.5b \\ a \\ -h \end{pmatrix} \qquad \mathbf{EF} = \begin{pmatrix} -0.5b \\ 0 \\ -h \end{pmatrix}$$

Guesses

$F_{AB} = 1\text{ lb}$ $F_{AC} = 1\text{ lb}$ $F_{AD} = 1\text{ lb}$

$F_{AE} = 1\text{ lb}$ $F_{AF} = 1\text{ lb}$ $F_{BC} = 1\text{ lb}$

$F_{BD} = 1\text{ lb}$ $F_{CD} = 1\text{ lb}$ $F_{CF} = 1\text{ lb}$

$F_{DE} = 1\text{ lb}$ $F_{DF} = 1\text{ lb}$ $F_{EF} = 1\text{ lb}$

Given

$$\mathbf{F} + F_{DE}\frac{-\mathbf{DE}}{|\mathbf{DE}|} + F_{AE}\frac{-\mathbf{AE}}{|\mathbf{AE}|} + F_{EF}\frac{\mathbf{EF}}{|\mathbf{EF}|} = \mathbf{0}$$

$$F_{CF}\frac{\mathbf{CF}}{|\mathbf{CF}|} + F_{CD}\frac{\mathbf{CD}}{|\mathbf{CD}|} + F_{BC}\frac{-\mathbf{BC}}{|\mathbf{BC}|} + F_{AC}\frac{-\mathbf{AC}}{|\mathbf{AC}|} = \mathbf{0}$$

$$F_{DE}\frac{\mathbf{DE}}{|\mathbf{DE}|} + F_{DF}\frac{\mathbf{DF}}{|\mathbf{DF}|} + F_{AD}\frac{-\mathbf{AD}}{|\mathbf{AD}|} + F_{BD}\frac{-\mathbf{BD}}{|\mathbf{BD}|} + F_{CD}\frac{-\mathbf{CD}}{|\mathbf{CD}|} + \begin{pmatrix} 0 \\ D_y \\ 0 \end{pmatrix} = \mathbf{0}$$

$$F_{AB}\frac{-\mathbf{AB}}{|\mathbf{AB}|} + F_{BC}\frac{\mathbf{BC}}{|\mathbf{BC}|} + F_{BD}\frac{\mathbf{BD}}{|\mathbf{BD}|} + \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} = \mathbf{0}$$

$$F_{AB}\frac{\mathbf{AB}}{|\mathbf{AB}|} + F_{AC}\frac{\mathbf{AC}}{|\mathbf{AC}|} + F_{AF}\frac{\mathbf{AF}}{|\mathbf{AF}|} + F_{AD}\frac{\mathbf{AD}}{|\mathbf{AD}|} + F_{AE}\frac{\mathbf{AE}}{|\mathbf{AE}|} + \begin{pmatrix} A_x \\ 0 \\ 0 \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \\ F_{AE} \\ F_{AF} \\ F_{BC} \\ F_{BD} \\ F_{CD} \\ F_{CF} \\ F_{DE} \\ F_{DF} \\ F_{EF} \end{pmatrix} = \text{Find}(F_{AB}, F_{AC}, F_{AD}, F_{AE}, F_{AF}, F_{BC}, F_{BD}, F_{CD}, F_{CF}, F_{DE}, F_{DF}, F_{EF})$$

$$\begin{pmatrix} F_{CF} \\ F_{EF} \\ F_{DF} \end{pmatrix} = \begin{pmatrix} 72.2 \\ -144.3 \\ 0 \end{pmatrix} \text{lb}$$

Positive (T)
Negative (C)

Problem 6-64

Determine the force developed in each member of the space truss and state if the members are in tension or compression. The crate has weight W.

Given:

$W = 150 \text{ lb}$

$a = 6 \text{ ft}$

$b = 6 \text{ ft}$

$c = 6 \text{ ft}$

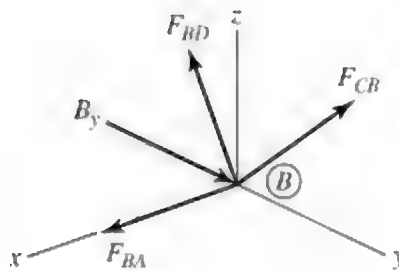
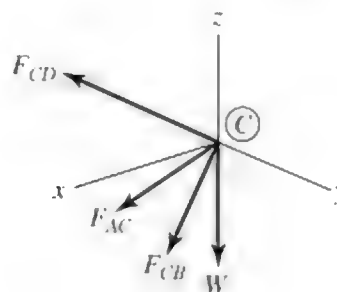
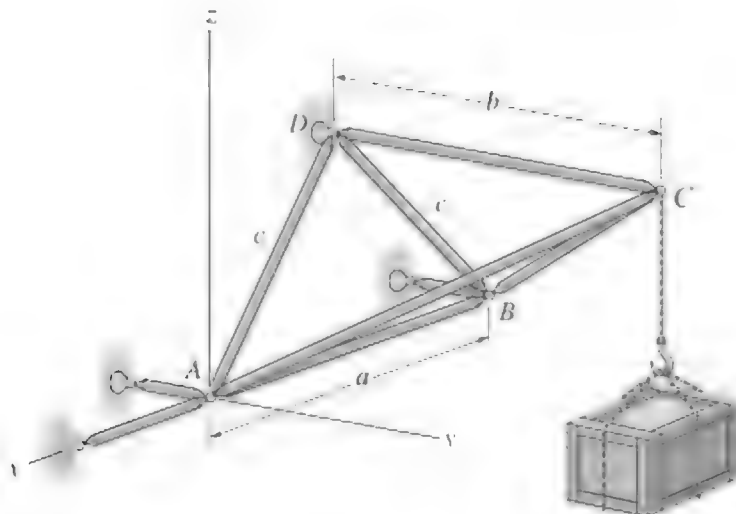
Solution: Unit Vectors

$$h = \sqrt{c^2 - \left(\frac{a}{2}\right)^2}$$

$$\mathbf{u_{AD}} = \frac{1}{\sqrt{h^2 + \left(\frac{a}{2}\right)^2}} \begin{pmatrix} \frac{-a}{2} \\ 0 \\ h \end{pmatrix}$$

$$\mathbf{u_{BD}} = \frac{1}{\sqrt{h^2 + \left(\frac{a}{2}\right)^2}} \begin{pmatrix} \frac{a}{2} \\ 0 \\ h \end{pmatrix}$$

$$\mathbf{u_{AC}} = \frac{1}{\sqrt{h^2 + b^2 + \left(\frac{a}{2}\right)^2}} \begin{pmatrix} -\frac{a}{2} \\ b \\ h \end{pmatrix}$$

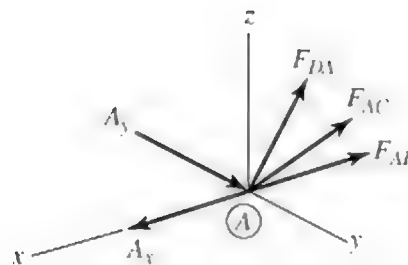
$$\mathbf{u_{BC}} = \frac{1}{\sqrt{h^2 + b^2 + \left(\frac{a}{2}\right)^2}} \begin{pmatrix} \frac{a}{2} \\ b \\ h \end{pmatrix}$$

Guesses $B_y = 1 \text{ lb}$ $A_x = 1 \text{ lb}$ $A_y = 1 \text{ lb}$

$F_{AB} = 1 \text{ lb}$ $F_{AC} = 1 \text{ lb}$ $F_{AD} = 1 \text{ lb}$ $F_{BC} = 1 \text{ lb}$ $F_{BD} = 1 \text{ lb}$ $F_{CD} = 1 \text{ lb}$

Given

$$\begin{pmatrix} 0 \\ -F_{CD} \\ -W \end{pmatrix} - F_{AC}\mathbf{u_{AC}} - F_{BC}\mathbf{u_{BC}} = \mathbf{0}$$

$$\begin{pmatrix} F_{AB} \\ B_y \\ 0 \end{pmatrix} + F_{BC}\mathbf{u_{BC}} + F_{BD}\mathbf{u_{BD}} = \mathbf{0}$$

$$\begin{pmatrix} A_x - F_{AB} \\ A_y \\ 0 \end{pmatrix} + F_{AC}\mathbf{u_{AC}} + F_{AD}\mathbf{u_{AD}} = \mathbf{0}$$

$$\begin{pmatrix} A_x \\ A_y \\ B_y \\ F_{AB} \\ F_{AC} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{CD} \end{pmatrix} = \text{Find}(A_x, A_y, B_y, F_{AB}, F_{AC}, F_{AD}, F_{BC}, F_{BD}, F_{CD})$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \\ F_{BC} \\ F_{BD} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} 0.0 \\ -122.5 \\ 86.6 \\ -122.5 \\ 86.6 \\ 173.2 \end{pmatrix} \text{lb}$$

Positive (T)
Negative (C)

Problem 6-65

The space truss is used to support vertical forces at joints B, C, and D. Determine the force in each member and state if the members are in tension or compression.

Units Used:

$$kN = 10^3 \text{ N}$$

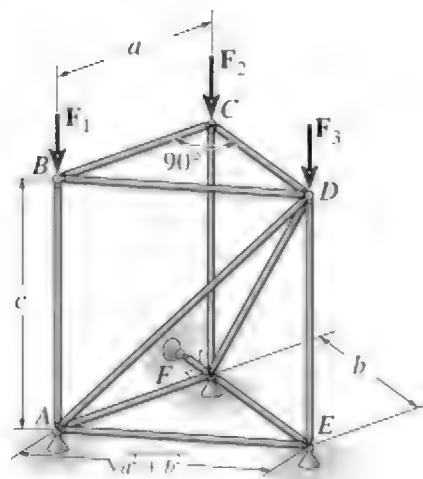
Given:

$F_1 = 6 \text{ kN}$ $a = 0.75 \text{ m}$

$F_2 = 8 \text{ kN}$ $b = 1.00 \text{ m}$

$F_3 = 9 \text{ kN}$ $c = 1.5 \text{ m}$

Solution:

Assume that the connections at A, E, and F are rollers

Guesses

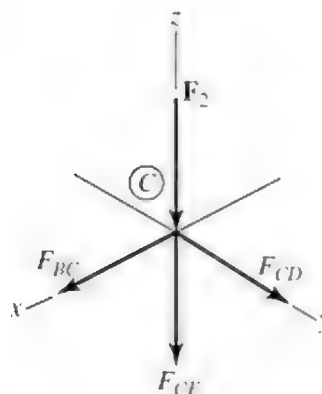
$F_{BC} = 1 \text{ kN}$ $F_{CF} = 1 \text{ kN}$

$F_{CD} = 1 \text{ kN}$ $F_{AD} = 1 \text{ kN}$

$F_{DF} = 1 \text{ kN}$ $F_{DE} = 1 \text{ kN}$

$F_{BD} = 1 \text{ kN}$ $F_{BA} = 1 \text{ kN}$

$F_{EF} = 1 \text{ kN}$ $F_{AE} = 1 \text{ kN}$

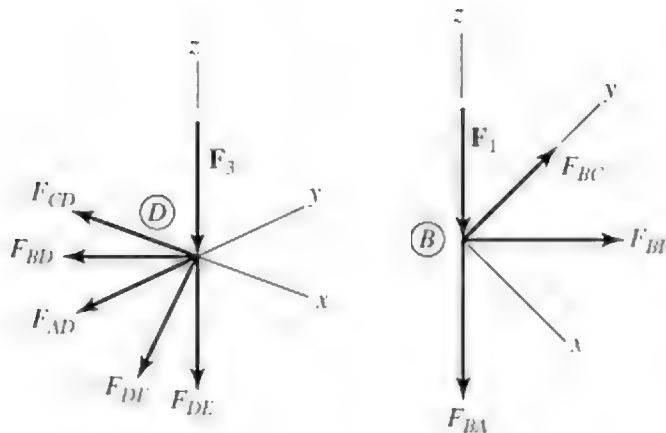
$F_{AF} = 1 \text{ kN}$

Given

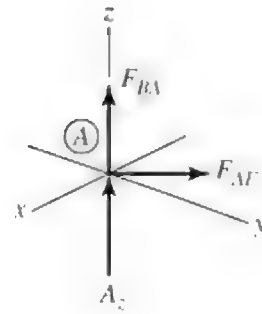
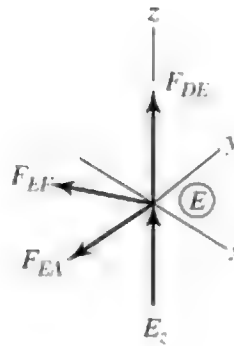
Joint C

$F_{BC} = 0$ $F_{CD} = 0$

$$-F_2 - F_{CF} = 0$$

Joint D

$$\frac{a}{\sqrt{a^2 + b^2}} F_{BD} + \frac{a}{\sqrt{a^2 + b^2 + c^2}} F_{AD} = 0$$

$$-F_{CD} - \frac{b}{\sqrt{a^2+b^2}}F_{BD} \cdots = 0$$
$$+ \frac{-b}{\sqrt{a^2+b^2+c^2}}F_{AD} - \frac{b}{\sqrt{b^2+c^2}}F_{DF}$$

$$-F_3 - F_{DE} - \frac{c}{\sqrt{b^2+c^2}}F_{DF} \cdots = 0$$
$$+ \frac{-c}{\sqrt{a^2+b^2+c^2}}F_{AD}$$

Joint B

$$-F_{BC} - \frac{a}{\sqrt{a^2+b^2}}F_{BD} = 0 \qquad \frac{b}{\sqrt{a^2+b^2}}F_{BD} = 0 \qquad -F_1 - F_{BA} = 0$$

Joint E

$$\frac{a}{\sqrt{a^2+b^2}}F_{AE} = 0 \qquad -F_{EF} - \frac{b}{\sqrt{a^2+b^2}}F_{AE} = 0$$

$$\begin{pmatrix} F_{BC} \\ F_{CF} \\ F_{CD} \\ F_{AD} \\ F_{DF} \\ F_{DE} \\ F_{BD} \\ F_{BA} \\ F_{EF} \\ F_{AE} \\ F_{AF} \end{pmatrix} = \mathrm{Find}\left(F_{BC}, F_{CF}, F_{CD}, F_{AD}, F_{DF}, F_{DE}, F_{BD}, F_{BA}, F_{EF}, F_{AE}, F_{AF}\right)$$

$$\begin{pmatrix} F_{BC} \\ F_{CF} \\ F_{CD} \\ F_{AD} \\ F_{DF} \\ F_{DE} \\ F_{BD} \\ F_{BA} \\ F_{EF} \\ F_{AE} \\ F_{AF} \end{pmatrix} = \begin{pmatrix} 0.00 \\ -8.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ -9.00 \\ 0.00 \\ -6.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-66

A force **P** is applied to the handles of the pliers. Determine the force developed on the smooth bolt *B* and the reaction that pin *A* exerts on its attached members.

Given:

$P = 8 \text{ lb}$

$a = 1.25 \text{ in}$

$b = 5 \text{ in}$

$c = 1.5 \text{ in}$

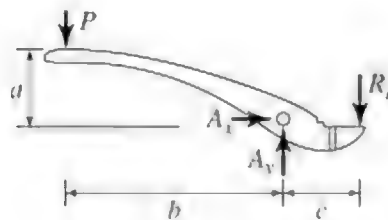
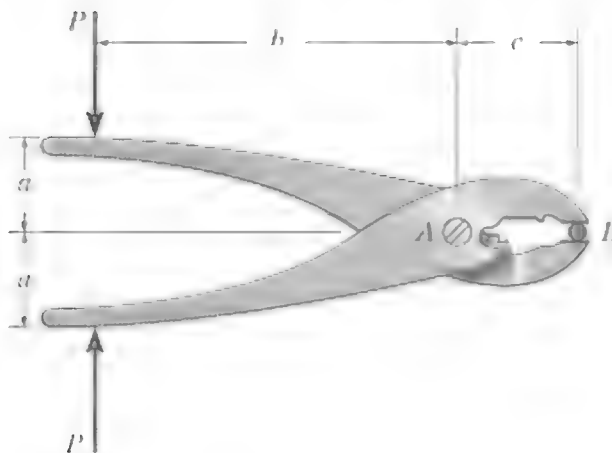
Solution:

$\Sigma M_A = 0;$ $\quad -R_B c + P b = 0$

$$R_B = P \frac{b}{c}$$

$$R_B = 26.7 \text{ lb}$$

$\Sigma F_x = 0;$ $\quad A_x = 0$

$\Sigma F_y = 0;$ $\quad A_y - P - R_B = 0$

$A_y = P + R_B$

$A_y = 34.7\,\text{lb}$

Problem 6-67

The eye hook has a positive locking latch when it supports the load because its two parts are pin-connected at A and they bear against one another along the smooth surface at B. Determine the resultant force at the pin and the normal force at B when the eye hook supports load **F**.

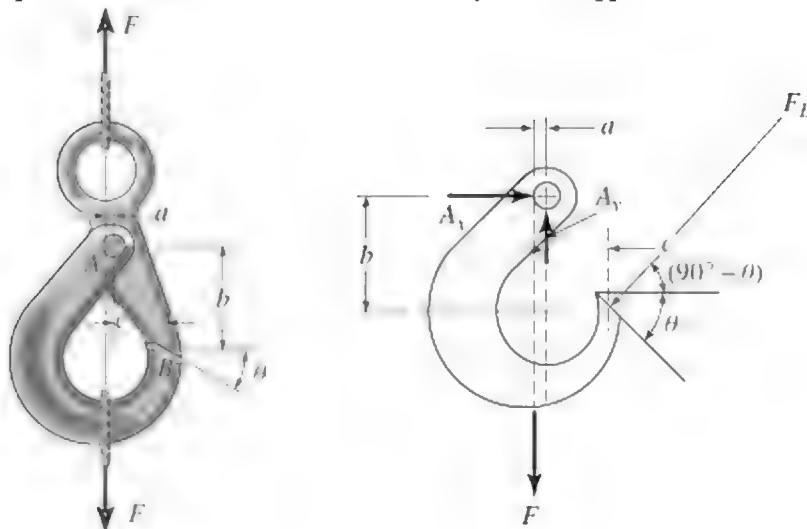
Given:

$F = 800\ \text{lb}$

$a = 0.25\ \text{in}$

$b = 3\ \text{in}$

$c = 2\ \text{in}$

$\theta = 30\ \text{deg}$

Solution:

$\Sigma M_A = 0;$ $\quad -F_B\cos(90\ \text{deg} - \theta)(b) - F_B\sin(90\ \text{deg} - \theta)(c) + F\,a = 0$

$$F_B = F\frac{a}{\cos(90\ \text{deg} - \theta)\,b + \sin(90\ \text{deg} - \theta)\,c} \qquad F_B = 61.9\,\text{lb}$$

$+\uparrow \Sigma F_y = 0;$ $\quad -F - F_B\sin(90\ \text{deg} - \theta) + A_y = 0$

$$A_y = F + F_B\sin(90\ \text{deg} - \theta) \qquad A_y = 854\,\text{lb}$$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - F_B\cos(90\ \text{deg} - \theta) = 0$

$$A_x = F_B\cos(90\ \text{deg} - \theta) \qquad A_x = 30.9\,\text{lb}$$

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} \qquad F_A = 854\,\text{lb}$$

Problem 6-68

Determine the force **P** needed to hold the block of mass F in equilibrium.

Given:

$F = 20\ \text{lb}$

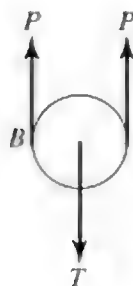
Solution:

Pulley B: $\Sigma F_y = 0;$ $2P - T = 0$

Pulley A: $\Sigma F_y = 0;$ $2T - F = 0$

$T = \frac{1}{2}F$ $T = 10\,\text{lb}$

$2P = T$ $P = \frac{1}{2}T$

$P = 5\,\text{lb}$

Problem 6-69

The link is used to hold the rod in place. Determine the required axial force on the screw at E if the largest force to be exerted on the rod at B, C or D is to be $\mathbf{F_{max}}$. Also, find the magnitude of the force reaction at pin A. Assume all surfaces of contact are smooth.

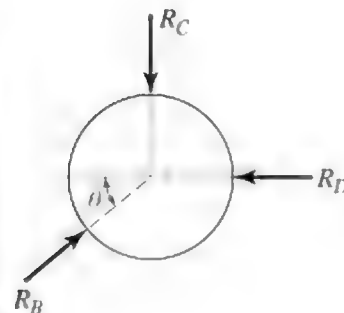
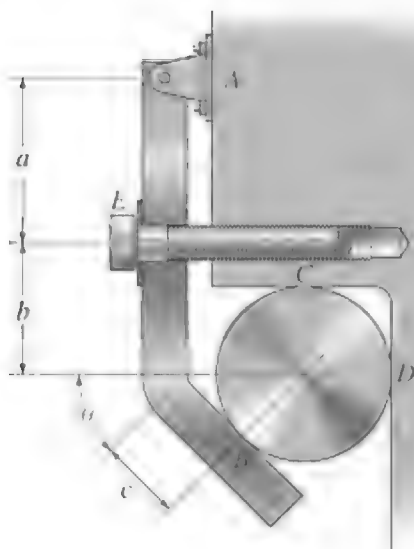
Given:

$F_{max} = 100\ \text{lb}$

$a = 100\ \text{mm}$

$b = 80\ \text{mm}$

$c = 50\ \text{mm}$

$\theta = 45\ \text{deg}$

Solution:

Assign an initial value for R_E. This will be scaled at the end of the problem.

Guesses $A_x = 1\ \text{lb}$ $A_y = 1\ \text{lb}$ $R_B = 1\ \text{lb}$

$R_C = 1\ \text{lb}$ $R_D = 1\ \text{lb}$ $R_E = 1\ \text{lb}$

Given $-A_x + R_E - R_B\cos(\theta) = 0$ $A_y - R_B\sin(\theta) = 0$

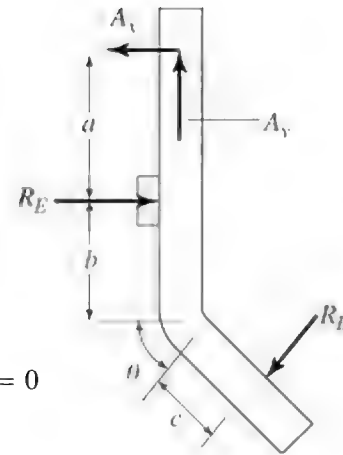
$$R_E a - R_B\cos(\theta)(a + b + c\cos(\theta)) - R_B\sin(\theta)c\sin(\theta) = 0$$

$R_B\cos(\theta) - R_D = 0$ $R_B\sin(\theta) - R_C = 0$

$$\begin{pmatrix} A_x \\ A_y \\ R_B \\ R_C \\ R_D \end{pmatrix} = \text{Find}(A_x, A_y, R_B, R_C, R_D) \qquad \begin{pmatrix} A_x \\ A_y \\ R_B \\ R_C \\ R_D \end{pmatrix} = \begin{pmatrix} 0.601 \\ 0.399 \\ 0.564 \\ 0.399 \\ 0.399 \end{pmatrix} \text{lb}$$

Now find the critical load and scale the problem

$$ans = \begin{pmatrix} R_B \\ R_C \\ R_D \end{pmatrix} \qquad F_{scale} = \frac{F_{max}}{\max(ans)} \qquad R_E = F_{scale} R_E \qquad R_E = 177.3\ \text{lb}$$

$$F_A = F_{scale}\sqrt{A_x^{\,2} + A_y^{\,2}} \qquad F_A = 127.9\ \text{lb}$$

Problem 6-70

The man of weight W_1 attempts to lift himself and the seat of weight W_2 using the rope and pulley system shown. Determine the force at A needed to do so, and also find his reaction on the seat.

Given:

$W_1 = 150$ lb

$W_2 = 10$ lb

Solution:

Pulley C:

$\Sigma F_y = 0; \quad 3T - R = 0$

Pulley B:

$\Sigma F_y = 0; \quad 3R - P = 0$

Thus, $\quad P = 9T$

Man and seat:

$\Sigma F_y = 0; \quad T + P - W_1 - W_2 = 0$

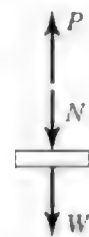
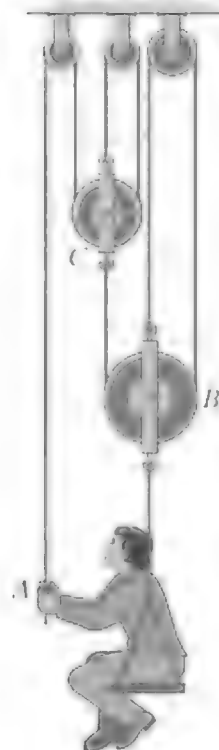
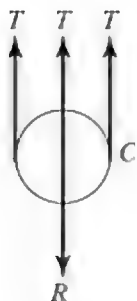
$10T = W_1 + W_2$

$T = \dfrac{W_1 + W_2}{10} \qquad T = 16\,\text{lb}$

$P = 9T \qquad P = 144\,\text{lb}$

Seat:

$\Sigma F_y = 0; \quad P - N - W_2 = 0$

$N = P - W_2 \qquad N = 134\,\text{lb}$

Problem 6-71

Determine the horizontal and vertical components of force that pins A and C exert on the frame.

Given:

$F = 500$ N

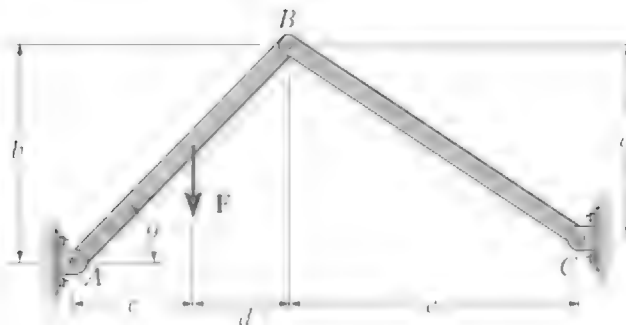
$a = 0.8$ m $\quad d = 0.4$ m

$b = 0.9\text{ m}$ $\quad e = 1.2\text{ m}$

$c = 0.5\text{ m}$ $\quad \theta = 45\text{deg}$

Solution:

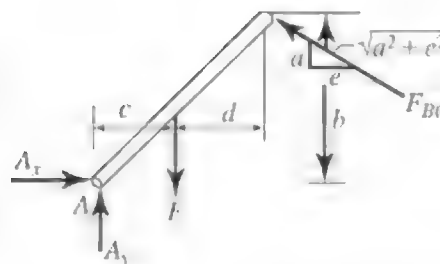
BC is a two-force member

Member AB :

$$\Sigma M_A = 0; \quad -F c + F_{BC}\frac{e}{\sqrt{a^2+e^2}}b + F_{BC}\frac{a}{\sqrt{a^2+e^2}}(c+d) = 0$$

$$F_{BC} = F c\frac{\sqrt{a^2+e^2}}{e\,b + a\,c + a\,d} \qquad F_{BC} = 200.3\text{ N}$$

Thus,

$$C_x = F_{BC}\frac{e}{\sqrt{a^2+e^2}} \qquad C_x = 167\text{ N}$$

$$C_y = F_{BC}\frac{a}{\sqrt{a^2+e^2}} \qquad C_y = 111\text{ N}$$

$$\Sigma F_x = 0; \quad A_x - F_{BC}\frac{e}{\sqrt{a^2+e^2}} = 0 \qquad A_x = F_{BC}\frac{e}{\sqrt{a^2+e^2}} \qquad A_x = 167\text{ N}$$

$$\Sigma F_y = 0; \quad A_y - F + F_{BC}\frac{a}{\sqrt{a^2+e^2}} = 0 \qquad A_y = F - F_{BC}\frac{a}{\sqrt{a^2+e^2}} \qquad A_y = 389\text{ N}$$

Problem 6-72

Determine the horizontal and vertical components of force that pins A and C exert on the frame.

Units Used:

$\text{kN} = 10^3\text{ N}$

Given:

$F_1 = 1\text{ kN}$

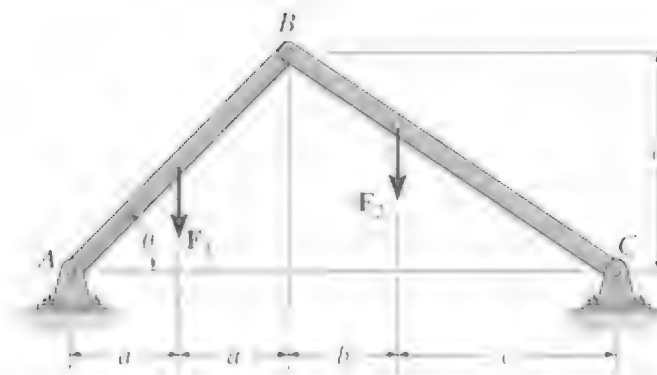
$F_2 = 500\ \text{N}$

$\theta = 45\ \text{deg}$

$a = 0.2\ \text{m}$

$b = 0.2\ \text{m}$

$c = 0.4\ \text{m}$

$d = 0.4\ \text{m}$

Solution:

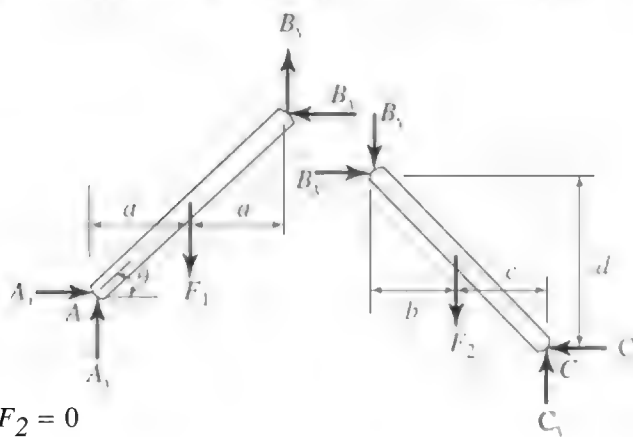
Guesses

$A_x = 1\ \text{N} \qquad A_y = 1\ \text{N}$

$C_x = 1\ \text{N} \qquad C_y = 1\ \text{N}$

Given

$A_x - C_x = 0 \qquad A_y + C_y - F_1 - F_2 = 0$

$F_1 a - A_y 2a + A_x d = 0 \qquad -F_2 b + C_y(b + c) - C_x d = 0$

$$\begin{pmatrix} A_x \\ A_y \\ C_x \\ C_y \end{pmatrix} = \text{Find}(A_x, A_y, C_x, C_y) \qquad \begin{pmatrix} A_x \\ A_y \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 500 \\ 1000 \\ 500 \\ 500 \end{pmatrix} \text{N}$$

Problem 6-73

The truck exerts the three forces shown on the girders of the bridge. Determine the reactions at the supports when the truck is in the position shown. The girders are connected together by a short vertical link *DC*.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

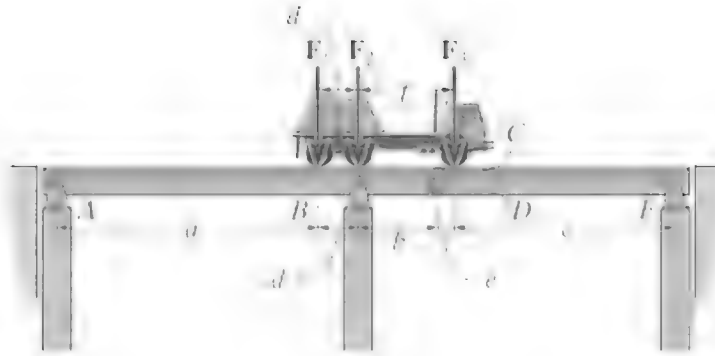
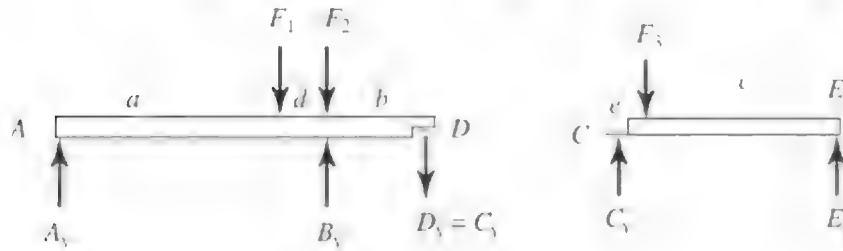
Given:

$a = 55\ \text{ft}$ $f = 12\ \text{ft}$

$b = 10\ \text{ft}$ $F_1 = 5\ \text{kip}$

$c = 48\ \text{ft}$ $F_2 = 4\ \text{kip}$

$d = 5\ \text{ft}$ $F_3 = 2\ \text{kip}$

$e = 2\ \text{ft}$

Solution:

Member CE:

$\Sigma M_C = 0;$ $-F_3 e + E_y(e + c) = 0$ $E_y = F_3 \dfrac{e}{e + c}$ $E_y = 80\ \text{lb}$

$\Sigma F_y = 0;$ $C_y - F_3 + E_y = 0$ $C_y = F_3 - E_y$ $C_y = 1920\ \text{lb}$

Member ABD:

$\Sigma M_A = 0;$ $-F_1 a - F_2(d + a) - C_y(a + d + b) + B_y(a + d) = 0$

$$B_y = \frac{F_1 a + F_2(d + a) + C_y(a + d + b)}{d + a} \qquad B_y = 10.8\ \text{kip}$$

$\Sigma F_y = 0;$ $A_y - F_1 + B_y - F_2 - C_y = 0$

$A_y = C_y + F_1 - B_y + F_2$ $A_y = 96.7\ \text{lb}$

Problem 6-74

Determine the greatest force **P** that can be applied to the frame if the largest force resultant acting at A can have a magnitude F_{max}.

Units Used:

$\text{kN} = 10^3\ \text{N}$

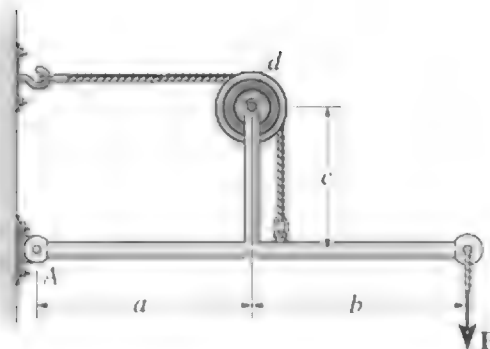
Given:

$F_{max} = 2\ \text{kN}$

$a = 0.75\ \text{m}$

$b = 0.75\ \text{m}$

$c = 0.5\ \text{m}$

$d = 0.1\ \text{m}$

Solution:

$\Sigma M_A = 0;\quad T(c+d) - P(a+b) = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - T = 0$

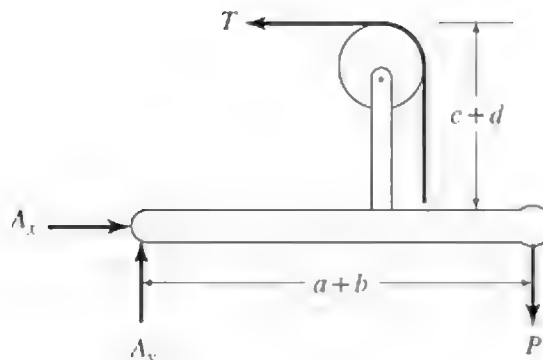
$+\uparrow \Sigma F_y = 0;\quad A_y - P = 0$

Thus, $T = \dfrac{a+b}{c+d}P \qquad A_y = P \qquad A_x = \dfrac{a+b}{c+d}P$

Require,

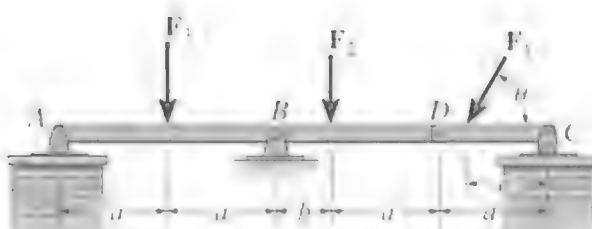
$$F_{max} = \sqrt{A_x^{\ 2} + A_y^{\ 2}}$$

$$P = \frac{F_{max}}{\sqrt{\left(\frac{a+b}{c+d}\right)^2 + 1}}$$

$P = 743\ \text{N}$

Problem 6-75

The compound beam is pin supported at B and supported by rockers at A and C. There is a hinge (pin) at D. Determine the reactions at the supports.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

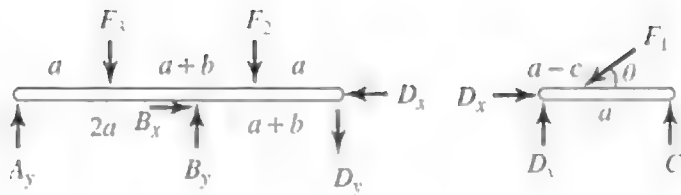
$F_1 = 7\ \text{kN}$ $a = 4\ \text{m}$

$F_2 = 6\ \text{kN}$ $b = 2\ \text{m}$

$F_3 = 16\ \text{kN}$ $c = 3\ \text{m}$

$\theta = 60\ \text{deg}$ $d = 4\ \text{m}$

Solution:

Member DC :

$\Sigma M_D = 0;$ $-F_1 \sin(\theta)(a - c) + C_y a = 0$

$$C_y = F_1 \sin(\theta) \frac{a - c}{a} \qquad C_y = 1.52\ \text{kN}$$

$\Sigma F_y = 0;$ $D_y - F_1 \sin(\theta) + C_y = 0$

$$D_y = F_1 \sin(\theta) - C_y \qquad D_y = 4.55\ \text{kN}$$

$\Sigma F_x = 0;$ $D_x - F_1 \cos(\theta) = 0$

$$D_x = F_1 \cos(\theta) \qquad D_x = 3.5\ \text{kN}$$

Member ABD :

$\Sigma M_A = 0;$ $-F_3 a - F_2(2\ a + b) - D_y(3\ a + b) + B_y 2\ a = 0$

$$B_y = \frac{F_3 a + F_2(2a + b) + D_y\ (3a + b)}{2\ a} \qquad B_y = 23.5\ \text{kN}$$

$\Sigma F_y = 0;$ $A_y - F_3 + B_y - F_2 - D_y = 0$

$$A_y = D_y + F_3 - B_y + F_2 \qquad A_y = 3.09\ \text{kN}$$

$\Sigma F_x = 0;$ $B_x - F_1 \cos(\theta) = 0$

$$B_x = F_1 \cos(\theta) \qquad B_x = 3.5\ \text{kN}$$

Problem 6-76

The compound beam is fixed supported at A and supported by rockers at B and C. If there are hinges (pins) at D and E, determine the reactions at the supports A, B, and C.

Units Used:

$kN = 10^3\ N$

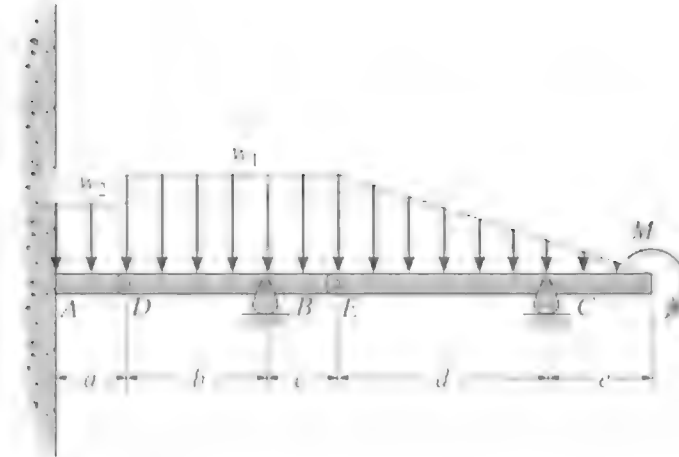
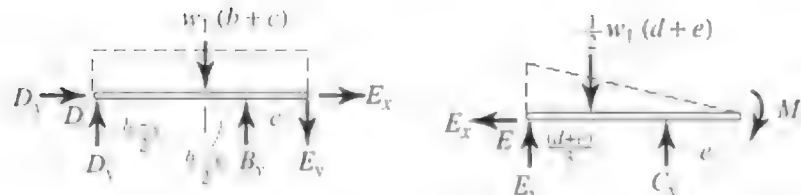
Given:

$a = 2\ m$ $M = 48\ kN\cdot m$

$b = 4\ m$ $w_1 = 8\ \frac{kN}{m}$

$c = 2\ m$

$d = 6\ m$ $w_2 = 6\ \frac{kN}{m}$

$e = 3\ m$

Solution:

Guesses

$A_x = 1\ N$ $A_y = 1\ N$ $M_A = 1\ Nm$

$D_x = 1\ N$ $D_y = 1\ N$ $B_y = 1\ N$

$E_y = 1\ N$ $E_x = 1\ N$ $C_y = 1\ N$

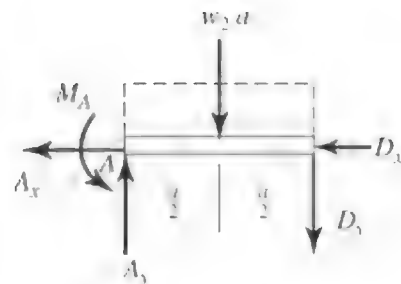
Given

$$A_y - w_2 a - D_y = 0 \qquad -A_x - D_x = 0$$

$$M_A - w_2 a \frac{a}{2} - D_y a = 0 \qquad D_y - w_1(b + c) + B_y - E_y = 0$$

$$-w_1 \frac{(b + c)^2}{2} + B_y b - E_y(b + c) = 0 \qquad D_x + E_x = 0$$

$$E_y - w_1 \frac{d + e}{2} + C_y = 0 \qquad -E_x = 0$$

$$-w_1\left(\frac{d + e}{2}\right)\left(\frac{d + e}{3}\right) + C_y d - M = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ M_A \\ D_x \\ D_y \\ B_y \\ E_y \\ E_x \\ C_y \end{pmatrix} = \text{Find}(A_x, A_y, M_A, D_x, D_y, B_y, E_y, E_x, C_y)$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 19 \end{pmatrix} \text{kN}$$

$M_A = 26\,\text{kN m}$

$B_y = 51\,\text{kN}$

$C_y = 26\,\text{kN}$

Problem 6-77

Determine the reactions at supports A and B.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$w_1 = 500\ \frac{\text{lb}}{\text{ft}}$

$w_2 = 700\ \frac{\text{lb}}{\text{ft}}$

$a = 6\ \text{ft}$

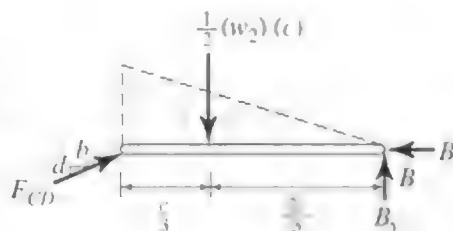
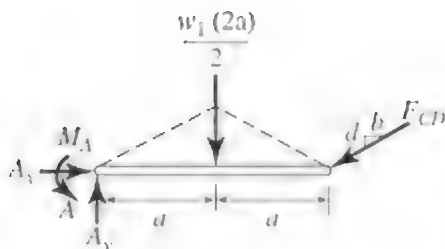
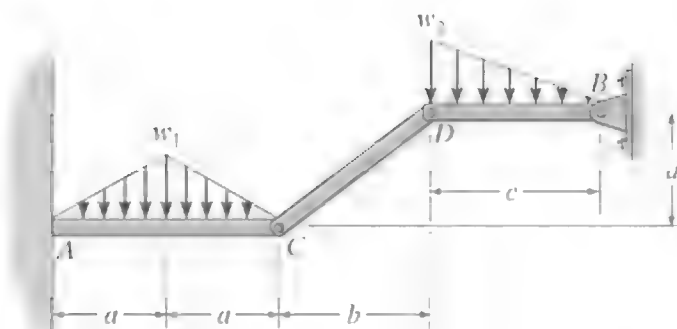
$b = 8\ \text{ft}$

$c = 9\ \text{ft}$

$d = 6\ \text{ft}$

Solution:

Guesses

$A_x = 1\ \text{lb} \quad A_y = 1\ \text{lb} \quad B_x = 1\ \text{lb} \quad B_y = 1\ \text{lb} \quad F_{CD} = 1\ \text{lb} \quad M_A = 1\ \text{lb}\cdot\text{ft}$

Given

$$A_y - w_1 a - \frac{d}{\sqrt{b^2 + d^2}} F_{CD} = 0 \qquad A_x - \frac{b}{\sqrt{b^2 + d^2}} F_{CD} = 0$$

$$M_A - w_1 a a - \frac{d}{\sqrt{d^2 + b^2}} F_{CD} 2\, a = 0 \qquad \frac{b}{\sqrt{b^2 + d^2}} F_{CD} - B_x = 0$$

$$\frac{d}{\sqrt{b^2 + d^2}} F_{CD} - w_2 \frac{c}{2} + B_y = 0 \qquad B_y c - w_2 \frac{c}{2} \frac{c}{3} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \\ F_{CD} \\ M_A \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y, F_{CD}, M_A) \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 2.8 \\ 5.1 \end{pmatrix} \text{kip}$$

$$M_A = 43.2 \text{ kip} \cdot \text{ft}$$

$$\begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 2.8 \\ 1.05 \end{pmatrix} \text{kip}$$

Problem 6-78

Determine the horizontal and vertical components of force at C which member ABC exerts on member CEF.

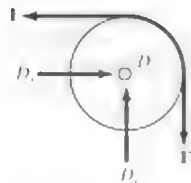
Given:

$F = 300$ lb

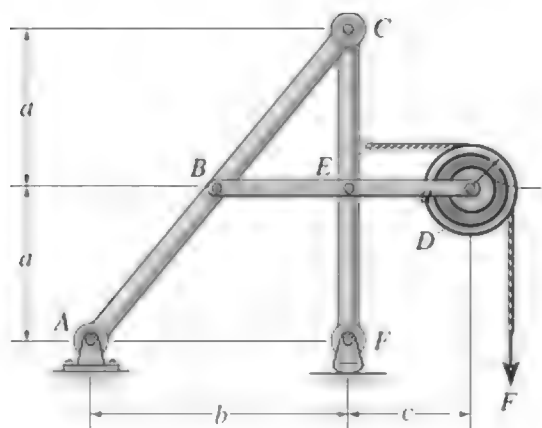
$a = 4$ ft

$b = 6$ ft

$c = 3$ ft

$r = 1$ ft

Solution:

Guesses

$A_x = 1$ lb $\qquad A_y = 1$ lb $\qquad F_y = 1$ lb

$C_x = 1\ \text{lb} \qquad C_y = 1\ \text{lb}$

Given

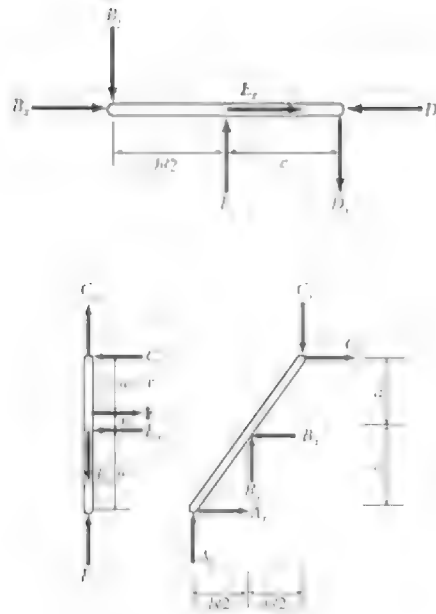
$$A_x a - A_y \frac{b}{2} - C_x a - C_y \frac{b}{2} = 0$$

$$C_x a - F r = 0$$

$$A_x = 0$$

$$A_y + F_y - F = 0$$

$$F_y b - F(b + c + r) = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ F_y \\ C_x \\ C_y \end{pmatrix} = \text{Find}(A_x, A_y, F_y, C_x, C_y) \qquad \begin{pmatrix} A_x \\ A_y \\ F_y \end{pmatrix} = \begin{pmatrix} 0 \\ -200 \\ 500 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 75 \\ 100 \end{pmatrix} \text{lb}$$

Problem 6-79

Determine the horizontal and vertical components of force that the pins at A, B, and C exert on their connecting members.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

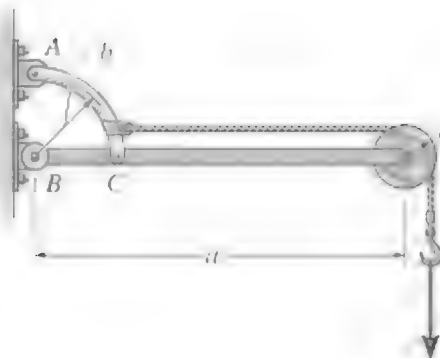
$F = 800\ \text{N}$

$a = 1\ \text{m}$

$r = 50\ \text{mm}$

$b = 0.2\ \text{m}$

Solution:

$$-F(a + r) + A_x b = 0$$

$A_x = F\frac{a+r}{b}$ $A_x = 4.2\,\text{kN}$

$-A_x + B_x = 0$

$B_x = A_x$ $B_x = 4.2\,\text{kN}$

$-F\,r - A_y b + A_x b = 0$

$A_y = \frac{-F\,r + A_x b}{b}$ $A_y = 4\,\text{kN}$

$A_y - B_y - F = 0$

$B_y = A_y - F$ $B_y = 3.2\,\text{kN}$

$-A_x + F + C_x = 0$

$C_x = A_x - F$ $C_x = 3.4\,\text{kN}$

$A_y - C_y = 0$

$C_y = A_y$ $C_y = 4\,\text{kN}$

Problem 6-80

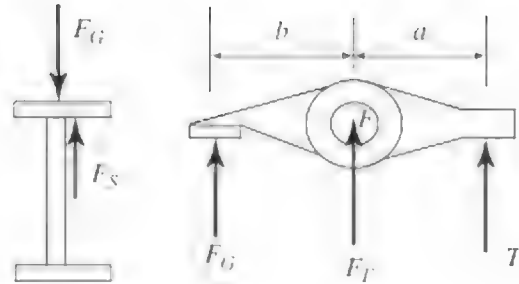
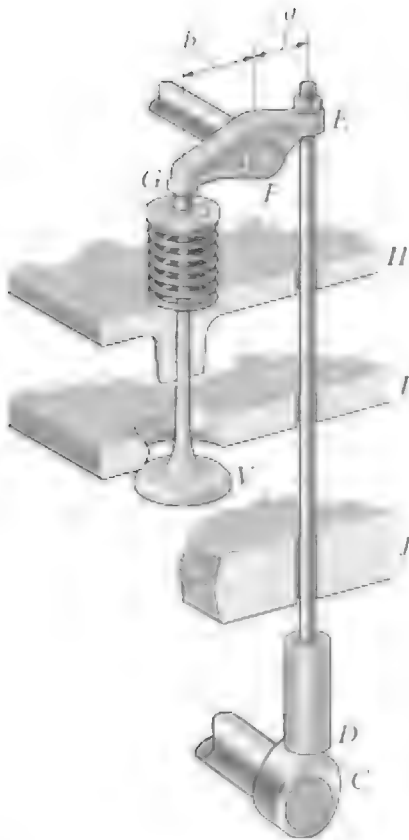
Operation of exhaust and intake valves in an automobile engine consists of the cam C, push rod DE, rocker arm EFG which is pinned at F, and a spring and valve, V. If the spring is compressed a distance δ when the valve is open as shown, determine the normal force acting on the cam lobe at C. Assume the cam and bearings at H, I, and J are smooth.The spring has a stiffness k.

Given:

$a = 25\text{ mm}$

$b = 40\text{ mm}$

$\delta = 20\text{ mm}$

$k = 300\,\frac{\text{N}}{\text{m}}$

Solution:

$F_s = k\delta \qquad F_s = 6 \text{ N}$

$\Sigma F_y = 0;$

$-F_G + F_s = 0$

$F_G = F_s \qquad F_G = 6 \text{ N}$

$\Sigma M_F = 0;$

$F_G b + T a = 0$

$T = F_G \dfrac{b}{a} \qquad T = 9.60 \text{ N}$

Problem 6-81

Determine the force **P** on the cord, and the angle θ that the pulley-supporting link AB makes with the vertical. Neglect the mass of the pulleys and the link. The block has weight W and the cord is attached to the pin at B. The pulleys have radii of r_1 and r_2.

Given:

$W = 200$ lb

$r_1 = 2$ in

$r_2 = 1$ in

$\phi = 45$ deg

Solution:

The initial guesses are

$\theta = 30$ deg $\quad F_{AB} = 30$ lb

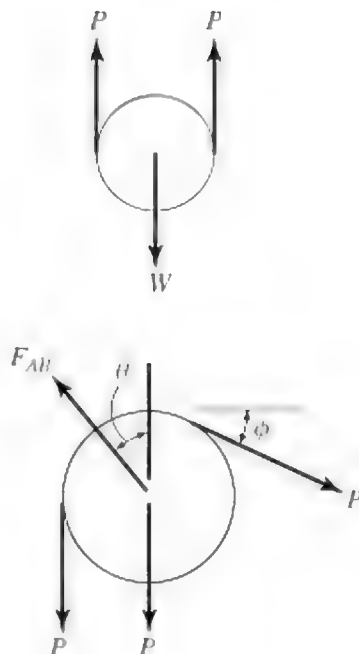
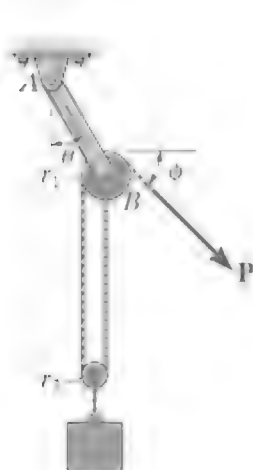
$+\uparrow \Sigma F_y = 0; \quad 2P - W = 0$

$$P = \frac{1}{2}W \qquad P = 100\,\text{lb}$$

Given

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad P\cos(\phi) - F_{AB}\sin(\theta) = 0$

$+\uparrow \Sigma F_y = 0; \quad F_{AB}\cos(\theta) - P - P - P\sin(\phi) = 0$

$$\begin{pmatrix} \theta \\ F_{AB} \end{pmatrix} = \text{Find}(\theta, F_{AB}) \qquad F_{AB} = 280\,\text{lb} \qquad \theta = 14.6\,\text{deg}$$

Problem 6-82

The nail cutter consists of the handle and the two cutting blades. Assuming the blades are pin connected at B and the surface at D is smooth, determine the normal force on the fingernail when a force **F** is applied to the handles as shown.The pin AC slides through a smooth hole at A and is attached to the bottom member at C.

Given:

$F = 1\ \text{lb}$

$a = 0.25\ \text{in}$

$b = 1.5\ \text{in}$

Solution:

Handle :

$\Sigma M_D = 0;\quad F_A a - F b = 0$

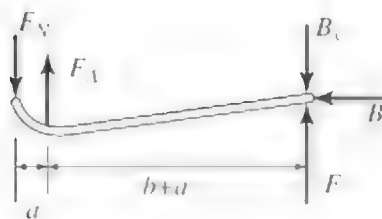
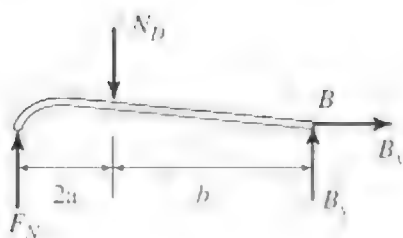
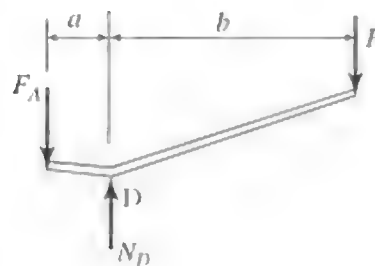
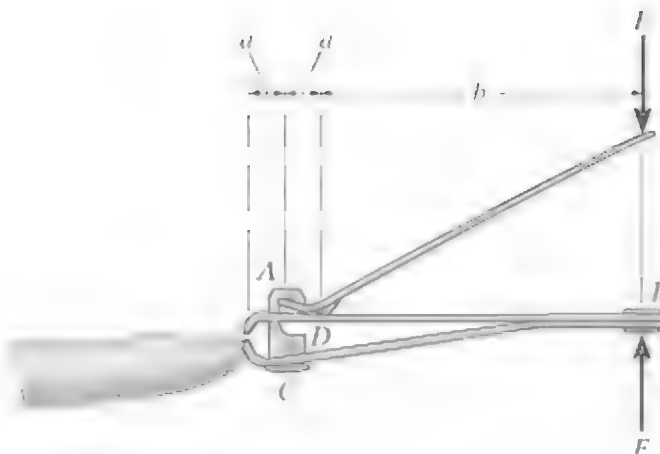
$F_A = F\left(\frac{b}{a}\right) \qquad F_A = 6\ \text{lb}$

$\Sigma F_y = 0;\quad N_D - F_A - F = 0$

$N_D = F_A + F \qquad N_D = 7\ \text{lb}$

Top blade :

$\Sigma M_B = 0;\quad N_D b - F_N(2a + b) = 0$

$F_N = N_D\left(\frac{b}{2a + b}\right) \qquad F_N = 5.25\ \text{lb}$

Problem 6-83

The wall crane supports load F. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W?

Units Used: $\text{kip} = 10^3\ \text{lb}$

Given:

$F = 700\ \text{lb}$

$a = 4 \text{ ft}$

$b = 4 \text{ ft}$

$c = 4 \text{ ft}$

$\theta = 60 \text{ deg}$

Solution:

Pulley E:

$+\uparrow \Sigma F_y = 0; \qquad 2T - F = 0$

$T = \frac{1}{2}F \qquad T = 350 \text{ lb}$

This is the force in the cable at the winch W

Member ABC: $\qquad \phi = \text{atan}\left(\frac{a}{b}\right)$

$\Sigma M_A = 0;$

$-F(b + c) + \left(T_{BD}\sin(\phi) - T\sin(\theta)\right)b = 0$

$$T_{BD} = \frac{F\left(\frac{b+c}{b}\right) + T\sin(\theta)}{\sin(\phi)}$$

$T_{BD} = 2.409 \times 10^3 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$

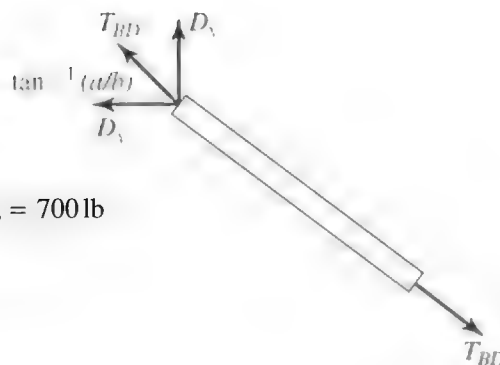
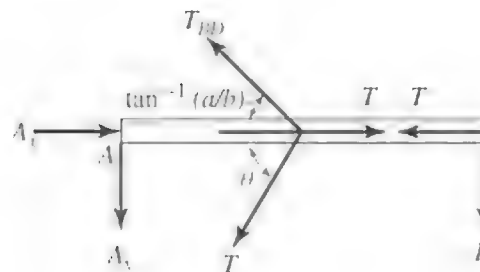
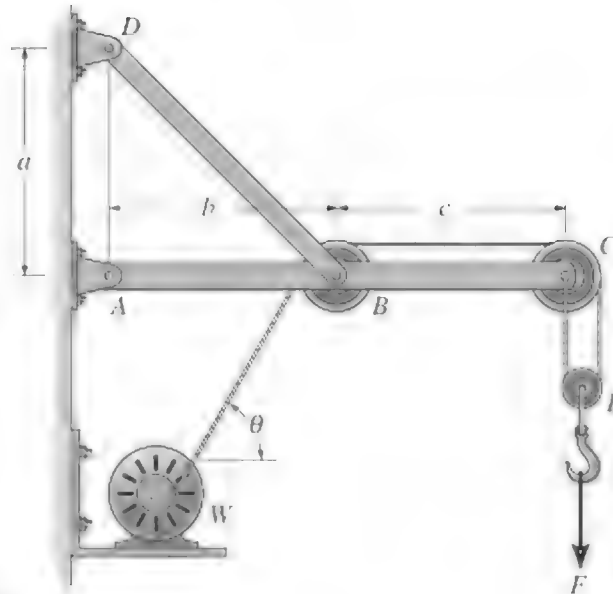
$-A_y + T_{BD}\sin(\phi) - T\sin(\theta) - F = 0$

$A_y = T_{BD}\sin(\phi) - T\sin(\theta) - F \qquad A_y = 700 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;$

$A_x - T_{BD}\cos(\phi) - T\cos(\theta) = 0$

$A_x = T_{BD}\cos(\phi) + T\cos(\theta) \qquad A_x = 1.878 \text{ kip}$

At D:

$D_x = T_{BD}\cos(\phi)$ $\qquad D_x = 1.703\,\text{kip}$

$D_y = T_{BD}\sin(\phi)$ $\qquad D_y = 1.703\,\text{kip}$

Problem 6-84

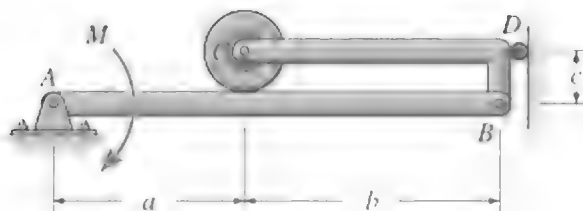
Determine the force that the smooth roller C exerts on beam AB. Also, what are the horizontal and vertical components of reaction at pin A? Neglect the weight of the frame and roller.

Given:

$M = 60\ \text{lb}\cdot\text{ft}$

$a = 3\ \text{ft}$

$b = 4\ \text{ft}$

$c = 0.5\ \text{ft}$

Solution:

$\Sigma M_A = 0;\qquad -M + D_x c = 0$

$D_x = \frac{M}{c}$

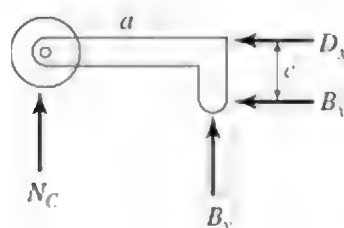
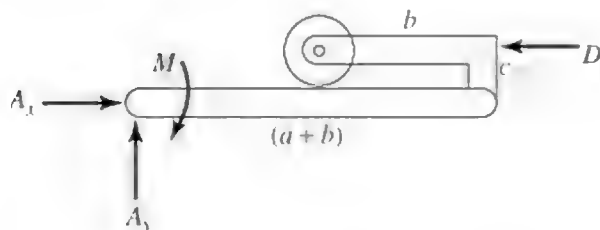
$D_x = 120\,\text{lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x - D_x = 0$

$A_x = D_x$ $\qquad A_x = 120\,\text{lb}$

$+\uparrow \Sigma F_y = 0;\qquad A_y = 0$ $\qquad A_y = 0\ \text{lb}$

$\Sigma M_B = 0;\qquad -N_C b + D_x c = 0$

$N_C = D_x \frac{c}{b}$ $\qquad N_C = 15.0\,\text{lb}$

Problem 6-85

Determine the horizontal and vertical components of force which the pins exert on member ABC.

Given:

$W = 80\ \text{lb}$

$a = 6\ \text{ft}$

$b = 9\ \text{ft}$

$c = 3\ \text{ft}$

$r = 0.5\ \text{ft}$

Solution:

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -A_x + W = 0$

$A_x = W \qquad A_x = 80\ \text{lb}$

$+\uparrow \Sigma F_y = 0;\quad A_y - W = 0$

$A_y = W \qquad A_y = 80\ \text{lb}$

$\Sigma M_C = 0;\quad A_y(a+b) - B_y b = 0$

$$B_y = A_y \frac{a+b}{b}$$

$B_y = 133\ \text{lb}$

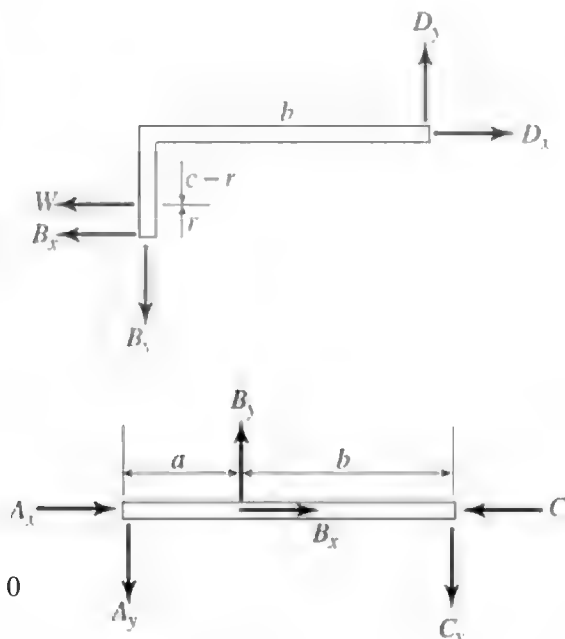
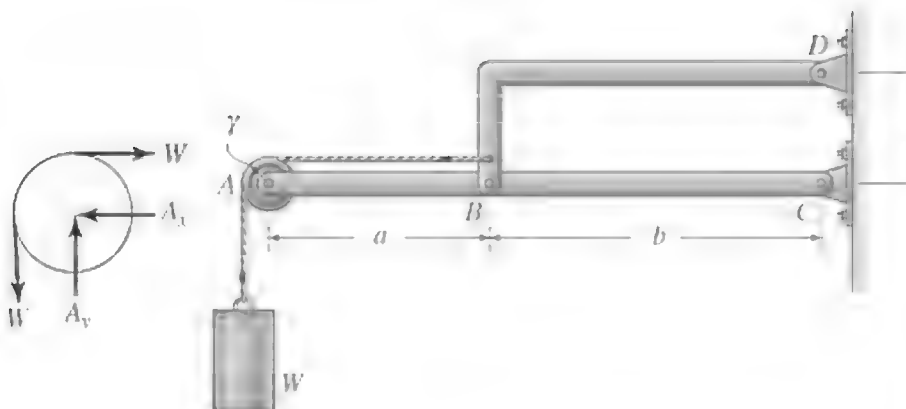
$\Sigma M_D = 0;\quad -W(c-r) + B_y b - B_x c = 0$

$$B_x = \frac{B_y\, b - W\,(c-r)}{c} \qquad B_x = 333\ \text{lb}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x + B_x - C_x = 0$

$C_x = A_x + B_x \qquad C_x = 413\ \text{lb}$

$+\uparrow \Sigma F_y = 0;\quad -A_y + B_y - C_y = 0$

$C_y = B_y - A_y \qquad C_y = 53.3\ \text{lb}$

Problem 6-86

The floor beams *AB* and *BC* are stiffened using the two tie rods *CD* and *AD*. Determine the force along each rod when the floor beams are subjected to a uniform load *w*. Assume the three contacting members at *B* are smooth and the joints at *A*, *C*, and *D* are pins. *Hint:*

Members AD, CD, and BD are two-force members

Units Used: $\text{kip} = 10^3 \text{ lb}$

Given:

$w = 80 \frac{\text{lb}}{\text{ft}}$

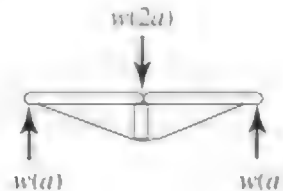
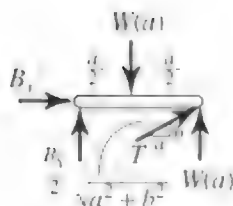
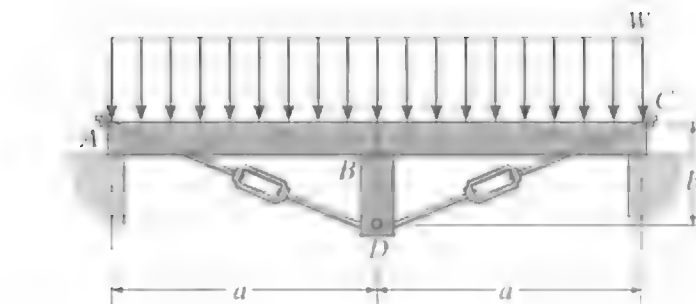
$b = 5 \text{ ft}$

$a = 12 \text{ ft}$

Solution:

Due to summetry:

$$C_y = \frac{w(2a)}{2}$$

$$C_y = 960 \text{ lb}$$

Member BC :

$\Sigma M_B = 0;$ $$C_y(a) - wa\left(\frac{a}{2}\right) - T\left(\frac{b}{\sqrt{a^2 + b^2}}\right)a = 0$$

$$T = \left(C_y - w\frac{a}{2}\right)\frac{\sqrt{a^2 + b^2}}{b}$$ $T = 1.248 \text{ kip}$

Problem 6-87

Determine the horizontal and vertical components of force at pins B and C.

Given:

$F = 50 \text{ lb}$ $c = 6 \text{ ft}$

$a = 4 \text{ ft}$ $d = 1.5 \text{ ft}$

$b = 4 \text{ ft}$ $r = 0.5 \text{ ft}$

Solution: Guesses

$C_x = 1 \text{ lb}$ $C_y = 1 \text{ lb}$

$B_x = 1 \text{ lb}$ $B_y = 1 \text{ lb}$

Given

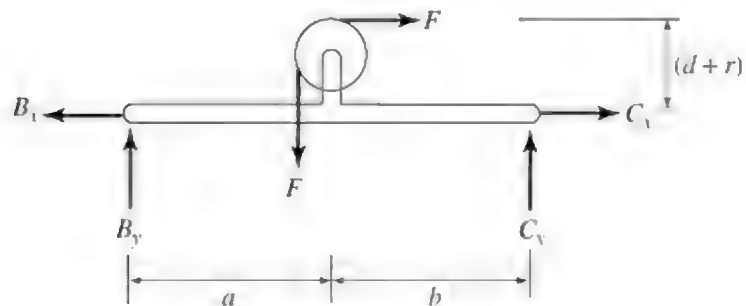
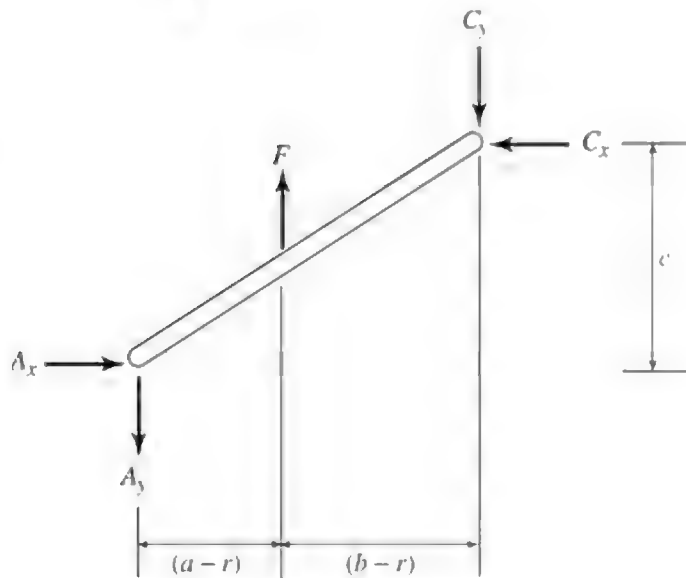
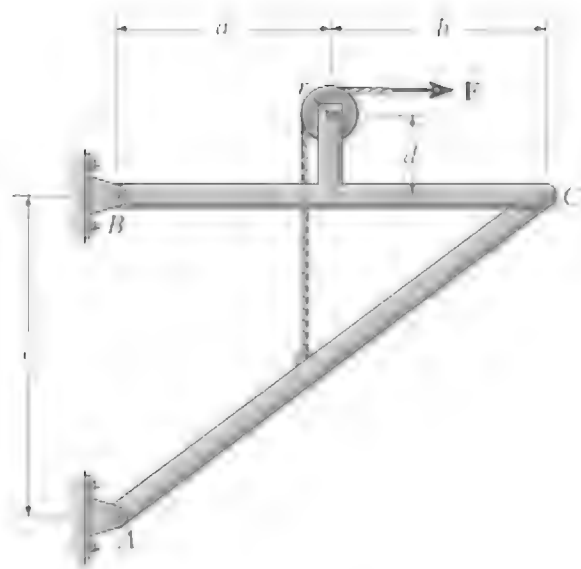
$$F(a-r) + C_x c - C_y(a+b) = 0$$

$$-F(a-r) - F(d+r) + C_y(a+b) = 0$$

$$-B_x + F + C_x = 0$$

$$B_y - F + C_y = 0$$

$$\begin{pmatrix} B_x \\ B_y \\ C_x \\ C_y \end{pmatrix} = \text{Find}(B_x, B_y, C_x, C_y)$$

$$\begin{pmatrix} B_x \\ B_y \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 66.667 \\ 15.625 \\ 16.667 \\ 34.375 \end{pmatrix} \text{lb}$$

Problem 6-88

The skid steer loader has a mass M_1, and in the position shown the center of mass is at G_1. If there is a stone of mass M_2 in the bucket, with center of mass at G_2 determine the reactions of each pair of wheels A and B on the ground and the force in the hydraulic cylinder CD and at the pin E. There is a similar linkage on each side of the loader.

Units Used:

$$\text{Mg} = 10^3 \text{ kg}$$

$$\text{kN} = 10^3 \text{ N}$$

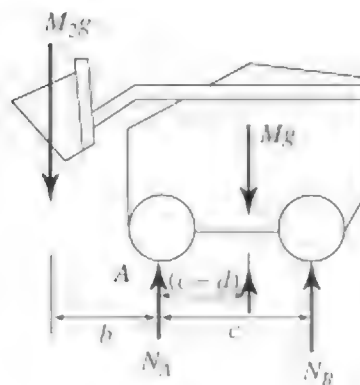
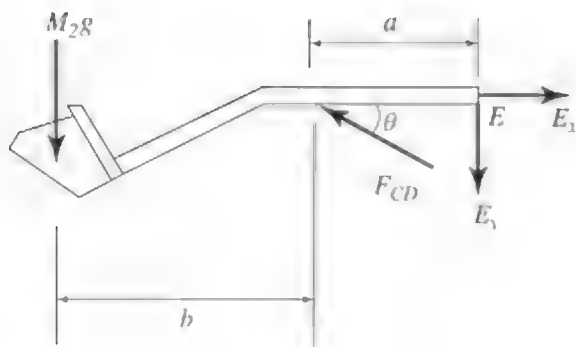
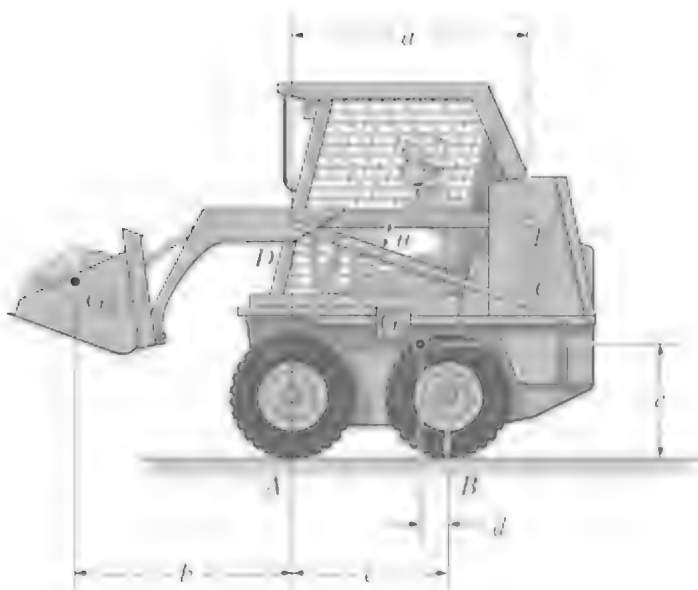
Given:

$M_1 = 1.18 \text{ Mg}$

$M_2 = 300 \text{ kg}$

$a = 1.25 \text{ m}$ $d = 0.15 \text{ m}$

$b = 1.5 \text{ m}$ $e = 0.5 \text{ m}$

$c = 0.75 \text{ m}$ $\theta = 30 \text{ deg}$

Solution: Entire System:

$\Sigma M_A = 0;$ $$M_2 g b - M_1 g(c-d) + N_B c = 0$$

$$N_B = \frac{M_1 g(c-d) - M_2 g b}{c} \qquad N_B = 3.37 \text{ kN} \quad \text{(Both wheels)}$$

$\Sigma F_y = 0;$ $$N_B - M_2 g - M_1 g + N_A = 0$$

$$N_A = -N_B + M_2 g + M_1 g \qquad N_A = 11.1 \text{ kN} \quad \text{(Both wheels)}$$

Upper member:

$\Sigma M_E = 0;$ $$M_2 g(a+b) - 2\ F_{CD} \sin(\theta) a = 0$$

$$F_{CD} = \frac{M_2 g(a+b)}{2\sin(\theta)a} \qquad F_{CD} = 6.5\,\text{kN}$$

$\Sigma F_x = 0;$ $\quad E_x = F_{CD}(\cos(\theta)) \qquad E_x = 5607\,\text{N}$

$\Sigma F_y = 0;$ $\quad E_y - \frac{M_2 g}{2} + F_{CD}\sin(\theta) = 0$

$$E_y = \frac{M_2 g}{2} - F_{CD}\sin(\theta) \qquad E_y = -1766\,\text{N}$$

$$F_R = \sqrt{E_x^2 + E_y^2} \qquad F_R = 5.879\,\text{kN}$$

Problem 6-89

Determine the horizontal and vertical components of force at each pin. The suspended cylinder has a weight W.

Given:

$W = 80$ lb $\quad d = 6$ ft

$a = 3$ ft $\quad e = 2$ ft

$b = 4$ ft $\quad r = 1$ ft

$c = 4$ ft

Solution:

Guesses

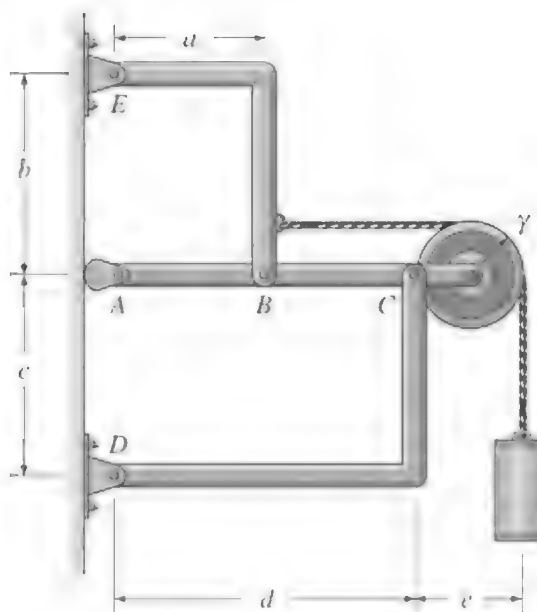
$A_x = 1$ lb $\quad B_x = 1$ lb

$B_y = 1$ lb $\quad F_{CD} = 1$ lb

$E_x = 1$ lb $\quad E_y = 1$ lb

Given

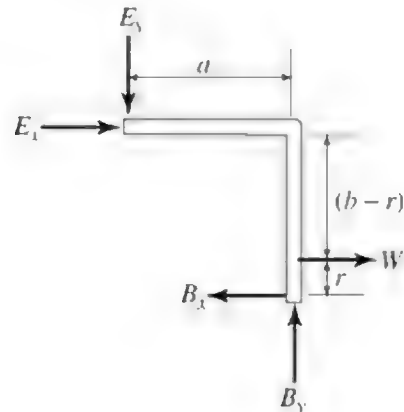
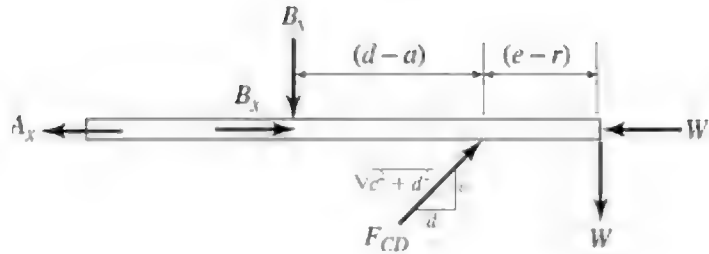
$$E_x - B_x + W = 0$$

$$-E_y + B_y = 0$$

$$-Wr + E_y a - E_x d = 0$$

$$-B_y + \frac{c}{\sqrt{c^2+d^2}} F_{CD} - W = 0$$

$$-B_y a + \frac{c}{\sqrt{c^2+d^2}} F_{CD} d - W(d+e-r) = 0$$

$$-A_x + B_x + \frac{d}{\sqrt{c^2+d^2}} F_{CD} - W = 0$$

$$\begin{pmatrix} A_x \\ B_x \\ B_y \\ F_{CD} \\ E_x \\ E_y \end{pmatrix} = \text{Find}(A_x, B_x, B_y, F_{CD}, E_x, E_y)$$

$$C_x = F_{CD}\frac{d}{\sqrt{c^2+d^2}} \qquad C_y = F_{CD}\frac{c}{\sqrt{c^2+d^2}} \qquad D_x = -C_x \qquad D_y = -C_y$$

$$\begin{pmatrix} A_x \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \\ E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 160 \\ 80 \\ 26.667 \\ 160 \\ 106.667 \\ -160 \\ -106.667 \\ -8.694 \times 10^{-13} \\ 26.667 \end{pmatrix} \text{lb}$$

Problem 6-90

The two-member frame is pin connected at *C*, *D*, and *E*. The cable is attached to *A*, passes over the smooth peg at *B*, and is attached to a load *W*. Determine the horizontal and vertical reactions at each pin.

Given:

$a = 2\ \text{ft}$

$b = 1\ \text{ft}$

$c = 0.75\ \text{ft}$

$W = 100\ \text{lb}$

Solution:

$$d = \frac{c}{b}(a + 2\ b)$$

Initial guesses:

$C_x = 1\ \text{lb}$ $C_y = 1\ \text{lb}$ $D_x = 1\ \text{lb}$ $D_y = 1\ \text{lb}$ $E_x = 1\ \text{lb}$ $E_y = 1\ \text{lb}$

Given

$$-D_x + C_x - W = 0$$

$$-D_y + C_y - W = 0$$

$$-C_x c + C_y b + W d - W(a + 2\ b) = 0$$

$$W - C_x + E_x = 0$$

$$E_y - C_y = 0$$

$$-W d + C_x c + C_y b = 0$$

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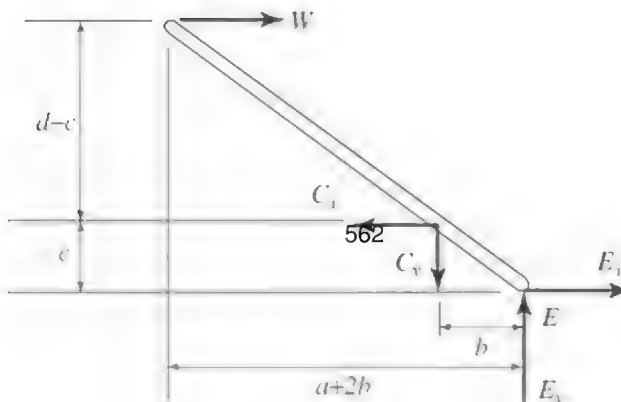
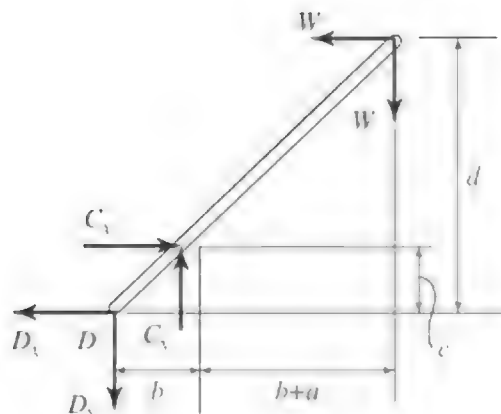
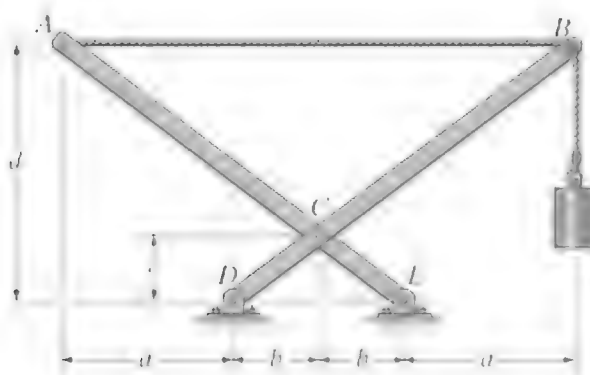

$$\begin{pmatrix} C_x \\ C_y \\ D_x \\ D_y \\ E_x \\ E_y \end{pmatrix} = \text{Find}(C_x, C_y, D_x, D_y, E_x, E_y) \qquad \begin{pmatrix} C_x \\ C_y \\ D_x \\ D_y \\ E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 133 \\ 200 \\ 33 \\ 100 \\ 33 \\ 200 \end{pmatrix} \text{lb}$$

Problem 6-91

Determine the horizontal and vertical components of force which the pins at A, B, and C exert on member ABC of the frame.

Given:

$F_1 = 400$ N

$F_2 = 300$ N

$F_3 = 300$ N

$a = 1.5$ m

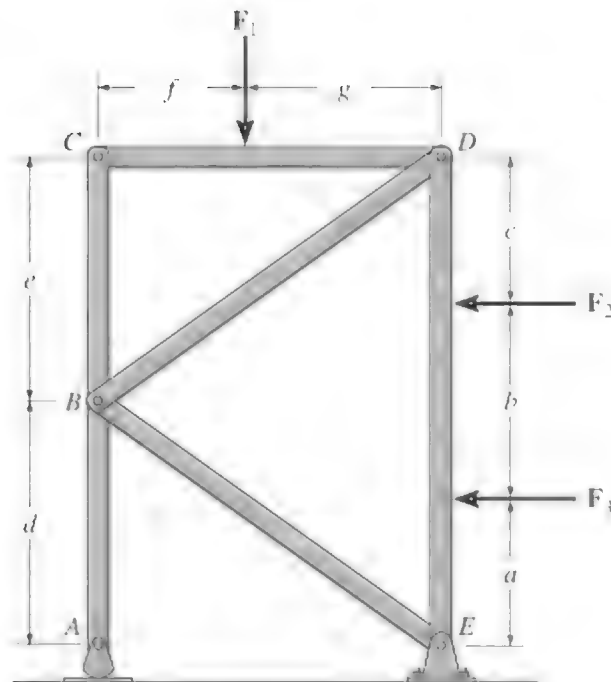
$b = 2$ m

$c = 1.5$ m

$d = 2.5$ m

$f = 1.5$ m

$g = 2$ m

$e = a + b + c - d$

Solution:

Guesses

$A_y = 1$ N $\quad C_x = 1$ N $\quad C_y = 1$ N

$F_{BD} = 1$ N $\quad F_{BE} = 1$ N

Given

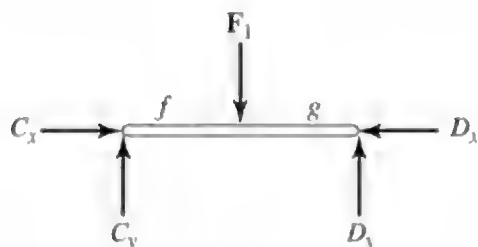
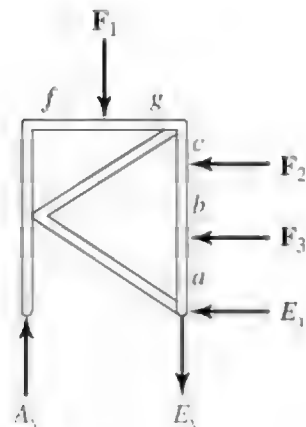
$$F_1 g + F_2(a+b) + F_3 a - A_y(f+g) = 0$$

$$F_1 g - C_y(f+g) = 0$$

$$C_x e = 0$$

$$-C_x - \frac{f+g}{\sqrt{e^2+(f+g)^2}}F_{BD} + \frac{f+g}{\sqrt{d^2+(f+g)^2}}F_{BE} = 0$$

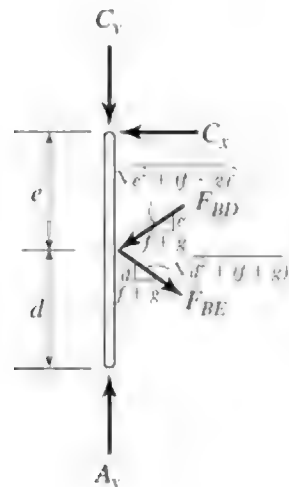
$$A_y - C_y - \frac{e}{\sqrt{e^2+(f+g)^2}}F_{BD} - \frac{d}{\sqrt{d^2+(f+g)^2}}F_{BE} = 0$$

$$\begin{pmatrix} A_y \\ C_x \\ C_y \\ F_{BD} \\ F_{BE} \end{pmatrix} = \text{Find}(A_y, C_x, C_y, F_{BD}, F_{BE})$$

$$B_x = -\frac{f+g}{\sqrt{e^2+(f+g)^2}}F_{BD} + \frac{f+g}{\sqrt{d^2+(f+g)^2}}F_{BE}$$

$$B_y = \frac{e}{\sqrt{e^2+(f+g)^2}}F_{BD} + \frac{d}{\sqrt{d^2+(f+g)^2}}F_{BE}$$

$A_y = 657$ N

$$\begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 0 \\ 429 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 0 \\ 229 \end{pmatrix} \text{N}$$

Problem 6-92

The derrick is pin-connected to the pivot at A. Determine the largest mass that can be supported by the derrick if the maximum force that can be sustained by the pin at A is F_{max}.

Units Used:

$$kN = 10^3\ N$$

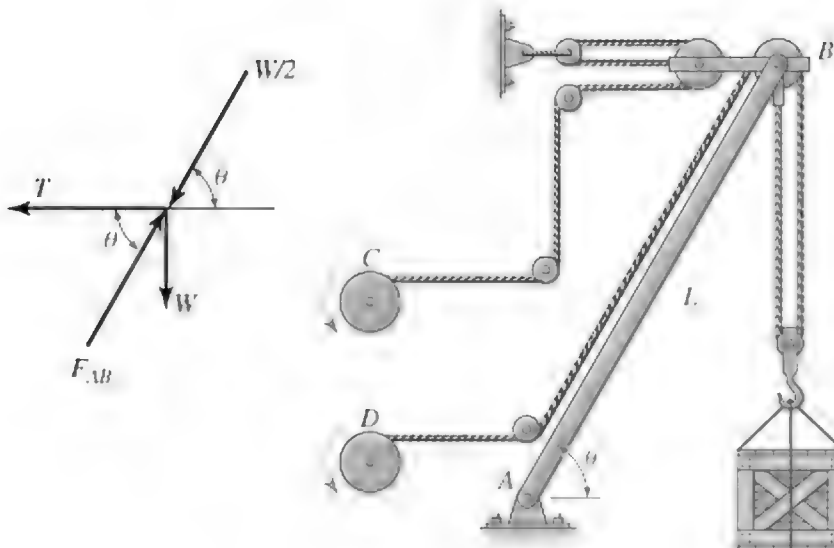
$$g = 9.81\ \frac{m}{s^2}$$

$$Mg = 10^3\ kg$$

Given:

$$F_{max} = 18\ kN$$

$$L = 5\ m$$

$$\theta = 60\ deg$$

Solution:

AB is a two-force member.

Require $F_{AB} = F_{max}$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\sin(\theta) - \frac{Mg}{2}\sin(\theta) - W = 0$$

$$M = 2\left(\frac{F_{AB}}{g}\right)\left(\frac{\sin(\theta)}{\sin(\theta)+2}\right) \qquad M = 5.439\ \frac{1}{s^2}\ Mg$$

Problem 6-93

Determine the required mass of the suspended cylinder if the tension in the chain wrapped around the freely turning gear is T. Also, what is the magnitude of the resultant force on pin A?

Units Used:

$$kN = 10^3\ N$$

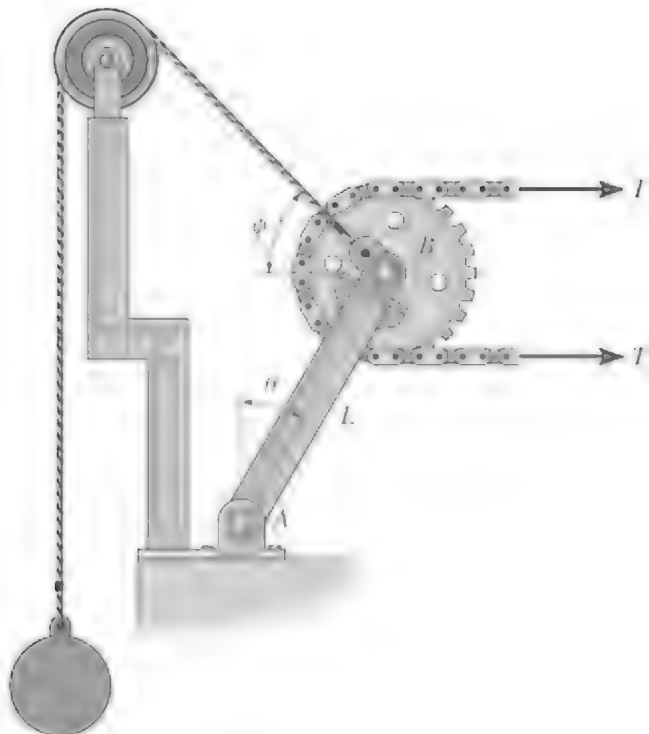
$$g = 9.8\ \frac{m}{s^2}$$

Given:

$T = 2\ \text{kN}$

$L = 2\ \text{ft}$

$\theta = 30\ \text{deg}$

$\phi = 45\ \text{deg}$

Solution:

$\Sigma M_A = 0;$ $\quad -2\,T L\cos(\theta) + M g\cos(\phi) L\cos(\theta) + M g\sin(\phi) L\sin(\theta) = 0$

$$M = \frac{2\,T\cos(\theta)}{(\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta))g}$$

$$M = 1793\ \frac{1}{s^2}\,\text{kg}$$

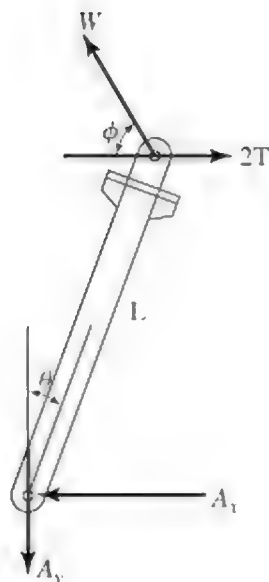
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 2\,T - M g\cos(\phi) - A_x = 0$

$$A_x = 2\,T - M g\cos(\phi)$$

$+\uparrow \Sigma F_y = 0;$ $\quad M g\sin(\phi) - A_y = 0$

$$A_y = M g\sin(\phi)$$

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}}$$

$$F_A = 2.928\ \text{kN}$$

Problem 6-94

The tongs consist of two jaws pinned to links at A, B, C, and D. Determine the horizontal and vertical components of force exerted on the stone of weight W at F and G in order to lift it.

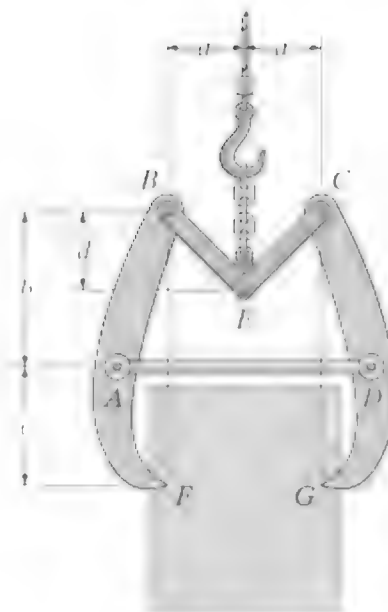
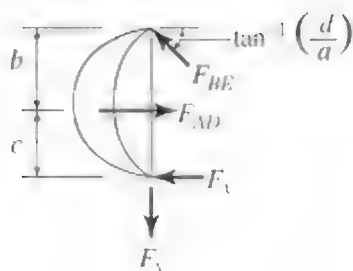
Given:

$a = 1\ \text{ft}$

$b = 2\ \text{ft}$

$c = 1.5\ \text{ft}$

$d = 1\ \text{ft}$

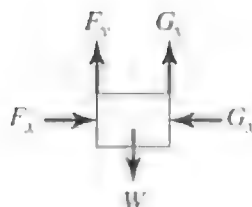
$W = 500\ \text{lb}$

Solution:

Guesses

$F_x = 1\ \text{lb}$

$F_y = 1\ \text{lb}$

$F_{AD} = 1\ \text{lb}$

$F_{BE} = 1\ \text{lb}$

Given $\quad 2\,F_y - W = 0 \qquad F_{AD}\,b - F_x(b+c) = 0$

$$F_{AD} - F_x - \left(\frac{a}{\sqrt{a^2+d^2}}\right)F_{BE} = 0 \qquad -F_y + \left(\frac{d}{\sqrt{a^2+d^2}}\right)F_{BE} = 0$$

$$\begin{pmatrix} F_x \\ F_y \\ F_{AD} \\ F_{BE} \end{pmatrix} = \text{Find}(F_x, F_y, F_{AD}, F_{BE}) \qquad \begin{pmatrix} G_x \\ G_y \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} \qquad \begin{pmatrix} F_x \\ F_y \\ G_x \\ G_y \end{pmatrix} = \begin{pmatrix} 333 \\ 250 \\ 333 \\ 250 \end{pmatrix} \text{lb}$$

Problem 6-95

Determine the force $\mathbf{P}$ on the cable if the spring is compressed a distance δ when the mechanism is in the position shown. The spring has a stiffness k.

Given:

$\delta = 0.5 \text{ in}$ $c = 6 \text{ in}$

$k = 800 \frac{\text{lb}}{\text{ft}}$ $d = 6 \text{ in}$

$a = 24 \text{ in}$ $e = 4 \text{ in}$

$b = 6 \text{ in}$ $\theta = 30 \text{ deg}$

Solution:

$F_E = k\delta$ $F_E = 33.333 \text{ lb}$

The initial guesses are

$P = 20 \text{ lb}$ $B_x = 11 \text{ lb}$

$B_y = 34 \text{ lb}$ $F_{CD} = 34 \text{ lb}$

Given

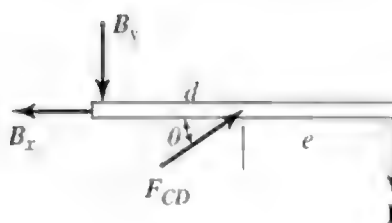
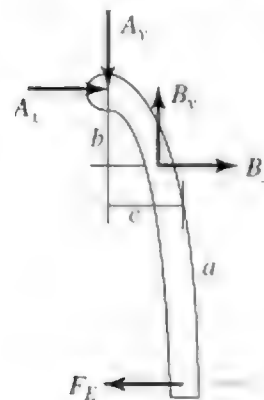
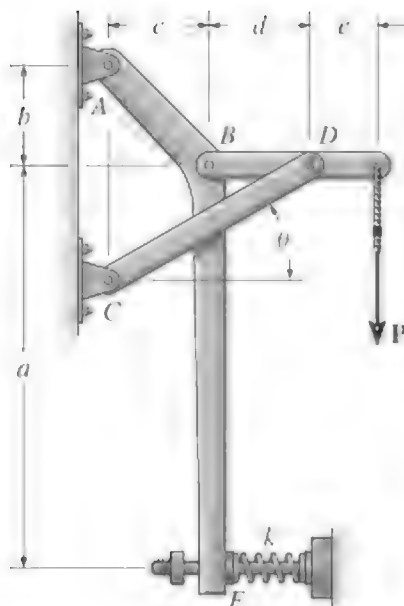
$\Sigma M_A = 0;$ $B_x b + B_y c - F_E(a+b) = 0$

$\Sigma M_D = 0;$ $B_y d - P e = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $-B_x + F_{CD}\cos(\theta) = 0$

$\Sigma M_B = 0;$ $F_{CD}\sin(\theta)d - P(d+e) = 0$

$$\begin{pmatrix} F_{CD} \\ B_x \\ B_y \\ P \end{pmatrix} = \text{Find}(F_{CD}, B_x, B_y, P)$$

$B_x = 135.398 \text{ lb}$ $B_y = 31.269 \text{ lb}$ $F_{CD} = 156.344 \text{ lb}$ $P = 46.903 \text{ lb}$

Problem 6-96

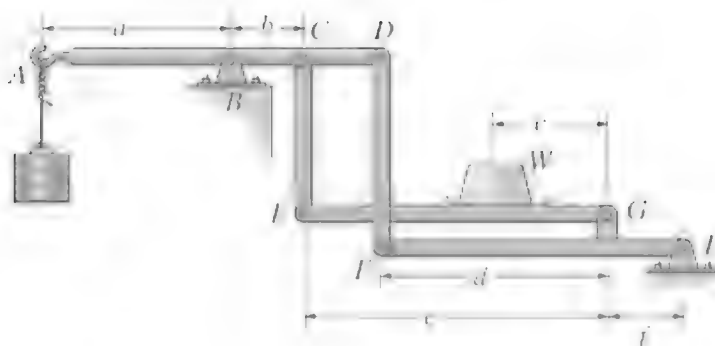
The scale consists of five pin-connected members. Determine the load W on the pan EG if a weight F is suspended from the hook at A.

Given:

$F = 3\text{ lb}$ $b = 3\text{ in}$

$a = 5\text{ in}$ $c = 4\text{ in}$

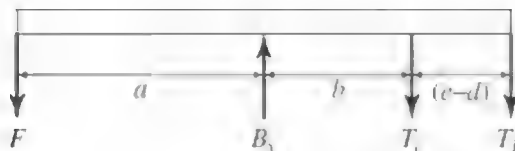
$d = 6\text{ in}$ $f = 2\text{ in}$

$e = 8\text{ in}$

Solution:

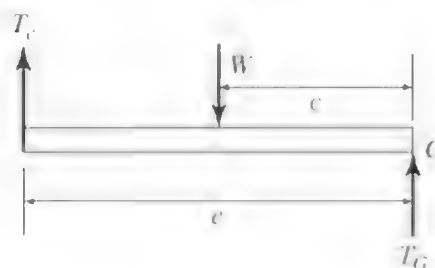
Guesses $T_C = 10\text{ lb}$ $T_D = 10\text{ lb}$ $T_G = 10\text{ lb}$ $W = 10\text{ lb}$

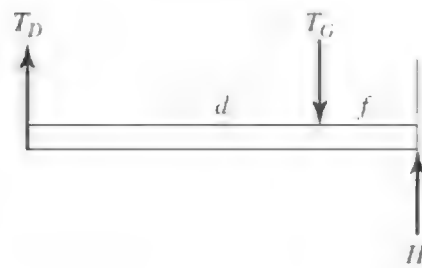
Given

Member $ABCD$: $\Sigma M_B = 0;$

$$F a - T_C b - T_D(b + e - d) = 0$$

Member EG:

$\Sigma M_G = 0;$ $-T_C e + W c = 0$

$\Sigma F_y = 0;$ $T_G - W + T_C = 0$

Member FH:

$\Sigma M_H = 0;$ $-T_D(d + f) + T_G f = 0$

$$\begin{pmatrix} T_C \\ T_D \\ T_G \\ W \end{pmatrix} = \text{Find}(T_C, T_D, T_G, W)$$

$W = 7.06\text{ lb}$

Problem 6-97

The machine shown is used for forming metal plates. It consists of two toggles ABC and DEF,

which are operated by the hydraulic cylinder H. The toggles push the movable bar G forward, pressing the plate p into the cavity. If the force which the plate exerts on the head is P, determine the force F in the hydraulic cylinder for the given angle θ.

Units Used:

$kN = 10^3\ N$

Given:

$P = 12\ kN$

$a = 200\ mm$

$\theta = 30\ deg$

Solution:

Member EF:

$\Sigma M_E = 0;\quad -F_y a\cos(\theta) + \frac{P}{2} a\sin(\theta) = 0$

$F_y = \frac{P}{2}\tan(\theta) \qquad F_y = 3.464\,kN$

$\Sigma F_x = 0;\quad E_x - \frac{P}{2} = 0$

$E_x = \frac{P}{2} \qquad E_x = 6\,kN$

$\Sigma F_y = 0;\quad E_y - F_y = 0$

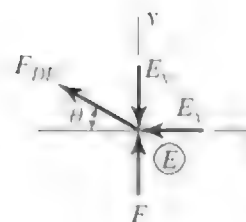
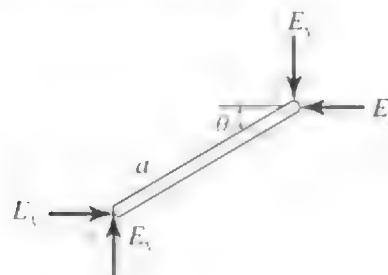
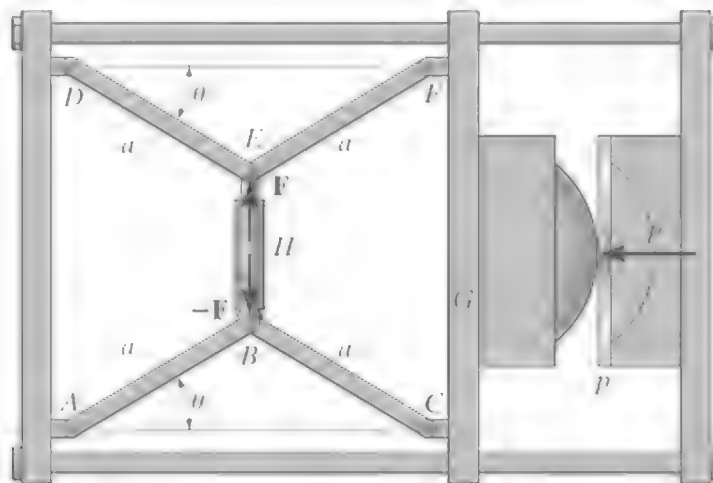
$E_y = F_y \qquad E_y = 3.464\,kN$

Joint E:

$\Sigma F_x = 0;\quad -F_{DE}\cos(\theta) - E_x = 0$

$F_{DE} = \frac{-E_x}{\cos(\theta)} \qquad F_{DE} = -6.928\,kN$

$\Sigma F_y = 0;\quad F - E_y + F_{DE}\sin(\theta) = 0$

$F = E_y - F_{DE}\sin(\theta) \qquad F = 6.93\,kN$

Problem 6-98

Determine the horizontal and vertical components of force at pins A and C of the two-member frame.

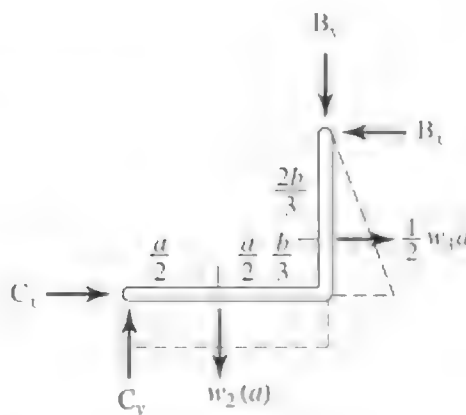
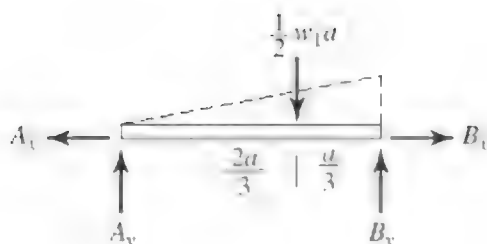
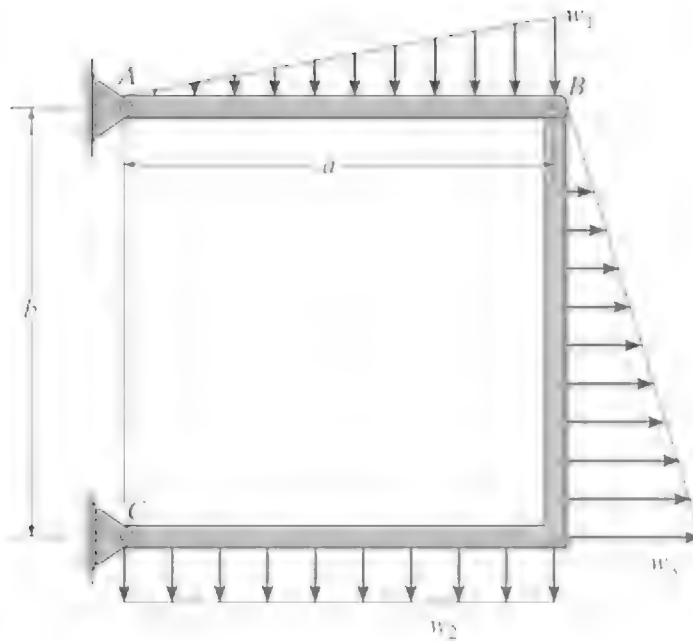
Given:

$$w_1 = 500\ \frac{\text{N}}{\text{m}}$$

$$w_2 = 400\ \frac{\text{N}}{\text{m}}$$

$$w_3 = 600\ \frac{\text{N}}{\text{m}}$$

$a = 3\text{ m}$

$b = 3\text{ m}$

Solution:

Guesses

$A_x = 1\text{ N}$ $A_y = 1\text{ N}$ $C_x = 1\text{ N}$ $C_y = 1\text{ N}$

Given

$$A_y + C_y - \frac{1}{2}w_1 a - w_2 a = 0 \qquad A_x a - \frac{1}{2}w_1 a\frac{2\ a}{3} - \frac{1}{2}w_3 b\frac{b}{3} - w_2 a\frac{a}{2} = 0$$

$$-A_x + C_x + \frac{1}{2}w_3 b = 0 \qquad -A_y a + \frac{1}{2}w_1 a\frac{a}{3} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ C_x \\ C_y \end{pmatrix} = \text{Find}(A_x, A_y, C_x, C_y) \qquad \begin{pmatrix} A_x \\ A_y \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 1400 \\ 250 \\ 500 \\ 1700 \end{pmatrix}\text{ N}$$

Problem 6-99

The truck rests on the scale, which consists of a series of compound levers. If a mass M_1 is placed on the pan P and it is required that the weight is located at a distance x to balance the "beam" ABC, determine the mass of the truck. There are pins at all lettered points. Is it necessary for the truck to be symmetrically placed on the scale? Explain.

Units Used:

$\text{Mg} = 10^3 \text{ kg} \qquad g = 9.81 \frac{\text{m}}{\text{s}^2}$

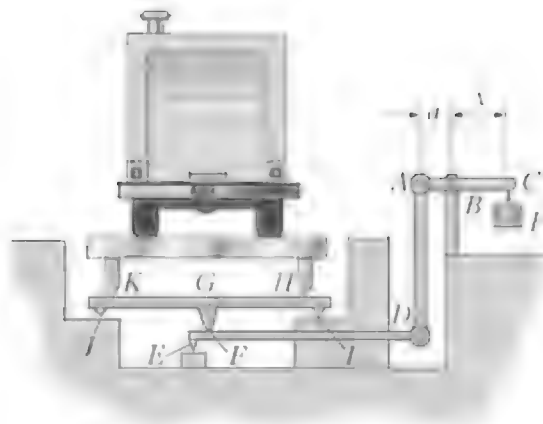
Given:

$M_1 = 15 \text{ kg} \qquad FD = 3 \text{ m}$

$x = 0.480 \text{ m} \qquad EF = 0.2 \text{ m}$

$a = 0.2 \text{ m}$

$HI = 0.1 \text{ m} \qquad GH = 2.5 \text{ m}$

$KJ = HI \qquad KG = GH$

Solution:

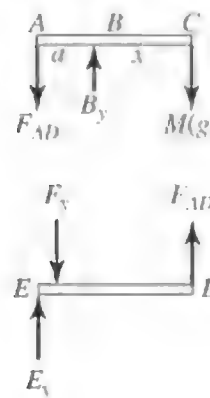
Member ABC :

$\Sigma M_B = 0; \qquad -M_1 g x + F_{AD} a = 0$

$$F_{AD} = M_1 g \frac{x}{a} \qquad F_{AD} = 72 \text{ s}^2 \text{ N}$$

Member EFD :

$\Sigma M_E = 0; \qquad -F_y EF + F_{AD}(FD + EF) = 0$

$$F_y = F_{AD}\left(\frac{FD + EF}{EF}\right) \qquad F_y = 1152 \text{ s}^2 \text{ N}$$

Member GHI :

$\Sigma M_I = 0;$ $H_y HI - G_Y(GH + HI) = 0$

Member JKG :

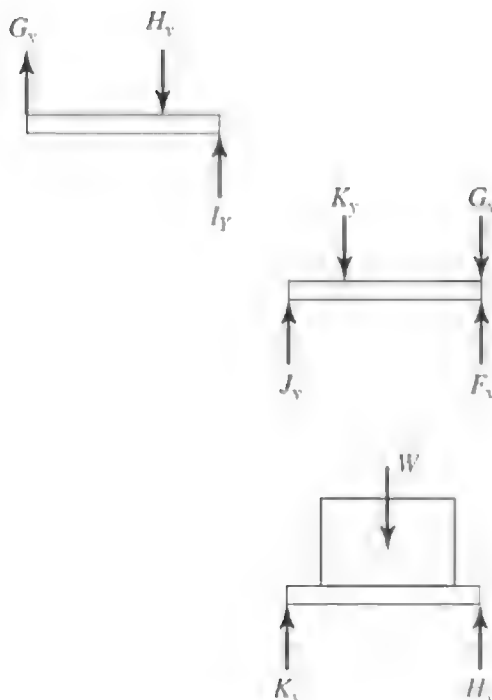
$\Sigma M_J = 0;$ $(F_y - G_y)(KJ + GH) - K_y(KJ) = 0$

$$K_y + H_y = F_y \frac{KJ + KG}{HI}$$

Scale Platform :

$\Sigma F_y = 0;$ $K_y + H_y = W$

$$W = F_y\left(\frac{KJ + KG}{HI}\right)$$

$$M = \frac{W}{g} \qquad M = 14.98\,\text{Mg}$$

Because $KJ = HI$ and $KG = GH$ it doesn't matter where the truck is on the scale.

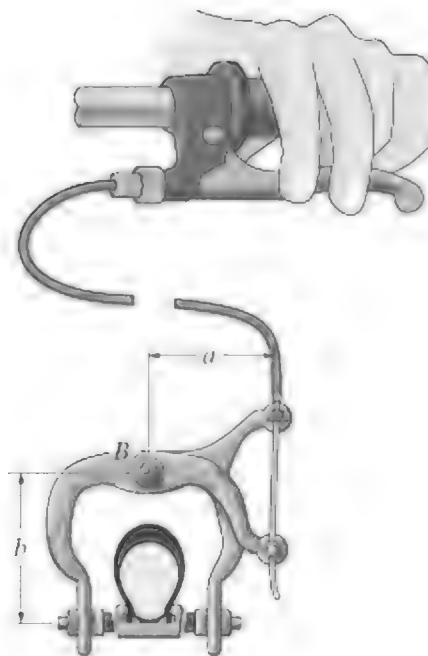
Problem 6-100

By squeezing on the hand brake of the bicycle, the rider subjects the brake cable to a tension **T** If the caliper mechanism is pin-connected to the bicycle frame at B, determine the normal force each brake pad exerts on the rim of the wheel. Is this the force that stops the wheel from turning? Explain.

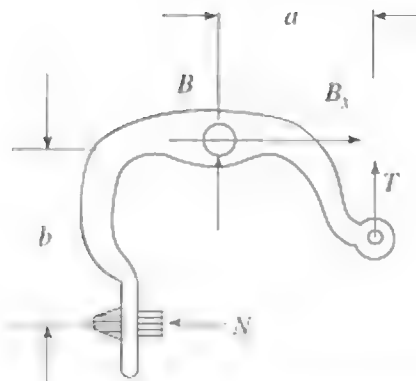
Given:

$T = 50$ lb

$a = 2.5$ in

$b = 3$ in

Solution:

$\Sigma M_B = 0;\quad -N b + T a = 0$

$$N = T\frac{a}{b} \qquad N = 41.7\,\text{lb}$$

This normal force **does not** stop the wheel from turning. A frictional force (see Chapter 8), which acts along the wheel's rim stops the wheel.

Problem 6-101

If a force of magnitude P is applied perpendicular to the handle of the mechanism, determine the magnitude of force **F** for equilibrium. The members are pin-connected at A, B, C, and D.

Given:

$P = 6\ \text{lb}$

$a = 25\ \text{in}$

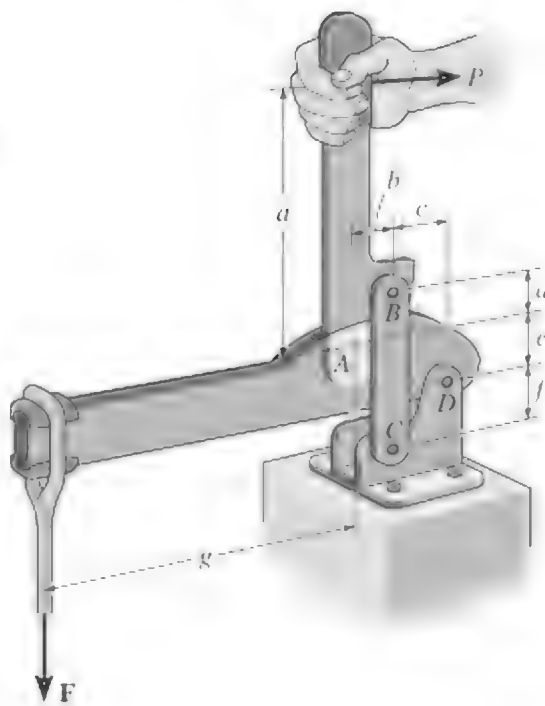
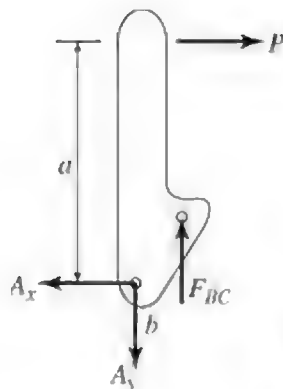
$b = 4\ \text{in}$

$c = 5\ \text{in}$

$d = 4\ \text{in}$

$e = 5\ \text{in}$

$f = 5\ \text{in}$

$g = 30\ \text{in}$

Solution:

$\Sigma M_A = 0;\quad F_{BC}\, b - P a = 0$

$$F_{BC} = \frac{P a}{b}$$

$F_{BC} = 37.5 \text{ lb}$

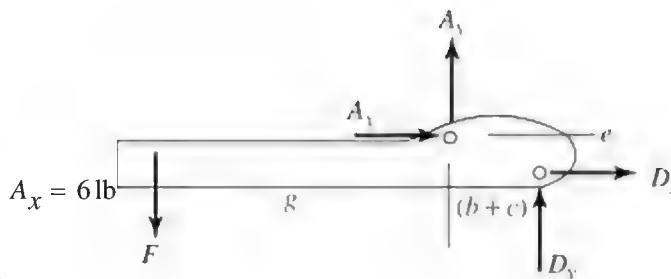
$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + P = 0$

$A_x = P \qquad A_x = 6 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad -A_y + F_{BC} = 0$

$A_y = F_{BC} \qquad A_y = 37.5 \text{ lb}$

$\Sigma M_D = 0; \quad -e A_x - A_y(b + c) + (g + b + c)F = 0$

$$F = \frac{e A_x + A_y(b + c)}{g + b + c} \qquad F = 9.423 \text{ lb}$$

Problem 6-102

The pillar crane is subjected to the load having a mass M. Determine the force developed in the tie rod AB and the horizontal and vertical reactions at the pin support C when the boom is tied in the position shown.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

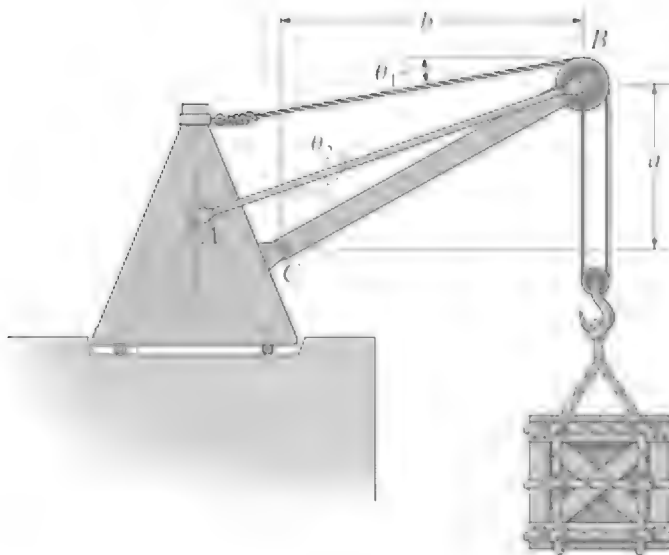
$M = 500 \text{ kg}$

$a = 1.8 \text{ m}$

$b = 2.4 \text{ m}$

$\theta_1 = 10 \text{ deg}$

$\theta_2 = 20 \text{ deg}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

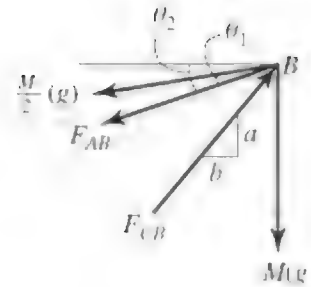
initial guesses: $F_{CB} = 10 \text{ kN} \qquad F_{AB} = 10 \text{ kN}$

Given

$$\frac{-M}{2}g\cos(\theta_1) - F_{AB}\cos(\theta_2) + F_{CB}\frac{b}{\sqrt{a^2+b^2}} = 0$$

$$\frac{-M}{2}g\sin(\theta_1) - F_{AB}\sin(\theta_2) + F_{CB}\frac{a}{\sqrt{a^2+b^2}} - Mg = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{CB} \end{pmatrix} = \text{Find}(F_{AB}, F_{CB}) \qquad \begin{pmatrix} C_x \\ C_y \end{pmatrix} = \frac{F_{CB}}{\sqrt{a^2+b^2}}\begin{pmatrix} b \\ a \end{pmatrix}$$

$$\begin{pmatrix} F_{AB} \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 9.7 \\ 11.53 \\ 8.65 \end{pmatrix} \text{kN}$$

Problem 6-103

The tower truss has a weight W and a center of gravity at G. The rope system is used to hoist it into the vertical position. If rope CB is attached to the top of the shear leg AC and a second rope CD is attached to the truss, determine the required tension in BC to hold the truss in the position shown. The base of the truss and the shear leg bears against the stake at A, which can be considered as a pin. Also, compute the compressive force acting along the shear leg.

Given:

$W = 575$ lb

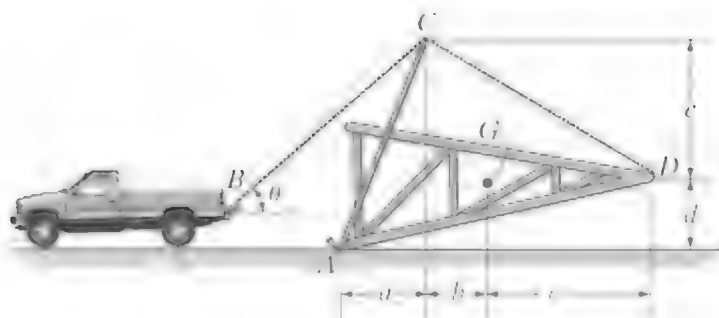
$\theta = 40$ deg

$a = 5$ ft

$b = 3$ ft

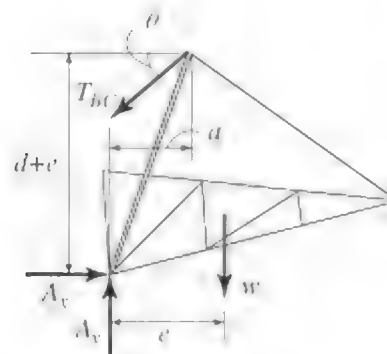
$c = 10$ ft

$d = 4$ ft

$e = 8$ ft

Solution: Entire system: $\Sigma M_A = 0;$

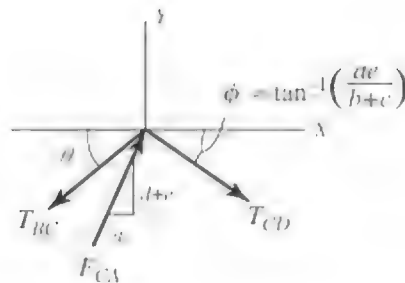
$$T_{BC}\cos(\theta)(d+e) - T_{BC}\sin(\theta)a - W(a+b) = 0$$

$$T_{BC} = \frac{W(a+b)}{\cos(\theta)(d+e) - \sin(\theta)a}$$

$$T_{BC} = 769 \text{ lb}$$

CA is a two-force member. At *C*:

$$\phi = \operatorname{atan}\left(\frac{e}{b+c}\right)$$

initial guesses: $F_{CA} = 500\text{ lb}$ $T_{CD} = 300\text{ lb}$

Given

$$\Sigma F_x = 0; \quad F_{CA}\frac{a}{\sqrt{a^2 + (d+e)^2}} + T_{CD}\cos(\phi) - T_{BC}\cos(\theta) = 0$$

$$\Sigma F_y = 0; \quad F_{CA}\frac{d+e}{\sqrt{a^2 + (d+e)^2}} - T_{CD}\sin(\phi) - T_{BC}\sin(\theta) = 0$$

$$\begin{pmatrix} F_{CA} \\ T_{CD} \end{pmatrix} = \operatorname{Find}(F_{CA}, T_{CD}) \qquad T_{CD} = 358\text{ lb} \qquad F_{CA} = 739\text{ lb}$$

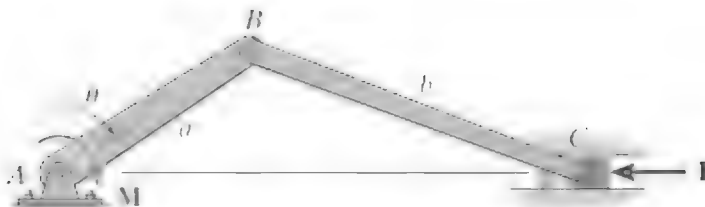
Problem 6-104

The constant moment *M* is applied to the crank shaft. Determine the compressive force *P* that is exerted on the piston for equilibrium as a function of θ. Plot the results of *P* (ordinate) versus θ (abscissa) for $0\text{ deg} \le \theta \le 90\text{ deg}$.

Given:

$a = 0.2\text{ m}$

$b = 0.45\text{ m}$

$M = 50\text{ N}\cdot\text{m}$

Solution:

$$a\cos(\theta) = b\sin(\phi) \qquad \phi = \operatorname{asin}\left(\frac{a}{b}\cos(\theta)\right)$$

$$-M + F_{BC}\cos(\theta - \phi)a = 0$$

$$F_{BC} = \frac{M}{a\cos(\theta - \phi)}$$

$$P = F_{BC}\cos(\phi) = \frac{M\cos(\phi)}{a\cos(\theta - \phi)}$$

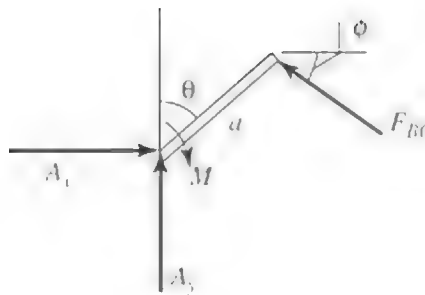
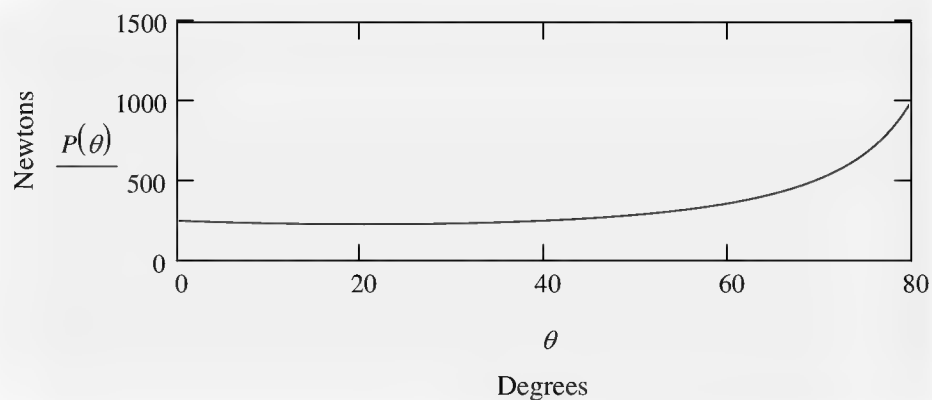
This function goes to infinity at $\theta = 90$ deg, so we will only plot it to $\theta = 80$ deg.

$\theta = 0, 0.1 .. 80$

$$\phi(\theta) = \operatorname{asin}\left(\frac{a}{b}\cos(\theta \text{ deg})\right) \qquad P(\theta) = \frac{M\cos(\phi(\theta))}{a\cos(\theta \text{ deg} - \phi(\theta))}$$

Newtons $\frac{P(\theta)}{}$ (axis: 0, 500, 1000, 1500)

θ (axis: 0, 20, 40, 60, 80)

Degrees

Problem 6-105

Five coins are stacked in the smooth plastic container shown. If each coin has weight W, determine the normal reactions of the bottom coin on the container at points A and B.

Given:

$W = 0.0235$ lb

$a = 3$

$b = 4$

Solution:

All coins :

$\Sigma F_y = 0;\quad N_B = 5W$

$N_B = 0.1175\ \text{lb}$

Bottom coin :

$\Sigma F_y = 0;\quad N_B - W - N\left(\frac{b}{\sqrt{a^2+b^2}}\right) = 0$

$N = \left(N_B - W\right)\left(\frac{\sqrt{a^2+b^2}}{b}\right)$

$N = 0.1175\ \text{lb}$

$\Sigma F_x = 0;\quad N_A = N\left(\frac{a}{\sqrt{a^2+b^2}}\right)$

$N_A = 0.0705\ \text{lb}$

Problem 6-106

Determine the horizontal and vertical components of force at pin *B* and the normal force the pin at *C* exerts on the smooth slot. Also, determine the moment and horizontal and vertical reactions of force at *A*. There is a pulley at *E*.

Given:

$F = 50\ \text{lb}$

$a = 4\ \text{ft}$

$b = 3\ \text{ft}$

Solution:

Guesses

$B_x = 1\ \text{lb}$

$B_y = 1\ \text{lb}$

$N_C = 1\ \text{lb}$

$A_x = 1\ \text{lb} \qquad A_y = 1\ \text{lb} \qquad M_A = 1\ \text{lb}\cdot\text{ft}$

Given

$$B_x + \left(\frac{a}{\sqrt{a^2+b^2}}\right)N_C - F = 0$$

$$B_y - \left(\frac{b}{\sqrt{a^2+b^2}}\right)N_C - F = 0$$

$$\left(F + B_x\right)a - \left(F + B_y\right)b = 0$$

$$F - \left(\frac{a}{\sqrt{a^2+b^2}}\right)N_C - A_x = 0$$

$$\left(\frac{b}{\sqrt{a^2+b^2}}\right)N_C - A_y = 0$$

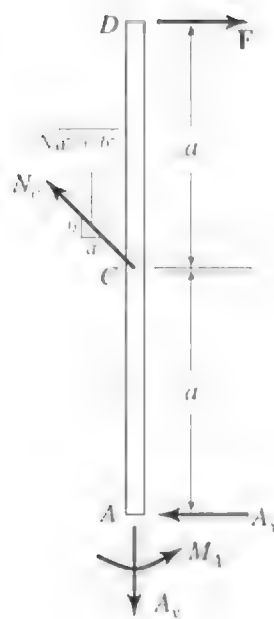
$$-F\ 2\ a + \left(\frac{a}{\sqrt{a^2+b^2}}\right)N_C a + M_A = 0$$

$$\begin{pmatrix} B_x \\ B_y \\ N_C \\ A_x \\ A_y \\ M_A \end{pmatrix} = \text{Find}\left(B_x, B_y, N_C, A_x, A_y, M_A\right)$$

$N_C = 20\,\text{lb}$

$$\begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 34 \\ 62 \end{pmatrix}\text{lb}$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 34 \\ 12 \end{pmatrix}\text{lb}$$

$M_A = 336\,\text{lb}\cdot\text{ft}$

Problem 6-107

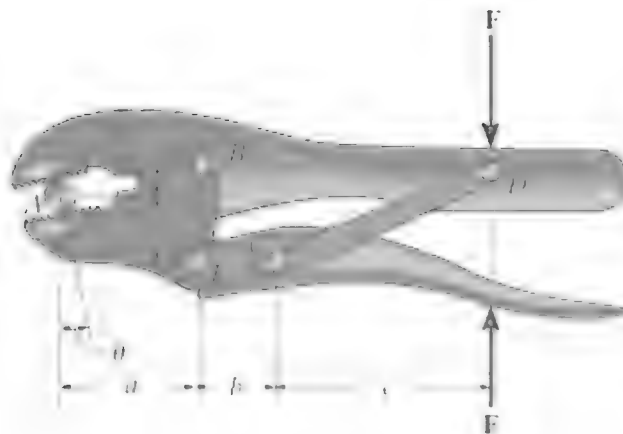
A force **F** is applied to the handles of the vise grip. Determine the compressive force developed on the smooth bolt shank A at the jaws.

Given:

$F = 5\text{ lb}$ $\quad b = 1\text{ in}$

$a = 1.5\text{ in}$ $\quad c = 3\text{ in}$

$d = 0.75\text{ in}$ $\quad e = 1\text{ in}$

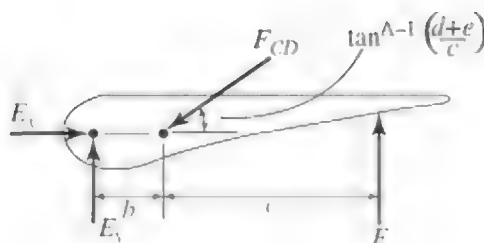
$\theta = 20\text{ deg}$

Solution:

From *FBD* (a)

$$\Sigma M_E = 0; \quad F(b+c) - F_{CD}\left[\frac{d+e}{\sqrt{c^2+(d+e)^2}}\right]b = 0$$

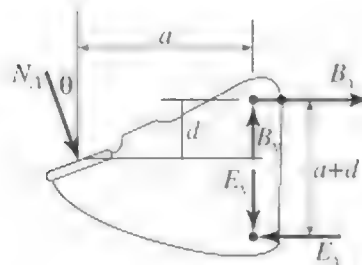
$$F_{CD} = F(b+c)\left[\frac{\sqrt{c^2+(d+e)^2}}{b(d+e)}\right] \qquad F_{CD} = 39.693\text{ lb}$$

$$\Sigma F_x = 0; \quad E_x = F_{CD}\left[\frac{c}{\sqrt{c^2+(d+e)^2}}\right]$$

$$E_x = 34.286\text{ lb}$$

From *FBD* (b)

$$\Sigma M_B = 0; \quad N_A\sin(\theta)d + N_A\cos(\theta)a - E_x(d+e) = 0$$

$$N_A = E_x\left(\frac{d+e}{\sin(\theta)\,d+\cos(\theta)\,a}\right)$$

$$N_A = 36.0\text{ lb}$$

Problem 6-108

If a force of magnitude P is applied to the grip of the clamp, determine the compressive force F that the wood block exerts on the clamp.

Given:

$P = 10 \text{ lb}$

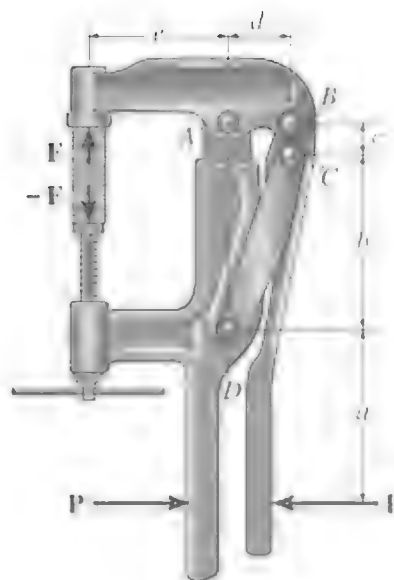
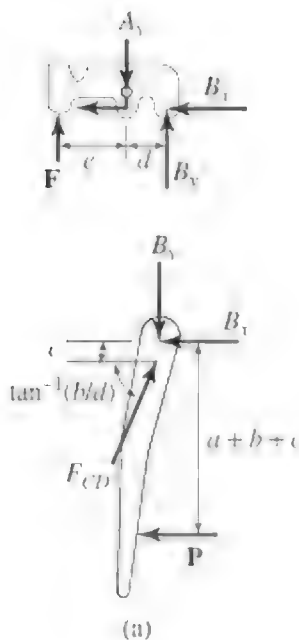
$a = 2 \text{ in}$

$b = 2 \text{ in}$

$c = 0.5 \text{ in}$

$d = 0.75 \text{ in}$

$e = 1.5 \text{ in}$

(a)

Solution:

Define $\phi = \operatorname{atan}\left(\frac{b}{d}\right)$ $\phi = 69.444 \text{ deg}$

From *FBD* (a),

$\Sigma M_B = 0;$ $F_{CD}\cos(\phi)c - P(a + b + c) = 0$

$$F_{CD} = \frac{P(a + b + c)}{\cos(\phi)(c)} \qquad F_{CD} = 256.32 \text{ lb}$$

$+\uparrow \Sigma F_y = 0;$ $F_{CD}\sin(\phi) - B_y = 0$

$$B_y = F_{CD}\sin(\phi) \qquad B_y = 240 \text{ lb}$$

From *FBD* (b),

$\Sigma M_A = 0;$ $B_y d - F e = 0$

$$F = \frac{B_y d}{e} \qquad F = 120 \text{ lb}$$

Problem 6-109

The hoist supports the engine of mass M. Determine the force in member DB and in the hydraulic cylinder H of member FB.

Units Used:

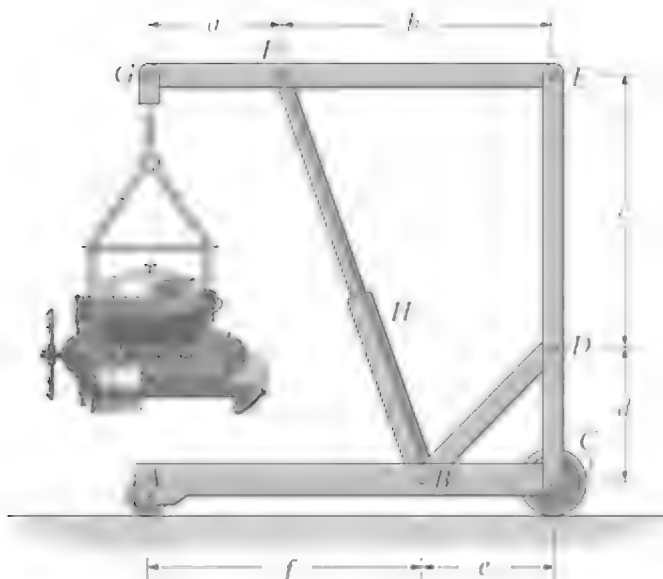
$$kN = 10^3\ N$$

Given:

$M = 125\ kg$ $\quad d = 1\ m$

$a = 1\ m$ $\quad e = 1\ m$

$b = 2\ m$ $\quad f = 2\ m$

$c = 2\ m$ $\quad g = 9.81\ \frac{m}{s^2}$

Solution:

Member GFE:

$$\Sigma M_E = 0; \quad -F_{FB}\left[\frac{c+d}{\sqrt{(c+d)^2+(b-e)^2}}\right]b + M g(a+b) = 0$$

$$F_{FB} = M\ g\left[\frac{a+b}{b(c+d)}\right]\sqrt{(c+d)^2+(b-e)^2}$$

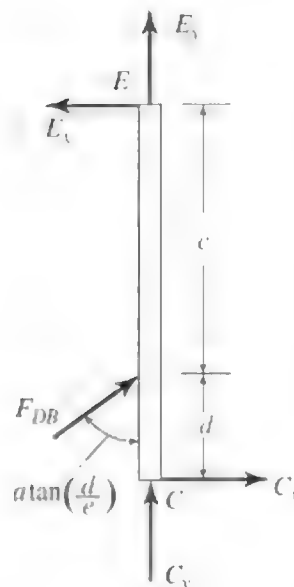
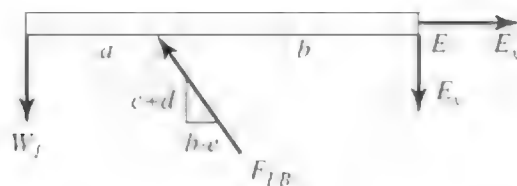
$$F_{FB} = 1.94\ kN$$

$$\Sigma F_x = 0; \quad E_x - F_{FB}\left[\frac{b-e}{\sqrt{(c+d)^2+(b-e)^2}}\right] = 0$$

$$E_x = F_{FB}\left[\frac{b-e}{\sqrt{(c+d)^2+(b-e)^2}}\right]$$

Member EDC:

$$\Sigma M_c = 0; \quad E_x(c+d) - F_{DB}\left(\frac{e}{\sqrt{e^2+d^2}}\right)d = 0$$

$$F_{DB} = E_x\left(\frac{c+d}{e\ d}\right)\sqrt{e^2+d^2} \qquad F_{DB} = 2.601\ kN$$

Problem 6-110

The flat-bed trailer has weight W_1 and center of gravity at G_T. It is pin-connected to the cab at D. The cab has a weight W_2 and center of gravity at G_C. Determine the range of values x for the position of the load L of weight W_3 so that no axle is subjected to a force greater than F_{Max}. The load has a center of gravity at G_L.

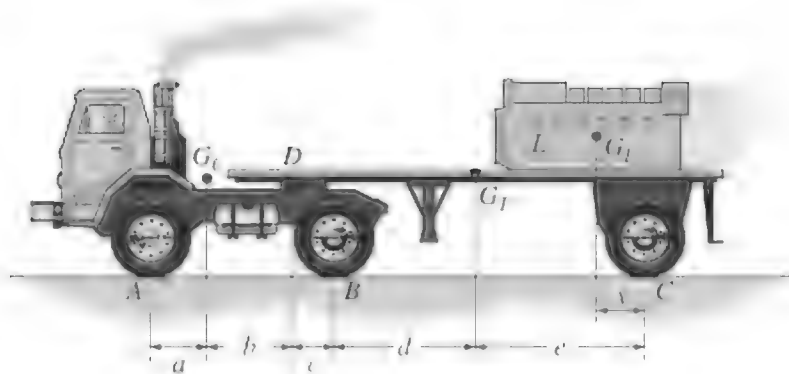
Given:

$W_1 = 7000\ \text{lb}$ $\quad a = 4\ \text{ft}$

$W_2 = 6000\ \text{lb}$ $\quad b = 6\ \text{ft}$

$W_3 = 2000\ \text{lb}$ $\quad c = 3\ \text{ft}$

$F_{max} = 5500\ \text{lb}$ $\quad d = 10\ \text{ft}$

$e = 12\ \text{ft}$

Solution:

Case 1: Assume $A_y = F_{max}$

Guesses $A_y = F_{max}$ $\quad B_y = F_{max}$ $\quad C_y = F_{max}$

$x = 1\ \text{ft}$ $\quad D_y = F_{max}$

Given $A_y + B_y - W_2 - D_y = 0$

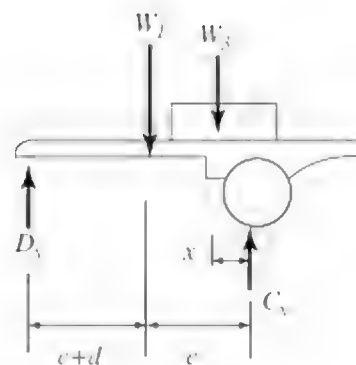
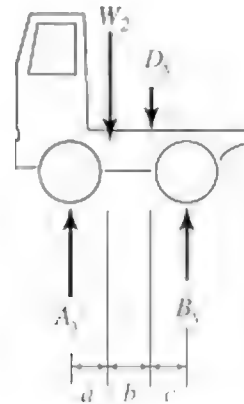
$$-W_2 a - D_y(a+b) + B_y(a+b+c) = 0$$

$$D_y - W_1 - W_3 + C_y = 0$$

$$W_3 x + W_1 e - D_y(c+d+e) = 0$$

$$\begin{pmatrix} B_y \\ C_y \\ D_y \\ x \end{pmatrix} = \text{Find}(B_y, C_y, D_y, x)$$

$$\begin{pmatrix} A_y \\ B_y \\ C_y \end{pmatrix} = \begin{pmatrix} 5.5 \times 10^3 \\ 6.333 \times 10^3 \\ 3.167 \times 10^3 \end{pmatrix} \text{lb} \quad x_1 = x \quad x_1 = 30.917\ \text{ft}$$

Since $B_y > F_{max}$ then this solution is no good.

Case 2: Assume $B_y = F_{max}$

Guesses $A_y = F_{max}$ $\quad B_y = F_{max}$ $\quad C_y = F_{max}$

$x = 1\ \text{ft} \qquad D_y = F_{max}$

Given $\quad A_y + B_y - W_2 - D_y = 0$

$-W_2 a - D_y(a + b) + B_y(a + b + c) = 0$

$D_y - W_1 - W_3 + C_y = 0$

$W_3 x + W_1 e - D_y(c + d + e) = 0$

$$\begin{pmatrix} A_y \\ C_y \\ D_y \\ x \end{pmatrix} = \text{Find}(A_y, C_y, D_y, x) \qquad \begin{pmatrix} A_y \\ B_y \\ C_y \end{pmatrix} = \begin{pmatrix} 5.25 \times 10^3 \\ 5.5 \times 10^3 \\ 4.25 \times 10^3 \end{pmatrix} \text{lb} \quad x_2 = x \qquad x_2 = 17.375\ \text{ft}$$

Since $A_y < F_{max}$ and $C_y < F_{max}$ then this solution is good.

Case 3: Assume $\quad C_y = F_{max}$

Guesses $\quad A_y = F_{max} \quad B_y = F_{max} \qquad C_y = F_{max}$

$x = 1\ \text{ft} \qquad D_y = F_{max}$

Given $\quad A_y + B_y - W_2 - D_y = 0$

$-W_2 a - D_y(a + b) + B_y(a + b + c) = 0$

$D_y - W_1 - W_3 + C_y = 0$

$W_3 x + W_1 e - D_y(c + d + e) = 0$

$$\begin{pmatrix} A_y \\ B_y \\ D_y \\ x \end{pmatrix} = \text{Find}(A_y, B_y, D_y, x) \qquad \begin{pmatrix} A_y \\ B_y \\ C_y \end{pmatrix} = \begin{pmatrix} 4.962 \times 10^3 \\ 4.538 \times 10^3 \\ 5.5 \times 10^3 \end{pmatrix} \text{lb} \quad x_3 = x \qquad x_3 = 1.75\ \text{ft}$$

Since $A_y < F_{max}$ and $B_y < F_{max}$ then this solution is good.

We conclude that $x_3 = 1.75\ \text{ft} < x < x_2 = 17.375\ \text{ft}$

Problem 6-111

Determine the force created in the hydraulic cylinders *EF* and *AD* in order to hold the shovel in equilibrium. The shovel load has a mass *W* and a center of gravity at *G*. All joints are pin connected.

Units Used:

$$\text{Mg} = 10^3 \text{ kg}$$

$$\text{kN} = 10^3 \text{ N}$$

Given:

$a = 0.25$ m $\theta_1 = 30$ deg

$b = 0.25$ m $\theta_2 = 10$ deg

$c = 1.5$ m $\theta_3 = 60$ deg

$d = 2$ m $W = 1.25$ Mg

$e = 0.5$ m

Solution:

Assembly *FHG* :

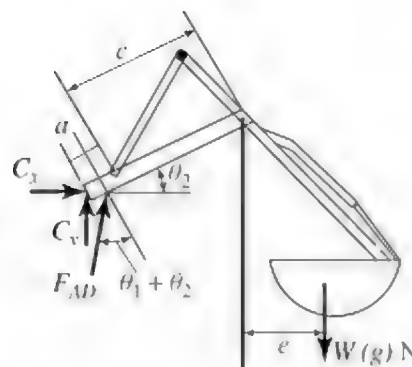
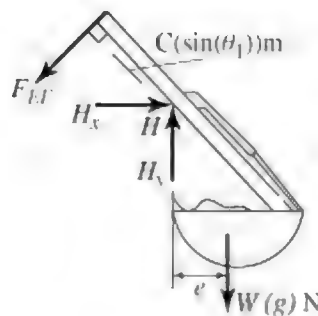
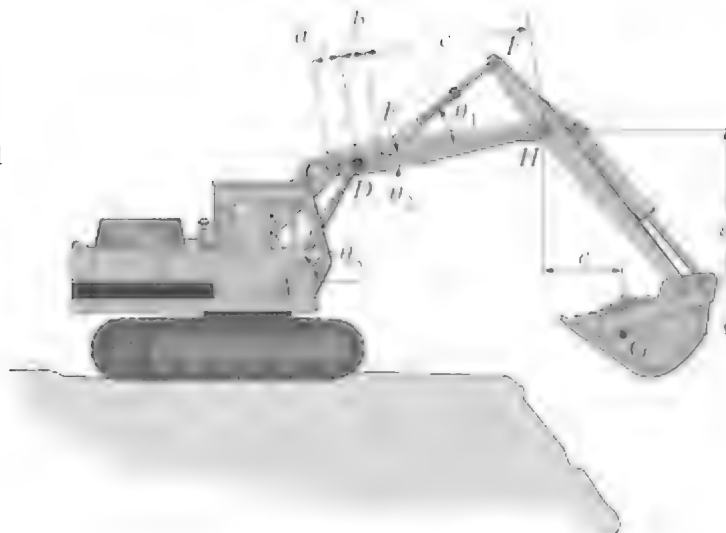
$$\Sigma M_H = 0; \quad -[Wg(e)] + F_{EF}\left(c\sin\left(\theta_1\right)\right) = 0$$

$$F_{EF} = W\ g\left(\frac{e}{c\ \sin\left(\theta_1\right)}\right) \qquad F_{EF} = 8.175\,\text{k(T)}$$

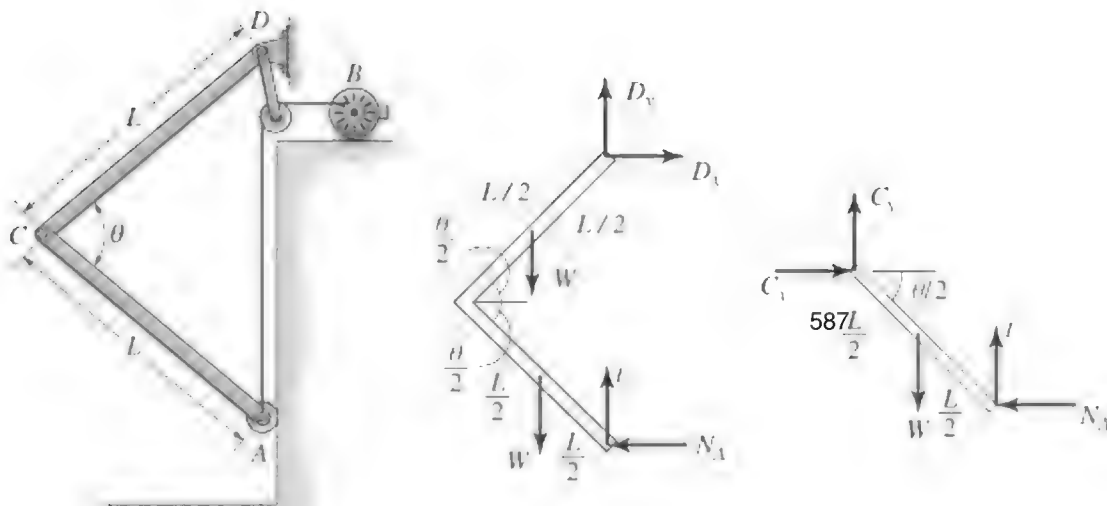
Assembly *CEFHG*:

$$\Sigma M_C = 0; \; F_{AD}\cos\left(\theta_1 + \theta_2\right)b - Wg\left[(a+b+c)\cos\left(\theta_2\right) + e\right] = 0$$

$$F_{AD} = W\ g\left(\frac{\cos\left(\theta_2\right)\ a + \cos\left(\theta_2\right)\ b + \cos\left(\theta_2\right)\ c + e}{\cos\left(\theta_1 + \theta_2\right)\ b}\right)$$

$$F_{AD} = 158\,\text{kN} \ \ \text{(C)}$$

Problem 6-112

The aircraft-hangar door opens and closes slowly by means of a motor which draws in the cable *AB*. If the door is made in two sections (bifold) and each section has a uniform weight *W* and length *L*, determine the force in the cable as a function of the door's position θ. The sections are pin-connected at *C* and *D* and the bottom is attached to a roller that travels along the vertical track.

587

Solution:

$\Sigma M_D = 0;\quad 2W\frac{L}{2}\cos\left(\frac{\theta}{2}\right) - 2L\sin\left(\frac{\theta}{2}\right)N_A = 0 \qquad N_A = \frac{W}{2}\cot\left(\frac{\theta}{2}\right)$

$\Sigma M_C = 0;\quad TL\cos\left(\frac{\theta}{2}\right) - N_A L\sin\left(\frac{\theta}{2}\right) - W\frac{L}{2}\cos\left(\frac{\theta}{2}\right) = 0 \qquad T = W$

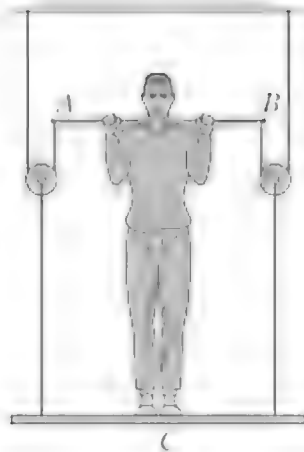
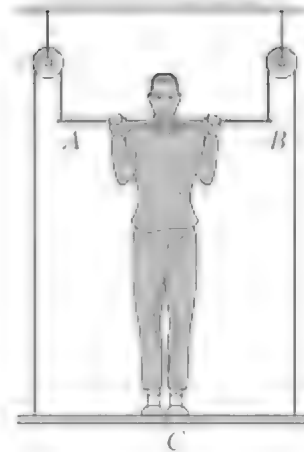
Problem 6-113

A man having weight *W* attempts to lift himself using one of the two methods shown. Determine the total force he must exert on bar *AB* in each case and the normal reaction he exerts on the platform at *C*. Neglect the weight of the platform.

Given:

$W = 175\ \text{lb}$

Solution:

(a) (b)

(a)

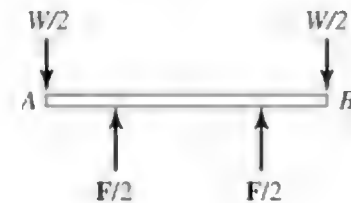
Bar:

$+\uparrow \Sigma F_y = 0;\quad 2\left(\frac{F}{2}\right) - 2\left(\frac{W}{2}\right) = 0$

$F = W \qquad F = 175\text{ lb}$

Man:

$+\uparrow \Sigma F_y = 0;\quad N_C - W - 2\left(\frac{F}{2}\right) = 0$

$N_C = W + F \qquad N_C = 350\text{ lb}$

(b)

Bar:

$+\uparrow \Sigma F_y = 0;\quad 2\left(\frac{W}{4}\right) - 2\,\frac{F}{2} = 0$

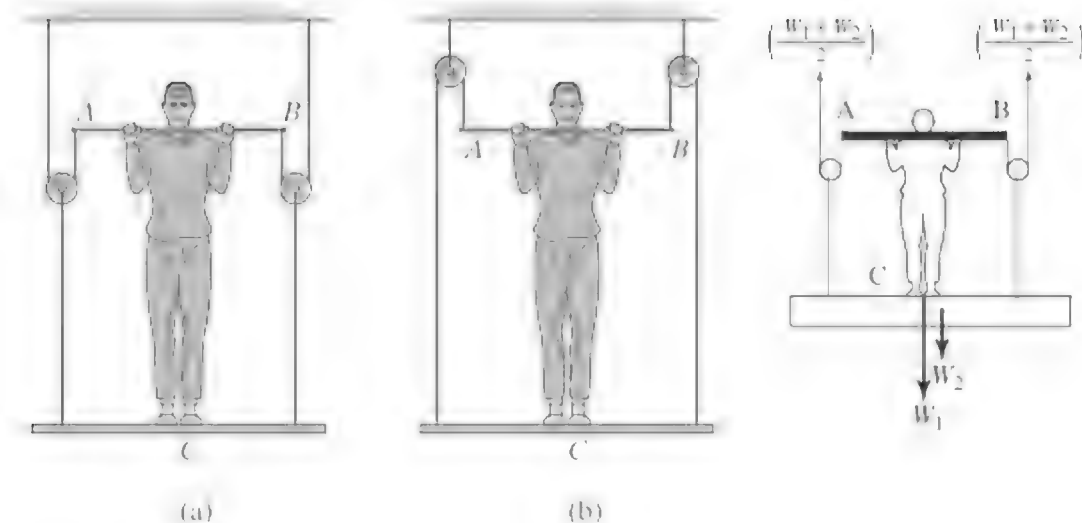
$F = \frac{W}{2} \qquad F = 87.5\text{ lb}$

Man:

$+\uparrow \Sigma F_y = 0;\quad N_C - W + 2\left(\frac{F}{2}\right) = 0$

$N_C = W - F \qquad N_C = 87.5\text{ lb}$

Problem 6-114

A man having weight W_1 attempts to lift himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. The platform has weight W_2.

Given:

$W_1 = 175 \text{ lb}$ $W_2 = 30 \text{ lb}$

Solution:

(a)

Bar:

$+\uparrow \Sigma F_y = 0;$ $2\frac{F}{2} - (W_1 + W_2) = 0$

$F = W_1 + W_2$

$F = 205 \text{ lb}$

Man:

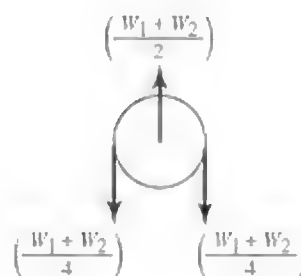
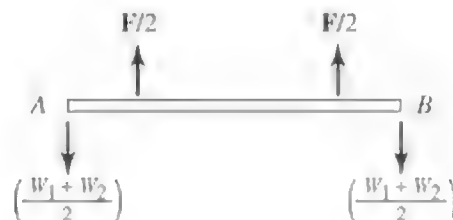
$+\uparrow \Sigma F_y = 0;$ $N_C - W_1 - 2\frac{F}{2} = 0$

$N_C = F + W_1$

$N_C = 380 \text{ lb}$

(b)

Bar:

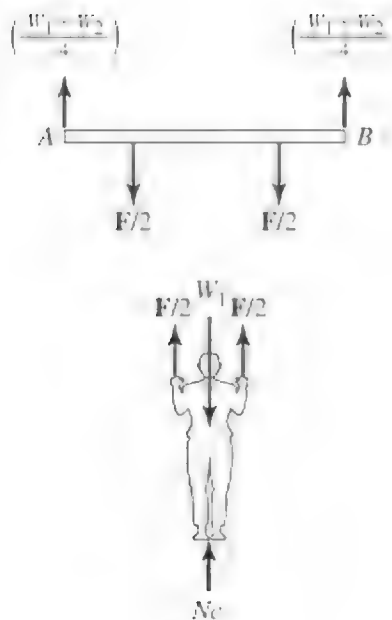
$+\uparrow \Sigma F_y = 0;$ $-2\left(\frac{F}{2}\right) + 2\left(\frac{W_1 + W_2}{4}\right) = 0$

$$F = \frac{W_1 + W_2}{2}$$

$$F = 102\,\text{lb}$$

Man:

$$+\uparrow \Sigma F_y = 0; \quad N_C - W_1 + 2\left(\frac{F}{2}\right) = 0$$

$$N_C = W_1 - F$$

$$N_C = 72.5\,\text{lb}$$

Problem 6-115

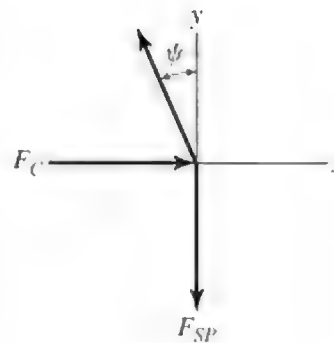
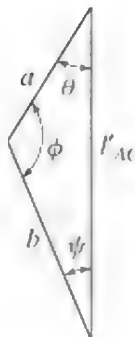
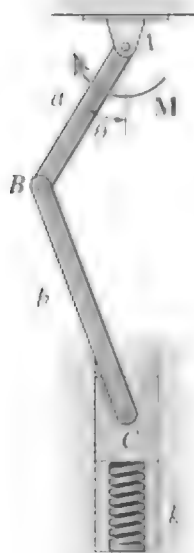
The piston C moves vertically between the two smooth walls. If the spring has stiffness k and is unstretched when $\theta = 0$, determine the couple **M** that must be applied to AB to hold the mechanism in equilibrium.

Given:

$$k = 15\,\frac{\text{lb}}{\text{in}}$$

$$\theta = 30\,\text{deg}$$

$$a = 8\,\text{in}$$

$$b = 12\,\text{in}$$

Solution:

Geometry:

$$b\sin(\psi) = a\sin(\theta)$$

$$\psi = \text{asin}\left(\sin(\theta)\,\frac{a}{b}\right) \qquad \psi = 19.471\,\text{deg}$$

$$\phi = 180\,\text{deg} - \psi - \theta \qquad \phi = 130.529\,\text{deg}$$

$$\frac{r_{AC}}{\sin(\phi)} = \frac{b}{\sin(\theta)} \qquad r_{AC} = b\left(\frac{\sin(\phi)}{\sin(\theta)}\right) \qquad r_{AC} = 18.242 \text{ in}$$

Free Body Diagram: The solution for this problem will be simplified if one realizes that member *CB* is a two force member. Since the spring stretches

$$x = (a+b) - r_{AC} \qquad x = 1.758 \text{ in}$$

the spring force is $F_{sp} = kx$ $\qquad F_{sp} = 26.371 \text{ lb}$

Equations of Equilibrium: Using the method of joints

$$+\uparrow \Sigma F_y = 0; \quad F_{CB}\cos(\psi) - F_{sp} = 0 \qquad F_{CB} = \frac{F_{sp}}{\cos(\psi)} \qquad F_{CB} = 27.971 \text{ lb}$$

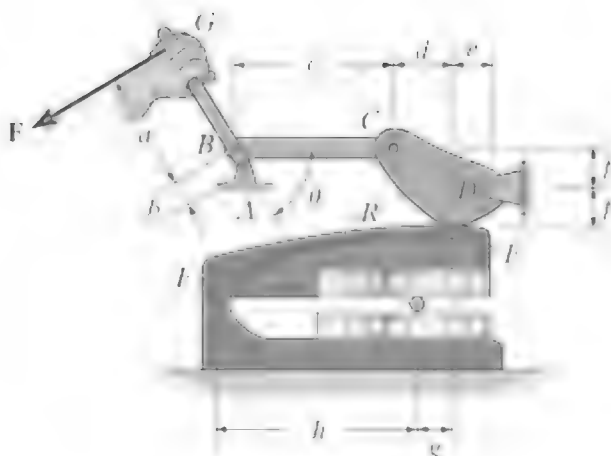
From *FBD* of bar *AB*

$$+\Sigma M_A = 0; \quad F_{CB}\sin(\phi)a - M = 0 \qquad M = F_{CB}\sin(\phi)a \qquad M = 14.2 \text{ lb}\cdot\text{ft}$$

Problem 6-116

The compound shears are used to cut metal parts. Determine the vertical cutting force exerted on the rod *R* if a force **F** is applied at the grip *G*. The lobe *CDE* is in smooth contact with the head of the shear blade at *E*.

Given:

$F = 20$ lb	$e = 0.5$ ft
$a = 1.4$ ft	$f = 0.5$ ft
$b = 0.2$ ft	$g = 0.5$ ft
$c = 2$ ft	$h = 2.5$ ft
$d = 0.75$ ft	$\theta = 60$ deg

Solution:

Member *AG*:

$$\Sigma M_A = 0; \quad F(a+b) - F_{BC}\,b\sin(\theta) = 0$$

$$F_{BC} = F\left(\frac{a+b}{b\sin(\theta)}\right) \qquad F_{BC} = 184.75\text{ lb}$$

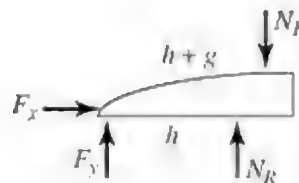
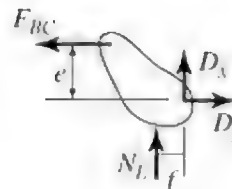
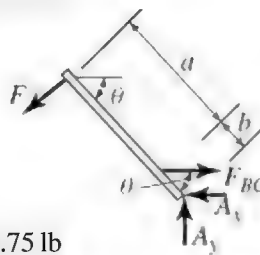
Lobe:

$\Sigma M_D = 0;\quad F_{BC}f - N_E e = 0$

$$N_E = F_{BC}\frac{f}{e} \qquad N_E = 184.75\text{ lb}$$

Head:

$\Sigma M_F = 0;\quad -N_E(h+g) + hN_R = 0$

$$N_R = N_E\left(\frac{h+g}{h}\right) \qquad N_R = 222\text{ lb}$$

Problem 6-117

The handle of the sector press is fixed to gear *G*, which in turn is in mesh with the sector gear *C*. Note that *AB* is pinned at its ends to gear *C* and the underside of the table *EF*, which is allowed to move vertically due to the smooth guides at *E* and *F*. If the gears exert tangential forces between them, determine the compressive force developed on the cylinder *S* when a vertical force **F** is applied to the handle of the press.

Given:

$F = 40\text{ N}$

$a = 0.5\text{ m}$

$b = 0.2\text{ m}$

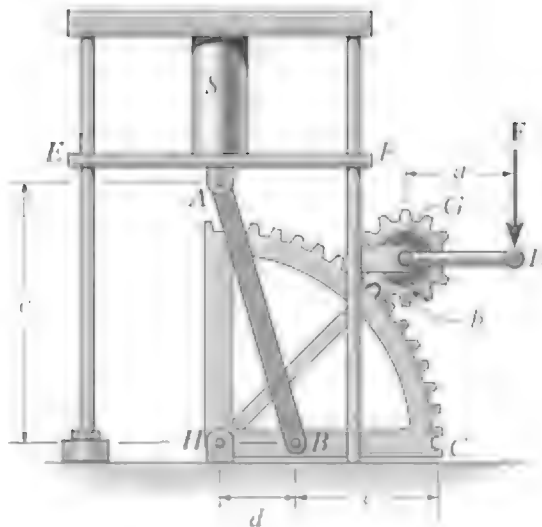
$c = 1.2\text{ m}$

$d = 0.35\text{ m}$

$e = 0.65\text{ m}$

Solution:

Member *GD*:

$\Sigma M_G = 0;\quad -Fa + F_{CG}b = 0$

$$F_{CG} = F\frac{a}{b} \qquad F_{CG} = 100\ \text{N}$$

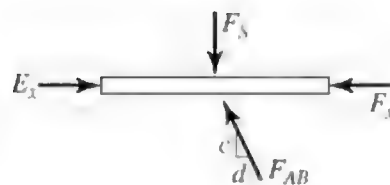
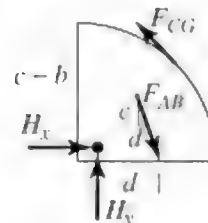
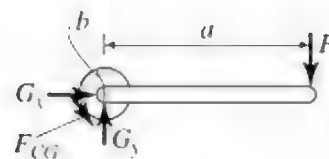
Sector gear :

$$\Sigma M_H = 0; \quad F_{CG}(d+e) - F_{AB}\left(\frac{c}{\sqrt{c^2+d^2}}\right)d = 0$$

$$F_{AB} = F_{CG}(d+e)\left(\frac{\sqrt{c^2+d^2}}{c\,d}\right) \qquad F_{AB} = 297.62\ \text{N}$$

Table:

$$\Sigma F_y = 0; \quad F_{AB}\left(\frac{c}{\sqrt{c^2+d^2}}\right) - F_s = 0$$

$$F_s = F_{AB}\left(\frac{c}{\sqrt{c^2+d^2}}\right) \qquad F_s = 286\ \text{N}$$

Problem 6-118

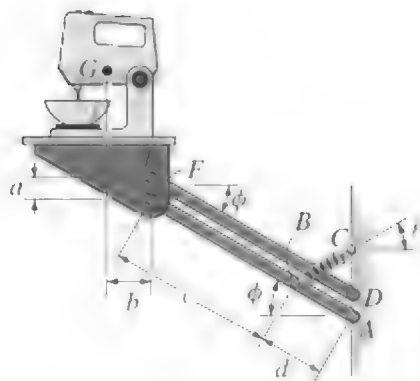
The mechanism is used to hide kitchen appliances under a cabinet by allowing the shelf to rotate downward. If the mixer has weight W, is centered on the shelf, and has a mass center at G, determine the stretch in the spring necessary to hold the shelf in the equilibrium position shown. There is a similar mechanism on each side of the shelf, so that each mechanism supports half of the load W. The springs each have stiffness k.

Given:

$W = 10\ \text{lb}$ $\qquad a = 2\ \text{in}$

$k = 4\ \frac{\text{lb}}{\text{in}}$ $\qquad b = 4\ \text{in}$

$\phi = 30\ \text{deg}$ $\qquad c = 15\ \text{in}$

$\theta = 30\ \text{deg}$ $\qquad d = 6\ \text{in}$

Solution:

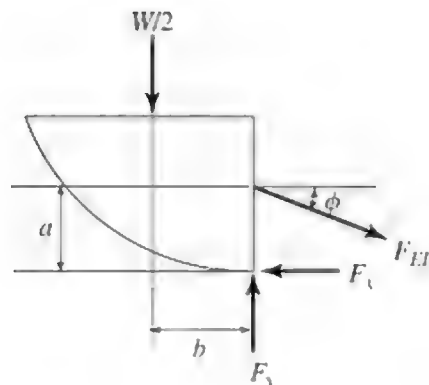
$$\Sigma M_F = 0; \qquad \frac{W}{2}b - aF_{ED}\cos(\phi) = 0$$

$$F_{ED} = \frac{Wb}{2\ a\cos(\phi)} \qquad F_{ED} = 11.547\ \text{lb}$$

$\overset{+}{\rightarrow}\ \Sigma\ F_x = 0; \quad -F_x + F_{ED}\cos(\phi) = 0$

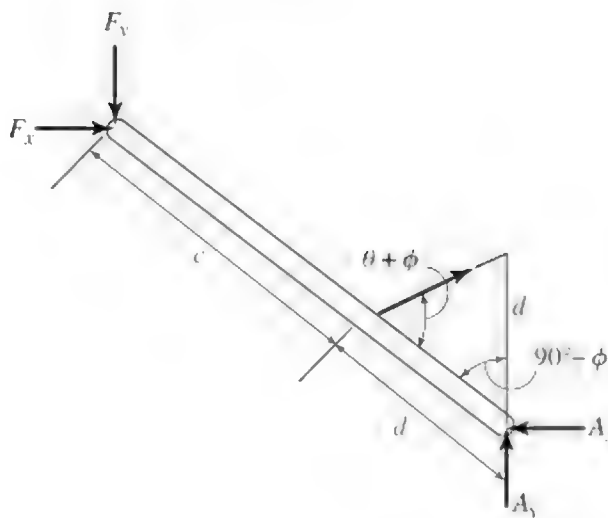
$$F_x = F_{ED}\cos(\phi) \qquad F_x = 10\ \text{lb}$$

$+\uparrow\Sigma\ F_y = 0; \quad \frac{-W}{2} + F_y - F_{ED}\sin(\phi) = 0$

$$F_y = \frac{W}{2} + F_{ED}\cdot\sin(\phi) \qquad F_y = 10.774\ \text{lb}$$

Member *FBA*:

$\Sigma M_A = 0; \quad F_y(c+d)\cos(\phi) - F_x(c+d)\sin(\phi) - F_s\sin(\theta+\phi)d = 0$

$$F_s = \frac{F_y(c+d)\cos(\phi) - F_x(c+d)\sin(\phi)}{d\sin(\theta+\phi)} \qquad F_s = 17.5\ \text{lb}$$

$F_s = ks; \quad F_s = kx \quad x = \frac{F_s}{k} \qquad x = 4.375\ \text{in}$

Problem 6-119

If each of the three links of the mechanism has a weight W, determine the angle θ for equilibrium.The spring, which always remains horizontal, is unstretched when $\theta = 0°$.

Given:

$W = 25 \text{ lb}$

$k = 60 \frac{\text{lb}}{\text{ft}}$

$a = 4 \text{ ft}$

$b = 4 \text{ ft}$

Solution:

Guesses $\theta = 30 \text{ deg}$

$B_x = 10 \text{ lb} \quad B_y = 10 \text{ lb}$

$C_x = 10 \text{ lb} \quad C_y = 10 \text{ lb}$

Given

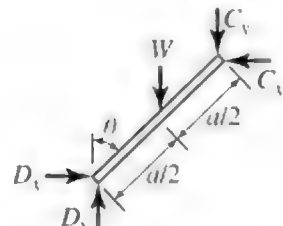
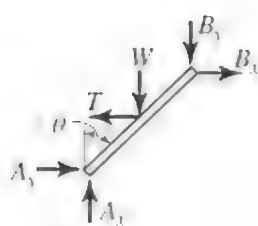
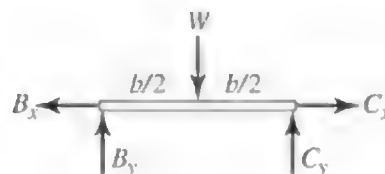
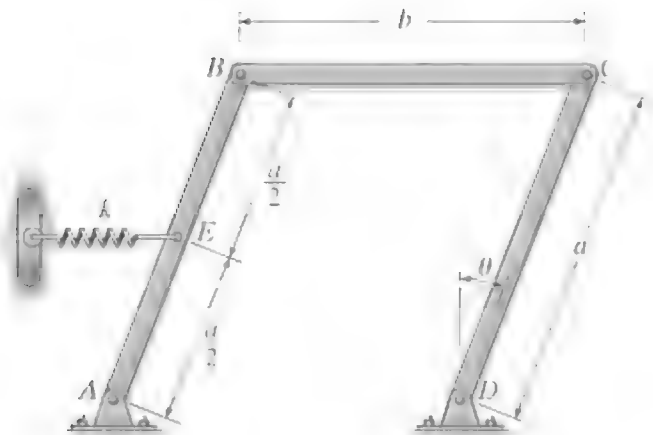
$$-W\left(\frac{a}{2}\right)\sin(\theta) - C_y a \sin(\theta) + C_x a \cos(\theta) = 0$$

$$-W\left(\frac{b}{2}\right) + C_y b = 0$$

$$B_y + C_y - W = 0$$

$$-B_x + C_x = 0$$

$$-B_x a \cos(\theta) - B_y a \sin(\theta) - W\left(\frac{a}{2}\right)\sin(\theta) + k\left(\frac{a}{2}\right)\sin(\theta)\left(\frac{a}{2}\right)\cos(\theta) = 0$$

$$\begin{pmatrix} B_x \\ B_y \\ C_x \\ C_y \\ \theta \end{pmatrix} = \text{Find}(B_x, B_y, C_x, C_y, \theta) \qquad \begin{pmatrix} B_x \\ B_y \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 16.583 \\ 12.5 \\ 16.583 \\ 12.5 \end{pmatrix} \text{lb} \qquad \theta = 33.6 \text{ deg}$$

Problem 6-120

Determine the required force **P** that must be applied at the blade of the pruning shears so that the blade exerts a normal force F on the twig at E.

Given:

$F = 20\ \text{lb}$

$a = 0.5\ \text{in}$

$b = 4\ \text{in}$

$c = 0.75\ \text{in}$

$d = 0.75\ \text{in}$

$e = 1\ \text{in}$

Solution:

initial guesses:

$A_x = 1\ \text{lb} \quad A_y = 1\ \text{lb} \quad D_x = 10\ \text{lb}$

$D_y = 10\ \text{lb} \quad P = 20\ \text{lb} \quad F_{CB} = 20\ \text{lb}$

Given

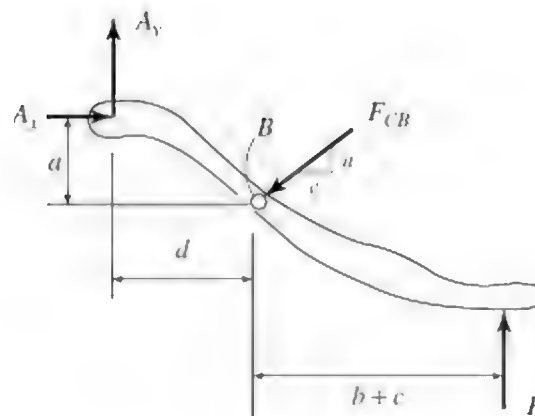
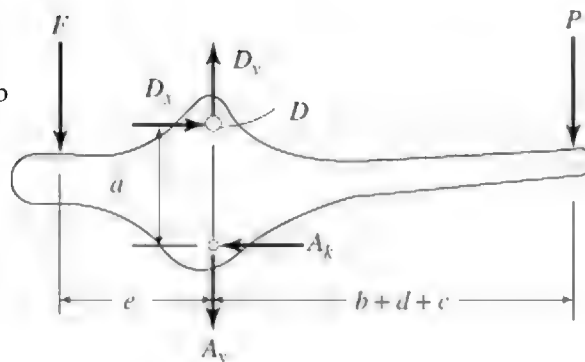
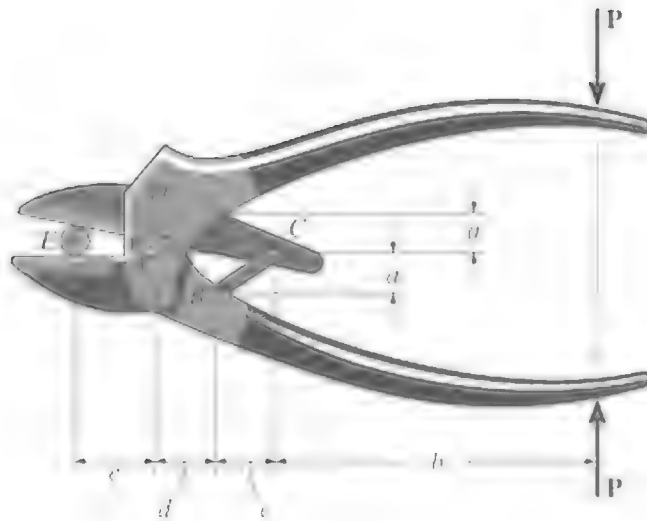
$$-P(b + c + d) - A_x a + F e = 0$$

$$D_y - P - A_y - F = 0$$

$$D_x - A_x = 0$$

$$-A_y(d) - A_x(a) + (b + c)P = 0$$

$$A_x - F_{CB}\left(\frac{c}{\sqrt{c^2 + a^2}}\right) = 0$$

$$A_y + P - F_{CB}\left(\frac{a}{\sqrt{a^2 + c^2}}\right) = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ D_x \\ D_y \\ P \\ F_{CB} \end{pmatrix} = \text{Find}(A_x, A_y, D_x, D_y, P, F_{CB}) \qquad \begin{pmatrix} A_x \\ A_y \\ D_x \\ D_y \\ F_{CB} \end{pmatrix} = \begin{pmatrix} 13.333 \\ 6.465 \\ 13.333 \\ 28.889 \\ 16.025 \end{pmatrix} \text{lb} \qquad P = 2.424\ \text{lb}$$

Problem 6-121

The three power lines exert the forces shown on the truss joints, which in turn are pin-connected to the poles *AH* and *EG*. Determine the force in the guy cable *AI* and the pin reaction at the support *H*.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

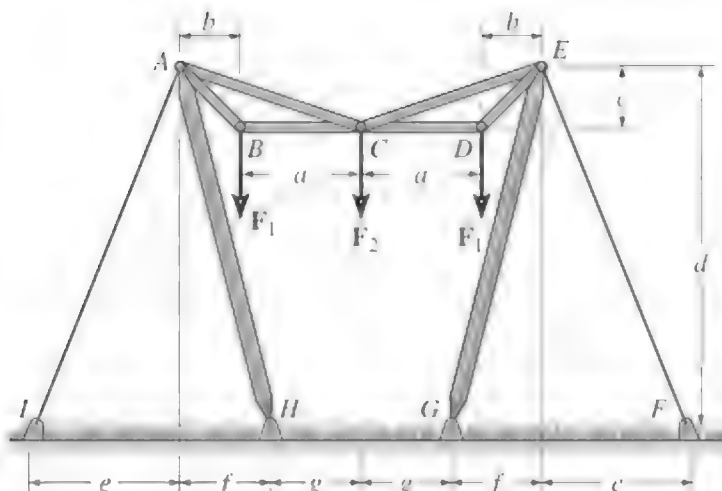
Given:

$F_1 = 800\ \text{lb}$ $\quad d = 125\ \text{ft}$

$F_2 = 800\ \text{lb}$ $\quad e = 50\ \text{ft}$

$a = 40\ \text{ft}$ $\quad f = 30\ \text{ft}$

$b = 20\ \text{ft}$ $\quad g = 30\ \text{ft}$

$c = 20\ \text{ft}$

Solution:

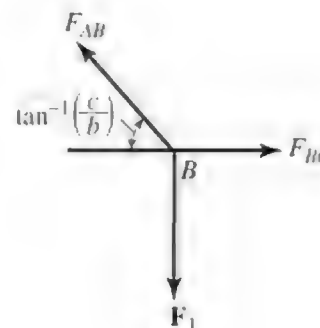
AH is a two-force member.

$\theta = \text{atan}\left(\dfrac{c}{b}\right)$ $\qquad \phi = \text{atan}\left(\dfrac{c}{a+b}\right)$

$\beta = \text{atan}\left(\dfrac{d}{f}\right)$ $\qquad \gamma = \text{atan}\left(\dfrac{e}{d}\right)$ $\qquad \alpha = 90\ \text{deg} - \beta + \gamma$

Guesses

$F_{AB} = 1\ \text{lb}$ $\qquad F_{BC} = 1\ \text{lb}$ $\qquad F_H = 1\ \text{lb}$

$F_{CA} = 1\ \text{lb}$ $\qquad F_{AI} = 1\ \text{lb}$

Given

$$F_{BC} - F_{AB}\cos(\theta) = 0$$

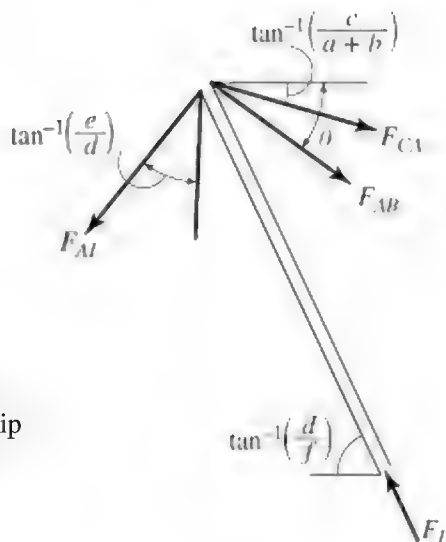
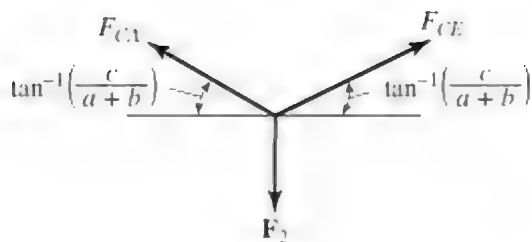
$$F_{AB}\sin(\theta) - F_1 = 0$$

$$2\ F_{CA}\sin(\phi) - F_2 = 0$$

$$-F_{AI}\sin(\alpha) + F_{AB}\sin(\beta - \theta) + F_{CA}\sin(\beta - \phi) = 0$$

$$-F_{AI}\cos(\alpha) - F_{AB}\cos(\beta - \theta) - F_{CA}\cos(\beta - \phi) + F_H = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{CA} \\ F_{BC} \\ F_{AI} \\ F_H \end{pmatrix} = \text{Find}(F_{AB}, F_{CA}, F_{BC}, F_{AI}, F_H)$$

$$\begin{pmatrix} F_{AB} \\ F_{CA} \\ F_{BC} \end{pmatrix} = \begin{pmatrix} 1.131 \\ 1.265 \\ 0.8 \end{pmatrix} \text{kip} \qquad \begin{pmatrix} F_{AI} \\ F_H \end{pmatrix} = \begin{pmatrix} 2.881 \\ 3.985 \end{pmatrix} \text{kip}$$

Problem 6-122

The hydraulic crane is used to lift the load of weight W. Determine the force in the hydraulic cylinder AB and the force in links AC and AD when the load is held in the position shown.

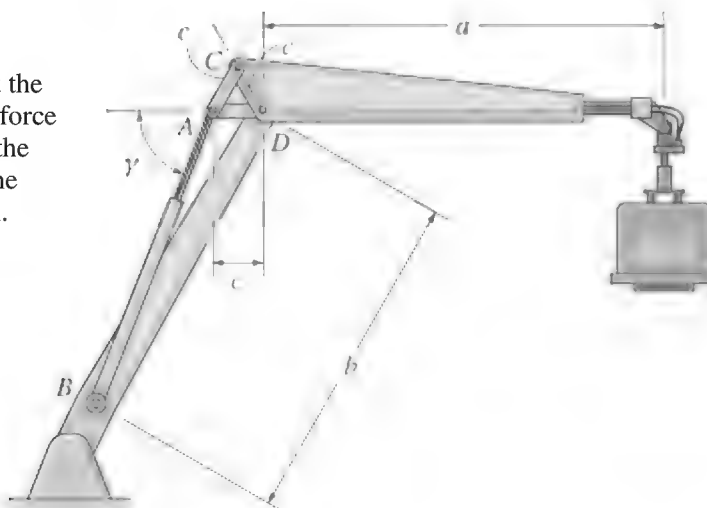
Units Used:

$$\text{kip} = 10^3\ \text{lb}$$

Given:

$$W = 1400\ \text{lb}$$

$$a = 8\ \text{ft} \qquad c = 1\ \text{ft}$$

$$b = 7\ \text{ft} \qquad \gamma = 70\ \text{deg}$$

Solution:

$\Sigma M_D = 0;$

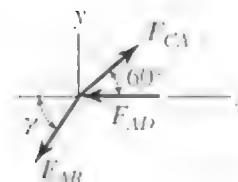
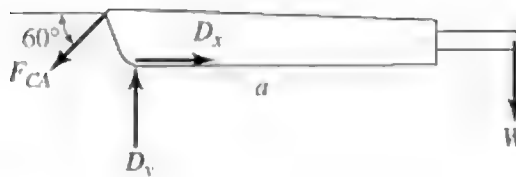
$$F_{CA}\sin(60\text{ deg})c - Wa = 0$$

$$F_{CA} = \frac{Wa}{\sin(60\text{ deg})\,c} \qquad F_{CA} = 12.9\text{ kip}$$

$+\uparrow \Sigma F_y = 0; \quad F_{CA}\sin(60\text{ deg}) - F_{AB}\sin(\gamma) = 0$

$$F_{AB} = F_{CA}\frac{\sin(60\text{ deg})}{\sin(\gamma)} \qquad F_{AB} = 11.9\text{ kip}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{AB}\cos(\gamma) + F_{CA}\cos(60\text{ deg}) - F_{AD} = 0$

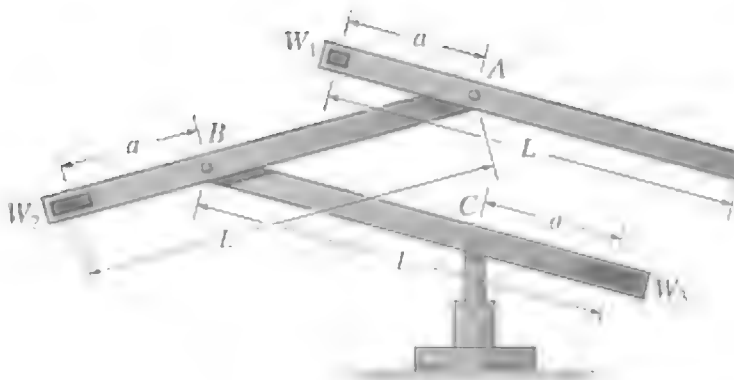
$$F_{AD} = -F_{AB}\cos(\gamma) + F_{CA}\cos(60\text{ deg}) \qquad F_{AD} = 2.39\text{ kip}$$

Problem 6-123

The kinetic sculpture requires that each of the three pinned beams be in perfect balance at all times during its slow motion. If each member has a uniform weight density γ and length L, determine the necessary counterweights W_1, W_2 and W_3 which must be added to the ends of each member to keep the system in balance for any position. Neglect the size of the counterweights.

Given:

$$\gamma = 2\ \frac{\text{lb}}{\text{ft}}$$

$$L = 3\text{ ft}$$

$$a = 1\text{ ft}$$

Solution:

$\Sigma M_A = 0; \quad W_1 a\cos(\theta) - \gamma L\cos(\theta)\left(\frac{L}{2} - a\right) = 0$

$$W_1 = \frac{\gamma L\left(\frac{L}{2} - a\right)}{a} \qquad W_1 = 3\,\text{lb}$$

$+\uparrow\Sigma F_y = 0;\qquad R_A - W_1 - \gamma L = 0$

$$R_A = W_1 + \gamma L \qquad R_A = 9\,\text{lb}$$

$\Sigma M_B = 0;$

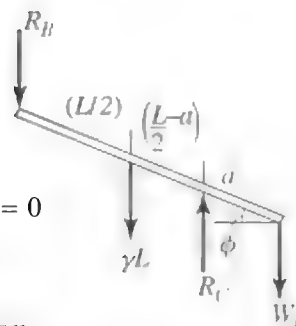
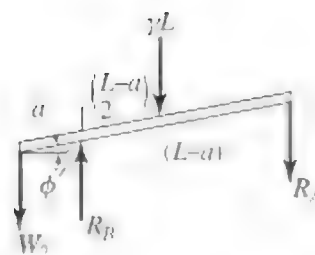
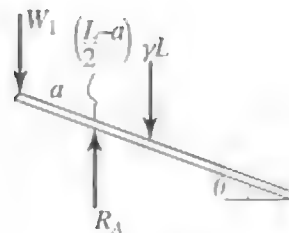
$$W_2 a\cos(\phi) - \gamma L\cos(\phi)\left(\frac{L}{2} - a\right) - R_A(L-a)\cos(\phi) = 0$$

$$W_2 = \frac{\gamma L\left(\frac{L}{2} - a\right) + R_A(L-a)}{a} \qquad W_2 = 21\,\text{lb}$$

$+\uparrow\Sigma F_y = 0;\quad R_B - W_2 - R_A - \gamma L = 0$

$$R_B = W_2 + R_A + \gamma L \qquad R_B = 36\,\text{lb}$$

$\Sigma M_C = 0;\quad R_B(L-a)\cos(\phi) + \gamma L\left(\frac{L}{2} - a\right)\cos(\phi) - W_3 a\cos(\phi) = 0$

$$W_3 = \frac{R_B(L-a) + \gamma L\left(\frac{L}{2} - a\right)}{a} \qquad W_3 = 75\,\text{lb}$$

Problem 6-124

The three-member frame is connected at its ends using ball-and-socket joints. Determine the *x*, *y*, *z* components of reaction at *B* and the tension in member *ED*. The force acting at *D* is **F**.

Given:

$$\mathbf{F} = \begin{pmatrix} 135 \\ 200 \\ -180 \end{pmatrix}\text{lb}$$

$a = 6\text{ ft} \qquad e = 3\text{ ft}$

$b = 4\text{ ft} \qquad f = 1\text{ ft}$

$d = 6 \text{ ft} \qquad g = 2 \text{ ft}$

$c = g + f$

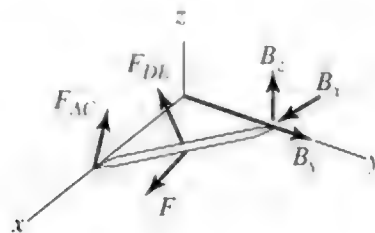
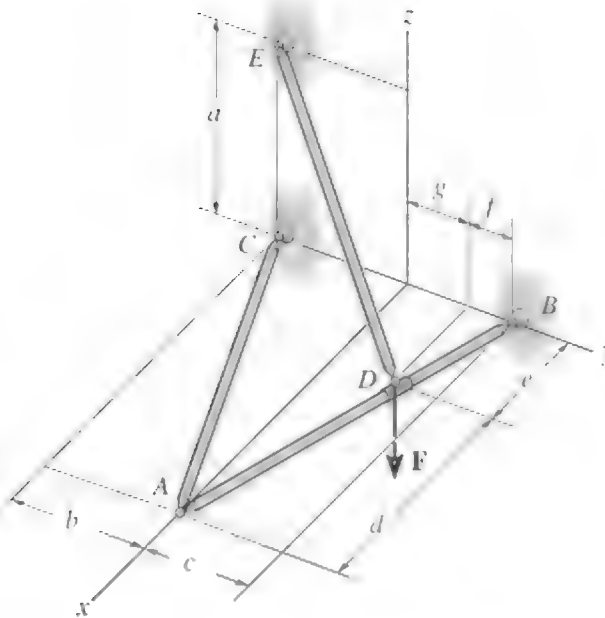
Solution:

AC and DE are two-force members.

Define some vectors

$$\mathbf{r_{DE}} = \begin{pmatrix} -e \\ -b-g \\ a \end{pmatrix} \qquad \mathbf{u_{DE}} = \frac{\mathbf{r_{DE}}}{|\mathbf{r_{DE}}|}$$

$$\mathbf{r_{AC}} = \begin{pmatrix} -d-e \\ -b \\ 0 \end{pmatrix} \qquad \mathbf{u_{AC}} = \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|}$$

$$\mathbf{r_{BD}} = \begin{pmatrix} e \\ -f \\ 0 \end{pmatrix} \qquad \mathbf{r_{BA}} = \begin{pmatrix} e+d \\ -c \\ 0 \end{pmatrix}$$

Guesses

$B_x = 1 \text{ lb} \quad B_y = 1 \text{ lb} \quad B_z = 1 \text{ lb} \quad F_{DE} = 1 \text{ lb} \quad F_{AC} = 1 \text{ lb}$

Given

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + F_{DE}\,\mathbf{u_{DE}} + F_{AC}\mathbf{u_{AC}} + \mathbf{F} = \mathbf{0} \qquad \mathbf{r_{BD}} \times \left(F_{DE}\mathbf{u_{DE}} + \mathbf{F}\right) + \mathbf{r_{BA}} \times \left(F_{AC}\mathbf{u_{AC}}\right) = \mathbf{0}$$

$$\begin{pmatrix} B_x \\ B_y \\ B_z \\ F_{DE} \\ F_{AC} \end{pmatrix} = \text{Find}\left(B_x, B_y, B_z, F_{DE}, F_{AC}\right) \qquad \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} -30 \\ -13.333 \\ 3.039 \times 10^{-12} \end{pmatrix} \begin{pmatrix} F_{DE} \\ F_{AC} \end{pmatrix} = \begin{pmatrix} 270 \\ 16.415 \end{pmatrix} \text{lb}$$

Problem 6-125

The four-member "A" frame is supported at A and E by smooth collars and at G by a pin. All the other joints are ball-and-sockets. If the pin at G will fail when the resultant force there is F_{max}, determine the largest vertical force P that can be supported by the frame. Also, what are the x, y, z force components which member BD exerts on members EDC and ABC? The collars at A and E and the pon at G only exert force components on the frame.

Given:

$F_{max} = 800 \text{ N}$

$a = 300 \text{ mm}$

$b = 600 \text{ mm}$

$c = 600 \text{ mm}$

Solution:

$\Sigma M_x = 0;$

$$-P\,2\,c + \frac{b}{\sqrt{b^2+c^2}} F_{max} c = 0$$

$$P = \frac{F_{max} b}{2\sqrt{b^2+c^2}} \qquad P = 282.843 \text{ N}$$

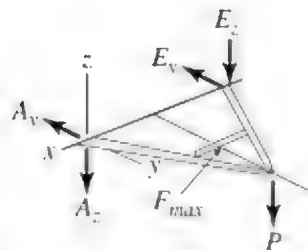
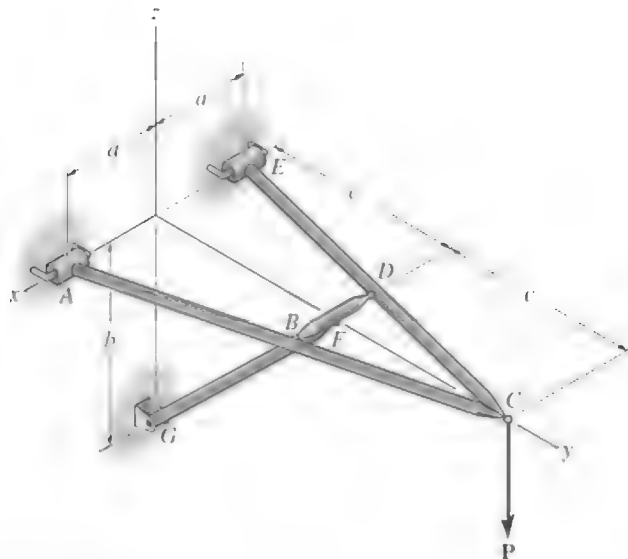
$$B_z + D_z - F_{max}\frac{c}{\sqrt{b^2+c^2}} = 0 \qquad D_z = B_z$$

$$B_z = \frac{F_{max} c}{2\sqrt{b^2+c^2}} \qquad D_z = B_z \qquad B_z = 283 \text{ N}$$

$$D_z = 283 \text{ N}$$

$$B_y + D_y - F_{max}\frac{b}{\sqrt{b^2+c^2}} = 0 \qquad D_y = B_y$$

$$B_y = \frac{F_{max} b}{2\sqrt{b^2+c^2}} \qquad D_y = B_y \qquad B_y = 283 \text{ N} \qquad D_y = 283 \text{ N}$$

$$B_x = D_x = 0$$

Problem 6-126

The structure is subjected to the loading shown. Member AD is supported by a cable AB and a roller at C and fits through a smooth circular hole at D. Member ED is supported by a roller at

D and a pole that fits in a smooth snug circular hole at E. Determine the x, y, z components of reaction at E and the tension in cable AB.

Units Used:

$$kN = 10^3\ N$$

Given:

$$\mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ -2.5 \end{pmatrix} kN$$

$a = 0.5$ m $\quad d = 0.3$ m

$b = 0.4$ m $\quad e = 0.8$ m

$c = 0.3$ m

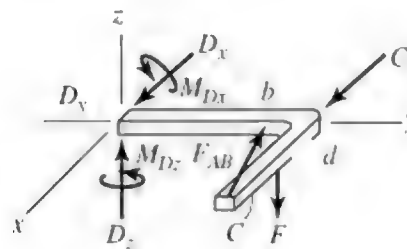
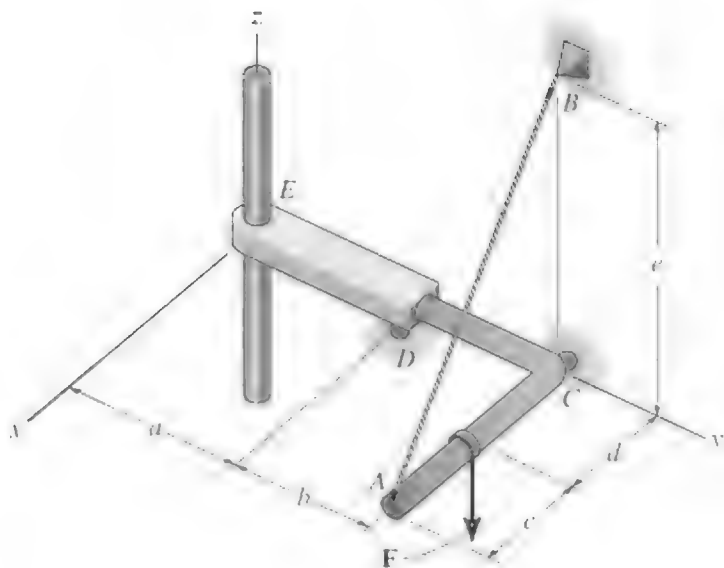
Solution: $\quad AB = \begin{pmatrix} -c-d \\ 0 \\ e \end{pmatrix}$

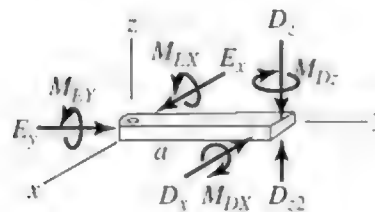
Guesses

$F_{AB} = 1$ kN $\quad D_x = 1$ kN $\quad D_z = 1$ kN

$D_{z2} = 1$ kN $\quad E_x = 1$ kN $\quad E_y = 1$ kN

$M_{Dx} = 1$ kN·m $\quad M_{Dz} = 1$ kN·m $\quad C_x = 1$ kN

$M_{Ex} = 1$ kN·m $\quad M_{Ey} = 1\cdot$kN m

Given

$$\mathbf{F} + F_{AB}\frac{AB}{|AB|} + \begin{pmatrix} C_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} D_x \\ 0 \\ D_z \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} M_{Dx} \\ 0 \\ M_{Dz} \end{pmatrix} + \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} C_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} d \\ b \\ 0 \end{pmatrix} \times \mathbf{F} + \begin{pmatrix} c+d \\ b \\ 0 \end{pmatrix} \times \left(F_{AB}\frac{AB}{|AB|}\right) = \mathbf{0}$$

$$\begin{pmatrix} -D_x \\ 0 \\ D_{z2} - D_z \end{pmatrix} + \begin{pmatrix} E_x \\ E_y \\ 0 \end{pmatrix} = \mathbf{0} \qquad \begin{pmatrix} -M_{Dx} \\ 0 \\ -M_{Dz} \end{pmatrix} + \begin{pmatrix} M_{Ex} \\ M_{Ey} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} D_x \\ 0 \\ D_z - D_{z2} \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} C_x \\ D_x \\ D_z \\ D_{z2} \\ E_x \\ E_y \\ F_{AB} \\ M_{Dx} \\ M_{Dz} \\ M_{Ex} \\ M_{Ey} \end{pmatrix} = \text{Find}\left(C_x, D_x, D_z, D_{z2}, E_x, E_y, F_{AB}, M_{Dx}, M_{Dz}, M_{Ex}, M_{Ey}\right)$$

$$\begin{pmatrix} C_x \\ D_x \\ D_z \\ D_{z2} \end{pmatrix} = \begin{pmatrix} 0.937 \\ 0 \\ 1.25 \\ 1.25 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} M_{Dx} \\ M_{Dz} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} \text{kN} \cdot \text{m}$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} M_{Ex} \\ M_{Ey} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} \text{kN} \cdot \text{m} \qquad F_{AB} = 1.562 \, \text{kN}$$

Problem 6-127

The structure is subjected to the loadings shown.Member *AB* is supported by a ball-and-socket at *A* and smooth collar at *B*. Member *CD* is supported by a pin at *C*. Determine the *x, y, z* components of reaction at *A* and *C*.

Given:

$a = 2\text{ m}$ $M = 800\text{ N·m}$

$b = 1.5\text{ m}$ $F = 250\text{ N}$

$c = 3\text{ m}$ $\theta_1 = 60\text{ deg}$

$d = 4\text{ m}$ $\theta_2 = 45\text{ deg}$

$\theta_3 = 60\text{ deg}$

Solution:

Guesses

$B_x = 1\text{ N}$ $B_y = 1\text{ N}$

$A_x = 1\text{ N}$ $A_y = 1\text{ N}$ $A_z = 1\text{ N}$

$C_x = 1\text{ N}$ $C_y = 1\text{ N}$ $C_z = 1\text{ N}$

$M_{Bx} = 1\text{ N·m}$ $M_{By} = 1\text{ N·m}$

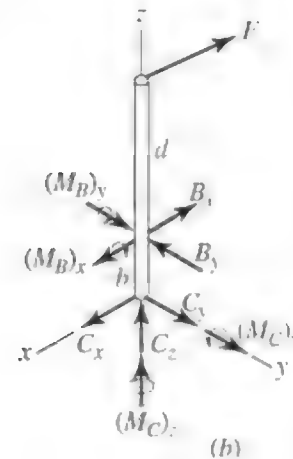
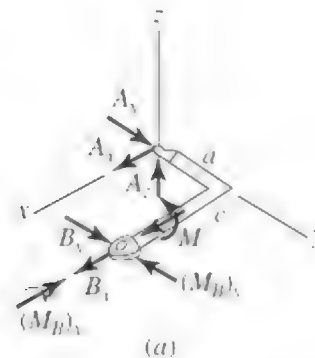
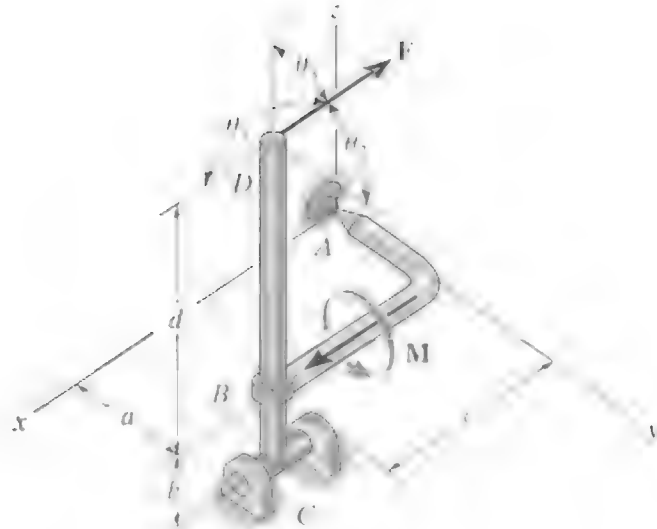
$M_{Cy} = 1\text{ N·m}$ $M_{Cz} = 1\text{ N·m}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} c \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} M \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -M_{Bx} \\ -M_{By} \\ 0 \end{pmatrix} = \mathbf{0}$$

$$F\begin{pmatrix} \cos(\theta_1) \\ \cos(\theta_2) \\ \cos(\theta_3) \end{pmatrix} + \begin{pmatrix} -B_x \\ -B_y \\ 0 \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} 0 \\ 0 \\ b+d \end{pmatrix} \times \left[F\begin{pmatrix} \cos(\theta_1) \\ \cos(\theta_2) \\ \cos(\theta_3) \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} -B_x \\ -B_y \\ 0 \end{pmatrix} + \begin{pmatrix} M_{Bx} \\ M_{By} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ M_{Cy} \\ M_{Cz} \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ C_x \\ C_y \\ C_z \\ B_x \\ B_y \\ M_{Bx} \\ M_{By} \\ M_{Cy} \\ M_{Cz} \end{pmatrix} = \text{Find}(A_x, A_y, A_z, C_x, C_y, C_z, B_x, B_y, M_{Bx}, M_{By}, M_{Cy}, M_{Cz})$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} -172.3 \\ -114.8 \\ 0 \\ 47.3 \\ -61.9 \\ -125 \end{pmatrix} \text{N} \qquad \begin{pmatrix} M_{Cy} \\ M_{Cz} \end{pmatrix} = \begin{pmatrix} -429 \\ 0 \end{pmatrix} \text{N}\cdot\text{m}$$

Problem 6-128

Determine the resultant forces at pins B and C on member ABC of the four-member frame.

Given:

$$w = 150 \ \frac{\text{lb}}{\text{ft}}$$

$$a = 5 \ \text{ft}$$

$$b = 2 \ \text{ft}$$

$c = 2\text{ ft}$

$e = 4\text{ ft}$

$d = a + b - c$

Solution:

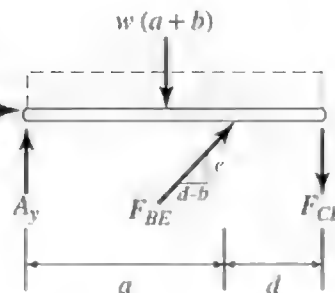
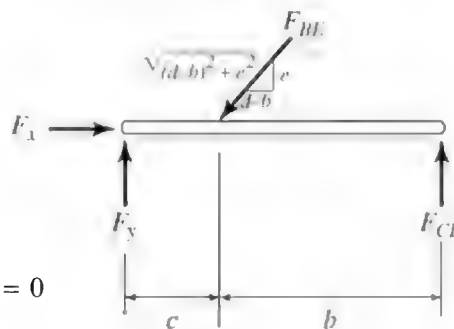
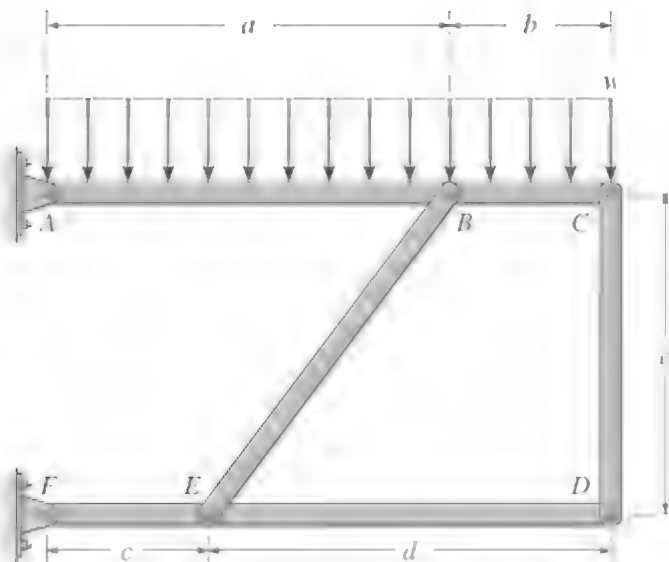
The initial guesses are

$F_{CD} = 20\text{ lb}$ $\quad F_{BE} = 40\text{ lb}$

Given

$$F_{CD}(c+d) - F_{BE}\frac{e\,c}{\sqrt{(d-b)^2+e^2}} = 0$$

$$-w(a+b)\left(\frac{a+b}{2}\right) + \frac{F_{BE}e}{\sqrt{(d-b)^2+e^2}}a - F_{CD}(a+b) = 0$$

$$\begin{pmatrix} F_{CD} \\ F_{BE} \end{pmatrix} = \text{Find}(F_{CD}, F_{BE}) \qquad \begin{pmatrix} F_{CD} \\ F_{BE} \end{pmatrix} = \begin{pmatrix} 350 \\ 1531 \end{pmatrix}\text{lb}$$

Problem 6-129

The mechanism consists of identical meshed gears A and B and arms which are fixed to the gears. The spring attached to the ends of the arms has an unstretched length δ and a stiffness k. If a torque M is applied to gear A, determine the angle θ through which each arm rotates. The gears are each pinned to fixed supports at their centers.

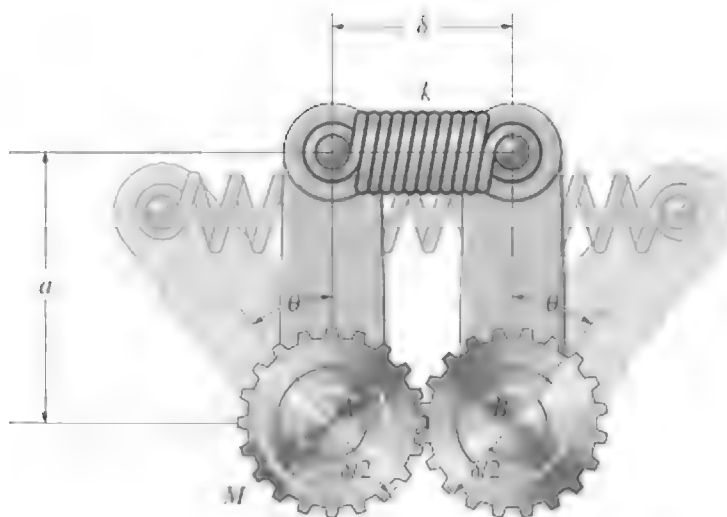
Given:

$\delta = 100 \text{ mm}$

$k = 250 \frac{\text{N}}{\text{m}}$

$M = 6 \text{ N}\cdot\text{m}$

$r = \frac{\delta}{2}$

$a = 150 \text{ mm}$

Solution:

$\Sigma M_A = 0;$ $\quad -F r - P a \cos(\theta) + M = 0$

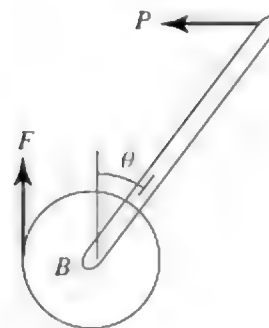
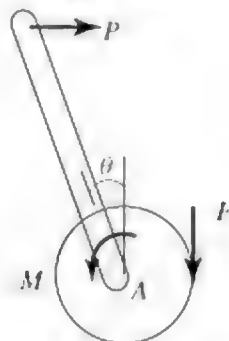
$\Sigma M_B = 0;$ $\quad P a \cos(\theta) - F r = 0$

$$2P a \cos(\theta) = M$$

$$2k(2a) \sin(\theta) a \cos(\theta) = M$$

$$2k a^2 \sin(2\theta) = M$$

$$\theta = \frac{1}{2} \operatorname{asin}\left(\frac{M}{2 k a^2}\right)$$

$$\theta = 16.1 \text{ deg}$$

Problem 6-130

Determine the force in each member of the truss and state if the members are in tension or compression.

Units Used:

$\text{kN} = 1000 \text{ N}$

Given:

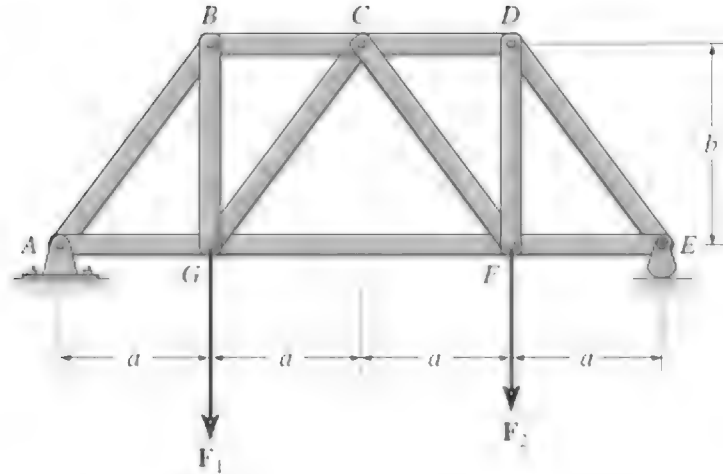
$F_1 = 20\ \text{kN}$

$F_2 = 10\ \text{kN}$

$a = 1.5\ \text{m}$

$b = 2\ \text{m}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{b}{a}\right)$$

Guesses $F_{AG} = 1\ \text{kN}$ $F_{BG} = 1\ \text{kN}$

$F_{GC} = 1\ \text{kN}$ $F_{GF} = 1\ \text{kN}$ $F_{AB} = 1\ \text{kN}$

$F_{BC} = 1\ \text{kN}$ $F_{CD} = 1\ \text{kN}$ $F_{CF} = 1\ \text{kN}$

$F_{DF} = 1\ \text{kN}$ $F_{DE} = 1\ \text{kN}$ $F_{EF} = 1\ \text{kN}$

Given

$$-F_{AB}\cos(\theta) + F_{BC} = 0$$

$$-F_{AB}\sin(\theta) - F_{BG} = 0$$

$$F_{GC}\cos(\theta) + F_{GF} - F_{AG} = 0$$

$$F_{GC}\sin(\theta) + F_{BG} - F_1 = 0$$

$$-F_{BC} + F_{CD} - F_{GC}\cos(\theta) + F_{CF}\cos(\theta) = 0$$

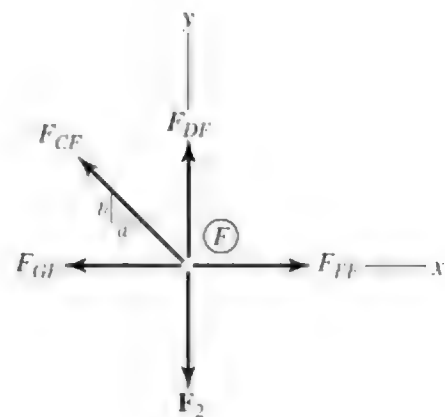
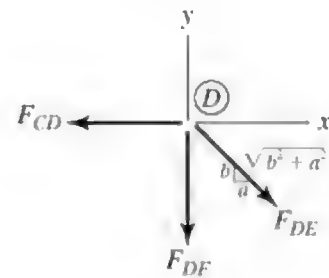
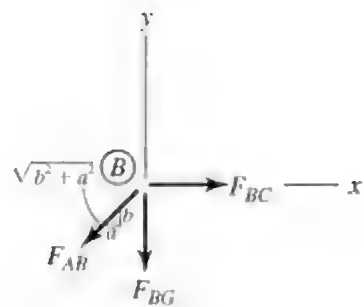
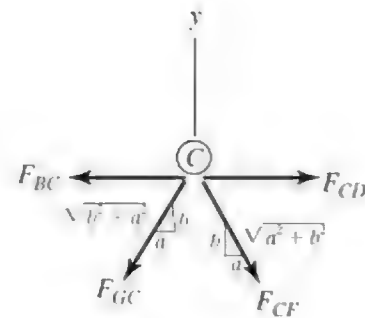
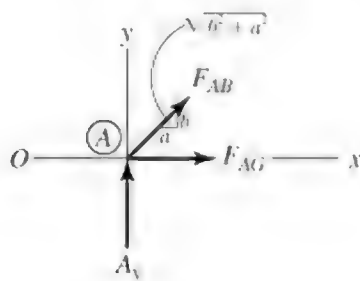
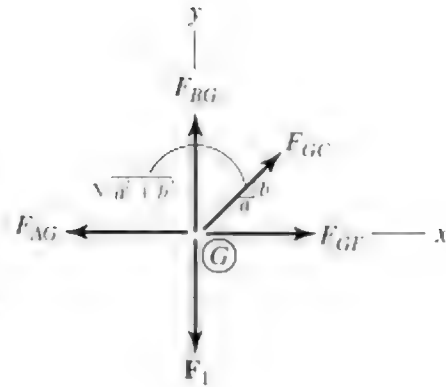
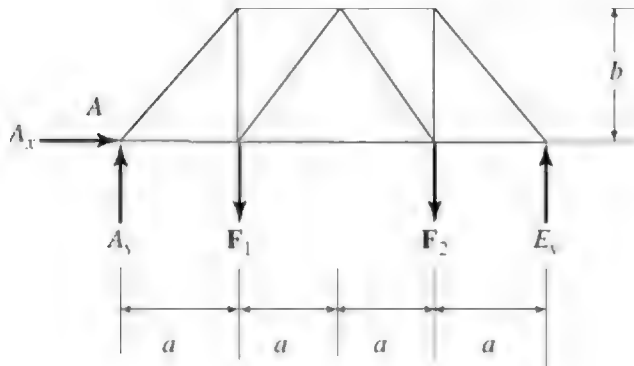
$$-F_{GC}\sin(\theta) - F_{CF}\sin(\theta) = 0$$

$$-F_{CD} + F_{DE}\cos(\theta) = 0$$

$$-F_{DF} - F_{DE}\sin(\theta) = 0$$

$$-F_{GF} - F_{CF}\cos(\theta) + F_{EF} = 0$$

$$F_{DF} + F_{CF}\sin(\theta) - F_2 = 0$$

$$-F_{DE}\cos(\theta) - F_{EF} = 0$$

$$\begin{pmatrix} F_{AG} \\ F_{BG} \\ F_{GC} \\ F_{GF} \\ F_{AB} \\ F_{BC} \\ F_{CD} \\ F_{CF} \\ F_{DF} \\ F_{DE} \\ F_{EF} \end{pmatrix} = \text{Find}(F_{AG}, F_{BG}, F_{GC}, F_{GF}, F_{AB}, F_{BC}, F_{CD}, F_{CF}, F_{DF}, F_{DE}, F_{EF})$$

$$\begin{pmatrix} F_{AG} \\ F_{BG} \\ F_{GC} \\ F_{GF} \\ F_{AB} \\ F_{BC} \\ F_{CD} \\ F_{CF} \\ F_{DF} \\ F_{DE} \\ F_{EF} \end{pmatrix} = \begin{pmatrix} 13.13 \\ 17.50 \\ 3.13 \\ 11.25 \\ -21.88 \\ -13.13 \\ -9.37 \\ -3.13 \\ 12.50 \\ -15.62 \\ 9.37 \end{pmatrix} \text{kN}$$

Positive (T)
Negative (C)

Problem 6-131

The spring has an unstretched length δ. Determine the angle θ for equilibrium if the uniform links each have a mass m_{link}.

Given:

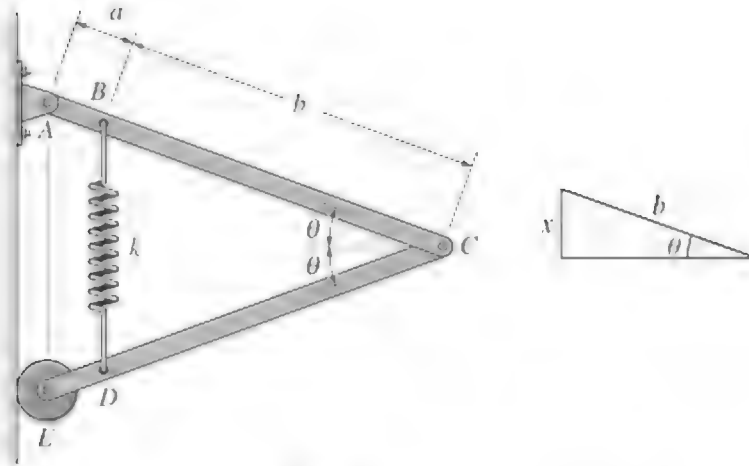
$m_{link} = 5 \text{ kg}$

$\delta = 0.3 \text{ m}$

$k = 400 \frac{\text{N}}{\text{m}}$

$a = 0.1 \text{ m}$

$b = 0.6 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

Guesses

$\theta = 10 \text{ deg}$

$F_{BD} = 1 \text{ N}$

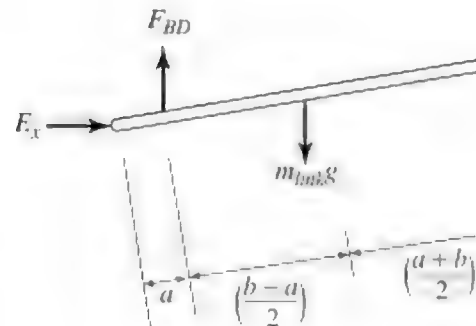
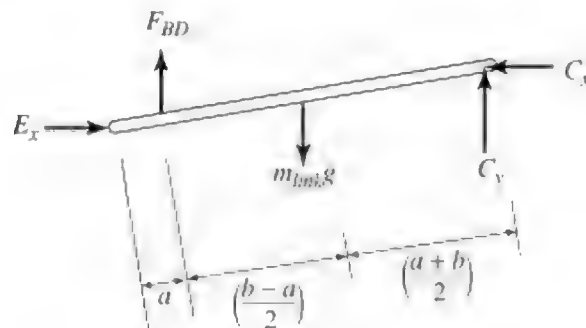
$E_x = 1 \text{ N}$

Given

$$m_{link}\, g \frac{a+b}{2} \cos(\theta) - F_{BD}\, b \cos(\theta) + E_x\, b \sin(\theta) = 0$$

$$-2\, m_{link}\, g \frac{a+b}{2} \cos(\theta) + E_x\, 2\, b \sin(\theta) = 0$$

$$F_{BD} = k(2\, b \sin(\theta) - \delta)$$

$$\begin{pmatrix} F_{BD} \\ E_x \\ \theta \end{pmatrix} = \text{Find}(F_{BD}, E_x, \theta) \qquad \theta = 21.7 \text{ deg}$$

Problem 6-132

The spring has an unstretched length δ. Determine the mass m_{link} of each uniform link if the angle for equilibrium is θ.

Given:

$\delta = 0.3 \text{ m}$

$\theta = 20 \text{ deg}$

$k = 400 \frac{\text{N}}{\text{m}}$

$a = 0.1 \text{ m}$

$b = 0.6 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

Guesses $E_y = 1 \text{ N}$ $m_{link} = 1 \text{ kg}$

$F_s = 1 \text{ N}$

Given

$$F_s = (2\,b\sin(\theta) - \delta)k$$

$$m_{link}\,g\,\frac{a+b}{2}\cos(\theta) - F_s b\cos(\theta) + E_y b\sin(\theta) = 0$$

$$-2m_{link}\,g\,\frac{a+b}{2}\cos(\theta) + E_y 2b\sin(\theta) = 0$$

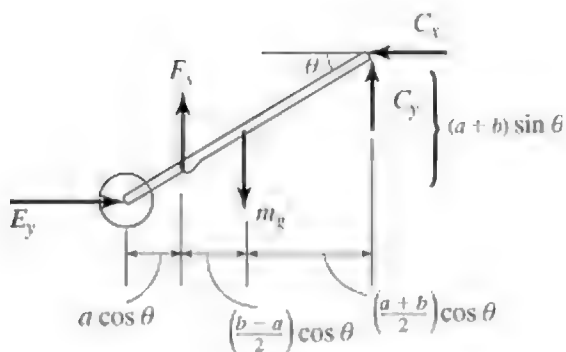
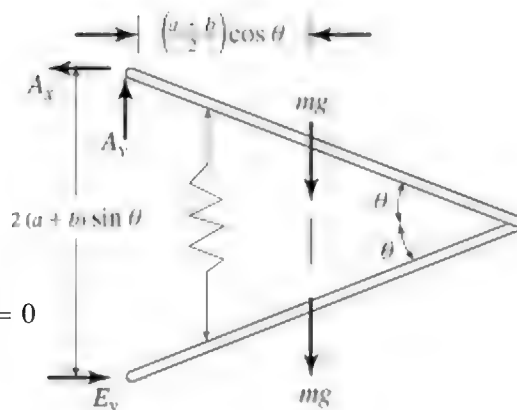
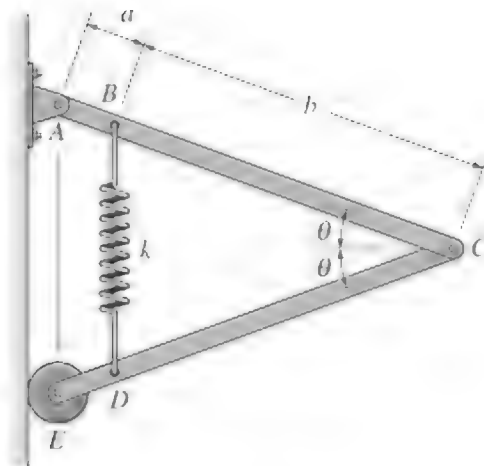
$$\begin{pmatrix} m_{link} \\ F_s \\ E_y \end{pmatrix} = \text{Find}(m_{link}, F_s, E_y)$$

$m_{link} = 3.859 \text{ kg}$

$$\frac{y}{2(b)} = \sin(\theta)$$

$$y = 2\,b\sin(\theta)$$

$$F_s = (y - \delta)(k)$$

$F_s = 44.17 \text{ N}$

$\Sigma M_A = 0;$ $\left[E_y(2)(a+b)\sin(\theta) - 2(M)(g)\right]\frac{a+b}{2}(\cos(\theta)) = 0$

$$\left[E_y(2)\sin(\theta) - 2\,(m)\,\frac{g}{2}\right](\cos(\theta) = 0$$

$$\Rightarrow \quad E_y(2\,\sin(\theta)) = m(g)(\cos(\theta))$$

$$E_y = \frac{m(g)(\cos(\theta))}{2\,\sin(\theta)}$$

$\Sigma M_C = 0;$ $\frac{m(g)(\cos(\theta))}{2\,\sin(\theta)}(a+b)\sin(\theta) + m(g)\left(\frac{a+b}{2}\right)\cos(\theta) - F_s(b\cos(\theta)) = 0$

$$m = F_s\,\frac{b}{g\,(a+b)}$$

$$m = 3.859 \text{ kg}$$

Problem 6-133

Determine the horizontal and vertical components of force that the pins A and B exert on the two-member frame.

Given:

$w = 400\ \frac{\text{N}}{\text{m}}$

$a = 1.5\text{ m}$

$b = 1\text{ m}$

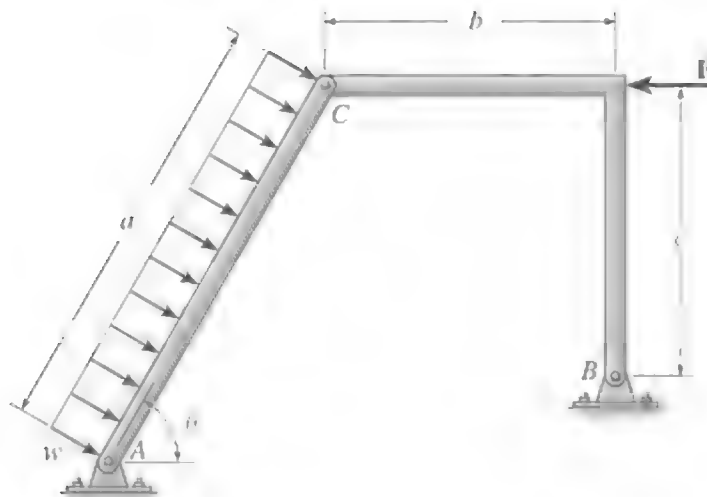
$c = 1\text{ m}$

$F = 0\text{ N}$

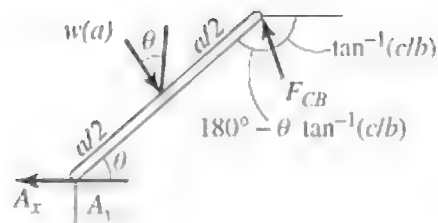
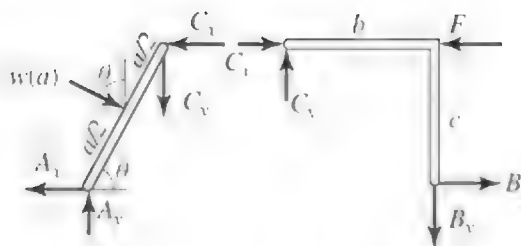
$\theta = 60\text{ deg}$

Solution:

Guesses

$A_x = 1\text{ N}$ $\quad A_y = 1\text{ N}$ $\quad B_x = 1\text{ N}$

$B_y = 1\text{ N} \qquad C_x = 1\text{ N} \qquad C_y = 1\text{ N}$

Given

$$-wa\frac{a}{2} + C_x a\sin(\theta) - C_y a\cos(\theta) = 0$$

$$Fc - C_x c - C_y b = 0 \qquad A_y - C_y - wa\cos(\theta) = 0$$

$$-A_x - C_x + wa\sin(\theta) = 0 \qquad C_x - B_x - F = 0$$

$$C_y + B_y = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y, C_x, C_y) \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 300.0 \\ 80.4 \end{pmatrix}\text{ N} \qquad \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 220 \\ 220 \end{pmatrix}\text{ N}$$

Problem 6-134

Determine the horizontal and vertical components of force that the pins A and B exert on the two-member frame.

Given:

$w = 400\ \frac{\text{N}}{\text{m}}$

$a = 1.5\text{ m}$

$b = 1\text{ m}$

$c = 1\text{ m}$

$F = 500\text{ N}$

$\theta = 60\text{ deg}$

Solution:

Guesses

$A_x = 1\ \text{N}$ $A_y = 1\ \text{N}$ $B_x = 1\ \text{N}$

$B_y = 1\ \text{N}$ $C_x = 1\ \text{N}$ $C_y = 1\ \text{N}$

Given

$$-wa\frac{a}{2} + C_x a\sin(\theta) - C_y a\cos(\theta) = 0$$

$$A_y - C_y - wa\cos(\theta) = 0$$

$$Fc - C_x c - C_y b = 0$$

$$C_x + B_x - F = 0$$

$$-A_x - C_x + wa\sin(\theta) = 0$$

$$C_y - B_y = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y, C_x, C_y)$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 117.0 \\ 397.4 \end{pmatrix}\ \text{N} \qquad \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 97.4 \\ 97.4 \end{pmatrix}\ \text{N}$$

Problem 6-135

Determine the force in each member of the truss and indicate whether the members are in tension or compression.

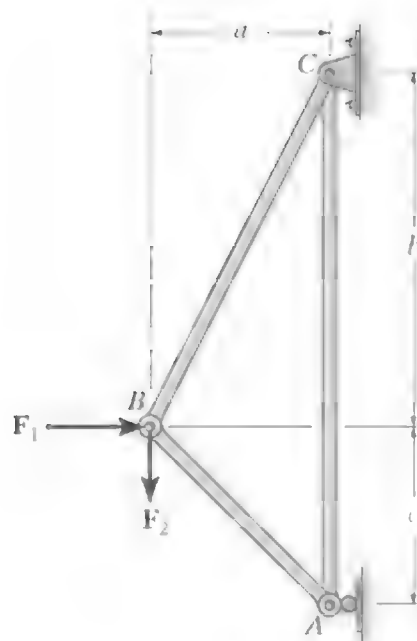
Units Used:

$\text{kip} = 1000\ \text{lb}$

Given:

$F_1 = 1000\ \text{lb}$ $b = 8\ \text{ft}$

$F_2 = 500\ \text{lb}$ $c = 4\ \text{ft}$

$a = 4\ \text{ft}$

Solution:

Joint B:

Initial Guesses:

Given

$F_{BC} = 100 \text{ lb} \quad F_{BA} = 150 \text{ lb}$

$\Sigma F_x = 0; \quad F_1 - F_{BC}\cos\left(\operatorname{atan}\left(\frac{b}{a}\right)\right) - F_{BA}\cos\left(\operatorname{atan}\left(\frac{c}{a}\right)\right) = 0$

$\Sigma F_y = 0; \quad -F_{BC}\sin\left(\operatorname{atan}\left(\frac{b}{a}\right)\right) + F_{BA}\sin\left(\operatorname{atan}\left(\frac{c}{a}\right)\right) - F_2 = 0$

$\begin{pmatrix} F_{BC} \\ F_{BA} \end{pmatrix} = \operatorname{Find}(F_{BC}, F_{BA})$

$F_{BC} = 373 \text{ lt(C)}$

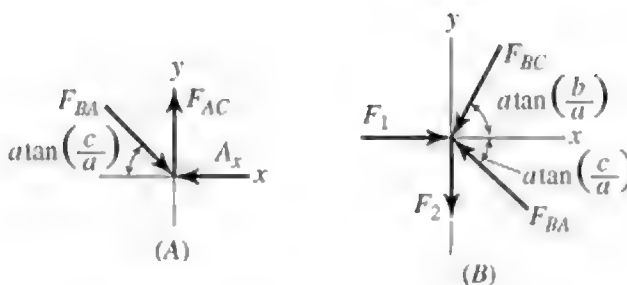
$F_{BA} = 1178.51 \text{ lb}$

$F_{BA} = 1.179 \text{ kip (C)}$

Joint A:

$\Sigma F_y = 0; \quad F_{AC} - (F_{BA})\sin\left(\operatorname{atan}\left(\frac{c}{a}\right)\right) = 0$

$F_{AC} = F_{BA} \dfrac{c}{a\sqrt{\dfrac{a^2 + c^2}{a^2}}}$

$F_{AC} = 833 \text{ lb (T)}$

Problem 6-136

Determine the force in each member of the truss and state if the members are in tension or compression.

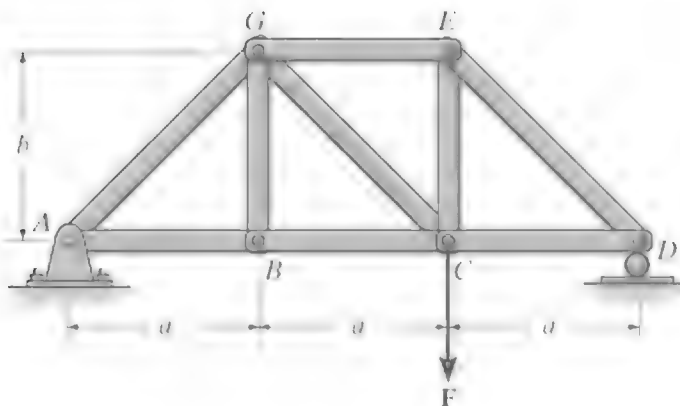
Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$F = 1000 \text{ lb}$

$a = 10 \text{ ft}$

$b = 10 \text{ ft}$

Solution: $\theta = \operatorname{atan}\left(\frac{b}{a}\right)$

Guesses

$F_{AB} = 1\ \text{lb}$ $F_{AG} = 1\ \text{lb}$ $F_{BC} = 1\ \text{lb}$ $F_{BG} = 1\ \text{lb}$

$F_{CD} = 1\ \text{lb}$ $F_{CE} = 1\ \text{lb}$ $F_{CG} = 1\ \text{lb}$ $F_{DE} = 1\ \text{lb}$

$F_{EG} = 1\ \text{lb}$

Given

$$F_{AB} + F_{AG}\cos(\theta) = 0$$

$$-F_{AB} + F_{BC} = 0$$

$$F_{BG} = 0$$

$$F_{CD} - F_{BC} - F_{CG}\cos(\theta) = 0$$

$$F_{CE} - F + F_{CG}\sin(\theta) = 0$$

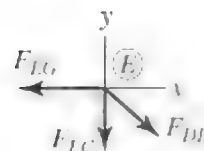
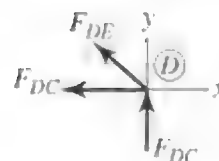
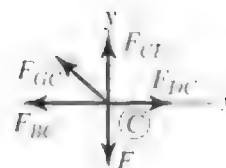
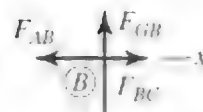
$$-F_{CD} - F_{DE}\cos(\theta) = 0$$

$$F_{DE}\cos(\theta) - F_{EG} = 0$$

$$-F_{CE} - F_{DE}\sin(\theta) = 0$$

$$F_{EG} - F_{AG}\cos(\theta) + F_{CG}\cos(\theta) = 0$$

$$-F_{AG}\sin(\theta) - F_{BG} - F_{CG}\sin(\theta) = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{AG} \\ F_{BC} \\ F_{BG} \\ F_{CD} \\ F_{CE} \\ F_{CG} \\ F_{DE} \\ F_{EG} \end{pmatrix} = \text{Find}(F_{AB}, F_{AG}, F_{BC}, F_{BG}, F_{CD}, F_{CE}, F_{CG}, F_{DE}, F_{EG})$$

$$\begin{pmatrix} F_{AB} \\ F_{AG} \\ F_{BC} \\ F_{BG} \\ F_{CD} \\ F_{CE} \\ F_{CG} \\ F_{DE} \\ F_{EG} \end{pmatrix} = \begin{pmatrix} 333 \\ -471 \\ 333 \\ 0 \\ 667 \\ 667 \\ 471 \\ -943 \\ -667 \end{pmatrix} \text{lb} \qquad \text{Positive (T), Negative (C)}$$

Problem 6-137

Determine the force in members *AB, AD*, and *AC* of the space truss and state if the members are in tension or compression. The force **F** is vertical.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$F = 600 \text{ lb}$

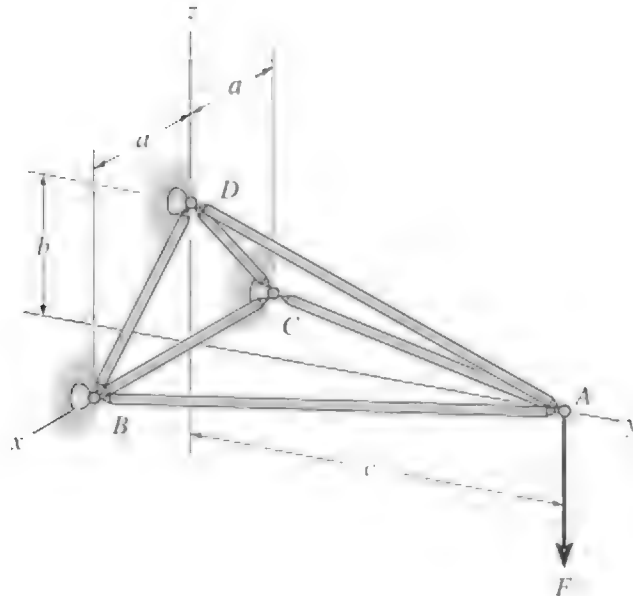
$a = 1.5 \text{ ft}$

$b = 2 \text{ ft}$

$c = 8 \text{ ft}$

Solution:

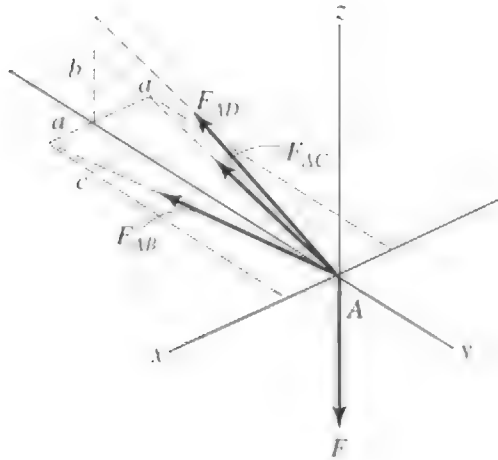
$$\mathbf{AB} = \begin{pmatrix} a \\ -c \\ 0 \end{pmatrix} \qquad \mathbf{AC} = \begin{pmatrix} -a \\ -c \\ 0 \end{pmatrix}$$

$$\mathbf{AD} = \begin{pmatrix} 0 \\ -c \\ b \end{pmatrix}$$

Guesses $\quad F_{AB} = 1 \text{ lb} \quad F_{AC} = 1 \text{ lb} \quad F_{AD} = 1 \text{ lb}$

Given $$F_{AB}\frac{\mathbf{AB}}{|\mathbf{AB}|} + F_{AC}\frac{\mathbf{AC}}{|\mathbf{AC}|} + F_{AD}\frac{\mathbf{AD}}{|\mathbf{AD}|} + \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \text{Find}\left(F_{AB}, F_{AC}, F_{AD}\right)$$

$$\begin{pmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{pmatrix} = \begin{pmatrix} -1.221 \\ -1.221 \\ 2.474 \end{pmatrix} \text{kip}$$

Positive (T)
Negative (C)

Problem 7-1

The column is fixed to the floor and is subjected to the loads shown. Determine the internal normal force, shear force, and moment at points *A* and *B*.

Units Used:

$kN = 10^3\ N$

Given:

$F_1 = 6\ kN$

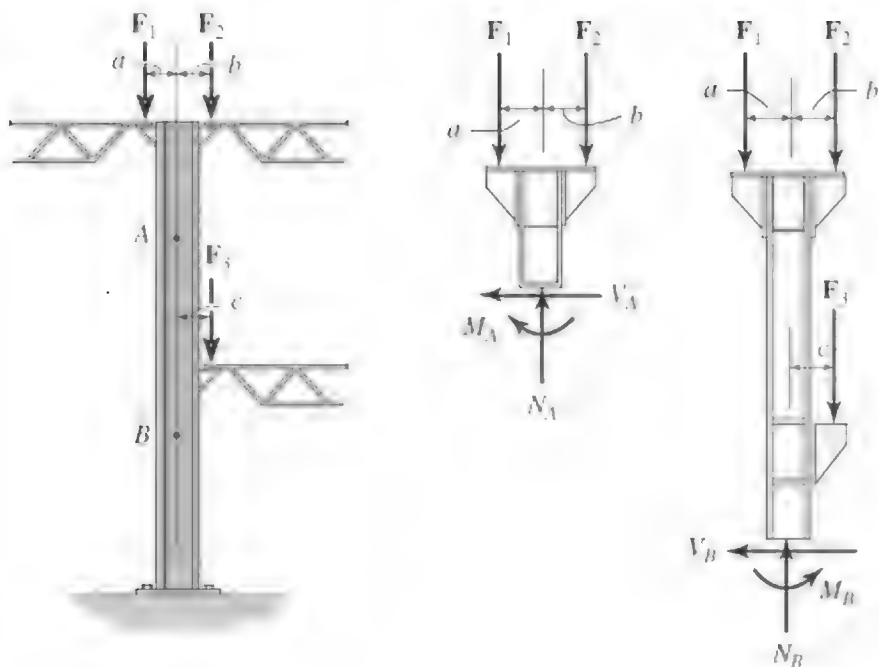
$F_2 = 6\ kN$

$F_3 = 8\ kN$

$a = 150\ mm$

$b = 150\ mm$

$c = 150\ mm$

Solution:

Free body Diagram: The support reaction need not be computed in this case.

Internal Forces: Applying equations of equillibrium to the top segment sectioned through point A, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad V_A = 0 \qquad\qquad V_A = 0$

$+\uparrow \Sigma F_y = 0;\quad N_A - F_1 - F_2 = 0 \qquad N_A = F_1 + F_2 \qquad N_A = 12.0\,\text{kN}$

$\Sigma M_A = 0;\quad F_1 a - F_2 b - M_A = 0 \qquad M_A = F_1 a - F_2 b \qquad M_A = 0\,\text{kN}\cdot\text{m}$

Applying equations of equillibrium to the top segment sectioned through point *B*, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad V_B = 0 \qquad\qquad V_B = 0$

$+\uparrow \Sigma F_y = 0;\quad N_B - F_1 - F_2 - F_3 = 0 \qquad N_B = F_1 + F_2 + F_3 \qquad N_B = 20.0\,\text{kN}$

$+\Sigma M_B = 0;\quad F_1\, a - F_2 b - F_3 c + M_B = 0 \qquad M_B = -F_1 a + F_2 b + F_3 c \qquad M_B = 1.20\,\text{kN}\cdot\text{m}$

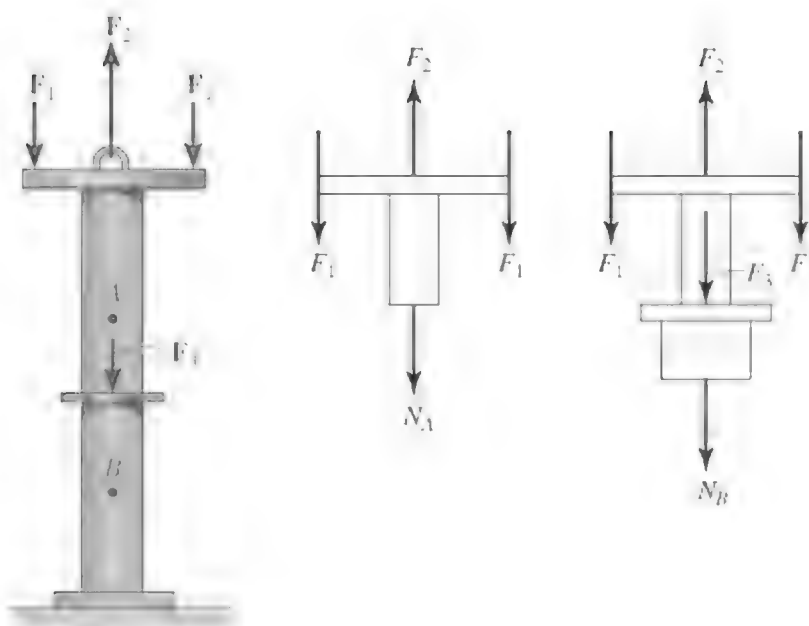
Problem 7-2

The axial forces act on the shaft as shown. Determine the internal normal forces at points *A* and *B*.

Given:

$F_1 = 20 \text{ lb}$

$F_2 = 50 \text{ lb}$

$F_3 = 10 \text{ lb}$

Solution:

Section A:

$\Sigma F_z = 0;\quad F_2 - 2F_1 - N_A = 0$

$$N_A = F_2 - 2\ F_1$$

$$N_A = 10.00 \text{ lb}$$

Section B:

$\Sigma F_z = 0;\quad F_2 - 2F_1 - N_A + N_B = 0$

$$N_B = -F_2 + 2F_1 + N_A$$

$$N_B = 0.00 \text{ lb}$$

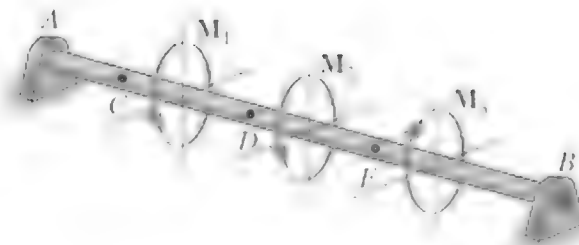
Problem 7-3

The shaft is supported by smooth bearings at A and B and subjected to the torques shown. Determine the internal torque at points C, D, and E.

Given:

$M_1 = 400 \text{ N}\cdot\text{m}$

$M_2 = 150 \text{ N}\cdot\text{m}$

$M_3 = 550 \text{ N}\cdot\text{m}$

Solution:

Section C:

$\Sigma M_x = 0;\quad T_C = 0$

Section D:

$\Sigma M_x = 0;\quad T_D - M_1 = 0$

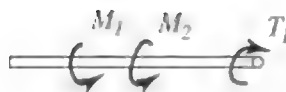
$T_D = M_1$

$T_D = 400.00\ \text{N}\cdot\text{m}$

Section E:

$\Sigma M_x = 0;\quad M_1 + M_2 - T_E = 0$

$T_E = M_1 + M_2$

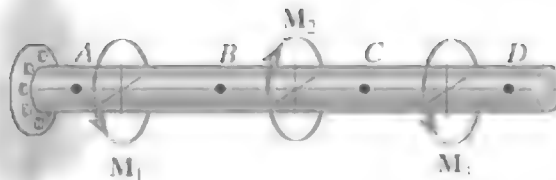
$T_E = 550.00\ \text{N}\cdot\text{m}$

Problem 7-4

Three torques act on the shaft. Determine the internal torque at points A, B, C, and D.

Given:

$M_1 = 300\ \text{N}\cdot\text{m}$

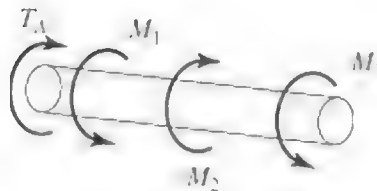
$M_2 = 400\ \text{N}\cdot\text{m}$

$M_3 = 200\ \text{N}\cdot\text{m}$

Solution:

Section A:

$\Sigma M_x = 0;\quad -T_A + M_1 - M_2 + M_3 = 0$

$T_A = M_1 - M_2 + M_3$

$T_A = 100.00\ \text{N}\cdot\text{m}$

Section B:

$\Sigma M_x = 0;\quad T_B + M_3 - M_2 = 0$

$$T_B = -M_3 + M_2$$

$$T_B = 200.00\,\text{N}\cdot\text{m}$$

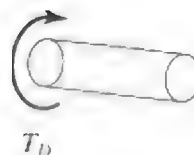
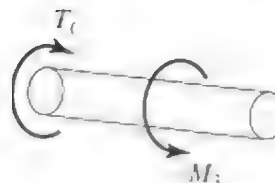
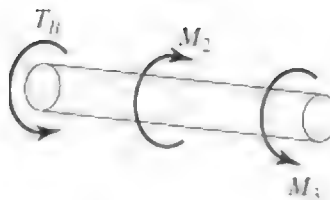
Section C:

$\Sigma M_x = 0;$ $\quad -T_C + M_3 = 0$

$$T_C = M_3$$

$$T_C = 200.00\,\text{N}\cdot\text{m}$$

Section D:

$\Sigma M_x = 0;$ $\quad T_D = 0$

Problem 7-5

The shaft is supported by a journal bearing at A and a thrust bearing at B. Determine the normal force, shear force, and moment at a section passing through (a) point C, which is just to the right of the bearing at A, and (b) point D, which is just to the left of the force F_2.

Units Used:

$$\text{kip} = 10^3\ \text{lb}$$

Given:

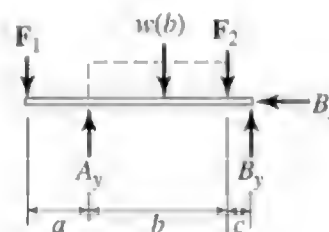
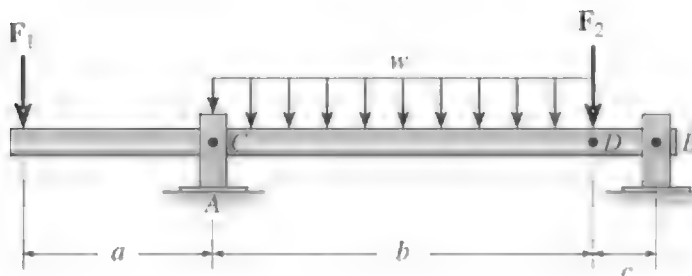
$$F_1 = 2.5\ \text{kip}$$

$$F_2 = 3\ \text{kip}$$

$$w = 75\ \frac{\text{lb}}{\text{ft}} \qquad b = 12\ \text{ft}$$

$$a = 6\ \text{ft} \qquad c = 2\ \text{ft}$$

Solution:

$\Sigma M_B = 0;$

$$-A_y(b+c) + F_1(a+b+c) + wb\left(c+\frac{b}{2}\right) + F_2c = 0$$

$$A_y = \frac{F_1(a+b+c) + wb\left(c+\frac{b}{2}\right) + F_2c}{b+c}$$

$A_y = 4514\,\text{lb}$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;\qquad B_x = 0\ \text{lb}$

$+\uparrow\ \Sigma F_y = 0;\qquad A_y - F_1 - wb - F_2 + B_y = 0$

$B_y = -A_y + F_1 + wb + F_2\qquad B_y = 1886\,\text{lb}$

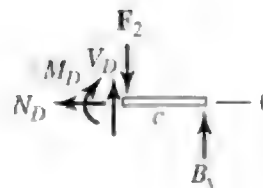
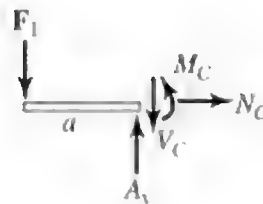
$\Sigma M_C = 0;\qquad F_1 a + M_c = 0\qquad M_C = -F_1 a\qquad M_C = -15.0\,\text{kip}\cdot\text{ft}$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;\qquad N_C = 0\ \text{lb}\qquad N_C = 0.00\,\text{lb}$

$+\uparrow\ \Sigma F_y = 0;\qquad -F_1 + A_y - V_C = 0\qquad V_C = A_y - F_1\qquad V_C = 2.01\,\text{kip}$

$\Sigma M_D = 0;\qquad -M_D + B_y c = 0\qquad M_D = B_y c\qquad M_D = 3.77\,\text{kip}\cdot\text{ft}$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;\qquad N_D = 0\ \text{lb}\qquad N_D = 0.00\,\text{lb}$

$+\uparrow\ \Sigma F_y = 0;\qquad V_D - F_2 + B_y = 0\qquad V_D = F_2 - B_y\qquad V_D = 1.11\,\text{kip}$

Problem 7-6

Determine the internal normal force, shear force, and moment at point C.

Given:

$M = 400\ \text{lb}\cdot\text{ft}$

$a = 4\ \text{ft}$

$b = 12\ \text{ft}$

Solution:

Beam:

$\Sigma M_B = 0;\quad M - A_y(a+b) = 0$

$A_y = \dfrac{M}{a+b} \qquad A_y = 25.00\ \text{lb}$

Segment AC:

$\Sigma F_x = 0;\quad N_C = 0$

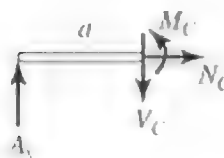
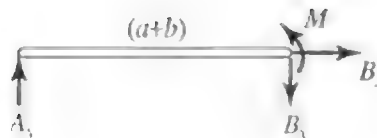
$\Sigma F_y = 0;\quad A_y - V_C = 0$

$V_C = A_y$

$V_C = 25.00\ \text{lb}$

$\Sigma M_C = 0;\quad -A_y a + M_C = 0$

$M_C = A_y a$

$M_C = 100.00\ \text{lb}\cdot\text{ft}$

Problem 7-7

Determine the internal normal force, shear force, and moment at point C.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

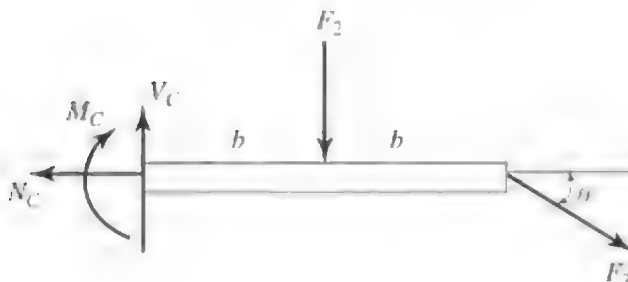
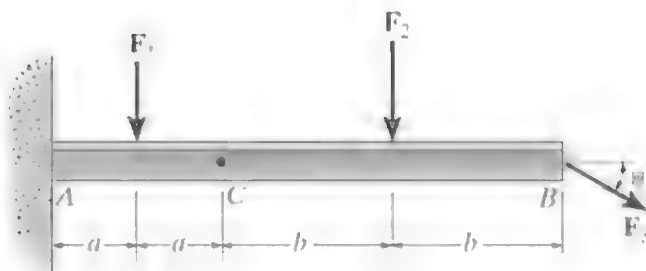
$F_1 = 30\ \text{kN}$

$F_2 = 50\ \text{kN}$

$F_3 = 25\ \text{kN}$

$a = 1.5\ \text{m}$

$b = 3\ \text{m}$

$\theta = 30\ \text{deg}$

Solution:

$\Sigma F_x = 0;\quad -N_C + F_3\cos(\theta) = 0$

$$N_C = F_3 \cos(\theta)$$

$$N_C = 21.7\,\text{kN}$$

$\Sigma F_y = 0;$ $\quad V_C - F_2 - F_3 \sin(\theta) = 0$

$$V_C = F_2 + F_3 \sin(\theta)$$

$$V_C = 62.50\,\text{kN}$$

$\Sigma M_C = 0;$ $\quad -M_C - F_2 b - F_3 \sin(\theta) 2b = 0$

$$M_C = -F_2 b - F_3 \sin(\theta)\, 2b$$

$$M_C = -225.00\,\text{kN}\cdot\text{m}$$

Problem 7-8

Determine the normal force, shear force, and moment at a section passing through point *C*. Assume the support at *A* can be approximated by a pin and *B* as a roller.

Units used:

$$\text{kip} = 10^3\ \text{lb}$$

Given:

$F_1 = 10\ \text{kip}$ $\quad a = 6\ \text{ft}$

$F_2 = 8\ \text{kip}$ $\quad b = 12\ \text{ft}$

$w = 0.8\ \dfrac{\text{kip}}{\text{ft}}$ $\quad c = 12\ \text{ft}$

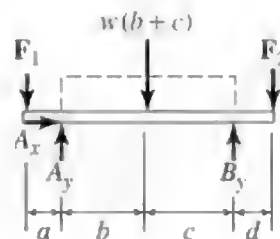
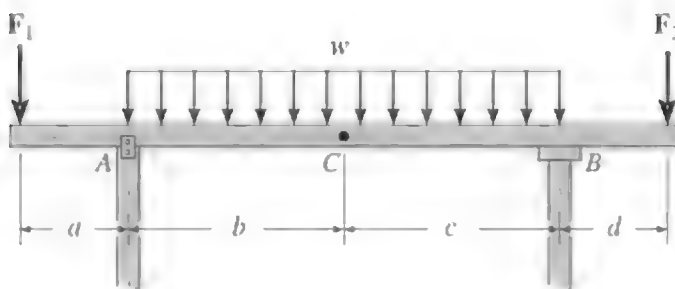
$d = 6\ \text{ft}$

Solution:

$\circlearrowleft\ \Sigma M_A = 0;$

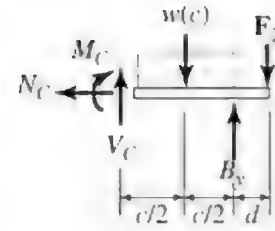
$$-w(b+c)\left(\frac{b+c}{2}\right) - F_2(b+c+d) + B_y(b+c) + F_1 a = 0$$

$$B_y = \frac{w\dfrac{(b+c)^2}{2} + F_2(b+c+d) - F_1 a}{b+c} \qquad B_y = 17.1\,\text{kip}$$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;$ $\quad N_C = 0$

$+\uparrow \Sigma F_y = 0; \qquad V_C - wc + B_y - F_2 = 0$

$V_C = wc - B_y + F_2 \qquad V_C = 0.5\,\text{kip}$

$\Sigma M_C = 0; \qquad -M_C - wc\left(\frac{c}{2}\right) + B_y c - F_2(c+d) = 0$

$M_C = -w\left(\frac{c^2}{2}\right) + B_y c - F_2(c+d) \qquad M_C = 3.6\,\text{kip}\cdot\text{ft}$

Problem 7-9

The beam AB will fail if the maximum internal moment at D reaches M_{max} or the normal force in member BC becomes P_{max}. Determine the largest load w it can support.

Given:

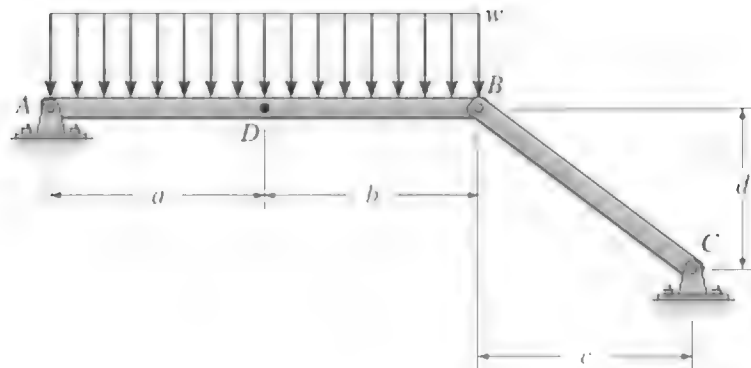
$M_{max} = 800\ \text{N}\cdot\text{m}$

$P_{max} = 1500\ \text{N}$

$a = 4\ \text{m}$

$b = 4\ \text{m}$

$c = 4\ \text{m}$

$d = 3\ \text{m}$

Solution:

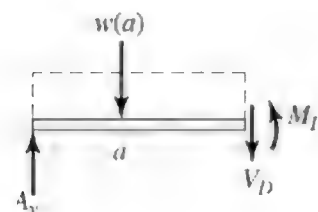
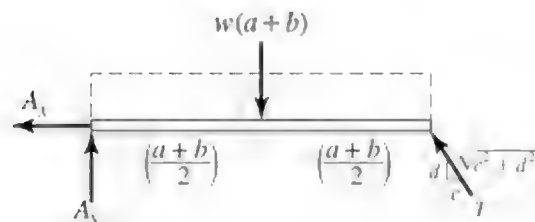
$$w\,(a+b)\left(\frac{a+b}{2}\right) - A_y(a+b) = 0$$

$$A_y = \frac{w(a+b)}{2}$$

$$w\,a\left(\frac{a}{2}\right) - A_y a + M_D = 0$$

$$M_D = w\left(\frac{ab}{2}\right)$$

$$A_y - w(a+b) + \left(\frac{d}{\sqrt{c^2+d^2}}\right)T = 0$$

$$T = \frac{w(a+b)\sqrt{c^2+d^2}}{2d}$$

Assume the maximum moment has been reached

$M_D = M_{max}$ $\quad w_1 = \dfrac{2M_D}{ab}$ $\quad w_1 = 100\,\dfrac{\text{N}}{\text{m}}$

Assume that the maximum normal force in BC has been reached

$T = P_{max}$ $\quad w_2 = \dfrac{T2d}{(a+b)\sqrt{c^2+d^2}}$ $\quad w_2 = 225\,\dfrac{\text{N}}{\text{m}}$

Now choose the critical load $\quad w = \min(w_1, w_2)$ $\quad w = 100\,\dfrac{\text{N}}{\text{m}}$

Problem 7-10

Determine the shear force and moment acting at a section passing through point C in the beam.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$w = 3\ \dfrac{\text{kip}}{\text{ft}}$

$a = 6\ \text{ft}$

$b = 18\ \text{ft}$

Solution:

$\Sigma M_B = 0;$ $\quad -A_y b + \dfrac{1}{2} w b \left(\dfrac{b}{3}\right) = 0$

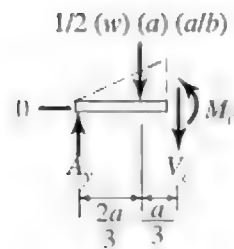
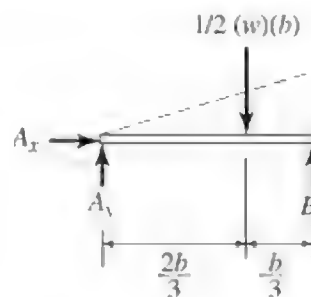
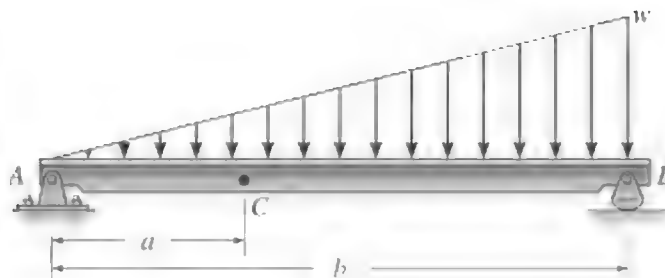
$A_y = \dfrac{1}{6} w b \qquad A_y = 9\,\text{kip}$

$\Sigma M_C = 0;$ $\quad -A_y a + \dfrac{1}{2}\left(w\dfrac{a}{b}\right)a\left(\dfrac{a}{3}\right) + M_C = 0$

$M_C = A_y a - \dfrac{w a^3}{6b}$

$M_C = 48\,\text{kip}\cdot\text{ft}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - \dfrac{1}{2}\left(w\dfrac{a}{b}\right)a - V_C = 0$

$V_C = A_y - \dfrac{w a^2}{2b}$

$V_C = 6\,\text{kip}$

Problem 7-11

Determine the internal normal force, shear force, and moment at points E and D of the compound beam.

Given:

$M = 200\ \text{N}\cdot\text{m}$ $\quad c = 4\ \text{m}$

$F = 800\ \text{N}$ $\quad d = 2\ \text{m}$

$a = 2\ \text{m}$ $\quad e = 2\ \text{m}$

$b = 2\ \text{m}$

Solution:

Segment BC :

$-M + C_y(d+e) = 0 \qquad C_y = \dfrac{M}{d+e}$

$-B_y + C_y = 0 \qquad B_y = C_y$

Segment EC :

$-N_E = 0 \qquad N_E = 0\text{N} \qquad N_E = 0.00$

$V_E + C_y = 0 \qquad V_E = -C_y \qquad V_E = -50.00\ \text{N}$

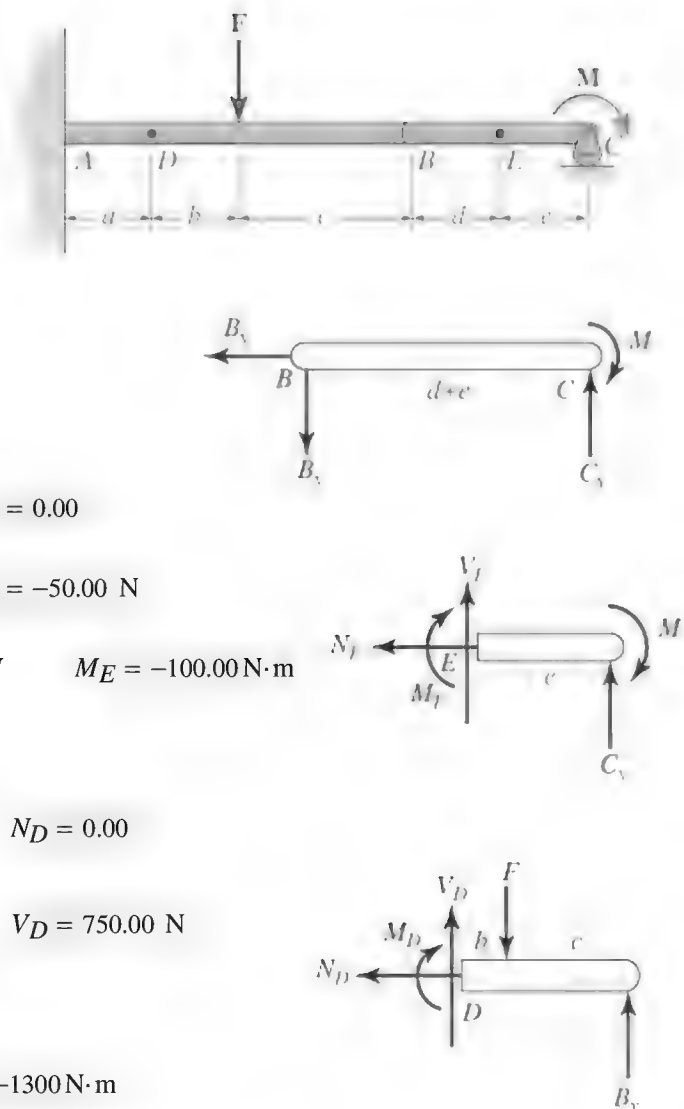
$-M_E - M + C_y e = 0 \quad M_E = C_y e - M \qquad M_E = -100.00\,\text{N}\cdot\text{m}$

Segment DB :

$-N_D = 0 \qquad N_D = 0\text{N} \qquad N_D = 0.00$

$V_D - F + B_y = 0 \qquad V_D = F - B_y \qquad V_D = 750.00\ \text{N}$

$-M_D - F\,b + B_y(b+c) = 0$

$M_D = -F\,b + B_y(b+c) \qquad M_D = -1300\,\text{N}\cdot\text{m}$

Problem 7-12

The boom DF of the jib crane and the column DE have a uniform weight density γ. If the hoist and load have weight W, determine the normal force, shear force, and moment in the crane at sections passing through points A, B, and C. Treat the boom tip, beyond the hoist, as weightless.

Given:

$W = 300$ lb

$\gamma = 50\ \frac{\text{lb}}{\text{ft}}$

$a = 7$ ft

$b = 5$ ft

$c = 2$ ft

$d = 8$ ft

$e = 3$ ft

Solution:

$\xrightarrow{+} \Sigma F_x = 0;\quad -N_A = 0 \qquad N_A = 0\text{lb} \qquad N_A = 0.00\,\text{lb}$

$+\uparrow \Sigma F_y = 0;\quad V_A - W - \gamma e = 0$

$V_A = W + \gamma\, e \qquad V_A = 450\,\text{lb}$

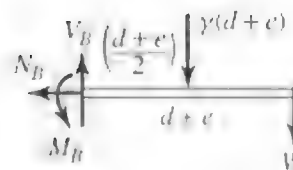
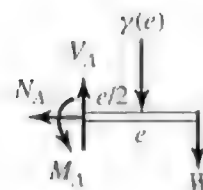
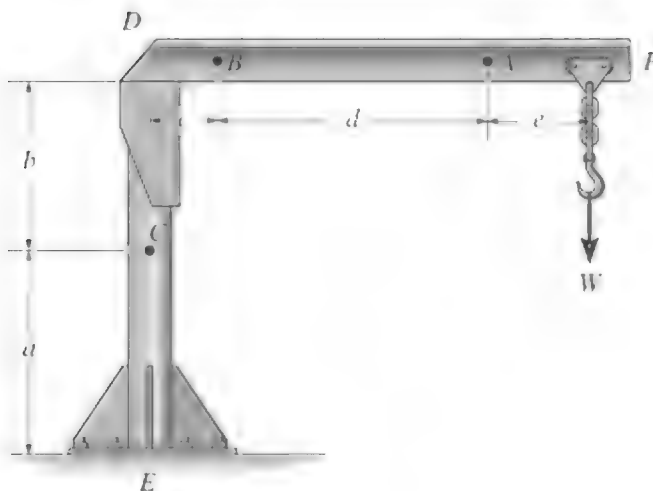
$\Sigma M_A = 0;\quad M_A - \gamma\, e\left(\frac{e}{2}\right) - We = 0$

$M_A = \gamma\left(\frac{e^2}{2}\right) + We \qquad M_A = 1125.00\,\text{lb}\cdot\text{ft}$

$\xrightarrow{+} \Sigma F_x = 0;\quad -N_B = 0 \qquad N_B = 0\text{lb} \qquad N_B = 0.00\,\text{lb}$

$+\uparrow \Sigma F_y = 0;\quad V_B - \gamma(d+e) - W = 0 \qquad V_B = \gamma(d+e) + W \qquad V_B = 850\,\text{lb}$

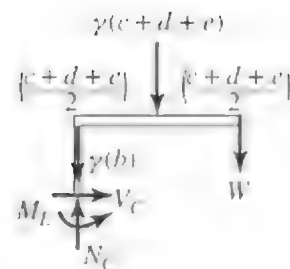
$\Sigma M_B = 0;\quad M_B - \gamma(d+e)\left(\frac{d+e}{2}\right) - W(d+e) = 0$

$M_B = \frac{1}{2}\,\gamma\,(d+e)^2 + W(d+e) \qquad M_B = 6325.00\,\text{lb}\cdot\text{ft}$

$\xrightarrow{+}$ $\Sigma F_x = 0;$ $V_C = 0$ $V_C = 0\text{lb}$ $V_C = 0.00\,\text{lb}$

$+\uparrow$ $\Sigma F_y = 0;$ $N_C - (c+d+e)\,\gamma - W - \gamma(b) = 0$

$N_C = \gamma(c+d+e+b) + W$ $N_C = 1200.00\,\text{lb}$

$\Sigma M_C = 0;$ $M_C - (c+d+e)\gamma\left(\frac{c+d+e}{2}\right) - W(c+d+e) = 0$

$M_C = (c+d+e)\gamma\left(\frac{c+d+e}{2}\right) + W(c+d+e)$ $M_C = 8125.00\,\text{lb}\cdot\text{ft}$

Problem 7-13

Determine the internal normal force, shear force, and moment at point C.

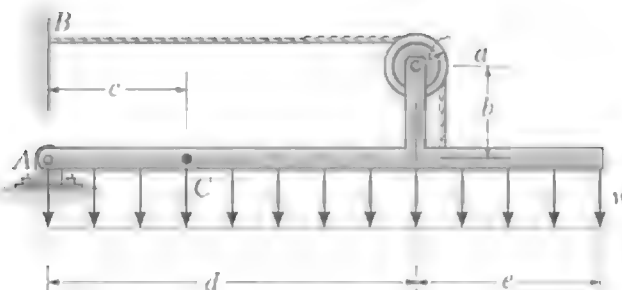
Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$a = 0.5\ \text{ft}$ $d = 8\text{ft}$

$b = 2\ \text{ft}$ $e = 4\ \text{ft}$

$c = 3\ \text{ft}$ $w = 150\ \frac{\text{lb}}{\text{ft}}$

Solution:

Entire beam:

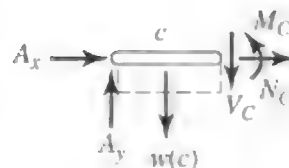
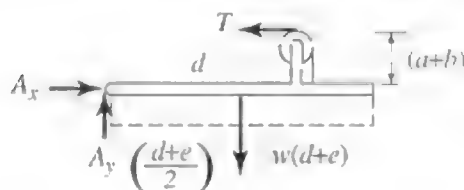
$\Sigma M_A = 0;$ $-w(d+e)\left(\frac{d+e}{2}\right) + T(a+b) = 0$

$T = \frac{w(d+e)^2}{2(a+b)}$ $T = 4.32\,\text{kip}$

$\Sigma F_x = 0;$ $A_x - T = 0$

$A_x = T$ $A_x = 4.32\,\text{kip}$

$\Sigma F_y = 0;$ $A_y - w(d+e) = 0$

$A_y = w(d+e)$ $A_y = 1.80\,\text{kip}$

Segment AC:

$\Sigma F_x = 0;\quad A_x + N_C = 0\qquad N_C = -A_x\qquad N_C = -4.32\text{ kip}$

$\Sigma F_y = 0;\quad A_y - wc - V_C = 0\qquad V_C = A_y - w\,c\qquad V_C = 1.35\text{ kip}$

$\Sigma M_C = 0;\quad -A_y c + wc\left(\frac{c}{2}\right) + M_C = 0\qquad M_C = A_y c - w\left(\frac{c^2}{2}\right)\qquad M_C = 4.72\text{ kip}\cdot\text{ft}$

Problem 7-14

Determine the normal force, shear force, and moment at a section passing through point D of the two-member frame.

Units Used:

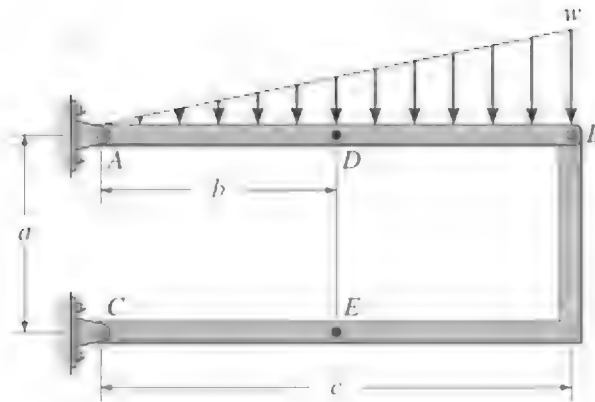
$$\text{kN} = 10^3\text{ N}$$

Given:

$$w = 400\ \frac{\text{N}}{\text{m}}$$

$$a = 2.5\text{ m}$$

$$b = 3\text{ m}$$

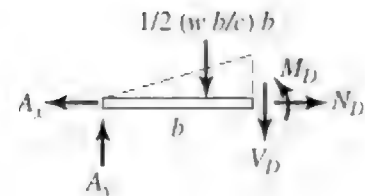
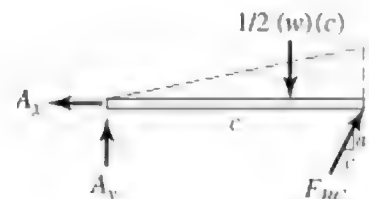
$$c = 6\text{ m}$$

Solution:

$\Sigma M_A = 0;\quad \frac{-1}{2}wc\left(\frac{2}{3}c\right) + F_{BC}\left(\frac{a}{\sqrt{a^2+c^2}}\right)c = 0$

$$F_{BC} = \frac{1}{3}wc\left(\frac{\sqrt{a^2+c^2}}{a}\right)\qquad F_{BC} = 2080\text{ N}$$

$\xrightarrow{+}\Sigma F_x = 0;\quad \left(\frac{c}{\sqrt{a^2+c^2}}\right)F_{BC} - A_x = 0$

$$A_x = \left(\frac{c}{\sqrt{a^2+c^2}}\right)F_{BC}\qquad A_x = 1920\text{ N}$$

$+\uparrow\Sigma F_y = 0;\quad A_y - \frac{1}{2}wc + \left(\frac{a}{\sqrt{a^2+c^2}}\right)F_{BC} = 0$

$$A_y = \frac{1}{2}wc - \left(\frac{a}{\sqrt{a^2 + c^2}}\right) F_{BC} \qquad A_y = 400\ \text{N}$$

$\xrightarrow{+}\ \Sigma F_x = 0;\quad N_D - A_x = 0 \qquad N_D = A_x \qquad N_D = 1.920\,\text{kN}$

$+\uparrow\ \Sigma F_y = 0;\quad A_y - \frac{1}{2}\left(w\frac{b}{c}\right)b - V_D = 0$

$$V_D = A_y - \frac{1}{2}\ w\left(\frac{b^2}{c}\right) \qquad V_D = 100\ \text{N}$$

$\Sigma F_y = 0;\quad -A_y b + \frac{1}{2}\left(w\frac{b}{c}\right)b\left(\frac{b}{3}\right) + M_D = 0$

$$M_D = A_y b - \frac{1}{6}\ w\left(\frac{b^3}{c}\right) \qquad M_D = 900\,\text{N}\,\text{m}$$

Problem 7-15

The beam has weight density γ. Determine the internal normal force, shear force, and moment at point C.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

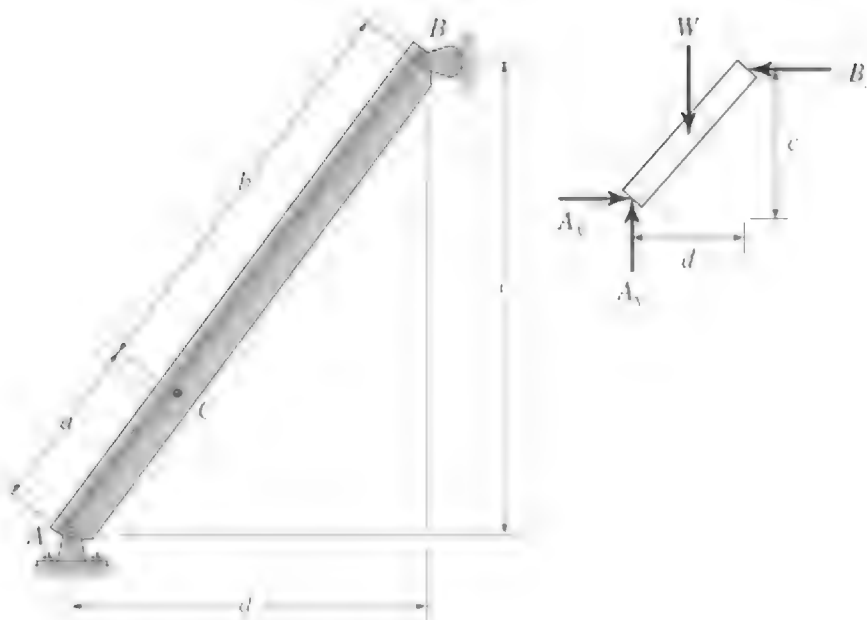
Given:

$\gamma = 280\ \frac{\text{lb}}{\text{ft}}$

$a = 3\ \text{ft}$

$b = 7\ \text{ft}$

$c = 8\ \text{ft}$

$d = 6\ \text{ft}$

Solution:

$\theta = \operatorname{atan}\left(\frac{c}{d}\right)$ $\quad W = \gamma\,(a+b)$

Guesses

$A_x = 1\ \text{lb}$ $\quad A_y = 1\ \text{lb}$ $\quad B_x = 1\ \text{lb}$

$N_C = 1\ \text{lb}$ $\quad V_C = 1\ \text{lb}$ $\quad M_C = 1\ \text{lb ft}$

Given

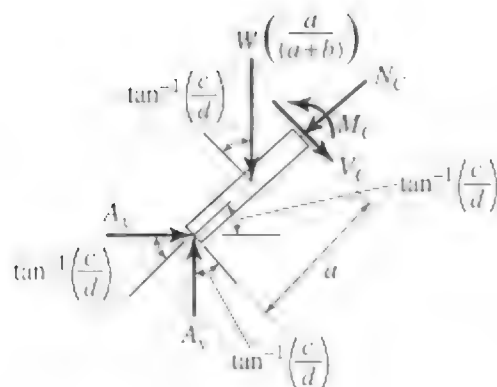
Entire beam:

$$A_x - B_x = 0 \qquad A_y - W = 0 \qquad B_x c - W\left(\frac{d}{2}\right) = 0$$

Bottom Section

$$A_x - N_C\cos(\theta) + V_C\sin(\theta) = 0$$

$$A_y - W\left(\frac{a}{a+b}\right) - N_C\sin(\theta) - V_C\cos(\theta) = 0$$

$$M_C - V_C a - W\left(\frac{a}{a+b}\right)\left(\frac{a}{2}\right)\cos(\theta) = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ N_C \\ V_C \\ M_C \end{pmatrix} = \text{Find}(A_x, A_y, B_x, N_C, V_C, M_C) \qquad \begin{pmatrix} A_x \\ A_y \\ B_x \end{pmatrix} = \begin{pmatrix} 1.05 \\ 2.80 \\ 1.05 \end{pmatrix}\text{kip} \qquad \begin{pmatrix} N_C \\ V_C \end{pmatrix} = \begin{pmatrix} 2.20 \\ 0.34 \end{pmatrix}\text{kip}$$

$$M_C = 1.76\,\text{kip}\cdot\text{ft}$$

Problem 7-16

Determine the internal normal force, shear force, and moment at points C and D of the beam.

Units Used:

$$kip = 10^3\ lb$$

Given:

$w_1 = 60\ \frac{lb}{ft}$ $\quad a = 12\ ft$

$b = 15\ ft$

$w_2 = 40\ \frac{lb}{ft}$ $\quad c = 10\ ft$

$F = 690\ lb$ $\quad d = 5\ ft$

$e = 12$ $\quad f = 5$

Solution: $\theta = \text{atan}\left(\frac{e}{f}\right)$

Guesses

$B_y = 1\ lb$ $\quad N_C = 1\ lb$

$V_C = 1\ lb$ $\quad M_C = 1\ lb\cdot ft$

$N_D = 1\ lb$ $\quad V_D = 1\ lb$

$M_D = 1\ lb\cdot ft$

Given

$$B_y b - F\sin(\theta)(b+c) - w_2 b\left(\frac{b}{2}\right) \dots = 0$$
$$+ \frac{-1}{2}(w_1 - w_2)b\left(\frac{b}{3}\right)$$

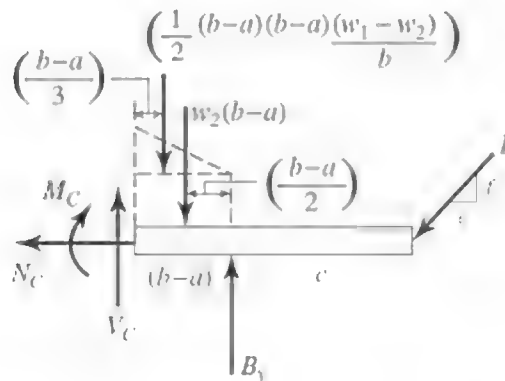
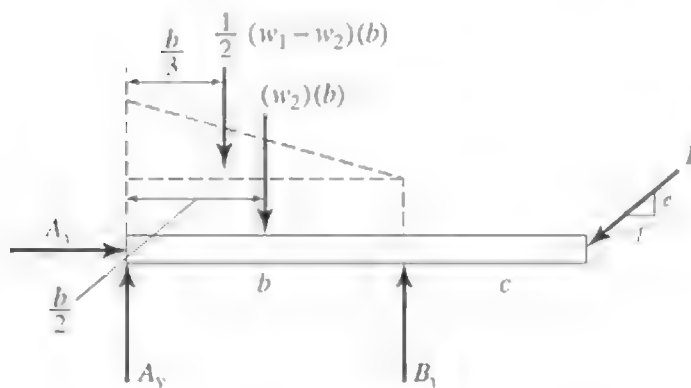
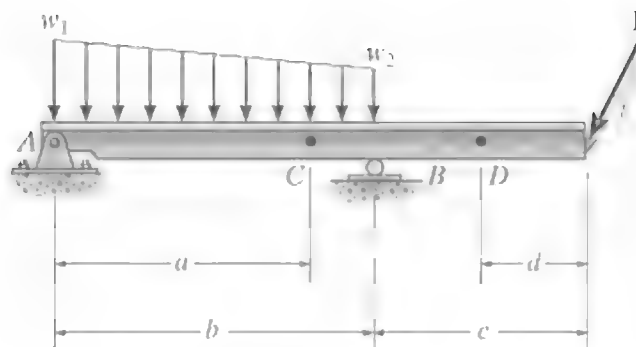
$$-N_C - F\cos(\theta) = 0$$

$$V_C - \frac{1}{2}(w_1 - w_2)\left(\frac{b-a}{b}\right)(b-a) \dots = 0$$
$$+ B_y - w_2(b-a) - F\sin(\theta)$$

$$-N_D - F\cos(\theta) = 0$$

$$V_D - F\sin(\theta) = 0$$

$$-M_D - F\sin(\theta)d = 0$$

$$-M_C - w_2\,(b-a)\left(\frac{b-a}{2}\right) - \frac{1}{2}(w_1 - w_2)\left(\frac{b-a}{b}\right)(b-a)\left(\frac{b-a}{3}\right) \dots = 0$$
$$+ B_y\,(b-a) - F\sin(\theta)(c+b-a)$$

$$\begin{pmatrix} B_y \\ N_C \\ V_C \\ M_C \\ N_D \\ V_D \\ M_D \end{pmatrix} = \text{Find}(B_y, N_C, V_C, M_C, N_D, V_D, M_D)$$

$$\begin{pmatrix} N_C \\ V_C \\ N_D \\ V_D \end{pmatrix} = \begin{pmatrix} -265 \\ -649 \\ -265 \\ 637 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} M_C \\ M_D \end{pmatrix} = \begin{pmatrix} -4.23 \\ -3.18 \end{pmatrix} \text{kip}\cdot\text{ft}$$

Problem 7-17

Determine the normal force, shear force, and moment acting at a section passing through point C.

Units Used: $\text{kip} = 10^3 \text{ lb}$

Given:

$F_1 = 800 \text{ lb}$

$F_2 = 700 \text{ lb}$

$F_3 = 600 \text{ lb}$

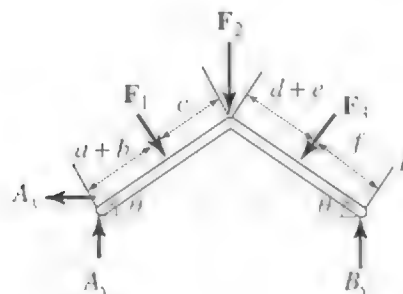
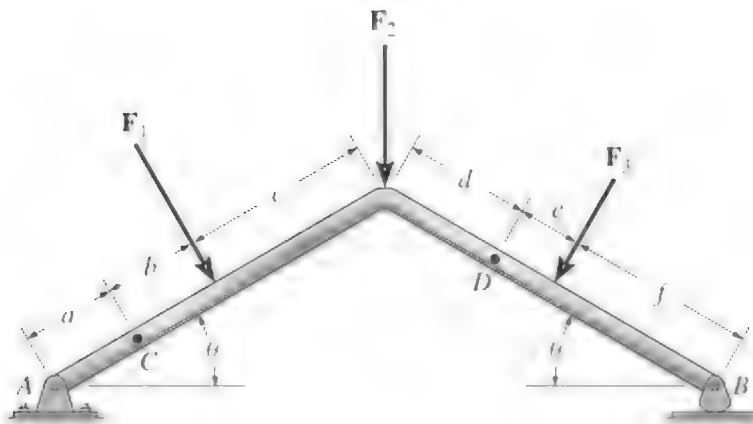
$\theta = 30 \text{ deg}$

$a = 1.5 \text{ ft}$

$b = 1.5 \text{ ft}$

$c = 3 \text{ ft}$

$d = 2 \text{ ft}$

$e = 1 \text{ ft}$

$f = a + b + c - d - e$

Solution:

Guesses $B_y = 1 \text{ lb}$ $A_x = 1 \text{ lb}$ $A_y = 1 \text{ lb}$

$N_C = 1 \text{ lb}$ $V_C = 1 \text{ lb}$ $M_C = 1 \text{ lb}\cdot\text{ft}$

Given

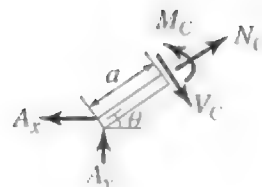
$$-A_x + V_C\sin(\theta) + N_C\cos(\theta) = 0$$

$$A_y - V_C\cos(\theta) + N_C\sin(\theta) = 0$$

$$M_C - A_x\,(a)\sin(\theta) - A_y\,(a)\cos(\theta) = 0$$

$$-A_x + F_1\sin(\theta) - F_3\sin(\theta) = 0$$

$$A_y + B_y - F_2 - F_1\cos(\theta) - F_3\cos(\theta) = 0$$

$$-F_1(a+b) - F_2(a+b+c)\cos(\theta) - F_3\cos(\theta)(a+b+c+d+e)\cos(\theta)\ ... = 0$$
$$+ F_3\sin(\theta)f\sin(\theta) + B_y2(a+b+c)\cos(\theta)$$

$$\begin{pmatrix} A_x \\ A_y \\ B_y \\ N_C \\ V_C \\ M_C \end{pmatrix} = \text{Find}(A_x, A_y, B_y, N_C, V_C, M_C)$$

$$\begin{pmatrix} A_x \\ A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 100 \\ 985 \\ 927 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} N_C \\ V_C \end{pmatrix} = \begin{pmatrix} -406 \\ 903 \end{pmatrix} \text{lb}$$

$$M_C = 1.355\ \text{kip}\cdot\text{ft}$$

Problem 7-18

Determine the normal force, shear force, and moment acting at a section passing through point *D*.

Units Used: kip $= 10^3$ lb

Given:

$F_1 = 800$ lb

$F_2 = 700$ lb

$F_3 = 600$ lb

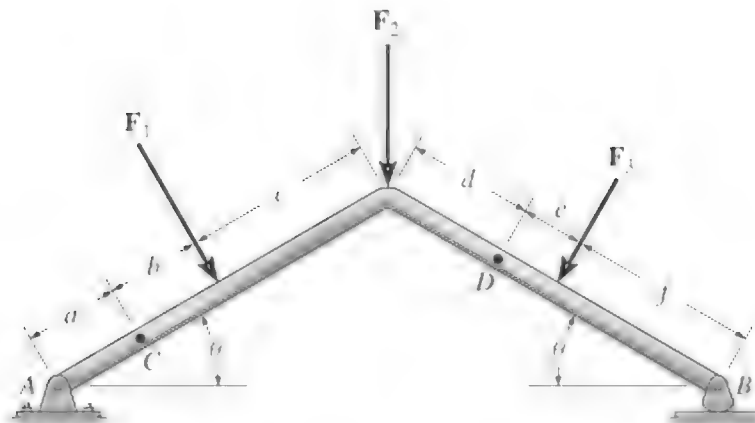
$\theta = 30$ deg

$a = 1.5$ ft

$b = 1.5$ ft

$c = 3$ ft

$d = 2$ ft

$e = 1$ ft

$f = a + b + c - d - e$

Solution:

Guesses $B_y = 1$ lb $A_x = 1$ lb $A_y = 1$ lb

$N_D = 1$ lb $V_D = 1$ lb $M_D = 1$ lb·ft

Given

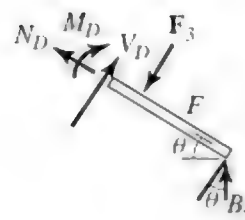
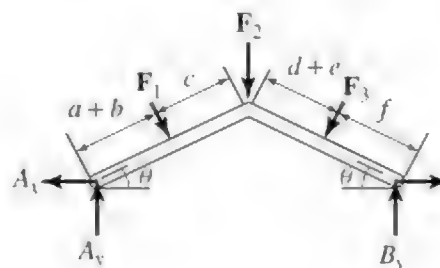
$$V_D \sin(\theta) - N_D \cos(\theta) - F_3 \sin(\theta) = 0$$

$$B_y + V_D \cos(\theta) + N_D \sin(\theta) - F_3 \cos(\theta) = 0$$

$$-M_D - F_3 e + B_y (e + f) \cos(\theta) = 0$$

$$A_y + B_y - F_2 - F_1 \cos(\theta) - F_3 \cos(\theta) = 0$$

$$-A_x + F_1 \sin(\theta) - F_3 \sin(\theta) = 0$$

$$-F_1 (a + b) - F_2 (a + b + c) \cos(\theta) - F_3 \cos(\theta)(a + b + c + d + e) \cos(\theta) \ldots = 0$$
$$+ F_3 \sin(\theta) f \sin(\theta) + B_y 2(a + b + c) \cos(\theta)$$

$$\begin{pmatrix} A_x \\ A_y \\ B_y \\ N_D \\ V_D \\ M_D \end{pmatrix} = \text{Find}(A_x, A_y, B_y, N_D, V_D, M_D)$$

$$\begin{pmatrix} A_x \\ A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 100 \\ 985 \\ 927 \end{pmatrix} \text{lb}$$

$$\begin{pmatrix} N_D \\ V_D \end{pmatrix} = \begin{pmatrix} -464 \\ -203 \end{pmatrix} \text{lb}$$

$$M_D = 2.61 \text{ kip·ft}$$

Problem 7-19

Determine the normal force, shear force, and moment at a section passing through point C.

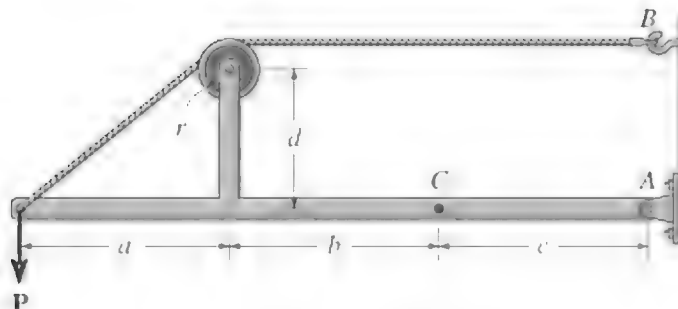
Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$P = 8\ \text{kN}$ $c = 0.75\ \text{m}$

$a = 0.75\text{m}$ $d = 0.5\ \text{m}$

$b = 0.75\text{m}$ $r = 0.1\ \text{m}$

Solution:

$\Sigma M_A = 0;$ $-T(d + r) + P(a + b + c) = 0$

$T = P\left(\frac{a+b+c}{d+r}\right)$ $T = 30\,\text{kN}$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0;$ $A_x = T$ $A_x = 30\,\text{kN}$

$+\uparrow\Sigma F_y = 0;$ $A_y = P$ $A_y = 8\,\text{kN}$

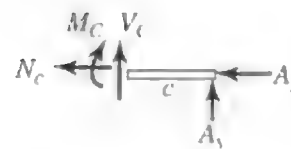
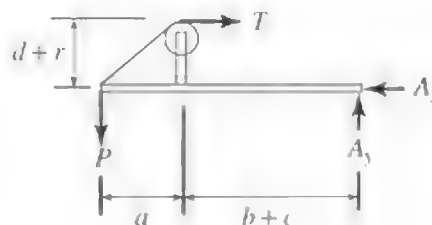
$\overset{+}{\rightarrow}\ \Sigma F_x = 0;$ $-N_C - T = 0$

$N_C = -T$ $N_C = -30\,\text{kN}$

$+\uparrow\Sigma F_y = 0;$ $V_C + P = 0$

$V_C = -P$ $V_C = -8\,\text{kN}$

$\Sigma M_C = 0;$ $-M_C + P\,c = 0$

$M_C = P\,c$ $M_C = 6\,\text{kN}\cdot\text{m}$

Problem 7-20

The cable will fail when subjected to a tension T_{max}. Determine the largest vertical load P the frame will support and calculate the internal normal force, shear force, and moment at a section passing through point C for this loading.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$T_{max} = 2 \text{ kN}$

$a = 0.75 \text{ m}$

$b = 0.75 \text{ m}$

$c = 0.75 \text{ m}$

$d = 0.5 \text{ m}$

$r = 0.1 \text{ m}$

Solution:

$\Sigma M_A = 0;$ $-T_{max}(r+d) + P(a+b+c) = 0$

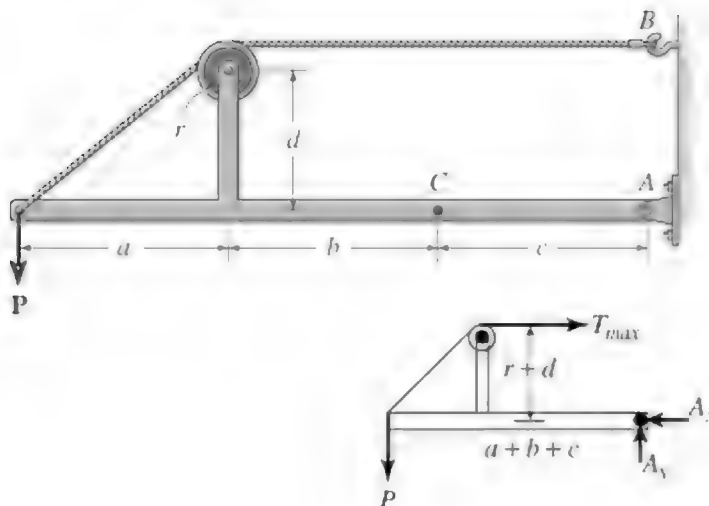
$P = T_{max}\left(\frac{d+r}{a+b+c}\right)$ $P = 0.533 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $T_{max} - A_x = 0$ $A_x = T_{max}$ $A_x = 2 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $A_y - P = 0$ $A_y = P$ $A_y = 0.533 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $-N_C - A_x = 0$ $N_C = -A_x$ $N_C = -2 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $-V_C + A_y = 0$ $V_C = A_y$ $V_C = 0.533 \text{ kN}$

$\Sigma M_C = 0;$ $-M_C + A_y c = 0$ $M_C = A_y c$ $M_C = 0.400 \text{ kN}\cdot\text{m}$

Problem 7-21

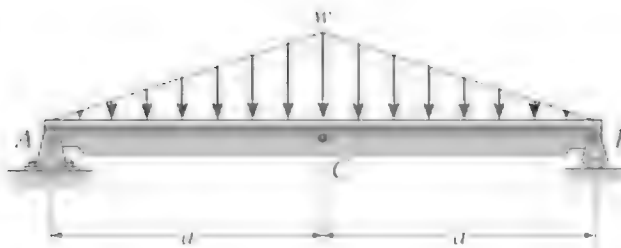
Determine the internal shear force and moment acting at point C of the beam.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

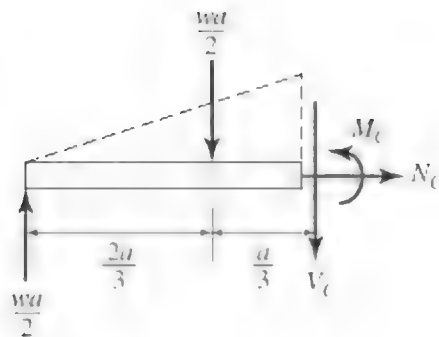
$w = 2 \frac{\text{kip}}{\text{ft}}$

$a = 9 \text{ ft}$

Solution:

$\Sigma F_x = 0;\quad N_C = 0 \qquad N_C = 0$

$\Sigma F_y = 0;\quad \frac{w a}{2} - \frac{w a}{2} - V_C = 0 \qquad V_C = 0$

$\Sigma M_C = 0;\quad M_C - \left(\frac{w a}{2}\right) a + \left(\frac{w a}{2}\right)\left(\frac{a}{3}\right) = 0$

$M_C = \frac{w a^2}{3} \qquad M_C = 54.00\ \text{kip}\cdot\text{ft}$

Problem 7-22

Determine the internal shear force and moment acting at point D of the beam.

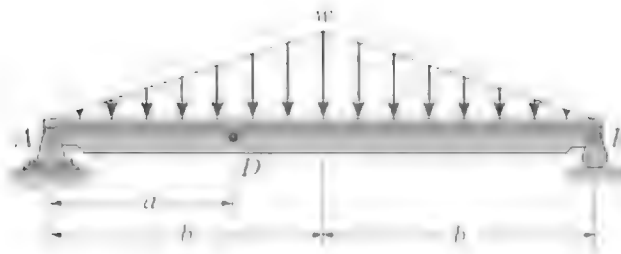
Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$w = 2\ \frac{\text{kip}}{\text{ft}}$

$a = 6\ \text{ft}$

$b = 9\ \text{ft}$

Solution:

$\Sigma F_x = 0;\quad N_D = 0$

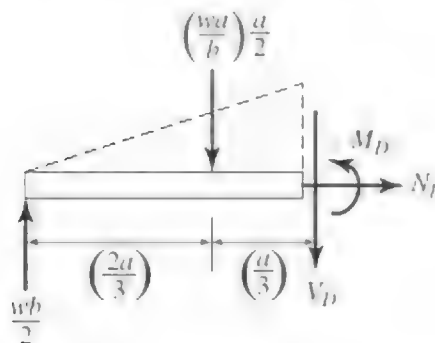
$\Sigma F_y = 0;\quad \frac{w b}{2} - w\left(\frac{a}{b}\right)\left(\frac{a}{2}\right) - V_D = 0$

$V_D = \frac{w b}{2} - w\left(\frac{a}{b}\right)\left(\frac{a}{2}\right)$

$V_D = 5.00\ \text{kip}$

$\Sigma M_D = 0;\quad M_D - \left(\frac{w b}{2}\right) a + \left(\frac{w a}{b}\right)\left(\frac{a}{2}\right)\left(\frac{a}{3}\right) = 0$

$M_D = \left(\frac{w b}{2}\right) a - \left(\frac{w a^3}{6b}\right)$

$M_D = 46.00\ \text{kip}\cdot\text{ft}$

Problem 7-23

The shaft is supported by a journal bearing at A and a thrust bearing at B. Determine the internal normal force, shear force, and moment at (a) point C, which is just to the right of the bearing at A, and (b) point D, which is just to the left of the $\mathbf{F_2}$ force.

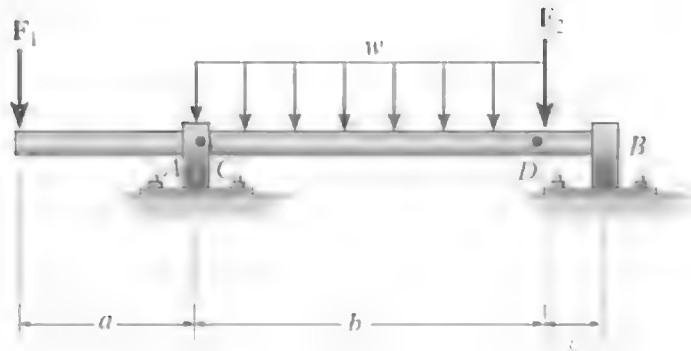
Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$F_1 = 2500 \text{ lb}$ $\quad a = 6 \text{ ft}$

$F_2 = 3000 \text{ lb}$ $\quad b = 12 \text{ ft}$

$w = 75 \dfrac{\text{lb}}{\text{ft}}$ $\quad c = 2\text{ft}$

Solution:

$\Sigma M_B = 0;$ $\quad -A_y(b+c) + F_1(a+b+c) + wb\left(\dfrac{b}{2}+c\right) + F_2 c = 0$

$$A_y = \frac{1}{2}\,\frac{2F_1(a+b+c) + w\left(b^2 + 2cb\right) + 2F_2 c}{b+c} \qquad A_y = 4514\,\text{lb}$$

$\Sigma F_x = 0;$ $\quad B_x = 0 \text{ lb}$

$\Sigma F_y = 0;$ $\quad A_y - F_1 - wb - F_2 + B_y = 0$

$B_y = -A_y + F_1 + wb + F_2 \qquad B_y = 1886\,\text{lb}$

Segment AC:

$\Sigma M_C = 0;$ $\quad F_1 a + M_C = 0$

$M_C = -F_1\, a \qquad M_C = -15\,\text{kip}\cdot\text{ft}$

$\Sigma F_x = 0;$ $\quad N_C = 0 \qquad N_C = 0$

$\Sigma F_y = 0;$ $\quad -F_1 + A_y - V_C = 0$

$V_C = A_y - F_1 \qquad V_C = 2.01\,\text{kip}$

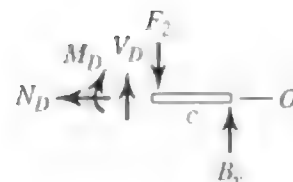
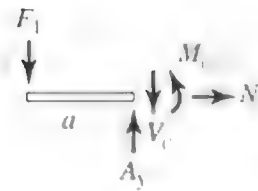
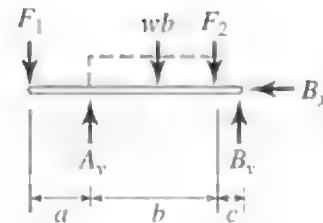
Segment BD:

$\Sigma M_D = 0;$ $\quad -M_D + B_y\, c = 0$

$M_D = B_y\, c \qquad M_D = 3.77\,\text{kip}\cdot\text{ft}$

$\Sigma F_x = 0;$ $\quad N_D = 0 \qquad N_D = 0$

$\Sigma F_y = 0;$ $\quad V_D - F_2 + B_y = 0$

$V_D = F_2 - B_y \qquad V_D = 1.11\,\text{kip}$

Problem 7-24

The jack AB is used to straighten the bent beam DE using the arrangement shown. If the axial compressive force in the jack is P, determine the internal moment developed at point C of the top beam. Neglect the weight of the beams.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$P = 5000\ \text{lb}$

$a = 2\ \text{ft}$

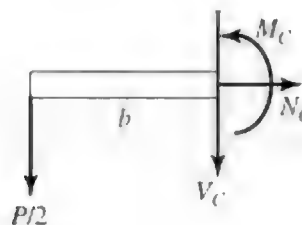
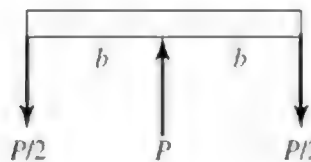
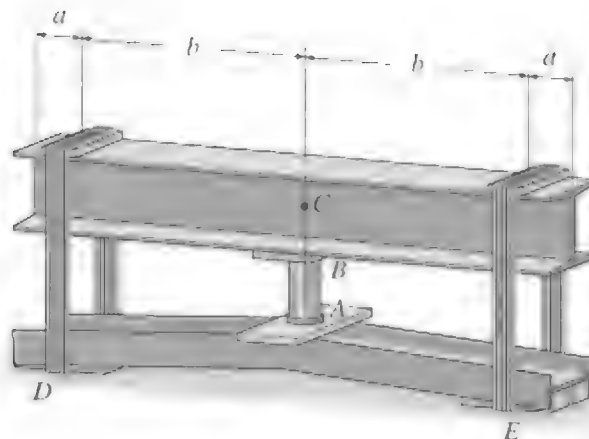
$b = 10\ \text{ft}$

Solution:

Segment:

$\Sigma M_C = 0;\quad M_C + \left(\frac{P}{2}\right)b = 0$

$M_C = -\frac{P}{2}b$

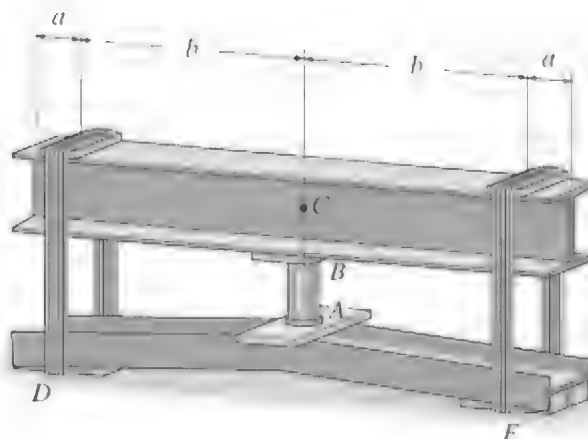
$M_C = -25.00\ \text{kip}\cdot\text{ft}$

Problem 7-25

The jack AB is used to straighten the bent beam DE using the arrangement shown. If the axial compressive force in the jack is P, determine the internal moment developed at point C of the top beam. Assume that each beam has a uniform weight density γ.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$P = 5000\ \text{lb}$

$$\gamma = 150\ \frac{\text{lb}}{\text{ft}}$$

$$a = 2\ \text{ft}$$

$$b = 10\ \text{ft}$$

Solution:

Beam:

$+\uparrow \Sigma F_y = 0;\quad P - 2\gamma(a+b) - 2R = 0$

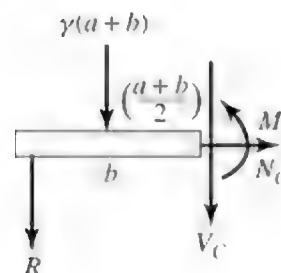
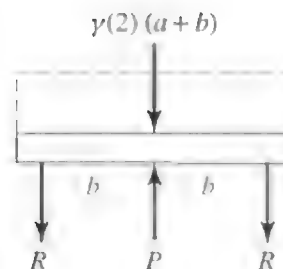
$$R = \frac{P}{2} - \gamma(a+b)$$

$$R = 700\,\text{lb}$$

Segment:

$\Sigma M_C = 0;\quad M_C + R\,b + \gamma\,(a+b)\left(\frac{a+b}{2}\right) = 0$

$$M_C = -R\,b - \gamma\,\frac{(a+b)^2}{2}$$

$$M_C = -17.8\,\text{kip}\cdot\text{ft}$$

Problem 7-26

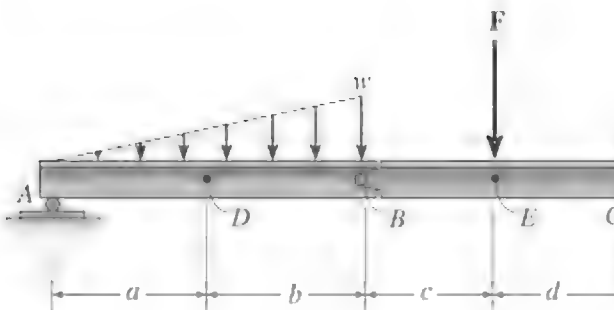
Determine the normal force, shear force, and moment in the beam at sections passing through points D and E. Point E is just to the right of the F load.

Units Used:

$$\text{kip} = 10^3\ \text{lb}$$

Given:

$$w = 1.5\ \frac{\text{kip}}{\text{ft}} \qquad F = 3\ \text{kip}$$

$$a = 6\ \text{ft} \qquad b = 6\ \text{ft} \qquad c = 4\ \text{ft} \qquad d = 4\ \text{ft}$$

Solution:

$\Sigma M_B = 0;$ $\frac{1}{2}w(a+b)\left(\frac{a+b}{3}\right) - A_y(a+b) = 0$

$$A_y = \frac{\frac{1}{2}w(a+b)\left(\frac{a+b}{3}\right)}{a+b}$$

$A_y = 3\,\text{kip}$

$\xrightarrow{+} \Sigma F_x = 0;$ $B_x = 0$

$+\uparrow \Sigma F_y = 0;$ $B_y + A_y - \frac{1}{2}w(a+b) = 0$

$$B_y = -A_y + \frac{1}{2}w(a+b)$$

$B_y = 6\,\text{kip}$

$\xrightarrow{+} \Sigma F_x = 0;$ $N_D = 0$

$+\uparrow \Sigma F_y = 0;$ $A_y - \frac{1}{2}\left(\frac{aw}{a+b}\right)a - V_D = 0$

$$V_D = A_y - \frac{1}{2}\left(\frac{aw}{a+b}\right)a$$

$V_D = 0.75\,\text{kip}$

$\Sigma M_D = 0;$ $M_D + \frac{1}{2}\left(\frac{aw}{a+b}\right)a\left(\frac{a}{3}\right) - A_y a = 0$

$$M_D = \frac{-1}{2}\left(\frac{aw}{a+b}\right)a\left(\frac{a}{3}\right) + A_y a$$

$M_D = 13.5\,\text{kip}\cdot\text{ft}$

$\xrightarrow{+} \Sigma F_x = 0;$ $N_E = 0$

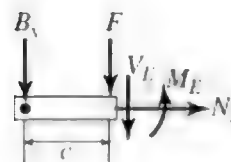
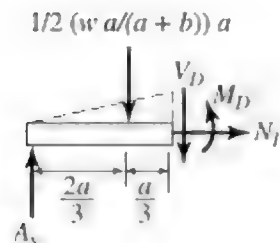
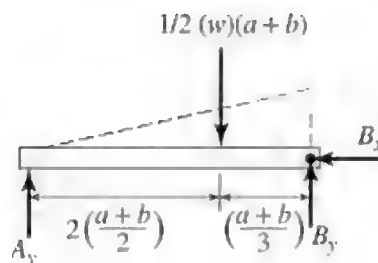
$+\uparrow \Sigma F_y = 0;$ $-V_E - F - B_y = 0$

$$V_E = -F - B_y$$

$V_E = -9\,\text{kip}$

$\Sigma M_E = 0;$ $M_E + B_y c = 0$

$$M_E = -B_y c$$

$M_E = -24.0\,\text{kip}\cdot\text{ft}$

Problem 7-27

Determine the normal force, shear force, and moment at a section passing through point D of the two-member frame.

Units Used:

$$kN = 10^3\ N$$

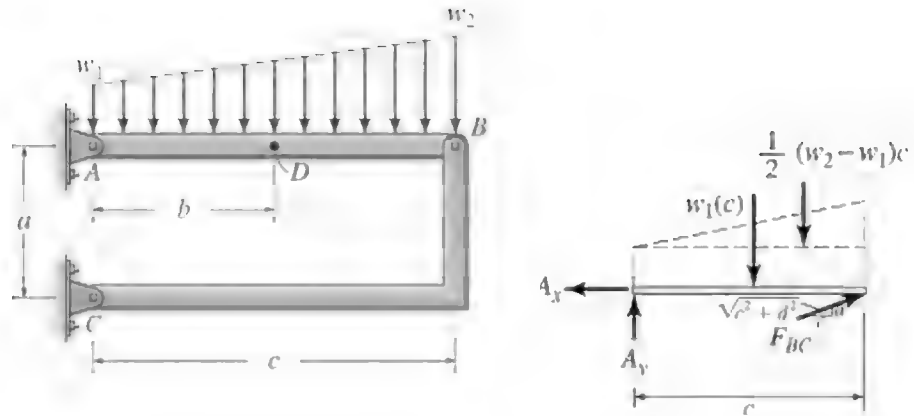
Given:

$$w_1 = 200\ \frac{N}{m}$$

$$w_2 = 400\ \frac{N}{m}$$

$$a = 2.5\ m$$

$$b = 3\ m$$

$$c = 6\ m$$

Solution:

$\Sigma M_A = 0;$

$$-w_1 c\left(\frac{c}{2}\right) - \frac{1}{2}(w_2 - w_1)c\left(\frac{2c}{3}\right) + \frac{a}{\sqrt{a^2 + c^2}}(F_{BC}\, c) = 0$$

$$F_{BC} = \left[w_1 \frac{c^2}{2} + (w_2 - w_1)\frac{c^2}{3}\right]\frac{\sqrt{a^2 + c^2}}{a\, c} \qquad F_{BC} = 2600\ N$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$

$$A_x = \left(\frac{c}{\sqrt{a^2 + c^2}}\right)F_{BC} \qquad A_x = 2400\ N$$

$+\uparrow \Sigma F_y = 0;$

$$A_y - w_1 c - \frac{1}{2}(w_2 - w_1)c + \left(\frac{a}{\sqrt{a^2 + c^2}}\right)F_{BC} = 0$$

$$A_y = w_1 c + \frac{1}{2}(w_2 - w_1)c - \left(\frac{a}{\sqrt{a^2 + c^2}}\right)F_{BC} \qquad A_y = 800\ N$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$

$$-A_x + N_D = 0 \qquad N_D = A_x \qquad N_D = 2.40\ kN$$

$+\uparrow \Sigma F_y = 0;$

$$A_y - w_1 b - \frac{1}{2}(w_2 - w_1)\left(\frac{b}{c}\right)b - V_D = 0$$

$$V_D = A_y - w_1 b - \frac{1}{2}(w_2 - w_1)\left(\frac{b^2}{c}\right) \qquad V_D = 50\ N$$

$\Sigma M_D = 0;$

$$-A_y b + w_1\, b\left(\frac{b}{2}\right) + \frac{1}{2}(w_2 - w_1)\left(\frac{b}{c}\right)b\left(\frac{b}{3}\right) + M_D = 0$$

$$M_D = A_y(b) - w_1\left(\frac{b^2}{2}\right) - \frac{1}{2}(w_2 - w_1)\left(\frac{b^3}{3c}\right) \qquad M_D = 1.35\ kN\cdot m$$

Problem 7-28

Determine the normal force, shear force, and moment at sections passing through points E and F. Member BC is pinned at B and there is a smooth slot in it at C. The pin at C is fixed to member CD.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

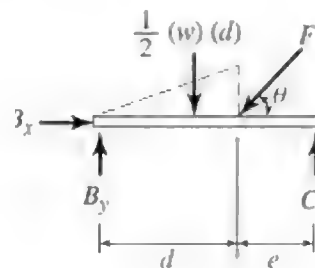
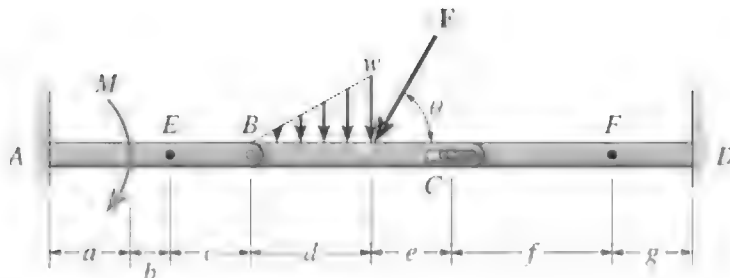
$M = 350 \text{ lb}\cdot\text{ft}$

$w = 80 \dfrac{\text{lb}}{\text{ft}}$ $\quad c = 2 \text{ ft}$

$F = 500 \text{ lb}$ $\quad d = 3 \text{ ft}$

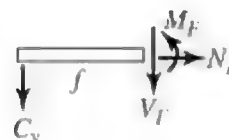
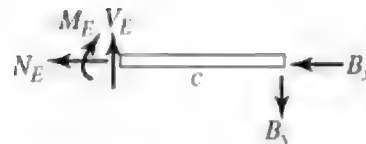
$\theta = 60 \text{ deg}$ $\quad e = 2 \text{ ft}$

$a = 2 \text{ ft}$ $\quad f = 4 \text{ ft}$

$b = 1 \text{ ft}$ $\quad g = 2 \text{ ft}$

Solution:

$\Sigma M_B = 0;$

$$\frac{-1}{2} w d \left(\frac{2d}{3}\right) - F \sin(\theta) d + C_y(d+e) = 0$$

$$C_y = \frac{\left(\frac{w d^2}{3}\right) + F \sin(\theta) d}{d+e} \qquad C_y = 307.8 \text{ lb}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x - F\cos(\theta) = 0 \qquad B_x = F\cos(\theta) \qquad B_x = 250 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad B_y - \frac{1}{2} w d - F\sin(\theta) + C_y = 0$

$$B_y = \frac{1}{2} w d + F\sin(\theta) - C_y \qquad B_y = 245.2 \text{ lb}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N_E - B_x = 0 \qquad N_E = -B_x \qquad N_E = -250 \text{ lb}$

$+\uparrow\Sigma F_y = 0;$	$V_E - B_y = 0$	$V_E = B_y$	$V_E = 245\text{ lb}$
$\Sigma M_E = 0$	$-M_E - B_y c = 0$	$M_E = -B_y c$	$M_E = -490\text{ lb}\cdot\text{ft}$
$\xrightarrow{+}\Sigma F_x = 0;$	$N_F = 0$	$N_F = 0\text{lb}$	$N_F = 0.00\text{ lb}$
$+\uparrow\Sigma F_y = 0;$	$-C_y - V_F = 0$	$V_F = -C_y$	$V_F = -308\text{ lb}$
$\Sigma M_F = 0;$	$C_y(f) + M_F = 0$	$M_F = -f C_y$	$M_F = -1.23\text{ kip}\cdot\text{ft}$

Problem 7-29

The bolt shank is subjected to a tension F. Determine the internal normal force, shear force, and moment at point C.

Given:

$F = 80\text{ lb}$

$a = 6\text{ in}$

Solution:

$\Sigma F_x = 0;$ $N_C + F = 0$

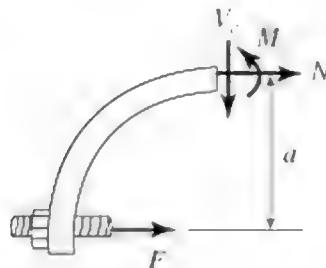
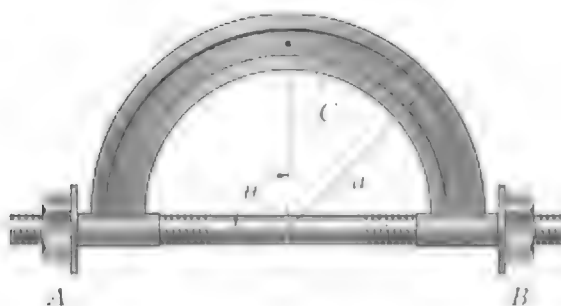
$N_C = -F$

$N_C = -80.00\text{ lb}$

$\Sigma F_y = 0;$ $V_C = 0$

$\Sigma M_C = 0;$ $M_C + F a = 0$

$M_C = -F a$

$M_C = -480.00\text{ lb}\cdot\text{in}$

Problem 7-30

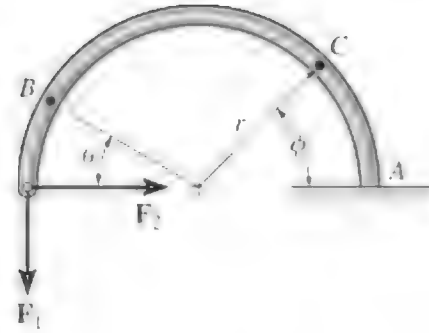
Determine the normal force, shear force, and moment acting at sections passing through points B and C on the curved rod.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$F_1 = 300\ \text{lb}$ $\quad \theta = 30\ \text{deg} \quad r = 2\ \text{ft}$

$F_2 = 400\ \text{lb}$ $\quad \phi = 45\ \text{deg}$

Solution:

$\nearrow^+ \Sigma F_x = 0;$ $\quad F_2 \sin(\theta) - F_1 \cos(\theta) + N_B = 0$

$N_B = -F_2 \sin(\theta) + F_1 \cos(\theta)$

$N_B = 59.8\ \text{lb}$

$^+\searrow \Sigma F_y = 0;$ $\quad V_B + F_2 \cos(\theta) + F_1 \sin(\theta) = 0$

$V_B = -F_2 \cos(\theta) - F_1 \sin(\theta)$

$V_B = -496\ \text{lb}$

$\Sigma M_B = 0;$ $\quad M_B + F_2 r \sin(\theta) + F_1 (r - r\cos(\theta)) = 0$

$M_B = -F_2 r \sin(\theta) - F_1 r(1 - \cos(\theta))$

$M_B = -480\ \text{lb}\cdot\text{ft}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_2 - A_x = 0 \quad A_x = F_2 \quad A_x = 400\ \text{lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - F_1 = 0 \quad A_y = F_1 \quad A_y = 300\ \text{lb}$

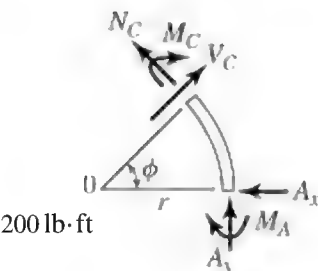
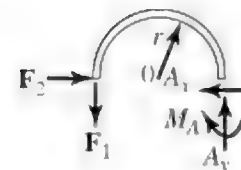
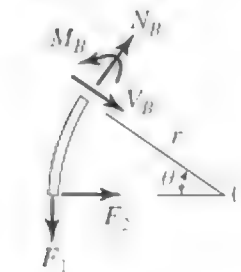
$\Sigma M_A = 0;$ $\quad -M_A + F_1 2r = 0 \quad M_A = 2 F_1 r \quad M_A = 1200\ \text{lb}\cdot\text{ft}$

$^+\nwarrow \Sigma F_x = 0;$ $\quad N_C + A_x \sin(\phi) + A_y \cos(\phi) = 0$

$N_C = -A_x \sin(\phi) - A_y \cos(\phi)$ $\quad N_C = -495\ \text{lb}$

$\nearrow^+ \Sigma F_y = 0;$ $\quad V_C - A_x \cos(\phi) + A_y \sin(\phi) = 0$

$V_C = A_x \cos(\phi) - A_y \sin(\phi)$ $\quad V_C = 70.7\ \text{lb}$

$\Sigma M_C = 0;$ $\quad -M_C - M_A - A_x r \sin(\phi) + A_y (r - r\cos(\phi)) = 0$

$$M_C = -M_A - A_x r \sin(\phi) + A_y r(1 - \cos(\phi)) \qquad M_C = -1.59 \text{ kip}\cdot\text{ft}$$

Problem 7-31

The cantilevered rack is used to support each end of a smooth pipe that has total weight W. Determine the normal force, shear force, and moment that act in the arm at its fixed support A along a vertical section.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$W = 300 \text{ lb}$

$r = 6 \text{ in}$

$\theta = 30 \text{ deg}$

Solution:

Pipe:

$+\uparrow \Sigma F_y = 0; \qquad N_B \cos(\theta) - \dfrac{W}{2} = 0$

$$N_B = \frac{1}{2}\left(\frac{W}{\cos(\theta)}\right) \qquad N_B = 173.205 \text{ lb}$$

Rack:

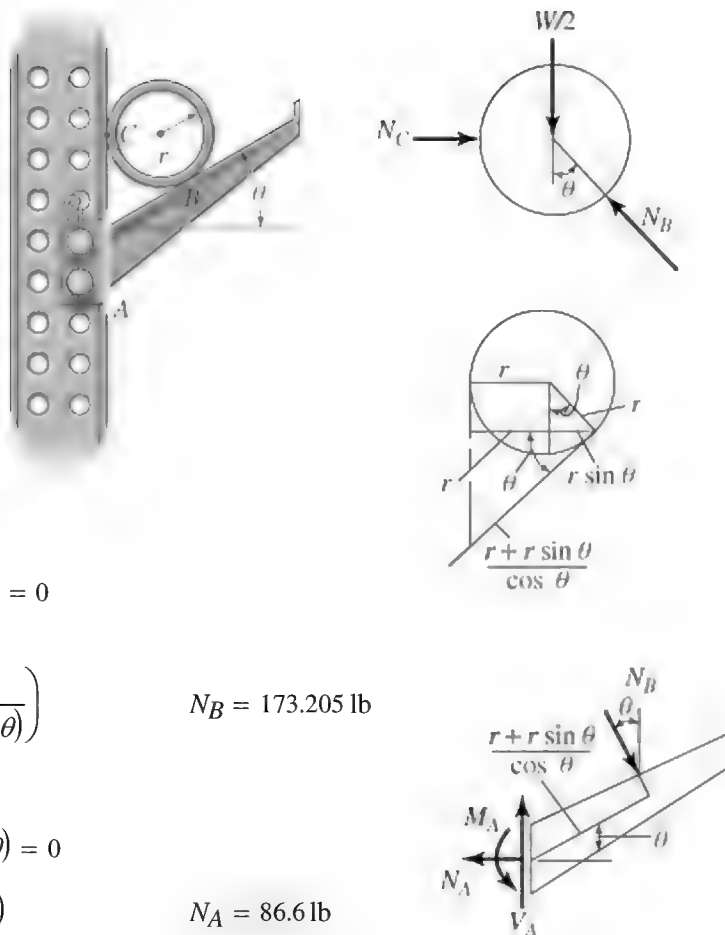
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N_A + N_B \sin(\theta) = 0$

$$N_A = N_B \sin(\theta) \qquad N_A = 86.6 \text{ lb}$$

$+\uparrow \Sigma F_y = 0; \qquad V_A - N_B \cos(\theta) = 0$

$$V_A = N_B \cos(\theta) \qquad V_A = 150 \text{ lb}$$

$\Sigma M_A = 0; \qquad M_A - N_B\left(\dfrac{r + r\sin(\theta)}{\cos(\theta)}\right) = 0$

$$M_A = N_B\left(\frac{r + r\sin(\theta)}{\cos(\theta)}\right) \qquad M_A = 1.800 \text{ kip}\cdot\text{in}$$

Problem 7-32

Determine the normal force, shear force, and moment at a section passing through point D of the two-member frame.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$w = 0.75\ \frac{\text{kN}}{\text{m}}$$

$$F = 4\ \text{kN}$$

$a = 1.5\ \text{m}$ $\quad d = 1.5\ \text{m}$

$b = 1.5\ \text{m}$ $\quad e = 3$

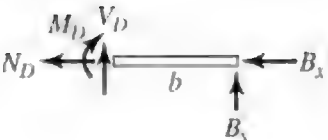
$c = 2.5\ \text{m}$ $\quad f = 4$

Solution:

$\Sigma M_C = 0;$

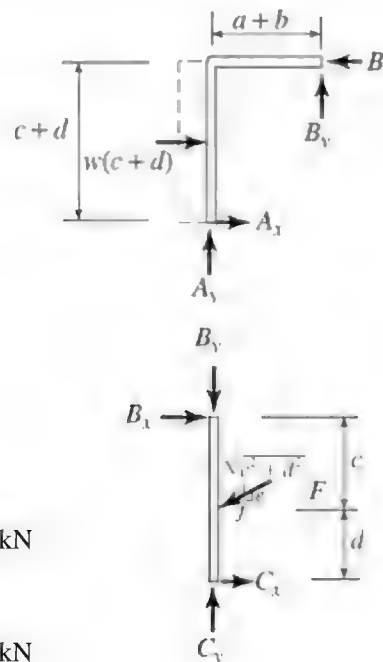
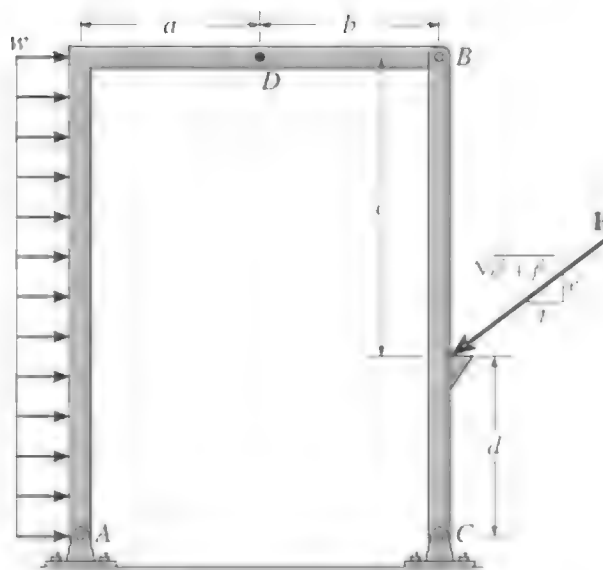
$$-B_x(c+d) + \left(\frac{f}{\sqrt{e^2+f^2}}\right) F\, d = 0$$

$$B_x = \frac{f\, d\, F}{\sqrt{e^2+f^2}\,(c+d)} \qquad B_x = 1.2\,\text{kN}$$

$\Sigma M_A = 0;$

$$-w(c+d)\left(\frac{c+d}{2}\right) + B_y(a+b) + B_x(c+d) = 0$$

$$B_y = \frac{w\left[\frac{(c+d)^2}{2}\right] - B_x(c+d)}{a+b} \qquad B_y = 0.40\,\text{kN}$$

$\overset{+}{\rightarrow}\ \Sigma F_x = 0; \quad -N_D - B_x = 0 \qquad N_D = -B_x \qquad N_D = -1.2\,\text{kN}$

$+\uparrow \Sigma F_y = 0; \quad V_D + B_y = 0 \qquad V_D = -B_y \qquad V_D = -0.4\,\text{kN}$

$\Sigma M_D = 0;$ $-M_D + B_y\, b = 0$ $M_D = B_y\, b$ $M_D = 0.6\,\text{kN}\cdot\text{m}$

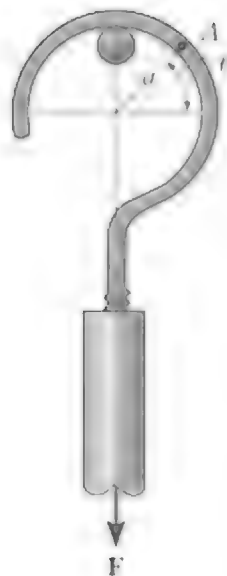
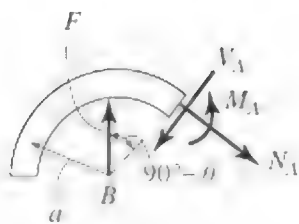
Problem 7-33

Determine the internal normal force, shear force, and moment acting at point A of the smooth hook.

Given:

$\theta = 45\ \text{deg}$

$a = 2\ \text{in}$

$F = 20\ \text{lb}$

Solution:

$\Sigma F_x = 0;$ $N_A - F\cos(\theta) = 0$

$N_A = F\cos(\theta)$ $N_A = 14.1\ \text{lb}$

$\Sigma F_y = 0;$ $V_A - F\sin(\theta) = 0$

$V_A = F\sin(\theta)$ $V_A = 14.1\ \text{lb}$

$\Sigma M_B = 0;$ $M_A - N_A\, a = 0$

$M_A = N_A\, a$ $M_A = 28.3\ \text{lb}\cdot\text{in}$

Problem 7-34

Determine the internal normal force, shear force, and moment acting at points B and C on the curved rod.

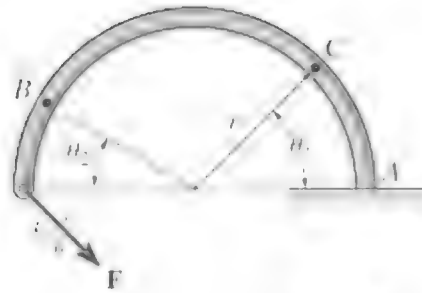
Units Used:

$$\text{kip} = 10^3\ \text{lb}$$

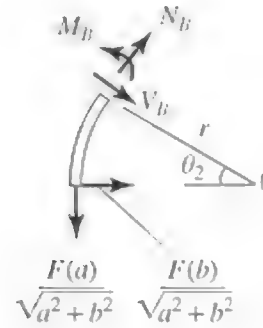
Given:

$F = 500$ lb $\qquad \theta_2 = 30$ deg

$r = 2$ ft $\qquad a = 3$

$\theta_1 = 45$ deg $\qquad b = 4$

Solution:

$\Sigma F_N = 0;$
$$\left(\frac{F\,b}{\sqrt{a^2+b^2}}\right)\sin(\theta_2) - \left(\frac{F\,a}{\sqrt{a^2+b^2}}\right)\cos(\theta_2) + N_B = 0$$

$$N_B = F\left[\frac{(a)\cos(\theta_2) - b\,\sin(\theta_2)}{\sqrt{a^2+b^2}}\right] \qquad N_B = 59.8\ \text{lb}$$

$\Sigma F_V = 0;$
$$V_B + \left(\frac{F\,b}{\sqrt{a^2+b^2}}\right)\cos(\theta_2) + \left(\frac{F\,a}{\sqrt{a^2+b^2}}\right)\sin(\theta_2) = 0$$

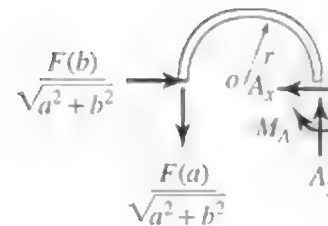
$$V_B = -F\left[\frac{b\cos(\theta_2) + (a)\sin(\theta_2)}{\sqrt{a^2+b^2}}\right] \qquad V_B = -496\ \text{lb}$$

$\Sigma M_B = 0;$
$$M_B + \left(\frac{F\,b}{\sqrt{a^2+b^2}}\right)r\sin(\theta_2) + F\left(\frac{a}{\sqrt{a^2+b^2}}\right)(r - r\cos(\theta_2)) = 0$$

$$M_B = F\,r\left[\frac{-b\sin(\theta_2) - a + (a)\cos(\theta_2)}{\sqrt{a^2+b^2}}\right] \qquad M_B = -480\ \text{lb}\cdot\text{ft}$$

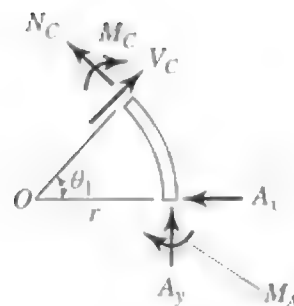
Also,

$\Sigma F_x = 0;$
$$-A_x + \frac{F\,b}{\sqrt{a^2+b^2}} = 0$$

$$A_x = F\left(\frac{b}{\sqrt{a^2+b^2}}\right) \qquad A_x = 400.00\ \text{lb}$$

$\Sigma F_y = 0;$
$$A_y - \frac{F\,a}{\sqrt{a^2+b^2}} = 0$$

$$A_y = F\left(\frac{a}{\sqrt{a^2+b^2}}\right) \qquad A_y = 300.00\,\text{lb}$$

$\Sigma M_A = 0;$ $$-M_A + \left(\frac{F\,a}{\sqrt{a^2+b^2}}\right)2r = 0$$

$$M_A = \frac{2F\,r\,a}{\sqrt{a^2+b^2}} \qquad M_A = 1200\,\text{lb}\cdot\text{ft}$$

$\Sigma F_x = 0;$ $$N_C + A_x\sin(\theta_1) + A_y\cos(\theta_1) = 0$$

$$N_C = -A_x\sin(\theta_1) - A_y\cos(\theta_1) \qquad N_C = -495\,\text{lb}$$

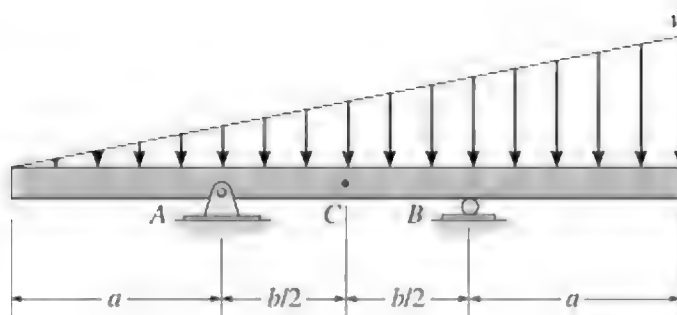
$\Sigma F_y = 0;$ $$V_C - A_x\cos(\theta_1) + A_y\sin(\theta_1) = 0$$

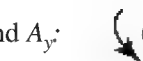
$$V_C = A_x\cos(\theta_1) - A_y\sin(\theta_1) \qquad V_C = 70.7\,\text{lb}$$

$\Sigma M_C = 0;$ $$-M_C - M_A + A_y(r - r\cos(\theta_1)) - A_x r\sin(\theta_1) = 0$$

$$M_C = -M_A + A_y(r - r\cos(\theta_1)) - A_x r\sin(\theta_1) \qquad M_C = -1.59\,\text{kip}\cdot\text{ft}$$

Problem 7-35

Determine the ratio *a/b* for which the shear force will be zero at the midpoint *C* of the beam.

Solution:

Find A_y: $\Sigma M_B = 0;$

$$\frac{1}{2}(2a+b)\ w\left[\frac{1}{3}(b-a)\right] - A_y b = 0$$

$$A_y = \frac{w}{6b}(2a+b)(b-a)$$

This problem requires $V_C = 0$.

Summing forces vertically for the section, we have

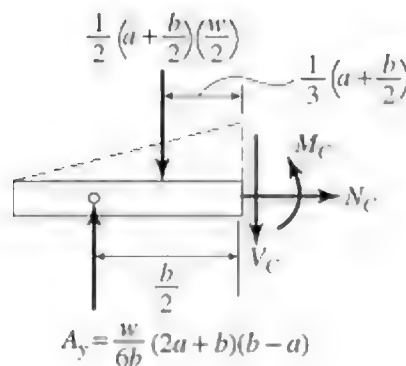
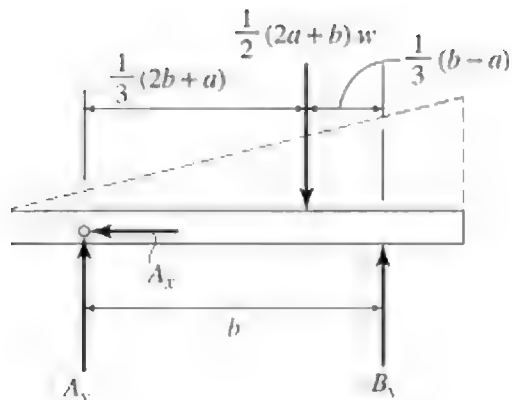
$+\uparrow \Sigma\ F_y = 0;$

$$\frac{w}{6b}(2a+b)(b-a) - \frac{1}{2}\left(a+\frac{b}{2}\right)\frac{w}{2} = 0$$

$$\frac{w}{6b}(2a+b)(b-a) = \frac{w}{8}(2a+b)$$

$$4(b-a) = 3b$$

$$b = 4a$$

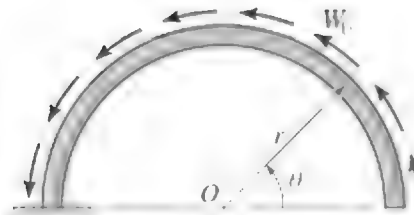
$$\frac{a}{b} = \frac{1}{4}$$

Problem 7-36

The semicircular arch is subjected to a uniform distributed load along its axis of w_0 per unit length. Determine the internal normal force, shear force, and moment in the arch at angle θ.

Given:

$\theta = 45\ \text{deg}$

Solution:

Resultants of distributed load:

$$F_{Rx} = \int_0^{\theta} w_0 r\, d\theta \sin(\theta) = r w_0 (1 - \cos(\theta))$$

$$F_{Rx} = r w_0 (1 - \cos(\theta))$$

$$F_{Ry} = \int_0^{\theta} w_0 r\, d\theta \cos(\theta) = r w_0 \sin(\theta)$$

$$F_{Rx} = r w_0 (\sin(\theta))$$

$$M_{Ro} = \int_0^{\theta} w_0 r\, d\theta r = r^2 w_0 \theta$$

$\Sigma F_x = 0;$ $\quad -V + F_{Rx}\cos(\theta) - F_{Ry}\sin(\theta) = 0$

$$V = [r w_0 (1 - \cos(\theta))]\cos(\theta) - [r w_0 (\sin(\theta))]\sin(\theta)$$

$$V = w_0 r (\cos(\theta) - 1)$$

$$a = \cos(\theta) - 1$$

$a = -0.293 \qquad V = a r w_0$

$\Sigma F_y = 0;$ $\quad N + F_{Ry}\cos(\theta) + F_{Rx}\sin(\theta) = 0$

$$N = -[r w_0 (1 - \cos(\theta))]\sin(\theta) - [r w_0 (\sin(\theta))]\cos(\theta)$$

$$N = -w_0 r \sin(\theta)$$

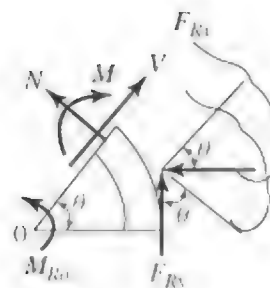
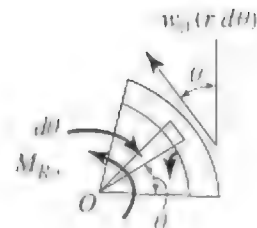
$$b = -\sin(\theta)$$

$b = -0.707 \qquad N = w_0\, r\, b$

$\Sigma M_o = 0;$ $\quad -M + r^2 w_0(\theta) + b r w_0 r = 0$

$$M = w_0 r^2 (\theta + b)$$

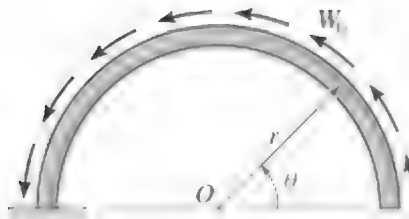
$$c = \theta + b$$

$c = 0.0783 \qquad M = c r^2 w_0$

Problem 7-37

The semicircular arch is subjected to a uniform distributed load along its axis of w_0 per unit length. Determine the internal normal force, shear force, and moment in the arch at angle θ.

Given:

$\theta = 120 \text{ deg}$

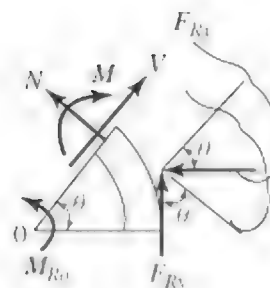
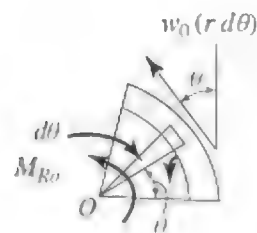
Solution:

Resultants of distributed load:

$$F_{Rx} = \int_0^{\theta} w_0 r\, d\theta \sin(\theta) = r w_0 (1 - \cos(\theta))$$

$$F_{Rx} = r w_0 (1 - \cos(\theta))$$

$$F_{Ry} = \int_0^{\theta} w_0 r\, d\theta \cos(\theta) = r w_0 \sin(\theta)$$

$$F_{Rx} = r w_0 (\sin(\theta))$$

$$M_{Ro} = \int_0^{\theta} w_0 r\, d\theta r = r^2 w_0 \theta$$

$\Sigma F_x = 0;$ $\quad -V + F_{Rx}\cos(\theta) - F_{Ry}\sin(\theta) = 0$

$$V = [r w_0 (1 - \cos(\theta))]\cos(\theta) - [r w_0 (\sin(\theta))]\sin(\theta)$$

$$V = w_0 r(\cos(\theta) - 1)$$

$$a = \cos(\theta) - 1$$

$a = -1.500 \qquad V = a r w_0$

$\Sigma F_y = 0;$ $\quad N + F_{Ry}\cos(\theta) + F_{Rx}\sin(\theta) = 0$

$$N = -[r w_0 (1 - \cos(\theta))]\sin(\theta) - [r w_0 (\sin(\theta))]\cos(\theta)$$

$$N = -w_0 r \sin(\theta)$$

$$b = -\sin(\theta)$$

$b = -0.866 \qquad N = w_0 \cdot r \cdot b$

$\Sigma M_o = 0; \quad -M + r^2 w_0(\theta) + b r w_0 r = 0$

$M = w_0\, r^2(\theta + b)$

$c = \theta + b$

$c = 1.2284 \qquad M = c r^2 w_0$

Problem 7-38

Determine the x, y, z components of internal loading at a section passing through point C in the pipe assembly. Neglect the weight of the pipe.

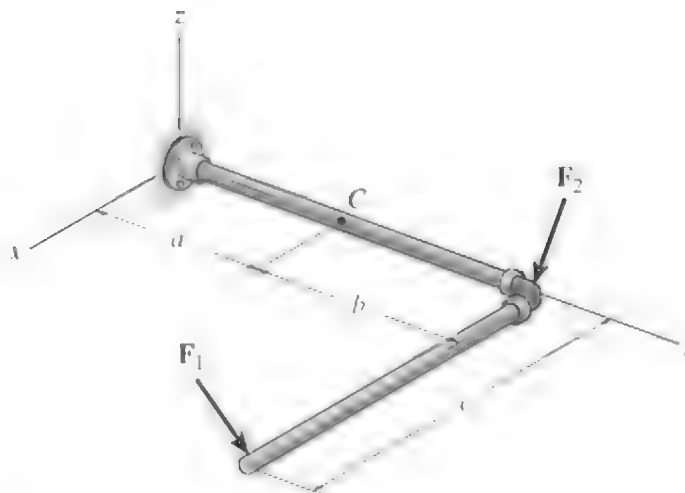
Units Used:

$\text{kip} = 10^3 \text{ lb}$

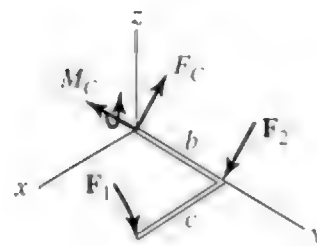
Given:

$\mathbf{F_1} = \begin{pmatrix} 0 \\ 350 \\ -400 \end{pmatrix} \text{lb}$

$\mathbf{F_2} = \begin{pmatrix} 150 \\ 0 \\ -300 \end{pmatrix} \text{lb}$

$a = 1.5 \text{ ft} \qquad b = 2 \text{ ft} \qquad c = 3 \text{ ft}$

Solution:

$\mathbf{r_1} = \begin{pmatrix} c \\ b \\ 0 \end{pmatrix} \quad \mathbf{r_2} = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}$

$\mathbf{F_C} = -\mathbf{F_1} - \mathbf{F_2} \qquad \mathbf{F_C} = \begin{pmatrix} -150.00 \\ -350.00 \\ 700.00 \end{pmatrix} \text{lb}$

$$\mathbf{M_C} = -\mathbf{r_1} \times \mathbf{F_1} - \mathbf{r_2} \times \mathbf{F_2} \qquad \mathbf{M_C} = \begin{pmatrix} 1400.00 \\ -1200.00 \\ -750.00 \end{pmatrix} \text{lb} \cdot \text{ft}$$

Problem 7-39

Determine the x, y, z components of internal loading at a section passing through point C in the pipe assembly. Neglect the weight of the pipe.

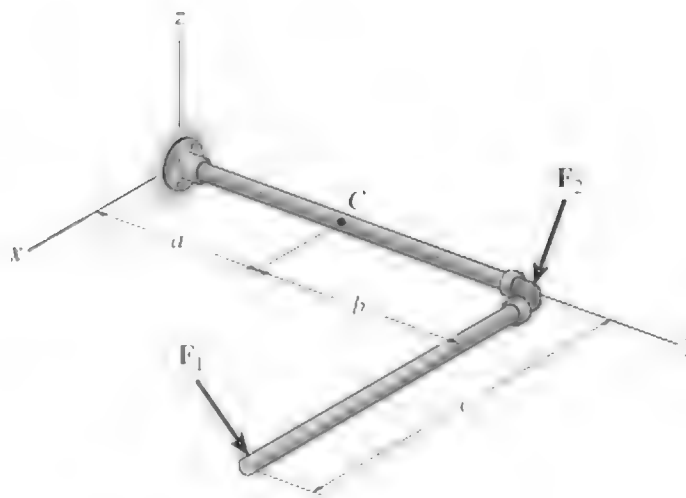
Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

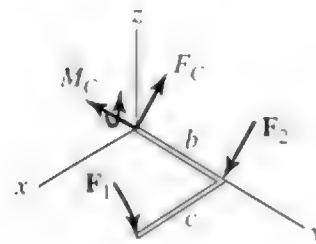
$$\mathbf{F_1} = \begin{pmatrix} -80 \\ 200 \\ -300 \end{pmatrix} \text{lb}$$

$$\mathbf{F_2} = \begin{pmatrix} 250 \\ -150 \\ -200 \end{pmatrix} \text{lb}$$

$a = 1.5 \text{ ft} \qquad b = 2 \text{ ft} \qquad c = 3 \text{ ft}$

Solution:

$$\mathbf{r_1} = \begin{pmatrix} c \\ b \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}$$

$$\mathbf{F_C} = -\mathbf{F_1} - \mathbf{F_2} \qquad \mathbf{F_C} = \begin{pmatrix} -170.00 \\ -50.00 \\ 500.00 \end{pmatrix} \text{lb}$$

$$\mathbf{M_C} = -\mathbf{r_1} \times \mathbf{F_1} - \mathbf{r_2} \times \mathbf{F_2} \qquad \mathbf{M_C} = \begin{pmatrix} 1000.00 \\ -900.00 \\ -260.00 \end{pmatrix} \text{lb} \cdot \text{ft}$$

Problem 7-40

Determine the x, y, z components of internal loading in the rod at point D.

Units Used:

$kN = 10^3\ N$

Given:

$M = 3\ kN{\cdot}m$

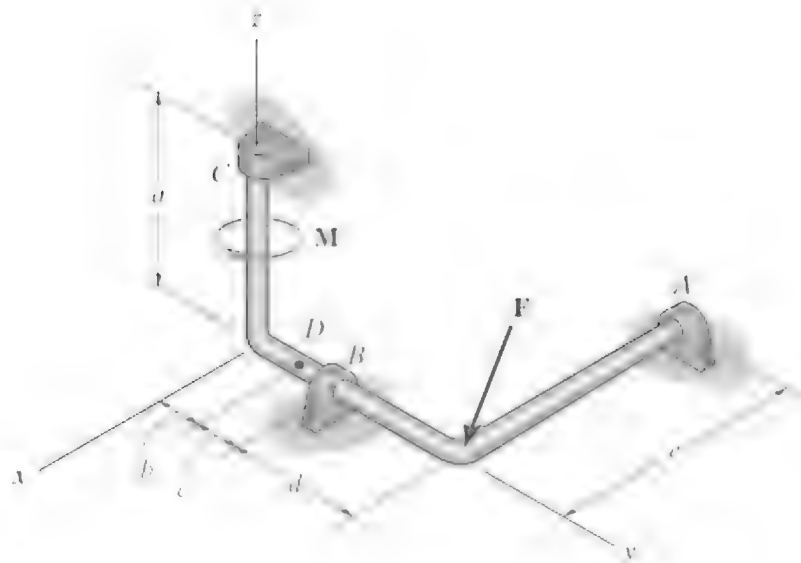
$\mathbf{F} = \begin{pmatrix} 7 \\ -12 \\ -5 \end{pmatrix} kN$

$a = 0.75\ m$

$b = 0.2\ m$

$c = 0.2\ m$

$d = 0.6\ m$

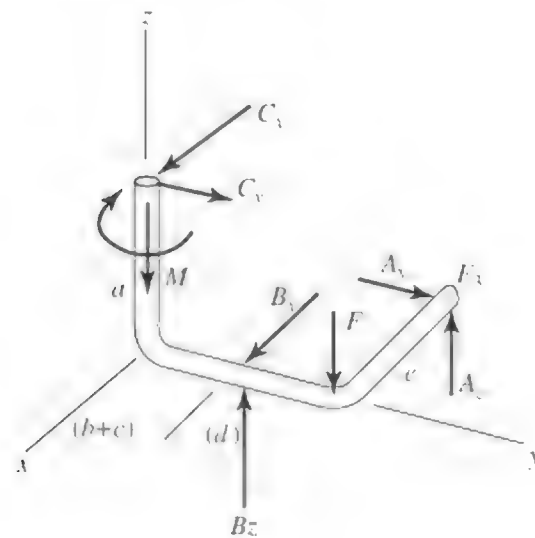
$e = 1\ m$

Solution:

Guesses

$C_x = 1\ N \quad C_y = 1\ N \quad B_x = 1\ N$

$B_z = 1\ N \quad A_y = 1\ N \quad A_z = 1\ N$

Given

$$\begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \mathbf{F} = \mathbf{0}$$

$$\begin{pmatrix} -e \\ b+c+d \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ b+c \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} \times \begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b+c+d \\ 0 \end{pmatrix} \times \mathbf{F} + \begin{pmatrix} 0 \\ 0 \\ -M \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} A_y \\ A_z \\ B_x \\ B_z \\ C_x \\ C_y \end{pmatrix} = \text{Find}(A_y, A_z, B_x, B_z, C_x, C_y) \qquad \begin{pmatrix} A_y \\ A_z \\ B_x \\ B_z \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} -53.60 \\ 87.00 \\ 109.00 \\ -82.00 \\ -116.00 \\ 65.60 \end{pmatrix} \text{kN}$$

Guesses

$V_{Dx} = 1\text{ N}$ $\quad N_{Dy} = 1\text{ N}$

$V_{Dz} = 1\text{N}$ $\quad M_{Dx} = 1\text{ N}\cdot\text{m}$

$M_{Dy} = 1\text{ N}\cdot\text{m}$ $\quad M_{Dz} = 1\text{ N}\cdot\text{m}$

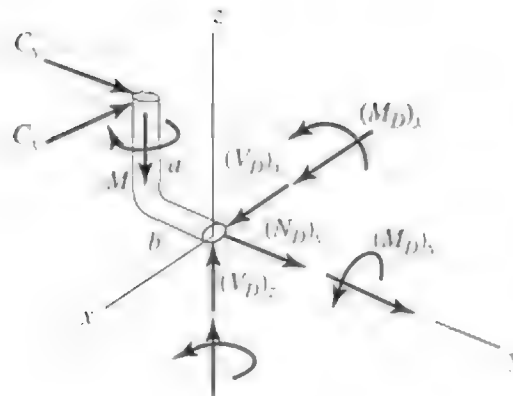
Given

$$\begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \begin{pmatrix} V_{Dx} \\ N_{Dy} \\ V_{Dz} \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} 0 \\ -b \\ a \end{pmatrix} \times \begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \begin{pmatrix} M_{Dx} \\ M_{Dy} \\ M_{Dz} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -M \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} V_{Dx} \\ N_{Dy} \\ V_{Dz} \\ M_{Dx} \\ M_{Dy} \\ M_{Dz} \end{pmatrix} = \text{Find}(V_{Dx}, N_{Dy}, V_{Dz}, M_{Dx}, M_{Dy}, M_{Dz})$$

$$\begin{pmatrix} V_{Dx} \\ N_{Dy} \\ V_{Dz} \end{pmatrix} = \begin{pmatrix} 116.00 \\ -65.60 \\ 0.00 \end{pmatrix} \text{kN}$$

$$\begin{pmatrix} M_{Dx} \\ M_{Dy} \\ M_{Dz} \end{pmatrix} = \begin{pmatrix} 49.20 \\ 87.00 \\ 26.20 \end{pmatrix} \text{kN}\cdot\text{m}$$

Problem 7-41

Determine the x, y, z components of internal loading in the rod at point E.

Units Used:

$$kN = 10^3\ N$$

Given:

$$M = 3\ kN \cdot m$$

$$\mathbf{F} = \begin{pmatrix} 7 \\ -12 \\ -5 \end{pmatrix} kN$$

$$a = 0.75\ m$$

$$b = 0.4\ m$$

$$c = 0.6\ m$$

$$d = 0.5\ m$$

$$e = 0.5\ m$$

Solution:

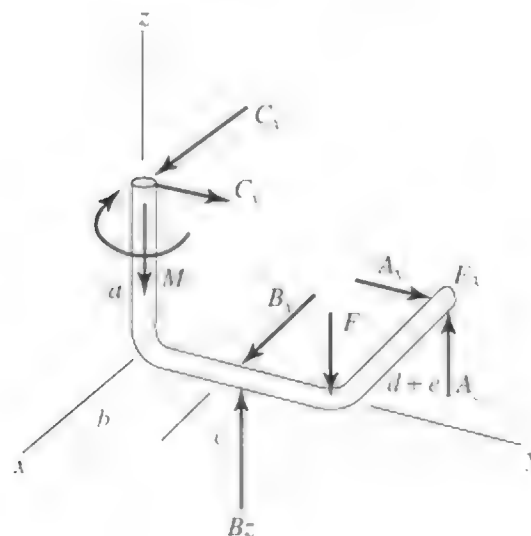
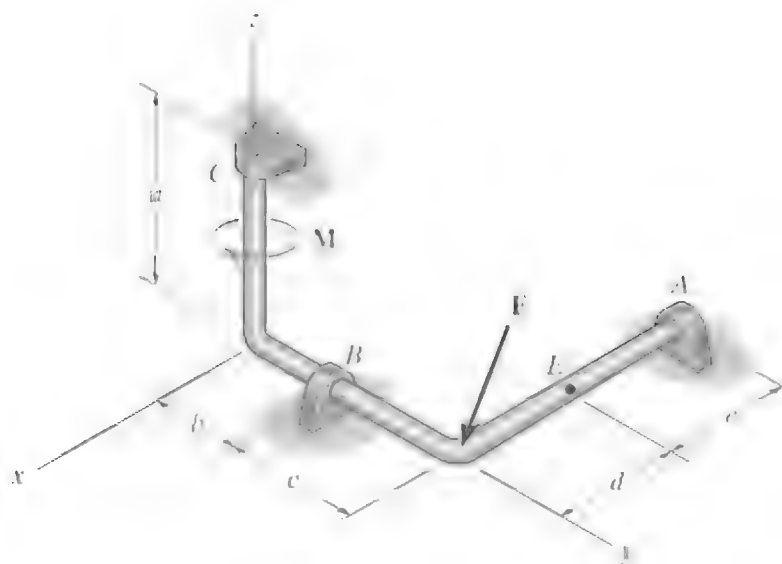
Guesses

$$C_x = 1\ N \quad C_y = 1\ N \quad B_x = 1\ N$$

$$B_z = 1\ N \quad A_y = 1\ N \quad A_z = 1\ N$$

Given

$$\begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \mathbf{F} = \mathbf{0}$$

$$\begin{pmatrix} -d-e \\ b+c \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} \times \begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b+c \\ 0 \end{pmatrix} \times \mathbf{F} + \begin{pmatrix} 0 \\ 0 \\ -M \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} A_y \\ A_z \\ B_x \\ B_z \\ C_x \\ C_y \end{pmatrix} = \text{Find}(A_y, A_z, B_x, B_z, C_x, C_y) \qquad \begin{pmatrix} A_y \\ A_z \\ B_x \\ B_z \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} -53.60 \\ 87.00 \\ 109.00 \\ -82.00 \\ -116.00 \\ 65.60 \end{pmatrix} \text{kN}$$

Guesses

$N_{Ex} = 1\ \text{N}$ $\quad V_{Ey} = 1\ \text{N}$

$V_{Ez} = 1\ \text{N}$ $\quad M_{Ex} = 1\ \text{N}\cdot\text{m}$

$M_{Ey} = 1\ \text{N}\cdot\text{m}$ $\quad M_{Ez} = 1\ \text{N}\cdot\text{m}$

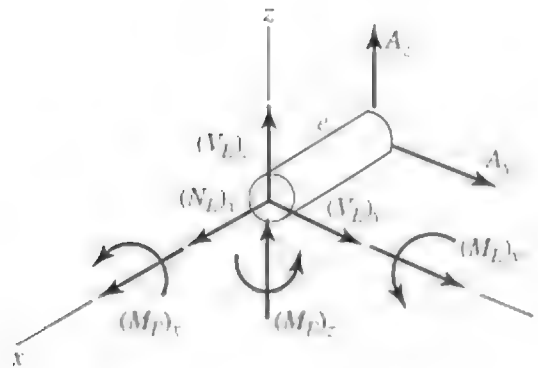
Given

$$\begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} N_{Ex} \\ V_{Ey} \\ V_{Ez} \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} -e \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} M_{Ex} \\ M_{Ey} \\ M_{Ez} \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} N_{Ex} \\ V_{Ey} \\ V_{Ez} \\ M_{Ex} \\ M_{Ey} \\ M_{Ez} \end{pmatrix} = \text{Find}(N_{Ex}, V_{Ey}, V_{Ez}, M_{Ex}, M_{Ey}, M_{Ez})$$

$$\begin{pmatrix} N_{Ex} \\ V_{Ey} \\ V_{Ez} \end{pmatrix} = \begin{pmatrix} 0.00 \\ 53.60 \\ -87.00 \end{pmatrix} \text{kN}$$

$$\begin{pmatrix} M_{Ex} \\ M_{Ey} \\ M_{Ez} \end{pmatrix} = \begin{pmatrix} 0.00 \\ -43.50 \\ -26.80 \end{pmatrix} \text{kN}\cdot\text{m}$$

Problem 7-42

Draw the shear and moment diagrams for the shaft in terms of the parameters shown; There is

a thrust bearing at A and a journal bearing at B.

Units Used:

$kN = 10^3\ N$

Given:

$P = 9\ kN$

$a = 2\ m$

$L = 6\ m$

Solution:

$P(L-a) - A_y L = 0 \qquad A_y = P\dfrac{L-a}{L}$

$x_1 = 0, 0.01a .. a$

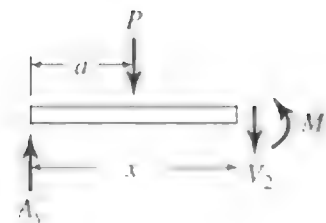
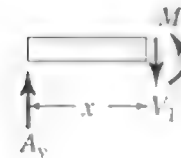
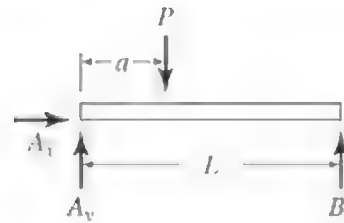
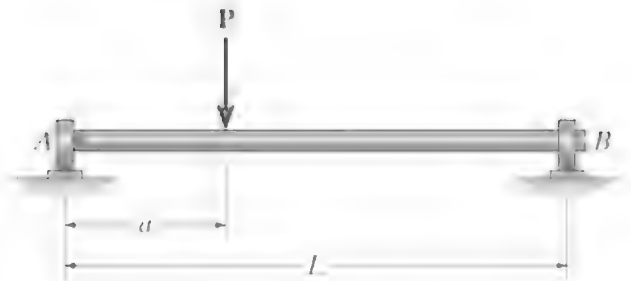
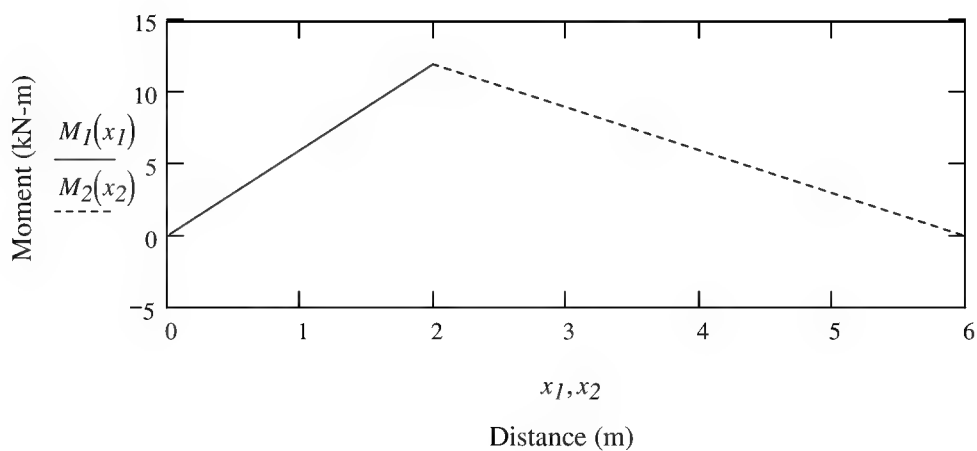
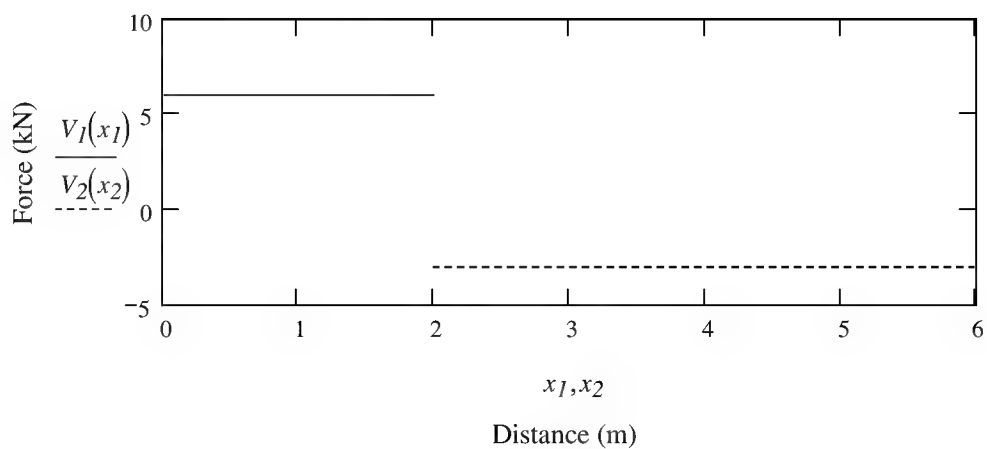
$A_y - V_1(x) = 0 \qquad V_1(x) = \dfrac{A_y}{kN}$

$M_1(x) - A_y x = 0 \qquad M_1(x) = \dfrac{A_y x}{kN \cdot m}$

$x_2 = a, 1.01a .. L$

$A_y - P - V_2(x) = 0 \qquad V_2(x) = \dfrac{A_y - P}{kN}$

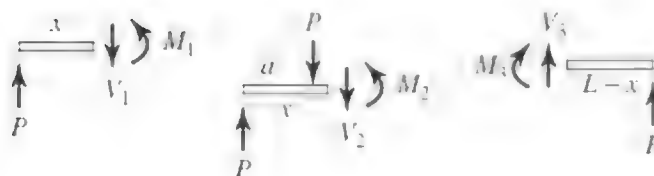
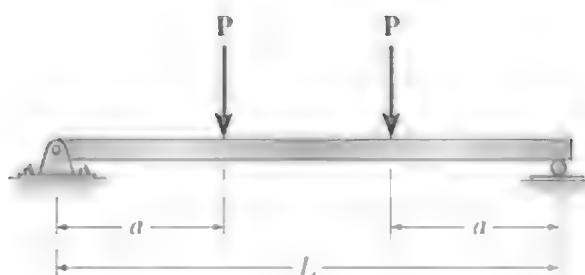
$M_2(x) - A_y x + P(x-a) = 0$

$M_2(x) = \dfrac{A_y x - P(x-a)}{kN \cdot m}$

Problem 7-43

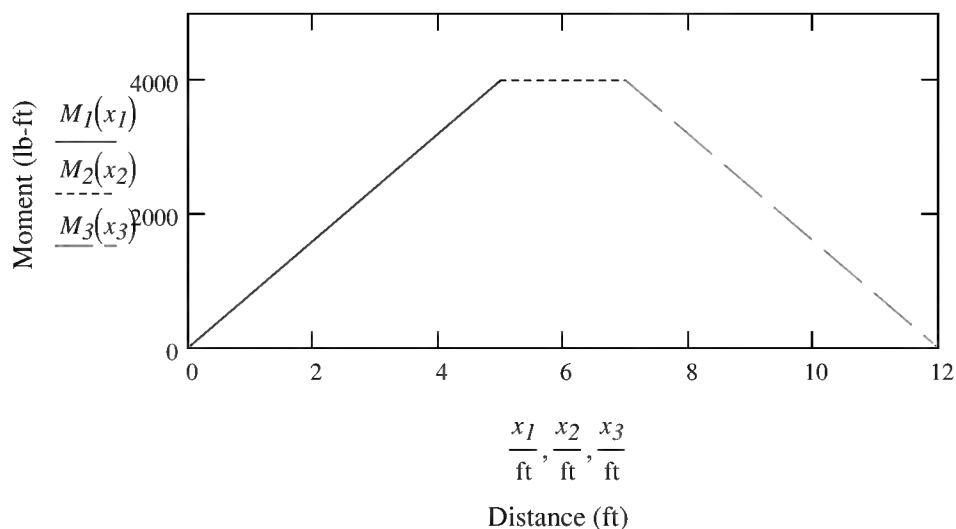
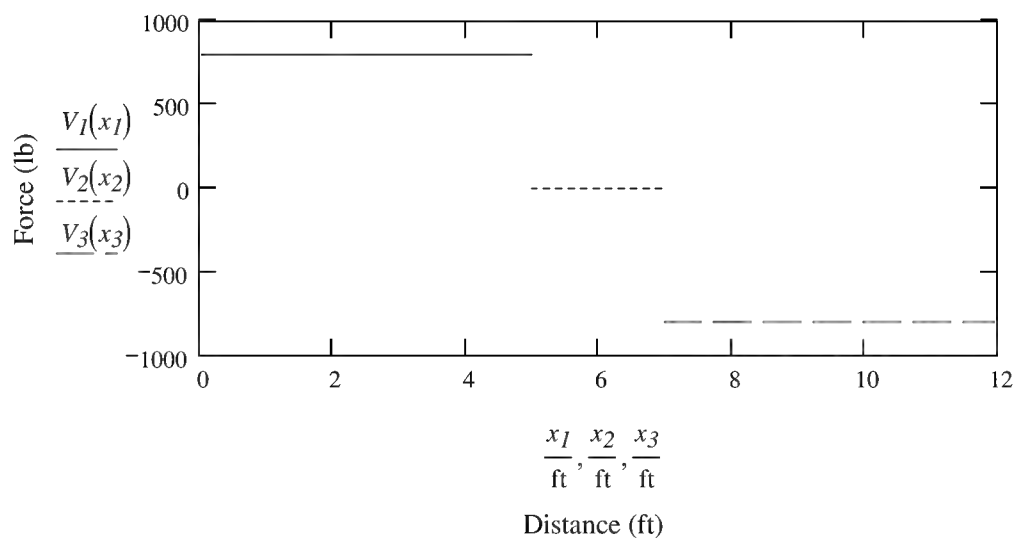
Draw the shear and moment diagrams for the beam in terms of the parameters shown.

Given:

$P = 800$ lb $\quad a = 5$ ft $\qquad L = 12$ ft

Solution:

$x_1 = 0, 0.01a .. a$ $\quad$ $x_2 = a, 1.01a .. L - a$ $\quad$ $x_3 = L - a, 1.01(L - a) .. L$

$V_1(x) = \frac{P}{\text{lb}}$ $\quad$ $V_2(x) = 0$ $\quad$ $V_3(x) = \frac{-P}{\text{lb}}$

$M_1(x) = \frac{P x}{\text{lb}\cdot\text{ft}}$ $\quad$ $M_2(x) = \frac{P a}{\text{lb}\cdot\text{ft}}$ $\quad$ $M_3(x) = \frac{P(L - x)}{\text{lb}\cdot\text{ft}}$

Problem 7-44

Draw the shear and moment diagrams for the beam (a) in terms of the parameters

shown; (b) set M_0 and L as given.

Given:

$M_0 = 500\ \text{N}\cdot\text{m}$

$L = 8\ \text{m}$

Solution:

For $0 \le x \le \frac{L}{3}$

$+\uparrow\Sigma F_y = 0;$ $V_1 = 0$

$\Sigma M_x = 0;$ $M_1 = 0$

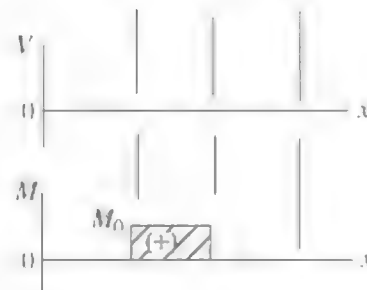
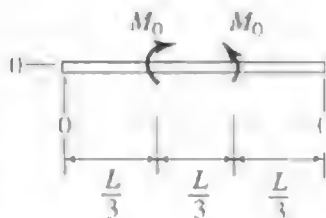
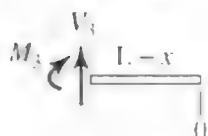
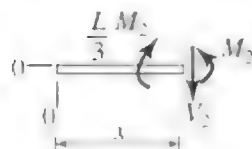
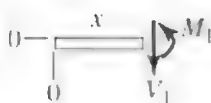
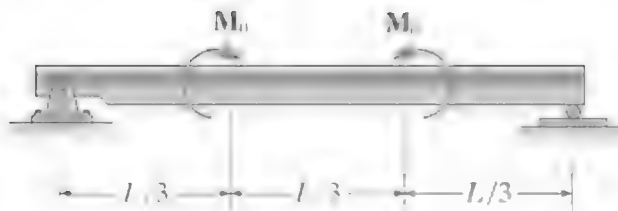
For $\frac{L}{3} \le x \le \frac{2L}{3}$

$+\uparrow\Sigma F_y = 0;$ $V_2 = 0$

$\Sigma M_x = 0;$ $M_2 = M_0$

For $\frac{2L}{3} \le x \le L$

$+\uparrow\Sigma F_y = 0;$ $V_3 = 0$

$\Sigma M_x = 0;$ $M_3 = 0$

(b) $x_1 = 0, 0.01L .. \frac{L}{3}$ $\quad x_2 = \frac{L}{3}, \frac{L}{3}1.01 .. \frac{2L}{3}$ $\quad x_3 = \frac{2L}{3}, \frac{2L}{3}1.01 .. L$

$V_1(x_1) = 0$ $\quad V_2(x_2) = 0$ $\quad V_3(x_3) = 0$

$M_1(x_1) = 0$ $\quad M_2(x_2) = M_0$ $\quad M_3(x_3) = 0$

Shear force in N

$V_1(x_1)$, $V_2(x_2)$, $V_3(x_3)$

1, 0.5, 0, −0.5, −1

0 1 2 3 4 5 6 7 8 9

x_1, x_2, x_3

Distance in m

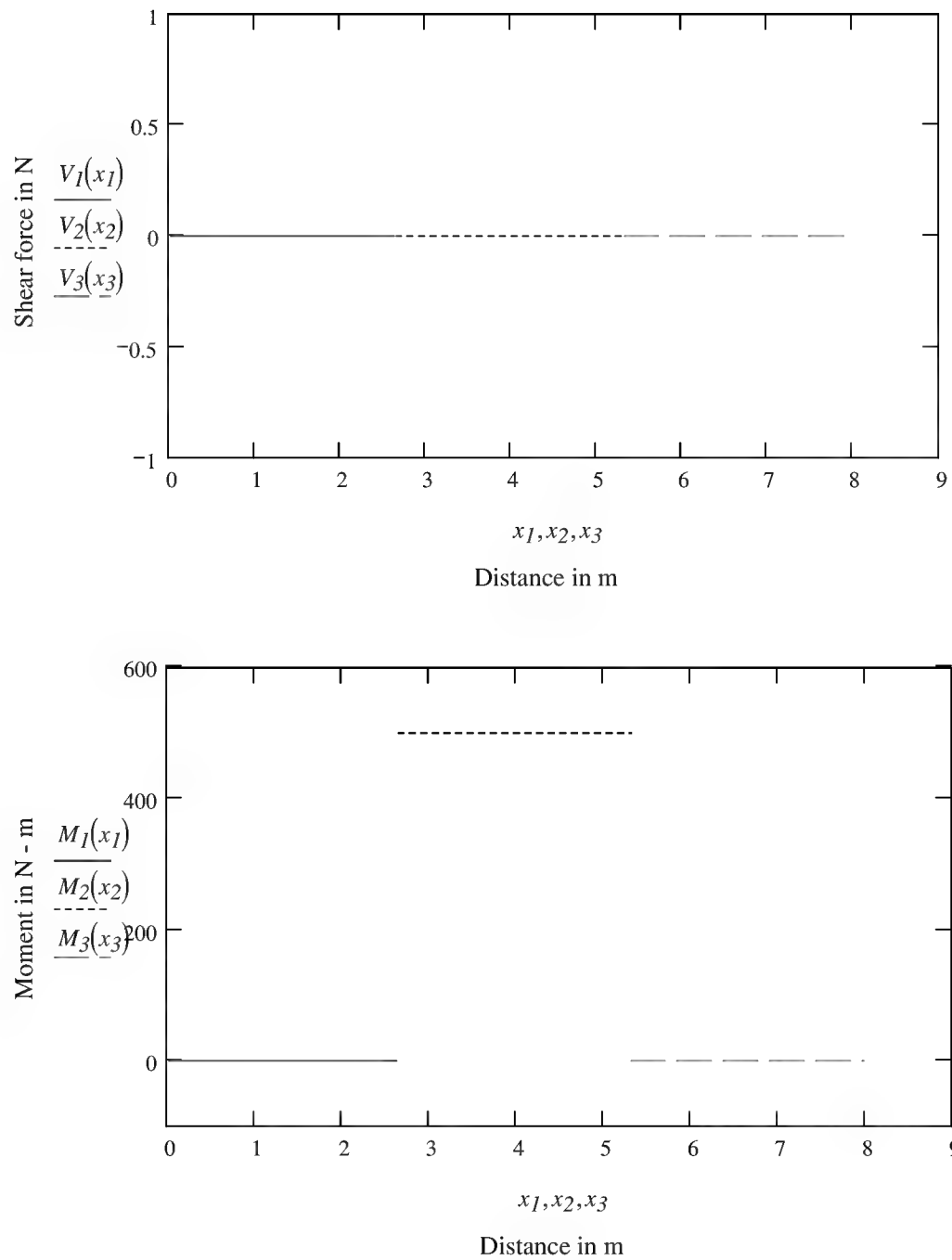
Moment in N - m

$M_1(x_1)$, $M_2(x_2)$, $M_3(x_3)$

600, 400, 200, 0

0 1 2 3 4 5 6 7 8 9

x_1, x_2, x_3

Distance in m

Problem 7-45

The beam will fail when the maximum shear force is V_{max} or the maximum bending moment is M_{max}. Determine the magnitude M_0 of the largest couple moments it will support.

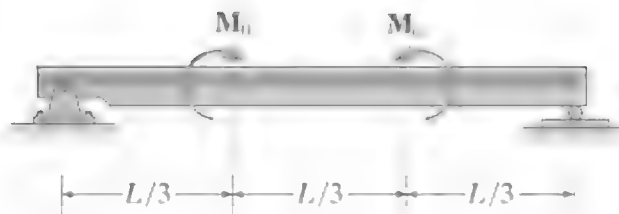
Units Used:

$$kN = 10^3\ N$$

Given:

$$L = 9\ m$$

$$V_{max} = 5\ kN$$

$$M_{max} = 2\ kN\cdot m$$

Solution:

The shear force is zero everywhere in the beam.

The moment is zero in the first third and the last third of the bam.

In the middle section of the beam the moment is $M = M_0$

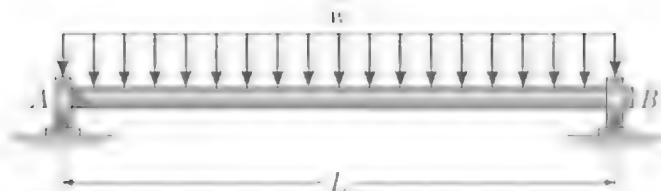
Thus the beam will fail when $M_0 = M_{max}$ $\quad M_0 = 2.00\,kN\cdot m$

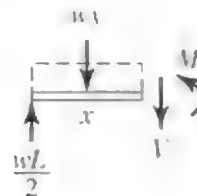
Problem 7-46

The shaft is supported by a thrust bearing at A and a journal bearing at B. Draw the shear and moment diagrams for the shaft in terms of the parameters shown.

Given:

$$w = 500\ \frac{lb}{ft}$$

$$L = 10\ ft$$

Solution: For $0 \le x < L$

$$\Sigma F_y = 0; \quad \frac{wL}{2} - wx - V = 0 \qquad V(x) = \frac{w}{2}(L - 2x)\frac{1}{lb}$$

$$\Sigma M = 0; \quad \frac{-wL}{2}x + wx\left(\frac{x}{2}\right) + M = 0 \qquad M(x) = \frac{w}{2}\left(Lx - x^2\right)\frac{1}{lb\cdot ft}$$

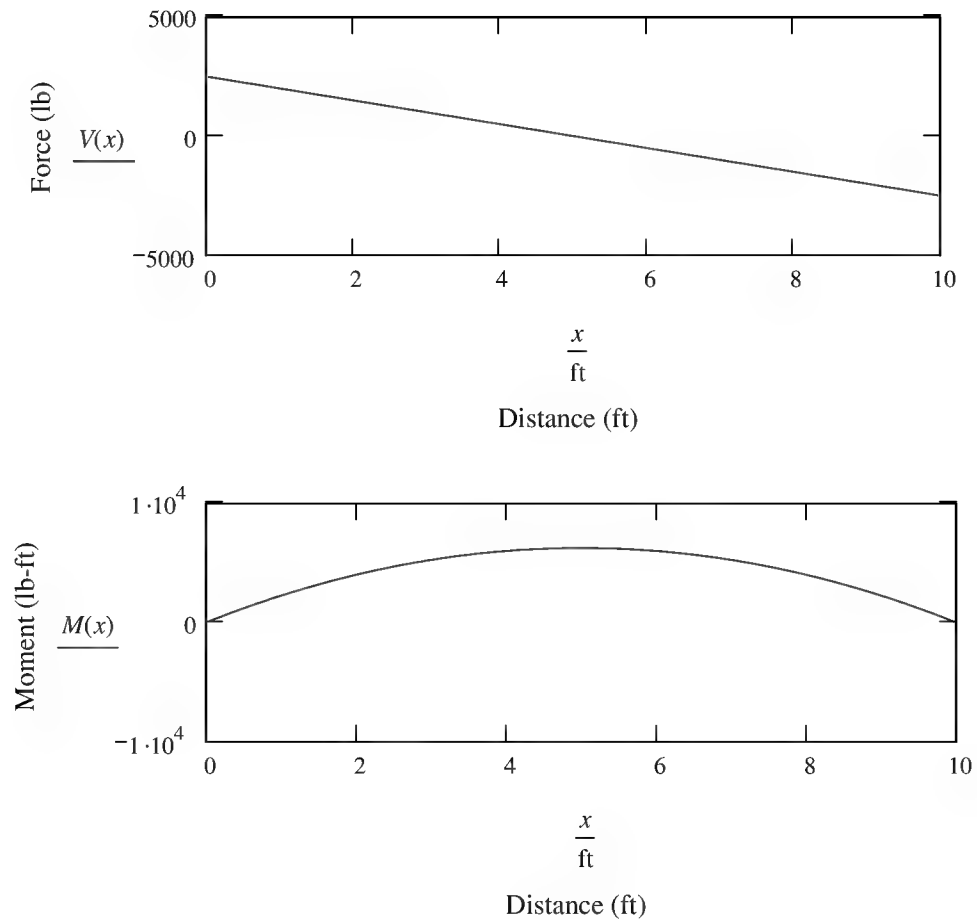
Force (lb) $\frac{V(x)}{}$ — 5000, 0, −5000

$\frac{x}{\text{ft}}$ — 0, 2, 4, 6, 8, 10

Distance (ft)

Moment (lb-ft) $\frac{M(x)}{}$ — $1 \cdot 10^4$, 0, $-1 \cdot 10^4$

$\frac{x}{\text{ft}}$ — 0, 2, 4, 6, 8, 10

Distance (ft)

Problem 7-47

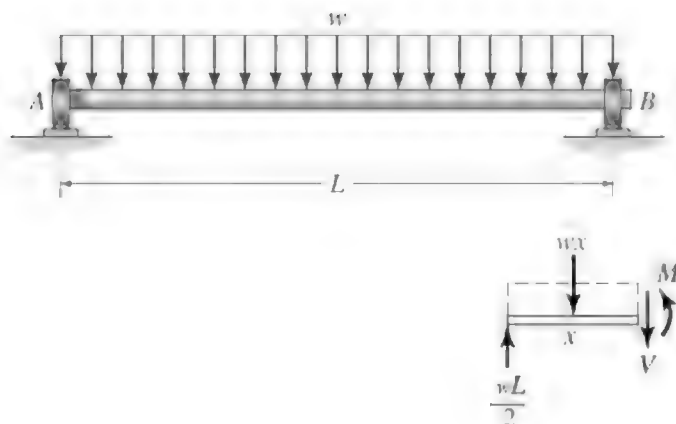
The shaft is supported by a thrust bearing at *A* and a journal bearing at *B*. The shaft will fail when the maximum moment is M_{max}. Determine the largest uniformly distributed load *w* the shaft will support.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

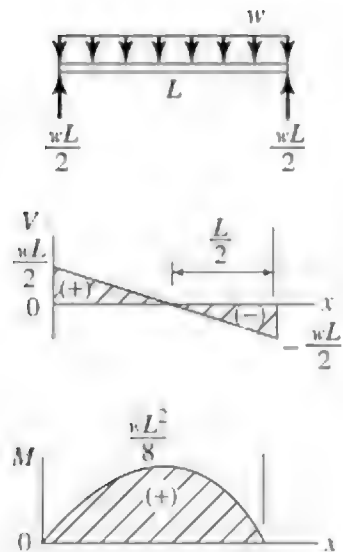
$$L = 10 \text{ ft}$$

$$M_{max} = 5 \text{ kip}\cdot\text{ft}$$

Solution:

$$\frac{wL}{2} - wx - V = 0 \qquad V = -wx + \frac{wL}{2}$$

$$-\frac{wL}{2}x + wx\left(\frac{x}{2}\right) + M = 0 \qquad M = \left(\frac{wL}{2}\right)x - \frac{wx^2}{2}$$

From the Moment Diagram,

$$M_{max} = \frac{wL^2}{8} \qquad w = \frac{8M_{max}}{L^2} \qquad w = 400.00\,\frac{\text{lb}}{\text{ft}}$$

Problem 7-48

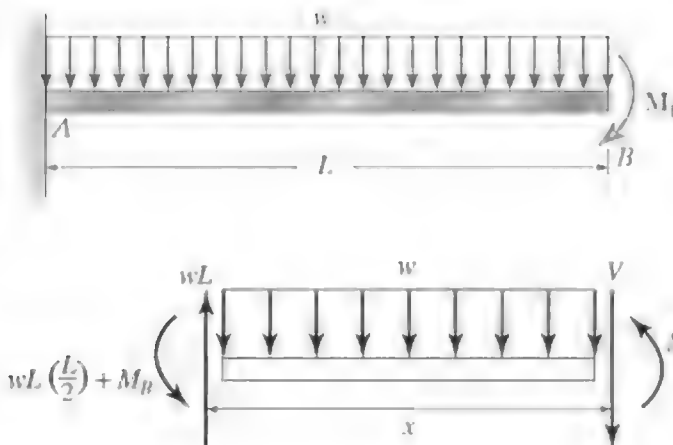
Draw the shear and moment diagrams for the beam.

Units Used: $\text{kN} = 10^3\ \text{N}$

Given:

$w = 2\ \frac{\text{kN}}{\text{m}}$

$L = 5\ \text{m}$

$M_B = 5\ \text{kN}\cdot\text{m}$

Solution:

$\Sigma F_y = 0;\quad -V(x) + \mathbf{w}L - \mathbf{w}x = 0 \qquad V(x) = (wL - wx)\frac{1}{\text{kN}}$

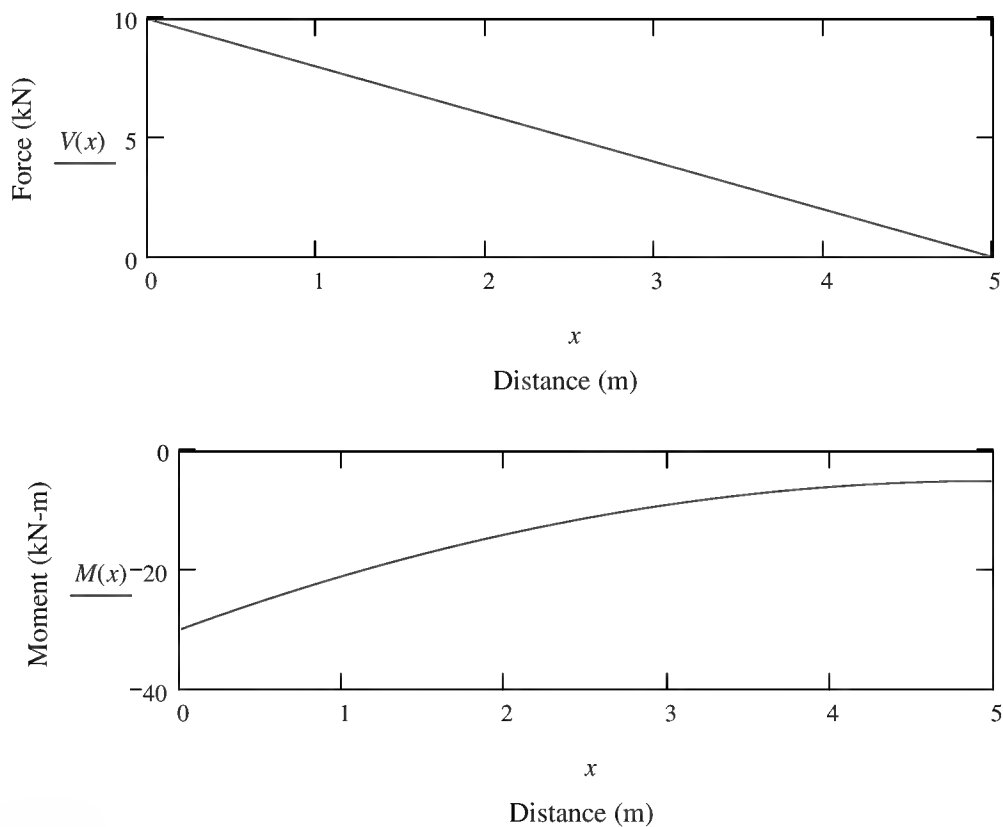
$$\Sigma M = 0;\quad M(x) + wL\left(\frac{L}{2}\right) + M_B - wLx + wx\left(\frac{x}{2}\right) = 0$$

$$M(x) = \left[wLx - wx\left(\frac{x}{2}\right) - wL\left(\frac{L}{2}\right) - M_B\right]\frac{1}{\text{kN}\cdot\text{m}}$$

Force (kN) $\frac{V(x)}{}$

Distance (m) x

Moment (kN-m) $\frac{M(x)}{}$

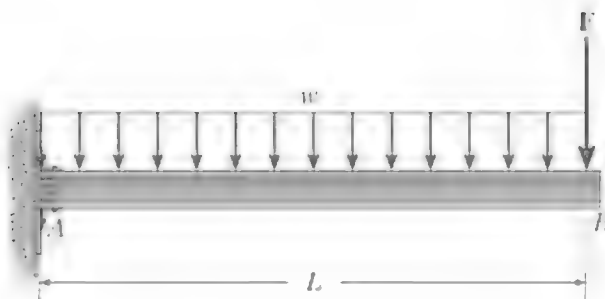
Distance (m) x

Problem 7-49

Draw the shear and moment diagrams for the beam.

Units Used: $\quad \text{kN} = 10^3 \text{ N}$

Given:

$$w = 3\ \frac{\text{kN}}{\text{m}} \qquad F = 10\ \text{kN}$$

$$L = 6\ \text{m}$$

Solution:

$$V(x) - w(L-x) - F = 0$$

$$V(x) = [w(L-x) + F]\frac{1}{\text{kN}}$$

$$-M(x) - w(L-x)\left(\frac{L-x}{2}\right) - F(L-x) = 0$$

$$M(x) = \left[-w\frac{(L-x)^2}{2} - F(L-x) \right] \frac{1}{\text{kN}\cdot\text{m}}$$

Force (kN)

$\frac{V(x)}{}$

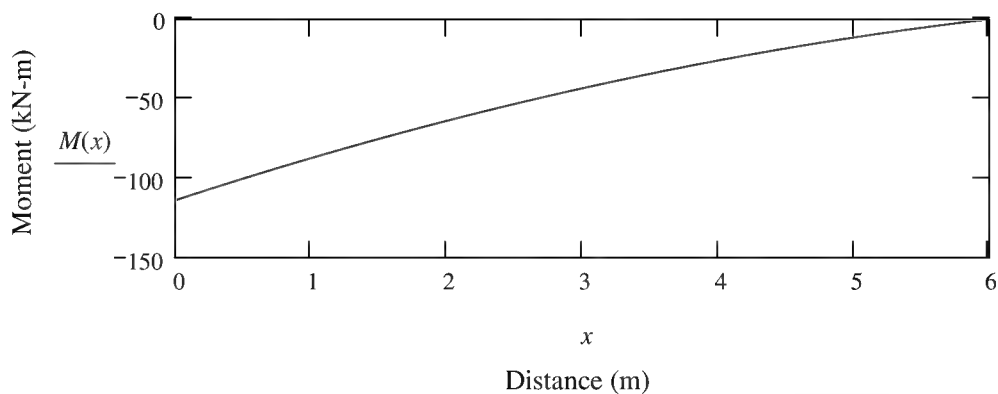
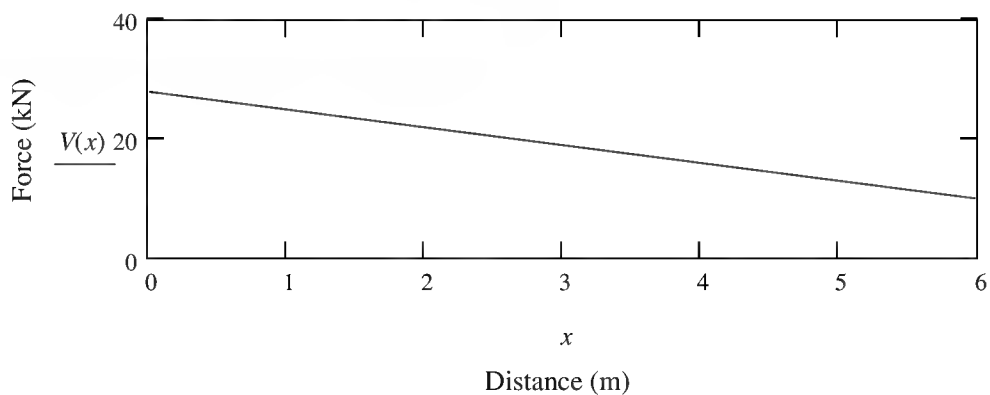
x

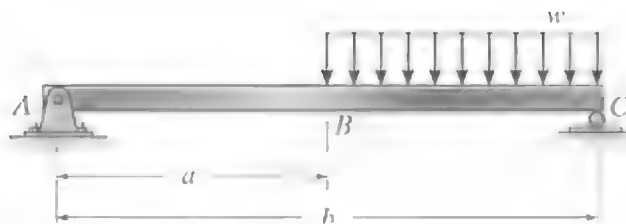
Distance (m)

Moment (kN-m)

$\frac{M(x)}{}$

x

Distance (m)

Problem 7-50

Draw the shear and moment diagrams for the beam.

Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$a = 2 \text{ m} \qquad b = 4 \text{ m} \qquad w = 1.5 \frac{\text{kN}}{\text{m}}$

Solution:

$$w(b-a)\left(\frac{b-a}{2}\right) - A_y b = 0 \qquad A_y = \frac{w(b-a)^2}{2b} \qquad A_y = 0.75\,\text{kN}$$

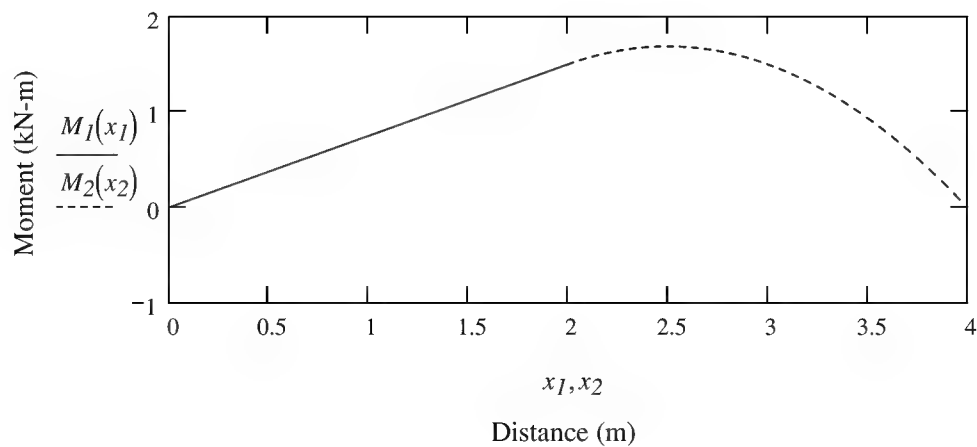
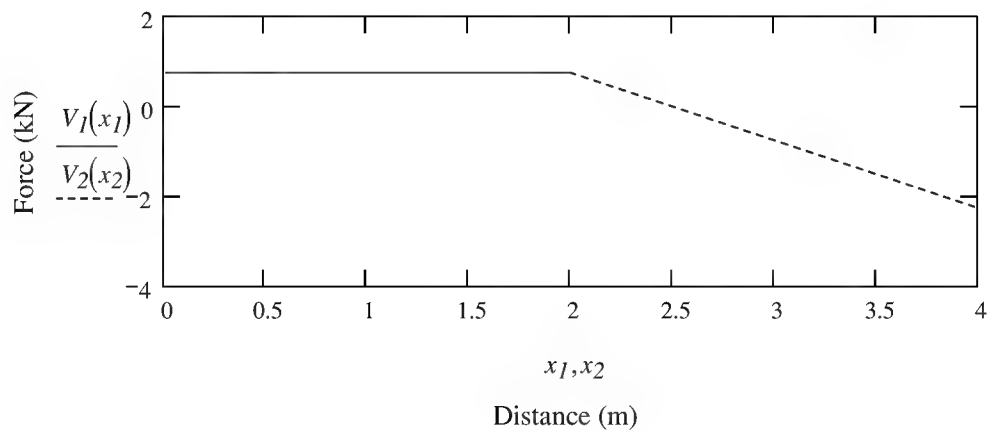
$x_1 = 0, 0.01a .. a$

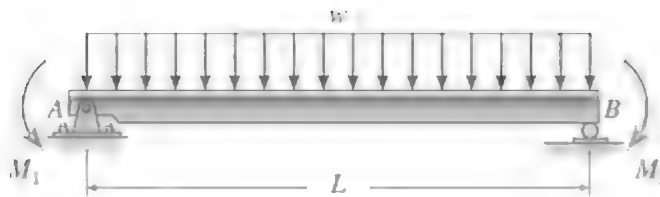
$$A_y - V_1(x) = 0 \qquad V_1(x) = A_y \frac{1}{\text{kN}}$$

$$-A_y x + M_1(x) = 0 \qquad M_1(x) = A_y x \frac{1}{\text{kN}\cdot\text{m}}$$

$$x_2 = b - a, 1.01(b-a) .. b$$

$$A_y - w(x-a) - V_2(x) = 0 \qquad V_2(x) = \left[A_y - w(x-a)\right]\frac{1}{\text{kN}}$$

$$-A_y x + w(x-a)\left(\frac{x-a}{2}\right) + M_2(x) = 0 \qquad M_2(x) = \left[A_y x - w\frac{(x-a)^2}{2}\right]\frac{1}{\text{kN}\cdot\text{m}}$$

Problem 7-51

Draw the shear and moment diagrams for the beam.

Given:

$L = 20\ \text{ft}$ $\quad w = 250\ \dfrac{\text{lb}}{\text{ft}}$ $\quad M_1 = 150\ \text{lb}\cdot\text{ft}$ $\quad M_2 = 150\ \text{lb}\cdot\text{ft}$

Solution:

$$M_1 + wL\left(\frac{L}{2}\right) - M_2 - A_y l = 0 \qquad A_y = \frac{M_1 - M_2 + w\left(\frac{L^2}{2}\right)}{L} \qquad A_y = 2500\ \text{lb}$$

$$V(x) = (A_y - wx)\frac{1}{\text{lb}} \qquad M(x) = \left[A_y x - w\left(\frac{x^2}{2}\right) - M_1\right]\frac{1}{\text{lb}\cdot\text{ft}}$$

Force in lb

$\dfrac{V(x)}{}$

2000
0
−2000

0 5 10 15 20

$\dfrac{x}{\text{ft}}$

Distance in ft

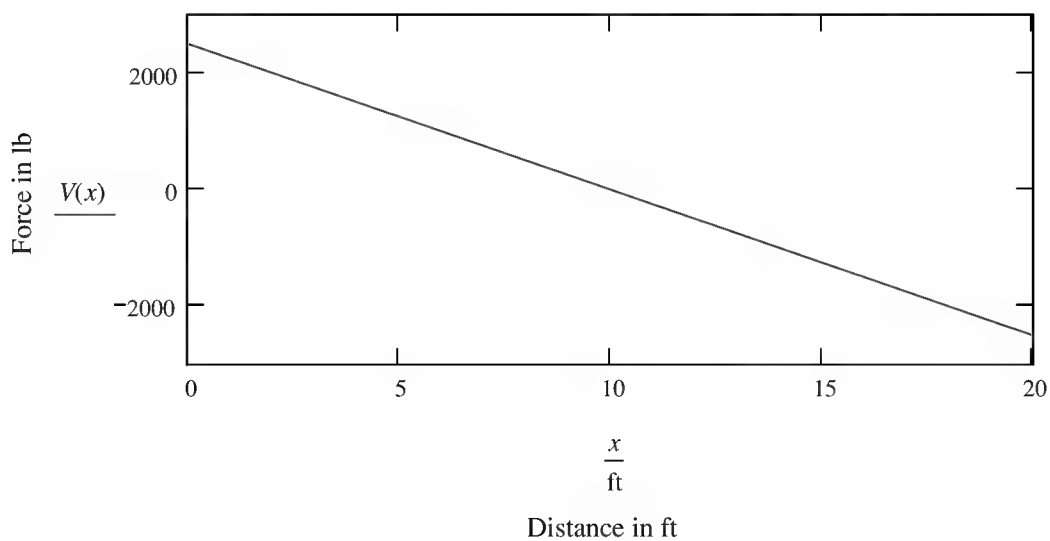
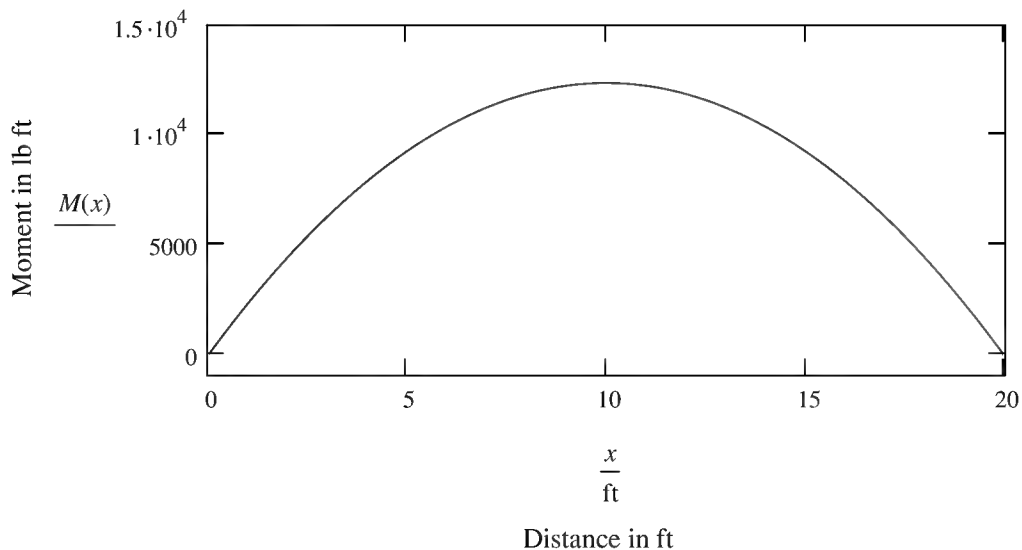
Moment in lb ft

$\frac{M(x)}{}$

1.5·10^4, 1·10^4, 5000, 0

0, 5, 10, 15, 20

$\frac{x}{\text{ft}}$

Distance in ft

Problem 7-52

Draw the shear and moment diagrams for the beam.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$w = 40\ \frac{\text{kN}}{\text{m}}$ $\quad F = 20\ \text{kN}$ $\quad M = 150\ \text{kN}\cdot\text{m}$ $\quad a = 8\ \text{m}$ $\quad b = 3\ \text{m}$

Solution:

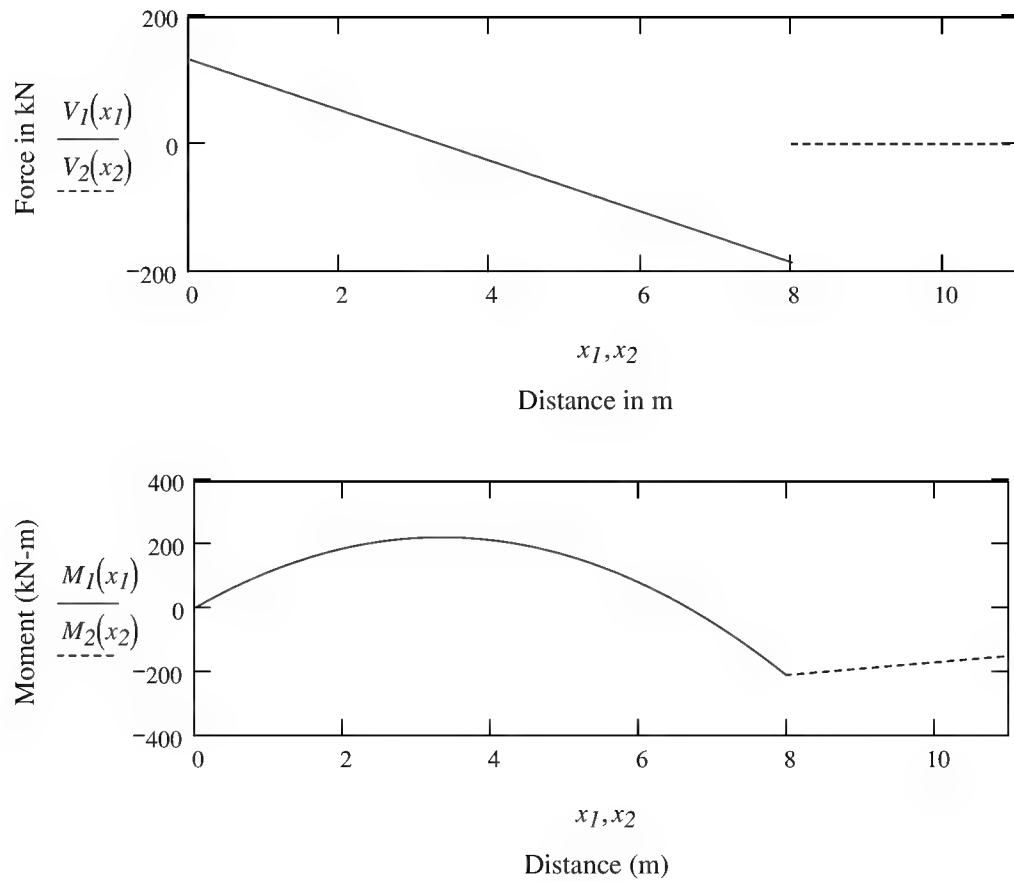
$$-A_y a + w a\left(\frac{a}{2}\right) - F b - M = 0 \qquad A_y = \left[w\left(\frac{a^2}{2}\right) - F b - M\right]\left(\frac{1}{a}\right) \qquad A_y = 133.75\,\text{kN}$$

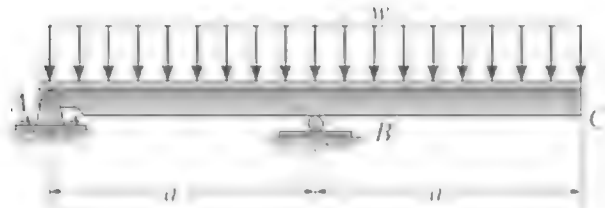
$x_1 = 0, 0.01a .. a$ $\qquad x_2 = a, 1.01a .. a + b$

$$V_1(x) = \left(A_y - w x\right)\frac{1}{\text{kN}} \qquad V_2(x) = \text{F}\,\frac{1}{\text{kN}}$$

$$M_1(x) = \left[A_y x - w\left(\frac{x^2}{2}\right)\right]\frac{1}{\text{kN}\cdot\text{m}} \qquad M_2(x) = [-F(a+b-x) - M]\,\frac{1}{\text{kN}\cdot\text{m}}$$

Force in kN: $\frac{V_1(x_1)}{V_2(x_2)}$ vs. x_1, x_2, Distance in m

Moment (kN-m): $\frac{M_1(x_1)}{M_2(x_2)}$ vs. x_1, x_2, Distance (m)

Problem 7-53

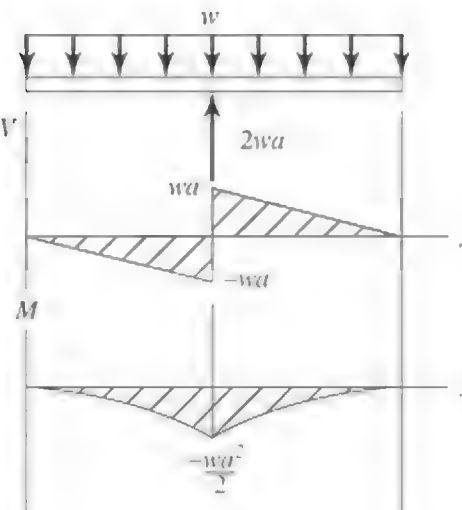
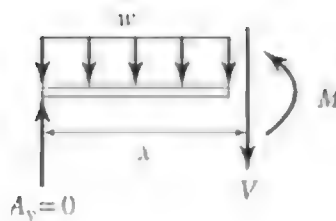
Draw the shear and moment diagrams for the beam.

Solution:

$0 \le x < a$

$\Sigma F_y = 0; \quad -V - wx = 0$

$$V = -wx$$

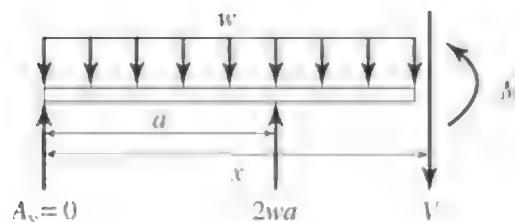
$\Sigma M = 0; \quad M + wx\left(\frac{x}{2}\right) = 0$

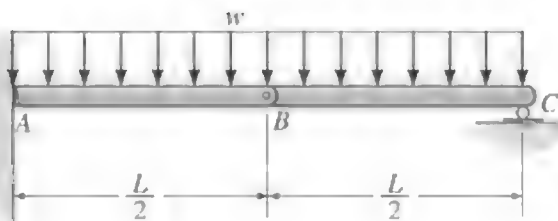
$$M = -w\frac{x^2}{2}$$

$a < x \le 2a$

$\Sigma F_y = 0; \quad -V + 2wa - wx = 0$

$$V = w(2a - x)$$

$\Sigma M = 0; \quad M + wx\left(\frac{x}{2}\right) - 2wa(x - a) = 0$

$$M = 2wa(x - a) - \frac{wx^2}{2}$$

Problem 7-54

Draw the shear and bending-moment diagrams for beam *ABC*. Note that there is a pin at *B*.

Solution:

Support Reactions: From *FBD* (b),

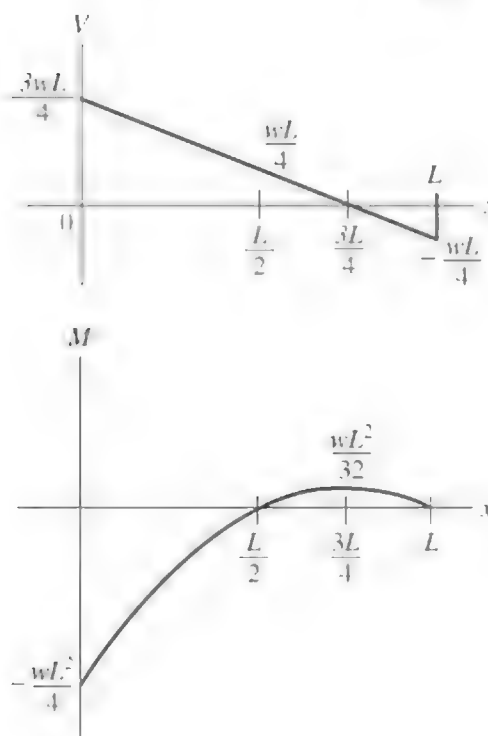
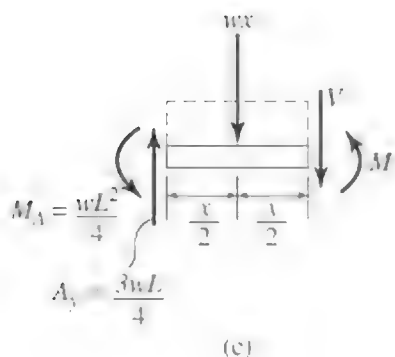
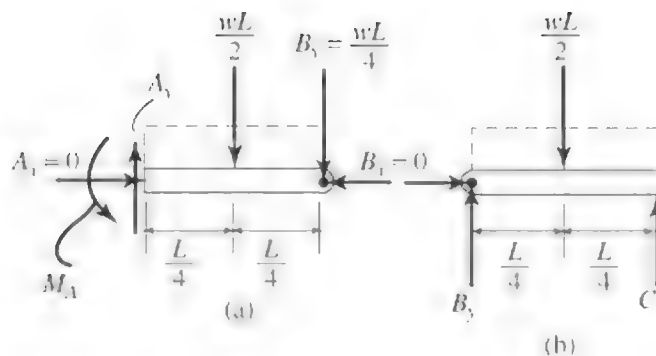
$$\frac{wL}{2}\left(\frac{L}{4}\right) - B_y\left(\frac{L}{2}\right) = 0$$

$$B_y = \frac{wL}{4}$$

From *FBD* (a), $A_y - \frac{wL}{2} - \frac{B}{y} = 0$

$$A_y = \frac{3wL}{4}$$

$$-B_y\frac{L}{2} - w\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + M_A = 0$$

$$M_A = w\left(\frac{L^2}{4}\right)$$

Shear and Moment Functions:
From *FBD* (c) For

$0 \le x \le L \qquad A_y - wx - V = 0$

$$V = \frac{w}{4}(3L - 4x)$$

$$M_A - A_y x + wx\left(\frac{x}{2}\right) + M = 0 \qquad M = \frac{w}{4}\left(3Lx - 2x^2 - L^2\right)$$

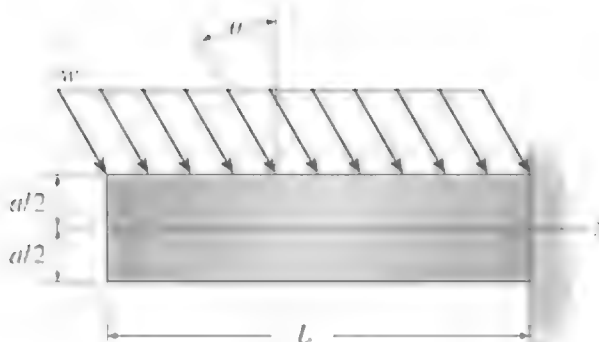
Problem 7-55

The beam has depth a and is subjected to a uniform distributed loading w which acts at an angle θ from the vertical as shown. Determine the internal normal force, shear force, and moment in the beam as a function of x. *Hint*:The moment loading is to be determined from a point along the centerline of the beam (x axis).

Given:

$a = 2\text{ ft}$

$L = 10\text{ ft}$

$\theta = 30\text{ deg}$

$w = 50\,\frac{\text{lb}}{\text{ft}}$

Solution:

$0 \le x \le L$

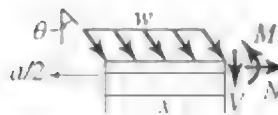
$\Sigma F_x = 0;\qquad N + w\sin(\theta)x = 0$

$$N(x) = -w\sin(\theta)x$$

$\Sigma F_y = 0;\qquad -V - w\cos(\theta)x = 0$

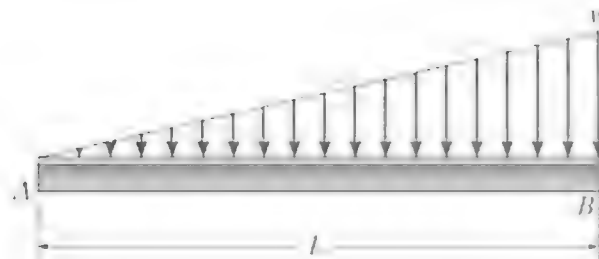
$$V = -w\cos(\theta)x$$

$\Sigma M = 0;\qquad w\cos(\theta)x\left(\frac{x}{2}\right) - w\sin(\theta)x\left(\frac{a}{2}\right) + M = 0$

$$M(x) = -w\cos(\theta)\left(\frac{x^2}{2}\right) + w\sin(\theta)\left(\frac{xa}{2}\right)$$

Problem 7-56

Draw the shear and moment diagrams for the beam.

Given:

$w = 250\,\frac{\text{lb}}{\text{ft}}\qquad L = 12\text{ ft}$

Solution:

$\Sigma F_y = 0;\quad -V - \frac{1}{2}x\frac{wx}{L} = 0 \qquad V(x) = -\frac{wx^2}{2L}\frac{1}{\text{lb}}$

$\Sigma M = 0;\quad M + \frac{x}{2}\left(\frac{wx}{L}\right)\left(\frac{1}{3}x\right) = 0 \qquad M(x) = \frac{-wx^3}{6L}\frac{1}{\text{lb}\cdot\text{ft}}$

Force (lb)

$V(x)$

0, −1000, −2000

0 2 4 6 8 10 12

$\frac{x}{\text{ft}}$

Distance (ft)

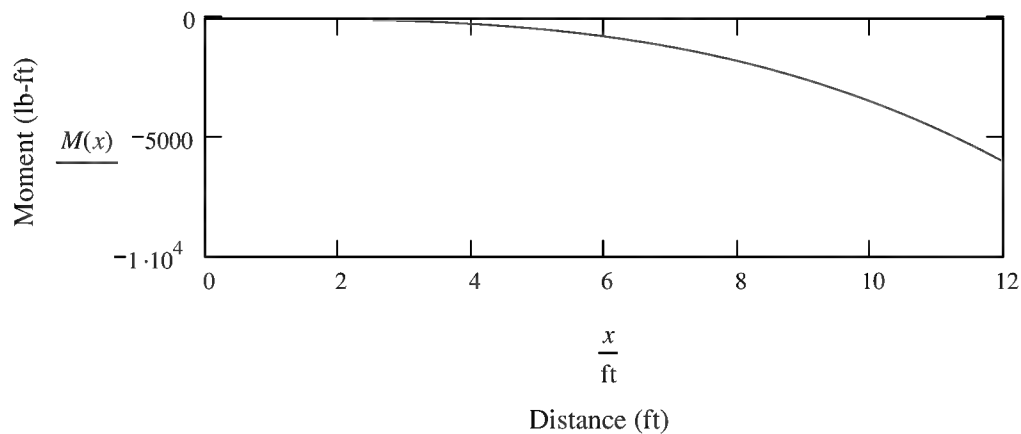
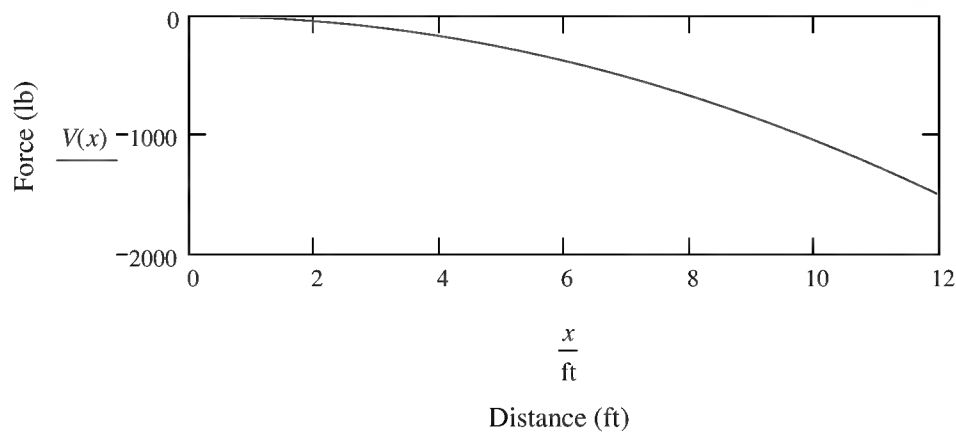
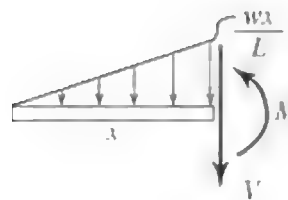
Moment (lb-ft)

$M(x)$

0, −5000, $-1\cdot10^4$

0 2 4 6 8 10 12

$\frac{x}{\text{ft}}$

Distance (ft)

Problem 7-57

The beam will fail when the maximum shear force is V_{max} or the maximum moment is M_{max}. Determine the largest intensity w of the distributed loading it will support.

Given:

$L = 18\ \text{ft}$

$V_{max} = 800\ \text{lb}$

$M_{max} = 1200\ \text{lb}\cdot\text{ft}$

Solution:

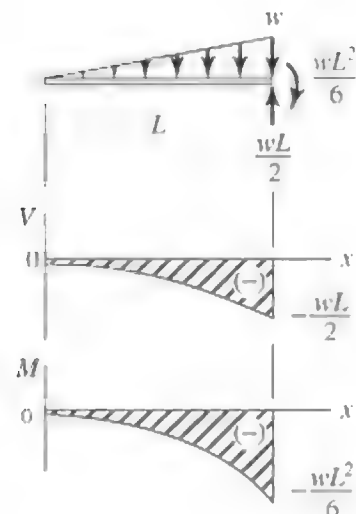
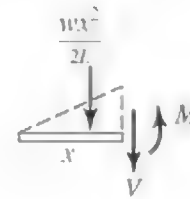
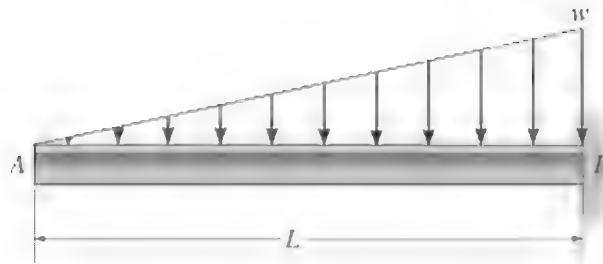
For $0 \le x \le L$

$$V = \frac{-wx^2}{2L} \qquad M = \frac{-wx^3}{6L}$$

$$\left|V_{max}\right| = \frac{wL}{2}$$

$$w_1 = 2\left(\frac{V_{max}}{L}\right) \qquad w_1 = 88.9\,\frac{\text{lb}}{\text{ft}}$$

$$\left|M_{max}\right| = \frac{w_2\,L^2}{6}$$

$$w_2 = 6\left(\frac{M_{max}}{L^2}\right) \qquad w_2 = 22.2\,\frac{\text{lb}}{\text{ft}}$$

Now choose the critical case $\quad w = \min(w_1, w_2) \qquad w = 22.22\,\frac{\text{lb}}{\text{ft}}$

Problem 7-58

The beam will fail when the maximum internal moment is M_{max}. Determine the position x of the concentrated force **P** and its smallest magnitude that will cause failure.

Solution:

For $\xi < x$, $M_1 = \dfrac{P\xi(L-x)}{L}$

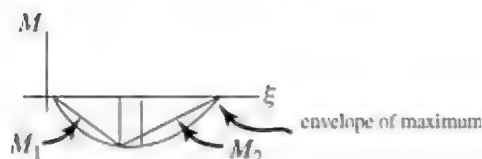
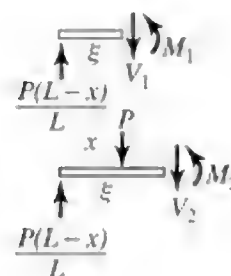
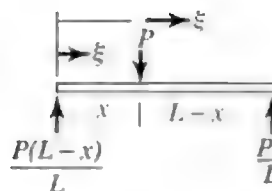
For $\xi > x$, $M_2 = \dfrac{Px(L-\xi)}{L}$

Note that $M_1 = M_2$ when $x = \xi$

$$M_{max} = M_1 = M_2 = \frac{Px(L-x)}{L} = \frac{P}{L}\left(Lx - x^2\right)$$

$$\frac{d}{dx}\left(Lx - x^2\right) = L - 2x \qquad x = \frac{L}{2}$$

Thus, $$M_{max} = \frac{P}{L}\left(\frac{L}{2}\right)\left(L - \frac{L}{2}\right) = \frac{P}{2}\left(\frac{L}{2}\right)$$

$$P = \frac{4M_{max}}{L}$$

Problem 7-59

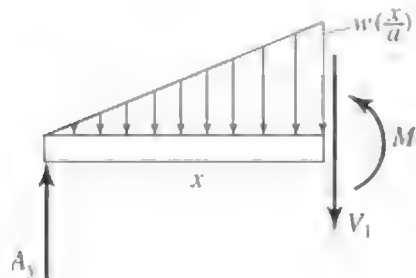
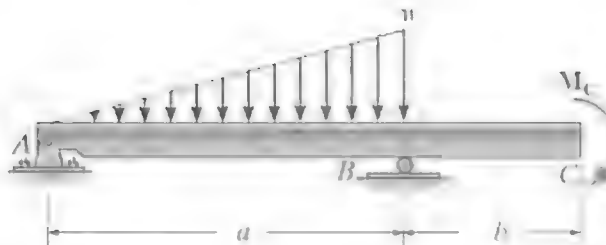
Draw the shear and moment diagrams for the beam.

Given:

$w = 30\ \dfrac{\text{lb}}{\text{ft}}$ $M_C = 180\ \text{lb}\cdot\text{ft}$

$a = 9\ \text{ft}$ $b = 4.5\ \text{ft}$

Solution:

$$-A_y a + \frac{1}{2}wa\left(\frac{a}{3}\right) - M_C = 0$$

$$A_y = \frac{wa}{6} - \frac{M_C}{a}$$

$$A_y + B_y - \frac{1}{2}wa = 0$$

$$B_y = \frac{wa}{2} - A_y$$

1/2 (w)(a)

$\left(\frac{2a}{3}\right)$ x M_2 V_2 A_y B_y

$$x_1 = 0, 0.01a .. a$$

$$A_y - \frac{1}{2}\, w\left(\frac{x}{a}\right)x - V_1(x) = 0 \qquad V_1(x) = \left(A_y - \frac{wx^2}{2a}\right)\frac{1}{\text{lb}}$$

$$-A_y x + \frac{1}{2}w\left(\frac{x}{a}\right)x\left(\frac{x}{3}\right) + M_1(x) = 0 \qquad M_1(x) = \left(A_y x - \frac{wx^3}{6a}\right)\frac{1}{\text{lb}\cdot\text{ft}}$$

$$x_2 = a, 1.01a .. a + b$$

$$A_y - \frac{1}{2}wa + B_y - V_2(x) = 0 \qquad V_2(x) = \left(A_y + B_y - \frac{wa}{2}\right)\frac{1}{\text{lb}}$$

$$-A_y x + \frac{1}{2}wa\left(x - \frac{2a}{3}\right) - B_y(x - a) + M_2(x) = 0$$

$$M_2(x) = \left[A_y x + B_y(x-a) - \frac{wa}{2}\cdot\left(x - \frac{2a}{3}\right)\right]\frac{1}{\text{lb}\cdot\text{ft}}$$

Force (lb) $\frac{V_1(x_1)}{V_2(x_2)}$

100, 0, −100, −200

0, 2, 4, 6, 8, 10, 12

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

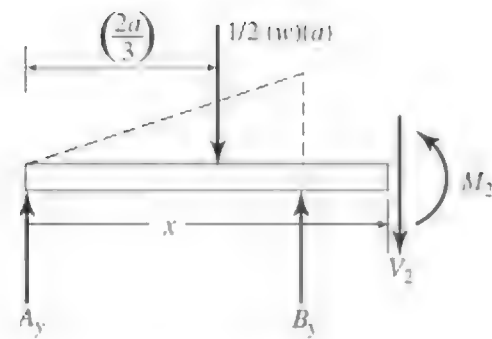
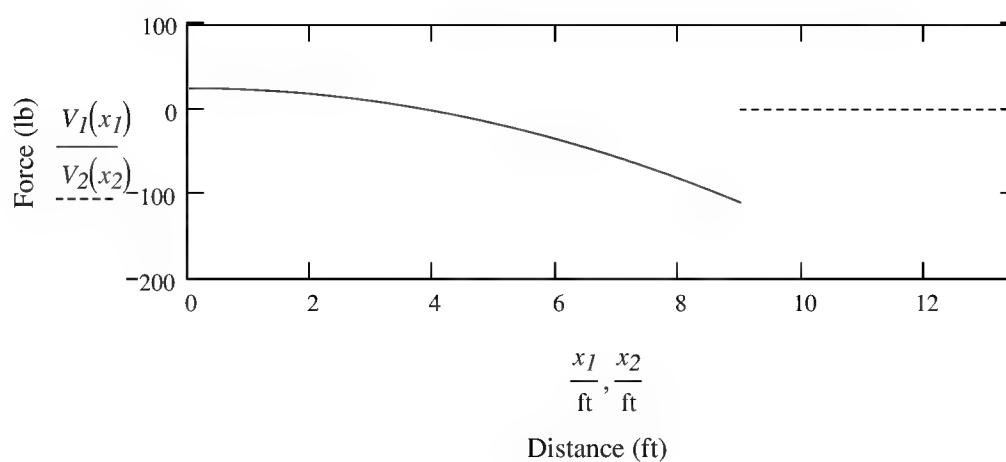
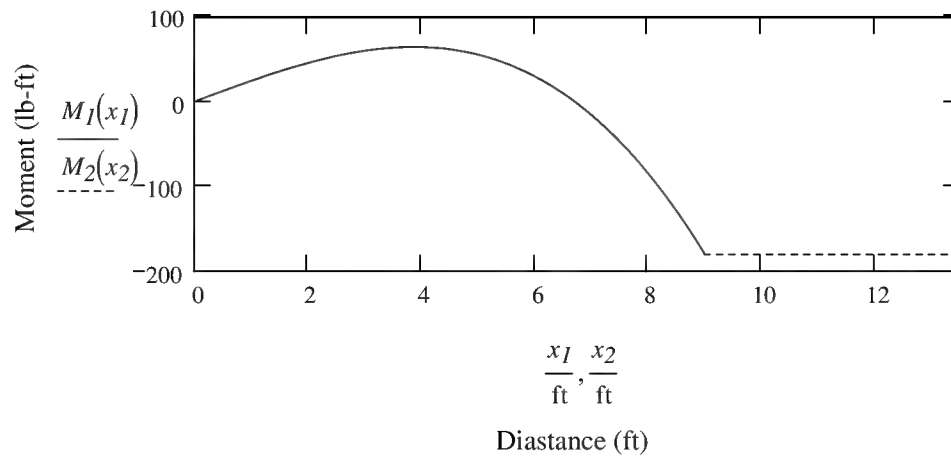
Distance (ft)

Moment (lb-ft)

$\frac{M_1(x_1)}{}$

$M_2(x_2)$

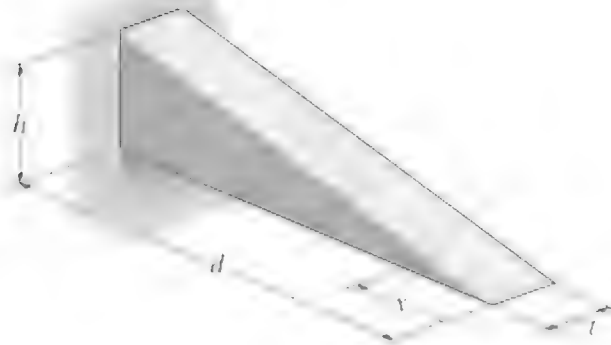
$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

Diastance (ft)

Problem 7-60

The cantilevered beam is made of material having a specific weight γ. Determine the shear and moment in the beam as a function of x.

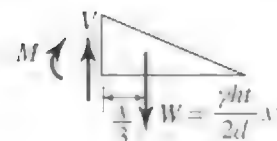
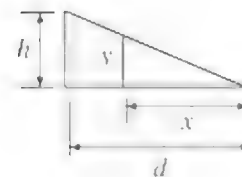
Solution:

By similar triangles

$$\frac{y}{x} = \frac{h}{d} \qquad y = \frac{h}{d}\,x$$

$$W = \gamma V = \gamma\left(\frac{1}{2}yxt\right) = \gamma\left[\frac{1}{2}\left(\frac{h}{d}x\right)xt\right] = \left(\frac{\gamma h t}{2d}\right)x^2$$

$$\Sigma F_y = 0; \qquad V - \left(\frac{\gamma h t}{2d}\right)x^2 = 0$$

$$V = \left(\frac{\gamma h t}{2d}\right)x^2$$

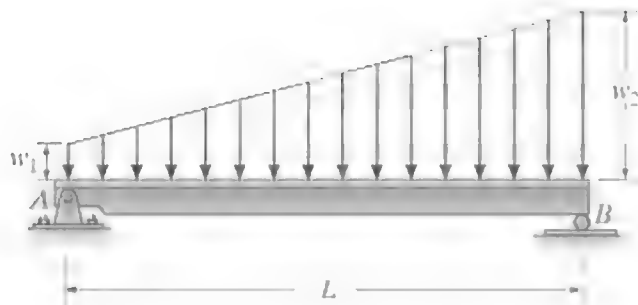
$$\Sigma M = 0; \qquad -M - \left(\frac{\gamma h t}{2d}\right)x^2\left(\frac{x}{3}\right) = 0$$

$$M = -\frac{\gamma h t}{6d} x^3$$

Problem 7-61

Draw the shear and moment diagrams for the beam.

Given: $\text{kip} = 10^3 \text{ lb}$

$w_1 = 30 \frac{\text{lb}}{\text{ft}}$ $\quad w_2 = 120 \frac{\text{lb}}{\text{ft}}$

$L = 12 \text{ ft}$

Solution:

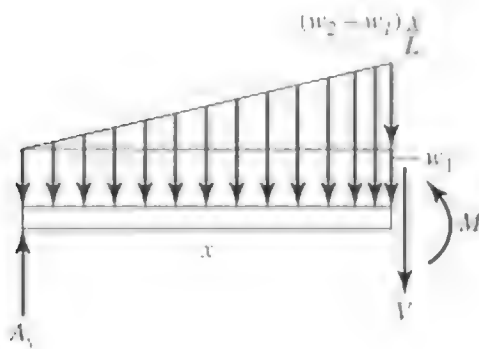
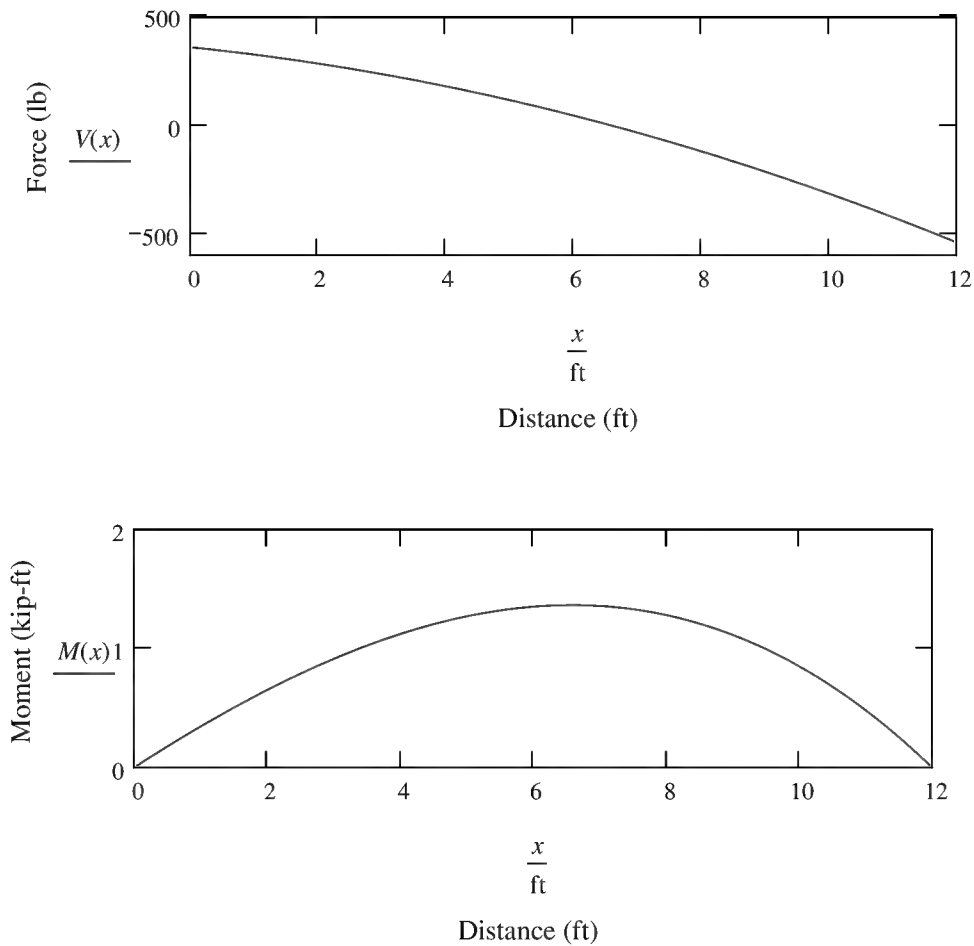
$$w_1 L \left(\frac{L}{2}\right) + \frac{1}{2}(w_2 - w_1)L\left(\frac{L}{3}\right) - A_y L = 0$$

$$A_y = w_1\left(\frac{L}{2}\right) + \left(\frac{w_2 - w_1}{2}\right)\left(\frac{L}{3}\right)$$

$$A_y - w_1 x - \frac{1}{2}(w_2 - w_1)\left(\frac{x}{L}\right)x - V(x) = 0$$

$$V(x) = \left[A_y - w_1 x - \frac{1}{2}(w_2 - w_1)\left(\frac{x^2}{L}\right)\right]\frac{1}{\text{lb}}$$

$$-A_y x + w_1 x\left(\frac{x}{2}\right) + \frac{1}{2}(w_2 - w_1)\left(\frac{x}{L}\right) x \left(\frac{x}{3}\right) + M(x) = 0$$

$$M(x) = \left[A_y x - w_1\left(\frac{x^2}{2}\right) - (w_2 - w_1)\left(\frac{x^3}{6L}\right)\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

Problem 7-62

Draw the shear and moment diagrams for the beam.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$F_1 = 20 \text{ kip}$ $F_2 = 20 \text{ kip}$ $w = 4 \frac{\text{kip}}{\text{ft}}$ $a = 15 \text{ ft}$ $b = 30 \text{ ft}$ $c = 15 \text{ ft}$

Solution:

$$F_1(a+b) + wb\left(\frac{b}{2}\right) - F_2 c - A_y b = 0 \qquad A_y = F_1\left(\frac{a+b}{b}\right) + \frac{wb}{2} - F_2\left(\frac{c}{b}\right)$$

$$A_y + B_y - F_1 - F_2 - wb = 0 \qquad B_y = F_1 + F_2 + wb - A_y$$

$x_1 = 0, 0.01a .. a$

$-F_1 - V_1(x) = 0 \qquad V_1(x) = -F_1 \frac{1}{\text{kip}}$

$F_1 x + M_1(x) = 0 \qquad M_1(x) = -F_1 x \frac{1}{\text{kip}\cdot\text{ft}}$

$x_2 = a, 1.01a .. a + b$

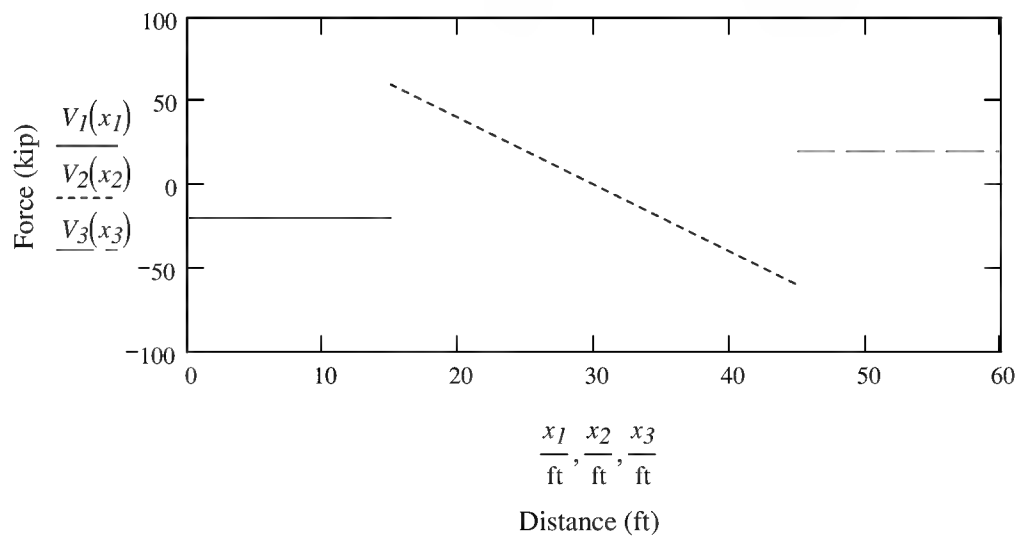
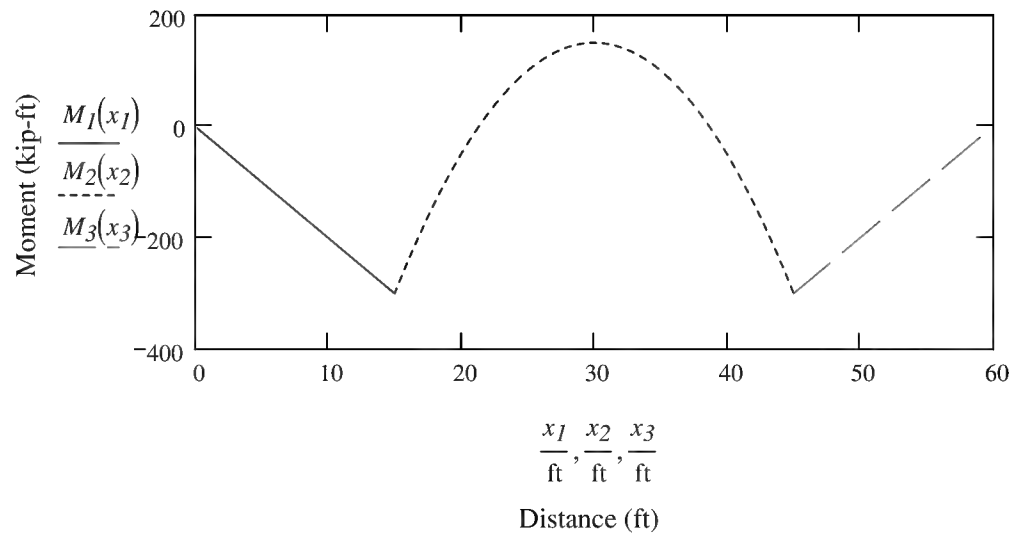
$-F_1 - w(x-a) + A_y - V_2(x) = 0 \qquad V_2(x) = \left[-F_1 - w(x-a) + A_y\right] \frac{1}{\text{kip}}$

$F_1 x - A_y(x-a) + w(x-a)\left(\frac{x-a}{2}\right) + M_2(x) = 0$

$$M_2(x) = \left[-F_1 x + A_y(x-a) - w\frac{(x-a)^2}{2}\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

$x_3 = a + b, 1.01(a+b) .. a + b + c$

$V_3(x) - F_2 = 0 \qquad V_3(x) = F_2 \frac{1}{\text{kip}}$

$-M_3(x) - F_2(a+b+c-x) = 0 \qquad M_3(x) = -F_2(a+b+c-x) \frac{1}{\text{kip}\cdot\text{ft}}$

Problem 7-63

Express the x, y, z components of internal loading in the rod at the specific value for y, where $0 < y < a$.

Units Used:

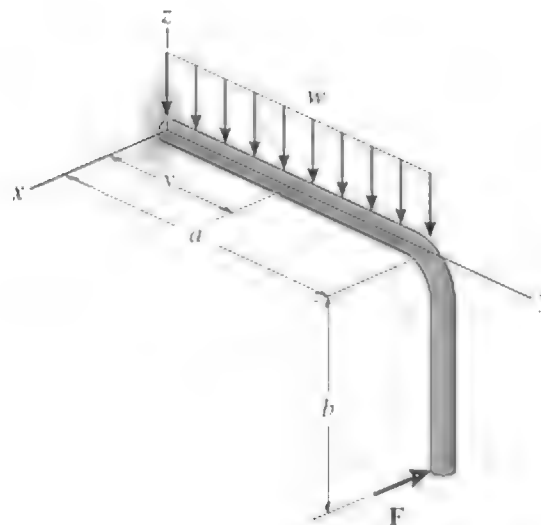
$$\text{kip} = 10^3 \text{ lb}$$

Given: $y = 2.5 \text{ ft}$

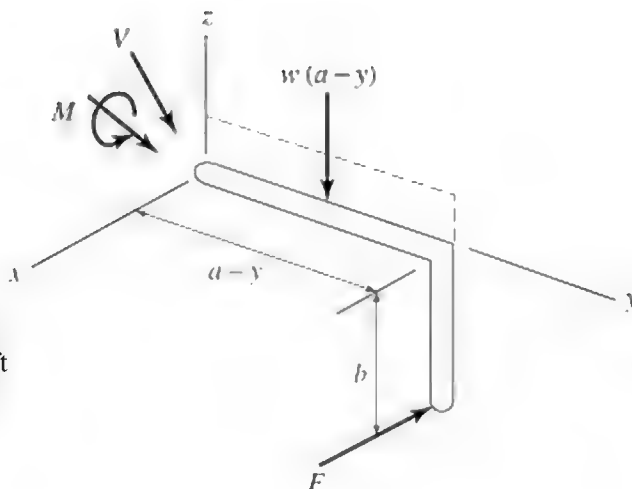
$w = 800 \frac{\text{lb}}{\text{ft}}$ $F = 1500 \text{ lb}$

$a = 4 \text{ ft}$ $b = 2 \text{ ft}$

Solution: In general we have

$$\mathbf{V} = \begin{bmatrix} F \\ 0 \\ w(a-y) \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} w(a-y)\left(\frac{a-y}{2}\right) \\ -F\,b \\ -F(a-y) \end{bmatrix}$$

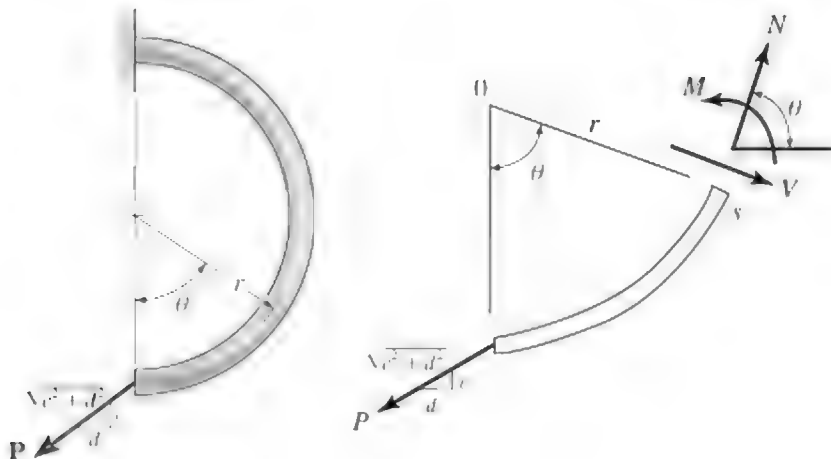
For these values we have

$$\mathbf{V} = \begin{pmatrix} 1500.00 \\ 0.00 \\ 1200.00 \end{pmatrix} \text{lb} \quad \mathbf{M} = \begin{pmatrix} 900.00 \\ -3000.00 \\ -2250.00 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 7-64

Determine the normal force, shear force, and moment in the curved rod as a function of θ.

Given:

$c = 3$

$d = 4$

Solution:

For $0 \le \theta \le \pi$

$\Sigma F_x = 0;$

$$N - \left(\frac{d}{\sqrt{c^2+d^2}}\right)P\cos(\theta) - \left(\frac{c}{\sqrt{c^2+d^2}}\right)P\sin(\theta) = 0$$

$$N = \left(\frac{P}{\sqrt{c^2+d^2}}\right)(d\cos(\theta) + c\sin(\theta))$$

$\Sigma F_y = 0;$

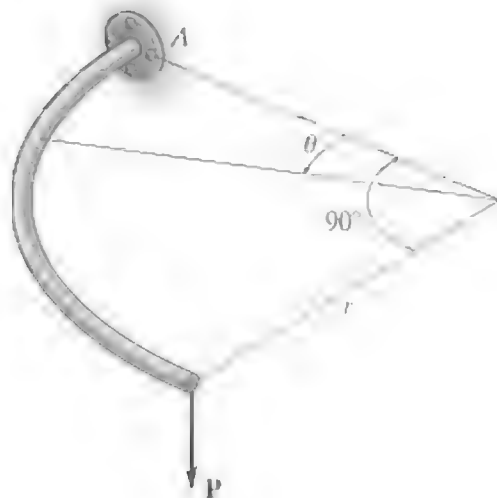
$$V - \left(\frac{d}{\sqrt{c^2+d^2}}\right)P\sin(\theta) + \left(\frac{c}{\sqrt{c^2+d^2}}\right)P\cos(\theta) = 0$$

$$V = \left(\frac{P}{\sqrt{c^2+d^2}}\right)(d\sin(\theta) - c\cos(\theta))$$

$\Sigma M = 0;$ $\left(\frac{-d}{\sqrt{c^2+d^2}}\right)P(r - r\cos(\theta)) + \left(\frac{c}{\sqrt{c^2+d^2}}\right)P\,r\sin(\theta) + M = 0$

$$M = \left(\frac{P\,r}{\sqrt{c^2+d^2}}\right)(d - d\cos(\theta) - c\sin(\theta))$$

Problem 7-65

The quarter circular rod lies in the horizontal plane and supports a vertical force **P** at its end. Determine the magnitudes of the components of the internal shear force, moment, and torque acting in the rod as a function of the angle θ.

Solution:

$\Sigma F_z = 0;$ $V = |P|$

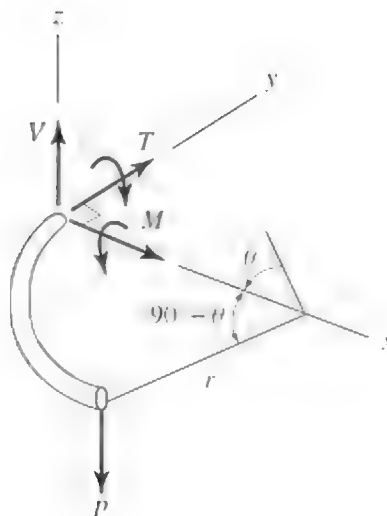
$\Sigma M_x = 0;$ $M + P\,r\cos(\theta) = 0$

$$M = -P\,r\cos(\theta)$$

$$M = |P\,r\cos(\theta)|$$

$\Sigma M_y = 0;$ $T + P\,r(1 - \sin(\theta)) = 0$

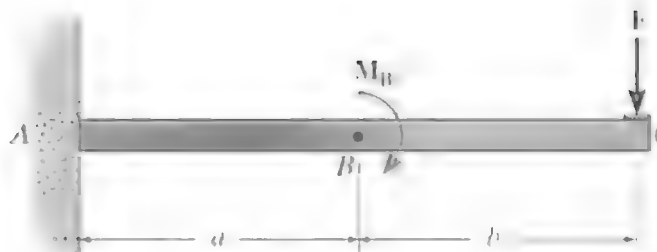
$$T = -P\,r(1 - \sin(\theta))$$

$$T = |P\,r(1 - \sin(\theta))|$$

Problem 7-66

Draw the shear and moment diagrams for the beam.

Given:

$M_B = 800\ \text{lb}\cdot\text{ft}$ $a = 5\ \text{ft}$

$F = 100\ \text{lb}$ $b = 5\ \text{ft}$

Solution:

$x_1 = 0, 0.01a .. a$ $V_1(x) = F\frac{1}{\text{lb}}$ $M_1(x) = \left[-F(a+b-x) - M_B\right]\frac{1}{\text{lb}\cdot\text{ft}}$

$x_2 = a, 1.01a .. a+b$ $V_2(x) = F\frac{1}{\text{lb}}$ $M_2(x) = -F(a+b-x)\frac{1}{\text{lb}\cdot\text{ft}}$

Force (lb)

$\frac{V_1(x_1)}{V_2(x_2)}$

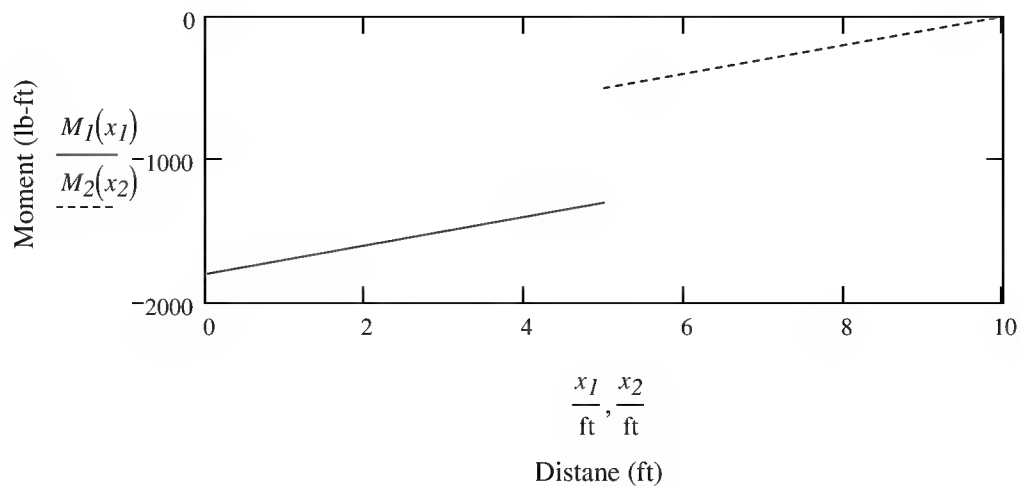
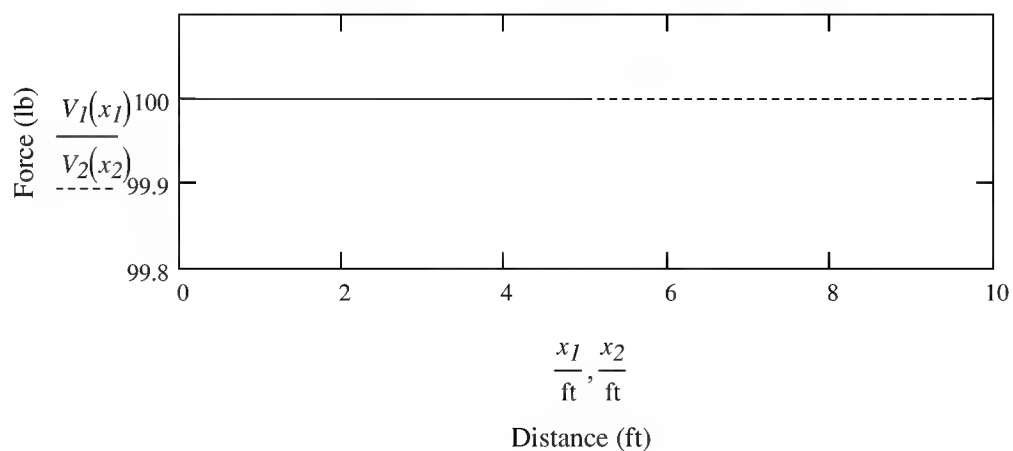
$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

Distance (ft)

Moment (lb-ft)

$\frac{M_1(x_1)}{M_2(x_2)}$

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

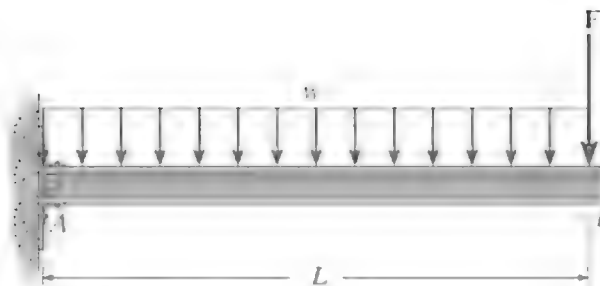
Distane (ft)

Problem 7-67

Draw the shear and moment diagrams for the beam.

Units Used: $kN = 10^3\ N$

Given:

$w = 3\ \frac{kN}{m}$ $F = 10\ kN$

$L = 6\ m$

Solution:

$$V(x) = [w(L-x) + F]\ \frac{1}{kN}$$

$$M(x) = \left[-w\frac{(L-x)^2}{2} - F(L-x)\right]\frac{1}{kN \cdot m}$$

Force (kN)

$V(x)$

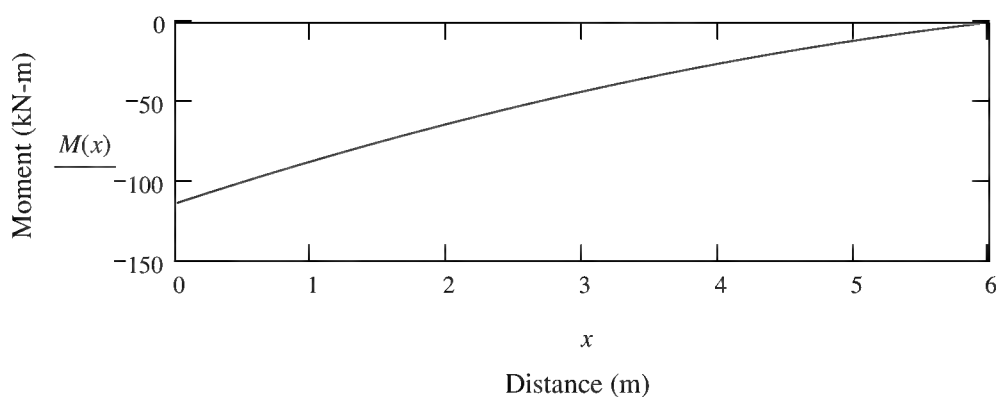
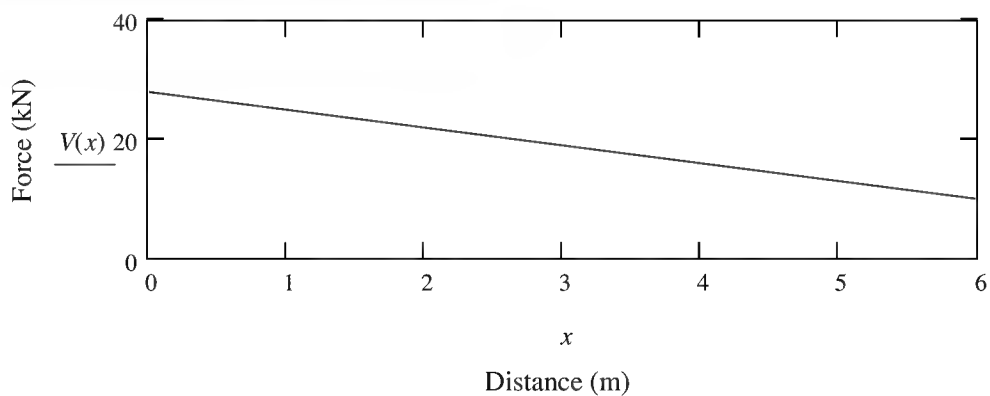
x

Distance (m)

Moment (kN-m)

$M(x)$

x

Distance (m)

Problem 7-68

Draw the shear and moment diagrams for the beam.

Units Used: $kN = 10^3\ N$

Given: $F = 7\ kN$ $M = 12\ kN\cdot m$ $a = 2\ m$ $b = 2\ m$ $c = 4\ m$

Solution: Guesses $A = 1\ N$ $B = 1\ N$

Given $A + B - F = 0$ $-F a - M + B(a + b + c) = 0$ $\begin{pmatrix} A \\ B \end{pmatrix} = \text{Find}(A, B)$

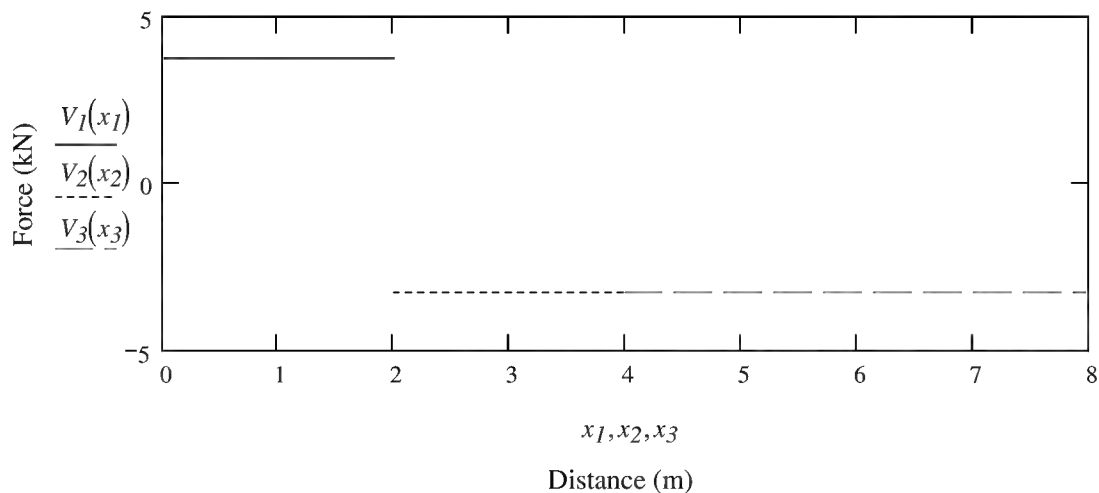
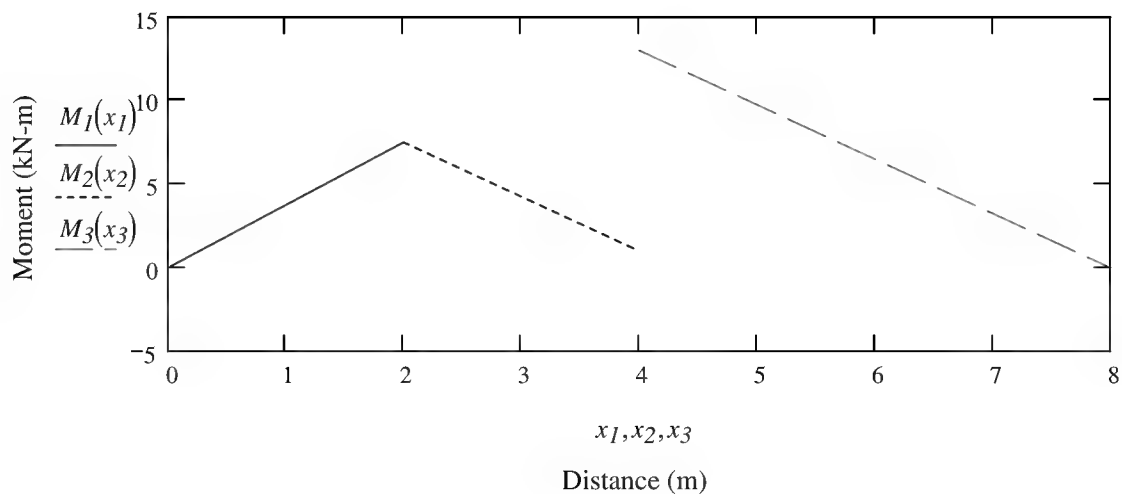
$x_1 = 0, 0.01a .. a$ $x_2 = a, 1.01a .. a + b$ $x_3 = a + b, 1.01(a + b) .. a + b + c$

$V_1(x_1) = A \dfrac{1}{kN}$ $V_2(x_2) = (A - F) \dfrac{1}{kN}$ $V_3(x_3) = -B \dfrac{1}{kN}$

$M_1(x_1) = \dfrac{A x_1}{kN\cdot m}$ $M_2(x_2) = \dfrac{A x_2 - F(x_2 - a)}{kN\cdot m}$ $M_3(x_3) = B(a + b + c - x_3)\dfrac{1}{kN\cdot m}$

Moment (kN-m): $M_1(x_1)$, $M_2(x_2)$, $M_3(x_3)$

x_1, x_2, x_3

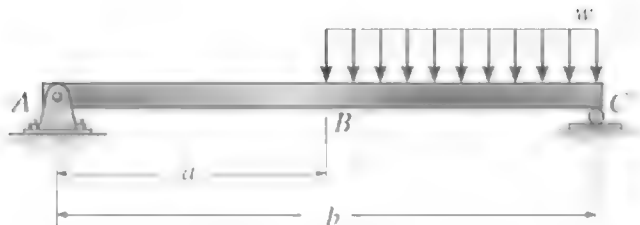
Distance (m)

Problem 7-69

Draw the shear and moment diagrams for the beam.

Units Used:

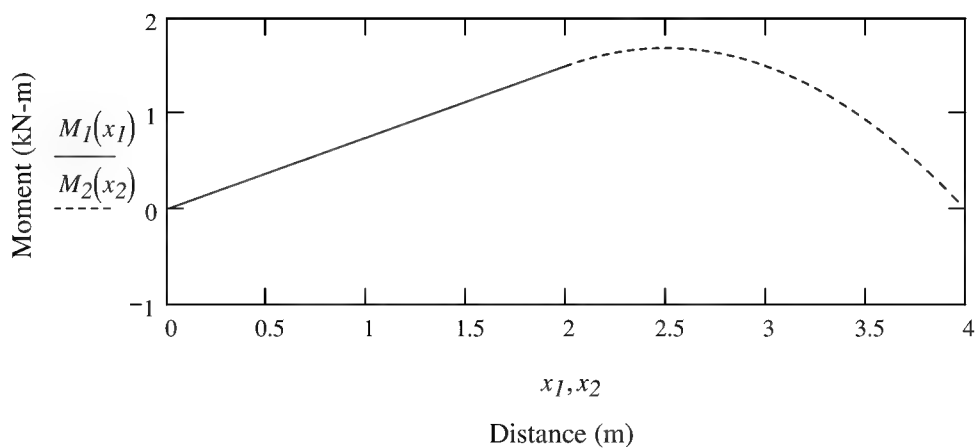
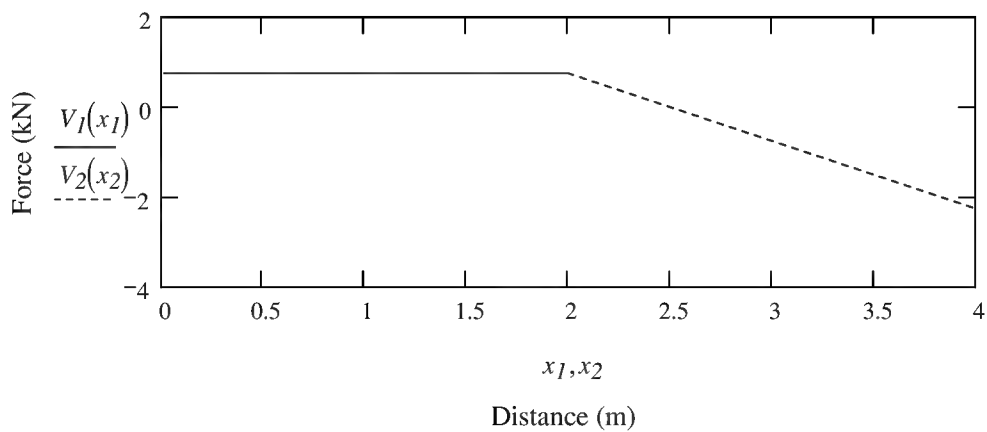
$$\text{kN} = 10^3 \text{ N}$$

Given:

$$a = 2 \text{ m} \qquad b = 4 \text{ m} \qquad w = 1.5 \frac{\text{kN}}{\text{m}}$$

Solution:

$$w(b-a)\left(\frac{b-a}{2}\right) - A_y b = 0 \qquad A_y = \frac{w(b-a)^2}{2b} \qquad A_y = 0.75 \text{ kN}$$

$$x_1 = 0, 0.01a \,..\, a \qquad V_1(x) = A_y \frac{1}{\text{kN}} \qquad M_1(x) = A_y x \frac{1}{\text{kN}\cdot\text{m}}$$

$$x_2 = b-a, 1.01(b-a) \,..\, b \qquad V_2(x) = \left[A_y - w(x-a)\right]\frac{1}{\text{kN}} \qquad M_2(x) = \left[A_y x - w\frac{(x-a)^2}{2}\right]\frac{1}{\text{kN}\cdot\text{m}}$$

Problem 7-70

Draw the shear and moment diagrams for the beam.

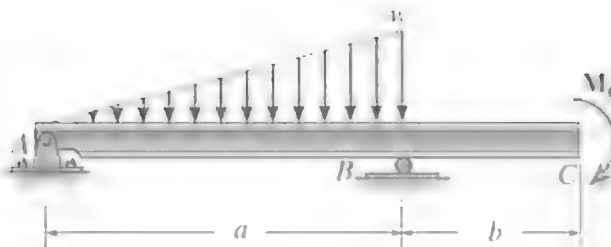
Given:

$w = 30\ \frac{\text{lb}}{\text{ft}}$ $\quad M_C = 180\ \text{lb}\cdot\text{ft}$

$a = 9\ \text{ft}$ $\quad b = 4.5\ \text{ft}$

Solution:

$$-A_y a + \frac{1}{2} w a\left(\frac{a}{3}\right) - M_C = 0$$

$$A_y = \frac{w a}{6} - \frac{M_C}{a}$$

$$A_y + B_y - \frac{1}{2} w a = 0$$

$$B_y = \frac{w a}{2} - A_y$$

$$x_1 = 0, 0.01a .. a$$

$$V_1(x) = \left(A_y - \frac{w x^2}{2a} \right) \frac{1}{\text{lb}} \qquad M_1(x) = \left(A_y x - \frac{w x^3}{6a} \right) \frac{1}{\text{lb} \cdot \text{ft}}$$

$$x_2 = a, 1.01a .. a + b$$

$$V_2(x) = \left(A_y + B_y - \frac{w a}{2} \right) \frac{1}{\text{lb}}$$

$$M_2(x) = \left[A_y x + B_y (x - a) - \frac{w a}{2} \left(x - \frac{2a}{3} \right) \right] \frac{1}{\text{lb} \cdot \text{ft}}$$

Force (lb)

$V_1(x_1)$

$V_2(x_2)$

100, 0, −100, −200

0, 2, 4, 6, 8, 10, 12

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

Distance (ft)

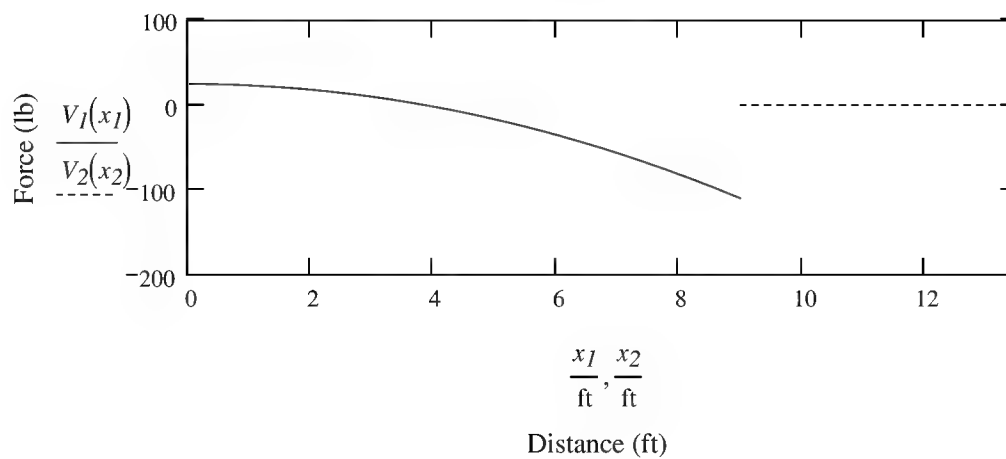
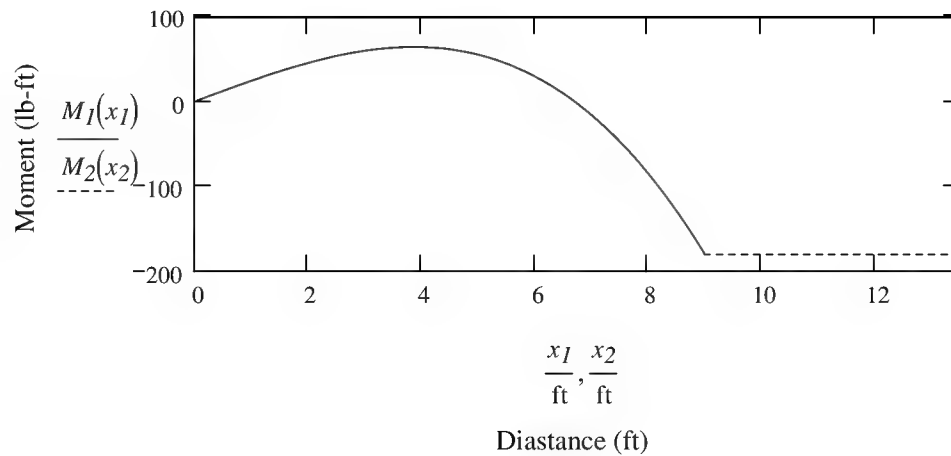
Moment (lb-ft)

$\frac{M_1(x_1)}{M_2(x_2)}$

100, 0, −100, −200

0, 2, 4, 6, 8, 10, 12

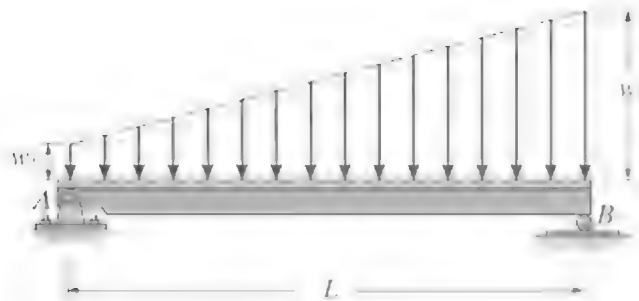
$\frac{x_1}{ft}, \frac{x_2}{ft}$

Diastance (ft)

Problem 7-71

Draw the shear and moment diagrams for the beam.

Given: $kip = 10^3\ lb$

$w_1 = 30\ \frac{lb}{ft}$ $\quad w_2 = 120\ \frac{lb}{ft}$

$L = 12\ ft$

Solution:

$$w_1\ L\left(\frac{L}{2}\right) + \frac{1}{2}(w_2 - w_1)L\left(\frac{L}{3}\right) - A_y L = 0$$

$$A_y = w_1\left(\frac{L}{2}\right) + \left(\frac{w_2 - w_1}{2}\right)\left(\frac{L}{3}\right)$$

$$V(x) = \left[A_y - w_1 x - \frac{1}{2}(w_2 - w_1)\frac{x^2}{L}\right]\frac{1}{lb}$$

$$M(x) = \left[A_y x - w_1 \frac{x^2}{2} - (w_2 - w_1)\frac{x^3}{6L}\right]\frac{1}{kip \cdot ft}$$

Force (lb)

$\frac{V(x)}{}$

500, 0, −500

0 2 4 6 8 10 12

$\frac{x}{\text{ft}}$

Distance (ft)

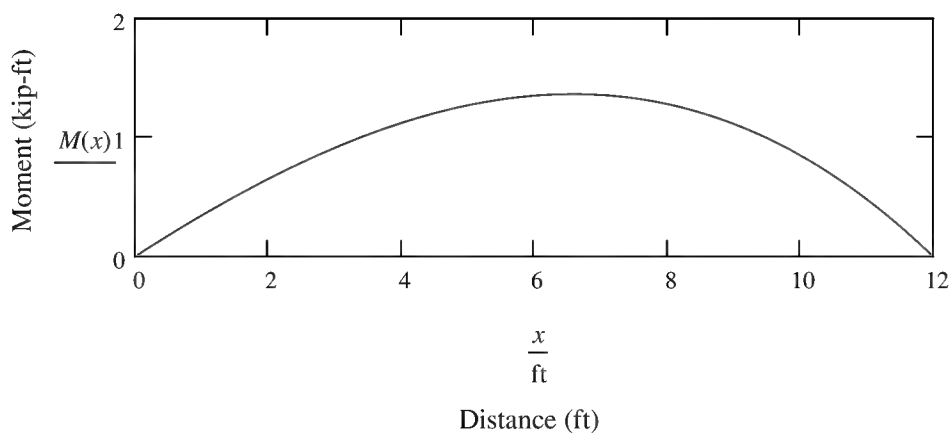
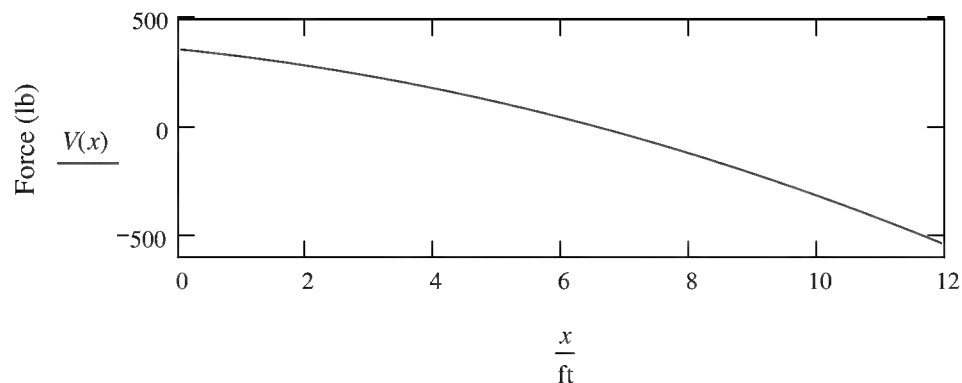
Moment (kip-ft)

$\frac{M(x)}{}$

2, 1, 0

0 2 4 6 8 10 12

$\frac{x}{\text{ft}}$

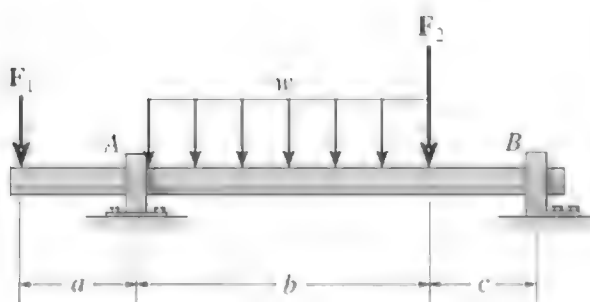
Distance (ft)

Problem 7-72

Draw the shear and moment diagrams for the shaft. The support at A is a journal bearing and at B it is a thrust bearing.

F_1, F_2, w, A, B, a, b, c

Given:

$F_1 = 400\ \text{lb}$ $F_2 = 800\ \text{lb}$

$w = 100\ \frac{\text{lb}}{\text{in}}$ $a = 4\ \text{in}$ $b = 12\ \text{in}$ $c = 4\ \text{in}$

Solution: $F_1 a - w b\left(\frac{b}{2}\right) - F_2 b + B(b+c) = 0$ $B = \dfrac{w\left(\frac{b^2}{2}\right) + F_2 b - F_1 a}{b+c}$ $B = 950.00\,\text{lb}$

$$x_1 = 0, 0.01a .. a \qquad V_1(x) = -F_1 \frac{1}{\text{lb}} \qquad M_1(x) = -F_1 x \frac{1}{\text{lb}\cdot\text{in}}$$

$$x_2 = a, 1.01a .. a+b \qquad V_2(x_2) = \left[-B + F_2 + w(a+b-x_2)\right]\frac{1}{\text{lb}}$$

$$M_2(x_2) = \left[B(a+b+c-x_2) - F_2(a+b-x_2) - w\cdot\frac{(a+b-x_2)^2}{2}\right]\frac{1}{\text{lb}\cdot\text{in}}$$

$$x_3 = a+b, 1.01(a+b) .. a+b+c \qquad V_3(x_3) = \frac{-B}{\text{lb}} \qquad M_3(x_3) = B(a+b+c-x_3)\frac{1}{\text{lb}\cdot\text{in}}$$

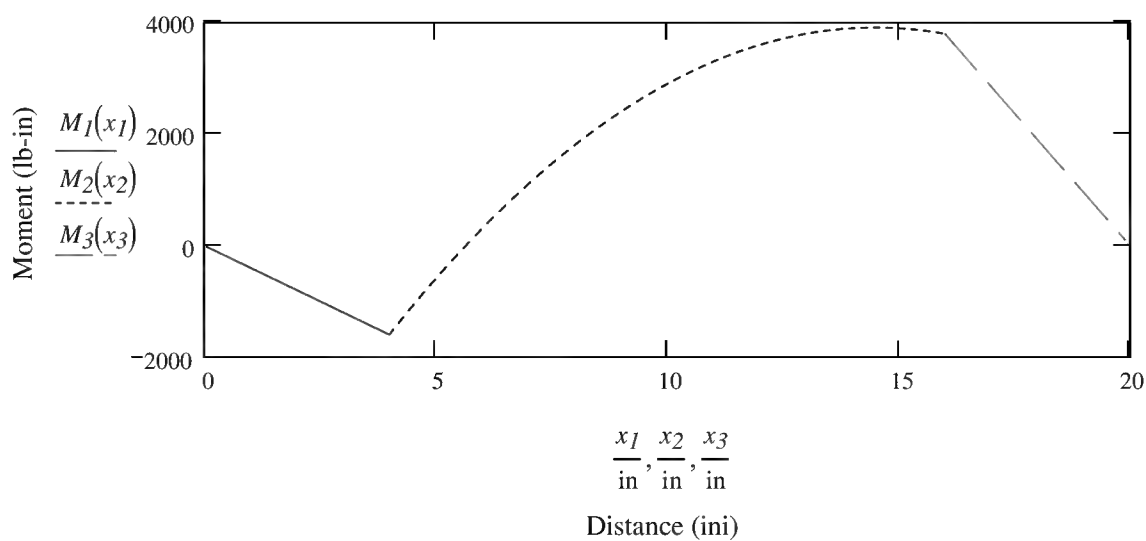
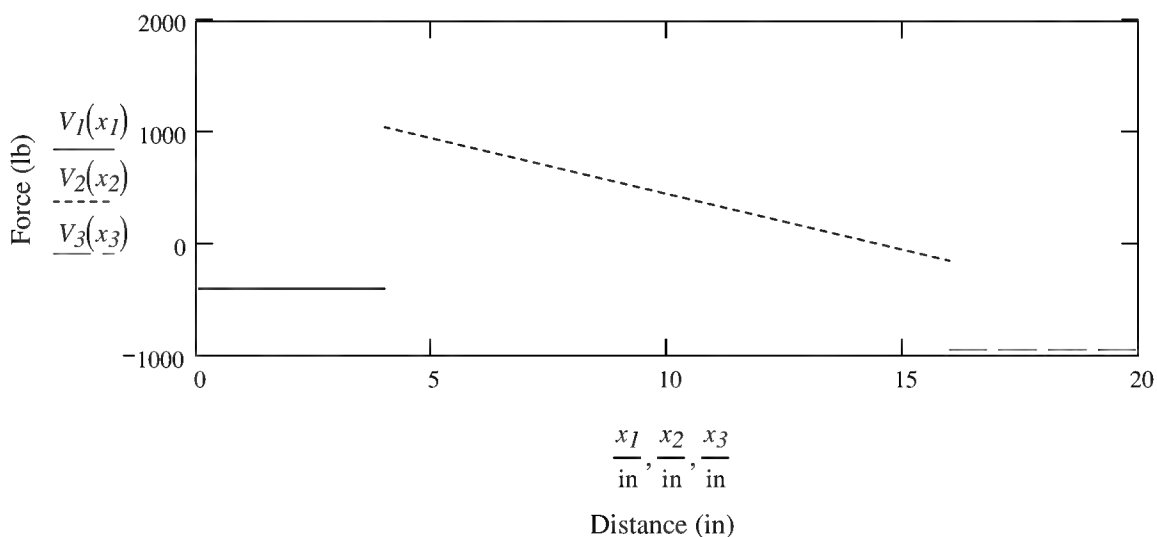
Force (lb): $V_1(x_1)$, $V_2(x_2)$, $V_3(x_3)$ vs. $\frac{x_1}{\text{in}}, \frac{x_2}{\text{in}}, \frac{x_3}{\text{in}}$

y-axis: −1000, 0, 1000, 2000; x-axis: 0, 5, 10, 15, 20

Distance (in)

Moment (lb-in): $M_1(x_1)$, $M_2(x_2)$, $M_3(x_3)$ vs. $\frac{x_1}{\text{in}}, \frac{x_2}{\text{in}}, \frac{x_3}{\text{in}}$

y-axis: −2000, 0, 2000, 4000; x-axis: 0, 5, 10, 15, 20

Distance (ini)

Problem 7-73

Draw the shear and moment diagrams for the beam.

Units Used:

$$kN = 10^3\ N$$

Given:

$F_1 = 10\ kN$ $F_2 = 10\ kN$ $M_0 = 12\ kN\cdot m$

$a = 2\ m$ $b = 2\ m$ $c = 2\ m$ $d = 2\ m$

Solution:

$$F_1(b+c+d) - M_0 + F_2 d - A_y(a+b+c+d) = 0$$

$$A_y = \frac{F_1(b+c+d) - M_0 + F_2 d}{a+b+c+d} \qquad A_y = 8.50\,\text{kN}$$

$$A_y + B_y - F_1 - F_2 = 0 \qquad B_y = F_1 + F_2 - A_y \qquad B_y = 11.50\,\text{kN}$$

$x_1 = 0, 0.01a\,..\,a$

$$V_1(x) = A_y \frac{1}{kN} \qquad M_1(x) = A_y x \frac{1}{kN\cdot m}$$

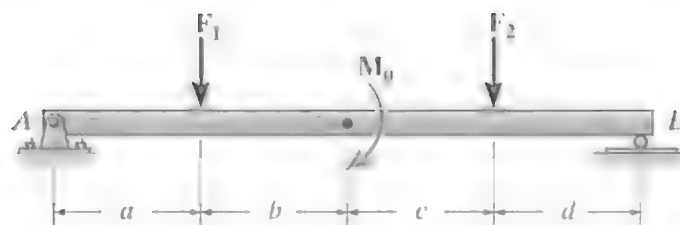
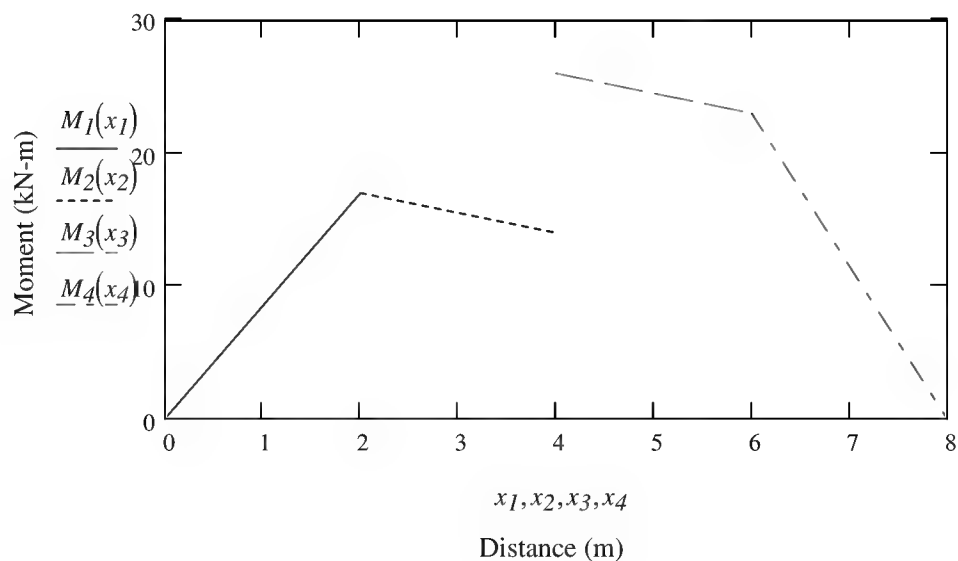
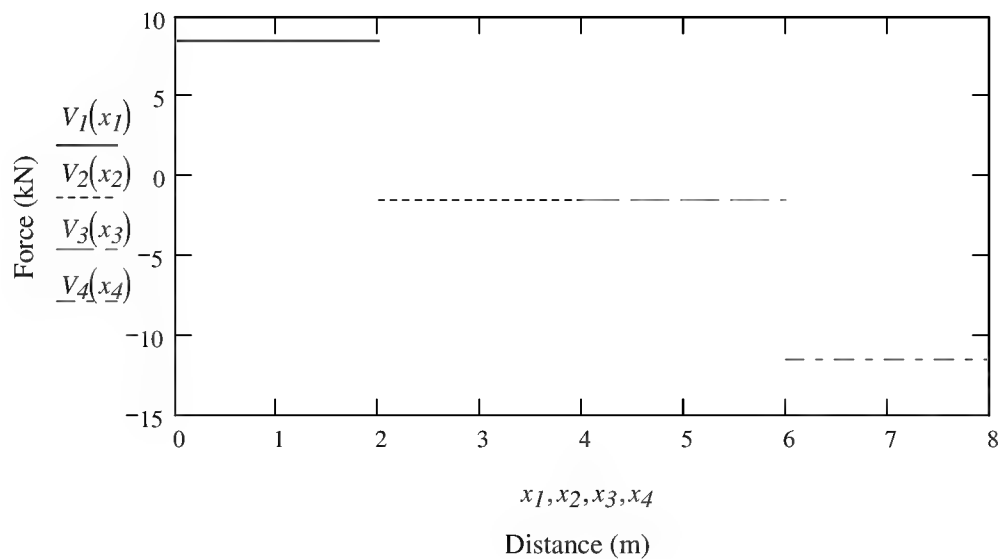
$x_2 = a, 1.01a\,..\,a + b$

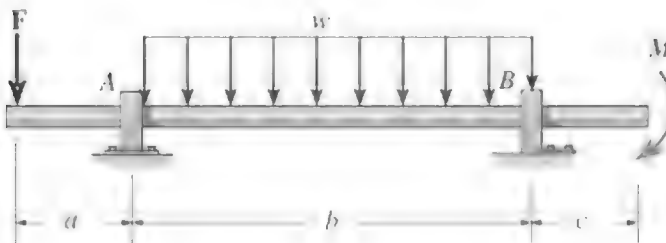
$$V_2(x) = \left(A_y - F_1\right)\frac{1}{kN} \qquad M_2(x) = \left[A_y x - F_1(x-a)\right]\frac{1}{kN\cdot m}$$

$x_3 = a + b, 1.01(a+b)\,..\,a+b+c$

$$V_3(x) = \left(A_y - F_1\right)\frac{1}{kN} \qquad M_3(x) = \left[A_y x - F_1(x-a) + M_0\right]\frac{1}{kN\cdot m}$$

$x_4 = a+b+c, 1.01(a+b+c)\,..\,a+b+c+d$

$$V_4(x) = -B_y \frac{1}{kN} \qquad M_4(x) = B_y(a+b+c+d-x)\frac{1}{kN\cdot m}$$

Problem 7-74

Draw the shear and moment diagrams for the shaft. The support at A is a journal bearing and at B it is a thrust bearing.

Given:

$F = 200 \text{ lb}$ $w = 100 \frac{\text{lb}}{\text{ft}}$ $M = 300 \text{ lb}\cdot\text{ft}$ $a = 1 \text{ ft}$ $b = 4 \text{ ft}$ $c = 1 \text{ ft}$

Solution:

$F(a+b) - A\,b + w\,b\left(\frac{b}{2}\right) - M = 0$ $A = \dfrac{F(a+b) + \left(\frac{w b^2}{2}\right) - M}{b}$ $A = 375.00 \text{ lb}$

$x_1 = 0, 0.01a .. a$ $V_1(x_1) = -F\frac{1}{\text{lb}}$ $M_1(x_1) = -F x_1 \frac{1}{\text{lb}\cdot\text{ft}}$

$x_2 = a, 1.01a .. a+b$ $V_2(x_2) = \left[-F + A - w(x_2 - a)\right]\frac{1}{\text{lb}}$

$$M_2(x_2) = \left[-F x_2 + A(x_2 - a) - w\frac{(x_2 - a)^2}{2}\right]\frac{1}{\text{lb}\cdot\text{ft}}$$

$x_3 = a+b, 1.01(a+b) .. a+b+c$ $V_3(x_3) = 0\frac{1}{\text{lb}}$ $M_3(x_3) = -M\frac{1}{\text{lb}\cdot\text{ft}}$

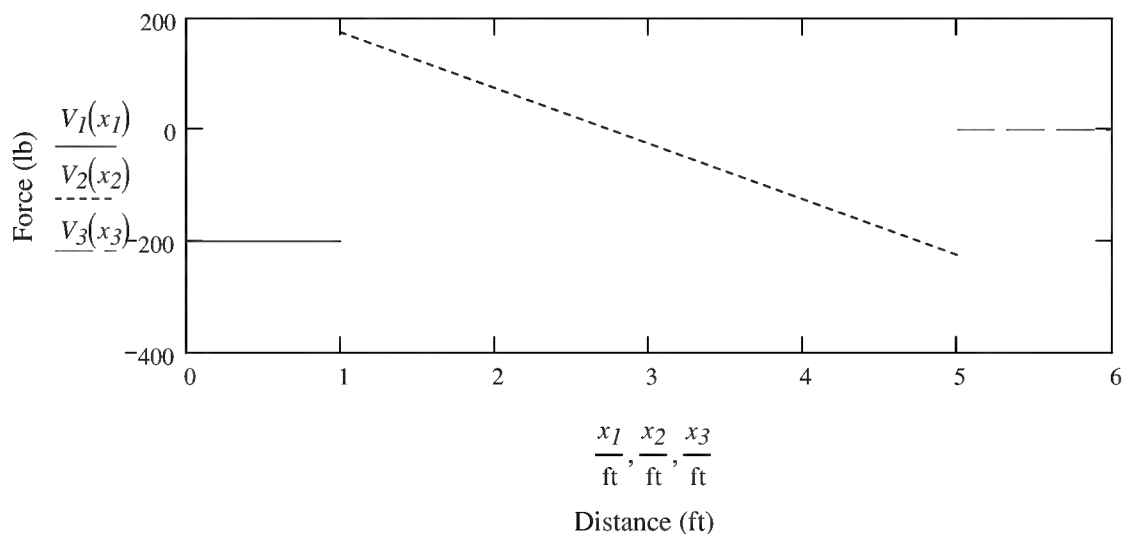
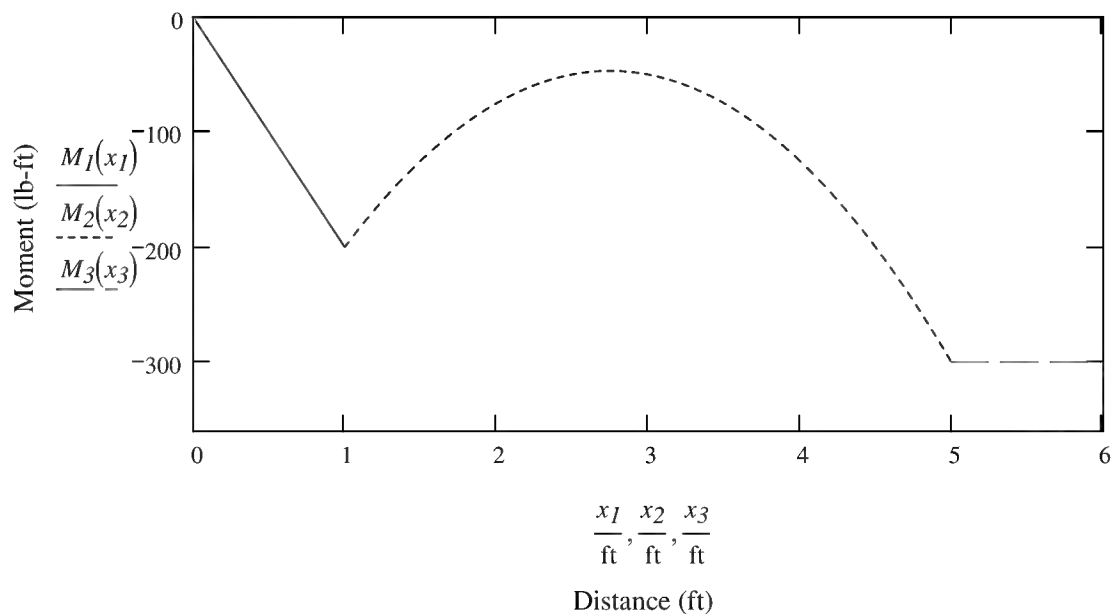
Moment (lb-ft)

$M_1(x_1)$, $M_2(x_2)$, $M_3(x_3)$

0, −100, −200, −300

0 1 2 3 4 5 6

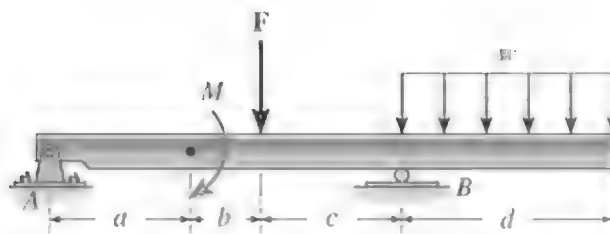
$\frac{x_1}{ft}, \frac{x_2}{ft}, \frac{x_3}{ft}$

Distance (ft)

Problem 7-75

Draw the shear and moment diagrams for the beam.

F, M, w, A, B, a, b, c, d

Units Used:

$$kN = 10^3\ N$$

Given: $F = 8\ kN$ $\quad M = 20\ kN\cdot m$ $\quad w = 15\ \frac{kN}{m}$ $\quad a = 2\ m$ $\quad b = 1\ m$

$c = 2\ m$ $\quad d = 3\ m$

Solution:

$$-A(a+b+c) - M + Fc - wd\left(\frac{d}{2}\right) = 0 \qquad A = \frac{Fc - M - w\left(\frac{d^2}{2}\right)}{a+b+c} \qquad A = -14.30\ kN$$

$$x_1 = 0, 0.01a .. a \qquad V_1(x_1) = A\frac{1}{kN} \qquad M_1(x_1) = Ax_1\frac{1}{kN\cdot m}$$

$$x_2 = a, 1.01a .. a+b \qquad V_2(x_2) = A\frac{1}{kN} \qquad M_2(x_2) = (Ax_2 + M)\frac{1}{kN\cdot m}$$

$$x_3 = a+b, 1.01(a+b) .. a+b+c \qquad V_3(x_3) = (A-F)\frac{1}{kN}$$

$$M_3(x_3) = [Ax_3 + M - F(x_3 - a - b)]\frac{1}{kN\cdot m}$$

$$x_4 = a + b + c, 1.01(a + b + c) .. a + b + c + d$$

$$V_4(x_4) = w(a + b + c + d - x_4)\frac{1}{\text{kN}} \qquad M_4(x_4) = -w\frac{(a + b + c + d - x_4)^2}{2}\frac{1}{\text{kN}\cdot\text{m}}$$

Force (kN)

$V_1(x_1)$ $V_2(x_2)$ $V_3(x_3)$ $V_4(x_4)$

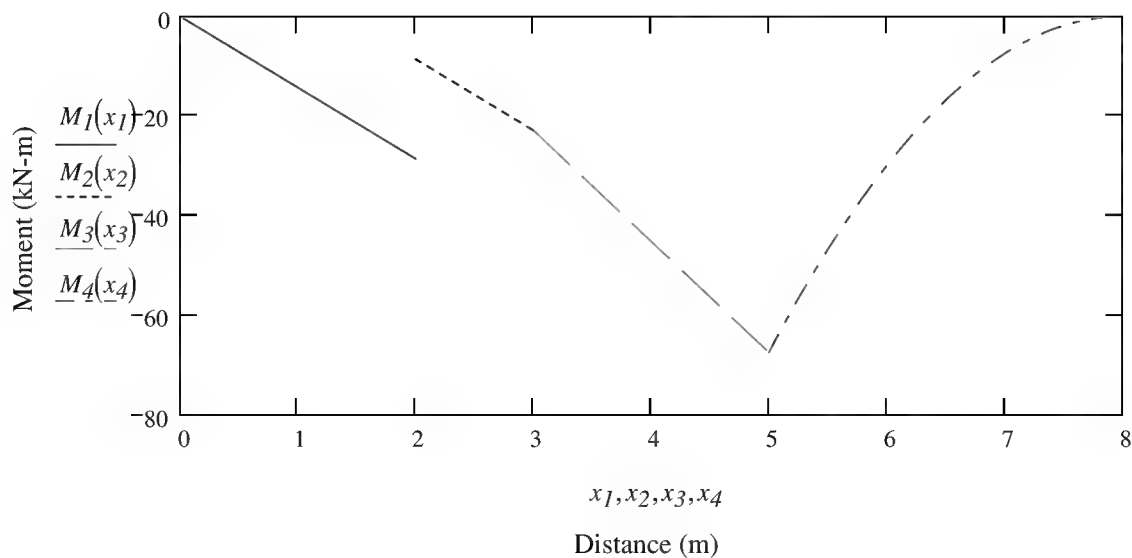
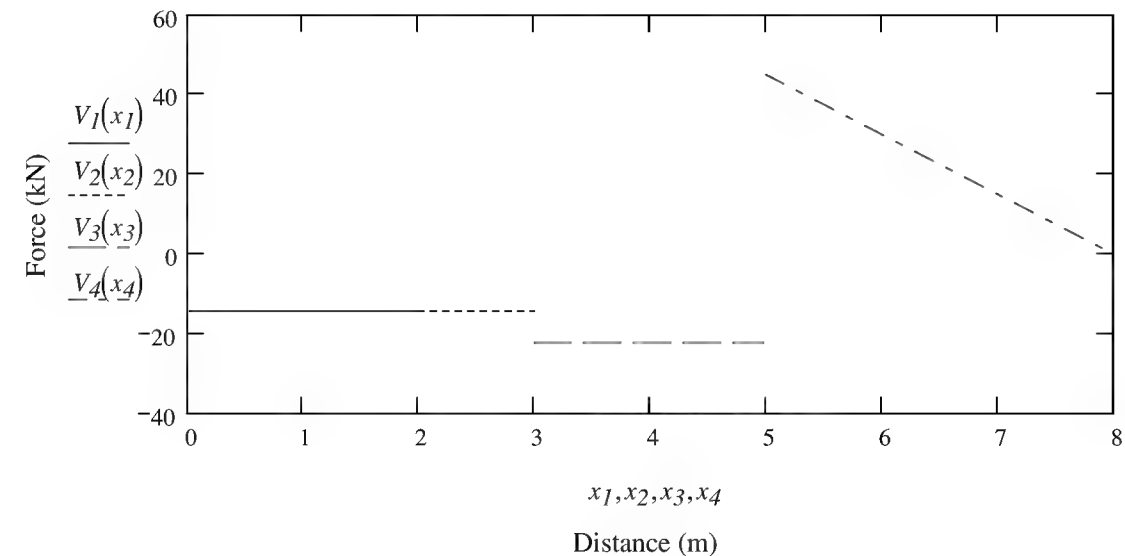
x_1, x_2, x_3, x_4

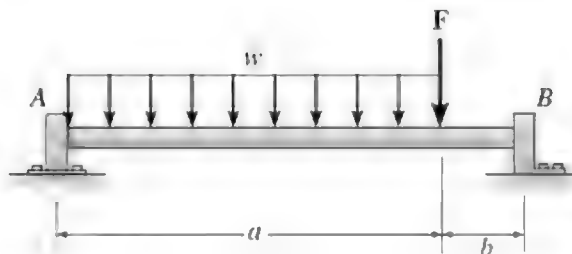
Distance (m)

Moment (kN-m)

$M_1(x_1)$ $M_2(x_2)$ $M_3(x_3)$ $M_4(x_4)$

x_1, x_2, x_3, x_4

Distance (m)

Problem 7-76

Draw the shear and moment diagrams for the shaft. The support at A is a thrust bearing and at B it is a journal bearing.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given: $w = 2\ \frac{\text{kN}}{\text{m}}$ $F = 4\ \text{kN}$ $a = 0.8\ \text{m}$ $b = 0.2\ \text{m}$

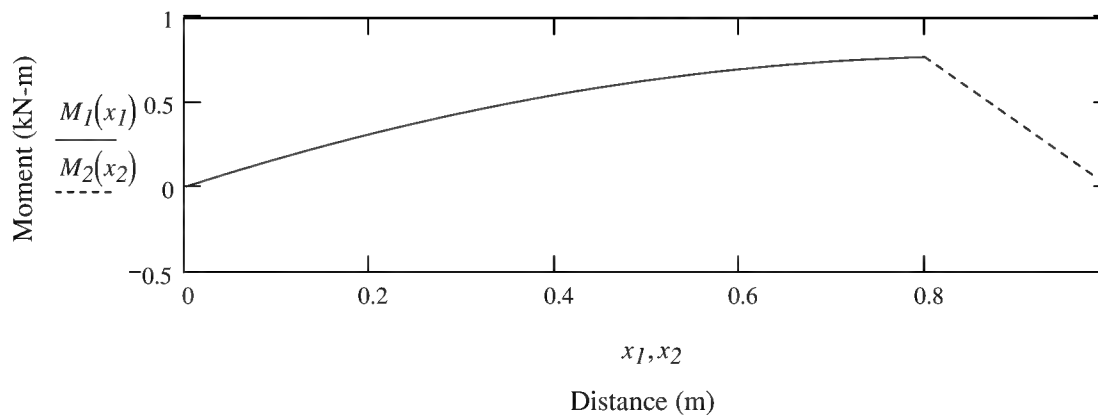
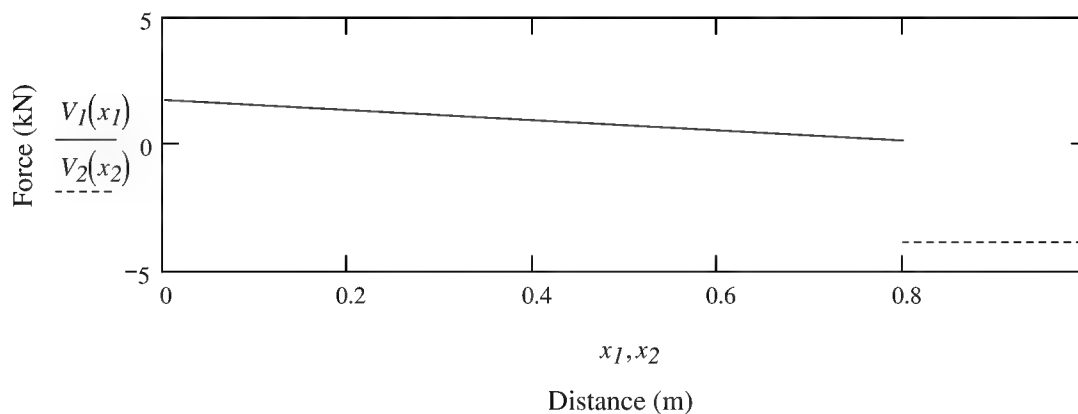
Solution: $B(a+b) - F\,a - w\,a\left(\frac{a}{2}\right) = 0$ $B = \dfrac{F\,a + w\left(\frac{a^2}{2}\right)}{a+b}$ $B = 3.84\,\text{kN}$

$A + B - w\,a - F = 0$ $A = w\,a + F - B$ $A = 1.76\,\text{kN}$

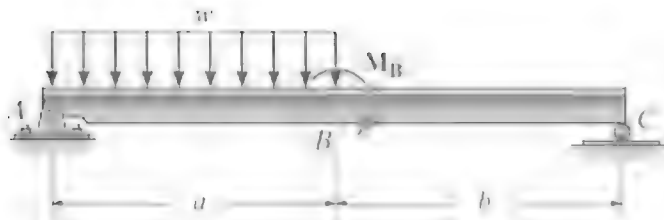
$x_1 = 0, 0.01a .. a$ $V_1(x_1) = (A - w\,x_1)\frac{1}{\text{kN}}$ $M_1(x_1) = \left[A\,x_1 - w\left(\frac{x_1^2}{2}\right)\right]\frac{1}{\text{kN}\cdot\text{m}}$

$x_2 = a, 1.01a .. a+b$ $V_2(x_2) = -B\frac{1}{\text{kN}}$ $M_2(x_2) = B(a+b-x_2)\frac{1}{\text{kN}\cdot\text{m}}$

Force (kN) vs. Distance (m) plot: $V_1(x_1)$, $V_2(x_2)$ (dashed) against x_1, x_2; axis 5, 0, −5; 0, 0.2, 0.4, 0.6, 0.8

Moment (kN-m) vs. Distance (m) plot: $M_1(x_1)$, $M_2(x_2)$ (dashed) against x_1, x_2; axis 1, 0.5, 0, −0.5; 0, 0.2, 0.4, 0.6, 0.8

Problem 7-77

Draw the shear and moment diagrams for the beam.

Given:

$w = 20\ \frac{\text{lb}}{\text{ft}}$ $M_B = 160\ \text{lb}\cdot\text{ft}$ $a = 20\ \text{ft}$ $b = 20\ \text{ft}$

Solution:

$-wa\left(\frac{a}{2}\right) - M_B + C_y(a+b) = 0$ $C_y = \left(w\frac{a^2}{2} + M_B\right)\left(\frac{1}{a+b}\right)$ $C_y = 104.00\ \text{lb}$

$A_y - wa + C_y = 0$ $A_y = wa - C_y$ $A_y = 296.00\ \text{lb}$

$x_1 = 0, 0.01a .. a$

$V_1(x) = (A_y - wx)\frac{1}{\text{lb}}$ $M_1(x) = \left(A_y x - w\frac{x^2}{2}\right)\frac{1}{\text{lb}\cdot\text{ft}}$

$x_2 = a, 1.01a .. a+b$

$V_2(x) = -C_y\frac{1}{\text{lb}}$ $M_2(x) = C_y(a+b-x)\frac{1}{\text{lb}\cdot\text{ft}}$

Force (lb): $V_1(x_1)$, $V_2(x_2)$ (dashed); Distance (ft): $\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

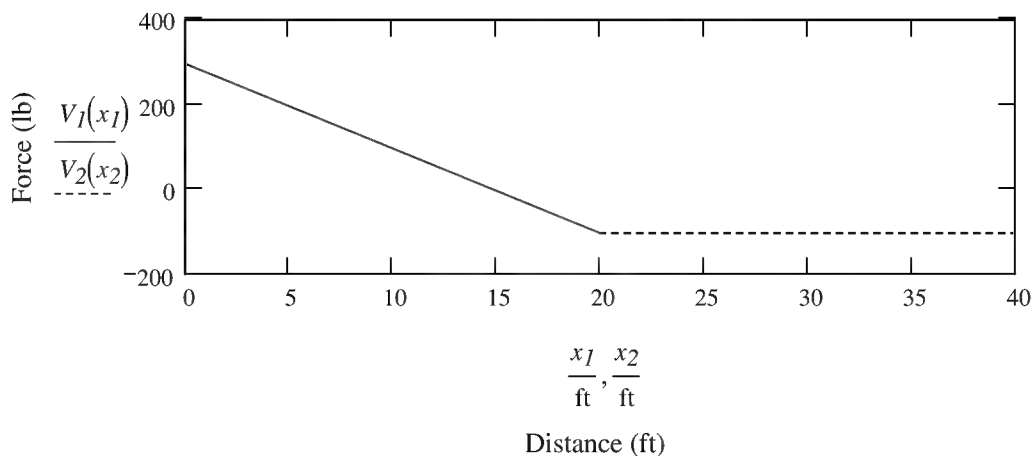
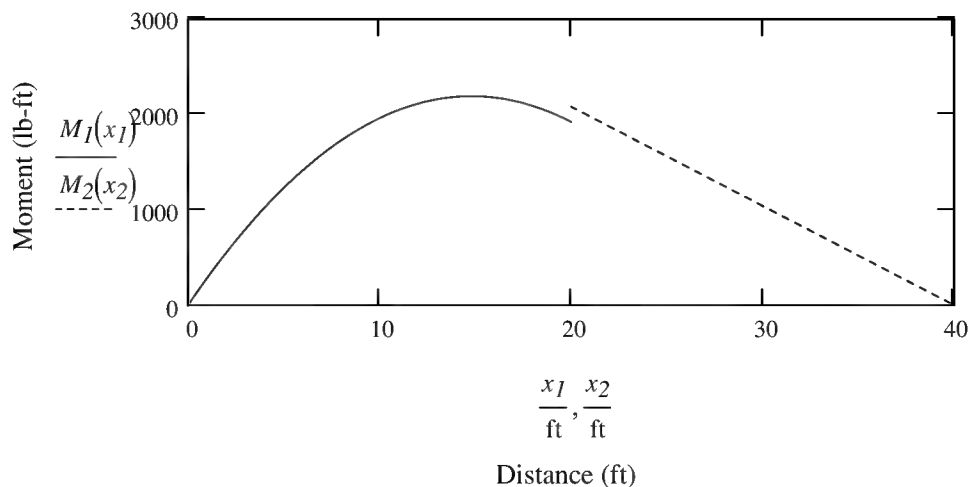
Moment (lb-ft)

$\frac{M_1(x_1)}{}$ — $M_2(x_2)$ - - - - -

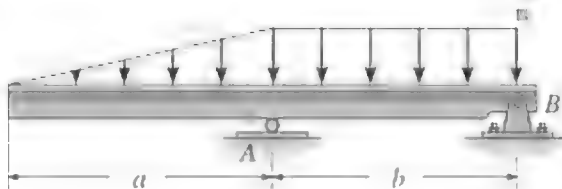
3000, 2000, 1000, 0

0 10 20 30 40

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

Distance (ft)

Problem 7-78

The beam will fail when the maximum moment is M_{max} or the maximum shear is V_{max}. Determine the largest distributed load w the beam will support.

B A a b

Units used: $\text{kip} = 10^3\ \text{lb}$

Given: $M_{max} = 30\ \text{kip}\cdot\text{ft}$ $V_{max} = 8\ \text{kip}$ $a = 6\ \text{ft}$ $b = 6\ \text{ft}$

Solution: Set $w = 1\ \frac{\text{kip}}{\text{ft}}$ and then scale the answer at the end

$$-A\,b + w\frac{a}{2}\left(\frac{a}{3} + b\right) + w\,b\frac{b}{2} = 0 \qquad A = \frac{w\frac{a}{2}\left(\frac{a}{3} + b\right) + w\frac{b^2}{2}}{b} \qquad A = 7.00\,\text{kip}$$

$$A + B - w\,b - w\frac{a}{2} = 0 \qquad B = w\left(b + \frac{a}{2}\right) - A \qquad B = 2.00\,\text{kip}$$

Shear limit - check critical points to the left and right of A and at B

$$V_{big} = \max\left(|B|, \left|w\frac{a}{2}\right|, \left|w\frac{a}{2} - A\right|\right) \qquad V_{big} = 4.00\,\text{kip}$$

$$w_{shear} = \left(\frac{V_{max}}{V_{big}}\right)w \qquad w_{shear} = 2.00\,\frac{\text{kip}}{\text{ft}}$$

Moment limit - check critical points at A and betwen A and B

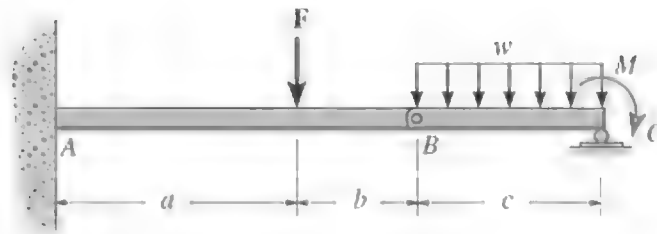
$$M_A = -w\left(\frac{a}{2}\right)\left(\frac{a}{3}\right) \quad x = \frac{B}{w} \quad M_{AB} = Bx - w\left(\frac{x^2}{2}\right) \quad \begin{pmatrix} M_A \\ M_{AB} \end{pmatrix} = \begin{pmatrix} -6.00 \\ 2.00 \end{pmatrix} \text{kip}\cdot\text{ft}$$

$$M_{big} = \max(|M_A|, |M_{AB}|) \qquad M_{big} = 6.00\,\text{kip}\cdot\text{ft}$$

$$w_{moment} = \left(\frac{M_{max}}{M_{big}}\right)w \qquad w_{moment} = 5.00\,\frac{\text{kip}}{\text{ft}}$$

Choose the critical case $\quad w_{ans} = \min(w_{shear}, w_{moment}) \qquad w_{ans} = 2.00\,\frac{\text{kip}}{\text{ft}}$

Problem 7-79

The beam consists of two segments pin connected at B. Draw the shear and moment diagrams for the beam.

Given:

$$F = 700\ \text{lb} \quad w = 150\ \frac{\text{lb}}{\text{ft}} \quad M = 800\ \text{lb ft} \quad a = 8\ \text{ft} \quad b = 4\ \text{ft} \quad c = 6\ \text{ft}$$

Solution:

$$-wc\left(\frac{c}{2}\right) - M + Cc = 0 \qquad C = \frac{w\left(\frac{c^2}{2}\right) + M}{c} \qquad C = 583.33\,\text{lb}$$

$$B + C - wc = 0 \qquad B = wc - C \qquad B = 316.67\,\text{lb}$$

$$x_1 = 0, 0.01a .. a \qquad V_1(x_1) = (B + F)\frac{1}{\text{lb}} \qquad M_1(x_1) = [-F(a - x_1) - B(a + b - x_1)]\frac{1}{\text{lb}\cdot\text{ft}}$$

$$x_2 = a, 1.01a .. a + b \qquad V_2(x_2) = B\frac{1}{\text{lb}} \qquad M_2(x_2) = -B(a + b - x_2)\frac{1}{\text{lb}\cdot\text{ft}}$$

$$x_3 = a + b, 1.01(a + b) .. a + b + c \qquad V_3(x_3) = [-C + w(a + b + c - x_3)]\frac{1}{\text{lb}}$$

$$M_3(x_3) = \left[C(a + b + c - x_3) - w\frac{(a + b + c - x_3)^2}{2} - M\right]\frac{1}{\text{lb}\cdot\text{ft}}$$

Force (lb)

$\frac{V_1(x_1)}{V_2(x_2)}$, $V_3(x_3)$

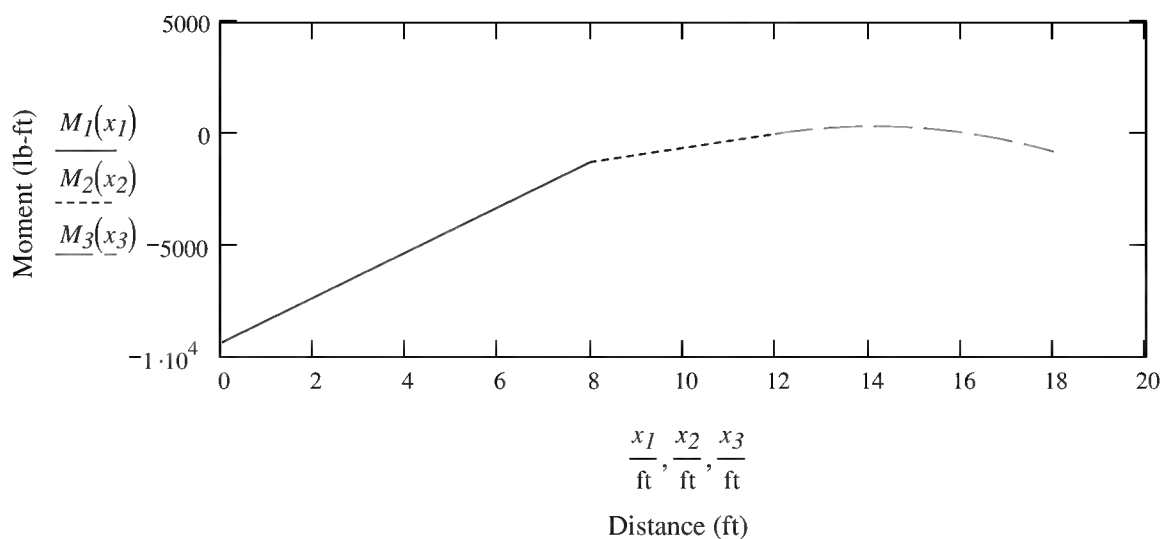
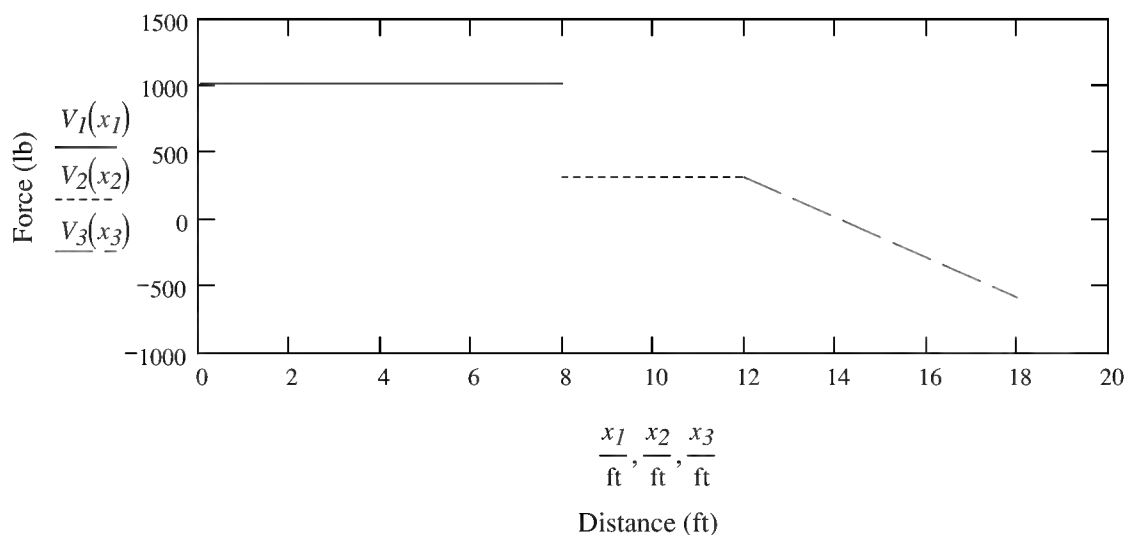
$\frac{x_1}{ft}, \frac{x_2}{ft}, \frac{x_3}{ft}$

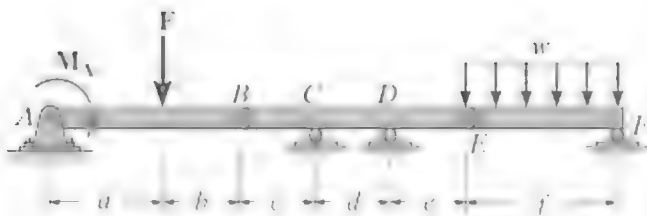
Distance (ft)

Moment (lb-ft)

$M_1(x_1)$, $M_2(x_2)$, $M_3(x_3)$

$\frac{x_1}{ft}, \frac{x_2}{ft}, \frac{x_3}{ft}$

Distance (ft)

Problem 7-80

The beam consists of three segments pin connected at B and E. Draw the shear and moment diagrams for the beam.

Units Used: $\quad kN = 10^3\ N$

Given: $\quad M_A = 8\ kN{\cdot}m \quad F = 15\ kN \quad w = 3\ \frac{kN}{m} \quad a = 3\ m \quad b = 2\ m$

$c = 2\ m \quad d = 2\ m \quad e = 2\ m \quad f = 4\ m$

Guesses $A_y = 1\ \text{N}$ $B_y = 1\ \text{N}$ $C_y = 1\ \text{N}$

$D_y = 1\ \text{N}$ $E_y = 1\ \text{N}$ $F_y = 1\ \text{N}$

Given

$$A_y + C_y + D_y + F_y - F - wf = 0 \qquad F b - M_A - A_y(a+b) = 0$$

$$-wf\left(\frac{f}{2}\right) + F_y f = 0 \qquad M_A + F a + B_y(a+b) = 0$$

$$B_y + C_y + D_y + E_y = 0 \qquad -B_y c + D_y d + E_y(d+e) = 0$$

$$\begin{pmatrix} A_y \\ B_y \\ C_y \\ D_y \\ E_y \\ F_y \end{pmatrix} = \text{Find}(A_y, B_y, C_y, D_y, E_y, F_y) \qquad \begin{pmatrix} A_y \\ B_y \\ C_y \\ D_y \\ E_y \\ F_y \end{pmatrix} = \begin{pmatrix} 4.40 \\ -10.60 \\ 15.20 \\ 1.40 \\ -6.00 \\ 6.00 \end{pmatrix} \text{kN}$$

$$x_1 = 0, 0.01a .. a$$

$$V_1(x) = A_y \frac{1}{\text{kN}} \qquad M_1(x) = (A_y x + M_A)\frac{1}{\text{kN}\cdot\text{m}}$$

$$x_2 = a, 1.01a .. a+b$$

$$V_2(x) = (A_y - F)\frac{1}{\text{kN}} \qquad M_2(x) = \left[A_y x + M_A - F(x-a)\right]\frac{1}{\text{kN}\cdot\text{m}}$$

$$x_3 = a+b, 1.01(a+b) .. a+b+c$$

$$V_3(x) = B_y \frac{1}{\text{kN}} \qquad M_3(x) = B_y(x-a-b)\frac{1}{\text{kN}\cdot\text{m}}$$

$$x_4 = a+b+c, 1.01(a+b+c) .. a+b+c+d$$

$$V_4(x) = (B_y + C_y)\frac{1}{\text{kN}} \qquad M_4(x) = \left[B_y(x-a-b) + C_y(x-a-b-c)\right]\frac{1}{\text{kN}\cdot\text{m}}$$

$$x_5 = a+b+c+d, 1.01(a+b+c+d) .. a+b+c+d+e$$

$$V_5(x) = -E_y \frac{1}{\text{kN}} \qquad M_5(x) = E_y(a+b+c+d+e-x)\frac{1}{\text{kN}\cdot\text{m}}$$

$$x_6 = a+b+c+d+e, 1.01(a+b+c+d+e) .. a+b+c+d+e+f$$

$$V_6(x) = \left[-F_y + w(a+b+c+d+e+f-x)\right]\frac{1}{\text{kN}}$$

$$M_6(x) = \left[F_y(a+b+c+d+e+f-x) - w\frac{(a+b+c+d+e+f-x)^2}{2}\right]\frac{1}{\text{kN}\cdot\text{m}}$$

Force (kN)

$V_1(x_1)$
$V_2(x_2)$
$V_3(x_3)$
$V_4(x_4)$
$V_5(x_5)$
$V_6(x_6)$

10, 5, 0, −5, −10, −15

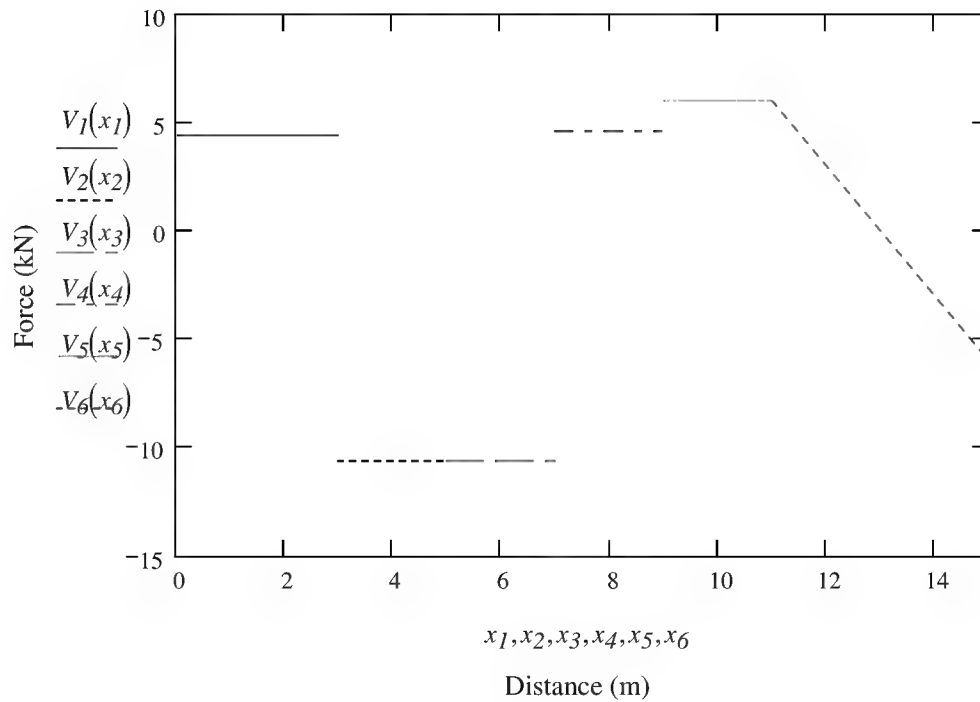
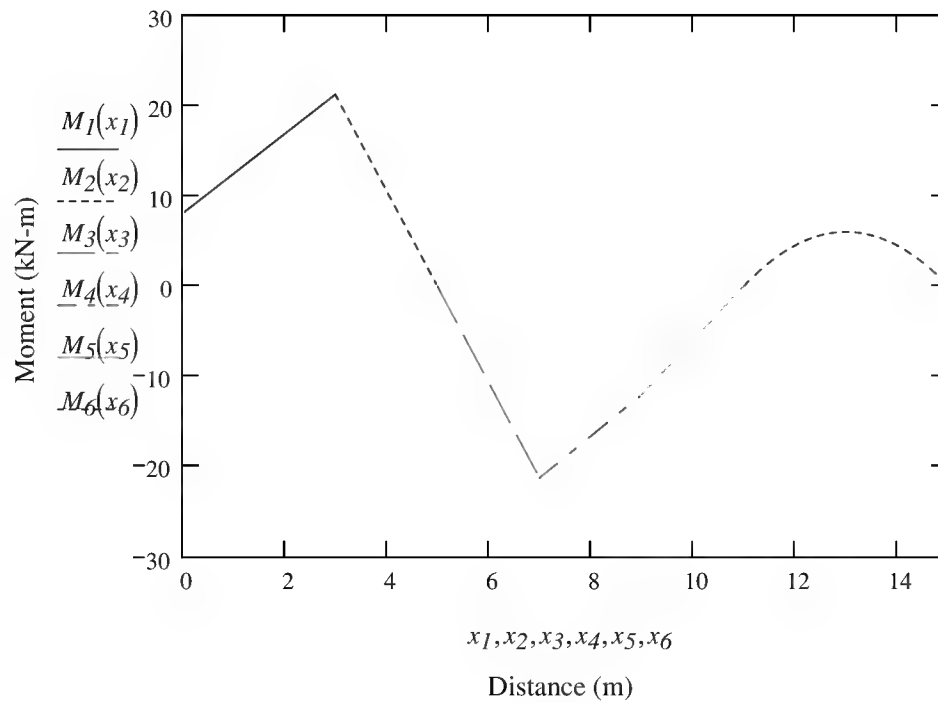
0, 2, 4, 6, 8, 10, 12, 14

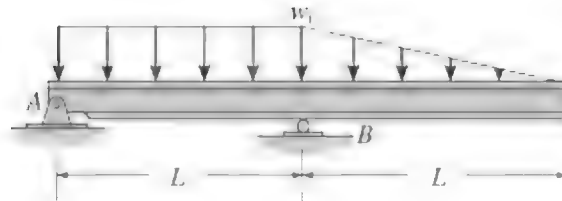
$x_1, x_2, x_3, x_4, x_5, x_6$

Distance (m)

$M_1(x_1)$, $M_2(x_2)$, $M_3(x_3)$, $M_4(x_4)$, $M_5(x_5)$, $M_6(x_6)$ — Moment (kN-m)

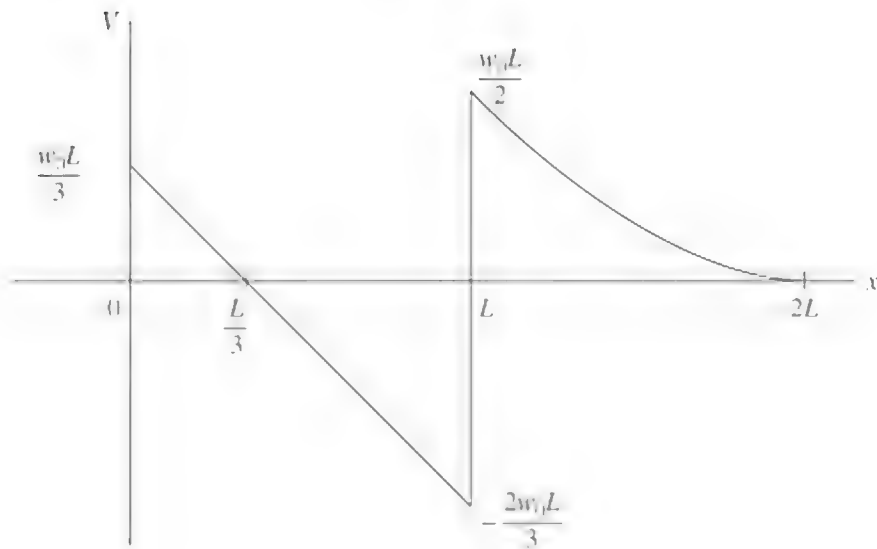
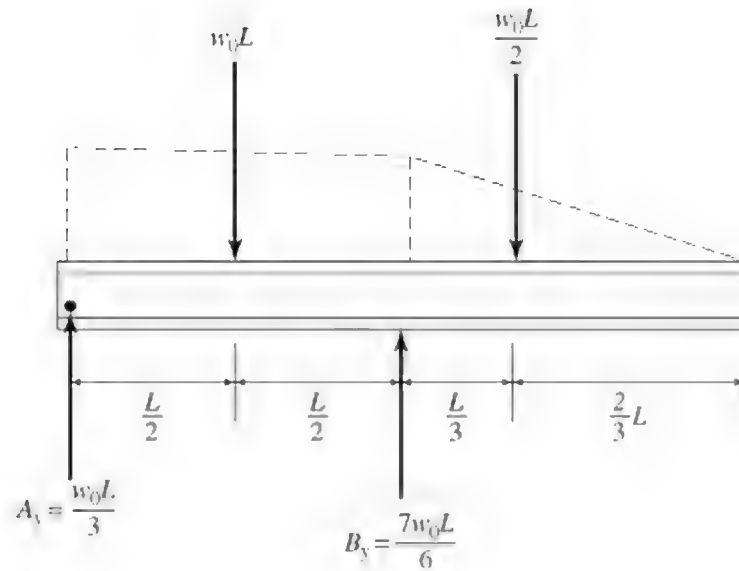
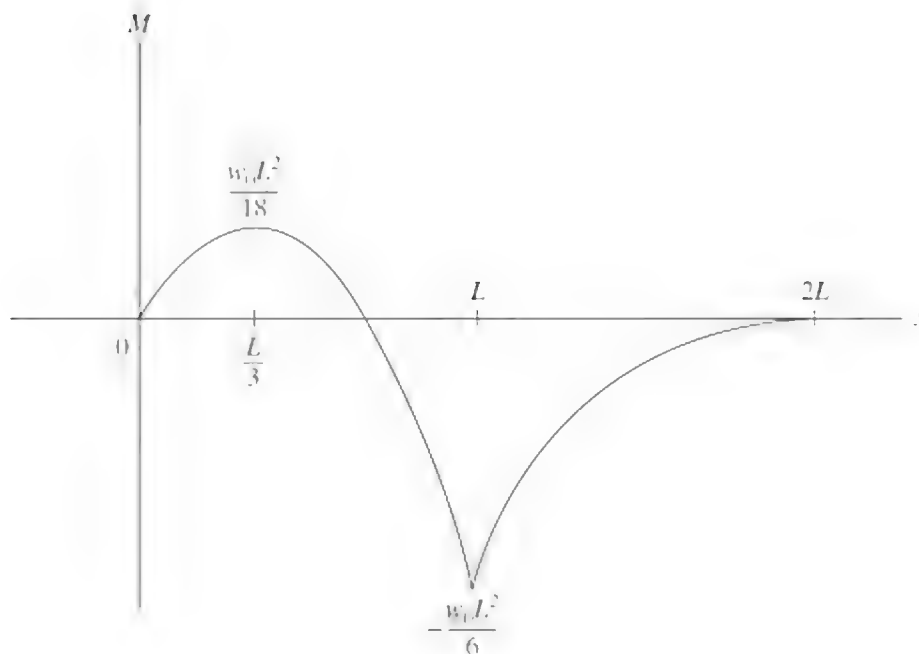
$x_1, x_2, x_3, x_4, x_5, x_6$

Distance (m)

Problem 7-81

Draw the shear and moment diagrams for the beam.

Solutions:

Support Reactions:

$$\Sigma M_x = 0; \qquad B_y L - w_0 L\left(\frac{L}{2}\right) - \frac{w_0 L}{2}\left(\frac{4L}{3}\right) = 0 \qquad B_y = \frac{7 w_0 L}{6}$$

$+\uparrow\Sigma F_y = 0;\qquad A_y + \left(\frac{7w_0 L}{6}\right) - w_0 L - \left(\frac{w_0 L}{2}\right) = 0 \qquad A_y = \frac{w_0 L}{3}$

Problem 7-82

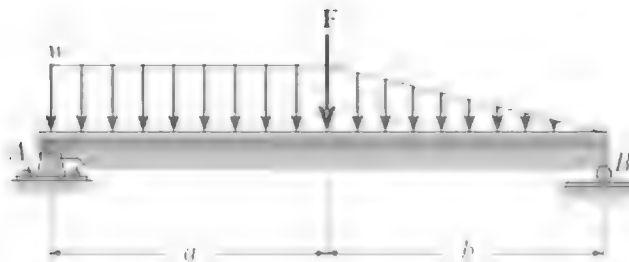
Draw the shear and moment diagrams for the beam.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$F = 2000 \text{ lb}$ $a = 9 \text{ ft}$

$w = 500 \dfrac{\text{lb}}{\text{ft}}$ $b = 9 \text{ ft}$

Solution:

$$\frac{1}{2} w b\left(\frac{2b}{3}\right) + F b + w a\left(b + \frac{a}{2}\right) - A_y(a+b) = 0 \qquad A_y = \frac{\left(\frac{w b^2}{3}\right) + F b + w a\left(b + \frac{a}{2}\right)}{a+b}$$

$$A_y + B_y - F - w a - \frac{1}{2} w b = 0 \qquad B_y = w a + \frac{1}{2} w b - A_y + F$$

$A_y = 5.13 \text{ kip}$ $B_y = 3.63 \text{ kip}$

$x_1 = 0, 0.01a .. a$

$$V_1(x) = (A_y - wx)\frac{1}{\text{kip}} \qquad M_1(x) = \left[A_y x - wx\left(\frac{x}{2}\right)\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

$$x_2 = a, 1.01a .. a + b$$

$$V_2(x) = \left[-B_y + \frac{1}{2}w\left(\frac{a+b-x}{b}\right)(a+b-x)\right]\frac{1}{\text{kip}}$$

$$M_2(x) = \left[B_y(a+b-x) - \frac{1}{2}w\left(\frac{a+b-x}{b}\right)(a+b-x)\left(\frac{a+b-x}{3}\right)\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

Force (kip)

$V_1(x_1)$ (solid), $V_2(x_2)$ (dashed)

Axis: 10, 5, 0, −5; 0, 5, 10, 15

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

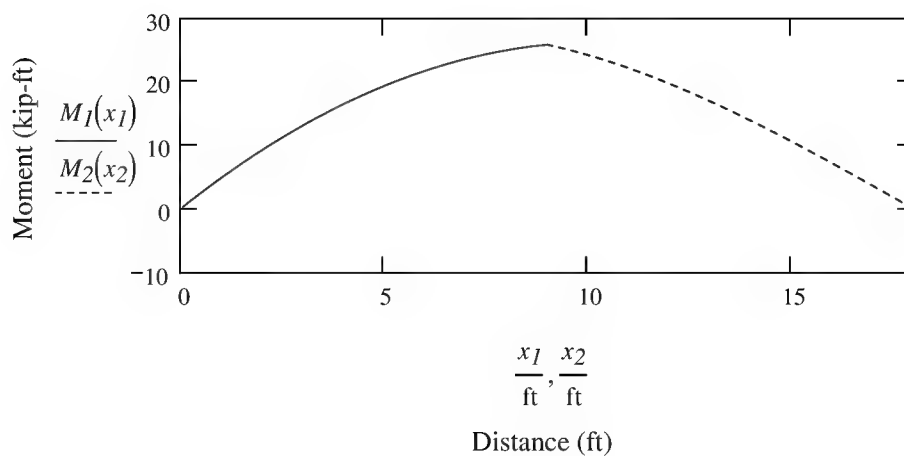
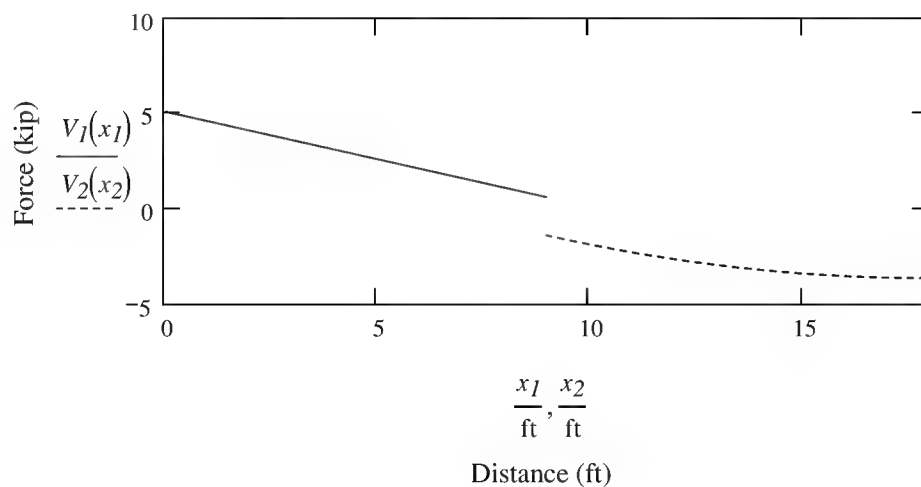
Distance (ft)

Moment (kip-ft)

$M_1(x_1)$ (solid), $M_2(x_2)$ (dashed)

Axis: 30, 20, 10, 0, −10; 0, 5, 10, 15

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}$

Distance (ft)

Problem 7-83

Draw the shear and moment diagrams for the beam.

Units Used:

$kN = 10^3\ N$

Given:

$w = 3\ \frac{kN}{m}$ $a = 3\ m$ $b = 3\ m$

Solution:

$$-A_y(a+b) + \left(\frac{wb}{2}\right)\left(\frac{2b}{3}\right) + \left(\frac{wa}{2}\right)\left(b + \frac{a}{3}\right) = 0 \qquad A_y = \frac{\frac{wb^2}{3} + \left(\frac{wa}{2}\right)\left(b + \frac{a}{3}\right)}{a+b}$$

$$A_y + B_y - \frac{w}{2}(a+b) = 0 \qquad B_y = \frac{w}{2}(a+b) - A_y$$

$x_1 = 0, 0.01a..a$

$$V_1(x) = \left[A_y - \frac{1}{2}w\left(\frac{x}{a}\right)x\right]\frac{1}{kN} \qquad M_1(x) = \left[A_y x - \frac{1}{2}w\left(\frac{x}{a}\right)x\left(\frac{x}{3}\right)\right]\frac{1}{kN\cdot m}$$

$x_2 = a, 1.01a..a+b$

$$V_2(x) = \left[-B_y + \frac{1}{2}w\left(\frac{a+b-x}{b}\right)(a+b-x)\right]\frac{1}{kN}$$

$$M_2(x) = \left[B_y(a+b-x) - \frac{1}{2}w\left(\frac{a+b-x}{b}\right)(a+b-x)\left(\frac{a+b-x}{3}\right)\right]\frac{1}{kN\cdot m}$$

Force (kN): $V_1(x_1)$, $V_2(x_2)$

x_1, x_2

Distance (m)

Moment (kN-m)

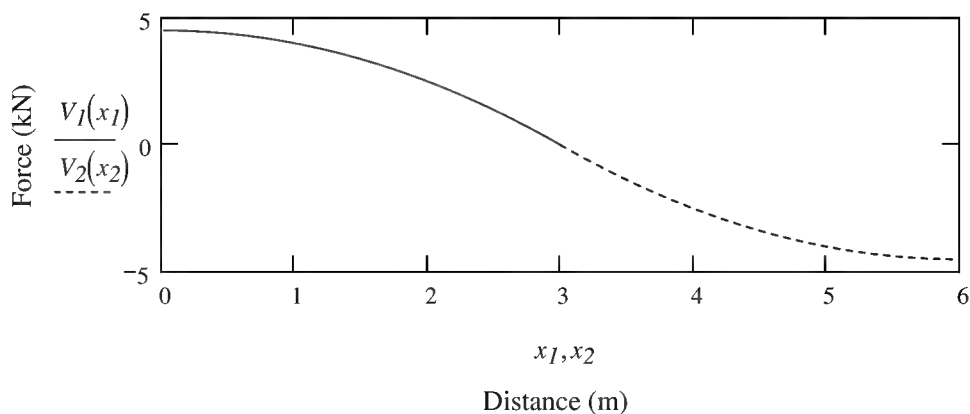
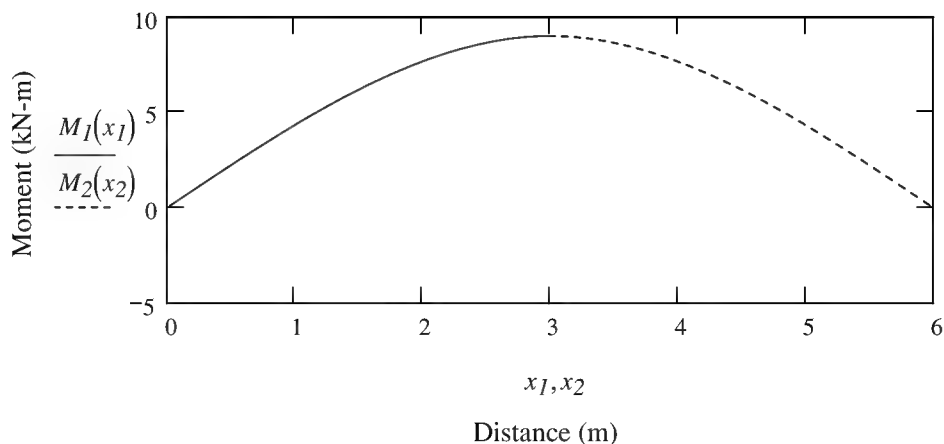
$M_1(x_1)$ ——

$M_2(x_2)$ - - - -

10, 5, 0, −5

0 1 2 3 4 5 6

x_1, x_2

Distance (m)

Problem 7-84

Draw the shear and moment diagrams for the beam.

Units Used:

$\text{kip} = 10^3 \text{ lb}$

Given:

$w = 100 \frac{\text{lb}}{\text{ft}}$ $\quad M_0 = 9 \text{ kip}\cdot\text{ft}$ $\quad a = 6 \text{ ft}$ $\quad b = 6 \text{ ft}$ $\quad c = 4 \text{ ft}$

Guesses: $A_y = 1 \text{ lb}$ $\quad B_y = 1 \text{ lb}$

Given $$-M_0 + B_y(a+b) - \frac{1}{2}wa\left(\frac{2a}{3}\right) - \frac{1}{2}wb\left(a + \frac{b}{3}\right) = 0$$

$$A_y + B_y - \frac{1}{2}w(a+b) = 0$$

$$\begin{pmatrix} A_y \\ B_y \end{pmatrix} = \text{Find}(A_y, B_y) \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} -0.45 \\ 1.05 \end{pmatrix} \text{kip}$$

$x_1 = 0, 0.01a .. a$

$$V_1(x) = \left[A_y - \frac{1}{2}w\left(\frac{x}{a}\right)x\right]\frac{1}{\text{lb}} \qquad M_1(x) = \left[A_y x - \frac{1}{2}w\left(\frac{x}{a}\right)x\left(\frac{x}{3}\right)\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

$x_2 = a, 1.01a .. a + b$

$$V_2(x) = \left[-B_y + \frac{1}{2}w\left(\frac{a+b-x}{b}\right)(a+b-x)\right]\frac{1}{\text{lb}}$$

$$M_2(x) = \left[B_y(a+b-x) - M_0 - \frac{1}{2}w\left(\frac{a+b-x}{b}\right)(a+b-x)\left(\frac{a+b-x}{3}\right)\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

$x_3 = a+b, 1.01(a+b) .. a+b+c$

$V_3(x) = 0 \qquad M_3(x) = -M_0\frac{1}{\text{kip}\cdot\text{ft}}$

Force (lb)

$V_1(x_1)$, $V_2(x_2)$, $V_3(x_3)$

0, −500, −1000, −1500

0 2 4 6 8 10 12 14 16

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}, \frac{x_3}{\text{ft}}$

Distance (ft)

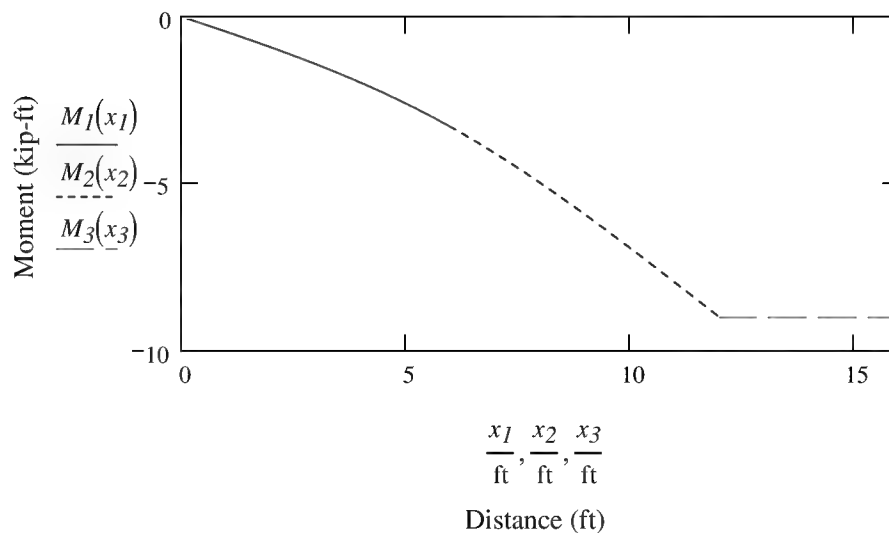
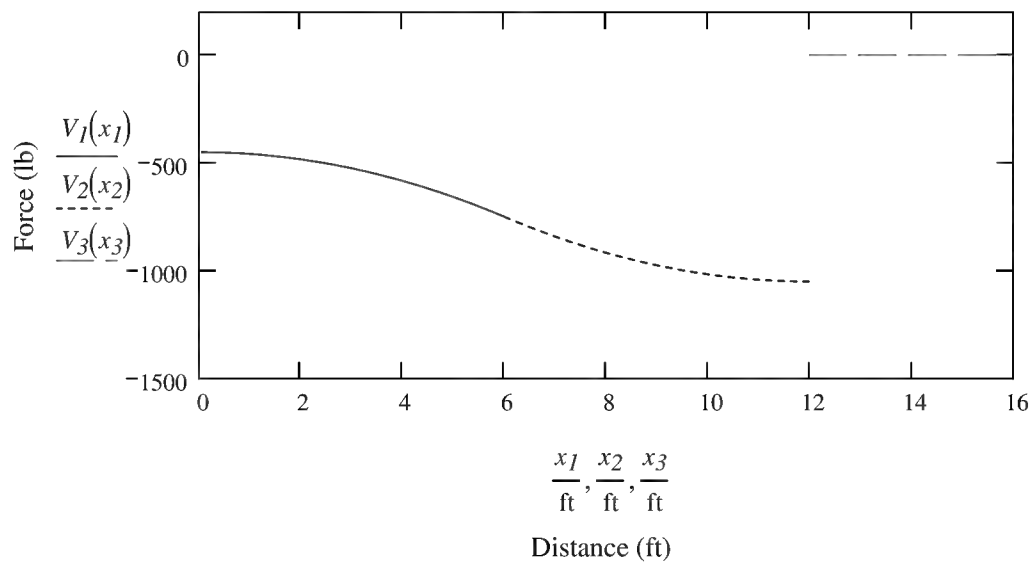
Moment (kip-ft)

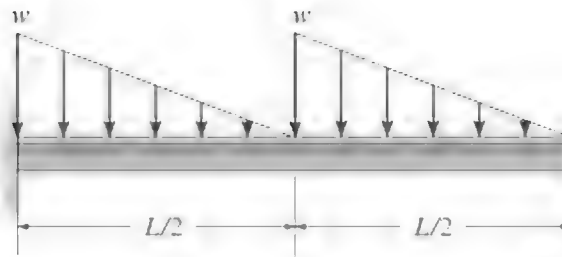
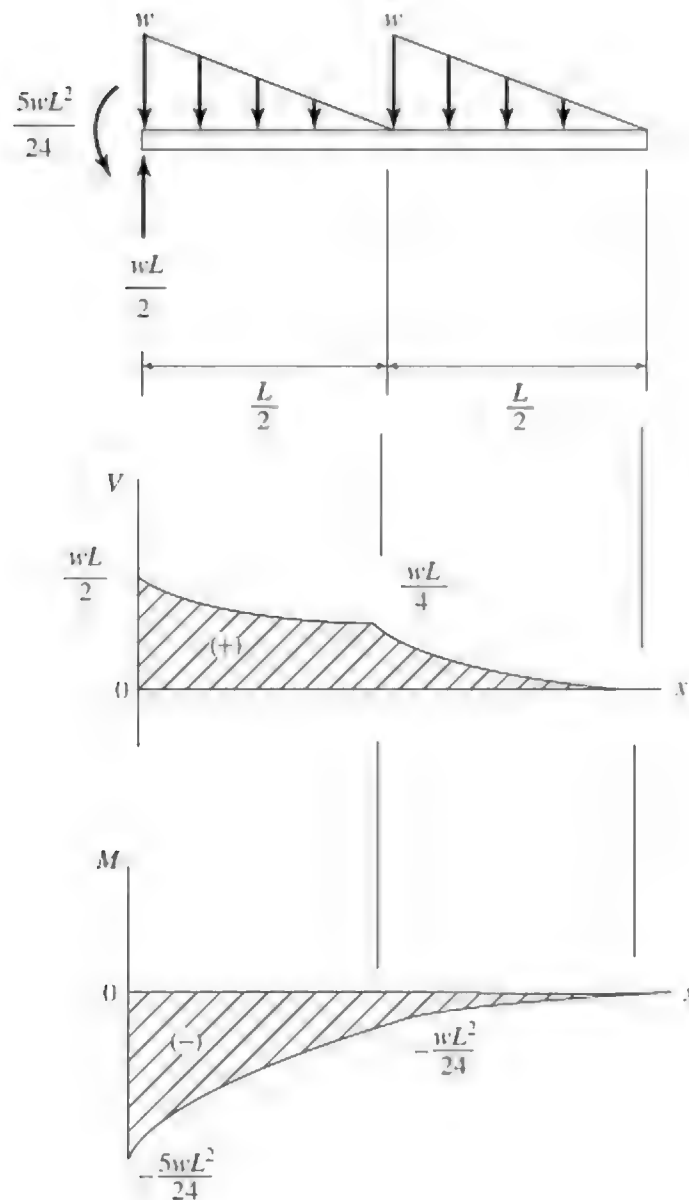
$M_1(x_1)$, $M_2(x_2)$, $M_3(x_3)$

0, −5, −10

0 5 10 15

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}, \frac{x_3}{\text{ft}}$

Distance (ft)

Problem 7-85

Draw the shear and moment diagrams for the beam.

Solution:

Problem 7-86

Draw the shear and moment diagrams for the beam.

Units Used:

$$kN = 10^3\ N$$

Given: $w = 2\ \frac{kN}{m}$ $\quad a = 3\ m$ $\quad b = 3\ m$

Solution:

$x_1 = 0, 0.01a .. a$

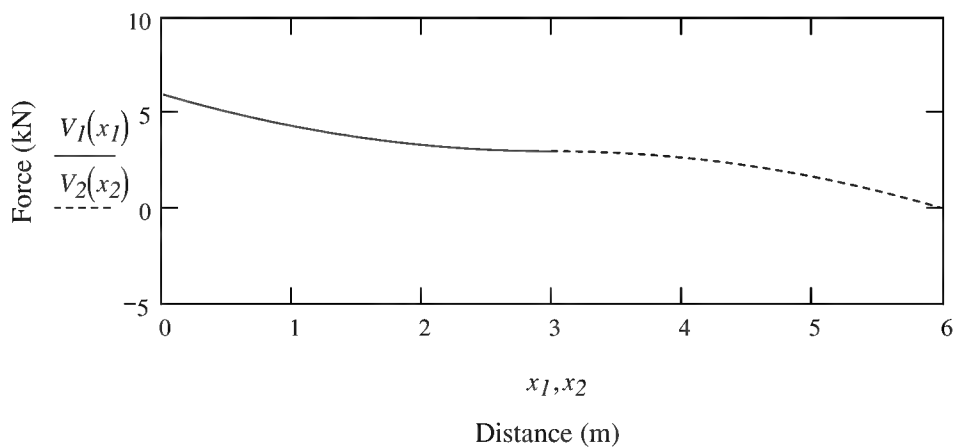
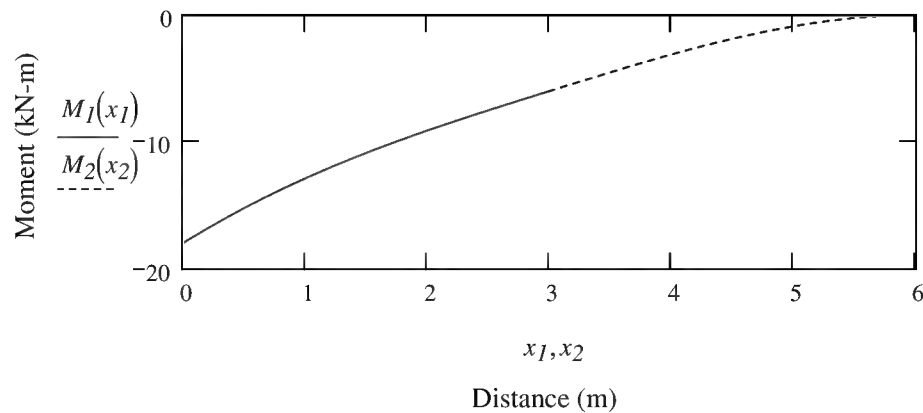
$$V_1(x) = \left[\frac{1}{2}wb + \frac{1}{2}w\left(\frac{a-x}{a}\right)(a-x)\right]\frac{1}{kN}$$

$$M_1(x) = \left[\frac{-1}{2}wb\left(\frac{2b}{3} + a - x\right) - \frac{1}{2}w\left(\frac{a-x}{a}\right)(a-x)\left(\frac{a-x}{3}\right)\right]\frac{1}{kN \cdot m}$$

$x_2 = a, 1.01a .. a + b$

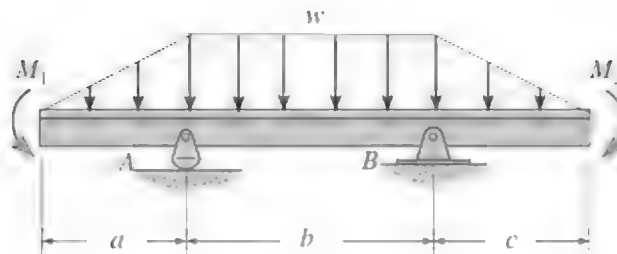
$$V_2(x) = \left[\frac{1}{2}wb - \frac{1}{2}w\left(\frac{x-a}{a}\right)(x-a)\right]\frac{1}{kN}$$

$$M_2(x) = \left[\frac{-1}{2}wb\left(a + \frac{2b}{3} - x\right) - \frac{1}{2}w\left(\frac{x-a}{b}\right)(x-a)\left(\frac{x-a}{3}\right)\right]\frac{1}{kN \cdot m}$$

Moment (kN-m) vs. Distance (m): $M_1(x_1)$, $M_2(x_2)$ plotted against x_1, x_2

Problem 7-87

Draw the shear and moment diagrams for the beam.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

$w = 5 \frac{\text{kip}}{\text{ft}}$ $\quad M_1 = 15 \text{ kip}\cdot\text{ft}$ $\quad M_2 = 15 \text{ kip}\cdot\text{ft}$ $\quad a = 6 \text{ ft}$ $\quad b = 10 \text{ ft}$ $\quad c = 6 \text{ ft}$

Solution:

$$M_1 - A\,b - M_2 + w\left(\frac{a}{2}\right)\left(b + \frac{a}{3}\right) + w\,b\left(\frac{b}{2}\right) - w\left(\frac{c}{2}\right)\left(\frac{c}{3}\right) = 0 \qquad A + B - w\,b - w\left(\frac{a+c}{2}\right) = 0$$

$$A = \frac{M_1 - M_2 + w\left(\frac{a}{2}\right)\left(b + \frac{a}{3}\right) + w\left(\frac{b^2}{2}\right) - w\left(\frac{c^2}{6}\right)}{b}$$

$$B = w\left(b + \frac{a+c}{2}\right) - A \qquad \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 40.00 \\ 40.00 \end{pmatrix} \text{kip}$$

$$x_1 = 0, 0.01a..a \qquad V_1(x) = \left(-w\frac{x}{a}\right)\frac{x}{2}\frac{1}{\text{kip}}$$

$$M_{1p}(x) = \left[\left(-w\frac{x}{a}\right)\frac{x}{2}\frac{x}{3} - M_1\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

$x_2 = a, 1.01a .. a + b$

$$V_2(x) = \left[A - w\frac{a}{2} - w(x-a)\right]\frac{1}{\text{kip}}$$

$$M_{2p}(x) = \left[-M_1 - w\frac{a}{2}\left(x - \frac{2a}{3}\right) + A(x-a) - w(x-a)\left(\frac{x-a}{2}\right)\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

$x_3 = a + b, 1.01(a+b) .. a + b + c$

$$V_3(x) = w\left(\frac{a+b+c-x}{c}\right)\left(\frac{a+b+c-x}{2}\right)\frac{1}{\text{kip}}$$

$$M_{3p}(x) = \left[-w\left(\frac{a+b+c-x}{c}\right)\left(\frac{a+b+c-x}{2}\right)\left(\frac{a+b+c-x}{3}\right) - M_2\right]\frac{1}{\text{kip}\cdot\text{ft}}$$

Force (lb)

$V_1(x_1)$, $V_2(x_2)$, $V_3(x_3)$

40, 20, 0, −20, −40

0, 5, 10, 15, 20

$\frac{x_1}{\text{ft}}, \frac{x_2}{\text{ft}}, \frac{x_3}{\text{ft}}$

Distance (ft)

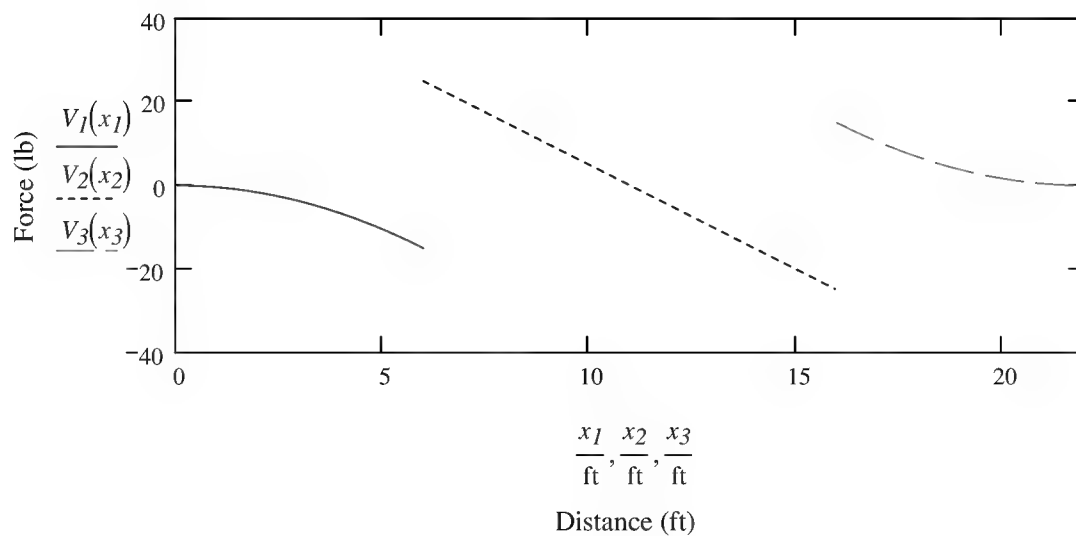
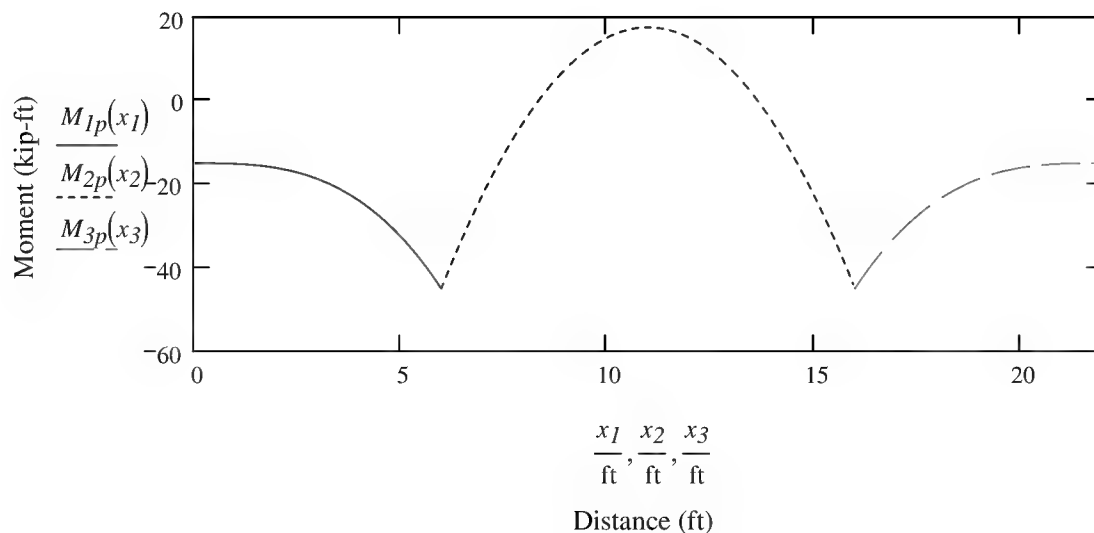
Moment (kip-ft)

$M_{1p}(x_1)$, $M_{2p}(x_2)$, $M_{3p}(x_3)$

20, 0, −20, −40, −60

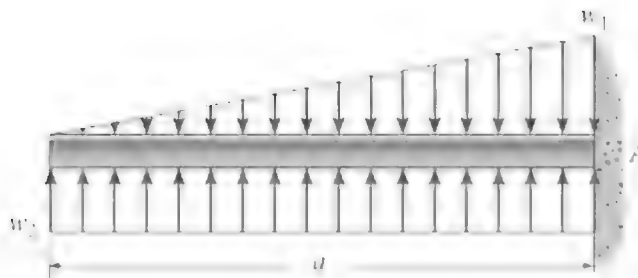
0, 5, 10, 15, 20

$\frac{x_1}{ft}, \frac{x_2}{ft}, \frac{x_3}{ft}$

Distance (ft)

Problem 7-88

Draw the shear and moment diagrams for the beam.

Units Used:

$$kip = 10^3 \text{ lb}$$

Given:

$$w_1 = 2\frac{kip}{ft} \qquad w_2 = 1\frac{kip}{ft} \qquad a = 15 \text{ ft}$$

Solution:

$$x = 0, 0.01a..a$$

$$V(x) = \left[w_2 x - \left(w_1 \frac{x}{a}\right)\frac{x}{2}\right]\frac{1}{kip} \qquad M(x) = \left[w_2 x \frac{x}{2} - \left(w_1 \frac{x}{a}\right)\frac{x}{2}\frac{x}{3}\right]\frac{1}{kip \cdot ft}$$

Force (kip)

$\frac{V(x)}{\text{kip}}$

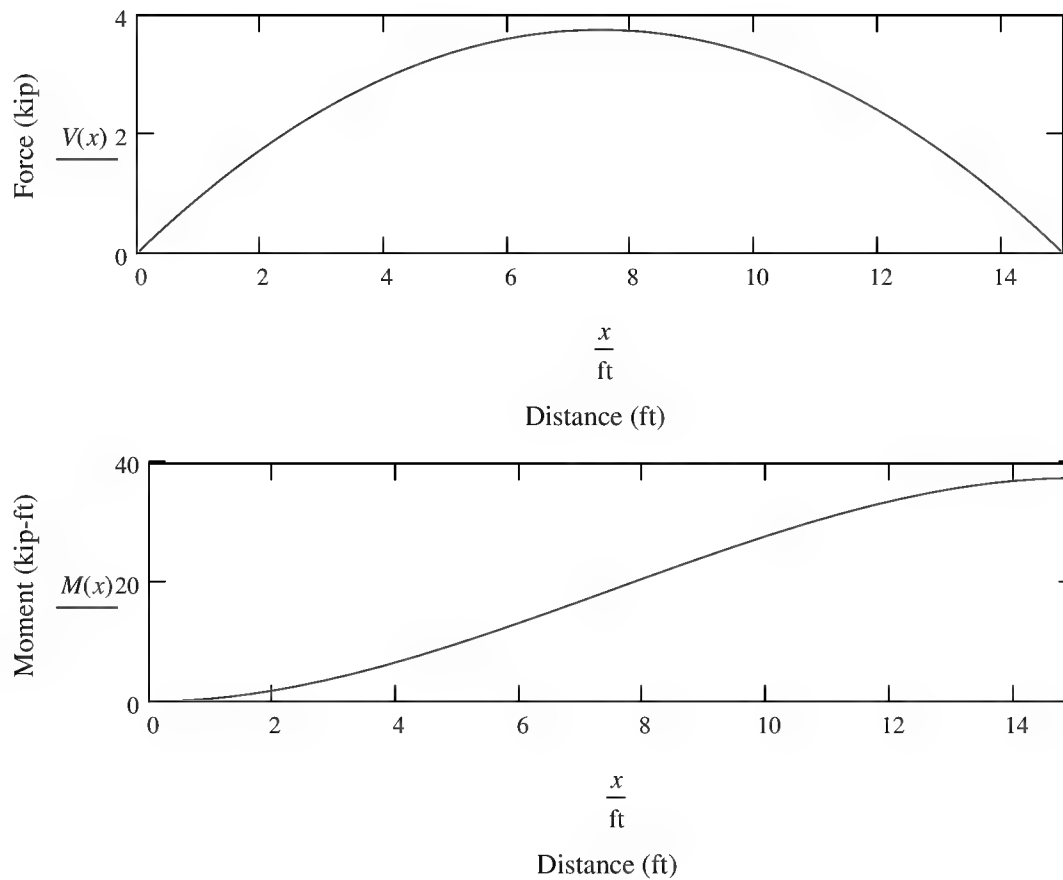
$\frac{x}{\text{ft}}$

Distance (ft)

Moment (kip-ft)

$\frac{M(x)}{\text{kip}\cdot\text{ft}}$

$\frac{x}{\text{ft}}$

Distance (ft)

Problem 7-89

Determine the force P needed to hold the cable in the position shown, i.e., so segment BC remains horizontal. Also, compute the sag y_B and the maximum tension in the cable.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$a = 4\text{ m}$ $\quad F_1 = 4\text{ kN}$

$b = 6\text{ m}$ $\quad F_2 = 6\text{ kN}$

$c = 3\text{ m}$

$d = 2\text{ m}$

$e = 3\text{ m}$

Solution:

Initial guesses:

$y_B = 1\text{ m}$ $\quad P = 1\text{ kN}$ $\quad T_{AB} = 1\text{ kN}$ $\quad T_{BC} = 1\text{ kN}$ $\quad T_{CD} = 1\text{ kN}$ $\quad T_{DE} = 1\text{ kN}$

Given

$$\left(\frac{-a}{\sqrt{a^2 + y_B^2}}\right)T_{AB} + T_{BC} = 0$$

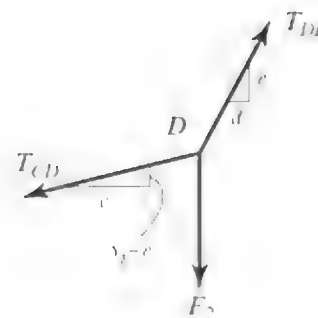
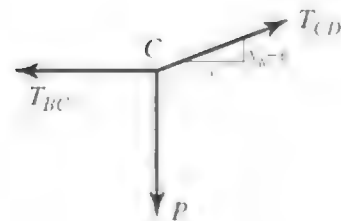
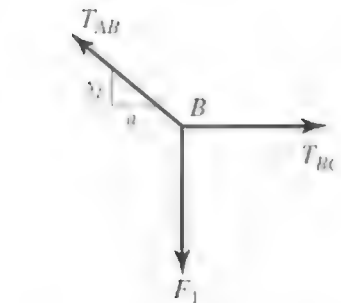
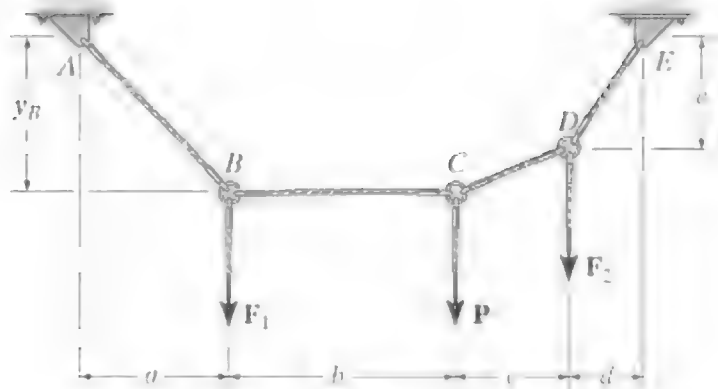
$$\left(\frac{y_B}{\sqrt{a^2 + y_B^2}}\right)T_{AB} - F_1 = 0$$

$$-T_{BC} + \left[\frac{c}{\sqrt{c^2 + (y_B - e)^2}}\right]T_{CD} = 0$$

$$\left[\frac{y_B - e}{\sqrt{c^2 + (y_B - e)^2}}\right]T_{CD} - P = 0$$

$$\left[\frac{-c}{\sqrt{c^2 + (y_B - e)^2}}\right]T_{CD} + \left(\frac{d}{\sqrt{d^2 + e^2}}\right)T_{DE} = 0$$

$$\left[\frac{-(y_B - e)}{\sqrt{c^2 + (y_B - e)^2}}\right]T_{CD} + \left(\frac{e}{\sqrt{e^2 + d^2}}\right)T_{DE} - F_2 = 0$$

$$\begin{pmatrix} y_B \\ P \\ T_{AB} \\ T_{BC} \\ T_{CD} \\ T_{DE} \end{pmatrix} = \text{Find}(y_B, P, T_{AB}, T_{BC}, T_{CD}, T_{DE})$$

$T_{max} = \max(T_{AB}, T_{BC}, T_{CD}, T_{DE})$

$y_B = 3.53$ m

$P = 800.00$ N

$T_{max} = 8.17$ kN

Problem 7-90

Cable *ABCD* supports the lamp of mass M_1 and the lamp of mass M_2. Determine the maximum tension in the cable and the sag of point *B*.

Given:

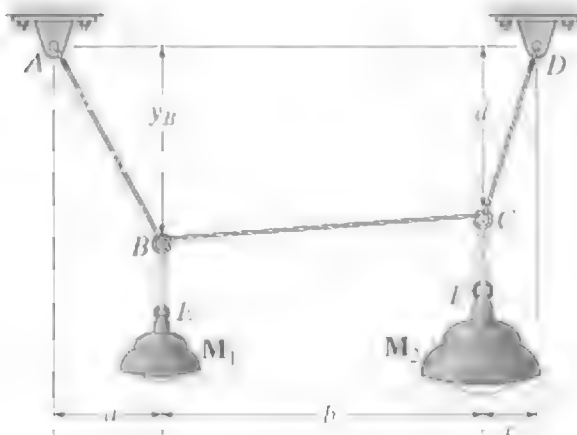
$M_1 = 10$ kg

$M_2 = 15$ kg

$a = 1$ m

$b = 3$ m

$c = 0.5$ m

$d = 2$ m

Solution:

Guesses $y_B = 1$ m $T_{AB} = 1$ N $T_{BC} = 1$ N $T_{CD} = 1$ N

Given

$$\left(\frac{-a}{\sqrt{a^2 + y_B^2}}\right) T_{AB} + \left[\frac{b}{\sqrt{b^2 + (y_B - d)^2}}\right] T_{BC} = 0$$

$$\left(\frac{y_B}{\sqrt{a^2 + y_B^2}}\right) T_{AB} + \left[\frac{y_B - d}{\sqrt{b^2 + (y_B - d)^2}}\right] T_{BC} - M_1 g = 0$$

$$\left[\frac{-b}{\sqrt{b^2 + (y_B - d)^2}}\right] T_{BC} + \left(\frac{c}{\sqrt{c^2 + d^2}}\right) T_{CD} = 0$$

$$\left[\frac{-(y_B - d)}{\sqrt{b^2 + (y_B - d)^2}}\right] T_{BC} + \left(\frac{d}{\sqrt{c^2 + d^2}}\right) T_{CD} - M_2 g = 0$$

$$\begin{pmatrix} y_B \\ T_{AB} \\ T_{BC} \\ T_{CD} \end{pmatrix} = \text{Find}(y_B, T_{AB}, T_{BC}, T_{CD}) \qquad \begin{pmatrix} T_{AB} \\ T_{BC} \\ T_{CD} \end{pmatrix} = \begin{pmatrix} 100.163 \\ 38.524 \\ 157.243 \end{pmatrix} \text{N}$$

$$T_{max} = \max(T_{AB}, T_{BC}, T_{CD}) \qquad T_{max} = 157.2\ \text{N} \qquad y_B = 2.43\ \text{m}$$

Problem 7-91

The cable supports the three loads shown. Determine the sags y_B and y_D of points B and D.

Given:

$a = 4$ ft $\quad e = 12$ ft

$b = 12$ ft $\quad f = 14$ ft

$c = 20$ ft $\quad P_1 = 400$ lb

$d = 15$ ft $\quad P_2 = 250$ lb

Solution:

Guesses $\quad y_B = 1$ ft $\quad y_D = 1$ ft

$T_{AB} = 1$ lb $\quad T_{BC} = 1$ lb

$T_{CD} = 1$ lb $\quad T_{DE} = 1$ lb

Given

$$\left(\frac{-b}{\sqrt{b^2 + y_B^2}}\right)T_{AB} + \left[\frac{c}{\sqrt{c^2 + (f - y_B)^2}}\right]T_{BC} = 0$$

$$\left(\frac{y_B}{\sqrt{b^2 + y_B^2}}\right)T_{AB} - \left[\frac{f - y_B}{\sqrt{c^2 + (f - y_B)^2}}\right]T_{BC} - P_2 = 0$$

$$\left[\frac{-c}{\sqrt{c^2 + (f - y_B)^2}}\right]T_{BC} + \left[\frac{d}{\sqrt{d^2 + (f - y_D)^2}}\right]T_{CD} = 0$$

$$\left[\frac{f-y_B}{\sqrt{c^2+(f-y_B)^2}}\right]T_{BC}+\left[\frac{f-y_D}{\sqrt{d^2+(f-y_D)^2}}\right]T_{CD}-P_1=0$$

$$\left[\frac{-d}{\sqrt{d^2+(f-y_D)^2}}\right]T_{CD}+\left[\frac{e}{\sqrt{e^2+(a+y_D)^2}}\right]T_{DE}=0$$

$$\left[\frac{-(f-y_D)}{\sqrt{d^2+(f-y_D)^2}}\right]T_{CD}+\left[\frac{a+y_D}{\sqrt{e^2+(a+y_D)^2}}\right]T_{DE}-P_2=0$$

$$\begin{pmatrix}T_{AB}\\T_{BC}\\T_{CD}\\T_{DE}\\y_B\\y_D\end{pmatrix}=\mathrm{Find}(T_{AB},T_{BC},T_{CD},T_{DE},y_B,y_D)$$

$$\begin{pmatrix}T_{AB}\\T_{BC}\\T_{CD}\\T_{DE}\end{pmatrix}=\begin{pmatrix}675.89\\566.90\\603.86\\744.44\end{pmatrix}\text{lb}$$

$$\begin{pmatrix}y_B\\y_D\end{pmatrix}=\begin{pmatrix}8.67\\7.04\end{pmatrix}\text{ft}$$

Problem 7-92

The cable supports the three loads shown. Determine the magnitude of $\mathbf{P_1}$ and find the sag y_D for the given data.

Given:

$P_2 = 300$ lb $\quad c = 20$ ft

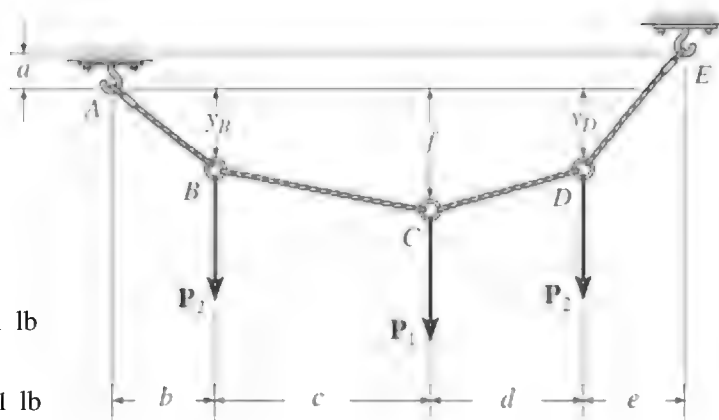
$y_B = 8$ ft $\quad d = 15$ ft

$a = 4$ ft $\quad e = 12$ ft

$b = 12$ ft $\quad f = 14$ ft

Solution:

Guesses $\quad P_1 = 1$ lb $\quad T_{AB} = 1$ lb

$T_{BC} = 1$ lb $\quad T_{CD} = 1$ lb

$T_{DE} = 1\ \text{lb} \qquad y_D = 1\ \text{ft}$

Given

$$\left(\frac{-b}{\sqrt{b^2 + y_B^2}}\right)T_{AB} + \left[\frac{c}{\sqrt{c^2 + (f - y_B)^2}}\right]T_{BC} = 0$$

$$\left(\frac{y_B}{\sqrt{b^2 + y_B^2}}\right)T_{AB} - \left[\frac{f - y_B}{\sqrt{c^2 + (f - y_B)^2}}\right]T_{BC} - P_2 = 0$$

$$\left[\frac{-c}{\sqrt{c^2 + (f - y_B)^2}}\right]T_{BC} + \left[\frac{d}{\sqrt{d^2 + (f - y_D)^2}}\right]T_{CD} = 0$$

$$\left[\frac{f - y_B}{\sqrt{c^2 + (f - y_B)^2}}\right]T_{BC} + \left[\frac{f - y_D}{\sqrt{d^2 + (f - y_D)^2}}\right]T_{CD} - P_1 = 0$$

$$\left[\frac{-d}{\sqrt{d^2 + (f - y_D)^2}}\right]T_{CD} + \left[\frac{e}{\sqrt{e^2 + (a + y_D)^2}}\right]T_{DE} = 0$$

$$\left[\frac{-(f - y_D)}{\sqrt{d^2 + (f - y_D)^2}}\right]T_{CD} + \left[\frac{a + y_D}{\sqrt{e^2 + (a + y_D)^2}}\right]T_{DE} - P_2 = 0$$

$$\begin{pmatrix} T_{AB} \\ T_{BC} \\ T_{CD} \\ T_{DE} \\ P_1 \\ y_D \end{pmatrix} = \text{Find}(T_{AB}, T_{BC}, T_{CD}, T_{DE}, P_1, y_D)$$

$$\begin{pmatrix} T_{AB} \\ T_{BC} \\ T_{CD} \\ T_{DE} \end{pmatrix} = \begin{pmatrix} 983.33 \\ 854.21 \\ 916.11 \\ 1084.68 \end{pmatrix} \text{lb}$$

$P_1 = 658\ \text{lb}$

$y_D = 6.44\ \text{ft}$

Problem 7-93

The cable supports the loading shown. Determine the distance x_B the force at point B acts from A.

Given:

$P = 40 \text{ lb}$ $\quad c = 2 \text{ ft}$

$F = 30 \text{ lb}$ $\quad d = 3 \text{ ft}$

$a = 5 \text{ ft}$ $\quad e = 3$

$b = 8 \text{ ft}$ $\quad f = 4$

Solution:

The initial guesses:

$T_{AB} = 10 \text{ lb}$ $\quad T_{CD} = 30 \text{ lb}$

$T_{BC} = 20 \text{ lb}$ $\quad x_B = 5 \text{ ft}$

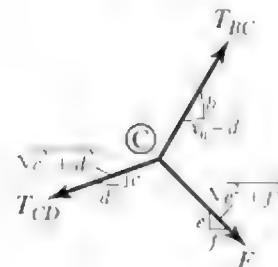
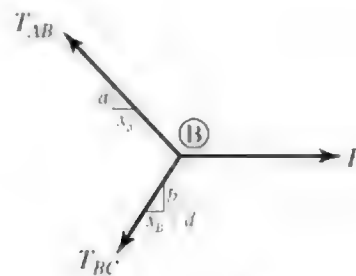
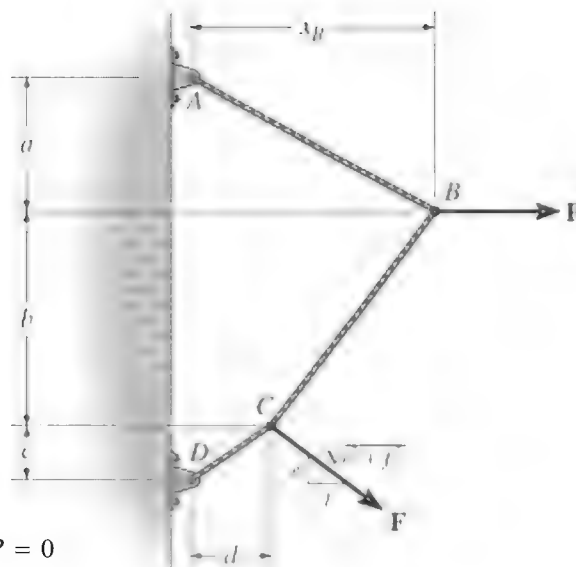
Given

$$\left(\frac{-x_B}{\sqrt{x_B^2 + a^2}}\right)T_{AB} - \left[\frac{x_B - d}{\sqrt{(x_B - d)^2 + b^2}}\right]T_{BC} + P = 0$$

$$\left(\frac{a}{\sqrt{x_B^2 + a^2}}\right)T_{AB} - \left[\frac{b}{\sqrt{(x_B - d)^2 + b^2}}\right]T_{BC} = 0$$

$$\left[\frac{x_B - d}{\sqrt{(x_B - d)^2 + b^2}}\right]T_{BC} - \left(\frac{d}{\sqrt{c^2 + d^2}}\right)T_{CD} + \left(\frac{f}{\sqrt{e^2 + f^2}}\right)F = 0$$

$$\left[\frac{b}{\sqrt{(x_B - d)^2 + b^2}}\right]T_{BC} - \left(\frac{c}{\sqrt{c^2 + d^2}}\right)T_{CD} - \left(\frac{e}{\sqrt{e^2 + f^2}}\right)F = 0$$

$$\begin{pmatrix} T_{AB} \\ T_{CD} \\ T_{BC} \\ x_B \end{pmatrix} = \text{Find}(T_{AB}, T_{CD}, T_{BC}, x_B) \qquad \begin{pmatrix} T_{AB} \\ T_{CD} \\ T_{BC} \end{pmatrix} = \begin{pmatrix} 50.90 \\ 36.70 \\ 38.91 \end{pmatrix} \text{lb} \qquad x_B = 4.36 \text{ ft}$$

Problem 7-94

The cable supports the loading shown. Determine the magnitude of the horizontal force **P**.

Given:

$F = 30 \text{ lb}$ $c = 2 \text{ ft}$

$x_B = 6 \text{ ft}$ $d = 3 \text{ ft}$

$a = 5 \text{ ft}$ $e = 3$

$b = 8 \text{ ft}$ $f = 4$

Solution:

The initial guesses:

$T_{AB} = 10 \text{ lb}$ $T_{CD} = 30 \text{ lb}$

$T_{BC} = 20 \text{ lb}$ $P = 10 \text{ lb}$

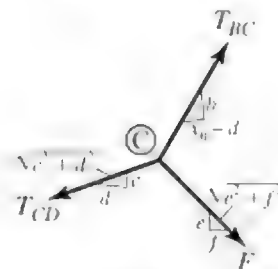
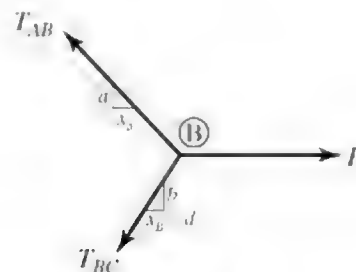
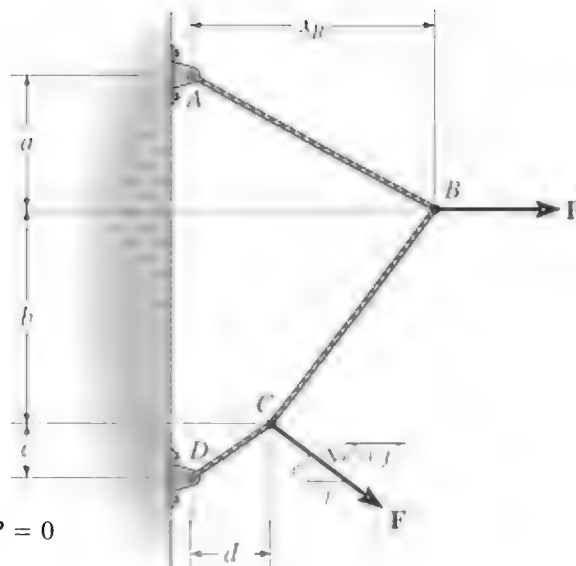
Given

$$\left(\frac{-x_B}{\sqrt{x_B^2 + a^2}}\right)T_{AB} - \left[\frac{x_B - d}{\sqrt{(x_B - d)^2 + b^2}}\right]T_{BC} + P = 0$$

$$\left(\frac{a}{\sqrt{a^2 + x_B^2}}\right)T_{AB} - \left[\frac{b}{\sqrt{b^2 + (x_B - d)^2}}\right]T_{BC} = 0$$

$$\left(\frac{-d}{\sqrt{c^2 + d^2}}\right)T_{CD} + \left[\frac{x_B - d}{\sqrt{b^2 + (x_B - d)^2}}\right]T_{BC} + \left(\frac{f}{\sqrt{e^2 + f^2}}\right)F = 0$$

$$\left(\frac{-c}{\sqrt{c^2 + d^2}}\right)T_{CD} + \left[\frac{b}{\sqrt{b^2 + (x_B - d)^2}}\right]T_{BC} - \left(\frac{e}{\sqrt{e^2 + f^2}}\right)F = 0$$

$$\begin{pmatrix} T_{AB} \\ T_{BC} \\ T_{CD} \\ P \end{pmatrix} = \text{Find}(T_{AB}, T_{BC}, T_{CD}, P) \qquad \begin{pmatrix} T_{AB} \\ T_{BC} \\ T_{CD} \end{pmatrix} = \begin{pmatrix} 70.81 \\ 48.42 \\ 49.28 \end{pmatrix} \text{lb} \qquad P = 71.40 \text{ lb}$$

Problem 7-95

Determine the forces P_1 and P_2 needed to hold the cable in the position shown, i.e., so segment CD remains horizontal. Also, compute the maximum tension in the cable.

Given: $\text{kN} = 10^3\ \text{N}$

$F = 5\ \text{kN}$ $\quad d = 4\ \text{m}$

$a = 1.5\ \text{m}$ $\quad e = 5\ \text{m}$

$b = 1\ \text{m}$ $\quad f = 4\ \text{m}$

$c = 2\ \text{m}$

Solution:

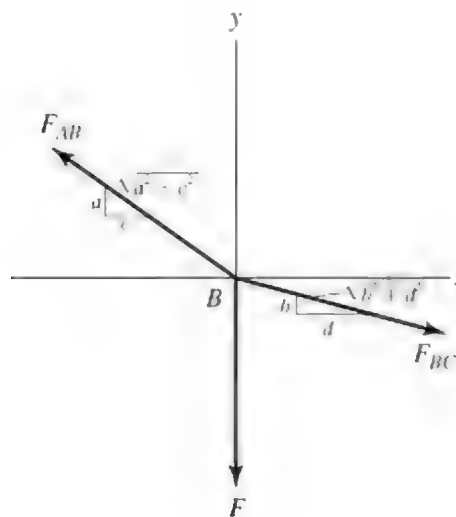
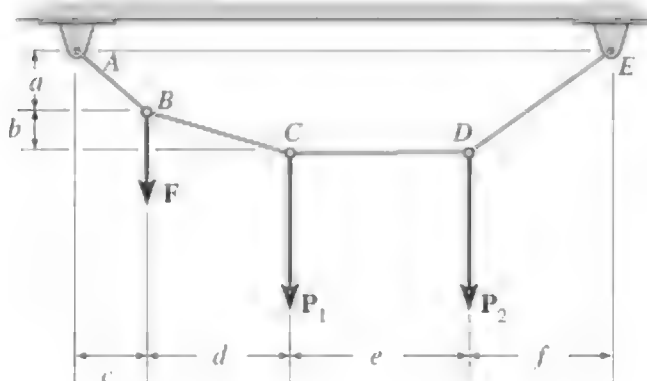
Guesses

$F_{AB} = 1\ \text{kN}$ $\quad F_{BC} = 1\ \text{kN}$

$F_{CD} = 1\ \text{kN}$ $\quad F_{DE} = 1\ \text{kN}$

$P_1 = 1\ \text{kN}$ $\quad P_2 = 1\ \text{kN}$

Given

$$\left(\frac{-c}{\sqrt{a^2+c^2}}\right)F_{AB} + \left(\frac{d}{\sqrt{b^2+d^2}}\right)F_{BC} = 0$$

$$\left(\frac{a}{\sqrt{a^2+c^2}}\right)F_{AB} - \left(\frac{b}{\sqrt{b^2+d^2}}\right)F_{BC} - F = 0$$

$$\left(\frac{-d}{\sqrt{b^2+d^2}}\right)F_{BC} + F_{CD} = 0$$

$$\left(\frac{b}{\sqrt{b^2+d^2}}\right)F_{BC} - P_1 = 0$$

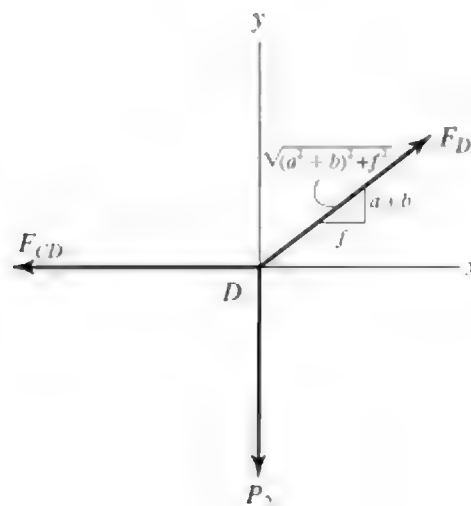
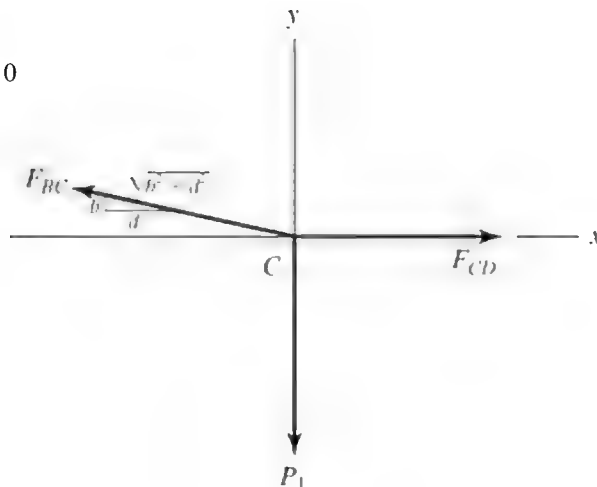
$$-F_{CD} + \left[\frac{f}{\sqrt{f^2+(a+b)^2}}\right]F_{DE} = 0$$

$$\left[\frac{a+b}{\sqrt{f^2+(a+b)^2}}\right]F_{DE} - P_2 = 0$$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \\ F_{DE} \\ P_1 \\ P_2 \end{pmatrix} = \text{Find}(F_{AB}, F_{BC}, F_{CD}, F_{DE}, P_1, P_2)$$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 12.50 \\ 10.31 \\ 10.00 \\ 11.79 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 2.50 \\ 6.25 \end{pmatrix} \text{kN}$$

$$T_{max} = \max(F_{AB}, F_{BC}, F_{CD}, F_{DE}) \qquad T_{max} = 12.50\ \text{kN}$$

$$F_{max} = \max(F_{AB}, F_{BC}, F_{CD}, F_{DE}) \qquad F_{max} = 12.50\ \text{kN}$$

Problem 7-96

The cable supports the loading shown. Determine the distance x_B from the wall to point B.

Given:

$W_1 = 8$ lb

$W_2 = 15$ lb

$a = 5$ ft

$b = 8$ ft

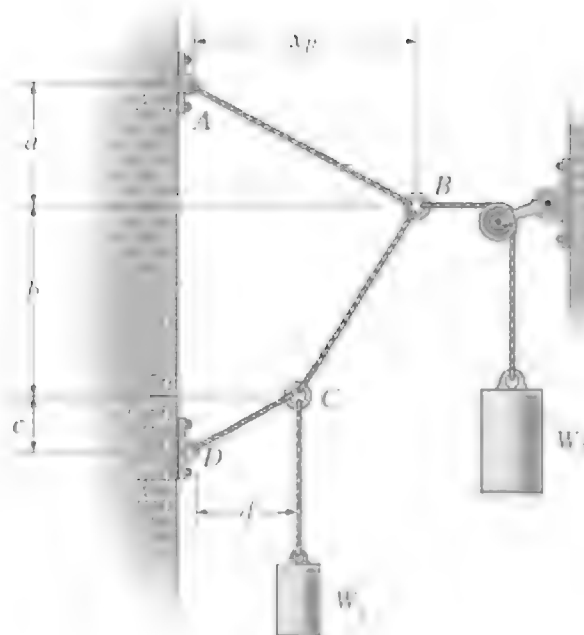
$c = 2$ ft

$d = 3$ ft

Solution:

Guesses

$T_{AB} = 1$ lb $\quad T_{BC} = 1$ lb

$T_{CD} = 1$ lb $\quad x_B = 1$ ft

Given

$$\left(\frac{-x_B}{\sqrt{a^2 + x_B^2}}\right)T_{AB} - \left[\frac{x_B - d}{\sqrt{b^2 + (x_B - d)^2}}\right]T_{BC} + W_2 = 0$$

$$\left(\frac{a}{\sqrt{a^2 + x_B^2}}\right)T_{AB} - \left[\frac{b}{\sqrt{b^2 + (x_B - d)^2}}\right]T_{BC} = 0$$

$$\left[\frac{x_B - d}{\sqrt{b^2 + (x_B - d)^2}}\right]T_{BC} - \left(\frac{d}{\sqrt{c^2 + d^2}}\right)T_{CD} = 0$$

$$\left[\frac{b}{\sqrt{b^2 + (x_B - d)^2}}\right]T_{BC} - \left(\frac{c}{\sqrt{c^2 + d^2}}\right)T_{CD} - W_1 = 0$$

$$\begin{pmatrix} T_{AB} \\ T_{BC} \\ T_{CD} \\ x_B \end{pmatrix} = \text{Find}(T_{AB}, T_{BC}, T_{CD}, x_B) \qquad \begin{pmatrix} T_{AB} \\ T_{BC} \\ T_{CD} \end{pmatrix} = \begin{pmatrix} 15.49 \\ 10.82 \\ 4.09 \end{pmatrix} \text{lb} \qquad x_B = 5.65 \text{ ft}$$

Problem 7-97

Determine the maximum uniform loading w, measured in lb/ft, that the cable can support if it is

capable of sustaining a maximum tension T_{max} before it will break.

Given:

$T_{max} = 3000\ \text{lb}$

$a = 50\ \text{ft}$

$b = 6\ \text{ft}$

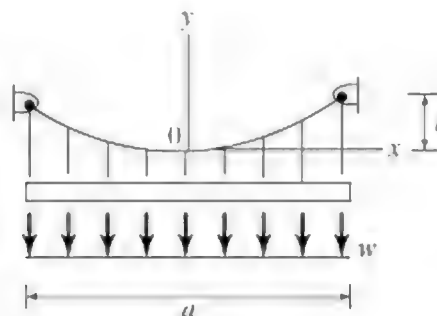
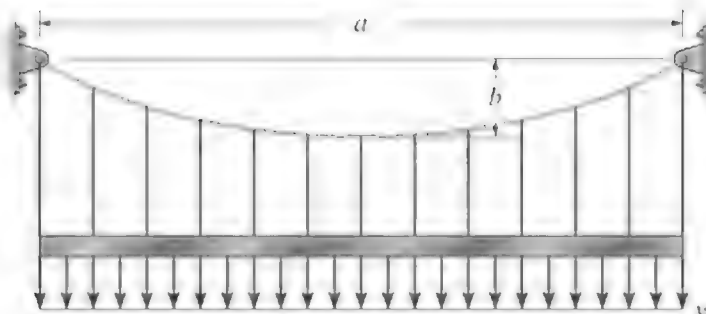
Solution:

$$y = \frac{1}{F_H}\int\int w\,dx\,dx = \frac{wx^2}{2F_H}$$

$$y = \left(\frac{w}{2F_H}\right)x^2$$

$$x = \frac{a}{2} \qquad y = b \qquad F_H = \frac{wa^2}{8b}$$

$$\left(\frac{dy}{dx}\right) = \tan(\theta_{max}) = \frac{w}{F_H}\left(\frac{a}{2}\right) = \frac{4b}{a} \qquad \theta_{max} = \operatorname{atan}\left(\frac{4b}{a}\right) \qquad \theta_{max} = 25.64\ \text{deg}$$

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} = \frac{wa^2}{8b\cos(\theta_{max})} \qquad w = \frac{T_{max}8b\cos(\theta_{max})}{a^2} \qquad w = 51.93\ \frac{\text{lb}}{\text{ft}}$$

Problem 7-98

The cable is subjected to a uniform loading w. Determine the maximum and minimum tension in the cable.

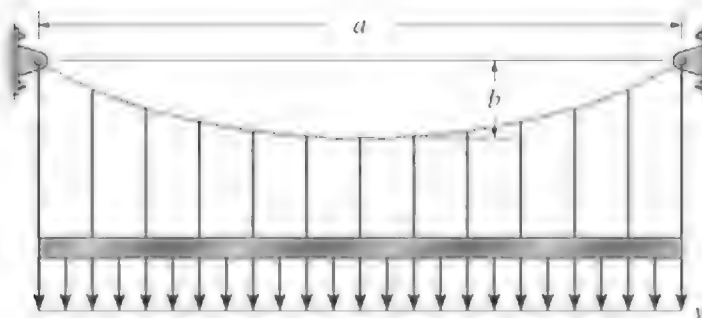
Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$w = 250\ \frac{\text{lb}}{\text{ft}}$

$a = 50\ \text{ft}$

$b = 6\ \text{ft}$

Solution:

$$y = \frac{wx^2}{2F_H} \qquad b = \frac{w}{2F_H}\left(\frac{a}{2}\right)^2 \qquad F_H = \frac{wa^2}{8b} \qquad F_H = 13021\text{ lb}$$

$$\tan(\theta_{max}) = \frac{d}{dx}y\left(x = \frac{a}{2}\right) = \frac{w}{F_H}\left(\frac{a}{2}\right) \qquad \theta_{max} = \operatorname{atan}\left(\frac{wa}{2F_H}\right) \qquad \theta_{max} = 25.64\text{ deg}$$

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} \qquad T_{max} = 14.44\text{ kip}$$

The minimum tension occurs at $\theta = 0$ deg

$$T_{min} = F_H \qquad T_{min} = 13.0\text{ kip}$$

Problem 7-99

The cable is subjected to the triangular loading. If the slope of the cable at A is zero, determine the equation of the curve $y = f(x)$ which defines the cable shape AB, and the maximum tension developed in the cable.

Units Used:

$\text{kip} = 10^3$ lb

Given:

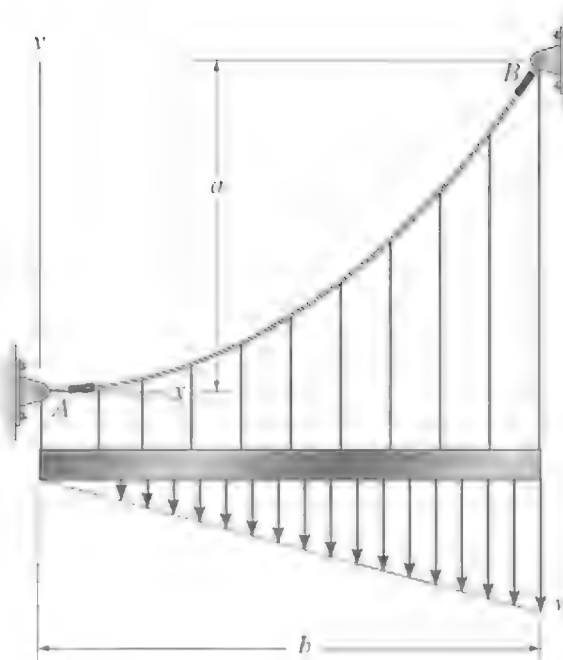
$w = 250 \frac{\text{lb}}{\text{ft}}$

$a = 20$ ft

$b = 30$ ft

Solution:

$$y = \frac{1}{F_H}\int\int \frac{wx}{b}\,dx\,dx$$

$$y = \frac{1}{F_H}\left(\frac{wx^3}{6b} + c_1 x + c_2\right)$$

Apply boundary conditions $y = x = 0$ and $\frac{d}{dx}y = 0, x = 0$ Thus $C_1 = C_2 = 0$

$y = \frac{wx^3}{6F_H b}$ set $y = a$ $x = b$ $a = \frac{wb^3}{6F_H b}$

$F_H = \frac{wb^2}{6a}$ $F_H = 1.875 \text{ kip}$ $\theta_{max} = \text{atan}\left(\frac{wb^2}{2F_H b}\right)$ $\theta_{max} = 63.43 \text{ deg}$

$T_{max} = \frac{F_H}{\cos(\theta_{max})}$ $T_{max} = 4.19 \text{ kip}$

Problem 7-100

The cable supports a girder which has weight density γ. Determine the tension in the cable at points A, B, and C.

Units used:

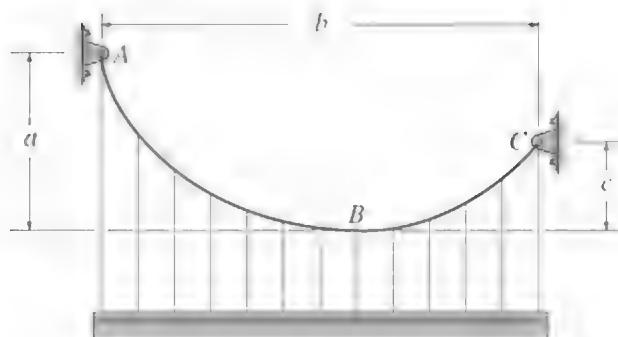
$\text{kip} = 10^3 \text{ lb}$

Given:

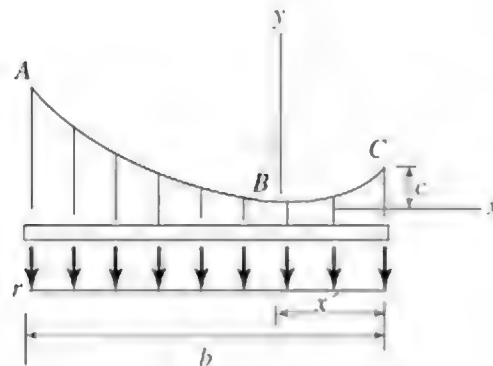
$\gamma = 850 \frac{\text{lb}}{\text{ft}}$

$a = 40 \text{ ft}$

$b = 100 \text{ ft}$

$c = 20 \text{ ft}$

Solution:

$y = \frac{1}{F_H}\int\int \gamma \, dx \, dx$

$y = \frac{\gamma x^2}{2F_H}$ $\frac{d}{dx}y = \frac{\gamma x}{F_H}$ Guesses $x_1 = 1 \text{ ft}$ $F_H = 1 \text{ lb}$

Given $c = \frac{\gamma x_1^{\,2}}{2F_H}$ $a = \frac{\gamma}{2F_H}(b - x_1)^2$ $\begin{pmatrix} x_1 \\ F_H \end{pmatrix} = \text{Find}(x_1, F_H)$ $F_H = 36.46\ \text{kip}$

$\tan(\theta_A) = \frac{\gamma}{F_H}(x_1 - b)$ $\theta_A = \text{atan}\left[\frac{\gamma}{F_H}(x_1 - b)\right]$ $\theta_A = -53.79\ \text{deg}$

$\tan(\theta_C) = \frac{\gamma}{F_H}x_1$ $\theta_C = \text{atan}\left(\frac{\gamma}{F_H}x_1\right)$ $\theta_C = 44.00\ \text{deg}$

$T_A = \frac{F_H}{\cos(\theta_A)}$ $T_B = F_H$ $T_C = \frac{F_H}{\cos(\theta_C)}$ $\begin{pmatrix} T_A \\ T_B \\ T_C \end{pmatrix} = \begin{pmatrix} 61.71 \\ 36.46 \\ 50.68 \end{pmatrix}\ \text{kip}$

Problem 7-101

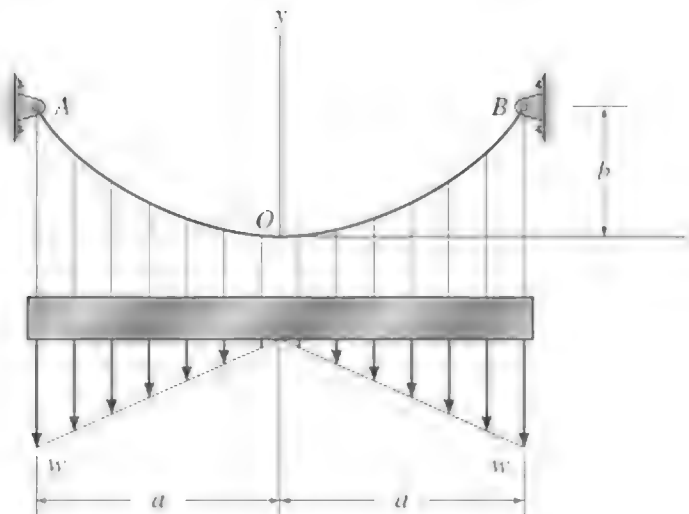
The cable is subjected to the triangular loading. If the slope of the cable at point O is zero, determine the equation of the curve $y = f(x)$ which defines the cable shape OB, and the maximum tension developed in the cable.

Units used:

$\text{kip} = 10^3\ \text{lb}$

Given:

$w = 500\ \frac{\text{lb}}{\text{ft}}$ $b = 8\ \text{ft}$

$a = 15\ \text{ft}$

Solution:

$$y = \frac{1}{F_H}\left(\int\int \frac{wx}{a}\,dx\,dx\right)$$

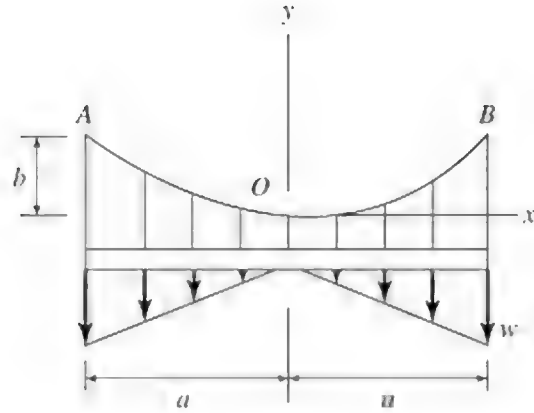
$$y = \frac{1}{F_H}\left[\frac{w}{a}\left(\frac{x^3}{6}\right) + C_1 x + C_2\right]$$

$$\frac{d}{dx}y = \frac{1}{F_H}\left(\frac{wx^2}{2a}\right) + \left(\frac{C_1}{F_H}\right)$$

At $x = 0$, $\frac{d}{dx}y = 0$, $C_1 = 0$

At $x = 0$, $y = 0$, $C_2 = 0$

$$y = \frac{wx^3}{6aF_H} \qquad \frac{d}{dx}y = \frac{wx^2}{2aF_H}$$

At $x = a$, $y = b$ $\quad b = \frac{wa^3}{6aF_H} \qquad F_H = \frac{1}{6}w\left(\frac{a^2}{b}\right) \qquad F_H = 2343.75\text{ lb}$

$$\frac{d}{dx}y = \tan(\theta_{max}) = \frac{wa^2}{2aF_H} \qquad (\theta_{max}) = \operatorname{atan}\left(\frac{wa^2}{2aF_H}\right) \qquad (\theta_{max}) = 57.99\ \text{deg}$$

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} \qquad T_{max} = 4.42\ \text{kip}$$

Problem 7-102

The cable is subjected to the parabolic loading $w = w_0(1-(2x/a)^2)$. Determine the equation $y = f(x)$ which defines the cable shape AB and the maximum tension in the cable.

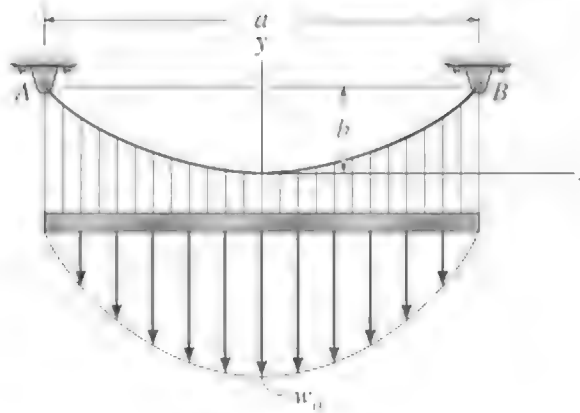
Units Used:

$$\text{kip} = 10^3\ \text{lb}$$

Given:

$$w = w_0\left[1-\left(\frac{2x}{a}\right)^2\right]$$

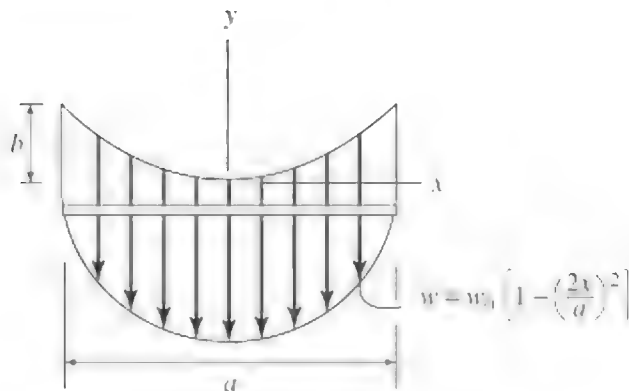
$$a = 100\ \text{ft} \qquad w_0 = 150\ \frac{\text{lb}}{\text{ft}}$$

$$b = 20\ \text{ft}$$

Solution:

$$y = \frac{1}{F_H}\int\int w(x)\ dx\ dx$$

$$y = \frac{1}{F_H}\int w_0\left(x - \frac{4x^3}{3a^2}\right) + C_1\ dx$$

$$y = \frac{1}{F_H}\left(\frac{w_0x^2}{2} - \frac{x^4 w_0}{3a^2} + C_1x + C_2\right)$$

$$\frac{dy}{dx} = \frac{1}{F_H}\left(w_0x - \frac{4w_0x^3}{3a^2} + C_1\right)$$

At $x = 0$ $\frac{dy}{dx} = 0$ $C_1 = 0$

At $x = 0$ $y = 0$ $C_2 = 0$

Thus

$$y = \frac{1}{F_H}\left(\frac{w_0 x^2}{2} - \frac{x^4 w_0}{3a^2}\right) \qquad \frac{dy}{dx} = \frac{1}{F_H}\left(w_0 x - \frac{4w_0 x^3}{3a^2}\right)$$

At $x = \frac{a}{2}$ we have

$$b = \frac{1}{F_H}\left[\frac{w_0}{2}\left(\frac{a}{2}\right)^2 - \frac{w_0}{3a^2}\left(\frac{a}{2}\right)^4\right] = \frac{5}{48}\left(\frac{w_0 a^2}{F_H}\right) \qquad F_H = \frac{5w_0 a^2}{48b} \qquad F_H = 7812.50 \text{ lb}$$

$$\tan(\theta_{max}) = \frac{d}{dx}y\left(\frac{a}{2}\right) = \frac{1}{F_H}\left[w_0\left(\frac{a}{2}\right) - \frac{4w_0}{3a^2}\left(\frac{a}{2}\right)^3\right] = \frac{w_0 a}{3F_H}$$

$$\theta_{max} = \operatorname{atan}\left(\frac{w_0 a}{3F_H}\right) \qquad \theta_{max} = 32.62 \text{ deg}$$

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} \qquad T_{max} = 9.28 \text{ kip}$$

Problem 7-103

The cable will break when the maximum tension reaches T_{max}. Determine the minimum sag h if it supports the uniform distributed load w.

Given: $\text{kN} = 10^3 \text{ N}$

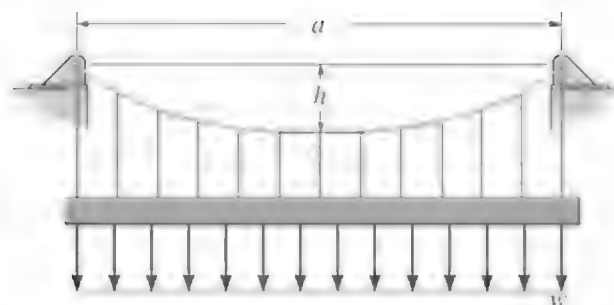
$T_{max} = 10 \text{ kN}$

$w = 600 \frac{\text{N}}{\text{m}}$

$a = 25 \text{ m}$

Solution:

The equation of the cable:

$$y = \frac{1}{F_H}\int\int w \, dx \, dx \qquad y = \frac{1}{F_H}\left(\frac{wx^2}{2} + C_1 x + C_2\right) \qquad \frac{dy}{dx} = \frac{1}{F_H}(wx + C_1)$$

Boundary Conditions:

$y = 0$ at $x = 0$, then from Eq.[1] $\quad 0 = \frac{1}{F_H}(C_2) \quad C_2 = 0$

$\frac{d}{dx}y = 0$ at $x = 0$, then from Eq.[2] $\quad 0 = \frac{1}{F_H}(C_1) \quad C_1 = 0$

Thus, $\quad y = \left(\frac{w}{2F_H}\right)x^2 \quad \frac{dy}{dx} = \frac{w}{F_H}x \quad h = \frac{w}{2F_H}\left(\frac{a}{2}\right)^2 \quad F_H = \frac{w}{2h}\left(\frac{a}{2}\right)^2$

$$\tan(\theta_{max}) = \frac{wa}{2F_H} \qquad \cos(\theta_{max}) = \frac{2F_H}{\sqrt{4F_H^2 + (wa)^2}}$$

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} = \sqrt{F_H^2 + \frac{(wa)^2}{4}} = \frac{wa}{2}\sqrt{\frac{a^2}{16h^2} + 1}$$

Guess $\quad h = 1 \text{ m}$

Given $\quad T_{max} = \frac{wa}{2}\sqrt{\frac{a^2}{16h^2} + 1} \qquad h = \text{Find}(h) \qquad h = 7.09 \text{ m}$

Problem 7-104

A fiber optic cable is suspended over the poles so that the angle at the supports is θ. Determine the minimum tension in the cable and the sag. The cable has a mass density ρ and the supports are at the same elevation.

Given:

$\theta = 22 \text{ deg}$

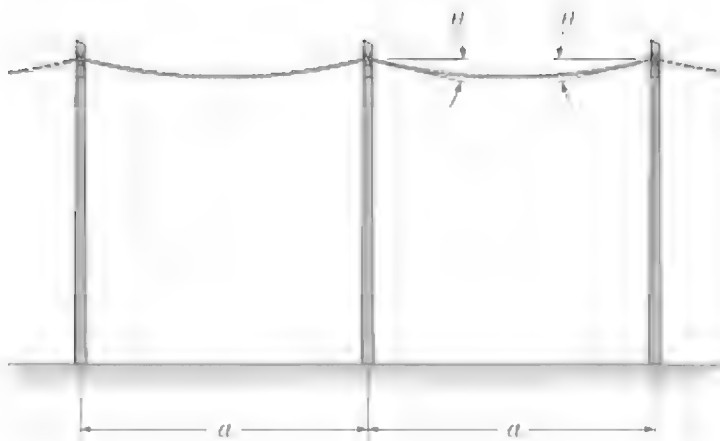
$\rho = 0.9 \frac{\text{kg}}{\text{m}}$

$a = 30 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

$\theta_{max} = \theta$

$$w_0 = \rho g$$

$$\frac{d}{dx}y = \tan(\theta) = \sinh\left(w_0 \frac{\frac{a}{2}}{F_H}\right)$$

$$F_H = \frac{w_0\left(\frac{a}{2}\right)}{\operatorname{asinh}(\tan(\theta))} \qquad F_H = 336 \text{ N}$$

$$T_{max} = \frac{F_H}{\cos(\theta)} \qquad T_{max} = 363 \text{ N}$$

$$h = \frac{F_H}{w_0}\left(\cosh\left(\frac{w_0 \frac{a}{2}}{F_H}\right) - 1\right) \qquad h = 2.99 \text{ m}$$

Problem 7-105

A cable has a weight density γ and is supported at points that are a distance d apart and at the same elevation. If it has a length L, determine the sag.

Given:

$$\gamma = 3 \frac{\text{lb}}{\text{ft}} \qquad d = 500 \text{ ft} \qquad L = 600 \text{ ft}$$

Solution:

Guess $F_H = 100 \text{ lb}$

Given $\frac{L}{2} - \left[\frac{F_H}{\gamma} \sinh\left[\frac{\gamma}{F_H}\left(\frac{d}{2}\right)\right]\right] = 0 \qquad F_H = \text{Find}(F_H) \qquad F_H = 704.3 \text{ lb}$

$$h = \frac{F_H}{\gamma}\left(\cosh\left(\frac{1}{2}\frac{\gamma}{F_H}d\right) - 1\right) \qquad h = 146 \text{ ft}$$

Problem 7-106

Show that the deflection curve of the cable discussed in Example 7.15 reduces to Eq. (4) in Example 7.14 when the *hyperbolic cosine function* is expanded in terms of a series and only the

first two terms are retained. (The answer indicates that the *catenary* may be replaced by a *parabola* in the analysis of problems in which the sag is small. In this case, the cable weight is assumed to be uniformly distributed along the horizontal.)

Solution:

$$\cosh(x) = 1 + \frac{x^2}{2!} + ..$$

Substituting into

$$y = \frac{F_H}{w_0}\left(\cosh\left(\frac{w_0}{F_H}x\right) - 1\right) = \frac{F_H}{w_0}\left(1 + \frac{w_0^2 x^2}{2F_H^2} + .. - 1\right) = \frac{w_0 x^2}{2F_H}$$

Using the boundary conditions $y = h$ at $x = \frac{L}{2}$

$$h = \frac{w_0}{2F_H}\left(\frac{L}{2}\right)^2 \qquad F_H = \frac{w_0 L^2}{8h} \qquad \text{We get} \qquad y = \frac{4h}{L^2}x^2$$

Problem 7-107

A uniform cord is suspended between two points having the same elevation. Determine the sag-to-span ratio so that the maximum tension in the cord equals the cord's total weight.

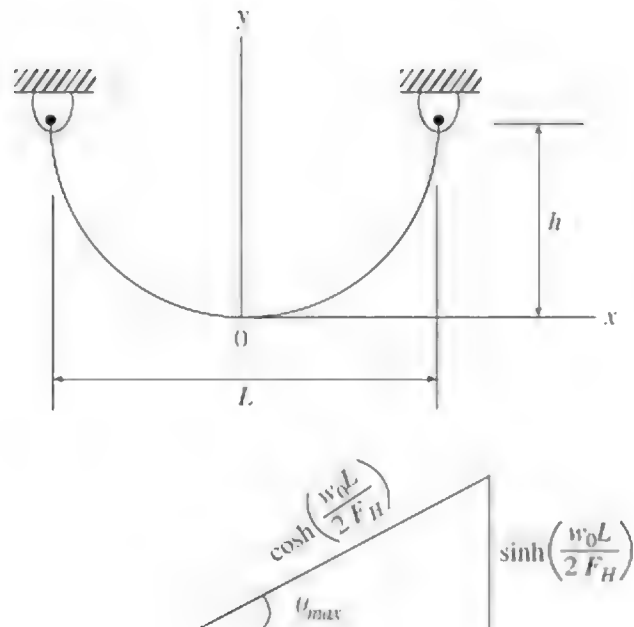
Solution:

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right)$$

$$y = \frac{F_H}{w_0}\left(\cosh\left(\frac{w_0}{F_H}x\right) - 1\right)$$

At $x = \frac{L}{2}$

$$\frac{d}{dx}y_{max} = \tan(\theta_{max}) = \sinh\left(\frac{w_0 L}{2F_H}\right)$$

$$\cos(\theta_{max}) = \frac{1}{\cosh\left(\frac{w_0 L}{2F_H}\right)}$$

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} \qquad w_0 2s = F_H \cosh\left(\frac{w_0 L}{2F_H}\right)$$

$$2F_H \sinh\left(\frac{w_0 L}{2F_H}\right) = F_H \cosh\left(\frac{w_0 L}{2F_H}\right) \qquad \tanh\left(\frac{w_0 L}{2F_H}\right) = \frac{1}{2}$$

$$k_1 = \text{atanh}(0.5) \qquad k_1 = 0.55 \qquad \frac{w_0 L}{2F_H} = k_1$$

when $x = \frac{L}{2}$ $\quad y = h \quad h = \frac{F_H}{W_0}(\cosh(k_1) - 1) \quad k_2 = \cosh(k_1) - 1 \quad k_2 = 0.15$

$$h = k_2\left(\frac{F_H}{w_0}\right) \qquad L = k_1\left(\frac{2F_h}{w_0}\right) \qquad ratio = \frac{h}{L} = \frac{k_2}{2k_1} \qquad ratio = \frac{k_2}{2k_1} \qquad ratio = 0.14$$

Problem 7-108

A cable has a weight denisty γ. If it can span a distance L and has a sag h determine the length of the cable. The ends of the cable are supported from the same elevation.

Given:

$$\gamma = 2\ \frac{\text{lb}}{\text{ft}} \qquad L = 100\ \text{ft} \qquad h = 12\ \text{ft}$$

Solution:

From Eq. (5) of Example 7-15 :

$$h = \frac{F_H}{\gamma}\left[\frac{\left(\frac{\gamma}{2}\frac{L}{F_H}\right)^2}{2}\right] \qquad F_H = \frac{1}{8}\gamma\left(\frac{L^2}{h}\right) \qquad F_H = 208.33\ \text{lb}$$

From Eq. (3) of Example 7-15:

$$\frac{l}{2} = \left(\frac{F_H}{\gamma}\right)\sinh\left[\frac{\gamma}{F_H}\left(\frac{L}{2}\right)\right] \qquad l = 2\left(\frac{F_H}{\gamma}\right)\sinh\left(\frac{1}{2}\gamma\frac{L}{F_H}\right) \qquad l = 104\ \text{ft}$$

Problem 7-109

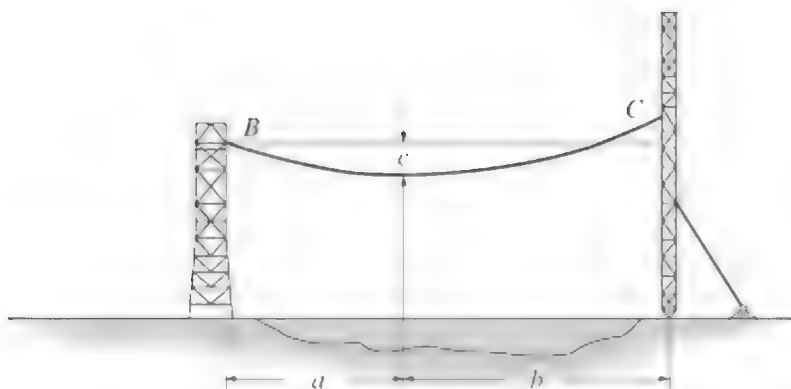
The transmission cable having a weight density γ is strung across the river as shown. Determine the required force that must be applied to the cable at its points of attachment to the towers at B and C.

Units Used:

$kip = 10^3\ lb$

Given:

$\gamma = 20\ \frac{lb}{ft}$ $\quad b = 75\ ft$

$a = 50\ ft$ $\quad c = 10\ ft$

Solution:

From Example 7-15,

$$y = \frac{F_H}{\gamma}\left[\cosh\left(\frac{\gamma}{F_H}x\right) - 1\right] \qquad \frac{dy}{dx} = \sinh\left(\frac{\gamma x}{F_H}\right)$$

Guess $\quad F_H = 1000\ lb$

Given $\quad$ At B: $\quad c = \frac{F_H}{\gamma}\left(\cosh\left(-\frac{\gamma a}{F_H}\right) - 1\right) \qquad F_H = \text{Find}(F_H) \qquad F_H = 2.53\ kip$

$$\tan(\theta_B) = \sinh\left(-\frac{\gamma a}{F_H}\right) \qquad \theta_B = \text{atan}\left(\sinh\left(\frac{-\gamma a}{F_H}\right)\right) \qquad \theta_B = -22.06\ deg$$

$$\tan(\theta_C) = \sinh\left(\frac{\gamma b}{F_H}\right) \qquad \theta_C = \text{atan}\left(\sinh\left(\frac{\gamma b}{F_H}\right)\right) \qquad \theta_C = 32.11\ deg$$

$$T_B = \frac{F_H}{\cos(\theta_B)} \qquad T_C = \frac{F_H}{\cos(\theta_C)} \qquad \begin{pmatrix} T_B \\ T_C \end{pmatrix} = \begin{pmatrix} 2.73 \\ 2.99 \end{pmatrix} kip$$

Problem 7-110

Determine the maximum tension developed in the cable if it is subjected to a uniform load w.

Units Used:

$MN = 10^6\ N$

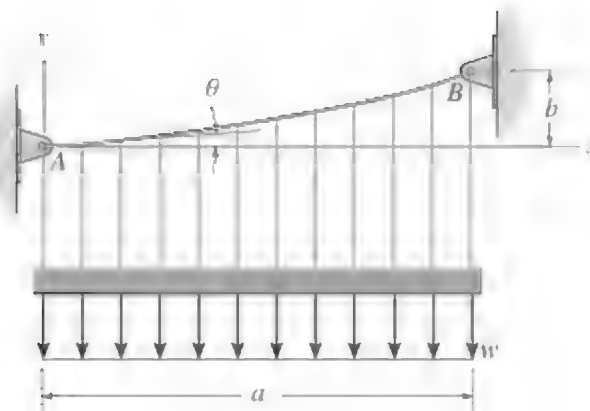
Given:

$w = 600\ \frac{N}{m}$

$a = 100\ m$

$b = 20\ m$

$\theta = 10\ deg$

Solution:

The Equation of the Cable:

$$y = \frac{1}{F_H}\int\int w(x)\,dx\,dx = \frac{1}{F_H}\left(\frac{wx^2}{2} + C_1 x + C_2\right)$$

$$\frac{dy}{dx} = \frac{1}{F_H}\left(wx + C_1\right)$$

Initial Guesses: $C_1 = 1\ N$ $C_2 = 1\ N\cdot m$ $F_H = 1\ N$

Given *Boundary Conditions:*

$x = 0$ $0 = \frac{1}{F_H}C_2$ $\tan(\theta) = \frac{1}{F_H}(C_1)$

$y = b$ at $x = a$ $b = \left(\frac{w}{2F_H}\right)a^2 + \left(\frac{C_1}{F_H}\right)a$

$$\begin{pmatrix} C_1 \\ C_2 \\ F_H \end{pmatrix} = \text{Find}(C_1, C_2, F_H) \qquad C_1 = 0.22\,MN \qquad C_2 = 0.00\,N\cdot m \qquad F_H = 1.27\,MN$$

$$\tan(\theta_{max}) = \frac{1}{F_H}(wa + C_1) \qquad \theta_{max} = \operatorname{atan}\left(\frac{wa + C_1}{F_H}\right) \qquad \theta_{max} = 12.61\ deg$$

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} \qquad T_{max} = 1.30\,MN$$

Problem 7-111

A chain of length L has a total mass M and is suspended between two points a distance d apart. Determine the maximum tension and the sag in the chain.

Given:

$L = 40\ \text{m}$ $M = 100\ \text{kg}$ $d = 10\ \text{m}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$w = M\frac{g}{L}$$

$$s = \left(\frac{F_H}{w}\right)\sinh\left(\frac{w}{F_H}x\right) \qquad y = \left(\frac{F_H}{w}\right)\left(\cosh\left(\frac{w}{F_H}x\right) - 1\right) \qquad \frac{d}{dx}y = \sinh\left(\frac{w}{F_H}x\right)$$

Guesses $F_H = 10\ \text{N}$ $h = 10\ \text{m}$

Given $$\frac{L}{2} = \left(\frac{F_H}{w}\right)\sinh\left(\frac{w}{F_H}\frac{d}{2}\right) \qquad h = \frac{F_H}{w}\left(\cosh\left(\frac{w}{F_H}\frac{d}{2}\right) - 1\right) \qquad \begin{pmatrix} F_H \\ h \end{pmatrix} = \text{Find}(F_H, h)$$

$$\theta_{max} = \text{atan}\left(\sinh\left(\frac{w}{F_H}\frac{d}{2}\right)\right) \qquad T_{max} = \frac{F_H}{\cos(\theta_{max})}$$

$F_H = 37.57\ \text{N}$ $h = 18.53\ \text{m}$ $T_{max} = 492\ \text{N}$

Problem 7-112

The cable has a mass density ρ and has length L. Determine the vertical and horizontal components of force it exerts on the top of the tower.

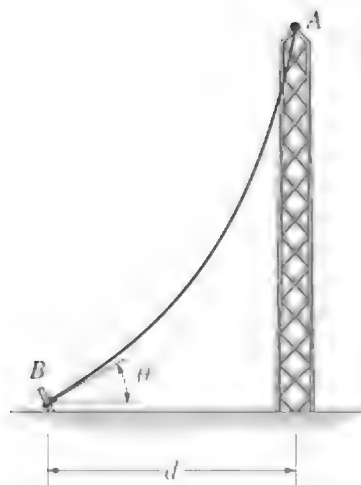
Given:

$\rho = 0.5\ \frac{\text{kg}}{\text{m}}$

$L = 25\ \text{m}$

$\theta = 30\ \text{deg}$

$d = 15\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$x = \int \frac{1}{\sqrt{1 + \frac{1}{F_H^2}\left(\int \rho g \, ds\right)^2}} \, ds$$

Performing the integration yields:

$$x = \frac{F_H}{\rho g}\left(\operatorname{asinh}\left(\frac{\rho g s + C_1}{F_H}\right) + C_2\right)$$

$$\frac{dy}{dx} = \frac{1}{F_H}\int \rho g \, ds = \frac{1}{F_H}(\rho g s + C_1)$$

At $s = 0$; $\frac{dy}{dx} = \tan(\theta)$ Hence $C_1 = F_H \tan(\theta)$

$$\frac{dy}{dx} = \frac{\rho g s}{F_H} + \tan(\theta)$$

Applying boundary conditions at $x = 0$; $s = 0$ to Eq.[1] and using the result $C_1 = F_H \tan(\theta)$ yields $C_2 = -\operatorname{asinh}(\tan(\theta))$. Hence

Guess $F_H = 1 \text{ N}$

Given

$$d = \left(\frac{F_H}{\rho g}\right)\left[\operatorname{asinh}\left[\left(\frac{1}{F_H}\right)(\rho g L + F_H \tan(\theta))\right] - (\operatorname{asinh}(\tan(\theta)))\right]$$

$F_H = \text{Find}(F_H)$ $F_H = 73.94 \text{ N}$

At A $\tan(\theta_A) = \frac{\rho g L}{F_H} + \tan(\theta)$ $\theta_A = \operatorname{atan}\left(\frac{\rho g L}{F_H} + \tan(\theta)\right)$ $\theta_A = 65.90 \text{ deg}$

$F_A = \frac{F_H}{\cos(\theta_A)}$ $F_{Ax} = F_A \cos(\theta_A)$ $F_{Ay} = F_A \sin(\theta_A)$ $\begin{pmatrix} F_{Ax} \\ F_{Ay} \end{pmatrix} = \begin{pmatrix} 73.94 \\ 165.31 \end{pmatrix} \text{ N}$

Problem 7-113

A cable of length L is suspended between two points a distance d apart and at the same elevation. If the minimum tension in the cable is T_{min}, determine the total weight of the cable and the maximum tension developed in the cable.

Units Used: $\text{kip} = 10^3\ \text{lb}$

Given: $L = 50\ \text{ft}$ $\quad d = 15\ \text{ft}$ $\quad T_{min} = 200\ \text{lb}$

Solution: $T_{min} = F_H$ $\quad F_H = T_{min}$ $\quad F_H = 200\,\text{lb}$

From Example 7-15: $$s = \left(\frac{F_H}{w_0}\right)\sinh\left(\frac{w_0 x}{F_H}\right)$$

Guess $\quad w_0 = 1\ \frac{\text{lb}}{\text{ft}}$

Given $$\frac{L}{2} = \left(\frac{F_H}{w_0}\right)\sinh\left(\frac{w_0}{F_H}\frac{d}{2}\right) \qquad w_0 = \text{Find}(w_0) \qquad w_0 = 79.93\ \frac{\text{lb}}{\text{ft}}$$

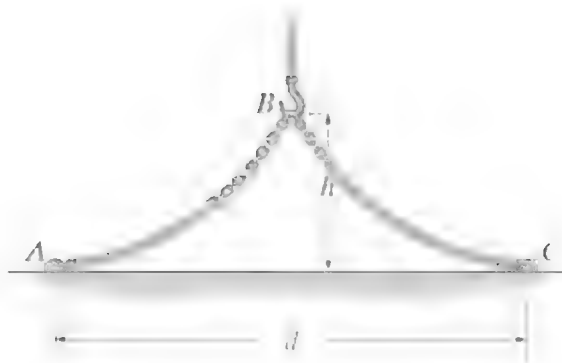
$$Totalweight = w_0 L \qquad Totalweight = 4.00\,\text{kip}$$

$$\tan(\theta_{max}) = \frac{w_0}{F_H}\frac{L}{2} \qquad \theta_{max} = \operatorname{atan}\left[\frac{w_0\left(\frac{L}{2}\right)}{F_H}\right] \qquad \theta_{max} = 84.28\ \text{deg}$$

Then,

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} \qquad T_{max} = 2.01\,\text{kip}$$

Problem 7-114

The chain of length L is fixed at its ends and hoisted at its midpoint B using a crane. If the chain has a weight density w, determine the minimum height h of the hook in order to lift the chain *completely* off the ground. What is the horizontal force at pin A or C when the chain is in this position? *Hint:* When h is a minimum, the slope at A and C is zero.

Given:

$L = 80\ \text{ft}$

$d = 60\ \text{ft}$

$w = 0.5\ \frac{\text{lb}}{\text{ft}}$

Solution:

Guesses $F_H = 10\ \text{lb}$ $h = 1\ \text{ft}$

Given $h = \frac{F_H}{w}\left(\cosh\left(\frac{w}{F_H}\frac{d}{2}\right) - 1\right)$ $\frac{L}{2} = \left(\frac{F_H}{w}\right)\sinh\left(\frac{w}{F_H}\frac{d}{2}\right)$

$\begin{pmatrix} h \\ F_H \end{pmatrix} = \text{Find}(h, F_H)$ $F_A = F_H$ $F_C = F_H$ $\begin{pmatrix} F_A \\ F_C \end{pmatrix} = \begin{pmatrix} 11.1 \\ 11.1 \end{pmatrix}\text{lb}$

$h = 23.5\ \text{ft}$

Problem 7-115

A steel tape used for measurement in surveying has a length L and a total weight W. How much horizontal tension must be applied to the tape so that the distance marked on the ground is a? In practice the calculation should also include the effects of elastic stretching and temperature changes on the tape's length.

Given:

$L = 100\ \text{ft}$

$W = 2\ \text{lb}$

$a = 99.90\ \text{ft}$

Solution:

$$w_0 = \frac{W}{L} \qquad w_0 = 0.02\,\frac{\text{lb}}{\text{ft}}$$

Guess $\quad F_H = 10\ \text{lb}$

Given $\quad \dfrac{L}{2} - \left(\dfrac{F_H}{w_0}\sinh\left(\dfrac{w_0}{F_H}\dfrac{a}{2}\right)\right) = 0 \qquad F_H = \text{Find}(F_H) \qquad F_H = 12.9\ \text{lb}$

Problem 7-116

A cable of weight W is attached between two points that are a distance d apart, having equal elevations. If the maximum tension developed in the cable is T_{max} determine the length L of the cable and the sag h.

Given: $\quad W = 100\ \text{lb} \qquad d = 50\ \text{ft} \qquad T_{max} = 75\ \text{lb}$

Solution:

Guesses $\quad F_H = 20\ \text{lb} \qquad L = 20\ \text{ft} \qquad \theta_{max} = 20\ \text{deg} \qquad h = 2\ \text{ft}$

Given

$$h = \frac{F_H L}{W}\left(\cosh\left(\frac{W}{F_H L}\frac{d}{2}\right) - 1\right) \qquad \tan(\theta_{max}) = \sinh\left(\frac{W}{F_H L}\frac{d}{2}\right)$$

$$T_{max} = \frac{F_H}{\cos(\theta_{max})} \qquad \frac{L}{2} = \left(\frac{F_H L}{W}\right)\sinh\left(\frac{W}{F_H L}\frac{d}{2}\right)$$

$$\begin{pmatrix} F_H \\ L \\ \theta_{max} \\ h \end{pmatrix} = \text{Find}(F_H, L, \theta_{max}, h) \qquad F_H = 55.90\ \text{lb} \qquad \begin{pmatrix} L \\ h \end{pmatrix} = \begin{pmatrix} 55.57 \\ 10.61 \end{pmatrix}\text{ft}$$

$$\theta_{max} = 41.81\ \text{deg}$$

Problem 7-117

Determine the distance a between the supports in terms of the beam's length L so that the moment in the *symmetric* beam is zero at the beam's center.

Solution:

Support Reactions:

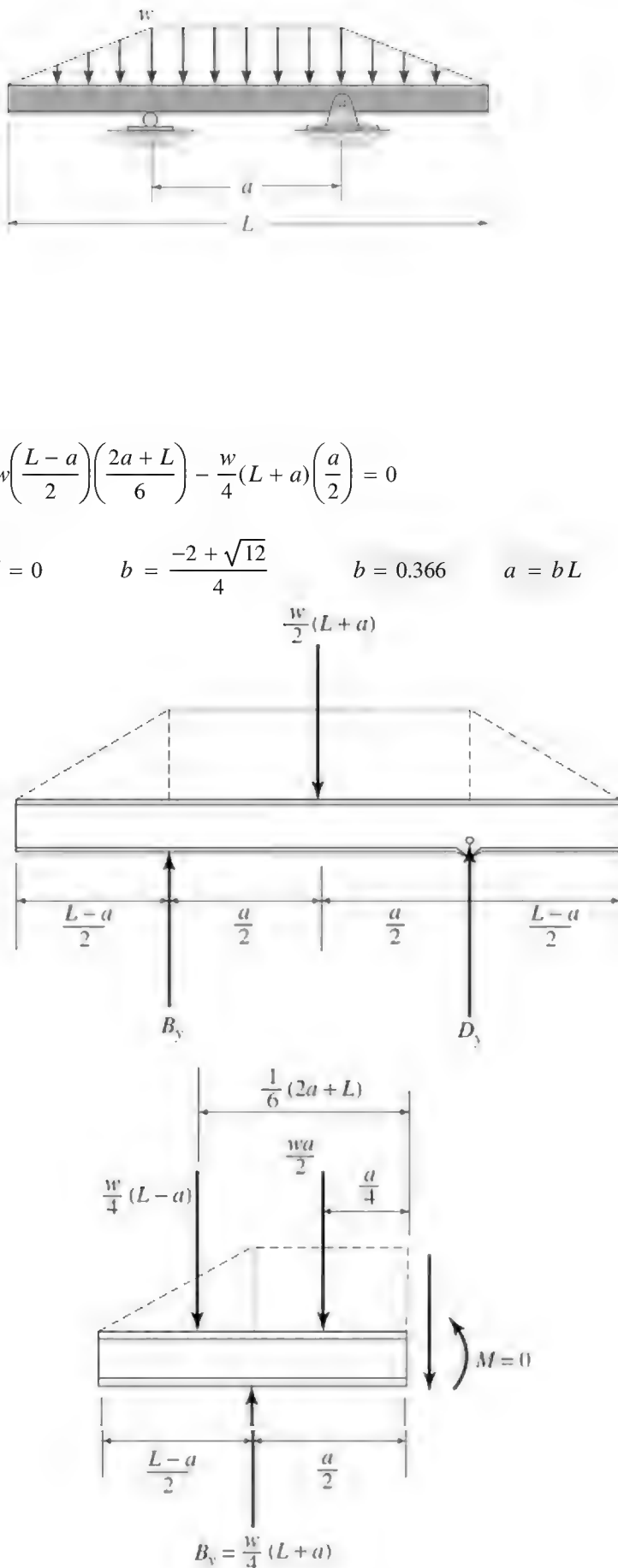
$\Sigma M_D = 0;$

$$\frac{w}{2}(L+a)\left(\frac{a}{2}\right) - B_y(a) = 0$$

$$B_y = \frac{w}{4}(L+a)$$

Internal Forces:

$\Sigma M_C = 0;$ $$w\left(\frac{a}{2}\right)\left(\frac{a}{4}\right) + \frac{1}{2}w\left(\frac{L-a}{2}\right)\left(\frac{2a+L}{6}\right) - \frac{w}{4}(L+a)\left(\frac{a}{2}\right) = 0$$

$$2a^2 + 2aL - L^2 = 0 \qquad b = \frac{-2+\sqrt{12}}{4} \qquad b = 0.366 \qquad a = bL$$

Problem 7-118

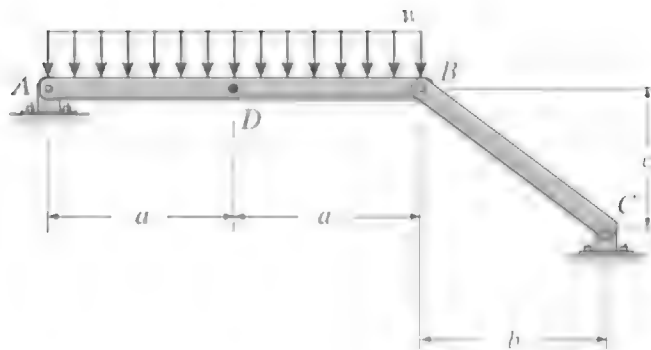
Determine the internal normal force, shear force, and moment at point D.

Given:

$$w = 150\ \frac{\text{N}}{\text{m}}$$

$$a = 4\ \text{m}$$

$$b = 4\ \text{m}$$

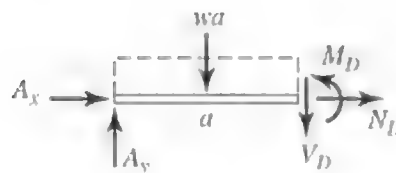
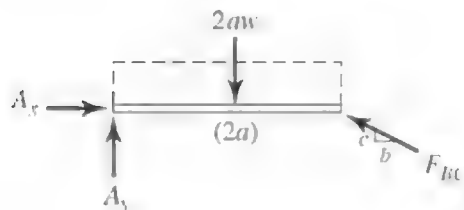
$$c = 3\ \text{m}$$

Solution:

Guesses $A_x = 1\ \text{N}$ $A_y = 1\ \text{N}$ $F_{BC} = 1\ \text{N}$

Given

$$A_x - \left(\frac{b}{\sqrt{b^2 + c^2}}\right) F_{BC} = 0$$

$$A_y - w(2a) + \left(\frac{c}{\sqrt{b^2 + c^2}}\right) F_{BC} = 0$$

$$w 2a\, a - A_y(2a) = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{BC} \end{pmatrix} = \text{Find}(A_x, A_y, F_{BC}) \qquad \begin{pmatrix} A_x \\ A_y \\ F_{BC} \end{pmatrix} = \begin{pmatrix} 800 \\ 600 \\ 1000 \end{pmatrix} \text{N}$$

Guesses $N_D = 1\ \text{N}$ $V_D = 1\ \text{N}$ $M_D = 1\ \text{N·m}$

Given $A_x + N_D = 0$ $A_y - w a - V_D = 0$ $-A_y a + w a\left(\frac{a}{2}\right) + M_D = 0$

$$\begin{pmatrix} N_D \\ V_D \\ M_D \end{pmatrix} = \text{Find}(N_D, V_D, M_D) \qquad \begin{pmatrix} N_D \\ V_D \end{pmatrix} = \begin{pmatrix} -800.00 \\ 0.00 \end{pmatrix} \text{N} \qquad M_D = 1200\,\text{N}\cdot\text{m}$$

Problem 7-119

The beam is supported by a pin at C and a rod AB. Determine the internal normal force, shear force, and moment at point D.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F = 4\ \text{kN}$

$a = 6\ \text{m}$

$b = 5\ \text{m}$

$c = 3\ \text{m}$

$d = 6\ \text{m}$

$\theta = 60\ \text{deg}$

Solution:

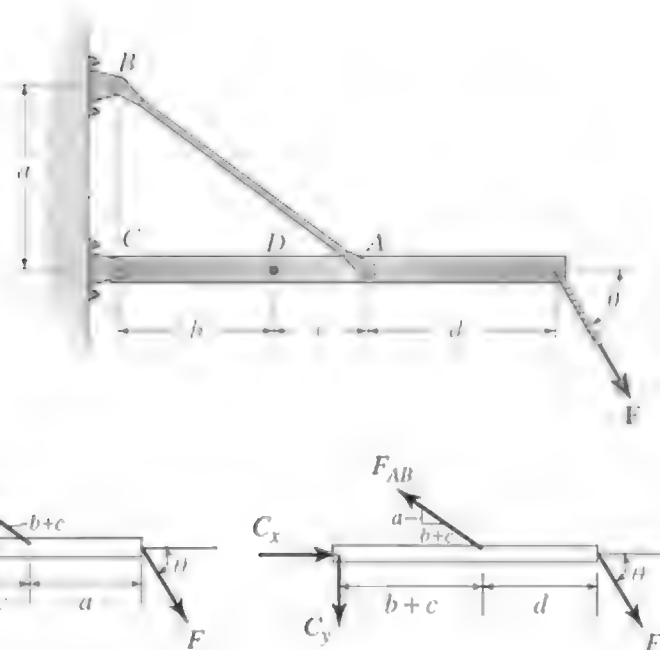
Guesses

$F_{AB} = 1\ \text{N} \quad N_D = 1\ \text{N} \quad V_D = 1\ \text{N} \quad M_D = 1\ \text{N}\cdot\text{m}$

Given $$-F\sin(\theta)(b+c+d) + \left[\frac{a}{\sqrt{a^2+(b+c)^2}}\right] F_{AB}(b+c) = 0$$

$$-N_D - \left[\frac{b+c}{\sqrt{a^2+(b+c)^2}}\right] F_{AB} + F\cos(\theta) = 0$$

$$V_D + \left[\frac{a}{\sqrt{a^2+(b+c)^2}}\right] F_{AB} - F\sin(\theta) = 0$$

$$\left[\frac{a}{\sqrt{a^2+(b+c)^2}}\right] F_{AB}\,c - F\sin(\theta)(c+d) - M_D = 0$$

$$\begin{pmatrix} F_{AB} \\ N_D \\ V_D \\ M_D \end{pmatrix} = \text{Find}(F_{AB}, N_D, V_D, M_D) \qquad \begin{pmatrix} N_D \\ V_D \end{pmatrix} = \begin{pmatrix} -6.08 \\ -2.6 \end{pmatrix} \text{kN} \qquad M_D = -12.99 \text{ kN}\cdot\text{m}$$

Problem 7-120

Express the shear and moment acting in the pipe as a function of y, where $0 \le y \le b$ ft.

Given:

$w = 4 \frac{\text{lb}}{\text{ft}}$

$a = 2$ ft

$b = 4$ ft

Solution:

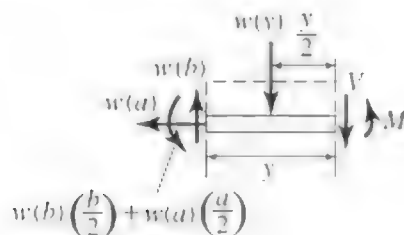
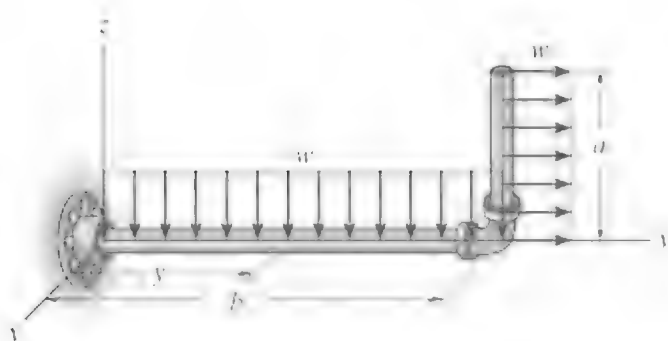
$\Sigma F_y = 0;$ $\quad wb - wy - V = 0$

$$V(y) = w(b - y)$$

$$V(y) = 4 \frac{\text{lb}}{\text{ft}} (4 \text{ ft} - y)$$

$\Sigma M = 0;$ $\quad M + wy\left(\frac{y}{2}\right) + wb\left(\frac{b}{2}\right) + wa\left(\frac{a}{2}\right) - wby = 0$

$$M(y) = w\, b\, y - \frac{1}{2} w\, y^2 - \frac{1}{2} w\, b^2 - \frac{1}{2} w\, a^2$$

$$M(y) = 16 \text{ lb } y - 2 \frac{\text{lb}}{\text{ft}} y^2 - 40 \text{ lb}\cdot\text{ft}$$

Problem 7-121

Determine the normal force, shear force, and moment at points B and C of the beam.

Given: $\quad \text{kN} = 10^3 \text{ N}$

$a = 5\ \text{m}$ $F_2 = 6\ \text{kN}$

$b = 5\ \text{m}$ $w_1 = 2\ \frac{\text{kN}}{\text{m}}$

$c = 1\ \text{m}$

$d = 3\ \text{m}$ $w_2 = 1\ \frac{\text{kN}}{\text{m}}$

$F_1 = 7.5\ \text{kN}$ $M = 40\ \text{kN}\cdot\text{m}$

Solution:

Guesses

$N_C = 1\ \text{N}$ $V_C = 1\ \text{N}$ $M_C = 1\ \text{N}\cdot\text{m}$

Given

$-N_C = 0$ $V_C - w_2 d - F_2 = 0$

$$-M_C - w_2 d\left(\frac{d}{2}\right) - F_2 d - M = 0$$

$$\begin{pmatrix} N_C \\ V_C \\ M_C \end{pmatrix} = \text{Find}(N_C, V_C, M_C)$$

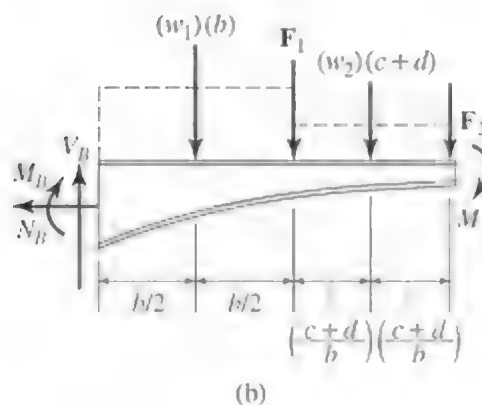
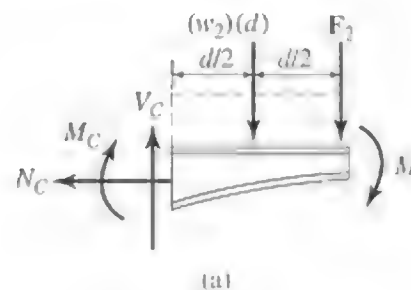
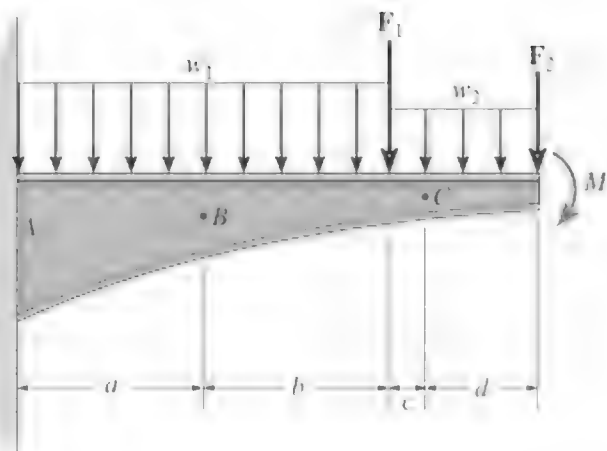
$$\begin{pmatrix} N_C \\ V_C \end{pmatrix} = \begin{pmatrix} 0.00 \\ 9.00 \end{pmatrix} \text{kN} \qquad M_C = -62.50\,\text{kN}\cdot\text{m}$$

Guesses $N_B = 1\ \text{N}$ $V_B = 1\ \text{N}$ $M_B = 1\ \text{N}\cdot\text{m}$

Given

$-N_B = 0$ $V_B - w_1 b - w_2(c+d) - F_1 - F_2 = 0$

$$-M_B - w_1 b\left(\frac{b}{2}\right) - F_1 b - w_2(c+d)\left(b + \frac{c+d}{2}\right) - F_2(b+c+d) - M = 0$$

$$\begin{pmatrix} N_B \\ V_B \\ M_B \end{pmatrix} = \text{Find}(N_B, V_B, M_B) \qquad \begin{pmatrix} N_B \\ V_B \end{pmatrix} = \begin{pmatrix} 0.00 \\ 27.50 \end{pmatrix} \text{kN} \qquad M_B = -184.50\,\text{kN}\cdot\text{m}$$

Problem 7-122

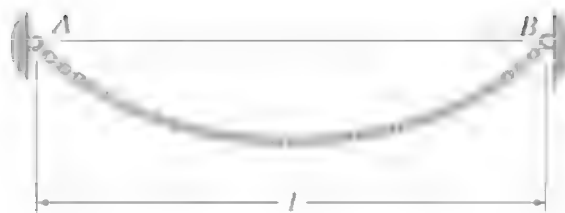
The chain is suspended between points A and B. If it has a weight weight density w and the

sag is h, determine the maximum tension in the chain.

Given:

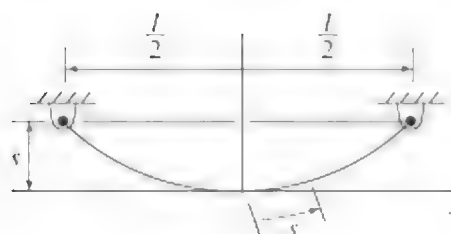
$$w = 0.5 \frac{\text{lb}}{\text{ft}}$$

$$L = 60 \text{ ft}$$

$$h = 3 \text{ ft}$$

Solution:

Form Example 7-15

$$y = \frac{F_H}{w}\left(\cosh\left(\frac{wx}{F_H}\right) - 1\right)$$

$$\frac{d}{dx}y = \sinh\left(\frac{wx}{F_H}\right)$$

Guess $F_H = 1 \text{ lb}$

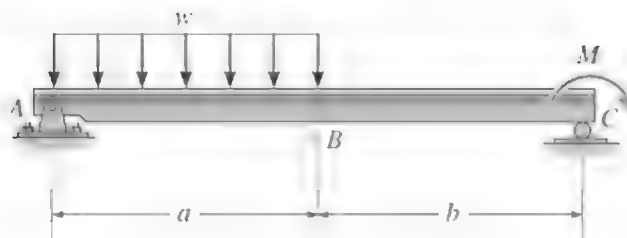
Given $h = \frac{F_H}{w}\left(\cosh\left(\frac{w}{F_H}\frac{L}{2}\right) - 1\right)$ $F_H = \text{Find}(F_H)$ $F_H = 75.2 \text{ lb}$

$$\theta_{max} = \text{atan}\left(\sinh\left(\frac{w}{F_H}\frac{L}{2}\right)\right) \qquad T_{max} = \frac{F_H}{\cos(\theta_{max})} \qquad T_{max} = 76.7 \text{ lb}$$

Problem 7-123

Draw the shear and moment diagrams for the beam.

Units Used:

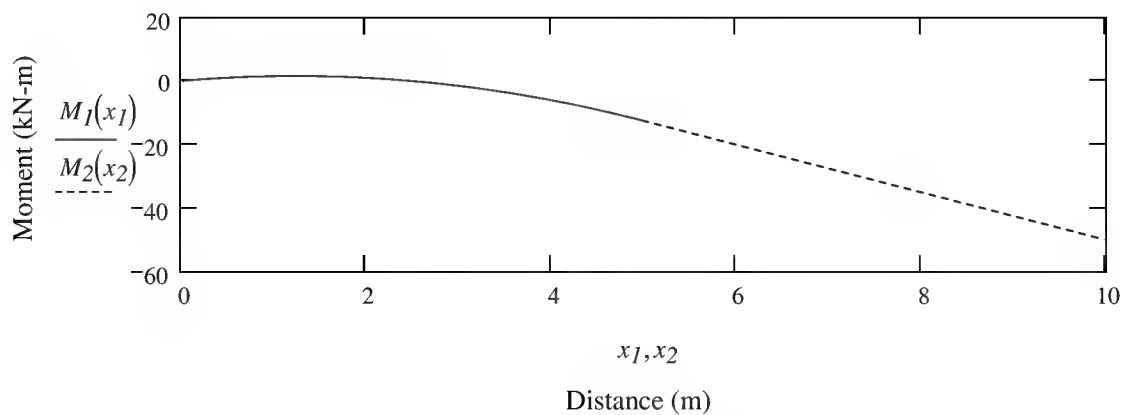
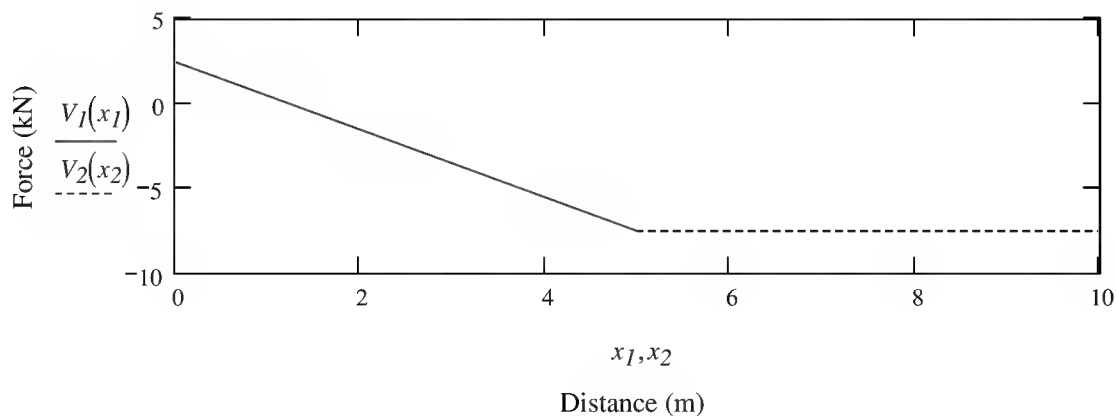
$$\text{kN} = 10^3 \text{ N}$$

Given:

$$w = 2 \frac{\text{kN}}{\text{m}} \qquad a = 5 \text{ m} \qquad b = 5 \text{ m} \qquad M = 50 \text{ kN}\cdot\text{m}$$

Solution: Guesses $A = 1 \text{ N}$ $C = 1 \text{ N}$

Given $\quad w a\left(b + \frac{a}{2}\right) - A(a+b) - M = 0 \qquad A + C - w a = 0 \qquad \begin{pmatrix} A \\ C \end{pmatrix} = \text{Find}(A, C)$

$x_1 = 0, 0.01a .. a \qquad V_1(x_1) = (A - w x_1)\frac{1}{\text{kN}} \qquad M_1(x_1) = \left(A x_1 - w x_1 \frac{x_1}{2}\right)\frac{1}{\text{kN}\cdot\text{m}}$

$x_2 = a, 1.01a .. a+b \qquad V_2(x_2) = -C\frac{1}{\text{kN}} \qquad M_2(x_2) = [-M + C(a + b - x_2)]\frac{1}{\text{kN}\cdot\text{m}}$

Problem 8-1

The horizontal force is **P**. Determine the normal and frictional forces acting on the crate of weight W. The friction coefficients are μ_k and μ_s.

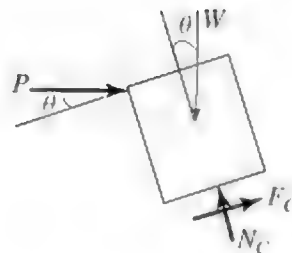
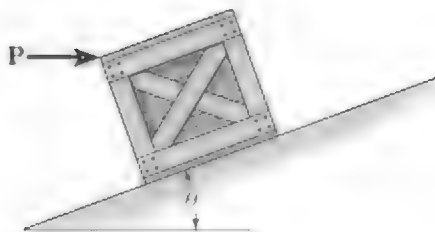
Given:

$W = 300$ lb

$P = 80$ lb

$\mu_s = 0.3$

$\mu_k = 0.2$

$\theta = 20$ deg

Solution:

Assume no slipping:

$\Sigma F_x = 0;$ $\quad P\cos(\theta) - W\sin(\theta) + F_C = 0$

$F_C = -P\cos(\theta) + W\sin(\theta)$ $\qquad F_C = 27.4$ lb

$\Sigma F_y = 0;$ $\quad N_C - W\cos(\theta) - P\sin(\theta) = 0$

$N_C = W\cos(\theta) + P\cdot\sin(\theta)$ $\qquad N_C = 309$ lb

Check $\quad F_{cmax} = \mu_s N_C$ $\qquad F_{cmax} = 92.8$ lb

$F_{cmax} > F_C$

Problem 8-2

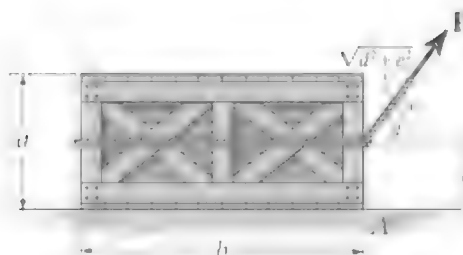
Determine the magnitude of force **P** needed to start towing the crate of mass M. Also determine the location of the resultant normal force acting on the crate, measured from point A.

Given:

$M = 40$ kg $\qquad c = 200$ mm

$\mu_s = 0.3$ $\qquad d = 3$

$a = 400$ mm $\qquad e = 4$

$b = 800$ mm

Solution:

Initial guesses: $N_C = 200 \text{ N}$ $P = 50 \text{ N}$

Given

$\Sigma F_x = 0;$ $$\left(\frac{d}{\sqrt{d^2+e^2}}\right)P - \mu_s N_C = 0$$

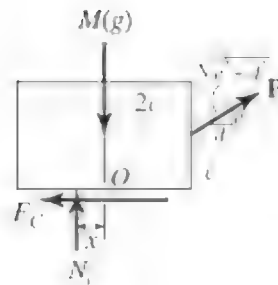
$\Sigma F_y = 0;$ $$N_C - M g + \frac{e P}{\sqrt{d^2+e^2}} = 0$$

$$\begin{pmatrix} N_C \\ P \end{pmatrix} = \text{Find}(N_C, P)$$

$N_C = 280.2 \text{ N}$ $P = 140 \text{ N}$

$\Sigma M_O = 0;$ $$-\mu_s N_C\left(\frac{a}{2}\right) - N_1 x + \left(\frac{e P}{\sqrt{d^2+e^2}}\right)\left(\frac{b}{2}\right) = 0$$

$$x = \frac{-1}{2}\,\frac{\mu_s N_C\, a\sqrt{d^2+e^2} - e\,P\,b}{N_C\sqrt{d^2+e^2}} \qquad x = 123.51 \text{ mm}$$

Thus, the distance from A is $A = x + \dfrac{b}{2}$ $A = 523.51 \text{ mm}$

Problem 8-3

Determine the friction force on the crate of mass M, and the resultant normal force and its position x, measured from point A, if the force is **P**.

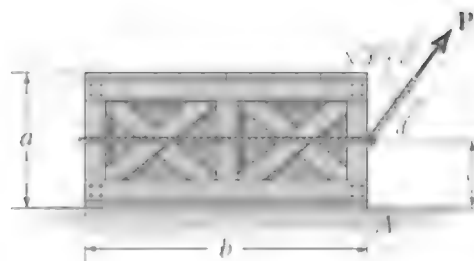
Given:

$M = 40 \text{ kg}$ $\mu_s = 0.5$

$a = 400 \text{ mm}$ $\mu_k = 0.2$

$b = 800 \text{ mm}$ $d = 3$

$c = 200 \text{ mm}$ $e = 4$

$P = 300 \text{ N}$

Solution:

Initial guesses: $F_C = 25 \text{ N}$ $N_C = 100 \text{ N}$

Given

$\Sigma F_x = 0;$ $$P\left(\frac{d}{\sqrt{d^2+e^2}}\right) - F_C = 0$$

$\Sigma F_y = 0;$ $$N_C - Mg + P\left(\frac{e}{\sqrt{d^2+e^2}}\right) = 0$$

$$\begin{pmatrix} F_C \\ N_C \end{pmatrix} = \text{Find}(F_C, N_C) \qquad F_{Cmax} = \mu_s N_C$$

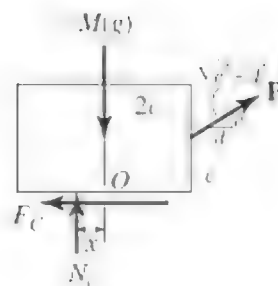
Since $F_C = 180.00$ N >
$F_{Cmax} = 76.13$ N then the crate slips

$$F_C = \mu_k N_C \qquad \begin{pmatrix} F_C \\ N_C \end{pmatrix} = \begin{pmatrix} 30.5 \\ 152.3 \end{pmatrix} \text{N}$$

$\Sigma M_O = 0;$ $$-N_C x + P\left(\frac{e}{\sqrt{d^2+e^2}}\right)a - P\left(\frac{d}{\sqrt{d^2+e^2}}\right)c = 0$$

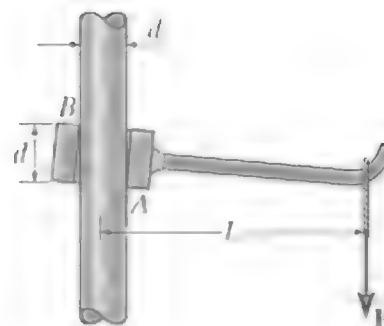
$$x = -P\left(\frac{-ea+dc}{N_C\sqrt{d^2+e^2}}\right)$$

Since $x = 0.39$ m $< \frac{b}{2} = 0.40$ m $\qquad x_1 = a + x \qquad x_1 = 0.79$ m

Then the block does not tip.

Problem 8-4

The loose-fitting collar is supported by the pipe for which the coefficient of static friction at the points of contact A and B is μ_s. Determine the smallest dimension d so the rod will not slip when the load **P** is applied.

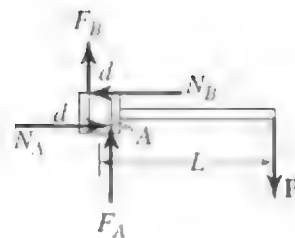
Given:

$$\mu_s = 0.2$$

Solution:

$\Sigma M_A = 0;\quad N_B\, d - P\left(L - \frac{d}{2}\right) - \mu_s\, N_B\, d = 0 \qquad N_B = \frac{P\left(L - \frac{d}{2}\right)}{\left(1 - \mu_s\right) d}$

$\Sigma M_B = 0;\quad N_A\, d + \mu_s\, N_A\, d - P\left(L + \frac{d}{2}\right) = 0 \qquad N_A = \frac{P\left(L + \frac{d}{2}\right)}{\left(1 + \mu_s\right) d}$

$\Sigma F_y = 0;\quad \mu_s\left(N_A + N_B\right) - P = 0 \qquad \mu_s \frac{P}{d}\left(\frac{L + \frac{d}{2}}{1 + \mu_s} + \frac{L - \frac{d}{2}}{1 - \mu_s}\right) = P$

Thus, $\quad d = 2\mu_s L \qquad k = 2\mu_s \qquad d = kL$

Problem 8-5

The spool of wire having a mass M rests on the ground at A and against the wall at B. Determine the force $\mathbf{P}$ required to begin pulling the wire horizontally off the spool. The coefficient of static friction between the spool and its points of contact is μ_s.

Units Used:

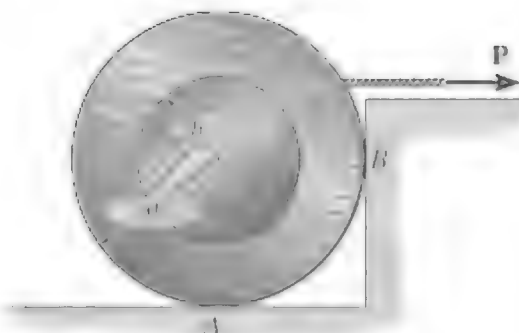
$kN = 10^3\ N$

Given:

$M = 150\ kg$

$\mu_s = 0.25$

$a = 0.45\ m$

$b = 0.25\ m$

Solution:

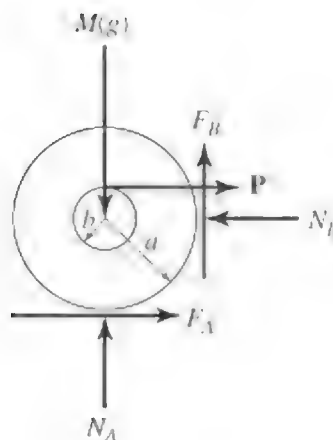
Initial guesses: $P = 100\ N \qquad F_A = 10\ N \qquad N_A = 20\ N \qquad N_B = 30\ N \qquad F_B = 10\ N$

Given

$\Sigma F_y = 0;$ $N_A + F_B - M\,g = 0$

$\Sigma F_x = 0;$ $F_A - N_B + P = 0$

$\Sigma M_B = 0;$ $-P\,b + M\,g\,a - N_A\,a + F_A\,a = 0$

$F_A = \mu_s\,N_A$ $F_B = \mu_s\,N_B$

$$\begin{pmatrix} P \\ F_A \\ F_B \\ N_A \\ N_B \end{pmatrix} = \text{Find}(P, F_A, F_B, N_A, N_B) \qquad \begin{pmatrix} F_A \\ N_A \\ F_B \\ N_B \end{pmatrix} = \begin{pmatrix} 0.28 \\ 1.12 \\ 0.36 \\ 1.42 \end{pmatrix} \text{kN} \qquad P = 1.14\,\text{kN}$$

Problem 8-6

The spool of wire having a mass M rests on the ground at A and against the wall at B. Determine the forces acting on the spool at A and B for the given force P. The coefficient of static friction between the spool and the ground at point A is μ_s. The wall at B is smooth.

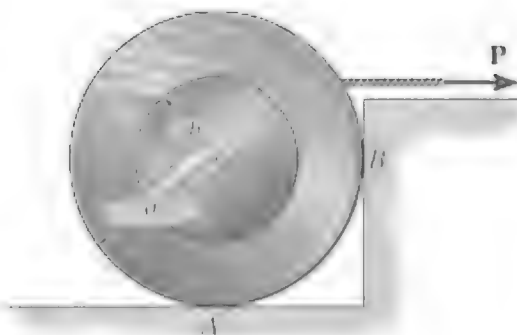
Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$P = 800\ \text{N}$ $a = 0.45\ \text{m}$

$M = 150\ \text{kg}$ $b = 0.25\ \text{m}$

$\mu_s = 0.35$

Solution: Assume no slipping

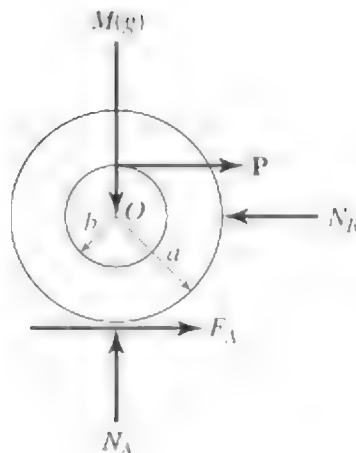
Initial guesses : $F_A = 10\text{N}$ $N_A = 10\text{N}$ $N_B = 10\text{N}$ $F_{Amax} = 10\text{N}$

Given

$\Sigma F_x = 0;$ $F_A - N_B + P = 0$

$\Sigma F_y = 0;$ $N_A - Mg = 0$

$\Sigma M_0 = 0;$ $-P\,b + F_A\,a = 0$

$F_{Amax} = \mu_s\,N_A$

$$\begin{pmatrix} F_A \\ F_{Amax} \\ N_A \\ N_B \end{pmatrix} = \text{Find}(F_A, F_{Amax}, N_A, N_B) \qquad \begin{pmatrix} F_A \\ F_{Amax} \end{pmatrix} = \begin{pmatrix} 444 \\ 515 \end{pmatrix} \text{N}$$

If $F_A = 444$ N $< F_{Amax} = 515$ N then our no-slip assumption is good.

$$\begin{pmatrix} N_A \\ F_A \end{pmatrix} = \begin{pmatrix} 1.47 \\ 0.44 \end{pmatrix} \text{kN} \qquad N_B = 1.24\,\text{kN}$$

Problem 8-7

The crate has a mass M and is subjected to a towing force **P** acting at an angle θ_1 with the horizontal. If the coefficient of static friction is μ_s, determine the magnitude of **P** to just start the crate moving down the plane.

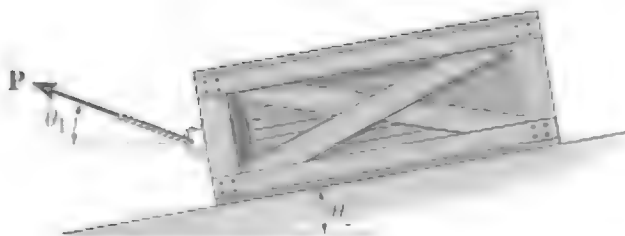
Given:

$M = 350$ kg

$\theta_1 = 20$ deg

$\theta_2 = 10$ deg

$\mu_s = 0.5$

$g = 9.81\,\frac{\text{m}}{\text{s}^2}$

Solution:

Initial guesses: $N_C = 10\text{N}$ $P = 20\text{N}$

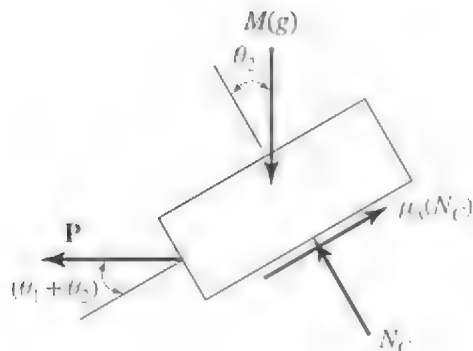
Given

$\Sigma F_x = 0;$ $P\cos(\theta_1 + \theta_2) - \mu_s N_C + M g \sin(\theta_2) = 0$

$\Sigma F_y = 0;$ $N_C - M g \cos(\theta_2) + P\sin(\theta_1 + \theta_2) = 0$

$\begin{pmatrix} N_C \\ P \end{pmatrix} = \text{Find}(N_C, P)$

$N_C = 2891\ \text{N}$

$P = 981\ \text{N}$

Problem 8-8

The winch on the truck is used to hoist the garbage bin onto the bed of the truck. If the loaded bin has weight W and center of gravity at G, determine the force in the cable needed to begin the lift. The coefficients of static friction at A and B are μ_A and μ_B respectively. Neglect the height of the support at A.

Units Used:

$\text{kip} = 10^3\ \text{lb}$

Given:

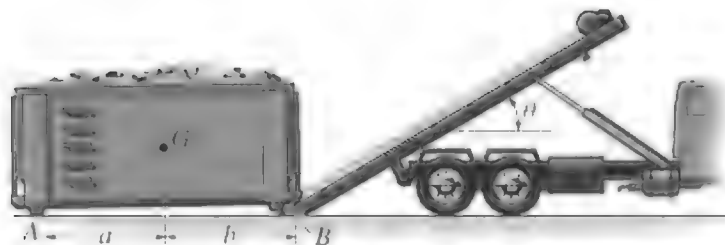
$W = 8.5\ \text{kip}$

$\mu_A = 0.3$

$\mu_B = 0.2$

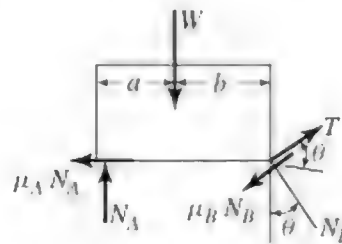
$a = 10\ \text{ft}$

$b = 12\ \text{ft}$

$\theta = 30\ \text{deg}$

Solution:

The initial guesses are $T = 1\ \text{lb}$ $N_B = 1\ \text{lb}$ $N_A = 1\ \text{lb}$

Given

$\Sigma M_B = 0;$ $W b - N_A (a + b) = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad T\cos(\theta) - \mu_B N_B \cos(\theta) - N_B \sin(\theta) - \mu_A N_A = 0$

$+\uparrow \Sigma F_y = 0; \quad N_A - W + T\sin(\theta) + N_B\cos(\theta) - \mu_B N_B \sin(\theta) = 0$

$$\begin{pmatrix} T \\ N_A \\ N_B \end{pmatrix} = \text{Find}(T, N_A, N_B) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 4.64 \\ 2.65 \end{pmatrix} \text{kip} \qquad T = 3.67\,\text{kip}$$

Problem 8-9

The motorcyclist travels with constant velocity along a straight, horizontal, banked road. If he aligns his bike so that the tires are perpendicular to the road at A, determine the frictional force at A. The man has a mass M_C and a mass center at G_C, and the motorcycle has a mass M_m and a mass center at G_m. If the coefficient of static friction at A is μ_A, will the bike slip?

Given:

$M_C = 60$ kg

$M_m = 120$ kg

$\mu_A = 0.4$

$\theta = 20$ deg

$g = 9.81\,\frac{\text{m}}{\text{s}^2}$

Solution: Assume no slipping

$\Sigma F_y = 0; \quad N_A - (M_m + M_C)g\cos(\theta) = 0$

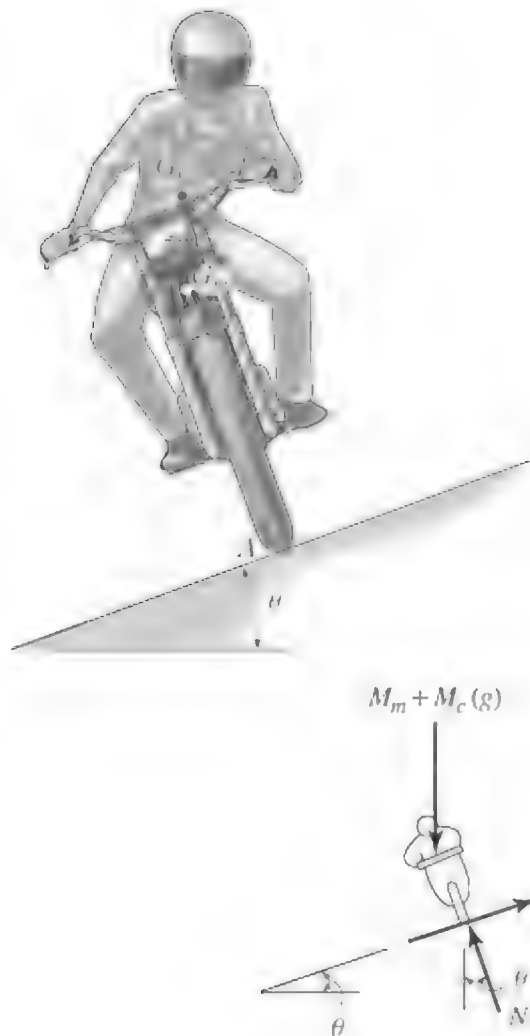
$N_A = (M_m + M_C)\, g\cos(\theta)$

$N_A = 1659$ N

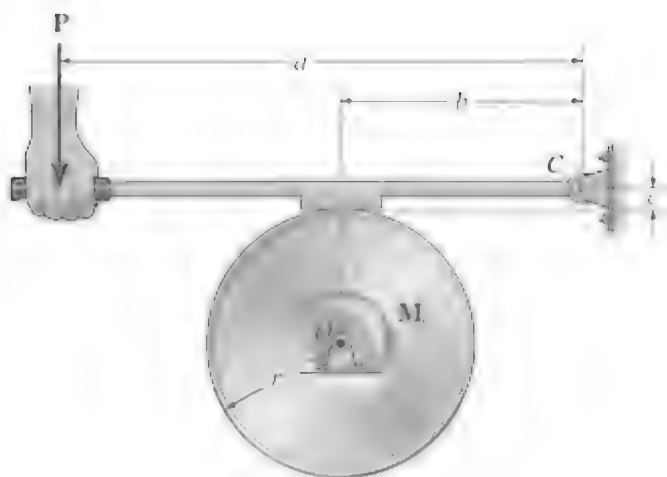
$\Sigma F_x = 0; \quad F_A - (M_m + M_C)\, g\sin(\theta)$

$F_A = (M_m + M_C)\, g\sin(\theta)$

$F_A = 604$ N

$F_{Amax} = \mu_A N_A$

Check: If $F_A = 604$ N $< F_{Amax} = 664$ N
then our no-slip assumption is good.

Problem 8-10

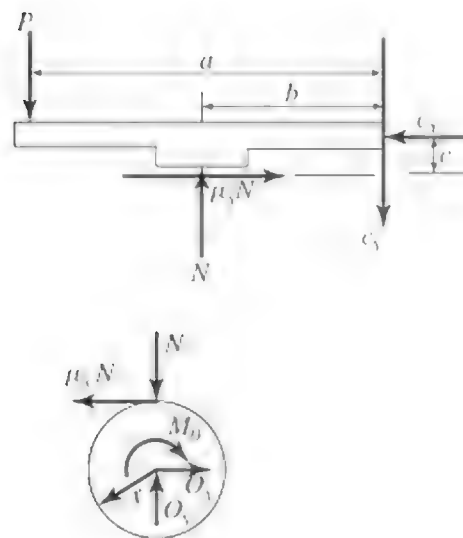
The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment M_0 If the coefficient of static friction between the wheel and the block is μ_s, determine the smallest force **P** that should be applied.

Solution:

$\Sigma M_C = 0;$ $\quad P\,a - N\,b + \mu_s\,Nc = 0$

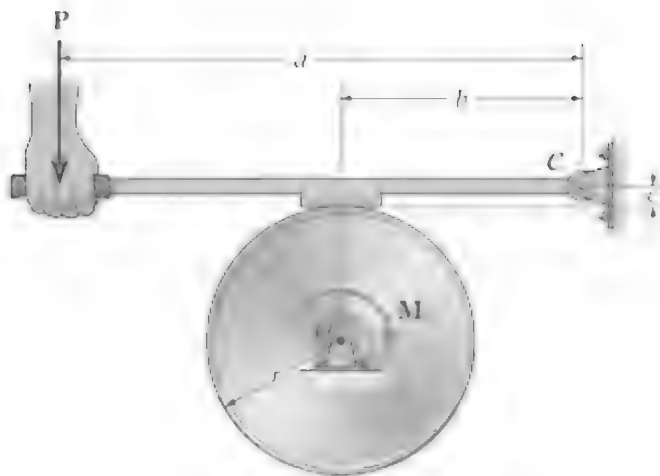
$$N = \frac{P\,a}{b - \mu_s\,c}$$

$\Sigma M_O = 0;$ $\quad \mu_s\,Nr - M_O = 0$

$$\frac{\mu_s\,P\,a\,r}{b - \mu_s\,c} = M_O$$

$$P = \frac{M_O\left(b - \mu_s c\right)}{\mu_s\,r\,a}$$

Problem 8-11

The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple

moment M_0 If the coefficient of static friction between the wheel and the block is μ_s , show that the brake is self locking, i. e., $P \le 0$, provided $\frac{b}{c} \le \mu_s$

Solution:

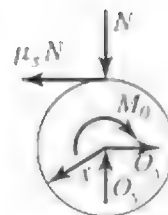
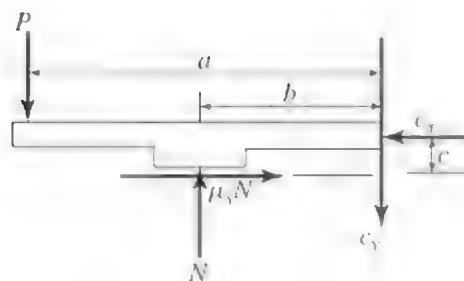
$\Sigma M_C = 0;\qquad P\,a - N\,b + \mu_s\,Nc = 0$

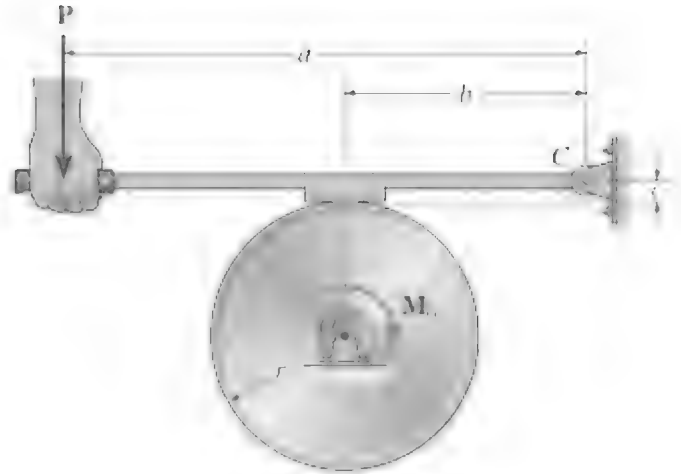
$$N = \frac{P\,a}{b - \mu_s\,c}$$

$\Sigma M_O = 0;\qquad \mu_s\,Nr - M_O = 0$

$$\frac{\mu_s\,P\,a\,r}{b - \mu_s\,c} = M_O$$

$$P = \frac{M_O\left(b - \mu_s\,c\right)}{\mu_s\,r\,a}$$

$$P < 0 \text{ if } \left(b - \mu_s c\right) < 0 \text{ i.e. if } \frac{b}{c} < \mu_s$$

Problem 8-12

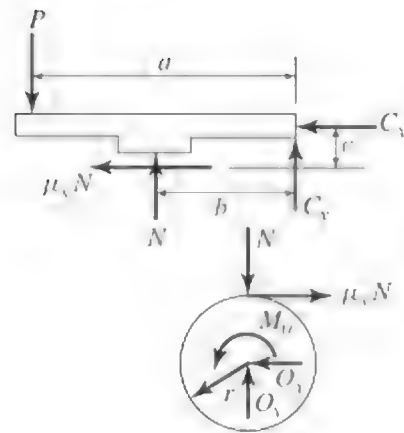
The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment M_0 If the coefficient of static friction between the wheel and the block is μ_s , determine the smallest force **P** that should be applied if the couple moment $\mathbf{M_O}$ is applied **counterclockwise**.

Solution:

$\Sigma M_C = 0;$ $P\ a - N\ b - \mu_s\ N\ c = 0$

$$N = \frac{P\ a}{b + \mu_s\ c}$$

$\Sigma M_O = 0;$ $-\mu_s\ N\ r + M_O = 0$

$$\frac{\mu_s\ P\ a\ r}{b + \mu_s\ c} = M_O$$

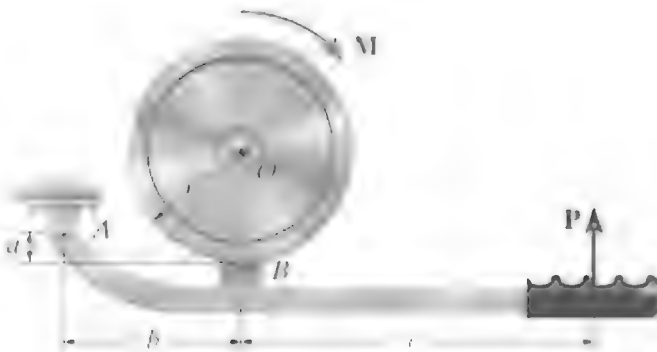
$$P = \frac{M_O\left(b + \mu_s\ c\right)}{\mu_s\ ra}$$

Problem 8-13

The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is μ_s and a torque **M** is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) P_1 (b) P_2.

Given:

$\mu_s = 0.3$

$M = 5\ \text{N}\cdot\text{m}$

$a = 50\ \text{mm}$

$b = 200\ \text{mm}$

$c = 400\ \text{mm}$

$r = 150\ \text{mm}$

$P_1 = 30\ \text{N}$

$P_2 = 70\ \text{N}$

Solution: To hold lever:

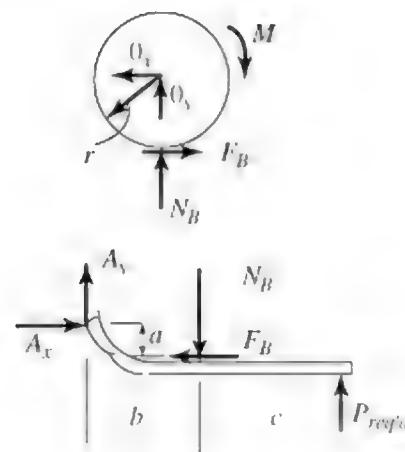
$\Sigma M_O = 0;\quad F_B\, r - M = 0$

$$F_B = \frac{M}{r} \qquad F_B = 33.333\ \text{N}$$

Require $$N_B = \frac{F_B}{\mu_s} \qquad N_B = 111.1\ \text{N}$$

Lever,

$\Sigma M_A = 0;\quad P_{Reqd}\,(b + c) - N_B\, b - F_B\, a = 0$

$$P_{Reqd} = \frac{N_B\, b + F_B\, a}{b + c} \qquad P_{Reqd} = 39.8\ \text{N}$$

(a) If $P_1 = 30.00\ \text{N} > P_{Reqd} = 39.81\ \text{N}$ then the break will hold the wheel

(b) If $P_2 = 70.00\ \text{N} > P_{Reqd} = 39.81\ \text{N}$ then the break will hold the wheel

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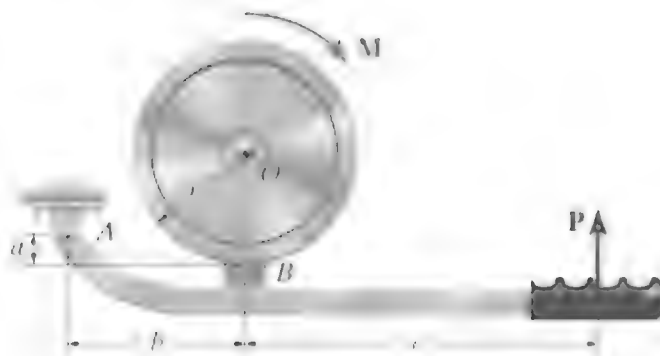
Problem 8-14

The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is μ_s and a torque **M** is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) P_1 (b) P_2. Assume that the torque **M** is applied **counter-clockwise.**

Given:

$\mu_s = 0.3$

$M = 5\ \text{N}\cdot\text{m}$

$a = 50\ \text{mm}$

$b = 200\ \text{mm}$

$c = 400\ \text{mm}$

$r = 150\ \text{mm}$

$P_1 = 30\ \text{N}$

$P_2 = 70\ \text{N}$

Solution: To hold lever:

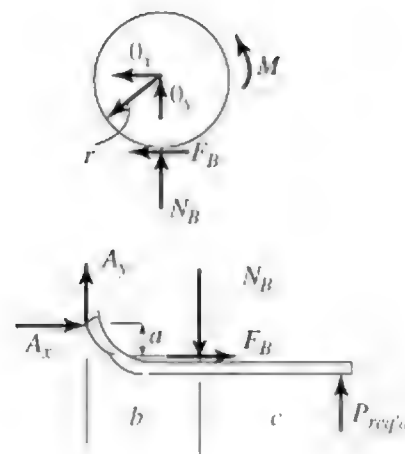
$\Sigma M_O = 0;\qquad F_B\, r - M = 0$

$$F_B = \frac{M}{r} \qquad F_B = 33.333\ \text{N}$$

Require $$N_B = \frac{F_B}{\mu_s} \qquad N_B = 111.1\ \text{N}$$

Lever,

$\Sigma M_A = 0;\qquad P_{Reqd}\,(b + c) - N_B\, b + F_B\, a = 0$

$$P_{Reqd} = \frac{N_B\, b - F_B\, a}{b + c} \qquad P_{Reqd} = 34.3\ \text{N}$$

(a) If $P_1 = 30.00\ \text{N} \;>\; P_{Reqd} = 34.26\ \text{N}$ then the break will hold the wheel

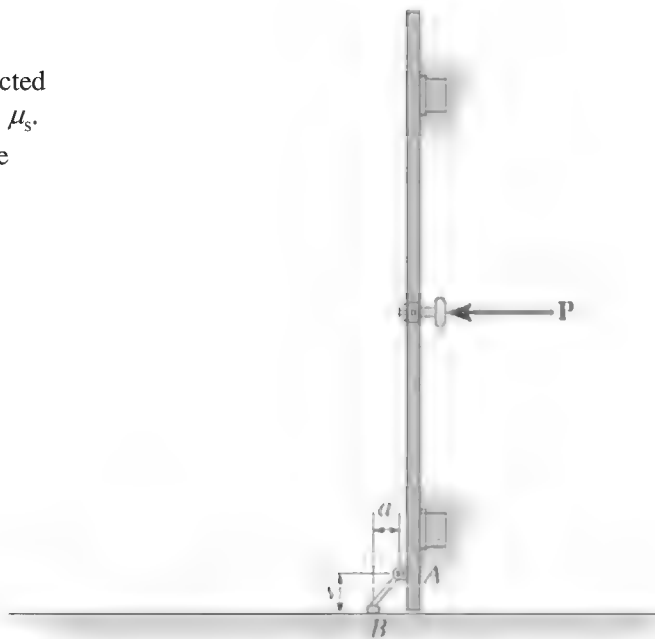
(b) If $P_2 = 70.00\ \text{N} \;>\; P_{Reqd} = 34.26\ \text{N}$ then the break will hold the wheel

Problem 8-15

The doorstop of negligible weight is pin connected at A and the coefficient of static friction at B is μ_s. Determine the required distance s from A to the floor so that the stop will resist opening of the door for any force **P** applied to the handle.

Given:

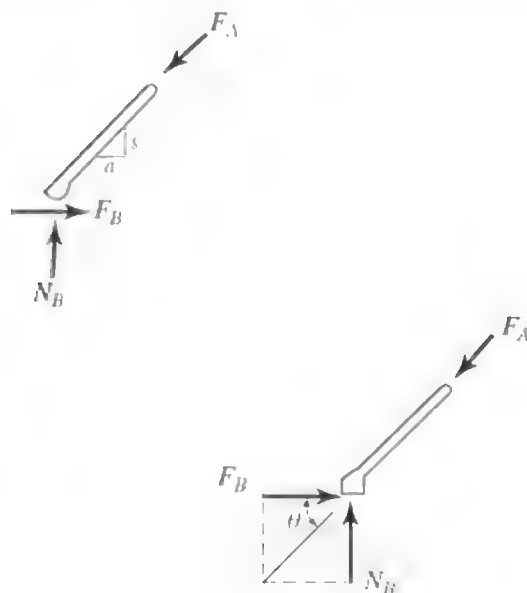
$\mu_s = 0.3$

$a = 1.5$ in

Solution:

$$\Sigma F_y = 0;\quad N_B - \left(\frac{s}{\sqrt{s^2 + a^2}}\right) F_A = 0$$

$$\Sigma F_x = 0;\quad \mu_s N_B - \left(\frac{a}{\sqrt{s^2 + a^2}}\right) F_A = 0$$

$$\left(\frac{\mu_s s}{\sqrt{s^2 + a^2}}\right) F_A - \left(\frac{a}{\sqrt{s^2 + a^2}}\right) F_A = 0$$

$$\mu_s s = a \qquad s = \frac{a}{\mu_s} \qquad s = 5.00 \text{ in}$$

Problem 8-16

The chair has a weight W and center of gravity at G. It is propped against the door as shown. If the coefficient of static friction at A is μ_A, determine the smallest force **P** that must be applied to the handle to open the door.

Given:

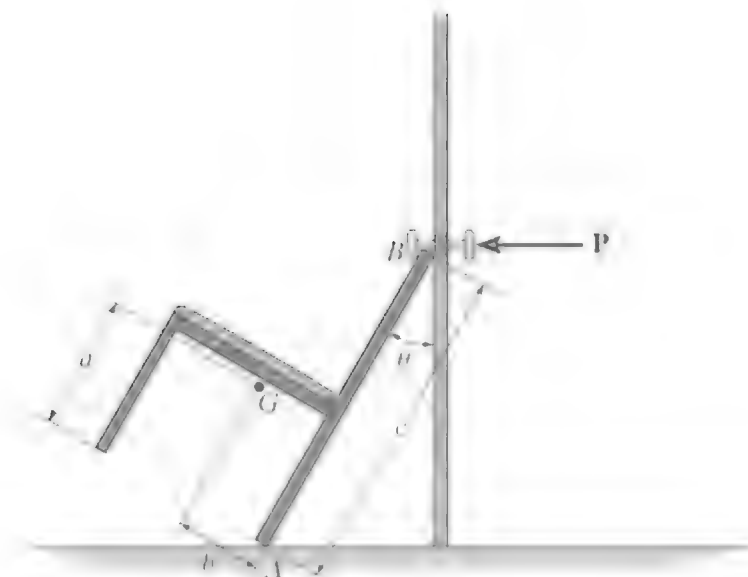
$\mu_A = 0.3$

$a = 1.20$ ft

$b = 0.75$ ft

$c = 3$ft

$\theta = 30$ deg

$W = 10$ lb

Solution:

Guesses $B_y = 1$ lb $N_A = 1$ lb $P = 1$ lb

Given

$\Sigma F_x = 0;\quad -P + \mu_A N_A = 0$

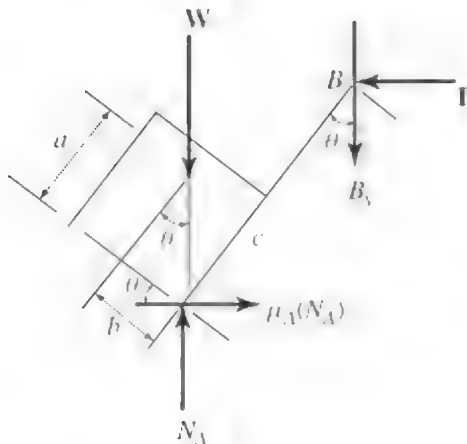
$\Sigma F_y = 0;\quad N_A - W - B_y = 0$

$\Sigma M_B = 0;\quad \mu_A N_A c \cos(\theta) - N_A c \sin(\theta) + W[(c-a)\sin(\theta) + b\cos(\theta)] = 0$

$$\begin{pmatrix} B_y \\ N_A \\ P \end{pmatrix} = \text{Find}(B_y, N_A, P)$$

$B_y = 11.5$ lb

$N_A = 21.5$ lb

$P = 6.45$ lb

Problem 8-17

The uniform hoop of weight W is suspended from the peg at A and a horizontal force **P** is slowly applied at B. If the hoop begins to slip at A when the angle is θ , determine the coefficient of static friction between the hoop and the peg.

Given:

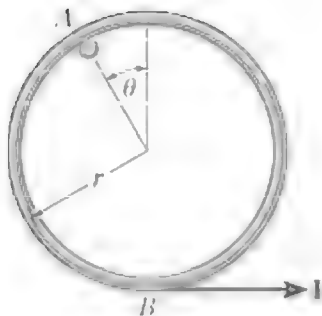
$\theta = 30 \text{ deg}$

Solution:

$\Sigma F_x = 0; \quad \mu N_A \cos(\theta) + P - N_A \sin(\theta) = 0$

$$P = (\mu \cos(\theta) - \sin(\theta))N_A$$

$\Sigma F_y = 0; \quad \mu N_A \sin(\theta) - W + N_A \cos(\theta) = 0$

$$W = (\mu \sin(\theta) + \cos(\theta))N_A$$

$\Sigma M_A = 0; \quad -W\, r \sin(\theta) + P\,(r + r\cos(\theta)) = 0$

$$W\sin(\theta) = P(1 + \cos(\theta))$$

$$(\mu \sin(\theta) + \cos(\theta))\sin(\theta) = (\sin(\theta) - \mu \cos(\theta))(1 + \cos(\theta))$$

$$\mu = \frac{\sin(\theta)}{1 + \cos(\theta)} \qquad \mu = 0.27$$

Problem 8-18

The uniform hoop of weight W is suspended from the peg at A and a horizontal force **P** is slowly applied at B. If the coefficient of static friction between the hoop and peg is μ_s, determine if it is possible for the hoop to reach an angle θ before the hoop begins to slip.

Given:

$\mu_s = 0.2$

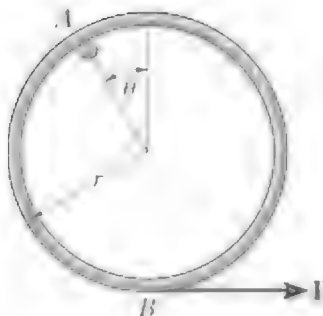
$\theta = 30 \text{ deg}$

Solution:

$\Sigma F_x = 0;\quad \mu N_A \cos(\theta) + P - N_A \sin(\theta) = 0$

$P = (\mu \cos(\theta) - \sin(\theta))\, N_A$

$\Sigma F_y = 0;\quad \mu N_A \sin(\theta) - W + N_A \cos(\theta) = 0$

$W = (\mu \sin(\theta) + \cos(\theta))\, N_A$

$\Sigma M_A = 0;\quad -W\, r \sin(\theta) + P\, (r + r\cos(\theta)) = 0$

$W \sin(\theta) = P\,(1 + \cos(\theta))$

$(\mu \sin(\theta) + \cos(\theta))\sin(\theta) = (\sin(\theta) - \mu \cos(\theta))(1 + \cos(\theta))$

$\mu = \dfrac{\sin(\theta)}{1 + \cos(\theta)} \qquad \mu = 0.27$

If $\mu_s = 0.20 < \mu = 0.27$ then it is not possible to reach $\theta = 30.00 \text{ deg}$.

Problem 8-19

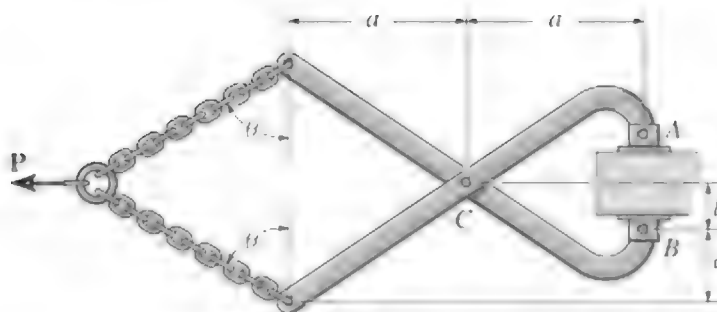
The coefficient of static friction between the shoes at A and B of the tongs and the pallet is μ_{s1} and between the pallet and the floor μ_{s2}. If a horizontal towing force **P** is applied to the tongs, determine the largest mass that can be towed.

Given:

$\mu_{s1} = 0.5$ $a = 75\ \text{mm}$

$\mu_{s2} = 0.4$ $b = 20\ \text{mm}$

$P = 300\ \text{N}$ $c = 30\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$ $\theta = 60\ \text{deg}$

Solution:

Assume that we are on the verge of slipping at every surface.

Guesses

$T = 1\text{N}$ $N_A = 1\text{N}$

$F = 1\text{N}$ $N_{ground} = 1\text{N}$

$F_A = 1\text{N}$ $mass = 1\text{kg}$

Given

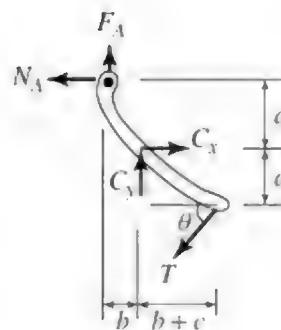
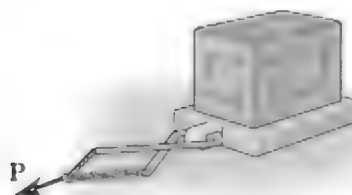
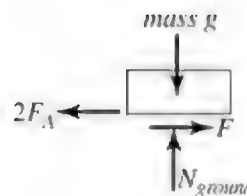
$$2\,T\sin(\theta) - P = 0$$

$$-T\sin(\theta)\,(b + c) - T\cos(\theta)\,a - F_A\,b + N_A\,a = 0$$

$$F_A = \mu_{s1}\,N_A$$

$$2\,F_A - F = 0$$

$$N_{ground} - mass\,g = 0$$

$$F = \mu_{s2}\,N_{ground}$$

$$\begin{pmatrix} T \\ N_A \\ F_A \\ F \\ N_{ground} \\ mass \end{pmatrix} = \text{Find}(T, N_A, F_A, F, N_{ground}, mass)$$

$$\begin{pmatrix} T \\ N_A \\ F_A \\ F \\ N_{ground} \end{pmatrix} = \begin{pmatrix} 173.21 \\ 215.31 \\ 107.66 \\ 215.31 \\ 538.28 \end{pmatrix} \text{N} \qquad mass = 54.9 \text{ kg}$$

Problem *8-20

The pipe is hoisted using the tongs. If the coefficient of static friction at A and B is μ_s, determine the smallest dimension b so that any pipe of inner diameter d can be lifted.

Solution:

$$W - 2\,F_B = 0$$

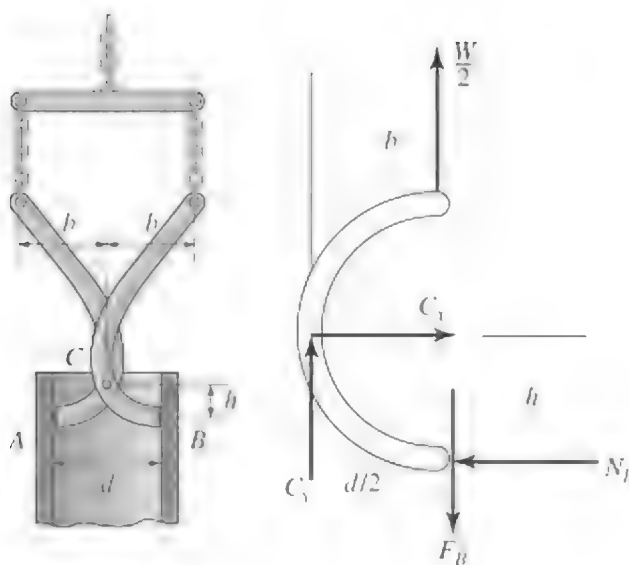
$$\left(\frac{W}{2}\right) b - N_B\,h - F_B\left(\frac{d}{2}\right) = 0$$

Thus

$$F_B = \frac{W}{2}$$

$$N_B = \frac{W\,(2\,b - d)}{4h}$$

Require

$$F_B \le \mu_s N_B \qquad \frac{W}{2} \le \frac{\mu_s\,W\,(2b-d)}{4\,h} \qquad 2\,h \le \mu_s\,(2b-d) \qquad b > \frac{h}{\mu_s} + \frac{d}{2}$$

Problem 8-21

A very thin bookmark having a width a. is in the middle of a dictionary of weight W. If the pages are b by c, determine the force **P** needed to start to pull the bookmark out.The coefficient of static friction between the bookmark and the paper is μ_s. Assume the pressure on each page and the bookmark is uniform.

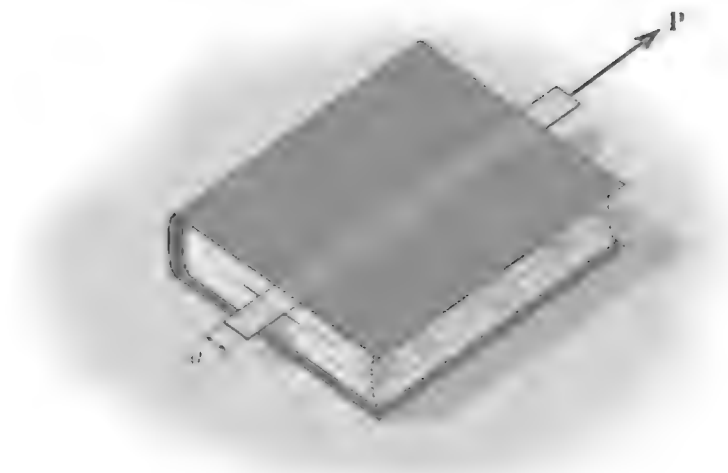
Given:

$a = 1 \text{ in}$

$W = 10 \text{ lb}$

$b = 8 \text{ in}$

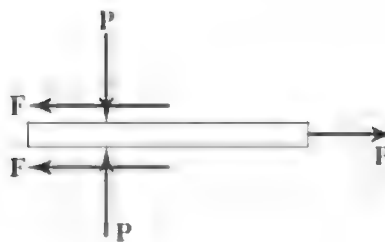
$c = 10 \text{ in}$

$\mu_s = 0.7$

Solution:

Pressure on book mark :

$P = \frac{1}{2}\frac{W}{b\,c}$ $\qquad P = 0.06 \frac{\text{lb}}{\text{in}^2}$

Normal force on bookmark: $\qquad N = P\,c\,a$

$F = \mu_s N$ $\qquad F = 0.44 \text{ lb}$

$\Sigma F_x = 0;$ $\qquad P - 2F = 0$ $\qquad P = 2F$ $\qquad P = 0.88 \text{ lb}$

Problem 8-22

The uniform dresser has weight W and rests on a tile floor for which the coefficient of friction is μ_s. If the man pushes on it in the direction θ, determine the smallest magnitude of force **F** needed to move the dresser. Also, if the man has a weight $W_{man,}$, determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.

Given:

$W = 90 \text{ lb}$

$\mu_s = 0.25$

$W_{man} = 150 \text{ lb}$

$\theta = 0 \text{ deg}$

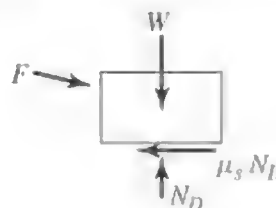
Solution:

Dresser: Guesses $N_D = 1\text{lb}$ $F = 1\text{lb}$

Given

$+\uparrow \Sigma F_y = 0;$ $N_D - W - F \sin(\theta) = 0$

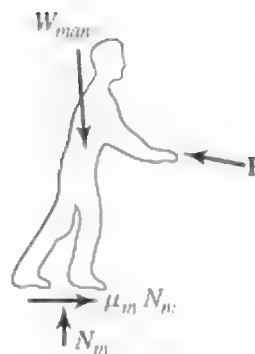
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $F \cos(\theta) - \mu_s N_D = 0$

$\begin{pmatrix} N_D \\ F \end{pmatrix} = \text{Find}(N_D, F)$ $F = 22.50 \text{ lb}$

Man: Guesses $N_m = 1\text{lb}$ $\mu_m = 0.2$

Given

$+\uparrow \Sigma F_y = 0;$ $N_m - W_{man} + F \sin(\theta) = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $-F \cos(\theta) + \mu_m N_m = 0$

$\begin{pmatrix} N_m \\ \mu_m \end{pmatrix} = \text{Find}(N_m, \mu_m)$ $\mu_m = 0.15$

Problem 8-23

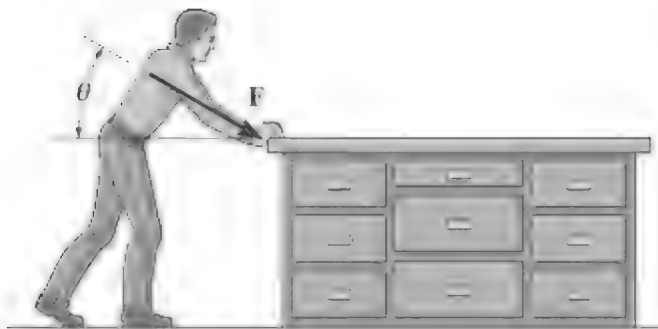
The uniform dresser has weight W and rests on a tile floor for which the coefficient of friction is μ_s. If the man pushes on it in the direction θ, determine the smallest magnitude of force **F** needed to move the dresser. Also, if the man has a weight W_{man}, determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.

Given:

$W = 90 \text{ lb}$

$\mu_s = 0.25$

$W_{man} = 150 \text{ lb}$

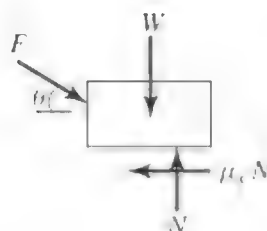
$\theta = 30 \text{ deg}$

Solution:

Dresser: Guesses $N_D = 1\text{lb}$ $F = 1\text{lb}$

Given

$+\uparrow \Sigma F_y = 0;$ $N_D - W - F \sin(\theta) = 0$

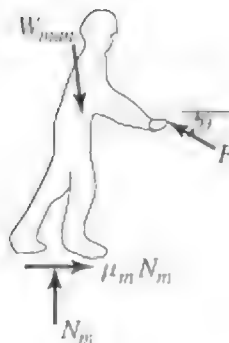
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $F \cos(\theta) - \mu_s N_D = 0$

$\begin{pmatrix} N_D \\ F \end{pmatrix} = \text{Find}(N_D, F)$ $F = 30.36 \text{ lb}$

Man: Guesses $N_m = 1\text{lb}$ $\mu_m = 0.2$

Given

$+\uparrow \Sigma F_y = 0;$ $N_m - W_{man} + F \sin(\theta) = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $-F \cos(\theta) + \mu_m N_m = 0$

$\begin{pmatrix} N_m \\ \mu_m \end{pmatrix} = \text{Find}(N_m, \mu_m)$ $\mu_m = 0.195$

Problem 8-24

The cam is subjected to a couple moment of **M**. Determine the minimum force **P** that should be applied to the follower in order to hold the cam in the position shown.The coefficient of static friction between the cam and the follower is μ_s. The guide at A is smooth.

Given:

$a = 10\text{ mm}$

$b = 60\text{ mm}$

$M = 5\text{ N}\cdot\text{m}$

$\mu_s = 0.4$

Solution:

$\Sigma M_O = 0;\quad M - \mu_s N_B b - a N_B = 0$

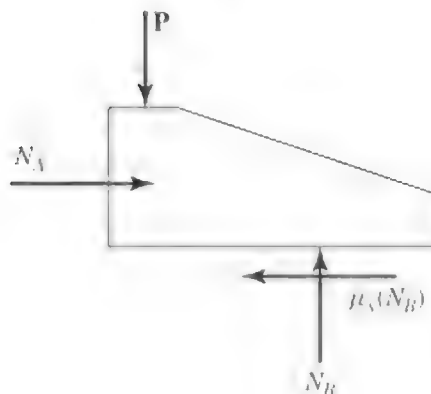
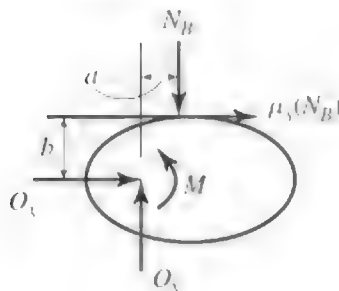
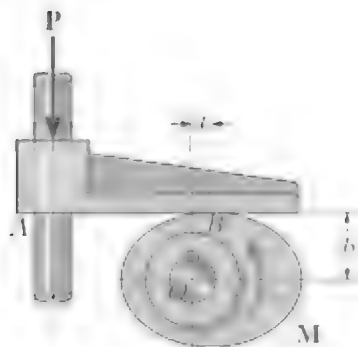
$$N_B = \frac{M}{\mu_s b + a}$$

$N_B = 147.06\text{ N}$

Follower:

$\Sigma F_y = 0;\quad N_B - P = 0$

$P = N_B$

$P = 147\text{ N}$

Problem 8-25

The board can be adjusted vertically by tilting it up and sliding the smooth pin A along the vertical guide G. When placed horizontally, the bottom C then bears along the edge of the guide, where the coefficient of friction is μ_s. Determine the largest dimension d which will support any applied force **F** without causing the board to slip downward.

Given:

$\mu_s = 0.4$

$a = 0.75\text{ in}$

$b = 6\text{ in}$

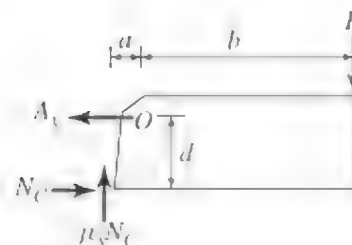
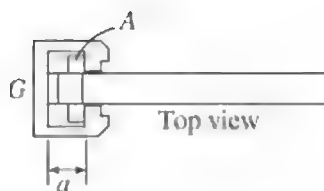
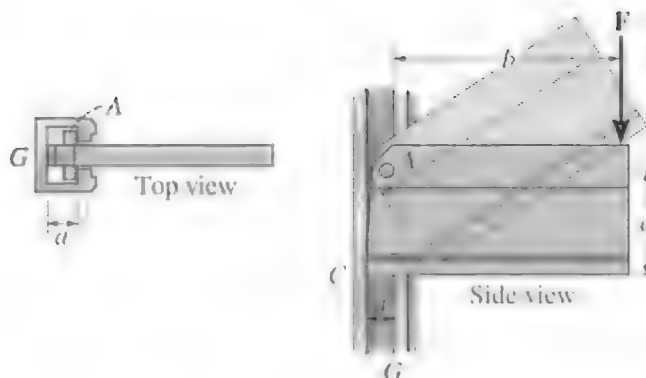
Solution:

$+\uparrow \Sigma F_y = 0;$

$\mu_S N_C - F = 0$

$\Sigma M_A = 0;$

$-F\, b + d\, N_C - \mu_S N_C\, a = 0$

Solving we find $\quad -\mu_s\, b + d - \mu_s\, a = 0 \quad d = \mu_s\,(a+b) \quad d = 2.70\,\text{in}$

Problem 8-26

The homogeneous semicylinder has a mass m and mass center at G. Determine the largest angle θ of the inclined plane upon which it rests so that it does not slip down the plane. The coefficient of static friction between the plane and the cylinder is μ_s. Also, what is the angle ϕ for this case?

Given:

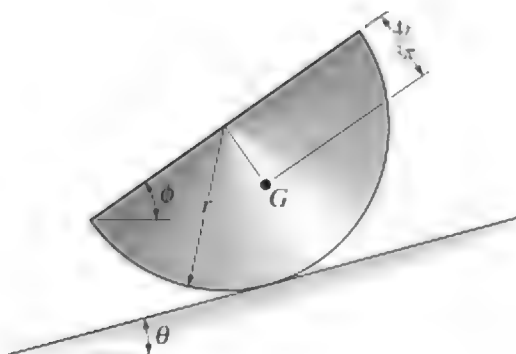
$\mu_s = 0.3$

Solution:

The semicylinder is a two-force member:

Since $\quad F = \mu\, N$

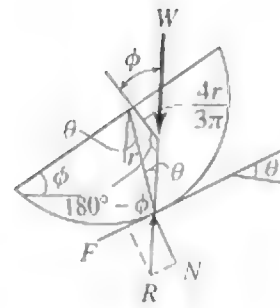
$$\tan(\theta) = \frac{\mu_s\, N}{N} = \mu_S$$

$$\theta = \operatorname{atan}(\mu_s)$$

$\theta = 16.7\,\text{deg}$

Law of sines

$$\frac{r}{\sin(180\ \text{deg} - \phi)} = \frac{\frac{4r}{3\pi}}{\sin(\theta)}$$

$$\phi = \operatorname{asin}\left(\frac{3\pi}{4}\sin(\theta)\right)$$

$\phi = 42.6\,\text{deg}$

Problem 8-27

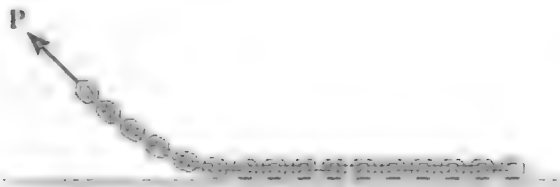
A chain having a length L and weight W rests on a street for which the coefficient of static friction is μ_s. If a crane is used to hoist the chain, determine the force **P** it applies to the chain if the length of chain remaining on the ground begins to slip when the horizontal component is P_x. What length of chain remains on the ground?

Given:

$L = 20\ \text{ft}$

$W = 8\ \frac{\text{lb}}{\text{ft}}$

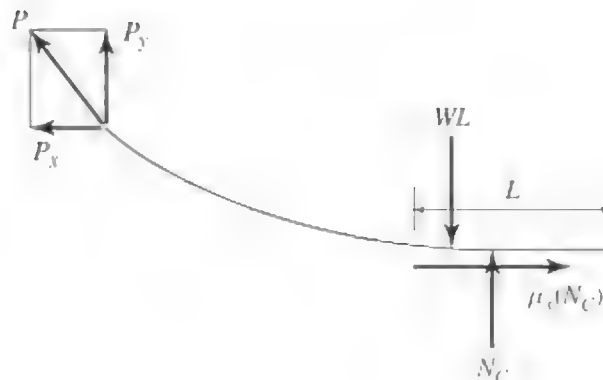
$\mu_s = 0.2$

$P_x = 10\ \text{lb}$

Solution:

$\Sigma F_x = 0;\quad -P_x + \mu_s N_c = 0$

$$N_c = \frac{P_x}{\mu_s}$$

$N_c = 50.00\,\text{lb}$

$\Sigma F_y = 0;\quad P_y - W\,L + N_c = 0$

$$P_y = W\,L - N_C$$

$$P_y = 110.00\,\text{lb}$$

$$P = \sqrt{P_x^{\;2} + P_y^{\;2}}$$

$$P = 110\,\text{lb}$$

The length on the ground is supported by $N_C = 50.00\,\text{lb}$thus

$$L = \frac{N_C}{W}$$

$$L = 6.25\,\text{ft}$$

Problem 8-28

The fork lift has a weight W_1 and center of gravity at G. If the rear wheels are powered, whereas the front wheels are free to roll, determine the maximum number of crates, each of weight W_2 that the fork lift can push forward. The coefficient of static friction between the wheels and the ground is μ_s and between each crate and the ground is μ'_s.

Given:

$W_1 = 2400\ \text{lb}$

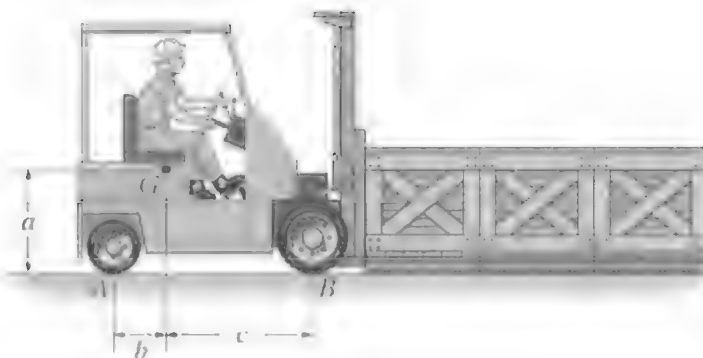
$W_2 = 300\ \text{lb}$

$\mu_s = 0.4$

$\mu'_s = 0.35$

$a = 2.5\ \text{ft}$

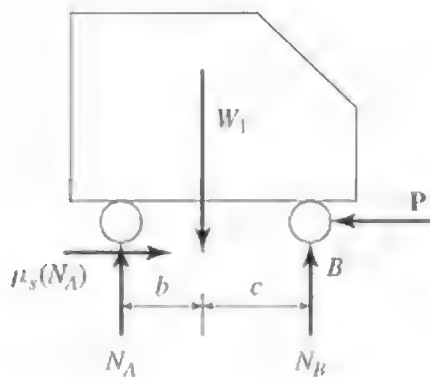
$b = 1.25\ \text{ft}$

$c = 3.50\ \text{ft}$

Solution:

Fork lift:

$\Sigma M_B = 0;\quad W_1\,c - N_A\,(b + c) = 0$

$$N_A = W_1\left(\frac{c}{b+c}\right) \qquad N_A = 1768.4\,\text{lb}$$

$\Sigma F_x = 0;\quad \mu_s\,N_A - P = 0$

$P = \mu_s N_A$ $P = 707.37\,\text{lb}$

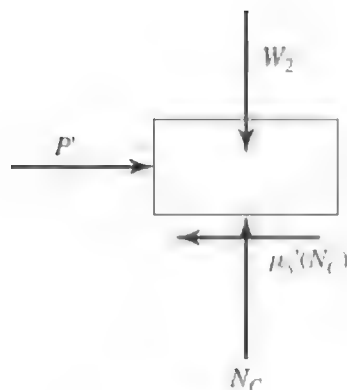
Crate:

$\Sigma F_y = 0;$ $N_c - W_2 = 0$

$N_c = W_2$ $N_c = 300.00\,\text{lb}$

$\Sigma F_x = 0;$ $P' - \mu'_s N_c = 0$

$P' = \mu'_s N_c$ $P' = 105.00\,\text{lb}$

Thus $n = \dfrac{P}{P'}$ $n = 6.74$ $n = \text{floor}(n)$ $n = 6.00$

Problem 8-29

The brake is to be designed to be self locking, that is, it will not rotate when no load P is applied to it when the disk is subjected to a clockwise couple moment $\mathbf{M_O}$. Determine the distance d of the lever that will allow this to happen. The coefficient of static friction at B is μ_s.

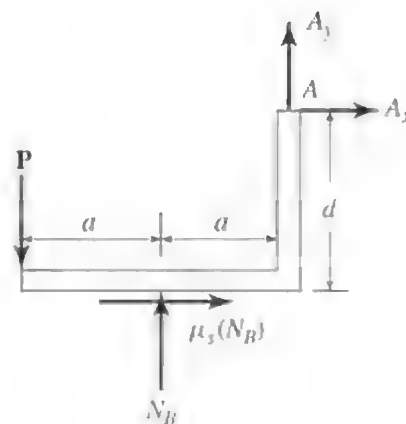
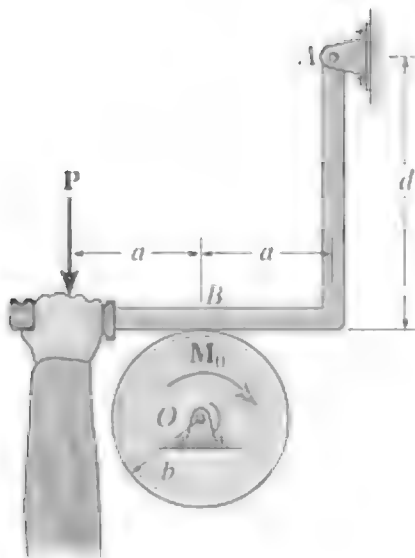
Given:

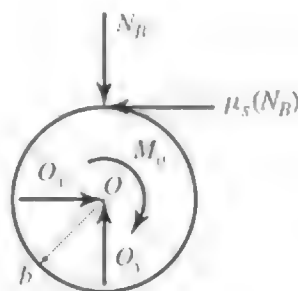
$a = 1.5\ \text{ft}$

$b = 1\ \text{ft}$

$\mu_s = 0.5$

Solution:

$\Sigma M_O = 0;$ $M_O - \mu_s N_B b = 0$

$$N_B = \frac{M_O}{\mu_s b}$$

$\Sigma M_A = 0;$ $P\,2\,a - N_B\,a + \mu_s N_B\,d = 0$

$$P = 0$$

$$d = \frac{a}{\mu_s}$$

$$d = 3.00\,\text{ft}$$

Problem 8-30

The concrete pipe of weight W is being lowered from the truck bed when it is in the position shown. If the coefficient of static friction at the points of support A and B is μ_s determine where it begins to slip first: at A or B, or both at A and B.

Given:

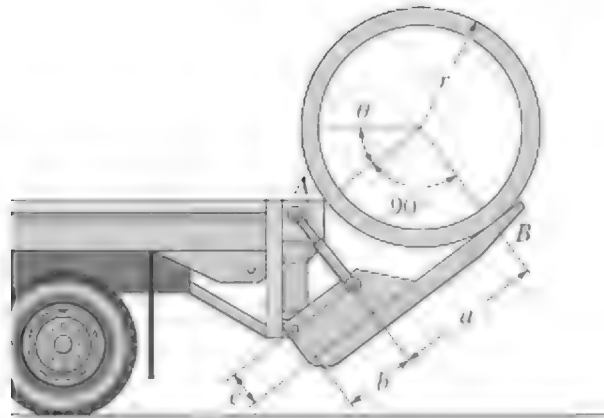
$W = 800$ lb $a = 30$ in

$\mu_s = 0.4$ $b = 18$ in

$\theta = 30$ deg $c = 5$ in

$r = 15$ in

Solution:

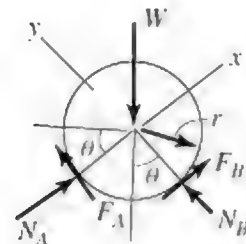
initial guesses are

$N_A = 10$ lb $N_B = 10$ lb $F_A = 10$ lb $F_B = 10$ lb

Given *Assume slipping at A:*

$\Sigma F_x = 0;$ $N_A + F_B - W \sin(\theta) = 0$

$\Sigma F_y = 0;$ $F_A + N_B - W \cos(\theta) = 0$

$\Sigma M_O = 0;$ $F_B\, r - F_A\, r = 0$

$F_A = \mu_s N_A$

$$\begin{pmatrix} N_A \\ N_B \\ F_A \\ F_B \end{pmatrix} = \text{Find}(N_A, N_B, F_A, F_B) \qquad \begin{pmatrix} N_A \\ N_B \\ F_A \\ F_B \end{pmatrix} = \begin{pmatrix} 285.71 \\ 578.53 \\ 114.29 \\ 114.29 \end{pmatrix} \text{lb}$$

At B,

$$F_{Bmax} = \mu_s N_B$$

Since $F_B = 114.29$ lb $<$ $F_{Bmax} = 231.41$ lb then we conclude that slipping begins at A.

Problem 8-31

A wedge of mass M is placed in the grooved slot of an inclined plane. Determine the maximum angle θ for the incline without causing the wedge to slip. The coefficient of static friction between the wedge and the surfaces of contact is μ_s.

Given:

$$M = 5 \text{ kg}$$

$$\mu_s = 0.2$$

$$\phi = 60 \text{ deg}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution:

Initial guesses:

$$N_W = 10 \text{ N} \qquad \theta = 10 \text{ deg}$$

Given

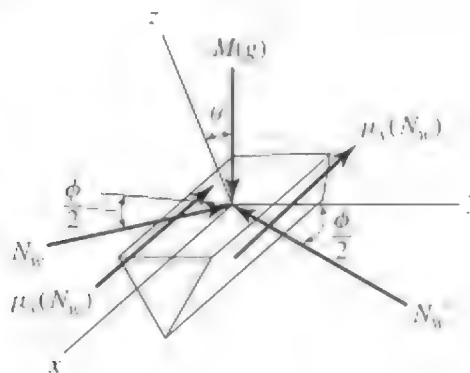
$\Sigma F_x = 0;$ $\quad M\ g \sin(\theta) - 2\ \mu_s\ N_W = 0$

$\Sigma F_z = 0;$ $\quad 2\ N_W \sin\left(\frac{\phi}{2}\right) - M\ g \cos(\theta) = 0$

Solving,

$$\begin{pmatrix} N_W \\ \theta \end{pmatrix} = \text{Find}(N_W, \theta)$$

$$N_W = 45.5 \text{ N}$$

$$\theta = 21.8 \text{ deg}$$

Problem 8-32

A roll of paper has a uniform weight W and is suspended from the wire hanger so that it rests against the wall. If the hanger has a negligible weight and the bearing at O can be considered frictionless, determine the force **P** needed to start turning the roll. The coefficient of static friction between the wall and the paper is μ_s.

Given:

$W = 0.75 \text{ lb}$

$\theta = 30 \text{ deg}$

$\phi = 30 \text{ deg}$

$\mu_s = 0.25$

$a = 3 \text{ in}$

Solution:

Initial guesses:

$R = 100 \text{ lb}$ $\quad N_A = 100 \text{ lb}$ $\quad P = 100 \text{ lb}$

Given

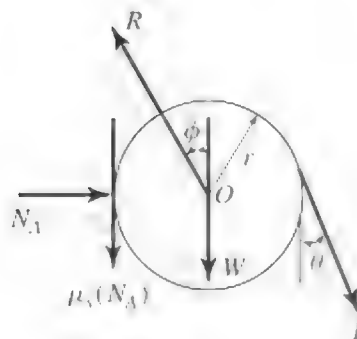
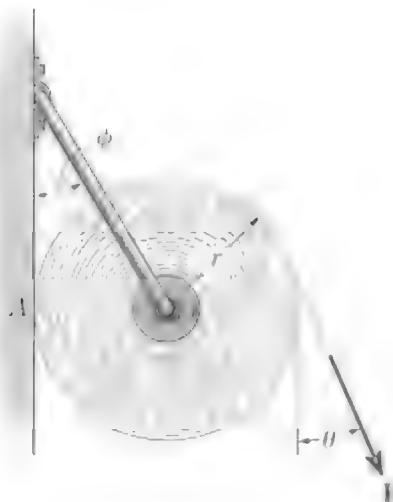
$\Sigma F_x = 0;$ $\quad N_A - R \sin(\phi) + P \sin(\theta) = 0$

$\Sigma F_y = 0;$ $\quad R \cos(\phi) - W - P \cos(\theta) - \mu_s N_A = 0$

$\Sigma M_0 = 0;$ $\quad \mu_s N_A a - P a = 0$

Solving for P,

$$\begin{pmatrix} R \\ N_A \\ P \end{pmatrix} = \text{Find}(R, N_A, P)$$

$R = 1.14 \text{ lb}$ $\quad N_A = 0.51 \text{ lb}$ $\quad P = 0.13 \text{ lb}$

Problem 8-33

A roll of paper has a uniform weight W and is suspended from the wire hanger so that it rests against the wall. If the hanger has a negligible weight and the bearing at O can be considered frictionless, determine the minimum force **P** and the associated angle θ needed to start turning the roll. The coefficient of static friction between the wall and the paper is μ_s.

Given:

$W = 0.75$ lb

$\phi = 30$ deg

$\mu_s = 0.25$

$r = 3$ in

Solution:

$\Sigma F_x = 0;$ $\quad N_A - R\sin(\phi) + P\sin(\theta) = 0$

$\Sigma F_y = 0;$ $\quad R\cos(\phi) - W - P\cos(\theta) - \mu_s N_A = 0$

$\Sigma M_O = 0;$ $\quad \mu_s N_A r - P r = 0$

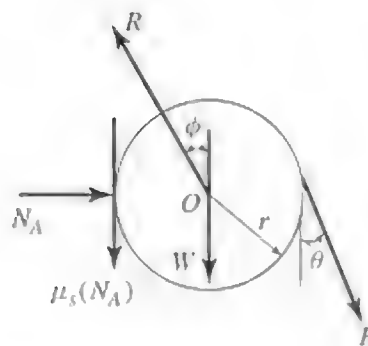
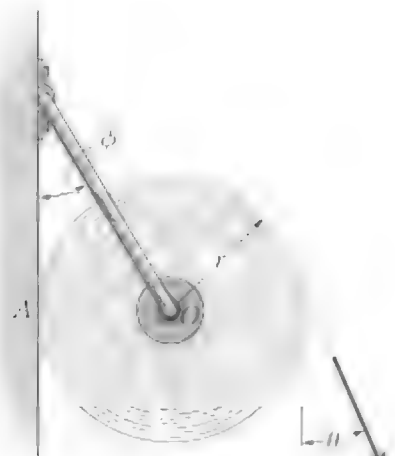
Solving for P,

$$P = \frac{\mu_s W \sin(\phi)}{\cos(\phi) + \mu \sin(\theta - \phi) - \mu \sin(\phi)}$$

For minimum P we must have

$$\frac{dP}{d\theta} = \frac{-\mu_s^2 W \sin(\phi)\cos(\theta - \phi)}{\left(\cos(\phi) + \mu_s \sin(\theta - \phi) - \mu_s \sin(\phi)\right)^2} = 0 \qquad \text{Implies} \qquad \cos(\theta - \phi) = 0$$

One answer is $\theta = \phi + 90$ deg $\qquad \theta = 120.00$ deg

$$P = \frac{\mu_s W \sin(\phi)}{\cos(\phi) + \mu_s \sin(\theta - \phi) - \mu_s \sin(\phi)} \qquad P = 0.0946 \text{ lb}$$

Problem 8-34

The door brace AB is to be designed to prevent opening the door. If the brace forms a pin connection under the doorknob and the coefficient of static friction with the floor is μ_s determine the largest length L the brace can have to prevent the door from being opened. Neglect the weight of the brace.

Given:

$\mu_s = 0.5$

$a = 3$ ft

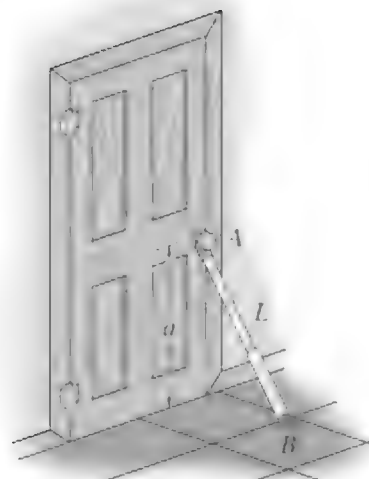
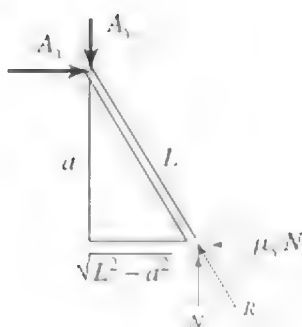
Solution:

The brace is a two-force member.

$$\frac{\mu_s N}{N} = \frac{\sqrt{L^2 - a^2}}{a}$$

$$\mu_s a = \sqrt{L^2 - a^2}$$

$$L = a\sqrt{1 + \mu_s^2}$$

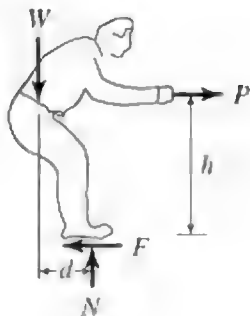
$$L = 3.35 \text{ ft}$$

Problem 8-35

The man has a weight W, and the coefficient of static friction between his shoes and the floor is μ_s. Determine where he should position his center of gravity G at d in order to exert the maximum horizontal force on the door. What is this force?

Given:

$W = 200$ lb

$\mu_s = 0.5$

$h = 3$ ft

Solution:

$N - W = 0 \quad N = W \quad N = 200.00 \text{ lb}$

$F_{max} = \mu_s N \quad F_{max} = 100 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad P - F_{max} = 0$

$P = F_{max} \quad P = 100 \text{ lb}$

$\Sigma M_O = 0; \quad W\, d - P\, h = 0 \quad d = P \frac{h}{W} \quad d = 1.50 \text{ ft}$

Problem 8-36

In an effort to move the two crates, each of weight W, which are stacked on top of one another, the man pushes horizontally on them at the bottom of crate A as shown. Determine the smallest force **P** that must be applied in order to cause impending motion. Explain what happens. The coefficient of static friction between the crates is μ_s and between the bottom crate and the floor is μ_s'.

Given:

$W = 100$ lb

$\mu_s = 0.8$

$\mu'_s = 0.3$

$a = 2$ ft

$b = 3$ ft

Solution:

Assume crate A slips:

$\Sigma F_y = 0;\quad N_A - W = 0 \qquad N_A = W \qquad N_A = 100.00\text{ lb}$

$\Sigma F_x = 0;\quad P - \mu_s N_A = 0 \qquad P_1 = \mu_s N_A \qquad P_1 = 80.00\text{ lb}$

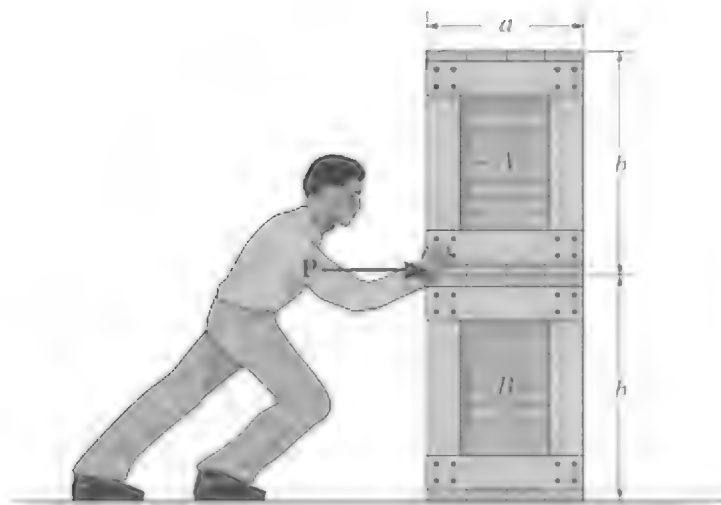
Assume crate B slips:

$\Sigma F_y = 0;\quad N_B - 2W = 0 \qquad N_B = 2W \qquad N_B = 200.00\text{ lb}$

$\Sigma F_x = 0;\quad P - \mu'_s N_B = 0 \qquad P_2 = \mu'_s N_B \qquad P_2 = 60.00\text{ lb}$

Assume both crates A and B tip:

$\Sigma M = 0;\quad 2W\left(\frac{a}{2}\right) - P b = 0 \qquad P_3 = W\left(\frac{a}{b}\right) \qquad P_3 = 66.7\text{ lb}$

$P = \min(P_1, P_2, P_3) \qquad P = 60.00\text{ lb}$

Problem 8-37

The man having a weight of W_1 pushes horizontally on the bottom of crate A, which is stacked on top of crate B. Each crate has a weight W_2. If the coefficient of static friction between each crate is μ_s and between the bottom crate, his shoes, and the floor is μ'_s, determine if he can cause impending motion.

Given:

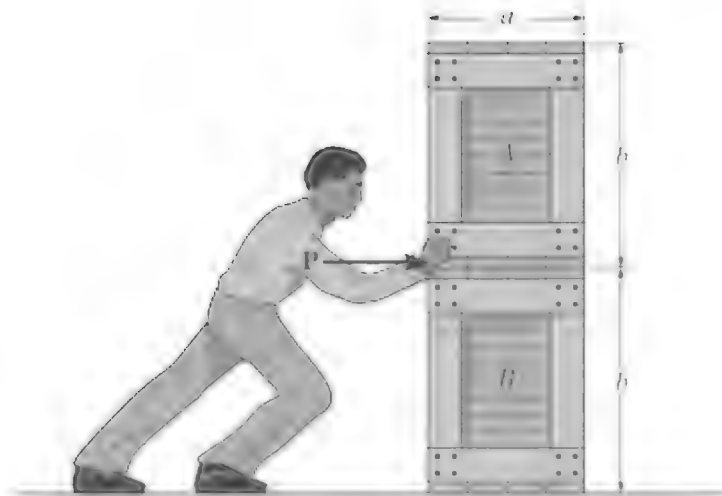
$W_1 = 150 \text{ lb}$

$W_2 = 100 \text{ lb}$

$a = 2 \text{ ft}$

$b = 3 \text{ ft}$

$\mu_s = 0.8$

$\mu'_s = 0.3$

Assume crate *A* slips:

$\Sigma F_y = 0; \quad N_A - W_2 = 0 \qquad N_A = W_2 \qquad N_A = 100.00 \text{ lb}$

$\Sigma F_x = 0; \quad P - \mu_s N_A = 0 \qquad P_1 = \mu_s N_A \qquad P_1 = 80.00 \text{ lb}$

Assume crate *B* slips:

$\Sigma F_y = 0; \quad N_B - 2\,W_2 = 0 \qquad N_B = 2\,W_2 \qquad N_B = 200.00 \text{ lb}$

$\Sigma F_x = 0; \quad P - \mu'_s N_B = 0 \qquad P_2 = \mu'_s N_B \qquad P_2 = 60.00 \text{ lb}$

Assume both crates *A* and *B* tip:

$\Sigma M = 0; \quad 2\,W_2 \left(\frac{a}{2}\right) - P\,b = 0 \qquad P_3 = W_2 \frac{a}{b} \qquad P_3 = 66.7 \text{ lb}$

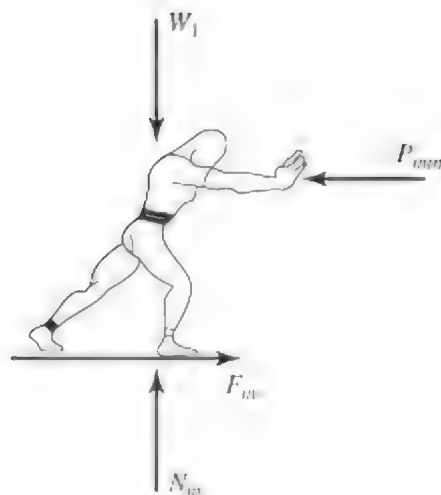
$P_{min} = \min(P_1, P_2, P_3) \qquad P_{min} = 60.00 \text{ lb}$

Now check to see if he can create this force

$\Sigma F_y = 0; \quad N_m - W_1 = 0 \qquad N_m = W_1$

$\Sigma F_x = 0; \quad F_m - P_{min} = 0 \qquad F_m = P_{min}$

$F_{mmax} = \mu'_s N_m$

Since $F_m = 60.00 \text{ lb} >$ $F_{mmax} = 45.00 \text{ lb}$ then the man cannot create the motion.

Problem 8-38

The crate has a weight W and a center of gravity at G. Determine the horizontal force **P** required to tow it. Also, determine the location of the resultant normal force measured from A.

Given:

$a = 3.5$ ft

$b = 3$ ft

$c = 2$ ft

$W = 200$ lb

$h = 4$ ft

$\mu_s = 0.4$

Solution:

$\Sigma F_x = 0;$ $P = F_O$

$\Sigma F_y = 0;$ $N_O = W$

$N_O = 200.00$ lb

$\Sigma M_o = 0;$ $-P\,h + W\,x = 0$

$F_O = \mu_s\,N_O$

$F_O = 80.00$ lb

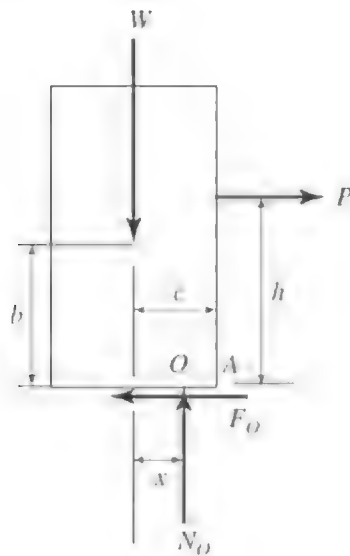
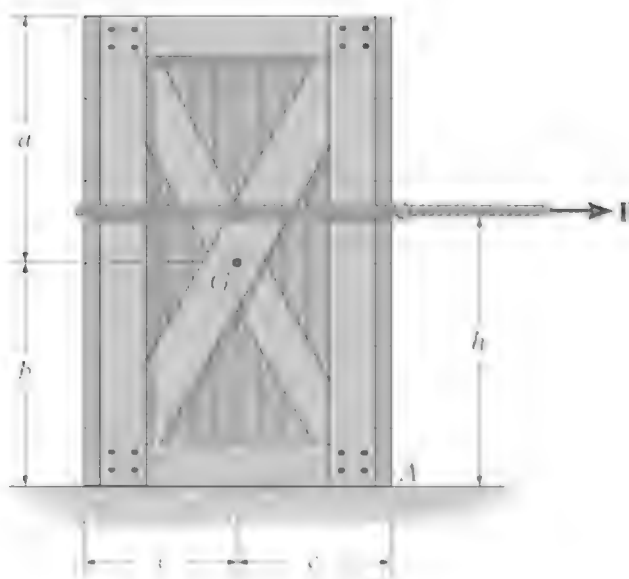
$P = F_O$

$P = 80.00$ lb

$$x = P\,\frac{h}{W}$$

$x = 1.60$ ft

The distance of N_O from A is

$c - x = 0.40$ ft

Problem 8-39

The crate has a weight W and a center of gravity at G. Determine the height h of the tow rope so that the crate slips and tips at the same time. What horizontal force **P** is required to do this?

Given:

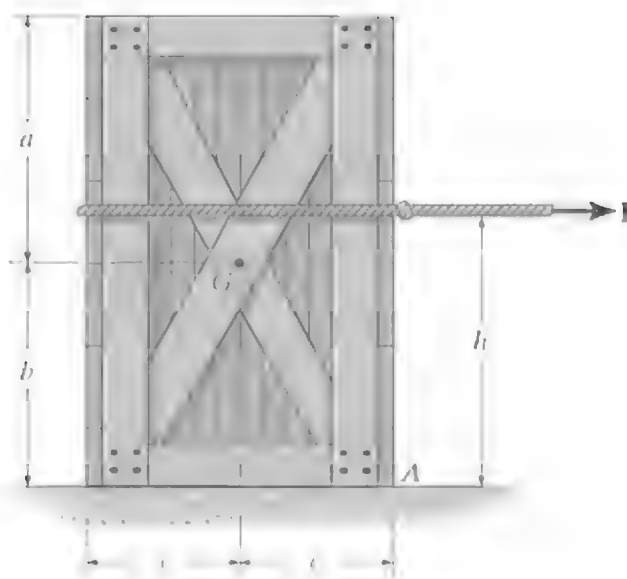
$a = 3.5$ ft

$b = 3$ ft

$c = 2$ ft

$W = 200$ lb

$h = 4$ ft

$\mu_s = 0.4$

Solution:

$\Sigma F_y = 0;$ $N_A = W$

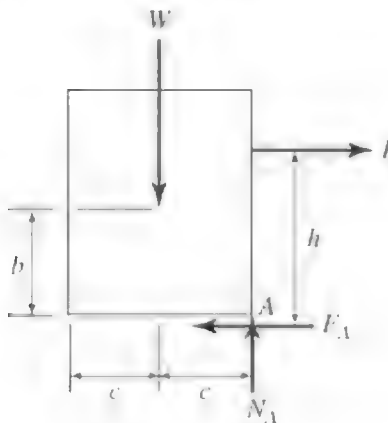
$N_A = 200.00$ lb

$\Sigma F_x = 0;$ $P = F_A$

$F_s = \mu_s N;$ $F_A = \mu_s W$

$F_A = 80.00$ lb $P = 80$ lb

$\Sigma M_A = 0;$ $-P\,h + W\,c = 0$

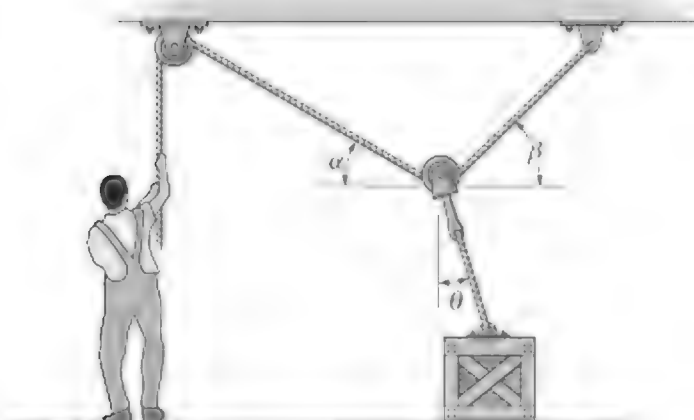
$h = W\dfrac{c}{P}$ $h = 5.00$ ft

Problem 8-40

Determine the smallest force the man must exert on the rope in order to move the crate of mass M. Also, what is the angle θ at this moment? The coefficient of static friction between the crate and the floor is μ_s.

Given:

$M = 80$ kg

$\mu_s = 0.3$

$\alpha = 30 \text{ deg}$

$\beta = 45 \text{ deg}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution : The initial guesses are $T_1 = 1 \text{ N}$ $N_C = 1 \text{ N}$ $\theta = 30 \text{ deg}$ $T = 1 \text{ N}$

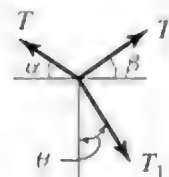
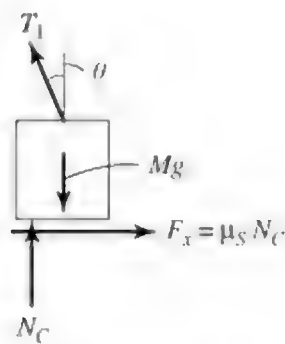
Given

$$N_C - M\, g + T_1 \cos(\theta) = 0$$

$$\mu_s N_C - T_1 \sin(\theta) = 0$$

$$T \cos(\beta) - T \cos(\alpha) + T_1 \sin(\theta) = 0$$

$$T \sin(\beta) + T \sin(\alpha) - T_1 \cos(\theta) = 0$$

$$\begin{pmatrix} T \\ T_1 \\ N_C \\ \theta \end{pmatrix} = \text{Find}(T, T_1, N_C, \theta) \qquad T = 451.86 \text{ N} \qquad \theta = 7.50 \text{ deg}$$

Problem 8-41

The symmetrical crab hook is used to lift packages by means of friction developed between the shoes $A$and B and a package. Determine the smallest coefficient of static friction at the shoes so that the package of weight W can be lifted.

Given:

$a = 1 \text{ ft}$

$b = 2 \text{ ft}$

$c = 0.8 \text{ ft}$

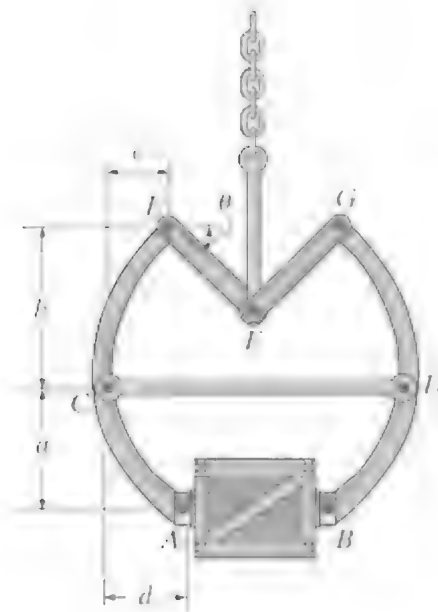
$d = 1$ ft

$\theta = 45$ deg

Solution:

From FBD (a)

$\Sigma F_y = 0; \quad W = 2\ F \sin(\theta)$

$$F = \frac{W}{2\sin(\theta)}$$

From FBD (b)

$\Sigma M_D = 0; \quad \mu\ N_B\ d + N_B\ a - F\ \sin(\theta)\ c - F\ \cos(\theta)\ b = 0$

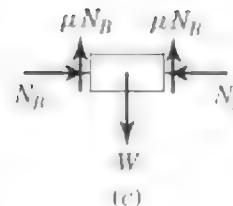
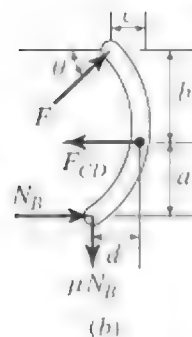
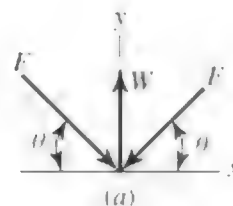
$$\mu\ N_B\ d + N_B\ a - \frac{W}{2\ \sin(\theta)}\ \sin(\theta)\ c - \frac{W}{2\ \sin(\theta)}\cos(\theta)\ b = 0$$

$$N_B = \frac{W}{2\ (\mu\ d + a)}(c + \cot(\theta)\ b)$$

From FBD (c)

$\Sigma F_y = 0; \quad 2\ \mu\ N_B - W = 0$

$$2\ \mu \left[\frac{W}{2(\mu\ d + a)}(c + \cot(\theta)\ b)\right] - W = 0$$

$$\mu = \frac{a}{c + b\ \cot(\theta) - d} \qquad \mu = 0.56$$

Problem 8-42

The friction hook is made from a fixed frame which is shown colored and a cylinder of negligible weight. A piece of paper is placed between the smooth wall and the cylinder. Determine the smallest coefficient of static friction μ at all points of contact so that any weight W of paper p

can be held.

Given:

$\theta = 20 \text{ deg}$

Solution:

Paper:

$+\uparrow \Sigma F_y = 0;$

$F + F - W = 0 \qquad F = \frac{W}{2}$

$F = \mu N \qquad N = \frac{W}{2\mu}$

Cylinder:

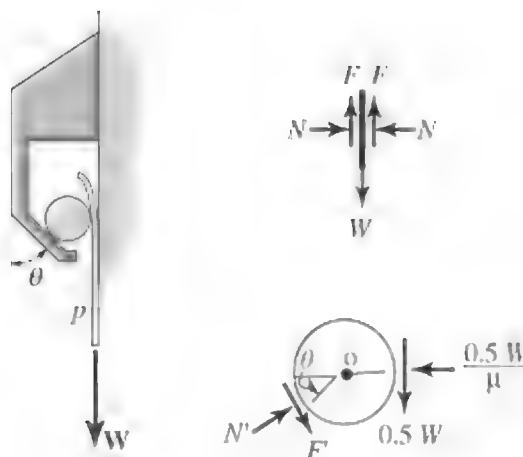
$F'\ r - \left(\frac{W}{2}\right) r = 0 \qquad F' = \frac{W}{2}$

$N' - \left(\frac{W}{2}\right) \sin(\theta) - \frac{W}{2\ \mu} \cos(\theta) = 0 \qquad N' = \frac{W}{2}\left(\sin(\theta) + \frac{1}{\mu}\cos(\theta)\right)$

$F' = \mu\ N' \qquad \frac{W}{2} = \mu \left(\frac{W}{2}\right)\left(\sin(\theta) + \frac{1}{\mu}\cos(\theta)\right)$

$1 = \mu\ \sin(\theta) + \cos(\theta)$

$\mu = \frac{1 - \cos(\theta)}{\sin(\theta)}$

$\mu = 0.176$

Problem 8-43

The crate has a weight W_1 and a center of gravity at G. If the coefficient of static friction between the crate and the floor is μ_s, determine if the man of weight W_2 can push the crate to the left. The coefficient of static friction between his shoes and the floor is μ'_s. Assume the man exerts only a horizontal force on the crate.

Given:

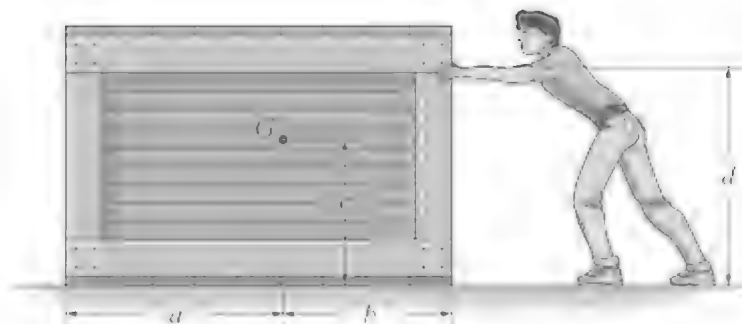
$W_1 = 300$ lb

$W_2 = 200$ lb

$\mu_s = 0.2$

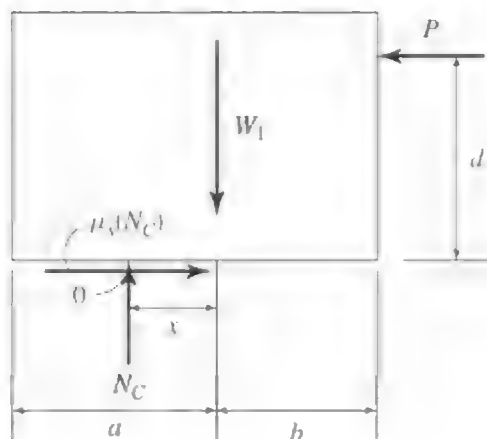
$\mu'_s = 0.35$

$a = 4.5$ ft $\quad c = 3$ ft

$b = 3.5$ ft $\quad d = 4.5$ ft

Solution:

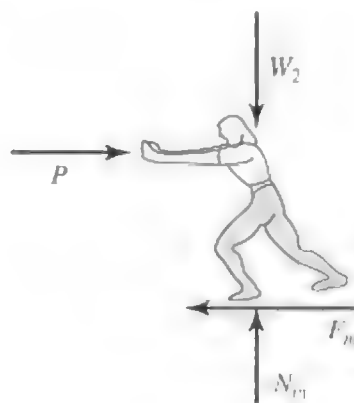
$\Sigma F_y = 0;\quad N_C - W_1 = 0 \quad N_C = W_1$

$\Sigma F_x = 0;\quad \mu_s N_C - P = 0 \quad P = \mu_s N_C$

$\Sigma M_O = 0;\quad -W_1 x + P d = 0 \quad x = \dfrac{P d}{W_1}$

Since $x = 0.90\,\text{ft} < a = 4.50\,\text{ft}$
there will not be any tipping.

$\Sigma F_y = 0;\quad N_m - W_2 = 0 \quad N_m = W_2 \quad N_m = 200.00\,\text{lb}$

$\Sigma F_x = 0;\quad P - F_m = 0 \quad F_m = P \quad F_m = 60.00\,\text{lb}$

$F_{mmax} = \mu'_s N_m \quad F_{mmax} = 70.00\,\text{lb}$

Since $F_m = 60.00\,\text{lb} < F_{mmax} = 70.00\,\text{lb}$ then the man can push the crate.

Problem 8-44

The crate has a weight W_1 and a center of gravity at G. If the coefficient of static friction between the crate and the floor is μ_s, determine the smallest weight of the man so that he can push the crate to the left. The coefficient of static friction between his shoes and the floor is μ'_s. Assume the man exerts only a horizontal force on the crate.

Given:

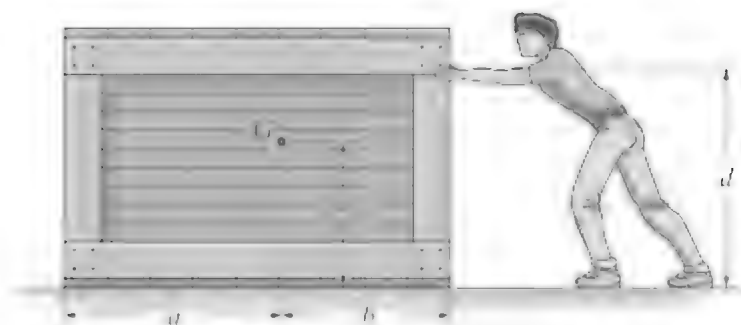
$W_1 = 300$ lb

$W_2 = 200$ lb

$\mu_s = 0.2$

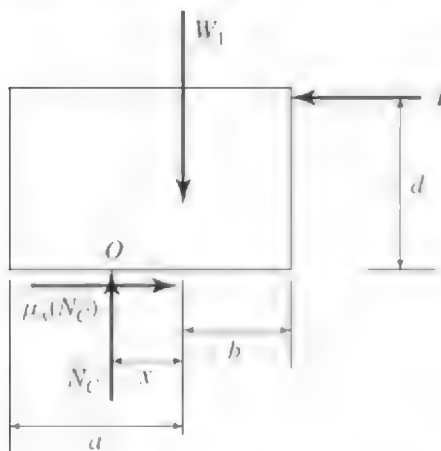
$\mu'_s = 0.35$

$a = 4.5$ ft $\quad c = 3$ ft

$b = 3.5$ ft $\quad d = 4.5$ ft

Solution:

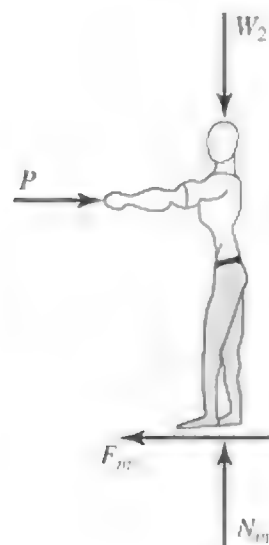
$\Sigma F_y = 0; \quad N_C - W_1 = 0 \quad N_C = W_1$

$\Sigma F_x = 0; \quad \mu_s N_C - P = 0 \quad P = \mu_s N_C$

$\Sigma M_O = 0; \quad -W_1 x + P d = 0 \quad x = \frac{P d}{W_1}$

Since $x = 0.90 \text{ ft} < a = 4.50 \text{ ft}$
there will not be any tipping.

$\Sigma F_x = 0; \quad P - F_m = 0 \quad F_m = P \quad F_m = 60.00 \text{ lb}$

$F_m = \mu'_s N_m \quad N_m = \frac{F_m}{\mu'_s} \quad N_m = 171.4 \text{ lb}$

$\Sigma F_y = 0; \quad N_m - W_2 = 0 \quad W_2 = N_m \quad W_2 = 171.4 \text{ lb}$

Problem 8-45

The wheel has weight W_A and rests on a surface for which the coefficient of friction is μ_B. A cord wrapped around the wheel is attached to the top of the homogeneous block of weight W_C.

If the coefficient of static friction at D is μ_D determine the smallest vertical force that can be applied tangentially to the wheel which will cause motion to impend.

Given:

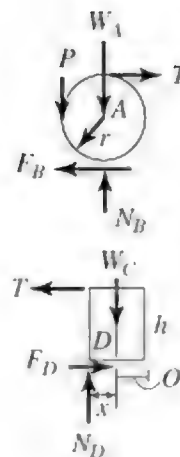
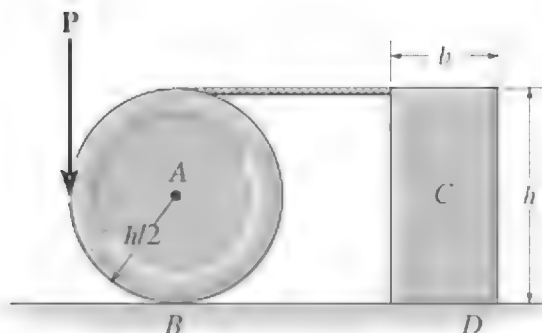
$W_A = 20$ lb

$\mu_B = 0.2$

$W_C = 30$ lb

$\mu_D = 0.3$

$h = 3$ ft

$b = 1.5$ ft

Solution: Assume that slipping occurs at B, but that the block does not move.

Guesses $P = 1$ lb $N_B = 1$ lb $F_B = 1$ lb $T = 1$ lb $N_D = 1$ lb $F_D = 1$ lb

$x = 1$ ft

Given $N_B - W_A - P = 0$ $T - F_B = 0$ $(P - T - F_B)\frac{h}{2} = 0$

$F_B = \mu_B N_B$ $-T + F_D = 0$ $N_D - W_C = 0$

$Th - N_D x = 0$

$$\begin{pmatrix} P \\ N_B \\ F_B \\ T \\ N_D \\ F_D \\ x \end{pmatrix} = \text{Find}(P, N_B, F_B, T, N_D, F_D, x) \qquad \begin{pmatrix} P \\ N_B \\ F_B \\ T \\ N_D \\ F_D \end{pmatrix} = \begin{pmatrix} 13.33 \\ 33.33 \\ 6.67 \\ 6.67 \\ 30.00 \\ 6.67 \end{pmatrix} \text{lb} \qquad x = 0.67 \text{ ft}$$

Now checke the assumptions $F_{Dmax} = \mu_D N_D$

Since $F_D = 6.67$ lb $<$ $F_{Dmax} = 9.00$ lb then the block does not slip

Since $x = 0.67\ \text{ft} < \frac{b}{2} = 0.75\ \text{ft}$ then the block does not tip.

So our original assumption is correct.

$P = 13.33\ \text{lb}$

Problem 8-46

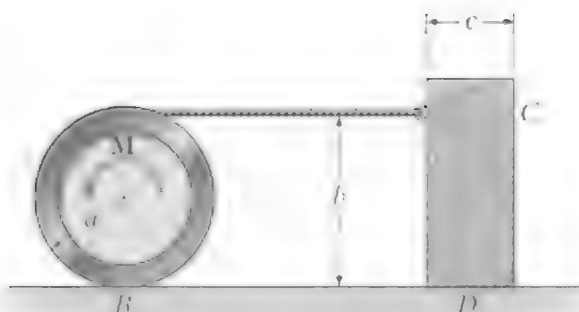
Determine the smallest couple moment which can be applied to the wheel of weight W_1 that will cause impending motion. The cord is attached to the block of weight W_2, and the coefficients of static friction are μ_B and μ_D.

Given:

$W_1 = 20\ \text{lb}$ $\quad a = 1.5\ \text{ft}$

$W_2 = 30\ \text{lb}$ $\quad b = 3\ \text{ft}$

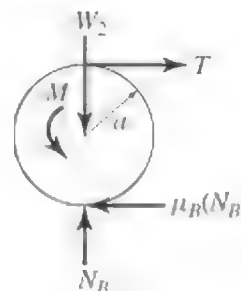
$\mu_B = 0.2$ $\quad c = 1.5\ \text{ft}$

$\mu_D = 0.3$

Solution:

For the wheel : Assume slipping occurs,

$\Sigma F_y = 0;\quad N_B - W_1 = 0 \quad N_B = W_1 \quad N_B = 20.00\ \text{lb}$

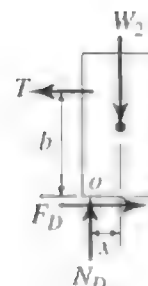
$\Sigma F_x = 0;\quad T - \mu_B N_B = 0 \quad T = \mu_B N_B \quad T = 4.00\ \text{lb}$

$\Sigma M_B = 0;\quad M - T2a = 0 \quad M = T2a \quad M = 12.00\ \text{lb}\cdot\text{ft}$

For block

$\Sigma F_y = 0;\quad N_D - W_2 = 0 \quad N_D = W_2 \quad N_D = 30.00\ \text{lb}$

$\Sigma F_x = 0;\quad F_D - T = 0 \quad F_D = T \quad F_D = 4.00\ \text{lb}$

$\Sigma M_O = 0;\quad Tb - N_D x = 0 \quad x = T\frac{b}{N_D} \quad x = 0.40\ \text{ft}$

$F_{Dmax} = \mu_D N_D \quad F_{Dmax} = 9.00\ \text{lb}$

Since $F_D = 4.00\ \text{lb} < F_{Dmax} = 9.00\ \text{lb}$ then the block doesn't slip

Since $x = 0.40\text{ ft} < \dfrac{c}{2} = 0.75\text{ ft}$ then the block doesn't tip.

Thus neither slipping nor tipping occurs for the block, and our assumption and answer are correct.

Problem 8-47

The beam AB has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass m_p and negligible thickness. Determine the minimum force **P** needed to move the post. The coefficients of static friction at B and C are μ_B and μ_C respectively.

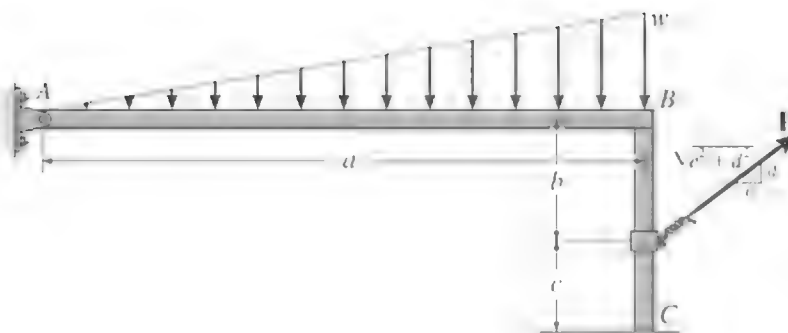
Given:

$m_p = 50\text{ kg}$ $\quad a = 2\text{ m}$

$\mu_B = 0.4$ $\quad b = 400\text{ mm}$

$\mu_C = 0.2$ $\quad c = 300\text{ mm}$

$w = 800\ \dfrac{\text{N}}{\text{m}}$ $\quad d = 3$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $\quad e = 4$

Solution:

Member AB:

$\Sigma M_A = 0;\quad -\left(\dfrac{1}{2}wa\right)\left(\dfrac{2a}{3}\right) + N_B a = 0 \qquad N_B = \dfrac{1}{3}wa \qquad N_B = 533.33\text{ N}$

Post:

Assume slipping occurs at C: $F_C = \mu_C N_C$

The initial guesses are

$P = 1\ \text{N}$ $N_C = 1\ \text{N}$ $F_B = 1\ \text{N}$

Given

$$\frac{-e}{\sqrt{d^2+e^2}} P\, c + F_B(b+c) = 0$$

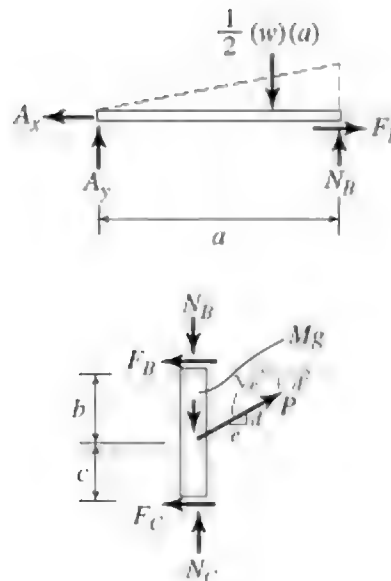
$$\frac{e}{\sqrt{d^2+e^2}} P - F_B - \mu_C N_C = 0$$

$$\frac{d}{\sqrt{e^2+d^2}} P + N_C - N_B - m_p\, g = 0$$

$$\begin{pmatrix} P \\ N_C \\ F_B \end{pmatrix} = \text{Find}(P, N_C, F_B) \qquad P = 354.79\ \text{N}$$

Now check to see if the post slips at B. $F_{Bmax} = \mu_B\, N_B$

Since $F_B = 122\ \text{N} < F_{Bmax} = 213\ \text{N}$ then our assumptions are correct

$P = 355\ \text{N}$

Problem 8-48

The beam AB has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass m_p and negligible thickness. Determine the two coefficients of static friction at B and at C so that when the magnitude of the applied force is increased to **P** the post slips at both B and C simultaneously.

Given:

$m_p = 50\ \text{kg}$

$P = 150\ \text{N}$

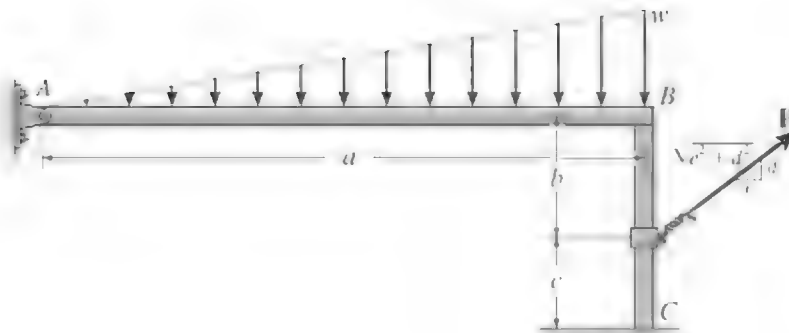
$w = 800\ \frac{\text{N}}{\text{m}}$

$a = 2\ \text{m}$

$b = 400\ \text{mm}$

$c = 300\ \text{mm}$

$d = 3$

$e = 4$

Solution: Member AB:

$$-\frac{1}{2}wa\frac{2a}{3} + N_B a = 0$$

$$N_B = \frac{1}{3}wa \qquad N_B = 533.33\ \text{N}$$

Post:

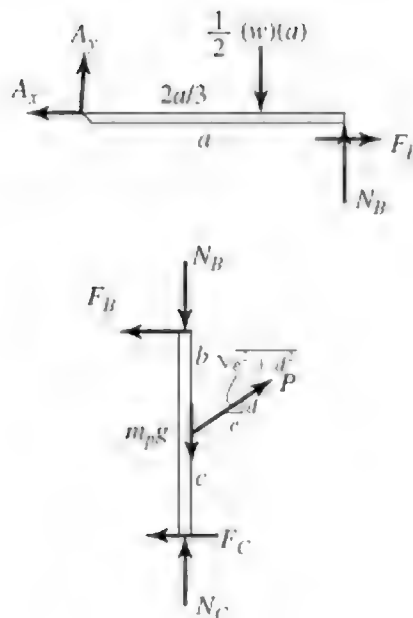
Guesses $N_C = 1\ \text{N}$ $\mu_B = 0.2$ $\mu_C = 0.2$

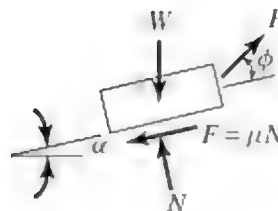
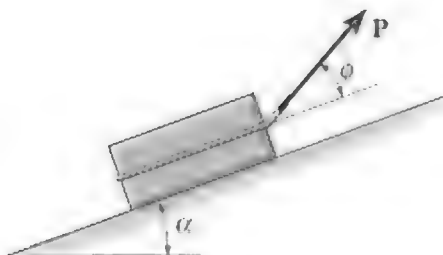
Given

$$N_C - N_B + P\left(\frac{d}{\sqrt{d^2 + e^2}}\right) - m_p g = 0$$

$$\left(\frac{e}{\sqrt{d^2 + e^2}}\right)P - \mu_C N_C - \mu_B N_B = 0$$

$$\left(\frac{-e}{\sqrt{d^2 + e^2}}\right)P c + \mu_B N_B(b + c) = 0$$

$$\begin{pmatrix} N_C \\ \mu_B \\ \mu_C \end{pmatrix} = \text{Find}(N_C, \mu_B, \mu_C) \qquad \begin{pmatrix} \mu_B \\ \mu_C \end{pmatrix} = \begin{pmatrix} 0.0964 \\ 0.0734 \end{pmatrix}$$

Problem 8-49

The block of weight W is being pulled up the inclined plane of slope α using a force **P**. If **P** acts at the angle ϕ as shown, show that for slipping to occur, $P = W \sin(\alpha + \theta)/ \cos(\phi - \theta)$ where θ is the angle of friction; $\theta = \tan^{-1} \mu$.

Solution: Let $\mu = \tan(\theta)$

$\Sigma F_x = 0;$ $$P\cos(\phi) - W\sin(\alpha) - \mu N = 0$$

$\Sigma F_y = 0;$ $$N - W\cos(\alpha) + P\sin(\phi) = 0$$

$$P\cos(\phi) - W\sin(\alpha) - \mu(W\cos(\alpha) - P\sin(\phi) = 0$$

$$P = W\left(\frac{\sin(\alpha) + \mu\cos(\alpha)}{\cos(\phi) + \mu\sin(\phi)}\right) = W\left(\frac{\sin(\alpha) + \tan(\theta)\cos(\alpha)}{\cos(\phi) + \tan(\theta)\sin(\phi)}\right)$$

$$P = W\left(\frac{\sin(\alpha)\cos(\theta) + \sin(\theta)\cos(\alpha)}{\cos(\phi)\cos(\theta) + \sin(\theta)\sin(\phi)}\right)$$

$$P = W\left(\frac{\sin(\alpha + \theta)}{\cos(\phi - \theta)}\right)$$ (QED)

Problem 8-50

Determine the angle ϕ at which **P** should act on the block so that the magnitude of **P** is as small as possible to begin pulling the block up the incline. What is the corresponding value of P? The block has weight W and the slope α is known.

Solution: Let $\mu = \tan(\theta)$

$\Sigma F_x = 0;$ $P\cos(\phi) - W\sin(\alpha) - \mu N = 0$

$\Sigma F_y = 0;$ $N - W\cos(\alpha) + P\sin(\phi) = 0$

$$P\cos(\phi) - W\sin(\alpha) - \mu(W\cos(\alpha) - P\sin(\phi) = 0$$

$$P = W\left(\frac{\sin(\alpha) + \mu\cos(\alpha)}{\cos(\phi) + \mu\sin(\phi)}\right) = W\left(\frac{\sin(\alpha) + \tan(\theta)\cos(\alpha)}{\cos(\phi) + \tan(\theta)\sin(\phi)}\right)$$

$$P = W\left(\frac{\sin(\alpha)\cos(\theta) + \sin(\theta)\cos(\alpha)}{\cos(\phi)\cos(\theta) + \sin(\theta)\sin(\phi)}\right) = W\frac{\sin(\alpha + \theta)}{\cos(\phi - \theta)}$$

$$\frac{dP}{d\phi} = W\left[\frac{\sin(\alpha + \theta)\sin(\phi - \theta)}{\cos^2(\phi - \theta)}\right] = 0$$

$$\sin(\alpha + \theta)\sin(\phi - \theta) = 0$$

$\sin(\phi - \theta) = 0$ $\phi = \theta$ $P = W\sin(\alpha + \phi)$

Problem 8-51

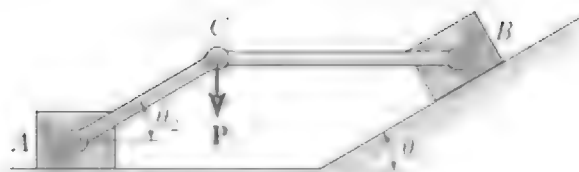
Two blocks A and B, each having a mass M, are connected by the linkage shown. If the coefficient of static friction at the contacting surfaces is μ_s determine the largest vertical force **P** that may be applied to pin C of the linkage without causing the blocks to move. Neglect the weight of the links.

Given:

$M = 6 \text{ kg}$

$\mu_s = 0.5$

$\theta_1 = 30 \text{ deg}$

$\theta_2 = 30 \text{ deg}$

Solution:

Guesses $N_A = 1 \text{ N}$ $F_A = 1 \text{ N}$ $F_{AC} = 1 \text{ N}$

$P = 1 \text{ N}$ $N_B = 1 \text{ N}$ $F_B = 1 \text{ N}$ $F_{BC} = 1 \text{ N}$

Assume that A slips first

Given

$$F_{AC}\cos(\theta_2) - F_{BC} = 0$$

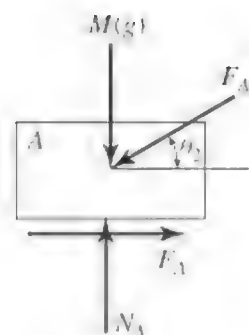
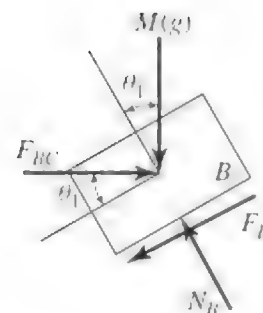
$$F_{AC}\sin(\theta_2) - P = 0$$

$$N_A - Mg - F_{AC}\sin(\theta_2) = 0$$

$$F_A - F_{AC}\cos(\theta_2) = 0$$

$$F_{BC}\cos(\theta_1) - Mg\sin(\theta_1) - F_B = 0$$

$$-F_{BC}\sin(\theta_1) - Mg\cos(\theta_1) + N_B = 0$$

$$F_A = \mu_s N_A$$

$$\begin{pmatrix} P_1 \\ N_A \\ F_A \\ N_B \\ F_B \\ F_{AC} \\ F_{BC} \end{pmatrix} = \text{Find}(P, N_A, F_A, N_B, F_B, F_{AC}, F_{BC}) \qquad P_1 = 23.9 \text{ N}$$

Assume that B slips first

Given

$$F_{AC}\cos(\theta_2) - F_{BC} = 0$$

$$F_{AC}\sin(\theta_2) - P = 0$$

$$N_A - M\,g - F_{AC}\sin(\theta_2) = 0$$

$$F_A - F_{AC}\cos(\theta_2) = 0$$

$$F_{BC}\cos(\theta_1) - M\,g\sin(\theta_1) - F_B = 0$$

$$-F_{BC}\sin(\theta_1) - M\,g\cos(\theta_1) + N_B = 0$$

$$F_B = \mu_s N_B$$

$$\begin{pmatrix} P_2 \\ N_A \\ F_A \\ N_B \\ F_B \\ F_{AC} \\ F_{BC} \end{pmatrix} = \text{Find}(P, N_A, F_A, N_B, F_B, F_{AC}, F_{BC}) \qquad P_2 = 51.5\ \text{N}$$

$$P = \min(P_1, P_2) \qquad P = 23.9\ \text{N}$$

Problem 8-52

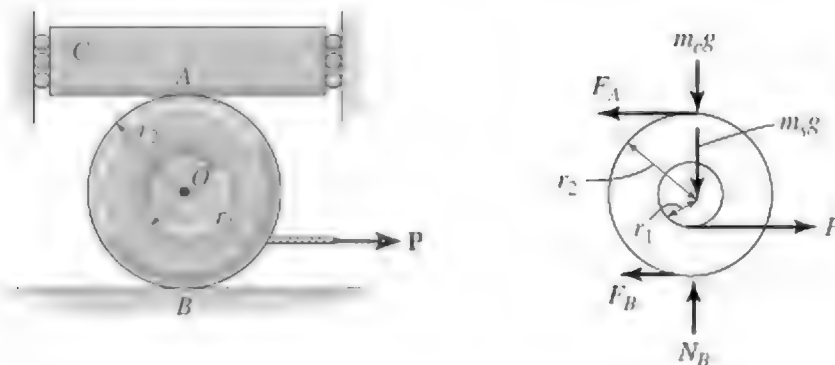
Block C has a mass m_c and is confined between two walls by smooth rollers. If the block rests on top of the spool of mass m_s, determine the minimum cable force **P** needed to move the spool. The cable is wrapped around the spool's inner core. The coefficients of static friction at μ_A and μ_B.

Given:

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

$$m_c = 50\ \text{kg}$$

$$m_s = 40\ \text{kg}$$

$$\mu_A = 0.3$$

$\mu_B = 0.6$ $r_1 = 0.2\ \text{m}$ $r_2 = 0.4\ \text{m}$

Solution: Assume that the spool slips at A but not at B.

The initial guesses are $F_B = 2\ \text{N}$ $P = 3\ \text{N}$ $N_B = 1\ \text{N}$

Given $P - F_B - \mu_A m_c g = 0$

$N_B - m_s g - m_c g = 0$

$P(r_1 + r_2) - F_B 2r_2 = 0$

$$\begin{pmatrix} F_B \\ P \\ N_B \end{pmatrix} = \text{Find}(F_B, P, N_B) \qquad \begin{pmatrix} F_B \\ P \\ N_B \end{pmatrix} = \begin{pmatrix} 441.45 \\ 588.60 \\ 882.90 \end{pmatrix} \text{N}$$

Now check the no slip assumption at B $F_{Bmax} = \mu_B N_B$ $F_{Bmax} = 529.74\ \text{N}$

Since $F_B = 441\ \text{N}$ $<$ $F_{Bmax} = 530\ \text{N}$ then our assumptions are correct.

$P = 589\ \text{N}$

Problem 8-53

A board of weight W_1 is placed across the channel and a boy of weight W_2 attempts to walk across. If the coefficient of static friction at A and B μ_s, determine if he can make the crossing; and if not, how far will he get from A before the board slips?

Given:

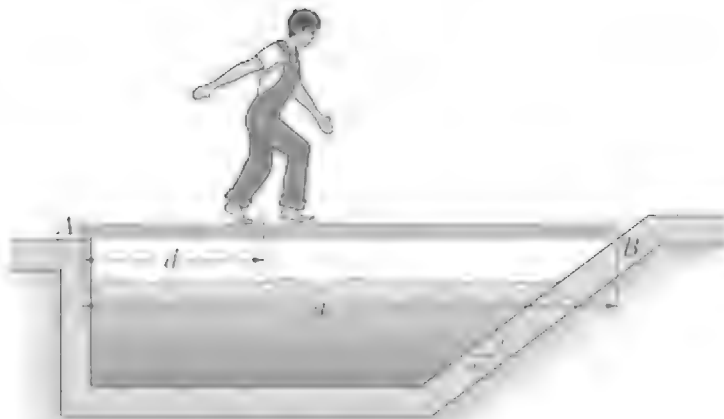
$W_1 = 50$ lb

$W_2 = 100$ lb

$\mu_s = 0.4$

$a = 10$ ft

$b = 4$

$c = 3$

Solution:

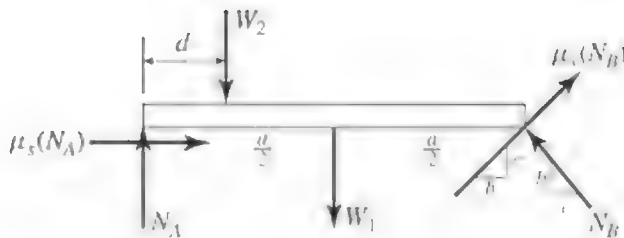
Initial guesses: $N_A = 10$ lb $\quad N_B = 20$ lb $\quad d = 3$ ft

Given

$\Sigma F_x = 0;$ $$\mu_s N_A + \mu_s N_B\left(\frac{b}{\sqrt{b^2 + c^2}}\right) - N_B\left(\frac{c}{\sqrt{b^2 + c^2}}\right) = 0$$

$\Sigma F_y = 0;$ $$N_A - W_2 - W_1 + N_B\left(\frac{b}{\sqrt{b^2 + c^2}}\right) + \mu_s N_B\left(\frac{c}{\sqrt{b^2 + c^2}}\right) = 0$$

$\Sigma M_B = 0;$ $$W_1\left(\frac{a}{2}\right) + W_2(a - d) - N_A a = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ d \end{pmatrix} = \text{Find}(N_A, N_B, d)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 60.34 \\ 86.21 \end{pmatrix} \text{lb} \qquad d = 6.47 \text{ ft}$$

Since $d = 6.47$ ft $< a = 10.00$ ft then the board will slip

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Problem 8-54

Determine the minimum force P needed to push the tube E up the incline. The tube has a mass of M_1 and the roller D has a mass of M_2. The force acts parallel to the plane, and the coefficients of static friction at the contacting surfaces are μ_A, μ_B and μ_C. Each cylinder has a radius of r.

Given:

$M_1 = 75$ kg

$M_2 = 100$ kg

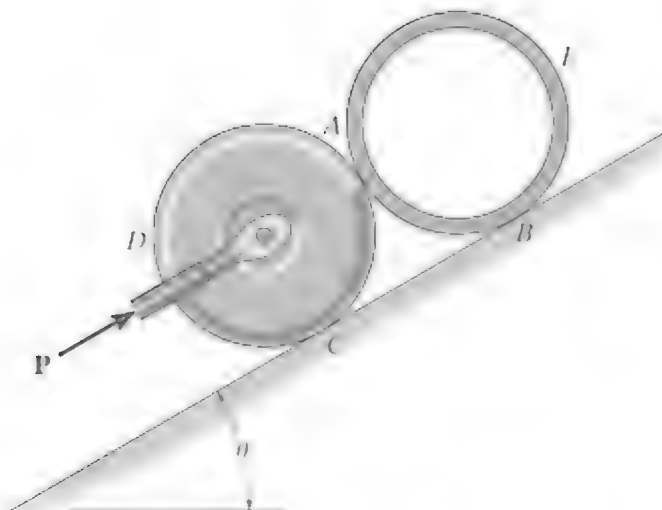
$\mu_A = 0.3$

$\mu_B = 0.25$

$\mu_C = 0.4$

$\theta = 30$ deg

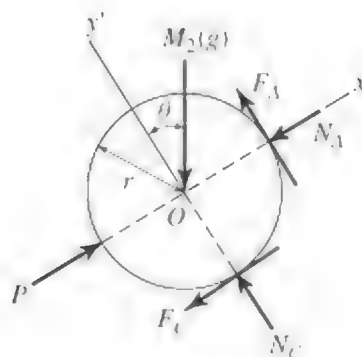
$r = 150$ mm

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

Initial guesses:

$N_A = 100$ N $F_A = 10$ N $P = 100$ N

$N_B = 200$ N $F_B = 20$ N

$N_C = 300$ N $F_C = 30$ N

Given

For roller D

$\Sigma F_{x'} = 0;$ $P - N_A - F_C - M_2 g \sin(\theta) = 0$

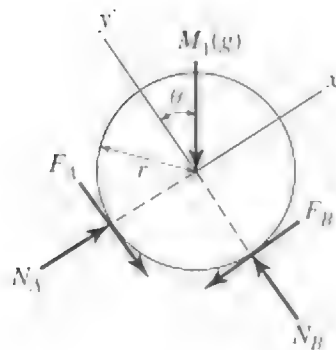
$\Sigma F_{y'} = 0;$ $N_C + F_A - M_2 g \cos(\theta) = 0$

$\Sigma M_{O'} = 0;$ $F_A r - F_C r = 0$

For tube E

$\Sigma F_{x'} = 0;$ $N_A - F_B - M_1 g \sin(\theta) = 0$

$\Sigma F_{y'} = 0;$ $N_B - F_A - M_1 g \cos(\theta) = 0$

$\Sigma M_{O'} = 0;$ $F_A r - F_B r = 0$

Assuming slipping occurs only at A. $F_A = \mu_A N_A$

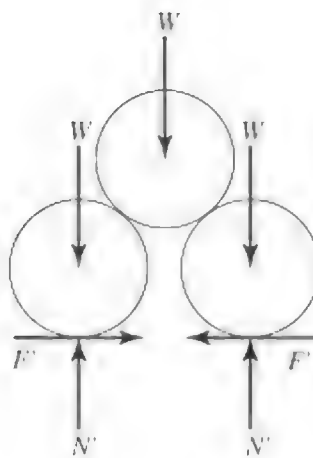
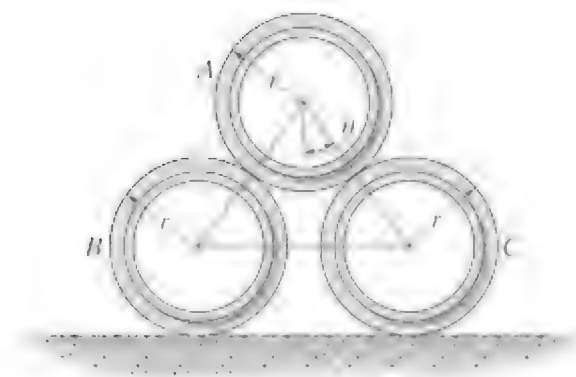
$$\begin{pmatrix} N_A \\ N_B \\ N_C \\ F_A \\ F_B \\ F_C \\ P \end{pmatrix} = \text{Find}(N_A, N_B, N_C, F_A, F_B, F_C, P) \qquad \begin{pmatrix} N_A \\ N_B \\ N_C \\ F_A \\ F_B \\ F_C \\ P \end{pmatrix} = \begin{pmatrix} 526 \\ 795 \\ 692 \\ 158 \\ 158 \\ 158 \\ 1174 \end{pmatrix} \text{N}$$

$$F_{Bmax} = \mu_B N_B \qquad F_{Cmax} = \mu_C N_C \qquad \begin{pmatrix} F_{Bmax} \\ F_{Cmax} \end{pmatrix} = \begin{pmatrix} 199 \\ 277 \end{pmatrix} \text{N}$$

Since $F_B = 158\text{ N} < F_{Bmax} = 199\text{ N}$ and $F_C = 158\text{ N} < F_{Cmax} = 277\text{ N}$ then our assumption is correct

$P = 1174\text{ N}$

Problem 8-55

The concrete pipe at A rests on top of B and C. If the coefficient of static friction between the pipes is μ_s and at the ground μ'_s, determine their smallest values so that the pipes will not slip. Each pipe has a radius r and weight W, and the angle between the centers as indicated is θ.

Solution:

Total System :

$\Sigma F_y = 0;\qquad 2N' - 3W = 0 \qquad N' = \frac{3}{2}W$

Pipe C:

$\Sigma F_x = 0;\qquad -F\cos(\theta) + N\sin(\theta) - F' = 0$

$\Sigma F_y = 0;\qquad N' - W - N\cos(\theta) - F\sin(\theta) = 0$

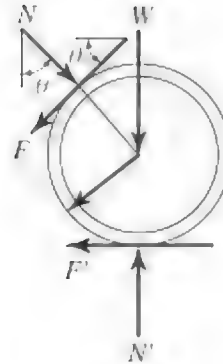
$\Sigma M_O = 0;\qquad F\,r - F'\,r = 0$

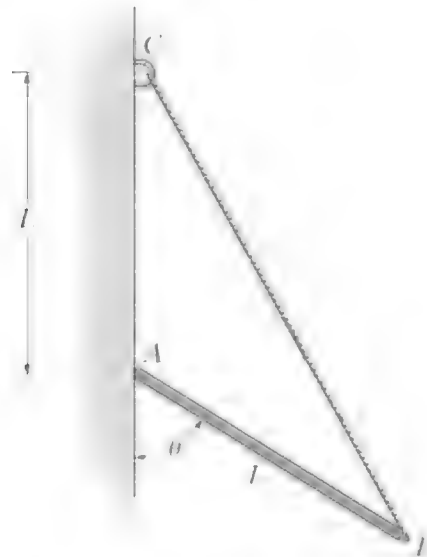
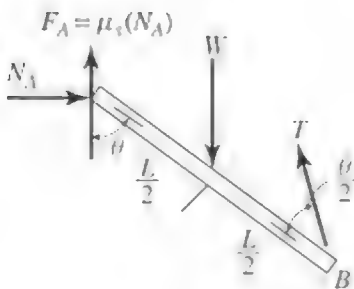
Solving, $\qquad F = F' = \frac{1}{2}\left(\frac{W\sin(\theta)}{\cos(\theta)+1}\right) \qquad N = \frac{W}{2}$

For Pipe A:

$$\mu_s N \ge \frac{1}{2}\left(\frac{W\sin(\theta)}{\cos(\theta)+1}\right) \qquad \mu_s \ge \frac{\sin(\theta)}{\cos(\theta)+1}$$

For Pipe C:

$$\mu'_s N' \ge \frac{1}{2}\left(\frac{W\sin(\theta)}{\cos(\theta)+1}\right) \qquad \mu'_s \ge \frac{1}{3}\,\frac{\sin(\theta)}{\cos(\theta)+1}$$

Problem 8-56

The uniform pole has a weight W and length L. Its end B is tied to a supporting cord, and end A is placed against the wall, for which the coefficient of static friction is μ_s. Determine the largest angle θ at which the pole can be placed without slipping.

Solution:

$\Sigma F_x = 0;\qquad N_A - T\sin\left(\frac{\theta}{2}\right) = 0$

$\Sigma F_y = 0;\qquad \mu_s N_A - W + T\cos\left(\frac{\theta}{2}\right) = 0$

$\Sigma M_B = 0;\qquad -N_A L\cos(\theta) - \mu_s N_A L\sin(\theta) + W\left(\frac{L}{2}\right)\sin(\theta) = 0$

Solving we find

$$N_A = T\sin\left(\frac{\theta}{2}\right)$$

$$\mu_s T\sin\left(\frac{\theta}{2}\right) - W + T\cos\left(\frac{\theta}{2}\right) = 0 \qquad W = T\left(\cos\left(\frac{\theta}{2}\right) + \mu_s\sin\left(\frac{\theta}{2}\right)\right)$$

$$T\sin\left(\frac{\theta}{2}\right)\left(L\cos(\theta) + \mu_s L\sin(\theta)\right) = T\left(\cos\left(\frac{\theta}{2}\right) + \mu_s\sin\left(\frac{\theta}{2}\right)\right)\frac{L}{2}\sin(\theta)$$

$$\sin\left(\frac{\theta}{2}\right)\left(\cos(\theta) + \mu_s\sin(\theta)\right) = \frac{1}{2}\sin(\theta)\left(\cos\left(\frac{\theta}{2}\right) + \mu_s\sin\left(\frac{\theta}{2}\right)\right)$$

$$\frac{\mu_s}{2}\sin\left(\frac{\theta}{2}\right)\sin(\theta) = \frac{1}{2}\sin(\theta)\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\cos(\theta)$$

$$\mu_s = \frac{\sin(\theta)\cos\left(\frac{\theta}{2}\right) - 2\sin\left(\frac{\theta}{2}\right)\cos(\theta)}{\sin\left(\frac{\theta}{2}\right)\sin(\theta)}$$

$$\mu_s = \frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} - \frac{2\left(\cos\left(\frac{\theta}{2}\right)^2 - \sin\left(\frac{\theta}{2}\right)^2\right)}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \cot\left(\frac{\theta}{2}\right) - \cos\left(\frac{\theta}{2}\right) + \tan\left(\frac{\theta}{2}\right)$$

$$\mu_s = \tan\left(\frac{\theta}{2}\right) \qquad \theta = 2\,\text{atan}(\mu)$$

Problem 8-57

The carpenter slowly pushes the uniform board horizontally over the top of the saw horse. The board has a uniform weight density γ and the saw horse has a weight W and a center of gravity at G. Determine if the saw horse will stay in position, slip, or tip if the board is pushed forward at the given distance d. The coefficients of static friction are shown in the figure.

Given:

$$\gamma = 3\ \frac{\text{lb}}{\text{ft}}$$

$L = 18$ ft

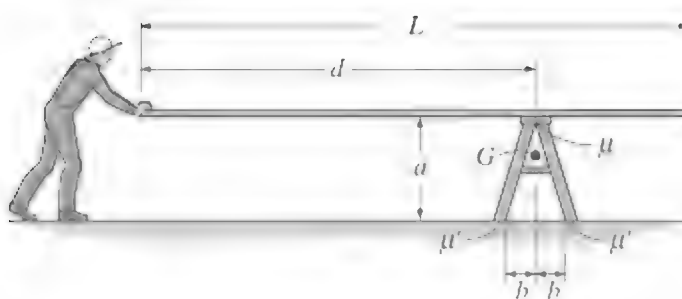
$W = 15$ lb

$a = 3$ ft

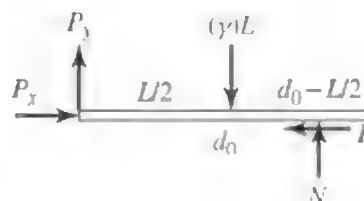
$b = 1$ ft

$\mu = 0.5$

$\mu' = 0.3$

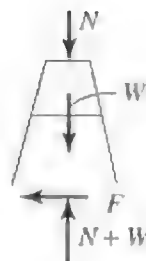
$d = 10$ ft

Solution:

Board:

$$-L\gamma\left(\frac{L}{2}\right) + Nd = 0 \quad N = \frac{1}{2}L^2\left(\frac{\gamma}{d}\right) \quad N = 48.60 \text{ lb}$$

To cause slipping of the board on the saw horse:

$$P_{xb} = \mu N \qquad P_{xb} = 24.30 \text{ lb}$$

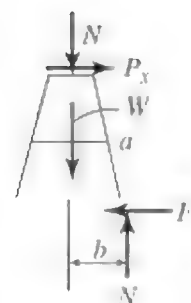
To cause slipping at the ground:

$$P_{xg} = \mu'(N + W) \qquad P_{xg} = 19.08 \text{ lb}$$

To cause tipping

$$(N + W)b - P_{xt}a = 0$$

$$P_{xt} = \frac{(N + W)b}{a} \qquad P_{xt} = 21.20 \text{ lb}$$

Choose the critical case

$$P_x = \min(P_{xb}, P_{xg}, P_{xt}) \qquad P_x = 19.08 \text{ lb}$$

Problem 8-58

The carpenter slowly pushes the uniform board horizontally over the top of the saw horse. The board has a uniform weight density γ and the saw horse has a weight W and a center of gravity at G. Determine if the saw horse will stay in position, slip, or tip if the board is pushed forward at the given distance d. The coefficients of static friction are shown in the figure.

Given:

$$\gamma = 3 \frac{\text{lb}}{\text{ft}}$$

$L = 18$ ft

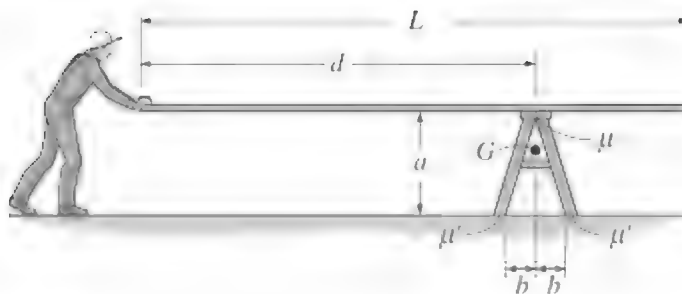
$W = 15$ lb

$a = 3$ ft

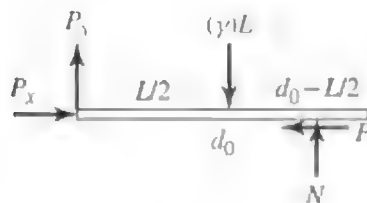
$b = 1$ ft

$\mu = 0.5$

$\mu' = 0.3$

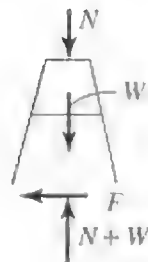
$d = 14$ ft

Solution:

Board:

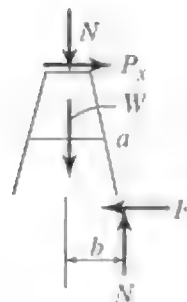
$$-L\gamma\left(\frac{L}{2}\right) + Nd = 0 \quad N = \frac{1}{2}L^2\left(\frac{\gamma}{d}\right) \quad N = 34.71 \text{ lb}$$

To cause slipping of the board on the saw horse:

$$P_{xb} = \mu N \qquad P_{xb} = 17.36 \text{ lb}$$

To cause slipping at the ground:

$$P_{xg} = \mu'(N + W) \qquad P_{xg} = 14.91 \text{ lb}$$

To cause tipping

$$(N + W)b - P_{xt}a = 0$$

$$P_{xt} = \frac{(N + W)b}{a} \qquad P_{xt} = 16.57 \text{ lb}$$

Choose the critical case

$$P_x = \min(P_{xb}, P_{xg}, P_{xt}) \qquad P_x = 14.91 \text{ lb}$$

Problem 8-59

The disk of mass m_o rests on the surface for which the coefficient of static friction is μ_A Determine the largest couple moment M that can be applied to the bar without causing motion.

Given:

$m_O = 45\ \text{kg}$

$\mu_A = 0.2$

$a = 300\ \text{mm}$

$b = 400\ \text{mm}$

$r = 125\ \text{mm}$

Solution:

Guesses

$M = 1\ \text{N m}$ $\quad N_A = 1\ \text{N}$

$B_x = 1\ \text{N}$ $\quad B_y = 1\ \text{N}$

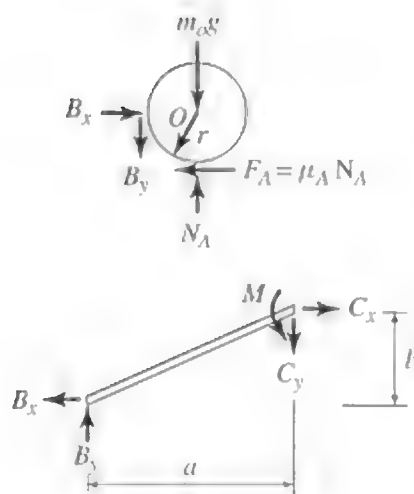
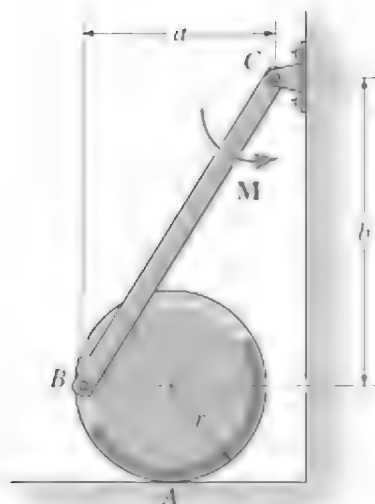
Given

$$M - B_x b - B_y a = 0$$
$$B_x - \mu_A N_A = 0$$

$$N_A - m_O g - B_y = 0$$

$$B_y r - \mu_A N_A r = 0$$

$$\begin{pmatrix} M \\ N_A \\ B_x \\ B_y \end{pmatrix} = \text{Find}(M, N_A, B_x, B_y)$$

$$\begin{pmatrix} N_A \\ B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 551.81 \\ 110.36 \\ 110.36 \end{pmatrix} \text{N} \qquad M = 77.3\ \text{N}\cdot\text{m}$$

Problem 8-60

The disk of mass m_0 rests on the surface for which the coefficient of static friction is μ_A Determine the friction force at A.

Given:

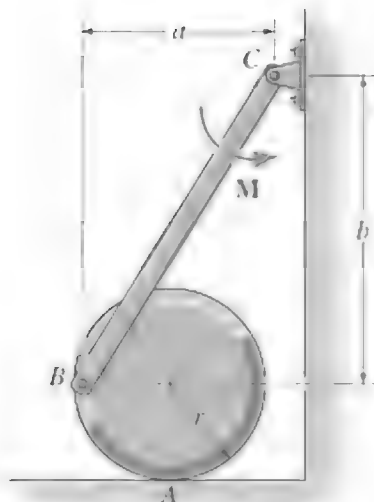
$M = 50\ \text{N}\cdot\text{m}$

$m_O = 45\ \text{kg}$

$\mu_A = 0.2$

$a = 300\ \text{mm}$

$b = 400\ \text{mm}$

$r = 125\ \text{mm}$

Solution: Assume no motion

Guesses

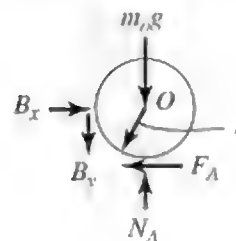
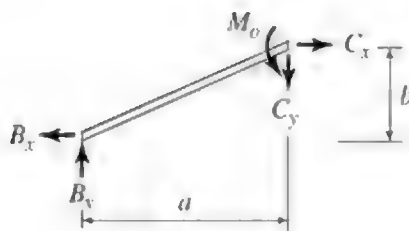
$B_x = 1\ \text{N} \quad B_y = 1\ \text{N} \quad N_A = 1\ \text{N} \quad F_A = 1\ \text{N}$

Given $M - B_y a - B_x b = 0$

$N_A - B_y - m_O g = 0$

$B_x - F_A = 0$

$B_y r - F_A r = 0$

$$\begin{pmatrix} B_x \\ B_y \\ N_A \\ F_A \end{pmatrix} = \text{Find}(B_x, B_y, N_A, F_A) \qquad \begin{pmatrix} N_A \\ F_A \end{pmatrix} = \begin{pmatrix} 512.88 \\ 71.43 \end{pmatrix} \text{N}$$

Check assumption: $F_{Amax} = \mu_A N_A$

Since $F_A = 71.4\ \text{N} \quad < \quad F_{Amax} = 102.6\ \text{N}$ then our assumption is good.

$F_A = 71.4\ \text{N}$

Problem 8-61

A block of weight W is attached to a light rod AD that pivots at pin A. If the coefficient of static friction between the plane and the block is μ_s, determine the minimum angle θ at which the block may be placed on the plane without slipping. Neglect the *size* of the block in the

calculation.

Given:

$a = 4\text{ ft}$

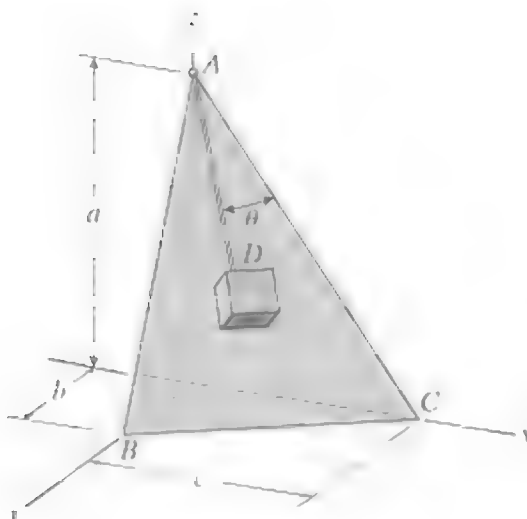
$b = 2\text{ ft}$

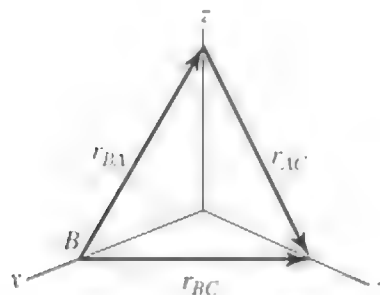
$c = 3\text{ ft}$

$\mu_s = 0.4$

$W = 6\text{ lb}$

Solution:

Establish a unit vector perpendicular to the plane.

$$r_{BC} = \begin{pmatrix} -b \\ c \\ 0 \end{pmatrix} \qquad r_{BA} = \begin{pmatrix} -b \\ 0 \\ a \end{pmatrix}$$

$$n_1 = \frac{r_{BC} \times r_{BA}}{|r_{BC} \times r_{BA}|} \qquad n_1 = \begin{pmatrix} 0.77 \\ 0.51 \\ 0.38 \end{pmatrix}$$

Establish 2 other unit vectors in this plane, and one in the z direction

$$k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad r_{CA} = \begin{pmatrix} 0 \\ -c \\ a \end{pmatrix} \qquad n_2 = \frac{r_{CA}}{|r_{CA}|} \qquad n_3 = n_1 \times n_2$$

Guess $T = 1\text{ lb}$ $N = 1\text{ lb}$ $\theta = 10\text{ deg}$

Given $N n_1 + T(\cos(\theta) n_2 - \sin(\theta) n_3) - \mu_s N(\sin(\theta) n_2 + \cos(\theta) n_3) - Wk = 0$

$$\begin{pmatrix} N \\ T \\ \theta \end{pmatrix} = \text{Find}(N, T, \theta) \qquad \begin{pmatrix} N \\ T \end{pmatrix} = \begin{pmatrix} 2.30 \\ 5.46 \end{pmatrix}\text{lb} \qquad \theta = 20.37\text{ deg}$$

Problem 8-62

Determine the force **P** needed to lift the load of weight W. Smooth rollers are placed between

the wedges. The coefficient of static friction between A and C and between B and D is μ_s. Neglect the weight of each wedge.

Given:

$\theta = 10 \text{ deg}$

$W = 100 \text{ lb}$

$\mu_s = 0.3$

Solution:

Initial guesses:

$N' = 10 \text{ lb}$ $\quad N_A = 15 \text{ lb}$

$N_B = 20 \text{ lb}$ $\quad P = 5 \text{ lb}$

Given

Wedge B:

$\Sigma F_x = 0;\quad N' \sin(\theta) - N_B = 0$

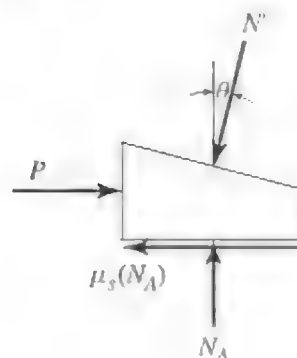
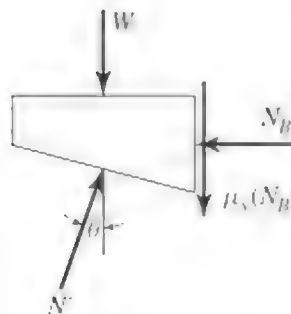
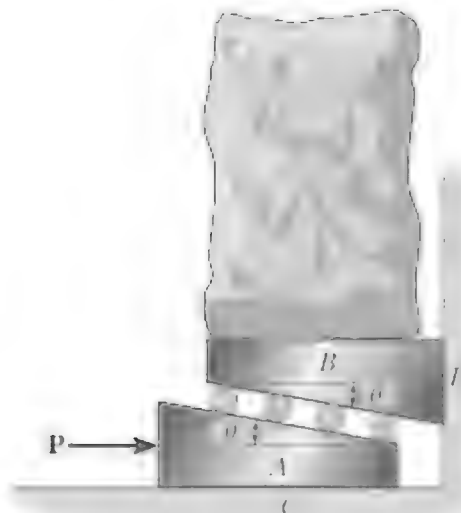
$\Sigma F_y = 0;\quad N' \cos(\theta) - W - \mu_s N_B = 0$

Wedge A:

$\Sigma F_x = 0;\quad P - N' \sin(\theta) - \mu_s N_A = 0$

$\Sigma F_y = 0;\quad N_A - N' \cos(\theta) = 0$

$$\begin{pmatrix} N' \\ N_B \\ N_A \\ P \end{pmatrix} = \text{Find}(N', N_B, N_A, P) \qquad \begin{pmatrix} N' \\ N_B \\ N_A \end{pmatrix} = \begin{pmatrix} 107.2 \\ 18.6 \\ 105.6 \end{pmatrix} \text{lb}$$

$P = 50.3 \text{ lb}$

Problem 8-63

The wedge is used to level the floor of a building. For the floor loading shown, determine the horizontal force **P** that must be applied to move the wedge forward. The coefficient of static friction between the wedge and the two surfaces of contact is μ_s. Neglect the size and weight

of the wedge and the thickness of the beam.

Units Used: $kN = 10^3\ N$

Given:

$F_1 = 2\ kN$ $a = 3\ m$

$F_2 = 4\ kN$ $b = 2\ m$

$F_3 = 4\ kN$ $c = 3\ m$

$F_4 = 2\ kN$ $\theta = 15\ deg$

$\mu_s = 0.25$

Solution:

Guesses

$N_1 = 1\ kN$ $N_2 = 1\ kN$ $P = 1\ kN$

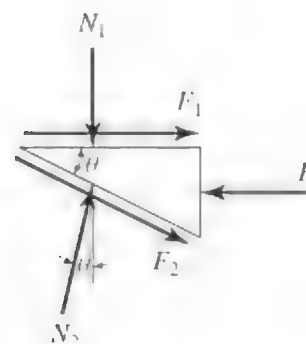
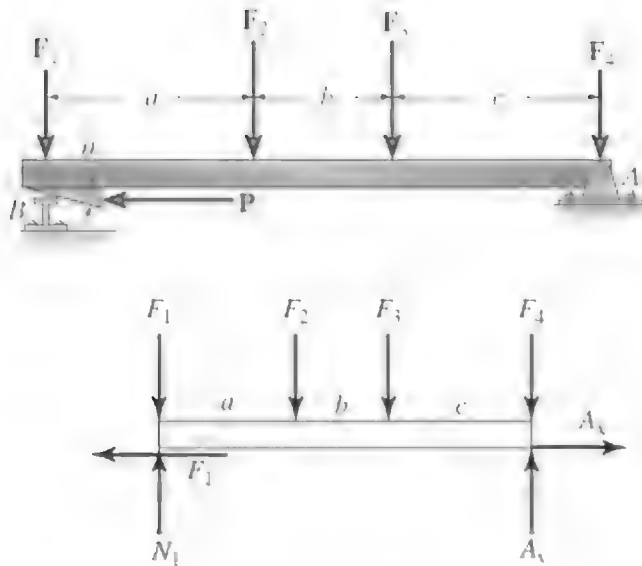
Given

$$(F_1 - N_1)(a + b + c) + F_2(b + c) + F_3 c = 0$$

$$N_2 \cos(\theta) - \mu_s N_2 \sin(\theta) - N_1 = 0$$

$$\mu_s N_1 + \mu_s N_2 \cos(\theta) + N_2 \sin(\theta) - P = 0$$

$$\begin{pmatrix} N_1 \\ N_2 \\ P \end{pmatrix} = \text{Find}(N_1, N_2, P) \qquad \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6.66 \end{pmatrix} kN$$

$P = 4.83\ kN$

Problem 8-64

The three stone blocks have weights W_A, W_B, and W_C. Determine the smallest horizontal force **P** that must be applied to block C in order to move this block.The coefficient of static friction between the blocks is μ_s, and between the floor and each block μ'_s.

Given:

$W_A = 600$ lb

$W_B = 150$ lb

$W_C = 500$ lb

$\mu_s = 0.3$

$\mu'_s = 0.5$

$\theta = 45$ deg

Solution:

Assume all blocks slip together

$N_1 = W_A + W_B + W_C$

$P_1 = \mu'_s N_1 \qquad P_1 = 625.00\,\text{lb}$

Assume that block A does not move and block B moves up

Guesses

$N_C = 1$ lb $\quad N' = 1$ lb $\quad N'' = 1$ lb $\quad P = 1$ lb

Given

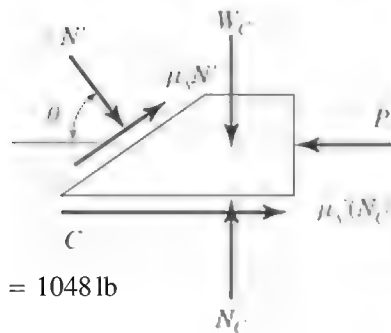
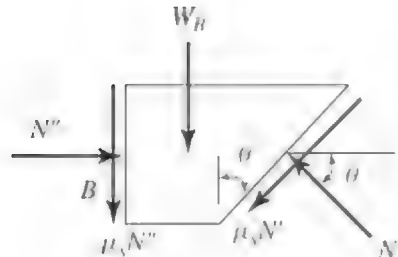
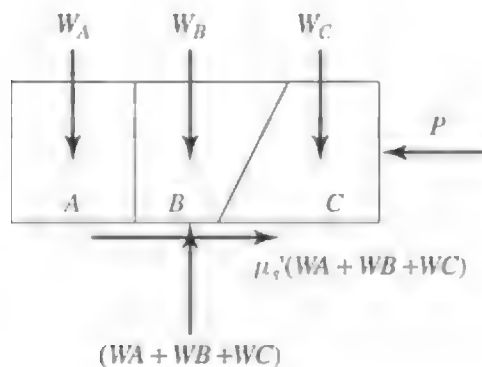
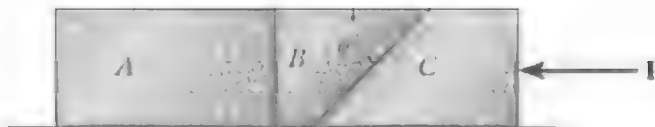
$N'' + \mu'_s N_C - P = 0$

$N_C - W_C - W_B - \mu_s N'' = 0$

$N' \sin(\theta) - \mu_s N' \cos(\theta) - \mu_s N'' - W_B = 0$

$N'' - \mu_s N' \sin(\theta) - N' \cos(\theta) = 0$

$$\begin{pmatrix} N_C \\ N' \\ N'' \\ P_2 \end{pmatrix} = \text{Find}(N_C, N', N'', P) \qquad \begin{pmatrix} N_C \\ N' \\ N'' \end{pmatrix} = \begin{pmatrix} 838.71 \\ 684.30 \\ 629.03 \end{pmatrix} \text{lb} \quad P_2 = 1048\,\text{lb}$$

Choose the critical case $\qquad P = \min(P_1, P_2) \qquad P = 625.00\,\text{lb}$

Problem 8-65

If the spring is compressed a distance δ and the coefficient of static friction between the tapered stub S and the slider A is μ_{sA}, determine the horizontal force **P** needed to move the slider forward. The stub is free to move without friction within the fixed collar C. The coefficient of static friction between A and surface B is μ_{AB}. Neglect the weights of the slider and stub.

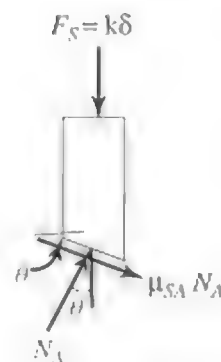
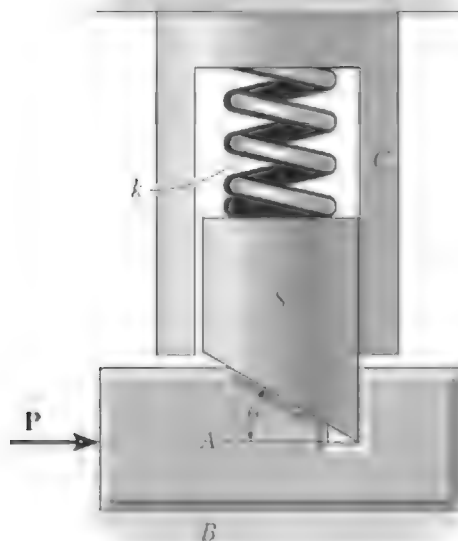
Given:

$\delta = 60\ \text{mm}$

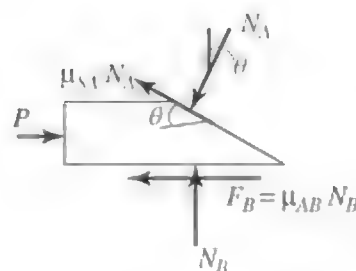
$\mu_{sA} = 0.5$

$\mu_{AB} = 0.4$

$k = 300\ \frac{\text{N}}{\text{m}}$

$\theta = 30\ \text{deg}$

Solution:

Stub:

$+\uparrow\Sigma F_y = 0;\qquad N_A\cos(\theta) - \mu_{sA}N_A\sin(\theta) - k\delta = 0$

$$N_A = \frac{k\delta}{\cos(\theta) - \mu_{sA}\sin(\theta)}\qquad N_A = 29.22\ \text{N}$$

Slider:

$+\uparrow\Sigma F_y = 0;\qquad N_B - N_A\cos(\theta) + \mu_{sA}N_A\sin(\theta) = 0$

$$N_B = N_A\cos(\theta) - \mu_{sA}N_A\sin(\theta)\qquad N_B = 18\ \text{N}$$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad P - \mu_{AB}N_B - N_A\sin(\theta) - \mu_{sA}N_A\cos(\theta) = 0$

$$P = \mu_{AB}N_B + N_A\sin(\theta) + \mu_{sA}N_A\cos(\theta)\qquad P = 34.5\ \text{N}$$

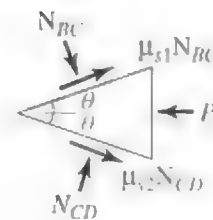
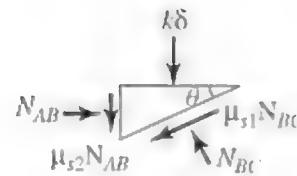
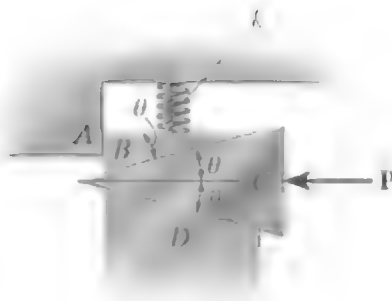
Problem 8-66

The coefficient of static friction between wedges B and C is μ_{s1} and between the surfaces of contact B and A and C and D, μ_{s2}. If the spring is compressed to a distance δ when in the position shown, determine the smallest force P needed to move wedge C to the left. Neglect the weight of the wedges.

Given:

$\mu_{s1} = 0.6 \qquad \theta = 15 \text{ deg}$

$\mu_{s2} = 0.4 \qquad k = 500 \frac{\text{N}}{\text{m}}$

$\delta = 200 \text{ mm}$

Solution:

The initial guesses:

$N_{AB} = 10 \text{ N} \qquad N_{BC} = 20 \text{ N} \qquad N_{CD} = 30 \text{ N} \qquad P = 40 \text{ N}$

Given

Wedge B:

$$N_{AB} - \mu_{s1} N_{BC} \cos(\theta) - N_{BC} \sin(\theta) = 0$$

$$N_{BC} \cos(\theta) - \mu_{s1} N_{BC} \sin(\theta) - \mu_{s2} N_{AB} - k\delta = 0$$

Wedge C:

$$N_{CD} \cos(\theta) - \mu_{s2} N_{CD} \sin(\theta) + \mu_{s1} N_{BC} \sin(\theta) - N_{BC} \cos(\theta) = 0$$

$$N_{CD} \sin(\theta) + \mu_{s2} N_{CD} \cos(\theta) + N_{BC} \sin(\theta) + \mu_{s1} N_{BC} \cos(\theta) - P = 0$$

$$\begin{pmatrix} N_{AB} \\ N_{BC} \\ N_{CD} \\ P \end{pmatrix} = \text{Find}(N_{AB}, N_{BC}, N_{CD}, P) \qquad \begin{pmatrix} N_{AB} \\ N_{BC} \\ N_{CD} \end{pmatrix} = \begin{pmatrix} 176.39 \\ 210.40 \\ 197.77 \end{pmatrix} \text{N} \quad P = 303.99 \text{ N}$$

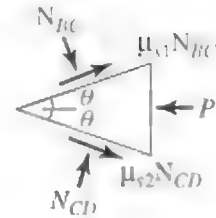
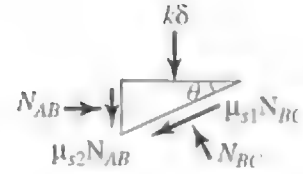
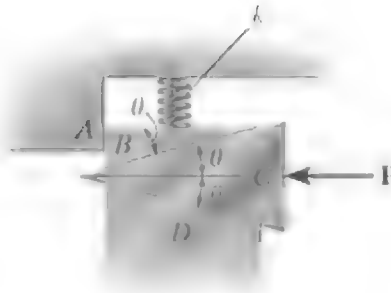
Problem 8-67

The coefficient of static friction between the wedges B and C is μ_{s1} and between the surfaces of contact B and A and C and D, μ_{s2}. Determine the smalles allowable compression of the spring δ without causing wedge C to move to the left. Neglect the weight of the wedges.

Given:

$\mu_{s1} = 0.6 \qquad \theta = 15 \text{ deg}$

$\mu_{s2} = 0.4 \qquad k = 500 \frac{\text{N}}{\text{m}}$

$P = 50 \text{ N}$

Solution:

The initial guesses:

$N_{AB} = 10 \text{ N} \quad N_{BC} = 20 \text{ N} \quad N_{CD} = 30 \text{ N} \quad \delta = 10 \text{ mm}$

Given

Wedge B:

$$N_{AB} - \mu_{s1} N_{BC}\cos(\theta) - N_{BC}\sin(\theta) = 0$$

$$N_{BC}\cos(\theta) - \mu_{s1} N_{BC}\sin(\theta) - \mu_{s2} N_{AB} - k\delta = 0$$

Wedge C:

$$N_{CD}\cos(\theta) - \mu_{s2} N_{CD}\sin(\theta) + \mu_{s1} N_{BC}\sin(\theta) - N_{BC}\cos(\theta) = 0$$

$$N_{CD}\sin(\theta) + \mu_{s2} N_{CD}\cos(\theta) + N_{BC}\sin(\theta) + \mu_{s1} N_{BC}\cos(\theta) - P = 0$$

$$\begin{pmatrix} N_{AB} \\ N_{BC} \\ N_{CD} \\ \delta \end{pmatrix} = \text{Find}(N_{AB}, N_{BC}, N_{CD}, \delta) \qquad \begin{pmatrix} N_{AB} \\ N_{BC} \\ N_{CD} \end{pmatrix} = \begin{pmatrix} 29.01 \\ 34.61 \\ 32.53 \end{pmatrix} \text{N} \qquad \delta = 32.90 \text{ mm}$$

Problem 8-68

The wedge blocks are used to hold the specimen in a tension testing machine. Determine the design angle θ of the wedges so that the specimen will not slip regardless of the applied load. The coefficients of static friction are μ_A at A and μ_B at B. Neglect the weight of the blocks.

Given:

$\mu_A = 0.1$

$\mu_B = 0.6$

Solution:

Specimen:

$+\uparrow \Sigma F_y = 0; \quad 2F_B - P = 0 \qquad F_B = \frac{P}{2}$

Wedge:

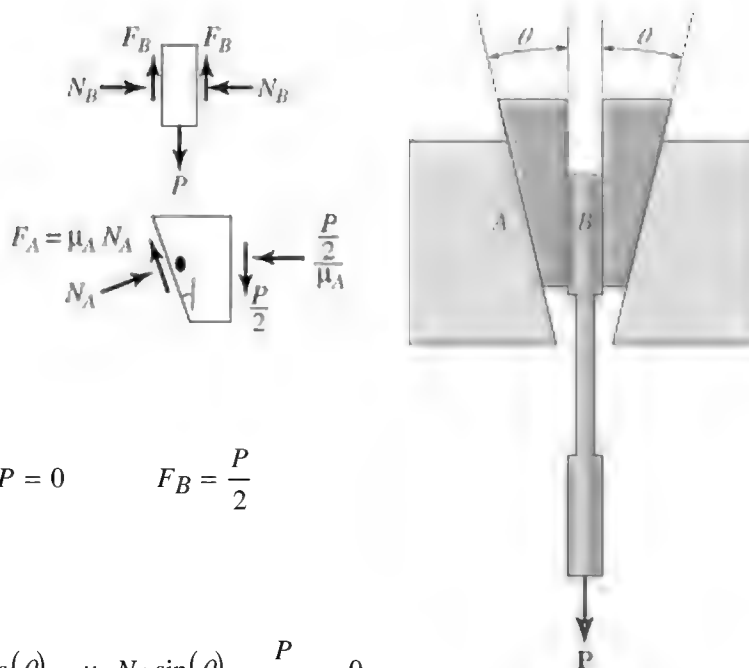
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_A \cos(\theta) - \mu_A N_A \sin(\theta) - \frac{P}{2\mu_B} = 0$

$+\uparrow \Sigma F_y = 0; \quad \mu_A N_A \cos(\theta) + N_A \sin(\theta) - \frac{P}{2} = 0$

If we eliminate P we have

$$\mu_B(N_A \cos(\theta) - \mu_A N_A \sin(\theta)) = \mu_A N_A \cos(\theta) + N_A \sin(\theta)$$

$$(1 + \mu_A\mu_B)\sin(\theta) = (\mu_B - \mu_A)\cos(\theta)$$

$$\theta = \text{atan}\left(\frac{\mu_B - \mu_A}{1 + \mu_A\mu_B}\right) \qquad \theta = 25.3\,\text{deg}$$

Problem 8-69

The wedge is used to level the member. Determine the reversed horizontal force -**P** that must be applied to pull the wedge out to the left. The coefficient of static friction between the wedge and the two surfaces of contact is μ_s. Neglect the weight of the wedge.

Units Used:

$kN = 10^3\ N$

Given:

$\mu_s = 0.15$

$\theta = 5\ deg$

$a = 600\ mm$

$b = 500\ mm$

$c = 250\ mm$

$d = 3$

$e = 4$

$F_1 = 8\ kN$

$F_2 = 15\ kN$

Solution:

Guesses

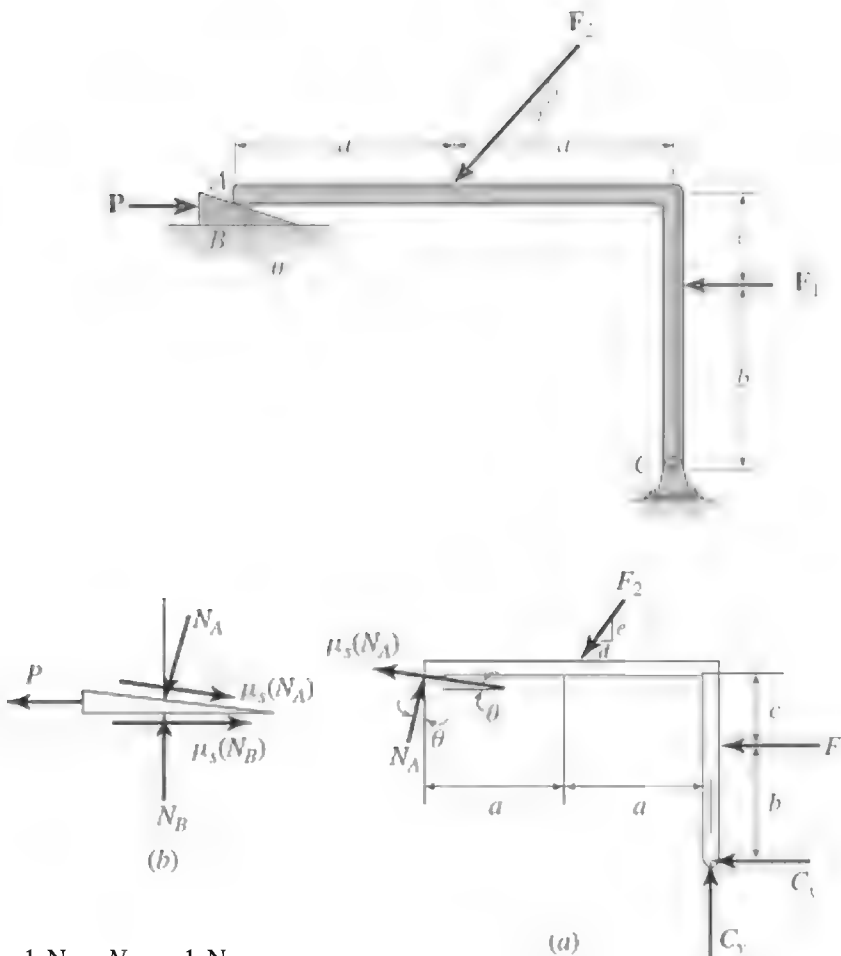
$P = 1\ N \quad N_A = 1\ N \quad N_B = 1\ N$

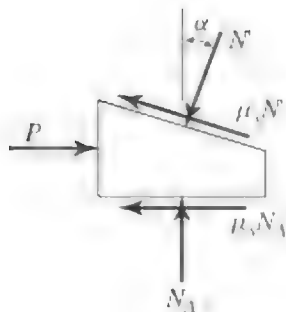
Given

$$-P + \mu_s N_B + \mu_s N_A \cos(\theta) - N_A \sin(\theta) = 0$$

$$N_B - N_A \cos(\theta) - \mu_s N_A \sin(\theta) = 0$$

$$F_1 b + \left(\frac{e}{\sqrt{e^2 + d^2}}\right) F_2 a + \left(\frac{d}{\sqrt{e^2 + d^2}}\right) F_2 (b + c) - \left(N_A \cos(\theta) + \mu_s N_A \sin(\theta)\right) 2a \ ... = 0$$
$$+ \left(\mu_s N_A \cos(\theta) - N_A \sin(\theta)\right)(b + c)$$

$$\begin{pmatrix} P \\ N_A \\ N_B \end{pmatrix} = \text{Find}(P, N_A, N_B) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 15.42 \\ 15.56 \end{pmatrix} kN \qquad P = 3.29\ kN$$

Problem 8-70

If the coefficient of static friction between all the surfaces of contact is μ_s, determine the force **P** that must be applied to the wedge in order to lift the brace that supports the load **F**.

Solution:

System:

$\Sigma F_x = 0;\qquad P - N_B - \mu_s N_A = 0 \qquad (1)$

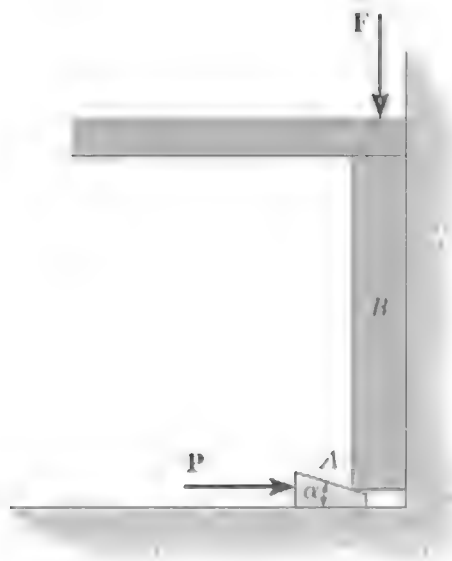
$\Sigma F_y = 0;\qquad N_A - \mu_s N_B - F = 0 \qquad (2)$

Wedge A:

$\Sigma F_x = 0;\qquad P - \mu_s N_A - \mu_s N' \cos(\alpha) - N' \sin(\alpha) = 0 \qquad (3)$

$\Sigma F_y = 0;\qquad N_A - N' \cos(\alpha) + \mu_s N' \sin(\alpha) = 0 \qquad (4)$

From Eqs. (3) and (4):

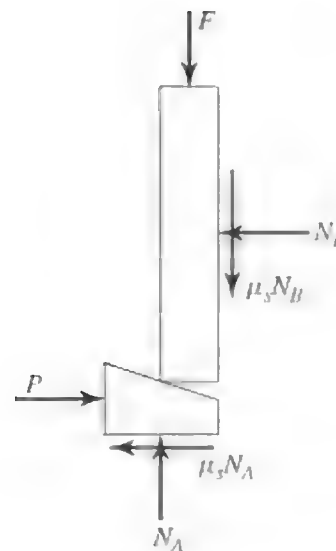
$$N' = \frac{P - \mu_s N_A}{\mu_s \cos(\alpha) + \sin(\alpha)} \qquad (5)$$

$$N_A = \frac{P(1 - \mu_s \tan(\alpha))}{2\mu_s + (1 - \mu_s^2)\tan(\alpha)} \qquad (6)$$

From Eqs. (1) and (2):

$$N_B = \frac{P - \mu_s N_A}{\mu_s}$$

$$P = \frac{(1 + \mu_s^2)N_A - F}{\mu_s}$$

Combining Eqs. (5) and (6) :

$$P = F\left[\frac{(1 - \mu_s^2)\tan(\alpha) + 2\mu_s}{1 - 2\mu_s \tan(\alpha) - \mu_s^2}\right]$$

Problem 8-71

The column is used to support the upper floor. If a force **F** is applied perpendicular to the handle to tighten the screw, determine the compressive force in the column. The square-threaded screw on the jack has a coefficient of static friction μ_s, mean diameter d, and a lead h.

Units Used:

$kN = 10^3\ N$

Given:

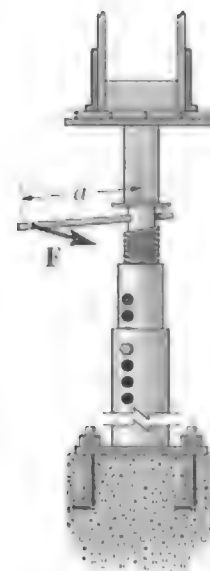
$F = 80\ N \qquad \mu_s = 0.4 \qquad d = 25\ mm$

$h = 3\ mm \qquad a = 0.5\ m$

Solution:

$\phi_s = \text{atan}(\mu_s) \qquad \phi_s = 21.80\ \text{deg}$

$\theta_p = \text{atan}\left(\frac{h}{\pi\, d}\right) \qquad \theta_p = 2.187\ \text{deg}$

$$M = F\,a = W\left(\frac{d}{2}\right)\tan(\phi_s + \theta_p)$$

$$W = 2F\left(\frac{a}{d\tan(\phi_s + \theta_p)}\right) \qquad W = 7.19\,\text{kN}$$

Problem 8-72

The column is used to support the upper floor. If the force **F** is removed from the handle of the jack, determine if the screw is self-locking.. The square-threaded screw on the jack has a coefficient of static friction μ_s, mean diameter d, and a lead h.

Given:

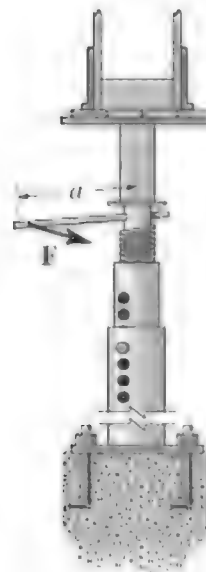
$F = 80\ \text{N} \quad \mu_s = 0.4 \quad d = 25\ \text{mm}$

$h = 3\ \text{mm} \quad a = 0.5\ \text{m}$

Solution:

$$\phi_s = \text{atan}(\mu_s) \qquad \phi_s = 21.8\,\text{deg}$$

$$\theta_p = \text{atan}\left(\frac{h}{\pi d}\right) \qquad \theta_p = 2.19\,\text{deg}$$

Since $\phi_s = 21.8\,\text{deg} > \ \theta_p = 2.19\,\text{deg}$, screw is self locking.

Problem 8-73

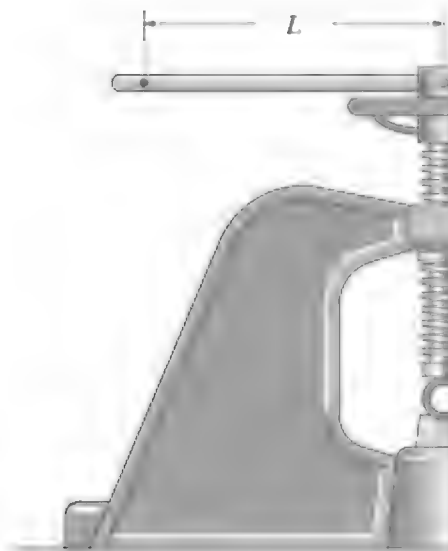
The vise is used to grip the pipe. If a horizontal force $\mathbf{F_1}$ is applied perpendicular to the end of the handle of length l, determine the compressive force **F** developed in the pipe. The square threads have a mean diameter d and a lead a. How much force must be applied perpendicular to the handle to loosen the vise?

Given:

$F_1 = 25\ \text{lb}$

$d = 1.5\ \text{in}$

$\mu_s = 0.3$

$L = 10\ \text{in}$

$a = 0.2 \text{ in}$

Solution:

$$r = \frac{d}{2}$$

$$\theta = \text{atan}\left(\frac{a}{2\pi r}\right) \qquad \theta = 2.43 \text{ deg}$$

$$\phi = \text{atan}(\mu_s) \qquad \phi = 16.70 \text{ deg}$$

$$F_1 L = F r \tan(\theta + \phi)$$

$$F = F_1 \left(\frac{L}{r \tan(\theta + \phi)}\right) \qquad F = 961 \text{ lb}$$

To loosen screw,

$$P L = F r \tan(\phi - \theta)$$

$$P = F r \frac{\tan(\phi - \theta)}{L} \qquad P = 18.3 \text{ lb}$$

Problem 8-74

Determine the couple forces **F** that must be applied to the handle of the machinist's vise in order to create a compressive force F_A in the block. Neglect friction at the bearing A. The guide at B is smooth so that the axial force on the screw is F_A. The single square-threaded screw has a mean radius b and a lead c, and the coefficient of static friction is μ_s.

Given:

$a = 125 \text{ mm}$

$F_A = 400 \text{ N}$

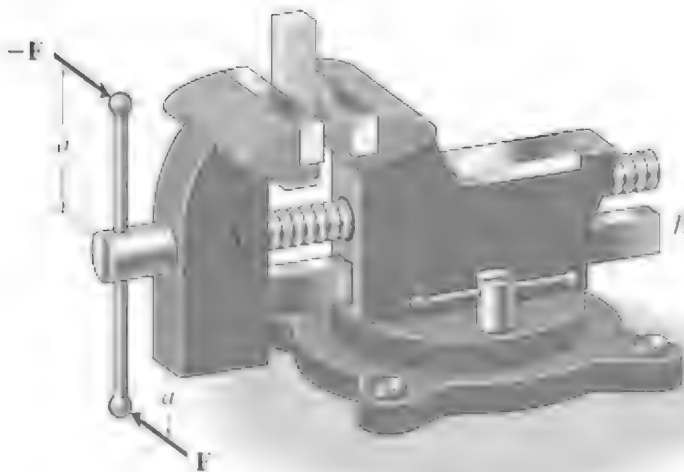
$b = 6 \text{ mm}$

$c = 8 \text{ mm}$

$\mu_s = 0.27$

Solution:

$\phi = \text{atan}(\mu_s)$ $\phi = 15.11\,\text{deg}$

$\theta = \text{atan}\left(\frac{c}{2\,\pi\,b}\right)$ $\theta = 11.98\,\text{deg}$

$F\,2\,a = F_A\,b\,\tan(\theta + \phi)$

$F = F_A\left(\frac{b}{2a}\right)\tan(\theta + \phi)$ $F = 4.91\ \text{N}$

Problem 8-75

If couple forces **F** are applied to the handle of the machinist's vise, determine the compressive force developed in the block. Neglect friction at the bearing *A*. The guide at *B* is smooth. The single square-threaded screw has a mean radius of r_1 and a lead of r_2, and the coefficient of static friction is μ_s.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$F = 35\ \text{N}$

$a = 125\ \text{mm}$

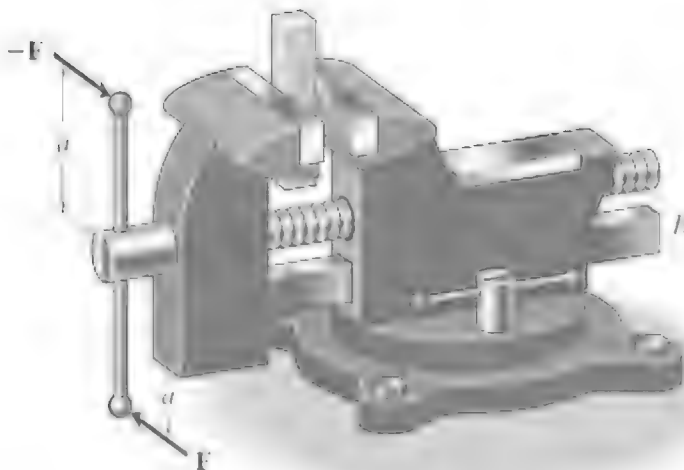
$r_1 = 6\ \text{mm}$

$r_2 = 8\ \text{mm}$

$\mu_s = 0.27$

Solution:

$\phi = \text{atan}(\mu_s)$ $\phi = 15.11\,\text{deg}$

$\theta = \text{atan}\left(\frac{r_2}{2\,\pi\,r_1}\right)$ $\theta = 11.98\,\text{deg}$

$F\,2\,a = P\,r_1\,\tan(\theta + \phi)$

$P = 2\,F\left(\frac{a}{r_1\,\tan(\theta + \phi)}\right)$ $P = 2.85\,\text{kN}$

Problem 8-76

The machine part is held in place using the double-end clamp.The bolt at B has square threads with a mean radius r and a lead r_l, and the coefficient of static friction with the nut is μ_s. If a torque M is applied to the nut to tighten it, determine the normal force of the clamp at the smooth contacts A and C.

Given:

$a = 260\text{ mm}$

$b = 90\text{ mm}$

$r = 4\text{ mm}$

$r_l = 2\text{ mm}$

$\mu_s = 0.5$

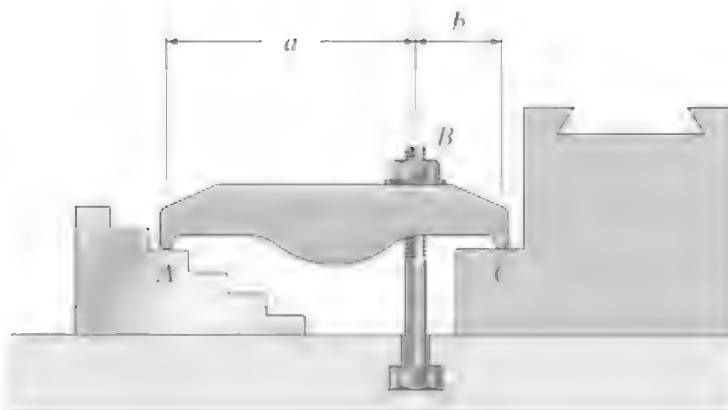
$M = 0.4\text{ N}\cdot\text{m}$

Solution:

$$\phi = \operatorname{atan}(\mu_s)$$

$$\phi = 26.57\text{ deg}$$

$$\theta = \operatorname{atan}\left(\frac{r_l}{2\pi r}\right)$$

$$\theta = 4.55\text{ deg}$$

$$M = W r \tan(\theta + \phi)$$

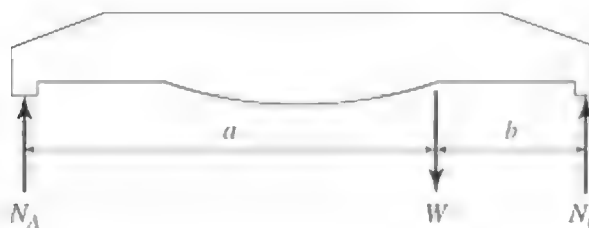
$$W = \frac{M}{r\tan(\theta + \phi)}$$

$$W = 165.67\text{ N}$$

$\Sigma M_A = 0;\quad N_C(a + b) - W a = 0$

$$N_C = W\frac{a}{a+b} \qquad N_C = 123\text{ N}$$

$\Sigma F_y = 0;\quad N_A - W + N_C = 0$

$$N_A = W - N_C \qquad N_A = 42.6\text{ N}$$

Problem 8-77

Determine the clamping force on the board A if the screw of the "C" clamp is tightened with a twist **M**. The single square-threaded screw has a mean radius r, a lead h, and the coefficient of static friction is μ_s.

Units Used:

$$kN = 10^3 \ N$$

Given:

$$M = 8 \ N \cdot m$$

$$r = 10 \ mm$$

$$h = 3 \ mm$$

$$\mu_s = 0.35$$

Solution:

$$\phi_s = \operatorname{atan}(\mu_s) \qquad \phi_s = 19.29 \ \deg$$

$$\theta_p = \operatorname{atan}\left(\frac{1}{2} \frac{h}{\pi r}\right) \qquad \theta_p = 2.734 \ \deg$$

$$M = P r \tan(\phi_s + \theta_p) \qquad P = \frac{M}{r \tan(\phi_s + \theta_p)} \qquad P = 1.98 \ kN$$

Problem 8-78

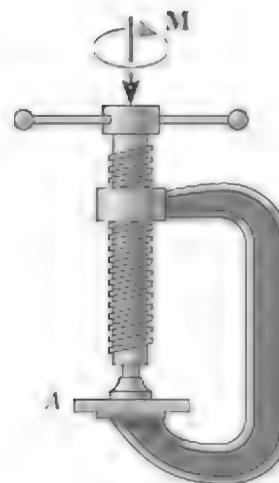
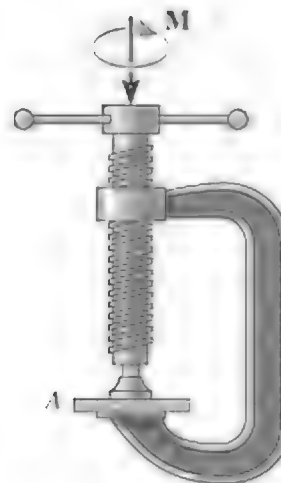
If the required clamping force at the board A is to be P, determine the torque **M** that must be applied to the handle of the "C" clamp to tighten it down. The single square-threaded screw has a mean radius r, a lead h, and the coefficient of static friction is μ_s.

Given:

$$P = 50 \ N$$

$$r = 10 \ mm$$

$$h = 3 \ mm$$

$$\mu_s = 0.35$$

Solution:

$\phi_S = \text{atan}(\mu_S)$ $\phi_S = 19.29\,\text{deg}$

$\theta_P = \text{atan}\left(\frac{1}{2}\frac{h}{\pi r}\right)$ $\theta_P = 2.73\,\text{deg}$

$M = P\ r\ \tan(\phi_S + \theta_P)$ $M = 0.202\,\text{N}\cdot\text{m}$

Problem 8-79

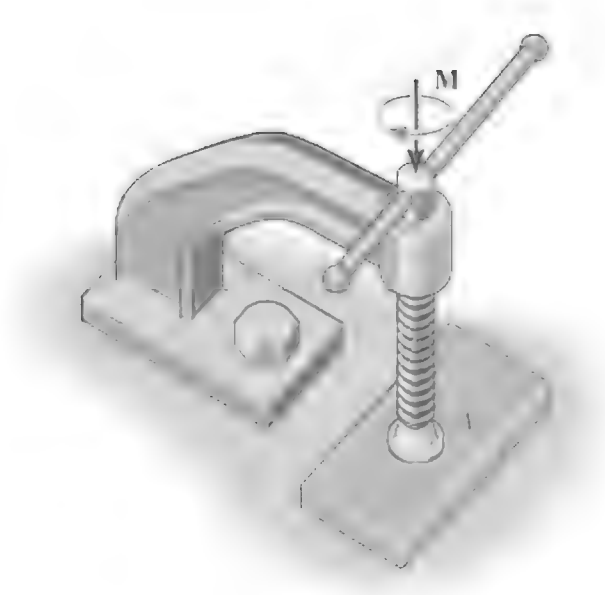
Determine the clamping force on the board at A if the screw of the hold-down clamp is tightened with a twist $\mathbf{M}$. The single square-threaded screw has a mean radius of r and a lead of r_l, and the coefficient of static friction is μ_s.

Given:

$M = 0.2\ \text{N m}$

$r = 8\ \text{mm}$

$r_l = 2\ \text{mm}$

$\mu_S = 0.38$

Solution:

$\phi = \text{atan}(\mu_S)$ $\phi = 20.81\,\text{deg}$

$\theta = \text{atan}\left(\frac{r_l}{2\pi r}\right)$ $\theta = 2.28\,\text{deg}$

$M = F\,r\tan(\theta + \phi)$

$F = \frac{M}{r\ \tan(\theta + \phi)}$ $F = 58.7\ \text{N}$

Problem 8-80

If the required clamping force at the board A is to be F, determine the torque M that must be applied to the handle of the hold-down clamp to tighten it down.The single square-threaded screw has a mean radius r and a lead r_l, and the coefficient of static friction is μ_s.

Given:

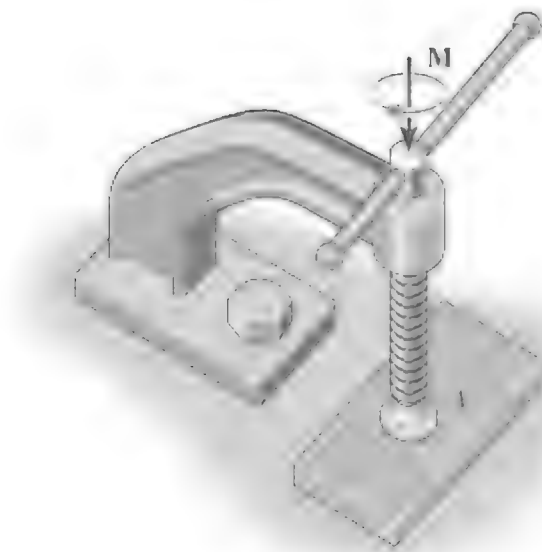
$F = 70\ \text{N}$

$r = 8\ \text{mm}$

$r_l = 2\ \text{mm}$

$\mu_s = 0.38$

Solution:

$\phi = \text{atan}(\mu_s)$ $\quad\phi = 20.81\ \text{deg}$

$\theta = \text{atan}\left(\dfrac{r_l}{2\pi r}\right)$ $\quad\theta = 2.2785\ \text{deg}$

$M = F\,r\tan(\theta + \phi)$ $\quad M = 0.24\ \text{N}\cdot\text{m}$

Problem 8-81

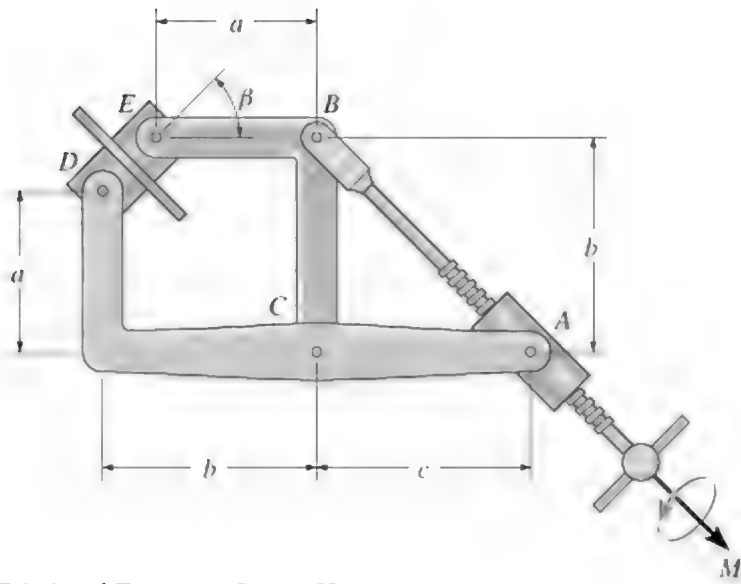
The fixture clamp consist of a square-threaded screw having a coefficient of static friction μ_s mean diameter d, and a lead h. The five points indicated are pin connections. Determine the clamping force at the smooth blocks D and E when a torque M is applied to the handle of the screw.

Given:

$\mu_s = 0.3$ $\quad a = 30\ \text{mm}$

$d = 3\ \text{mm}$ $\quad b = 40\ \text{mm}$

$h = 1\ \text{mm}$ $\quad c = 40\ \text{mm}$

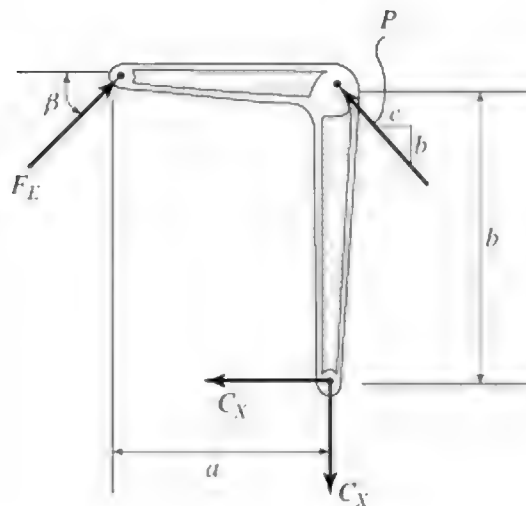
$M = 0.08\ \text{N}\cdot\text{m}$ $\quad \beta = 45\ \text{deg}$

Frictional Forces on Screw: Here

$$\theta = \text{atan}\left[\frac{h}{2\ \pi\left(\frac{d}{2}\right)}\right] \qquad \theta = 6.06\,\text{deg}$$

$$\phi_s = \text{atan}(\mu_s) \qquad \phi_s = 16.70\,\text{deg}$$

Applying Eq.8-3, we have

$$M = P\left(\frac{d}{2}\right)\tan(\theta + \phi_s)$$

$$P = 2\left(\frac{M}{d\ \tan(\theta + \phi_s)}\right) \qquad P = 127.15\ \text{N}$$

Note since $\phi_s = 16.70\,\text{deg} > \theta = 6.06\,\text{deg}$, the screw is self-locking. It will not unscrew even if the moment M is removed.

Equations of Equilibrium and Friction:

$\Sigma M_c = 0;$

$$\left(\frac{c}{\sqrt{b^2 + c^2}}\right)P\,b - F_E\cos(\beta)b - F_E\sin(\beta)a = 0$$

$$F_E = \frac{P c b}{\sqrt{b^2 + c^2}\left(\cos(\beta) b + \sin(\beta) a\right)} \qquad F_E = 72.7 \text{ N}$$

The equilibrium of clamped block requires that

$$F_D = F_E \qquad F_D = 72.7 \text{ N}$$

Problem 8-82

The clamp provides pressure from several directions on the edges of the board. If the square-threaded screw has a lead h, radius r, and the coefficient of static friction is μ_s, determine the horizontal force developed on the board at A and the vertical forces developed at B and C if a torque **M** is applied to the handle to tighten it further. The blocks at B and C are pin-connected to the board.

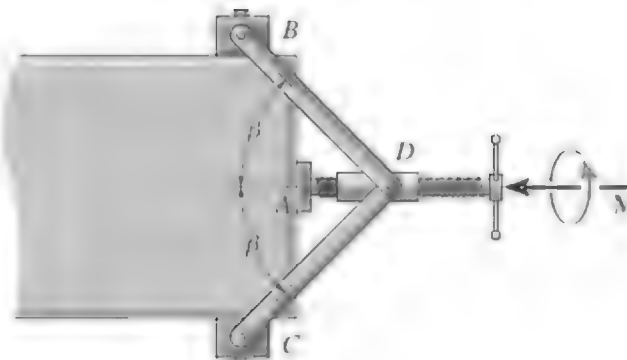
Given:

$h = 3$ mm

$r = 10$ mm

$\mu_s = 0.4$

$M = 1.5$ N·m

$\beta = 45$ deg

Solution:

$\phi_s = \operatorname{atan}(\mu_s) \qquad \phi_s = 21.801$ deg

$\theta = \operatorname{atan}\left(\frac{h}{2\pi r}\right) \qquad \theta = 2.734$ deg

$M = A_x r \tan(\phi_s + \theta) \qquad A_x = \frac{M}{r \tan(\phi_s + \theta)} \qquad A_x = 329$ N

$\overset{+}{\rightarrow} \Sigma F_x = 0;$

$A_x - 2T\cos(\beta) = 0 \qquad T = \frac{1}{2}\left(\frac{A_x}{\cos(\beta)}\right) \qquad T = 232.36$ N

$C_y = T\sin(\beta) \qquad B_y = C_y \qquad \begin{pmatrix} B_y \\ C_y \end{pmatrix} = \begin{pmatrix} 164.3 \\ 164.3 \end{pmatrix}$ N

Problem 8-83

The two blocks under the double wedge are brought together using a left and right square-threaded screw. If the mean diameter is d, the lead is r_l, and the coefficient of static friction is μ_s, determine the torque needed to draw the blocks together. The coefficient of static friction between each block and its surfaces of contact is μ'_s.

Units Used:

$$kN = 10^3\ N$$

Given:

$$F = 5\ kN$$

$$\theta = 20\ deg$$

$$d = 20\ mm$$

$$r_l = 5\ mm$$

$$\mu_s = 0.4$$

$$\mu'_s = 0.4$$

Solution:

Top block:

$$-F + 2N_1 \cos(\theta) - 2\mu'_s N_1 \sin(\theta) = 0$$

$$N_1 = \frac{F}{2(\cos(\theta) - \mu'_s \sin(\theta))}$$

$$N_1 = 3.1138\ kN$$

Bottom block:

$$N' - N_1 \cos(\theta) + \mu'_s N_1 \sin(\theta) = 0$$

$$N' = N_1 \cos(\theta) - \mu'_s N_1 \sin(\theta)$$

$$N' = 2.50\ kN$$

$$-N_1 \sin(\theta) - \mu'_s N_1 \cos(\theta) + T - \mu'_s N' = 0$$

$$T = N_1 \sin(\theta) + \mu'_s N_1 \cos(\theta) + \mu'_s N'$$

$$T = 3.2354\ kN$$

$$\phi = \text{atan}(\mu_s) \qquad \phi = 21.80\,\text{deg}$$

$$\theta = \text{atan}\left(\frac{r_l}{\pi d}\right) \qquad \theta = 4.55\,\text{deg}$$

Since there are two blocks,

$$M = 2T\frac{d}{2}\tan(\theta + \phi) \qquad M = 32\,\text{N}\cdot\text{m}$$

Problem 8-84

The two blocks under the double wedge are brought together using a left and right square-threaded screw. If the mean diameter is d, the lead is r_l, and the coefficient of static friction is μ_s, determine the torque needed to spread the blocks apart. The coefficient of static friction between each block and its surfaces of contact is μ'_s.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$F = 5\ \text{kN}$$

$$\theta = 20\ \text{deg}$$

$$d = 20\ \text{mm}$$

$$r_l = 5\ \text{mm}$$

$$\mu_s = 0.4$$

$$\mu'_s = 0.4$$

Solution:

Top block:

$$-F + 2N_1\cos(\theta) + 2\mu'_s N_1 \sin(\theta) = 0$$

$$N_1 = \frac{F}{2(\cos(\theta) + \mu'_s\sin(\theta))}$$

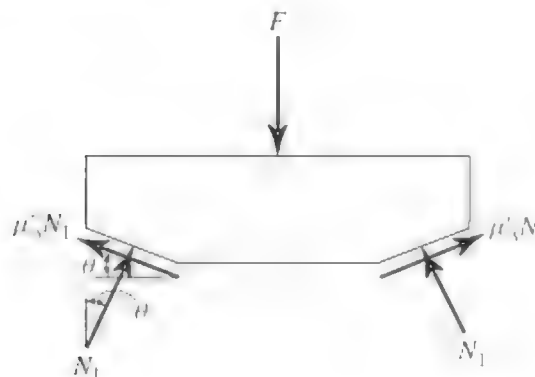
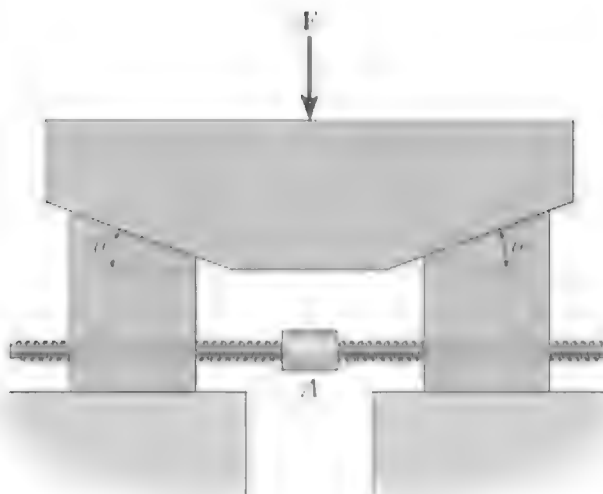
$$N_1 = 2.3223\,\text{kN}$$

Bottom block :

$$N' - N_1 \cos(\theta) - \mu'_s N_1 \sin(\theta) = 0$$

$$N' = N_1 \cos(\theta) + \mu'_s N_1 \sin(\theta)$$

$$N' = 2.50 \text{ kN}$$

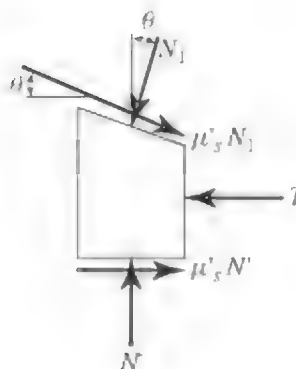
$$-N_1 \sin(\theta) + \mu'_s N_1 \cos(\theta) - T + \mu'_s N' = 0$$

$$T = -N_1 \sin(\theta) + \mu'_s N_1 \cos(\theta) + \mu'_s N'$$

$$T = 1.0786 \text{ kN}$$

$$\phi = \operatorname{atan}(\mu_s) \qquad \phi = 21.80 \text{ deg}$$

$$\theta = \operatorname{atan}\left(\frac{r_l}{\pi d}\right) \qquad \theta = 4.55 \text{ deg}$$

Since there are two blocks,

$$M = 2T\frac{d}{2}\tan(\phi - \theta) \qquad M = 6.7 \text{ N}\cdot\text{m}$$

Problem 8-85

The cord supporting the cylinder of mass M passes around three pegs, A, B, C, where the coefficient of friction is μ_s. Determine the range of values for the magnitude of the horizontal force **P** for which the cylinder will not move up or down.

Given:

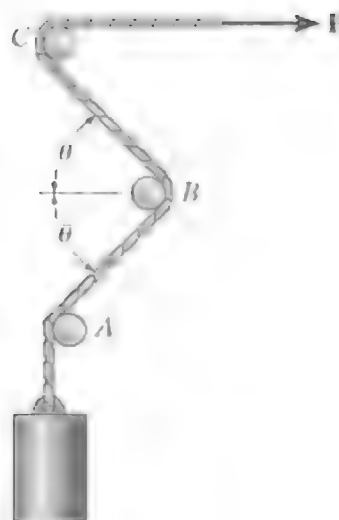
$$M = 6 \text{ kg}$$

$$\theta = 45 \text{ deg}$$

$$\mu_s = 0.2$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution:

Total angle $\quad \beta = \frac{5}{2}\pi - 4\theta \qquad \beta = 270.00 \text{ deg}$

Forces $P_{min} = M g e^{-\mu_s \beta}$ $P_{max} = M g e^{\mu_s \beta}$

Answer $P_{min} = 15.9 \text{ N} < P < P_{max} = 217.4 \text{ N}$

Problem 8-86

The truck, which has mass m_t, is to be lowered down the slope by a rope that is wrapped around a tree. If the wheels are free to roll and the man at A can resist a pull P, determine the minimum number of turns the rope should be wrapped around the tree to lower the truck at a constant speed. The coefficient of kinetic friction between the tree and rope is μ_k.

Units Used:

$\text{Mg} = 1000 \text{ kg}$

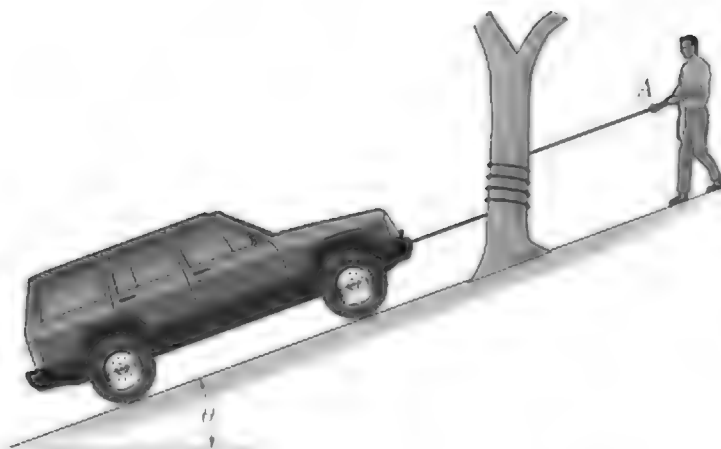
Given:

$m_t = 3.4 \text{ Mg}$

$P = 300 \text{ N}$

$\theta = 20 \text{ deg}$

$\mu_k = 0.3$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

$\Sigma F_x = 0;$

$T_2 - m_t g \sin(\theta) = 0$ $T_2 = m_t g \sin(\theta)$ $T_2 = 11407.74 \text{ N}$

$T_2 = P e^{\mu_k \beta}$ $\beta = \dfrac{\ln\left(\frac{T_2}{P}\right)}{\mu_k}$ $\beta = 694.86 \text{ deg}$

Use $\text{ceil}\left(\frac{\beta}{360 \text{ deg}}\right) = 2.00$ turns

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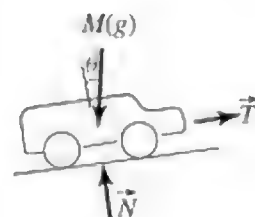

Problem 8-87

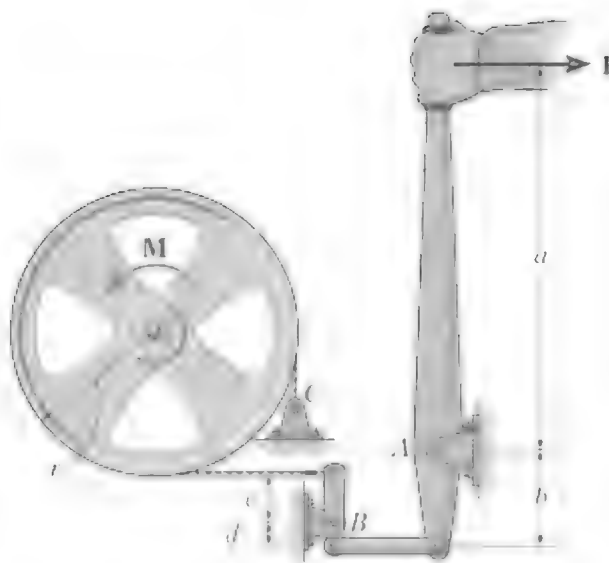
The wheel is subjected to a torque **M**. If the coefficient of kinetic friction between the band brake and the rim of the wheel is μ_k, determine the smallest horizontal force **P** that must be applied to the lever to stop the wheel.

Given:

$a = 400\text{ mm}$ $d = 25\text{ mm}$

$b = 100\text{ mm}$ $r = 150\text{ mm}$

$c = 50\text{ mm}$ $M = 50\text{ N}\cdot\text{m}$

$\mu_k = 0.3$

Solution:

Initial guesses: $T_1 = 5\text{ N}$ $T_2 = 10\text{ N}$

Given

Wheel:

$\Sigma M_0 = 0;$ $-T_2 r + T_1 r + M = 0$ $T_2 = T_1 e^{\mu_k\left(\frac{3\pi}{2}\right)}$

$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \text{Find}(T_1, T_2)$ $T_1 = 54.66\text{ N}$

Link:

$\Sigma M_B = 0;$ $T_1 c - F d = 0$ $F = T_1\left(\frac{c}{d}\right)$ $F = 109.32\text{ N}$

Lever:

$\Sigma M_A = 0;$ $-P a + F b = 0$ $P = F\left(\frac{b}{a}\right)$ $P = 27.3\text{ N}$

Problem 8-88

A cylinder A has a mass M. Determine the smallest force **P** applied to the handle of the lever required for equilibrium. The coefficient of static friction between the belt and the wheel is μ_s. The drum is pin connected at its center, B.

Given:

$M = 75$ kg

$a = 700$ mm

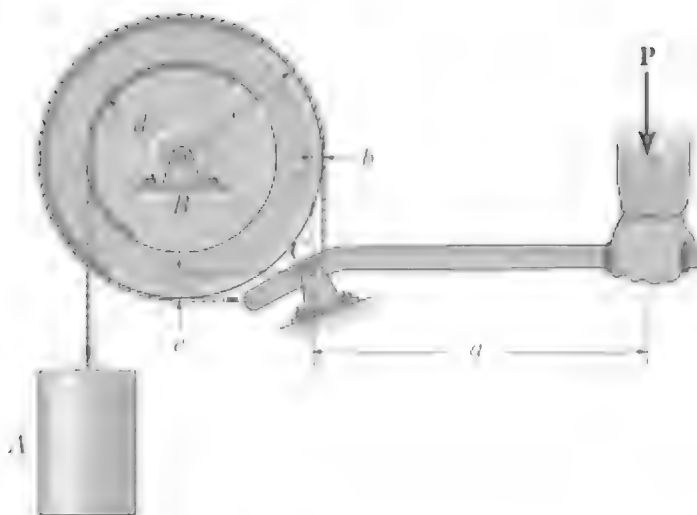
$b = 25$ mm

$c = 300$ mm

$d = 200$ mm

$e_1 = 60$ mm

$\mu_s = 0.3$

$e = 2.718$

Solution:

Initial guesses: $T_1 = 1$ N $T_2 = 1$ N $P = 1$ N

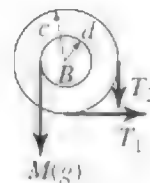
Given

Drum:

$$T_2 = T_1 e^{\mu_s \left(\frac{3\pi}{2}\right)}$$

$$-T_2 c + T_1 c + M g d = 0$$

Lever:

$$-T_1 e_1 + T_2 b - P a = 0$$

$$\begin{pmatrix} T_1 \\ T_2 \\ P \end{pmatrix} = \text{Find}(T_1, T_2, P) \qquad \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 157.7 \\ 648.2 \end{pmatrix} \text{ N} \qquad P = 9.63 \text{ N}$$

Problem 8-89

Determine the largest mass of cylinder A that can be supported from the drum if a force **P** is applied to the handle of the lever. The coefficient of static friction between the belt and the wheel is μ_s. The drum is pin supported at its center, B.

Given:

$P = 20$ N

$a = 700\text{ mm}$

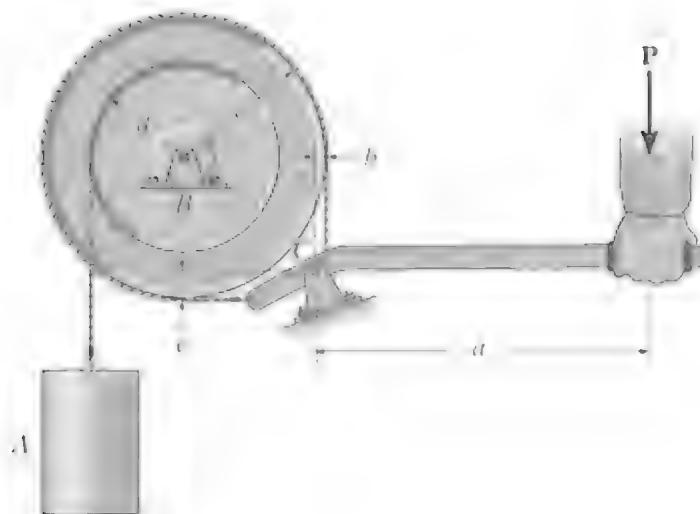
$b = 25\text{ mm}$

$c = 300\text{ mm}$

$d = 200\text{ mm}$

$e_1 = 60\text{ mm}$

$\mu_s = 0.3$

$e = 2.718$

Solution:

Initial guesses: $T_1 = 1\text{ N}$ $T_2 = 1\text{ N}$ $M = 1\text{ kg}$

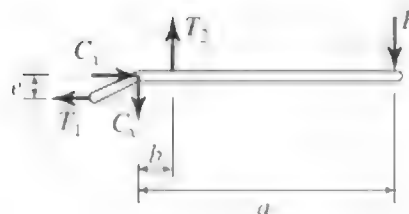
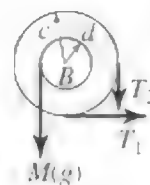
Given

Drum:

$$T_2 = T_1 e^{\mu_s\left(\frac{3\pi}{2}\right)}$$

$$-T_2 c + T_1 c + M g d = 0$$

Lever:

$$-T_1 e_1 + T_2 b - P a = 0$$

$$\begin{pmatrix} T_1 \\ T_2 \\ M \end{pmatrix} = \text{Find}(T_1, T_2, M) \qquad \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 327.4 \\ 1.3 \times 10^3 \end{pmatrix} \text{N} \qquad M = 155.7\text{ kg}$$

Problem 8-90

The uniform bar *AB* is supported by a rope that passes over a frictionless pulley at *C* and a fixed peg at *D*. If the coefficient of static friction between the rope and the peg is μ_D, determine the smallest distance *x* from the end of the bar at which a force **F** may be placed and not cause the bar to move.

Given:

$F = 20\text{ N}$ $\qquad a = 1\text{ m}$

$\mu_D = 0.3$

Solution:

Initial guesses:

$T_A = 5 \text{ N}$ $\quad T_B = 10 \text{ N}$ $\quad x = 10 \text{ m}$

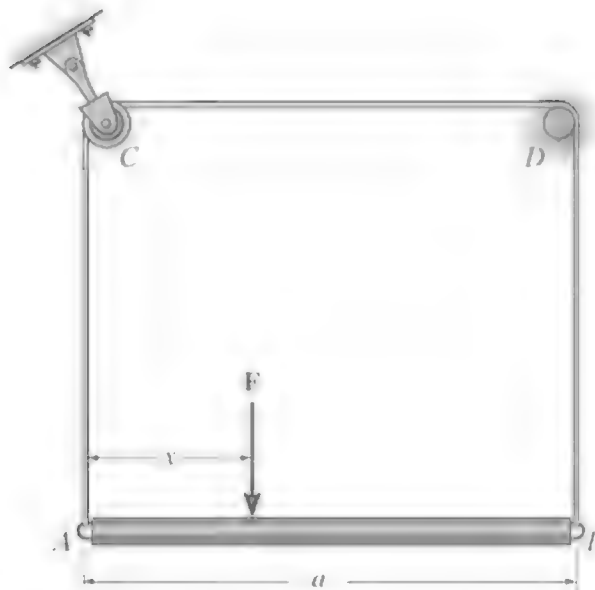
Given

$\Sigma M_A = 0;$ $\quad -F x + T_B a = 0$

$\Sigma F_y = 0;$ $\quad T_A + T_B - F = 0$

$$T_A = T_B e^{\mu_D \left(\frac{\pi}{2}\right)}$$

$$\begin{pmatrix} T_A \\ T_B \\ x \end{pmatrix} = \text{Find}(T_A, T_B, x)$$

$x = 0.38 \text{ m}$

Problem 8-91

Determine the smallest lever force **P** needed to prevent the wheel from rotating if it is subjected to a torque **M**. The coefficient of static friction between the belt and the wheel is μ_s. The wheel is pin-connected at its center, B.

Given:

$M = 250 \text{ N m}$

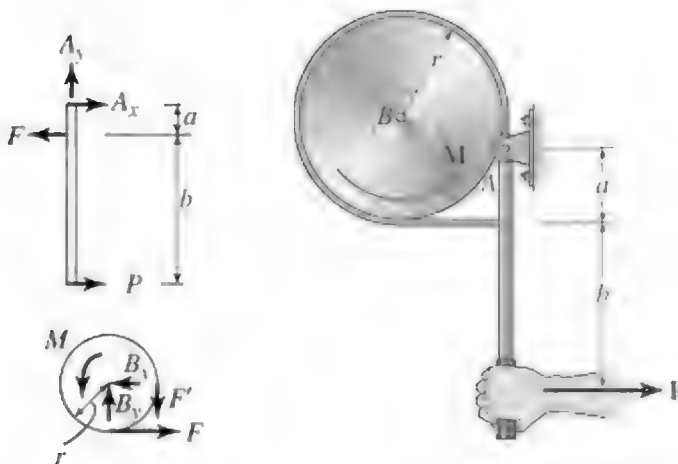
$\mu_s = 0.3$

$r = 400 \text{ mm}$

$a = 200 \text{ mm}$

$b = 750 \text{ mm}$

Solution:

$\Sigma M_A = 0;$ $\quad -F a + P(a + b) = 0$

$$F = P\left(\frac{a+b}{a}\right)$$

$$\beta = \frac{3\pi}{2}$$

$$F' = F e^{\mu_s \beta}$$

$\Sigma M_B = 0;$ $$-P\left(\frac{a+b}{a}\right)e^{\mu_s \beta} r + M + P\left(\frac{a+b}{a}\right)r = 0$$

$$P = \frac{M a}{(a+b) r \left(e^{\mu_s \beta} - 1\right)}$$ $P = 42.3\ \text{N}$

Problem 8-92

Determine the torque M that can be resisted by the band brake if a force **P** is applied to the handle of the lever. The coefficient of static friction between the belt and the wheel is μ_s. The wheel is pin-connected at its center, B.

Given:

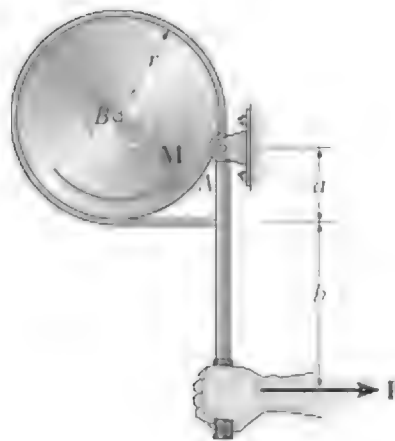
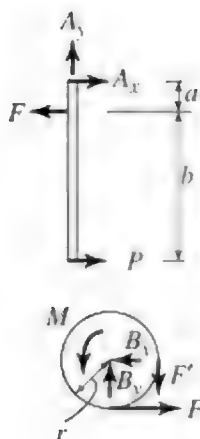
$P = 30\ \text{N}$

$\mu_s = 0.3$

$r = 400\ \text{mm}$

$a = 200\ \text{mm}$

$b = 750\ \text{mm}$

Solution:

$\Sigma M_A = 0;$ $$-F a + P(a+b) = 0$$

$$F = P\left(\frac{a+b}{a}\right)$$ $F = 142.5\ \text{N}$

$$F' = F e^{\mu_s \frac{3\pi}{2}}$$ $F' = 585.8\ \text{N}$

$\Sigma M_B = 0; \qquad -F' r + F r + M = 0$

$M = F' r - F r \qquad M = 177\,\text{N}\cdot\text{m}$

Problem 8-93

Blocks A and B weigh W_A and W_B and respectively. Using the coefficients of static friction indicated, determine the greatest weight of block D without causing motion.

Given:

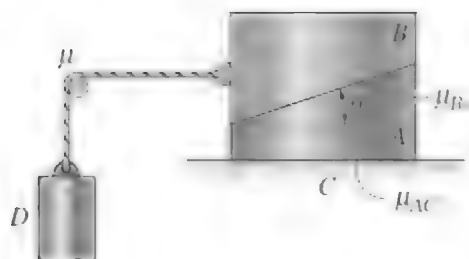
$W_A = 50\text{ lb}$

$W_B = 30\text{ lb}$

$\mu = 0.5$

$\mu_{BA} = 0.6$

$\mu_{AC} = 0.4$

$\theta = 20\text{ deg}$

Assume that B slips on A, but A does not move.

Guesses $W_D = 1\text{ lb} \quad T_B = 1\text{ lb}$

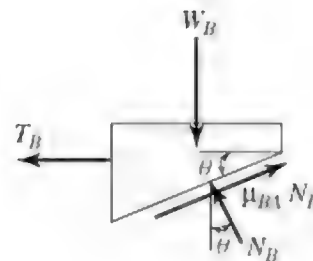
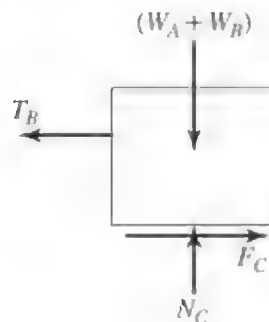
$N_B = 1\text{ lb} \quad N_C = 1\text{ lb} \quad F_C = 1\text{ lb}$

Given $W_D = T_B e^{\mu \frac{\pi}{2}}$

$-T_B + F_C = 0$

$N_C - W_A - W_B = 0$

$-T_B + \mu_{BA} N_B \cos(\theta) - N_B \sin(\theta) = 0$

$N_B \cos(\theta) + \mu_{BA} N_B \sin(\theta) - W_B = 0$

$$\begin{pmatrix} W_D \\ T_B \\ N_B \\ N_C \\ F_C \end{pmatrix} = \text{Find}(W_D, T_B, N_B, N_C, F_C) \qquad \begin{pmatrix} W_D \\ T_B \\ N_B \\ N_C \\ F_C \end{pmatrix} = \begin{pmatrix} 12.75 \\ 5.81 \\ 26.20 \\ 80.00 \\ 5.81 \end{pmatrix} \text{lb}$$

Now check the assumption that A does not move

$$F_{Cmax} = \mu_{AC} N_C \qquad F_{Cmax} = 32.00 \text{ lb}$$

Since $F_C = 5.81 \text{ lb} < F_{Cmax} = 32.00 \text{ lb}$ then our assumption is good. $W_D = 12.75 \text{ lb}$

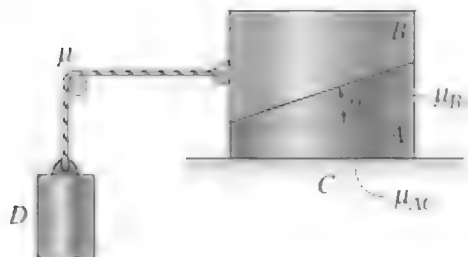
Problem 8-94

Blocks *A* and *B* have weight *W*, and *D* weighs W_D. Using the coefficients of static friction indicated, determine the frictional force between blocks *A* and *B* and between block *A* and the floor *C*.

Given:

$W = 75 \text{ lb}$ $\quad \mu_{BA} = 0.6$

$W_D = 30 \text{ lb}$ $\quad \mu_{AC} = 0.4$

$\mu = 0.5$ $\quad \theta = 20 \text{ deg}$

Solution:

$$W_D = T_B e^{\mu \frac{\pi}{2}} \qquad T_B = \frac{W_D}{e^{\frac{1}{2}\mu\pi}} \qquad T_B = 13.679 \text{ lb}$$

$$F_C = T_B \qquad F_C = 13.68 \text{ lb}$$

Check $\quad N_C - 2W = 0 \qquad N_C = 2W \qquad N_C = 150.00 \text{ lb}$

$$F_{Cmax} = \mu_{AC} N_C \qquad F_{Cmax} = 60.00 \text{ lb}$$

Since $F_C = 13.68 \text{ lb} < F_{Cmax} = 60.00 \text{ lb}$ then the system does not slip at C.

For block B:

The initial guessess: $N_B = 1\text{ lb}$ $F_B = 1\text{ lb}$

Given

$+\uparrow \Sigma F_y = 0;$ $N_B\cos(\theta) + F_B\sin(\theta) - W = 0$

$\xrightarrow{+} \Sigma F_x = 0;$ $F_B\cos(\theta) - N_B\sin(\theta) - T_B = 0$

$\begin{pmatrix} N_B \\ F_B \end{pmatrix} = \text{Find}(N_B, F_B)$ $N_B = 65.80\text{ lb}$ $F_B = 38.51\text{ lb}$

Check

$F_{Bmax} = \mu_{BA} N_B$ $F_{Bmax} = 39.48\text{ lb}$

Since $F_B = 38.51\text{ lb} < F_{Bmax} = 39.48\text{ lb}$ then no slipping occurs between the blocks

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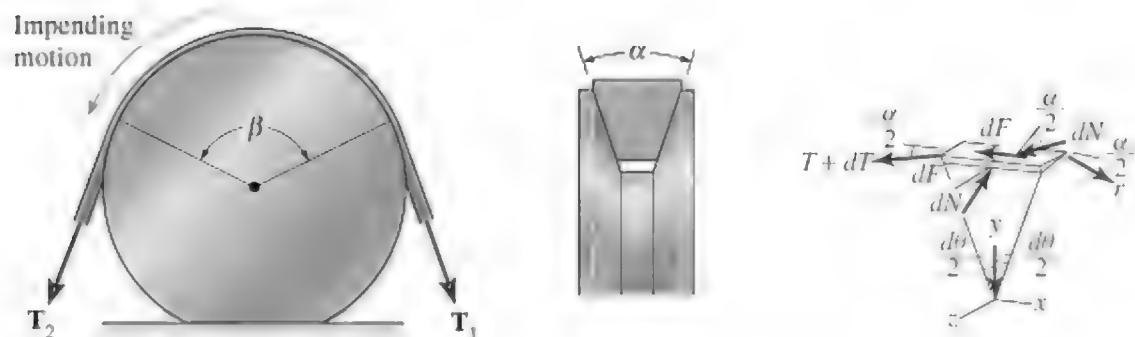
Problem 8-95

Show that the frictional relationship between the belt tensions, the coefficient of friction μ, and the angular contacts α and β for the V-belt is $T_2 = T_1 e^{\mu\beta/\sin(\alpha/2)}$ when the belt is on the verge of slipping.

Solution:

FBD of a section of the belt is shown.

Proceeding in the general manner:

$$\Sigma F_x = 0; \qquad -(T + dT)\cos\left(\frac{d\theta}{2}\right) + T\cos\left(\frac{d\theta}{2}\right) + 2\mu dN = 0$$

$\Sigma F_y = 0;$ $\quad -(T + dT)\sin\left(\frac{d\theta}{2}\right) - T\sin\left(\frac{d\theta}{2}\right) + 2dN\sin\left(\frac{\alpha}{2}\right) = 0$

Since $d\theta$, dN, and dT are small, these become

$$dT = 2\mu dN \qquad T d\theta = 2dN\sin\left(\frac{\alpha}{2}\right)$$

Combine $\quad \dfrac{dT}{T} = \mu\dfrac{d\theta}{\sin\left(\frac{\alpha}{2}\right)}$

Integrate from $\theta = 0, T = T_1$ to $\theta = \beta, T = T_2$

We get, $\quad T_2 = T_1 e^{\frac{\mu\beta}{\sin\left(\frac{\alpha}{2}\right)}}$ Q.E.D

Problem 8-96

A V-fan-belt (V-angle θ) of an automobile engine passes around the hub H of a generator G and over the housing F to a fan. If the generator locks, and the maximum tension the belt can sustain is T_{max}, determine the maximum possible torque **M** resisted by the axle as the belt slips over the hub. Assume that slipping of the belt occurs only at H and that the coefficient of kinetic friction for the hub is μ_s.

Given:

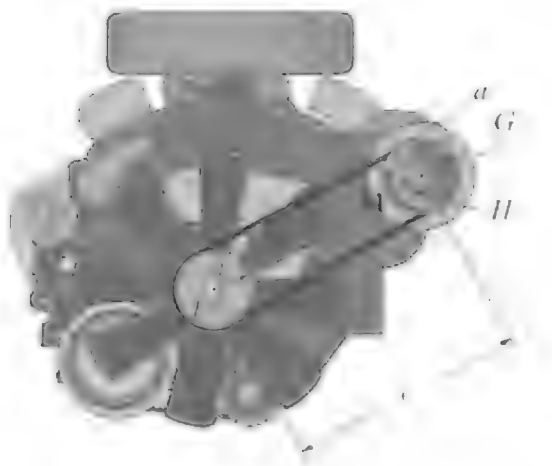
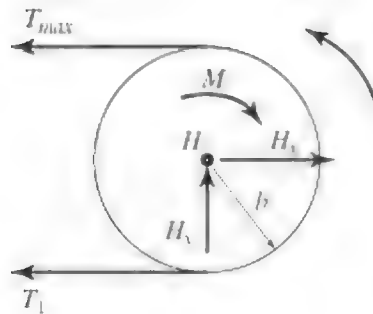
$\theta = 60$ deg $\qquad b = 2$ in

$a = 2$ in $\qquad c = 1.25$ ft

$T_{max} = 175$ lb $\qquad \mu_s = 0.25$

Solution:

$$-T_1 a + T_{max} a - M = 0$$

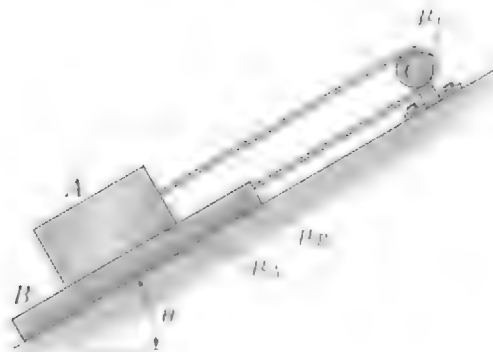
$$T_1 = T_{max} e^{-\mu_s\left(\frac{\pi}{\sin\left(\frac{1}{2}\theta\right)}\right)} \qquad T_1 = 36.4 \text{ lb}$$

$M = -T_1 a + T_{max} a$ $\qquad$ $M = 23.1\ \text{lb}\cdot\text{ft}$

Problem 8-97

A cable is attached to the plate B of mass M_B, passes over a fixed peg at C, and is attached to the block at A. Using the coefficients of static friction shown, determine the smallest mass of block A so that it will prevent sliding motion of B down the plane.

Given:

$M_B = 20\ \text{kg}$ $\qquad$ $\mu_A = 0.2$

$\theta = 30\ \text{deg}$ $\qquad$ $\mu_B = 0.3$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $\qquad$ $\mu_C = 0.3$

Solution:

Iniitial guesses: $T_1 = 1\ \text{N}$ $\quad T_2 = 1\ \text{N}$ $\quad N_A = 1\ \text{N}$ $\quad N_B = 1\ \text{N}$ $\quad M_A = 1\ \text{kg}$

Given

Block A:

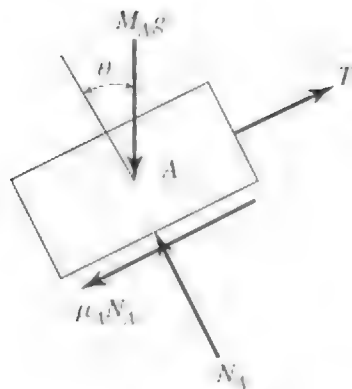
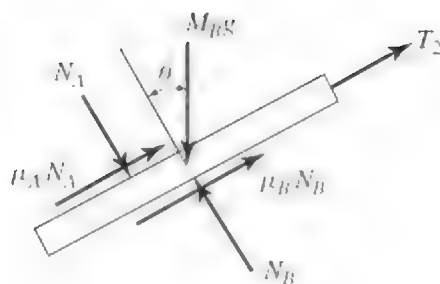
$\Sigma F_x = 0;$ $\qquad T_1 - \mu_A N_A - M_A g \sin(\theta) = 0$

$\Sigma F_y = 0;$ $\qquad N_A - M_A g \cos(\theta) = 0$

Plate B:

$\Sigma F_x = 0;$ $\qquad T_2 - M_B g \sin(\theta) + \mu_B N_B + \mu_A N_A = 0$

$\Sigma F_y = 0;$ $\qquad N_B - N_A - M_B g \cos(\theta) = 0$

Peg C: $\qquad T_2 = T_1 e^{\mu_C \pi}$

$$\begin{pmatrix} T_1 \\ T_2 \\ N_A \\ N_B \\ M_A \end{pmatrix} = \text{Find}(T_1, T_2, N_A, N_B, M_A)$$

$M_A = 2.22 \text{ kg}$

Problem 8-98

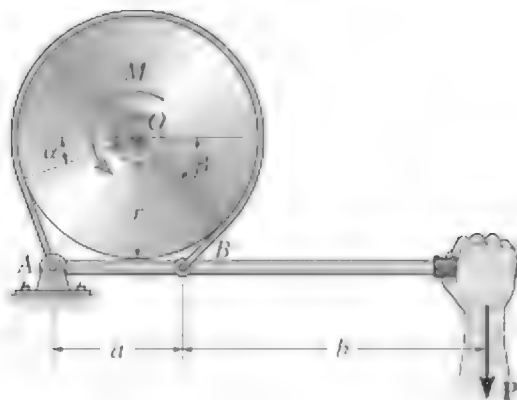
The simple band brake is constructed so that the ends of the friction strap are connected to the pin at A and the lever arm at B. If the wheel is subjected to a torque M, determine the smallest force **P** applied to the lever that is required to hold the wheel stationary. The coefficient of static friction between the strap and wheel is μ_s.

Given:

$M = 80 \text{ lb}\cdot\text{ft}$ $\beta = 45 \text{ deg}$

$\mu_s = 0.5$ $r = 1.25 \text{ ft}$

$\alpha = 20 \text{ deg}$ $a = 1.5 \text{ ft}$

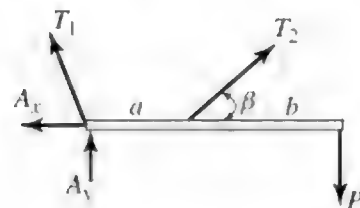
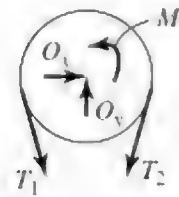
$b = 3 \text{ ft}$

Solution:

The initial guesses: $T_1 = 10 \text{ lb}$ $T_2 = 20 \text{ lb}$ $P = 30 \text{ lb}$

Given

$$T_1 r + M - T_2 r = 0$$

$$T_2 = T_1 e^{\mu_s(\pi+\alpha+\beta)}$$

$$T_2 \sin(\beta) a - (a + b) P = 0$$

$$\begin{pmatrix} T_1 \\ T_2 \\ P \end{pmatrix} = \text{Find}(T_1, T_2, P)$$

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 8.56 \\ 72.56 \end{pmatrix} \text{lb} \qquad P = 17.10\,\text{lb}$$

Problem 8-99

The uniform beam of weight W_1 is supported by the rope which is attached to the end of the beam, wraps over the rough peg, and is then connected to the block of weight W_2. If the coefficient of static friction between the beam and the block, and between the rope and the peg, is μ_s, determine the maximum distance that the block can be placed from A and still remain in equilibrium. Assume the block will not tip.

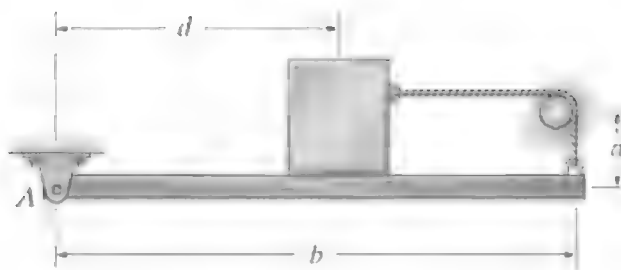
Given:

$W_1 = 50$ lb

$W_2 = 100$ lb

$\mu_s = 0.4$

$a = 1$ ft

$b = 10$ ft

Solution:

Block:

$\Sigma F_y = 0;\quad N - W_2 = 0$

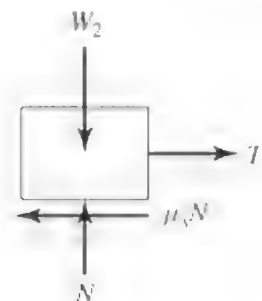
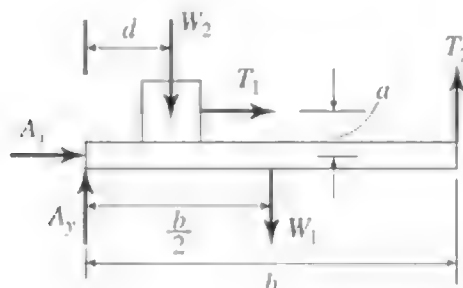
$N = W_2 \qquad N = 100.00\,\text{lb}$

$\Sigma F_x = 0;\quad T_1 - \mu_s N = 0$

$T_1 = \mu_s N \qquad T_1 = 40.00\,\text{lb}$

$T_2 = T_1 e^{\mu_s\left(\frac{\pi}{2}\right)} \qquad T_2 = 74.97\,\text{lb}$

System:

$\Sigma M_A = 0;\quad -W_2 d - T_1 a - W_1\left(\frac{b}{2}\right) + T_2 b = 0$

$$d = \frac{T_2 b - T_1 a - W_1\left(\frac{b}{2}\right)}{W_2}$$

$d = 4.6\ \text{ft}$

Problem 8-100

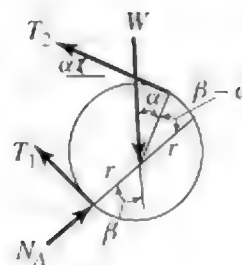
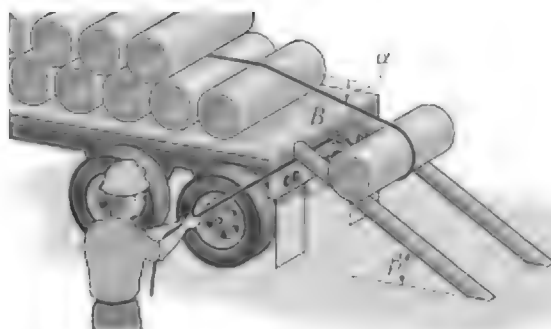
The uniform concrete pipe has weight W and is unloaded slowly from the truck bed using the rope and skids shown. If the coefficient of kinetic friction between the rope and pipe is μ_k, determine the force the worker must exert on the rope to lower the pipe at constant speed. There is a pulley at B, and the pipe does not slip on the skids. The lower portion of the rope is parallel to the skids.

Given:

$W = 800\ \text{lb}$

$\mu_k = 0.3$

$\alpha = 15\ \text{deg}$

$\beta = 30\ \text{deg}$

Solution:

$$-W r\sin(\beta) + T_2\cos(\alpha)(r\cos(\alpha) + r\cos(\beta)) + T_2\sin(\alpha)(r\sin(\alpha) + r\sin(\beta)) = 0$$

$$T_2 = \frac{W\sin(\beta)}{1 + \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)} \qquad T_2 = 203.47\ \text{lb}$$

$$T_1 = T_2 e^{-\mu_k(\pi+\beta-\alpha)} \qquad T_1 = 73.3\ \text{lb}$$

Problem 8-101

A cord having a weight density γ and a total length L is suspended over a peg P as shown. If the coefficient of static friction between the peg and cord is μ_s, determine the longest length h which one side of the suspended cord can have without causing motion. Neglect the size of the peg and the length of cord draped over it.

Given:

$\gamma = 0.5 \frac{\text{lb}}{\text{ft}}$

$L = 10 \text{ ft}$

$\mu_s = 0.5$

Solution:

$T_2 = T_1 e^{\mu\beta}$ $\quad T_1 = \gamma(L - h)$ $\quad T_2 = \gamma h$

$\gamma h = \gamma(L - h)\, e^{\mu_s \pi}$ $\quad h = L\left(\frac{e^{\mu_s \pi}}{1 + e^{\mu_s \pi}}\right)$ $\quad h = 8.28 \text{ ft}$

Problem 8-102

Granular material, having a density ρ is transported on a conveyor belt that slides over the fixed surface, having a coefficient of kinetic friction of μ_k. Operation of the belt is provided by a motor that supplies a torque **M** to wheel A.The wheel at B is free to turn, and the coefficient of static friction between the wheel at A and the belt is μ_A. If the belt is subjected to a pretension T when no load is on the belt, determine the greatest volume V of material that is permitted on the belt at any time without allowing the belt to stop. What is the torque **M** required to drive the belt when it is subjected to this maximum load?

Units used:

$\text{Mg} = 10^6 \text{ g}$

Given:

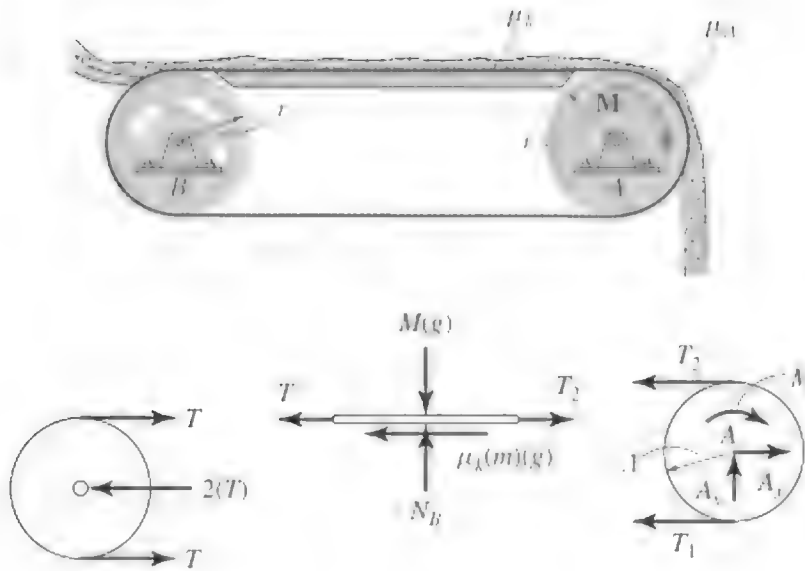
$r = 100 \text{ mm}$

$\mu_A = 0.4$

$\mu_k = 0.3$

$\rho = 1500 \frac{\text{kg}}{\text{m}^3}$

$T = 300 \text{ N}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

$T_2 = T e^{(\mu_A)\pi}$ $\quad T_2 = 1053.9\ \text{N}$

Wheel A :

$\Sigma M_A = 0;$ $\quad -M - T r + T_2 r = 0$ $\quad M = -T r + T_2 r$ $\quad M = 75.4\ \text{N}\cdot\text{m}$

Belt

$\Sigma F_x = 0;$ $\quad T_2 - \mu_k m_1 g - T = 0$ $\quad m_1 = \dfrac{T_2 - T}{\mu_k g}$ $\quad m_1 = 256.2\ \text{kg}$

$V = \dfrac{m_1}{\rho}$ $\quad V = 0.17\ \text{m}^3$

Problem 8-103

Blocks A and B have a mass M_A and M_B, respectively. If the coefficient of static friction between A and B and between B and C is μ_s and between the ropes and the pegs D and E μ'_s, determine the smallest force **F** needed to cause motion of block B .

Units Used:

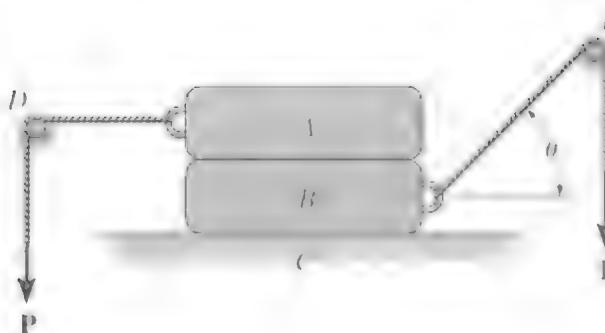
$\text{kN} = 10^3\ \text{N}$

Given:

$\theta = 45\ \text{deg}$ $\quad \mu_s = 0.25$

$M_A = 100\ \text{kg}$ $\quad \mu'_s = 0.5$

$M_B = 150\ \text{kg}$ $\quad P = 30\ \text{N}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

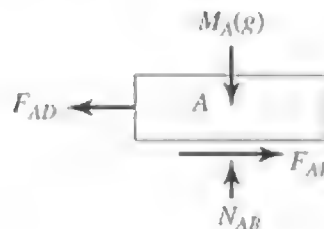
Assume no slipping between A & B.

Guesses

$F = 1\ \text{N}$ $\quad N_{AB} = 1\ \text{N}$ $\quad F_{AB} = 1\ \text{N}$

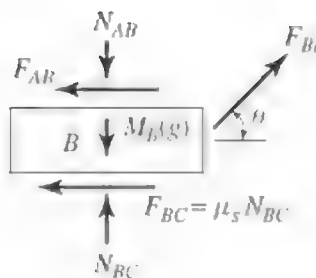
$N_{BC} = 1\ \text{N}$ $\quad F_{BE} = 1\ \text{N}$ $\quad F_{AD} = 1\ \text{N}$

Given

$F = F_{BE} e^{\mu'_s\left(\frac{\pi}{2}+\theta\right)}$ $\quad F_{AD} = P e^{\mu'_s \frac{\pi}{2}}$

$$F_{BE}\cos(\theta) - \mu_s N_{BC} - F_{AB} = 0$$

$$F_{BE}\sin(\theta) - N_{AB} - M_B g + N_{BC} = 0$$

$$-F_{AD} + F_{AB} = 0$$

$$N_{AB} - M_A g = 0$$

$$\begin{pmatrix} F \\ N_{AB} \\ F_{AB} \\ N_{BC} \\ F_{BE} \\ F_{AD} \end{pmatrix} = \text{Find}(F, N_{AB}, F_{AB}, N_{BC}, F_{BE}, F_{AD})$$

$$\begin{pmatrix} F \\ N_{AB} \\ F_{AB} \\ N_{BC} \\ F_{BE} \\ F_{AD} \end{pmatrix} = \begin{pmatrix} 2.49 \\ 0.98 \\ 0.07 \\ 1.91 \\ 0.77 \\ 0.07 \end{pmatrix} \text{kN}$$

Now check assumption $F_{ABmax} = \mu_s N_{AB}$ $F_{ABmax} = 245.25 \text{ N}$

Since $F_{AB} = 65.8 \text{ N} < F_{ABmax} = 245.3 \text{ N}$ then our assumption is correct

$F = 2.49 \text{kN}$

Problem 8-104

Blocks A and B weigh W_1 and W_2, respectively. Using the coefficients of static friction indicated, determine the greatest weight W of block E without causing motion.

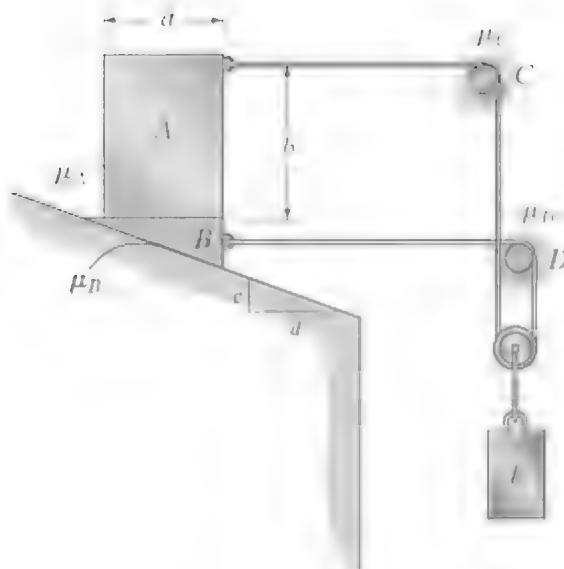
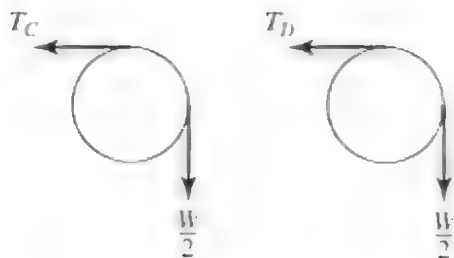
Given:

$W_1 = 50 \text{ lb}$ $d = 12$

$W_2 = 30 \text{ lb}$ $\mu_A = 0.3$

$a = 1.5 \text{ ft}$ $\mu_B = 0.5$

$b = 2 \text{ ft}$ $\mu_C = 0.2$

$c = 5$ $\mu_D = 0.3$

Solution: Assume that the wedge slips on the ground, but the block does not slip on the wedge and the block does not tip.

Guesses $W = 1\ \text{lb}$ $T_D = 1\ \text{lb}$ $T_C = 1\ \text{lb}$

$N_A = 1\ \text{lb}$ $F_A = 1\ \text{lb}$ $N_B = 1\ \text{lb}$

$x = 1\ \text{ft}$ $F_{Amax} = 1\ \text{lb}$

Given

$$\frac{W}{2} = T_C e^{\mu_C \frac{\pi}{2}} \qquad \frac{W}{2} = T_D e^{\mu_D \frac{\pi}{2}}$$

$$F_A - \mu_B N_B\left(\frac{d}{\sqrt{c^2 + d^2}}\right) + N_B\left(\frac{c}{\sqrt{c^2 + d^2}}\right) + T_D = 0$$

$$N_B\left(\frac{d}{\sqrt{c^2 + d^2}}\right) + \mu_B N_B\left(\frac{c}{\sqrt{c^2 + d^2}}\right) - W_2 - N_A = 0$$

$$T_C - F_A = 0 \qquad N_A - W_1 = 0 \qquad -T_C b + N_A x = 0 \qquad F_{Amax} = \mu_A N_A$$

$$\begin{pmatrix} W \\ T_D \\ T_C \\ N_A \\ F_A \\ N_B \\ x \\ F_{Amax} \end{pmatrix} = \text{Find}(W, T_D, T_C, N_A, F_A, N_B, x, F_{Amax}) \qquad W = 8.15\ \text{lb}$$

Check assumptions

Since $F_A = 2.97\text{ lb} < F_{Amax} = 15.00\text{ lb}$ then the block does not slip relative to the wedge.

Since $x = 0.12\text{ ft} < \frac{a}{2} = 0.75\text{ ft}$ then the block does not tip.

Problem 8-105

Block A has mass m_A and rests on surface B for which the coefficient of static friction is μ_{sAB}. If the coefficient of static friction between the cord and the fixed peg at C is μ_{sC}, determine the greatest mass m_D of the suspended cylinder D without causing motion.

Given:

$m_A = 50\text{ kg}$

$\mu_{sAB} = 0.25$

$\mu_{sC} = 0.3$

$a = 0.3\text{ m}$

$b = 0.25\text{ m}$

$c = 0.4\text{ m}$

$d = 3$

$f = 4$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Assume block A slips but does not tip. $\beta = \pi - \text{atan}\left(\frac{f}{d}\right)$

The initial guesses: $N_B = 100\text{ N}$ $T = 50\text{ N}$ $m_D = 1\text{ kg}$ $x = 10\text{ mm}$

Given $m_D g = T e^{\mu_{sC}\beta}$

$$\left(\frac{d}{\sqrt{f^2 + d^2}}\right)T - m_A g + N_B = 0$$

$$\left(\frac{-f}{\sqrt{f^2 + d^2}}\right)T + \mu_{sAB} N_B = 0$$

$$\left(\frac{f}{\sqrt{f^2 + d^2}}\right)T a - \left(\frac{d}{\sqrt{f^2 + d^2}}\right)T\left(\frac{b}{2}\right) - N_B x = 0$$

$$\begin{pmatrix} N_B \\ T \\ m_D \\ x \end{pmatrix} = \text{Find}(N_B, T, m_D, x) \qquad \begin{pmatrix} N_B \\ T \end{pmatrix} = \begin{pmatrix} 413.05 \\ 129.08 \end{pmatrix} \text{N} \qquad m_D = 25.6 \text{ kg}$$

$$x = 0.052 \text{ m}$$

Since $x = 51.6\,\text{mm} < \dfrac{b}{2} = 125\,\text{mm}$ our assumption is correct $\quad m_D = 25.6 \text{ kg}$

Problem 8-106

Block A rests on the surface for which the coefficient of friction is μ_{sAB}. If the mass of the suspended cylinder is m_D, determine the smallest mass m_A of block A so that it does not slip or tip. The coefficient of static friction between the cord and the fixed peg at C is μ_{sC}.

Units Used:

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Given:

$\mu_{sAB} = 0.25$

$m_D = 4 \text{ kg}$

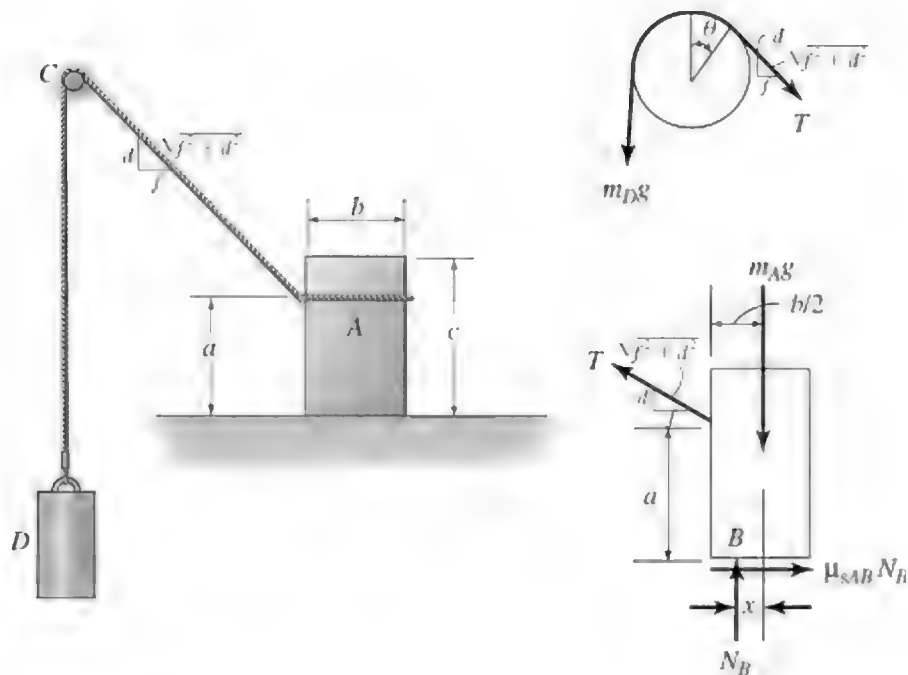
$\mu_{sC} = 0.3$

$a = 0.3 \text{ m}$

$b = 0.25 \text{ m}$

$c = 0.4 \text{ m}$

$d = 3$

$f = 4$

Solution: Assume that slipping is the crtitical motion $\quad \beta = \pi - \text{atan}\left(\dfrac{f}{d}\right)$

The initial guesses: $\quad N_B = 100 \text{ N} \quad T = 50 \text{ N} \quad m_A = 1 \text{ kg} \quad x = 10 \text{ mm}$

Given $\quad m_D g = T e^{\mu_{sC}\beta} \qquad \left(\dfrac{d}{\sqrt{f^2 + d^2}}\right)T - m_A g + N_B = 0$

$$\left(\frac{-f}{\sqrt{f^2+d^2}}\right)T + \mu_{sAB} N_B = 0 \qquad \left(\frac{f}{\sqrt{f^2+d^2}}\right)T a - \left(\frac{d}{\sqrt{f^2+d^2}}\right)T\frac{b}{2} - N_B x = 0$$

$$\begin{pmatrix} N_B \\ T \\ m_A \\ x \end{pmatrix} = \text{Find}(N_B, T, m_A, x) \qquad \begin{pmatrix} N_B \\ T \end{pmatrix} = \begin{pmatrix} 64.63 \\ 20.20 \end{pmatrix} \text{N} \qquad m_A = 7.82 \text{ kg}$$

$$x = 0.052 \text{ m}$$

Since $x = 51.6\,\text{mm} < \frac{b}{2} = 125\,\text{mm}$ our assumption is correct $\qquad m_A = 7.82 \text{ kg}$

Problem 8-107

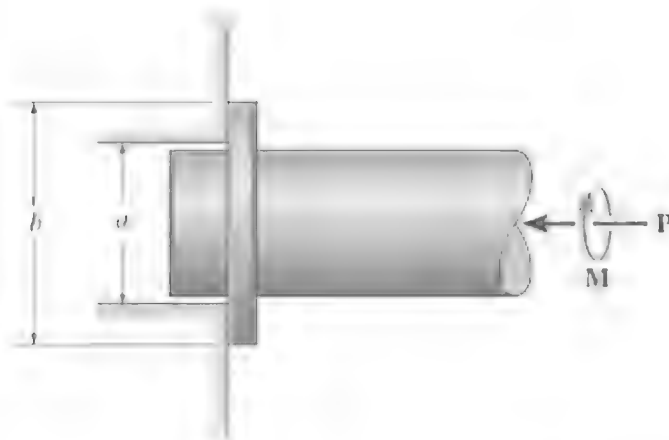
The collar bearing uniformly supports an axial force **P**. If the coefficient of static friction is μ_s, determine the torque **M** required to overcome friction.

Given:

$a = 2$ in

$b = 3$ in

$P = 800$ lb

$\mu_s = 0.3$

Solution:

$$M = \frac{a}{b}\mu_s P\left[\frac{\left(\frac{b}{2}\right)^3 - \left(\frac{a}{2}\right)^3}{\left(\frac{b}{2}\right)^2 - \left(\frac{a}{2}\right)^2}\right] \qquad M = \frac{a}{b}\mu_s P\left[\frac{1}{2}\left(\frac{a^2 + ab + b^2}{a+b}\right)\right] \qquad M = 304.00\,\text{lb}\cdot\text{in}$$

Problem 8-108

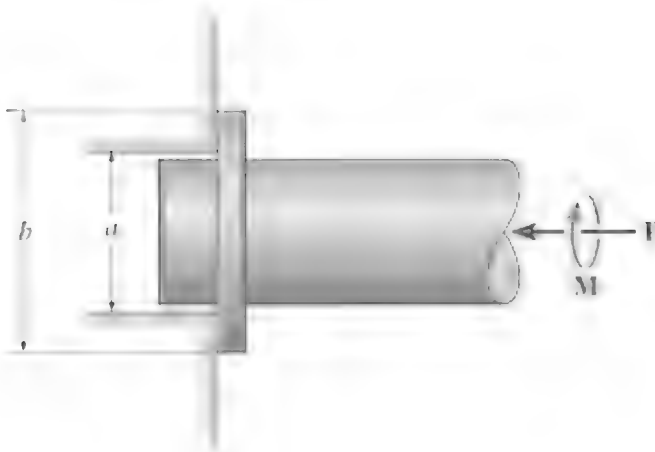
The collar bearing uniformly supports an axial force **P**. If a torque **M** is applied to the shaft and causes it to rotate at constant velocity, determine the coefficient of kinetic friction at the surface of contact.

Given:

$a = 2\text{ in}$

$b = 3\text{ in}$

$P = 500\text{ lb}$

$M = 3\text{ lb ft}$

Solution:

$$M = \frac{a}{b}\mu_k P \left[\frac{\left(\frac{b}{2}\right)^3 - \left(\frac{a}{2}\right)^3}{\left(\frac{b}{2}\right)^2 - \left(\frac{a}{2}\right)^2}\right] = \left(\frac{a\mu_k P}{2b}\right)\left(\frac{b^3 - a^3}{b^2 - a^2}\right)$$

$$\mu_k = \frac{2Mb\left(b^2 - a^2\right)}{aP\left(b^3 - a^3\right)} \qquad \mu_k = 0.0568$$

Problem 8-109

The *double-collar bearing* is subjected to an axial force **P**. Assuming that collar A supports kP and collar B supports $(1 - k)P$, both with a uniform distribution of pressure, determine the maximum frictional moment M that may be resisted by the bearing.

Units Used:

$\text{kN} = 10^3\text{ N}$

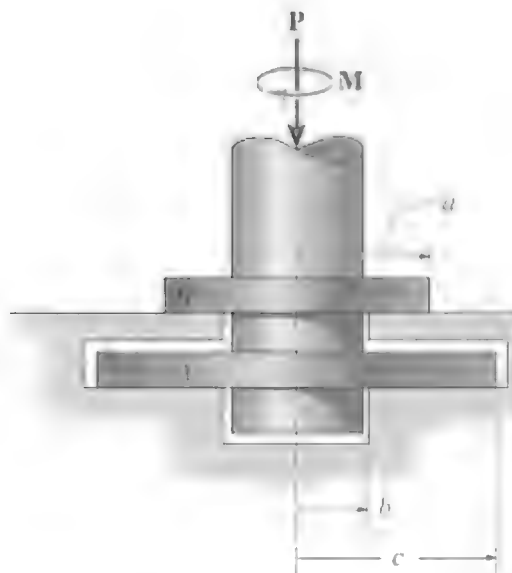
Given:

$P = 4\text{ kN}$

$a = 20\text{ mm}$

$b = 10\text{ mm}$

$c = 30\text{ mm}$

$\mu_s = 0.2$

$k = 0.75$

Solution:

$$M = \frac{2}{3}\mu_s\left[\frac{c^3 - b^3}{c^2 - b^2}kP + \frac{a^3 - b^3}{a^2 - b^2}(1 - k)P\right]$$

$M = 16.1\ \text{N}\cdot\text{m}$

Problem 8-110

The annular ring bearing is subjected to a thrust P. If the coefficient of static friction is μ_s, determine the torque **M** that must be applied to overcome friction.

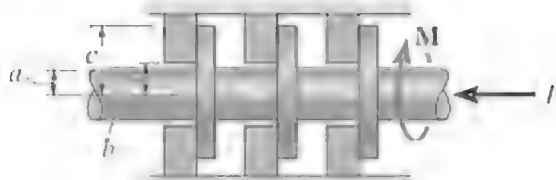
Given:

$P = 800\ \text{lb}$

$\mu_s = 0.35$

$a = 0.75\ \text{in}$

$b = 1\ \text{in}$

$c = 2\ \text{in}$

Solution:

$$M = \frac{2}{3}\mu_s P\left(\frac{b^3 - c^3}{b^2 - c^2}\right) \qquad M = 36.3\ \text{lb}\cdot\text{ft}$$

Problem 8-111

The floor-polishing machine rotates at a constant angular velocity. If it has weight W, determine the couple forces **F** the operator must apply to the handles to hold the machine stationary. The coefficient of kinetic friction between the floor and brush is μ_k. Assume the brush exerts a uniform pressure on the floor.

Given:

$W = 80\ \text{lb}$

$\mu_k = 0.3$

$a = 1.5$ ft

$b = 2$ ft

Solution:

$$F a = \frac{2}{3}\mu_k W\left(\frac{b}{2}\right)$$

$$F = \frac{1}{3}\ \mu_k\ W\left(\frac{b}{a}\right)$$

$F = 10.7$ lb

Problem 8-112

The plate clutch consists of a flat plate A that slides over the rotating shaft S. The shaft is fixed to the driving plate gear B. If the gear C, which is in mesh with B, is subjected to a torque M, determine the smallest force P, that must be applied via the control arm, to stop the rotation. The coefficient of static friction between the plates A and D is μ_s. Assume the bearing pressure between A and D to be uniform.

Given:

$M = 0.8$ N·m

$\mu_s = 0.4$

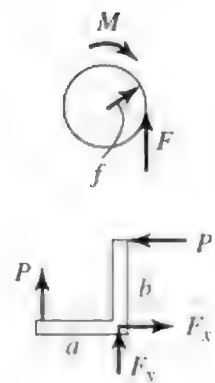
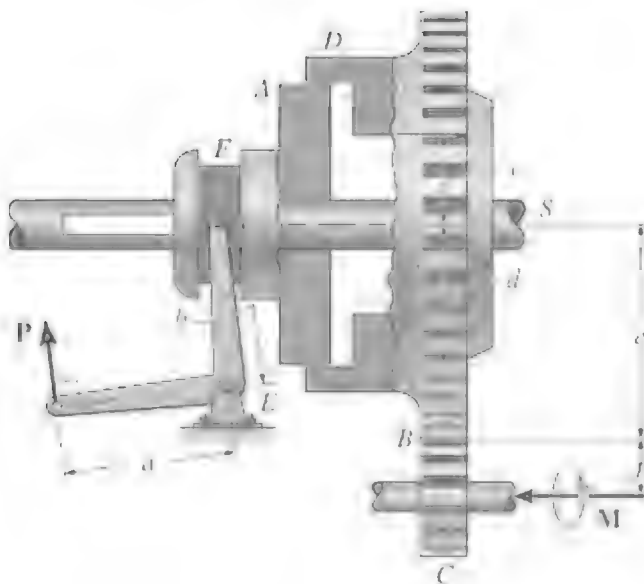
$a = 150$ mm

$b = 200$ mm

$c = 100$ mm

$d = 125$ mm

$e = 150$ mm

$f = 30$ mm

Solution:

$F = \frac{M}{f}$ $\qquad F = 26.667$ N

$M_2 = F e$ $\qquad M_2 = 4.00$ N·m

$$M_2 = \frac{2}{3}\mu_s P'\left(\frac{d^3 - c^3}{d^2 - c^2}\right) \qquad P' = \frac{3M_2}{2\mu_s}\left(\frac{d^2 - c^2}{d^3 - c^3}\right) \qquad P' = 88.5 \text{ N}$$

$$P' b - P a = 0 \qquad P = P'\left(\frac{b}{a}\right) \qquad P = 118 \text{ N}$$

Problem 8-113

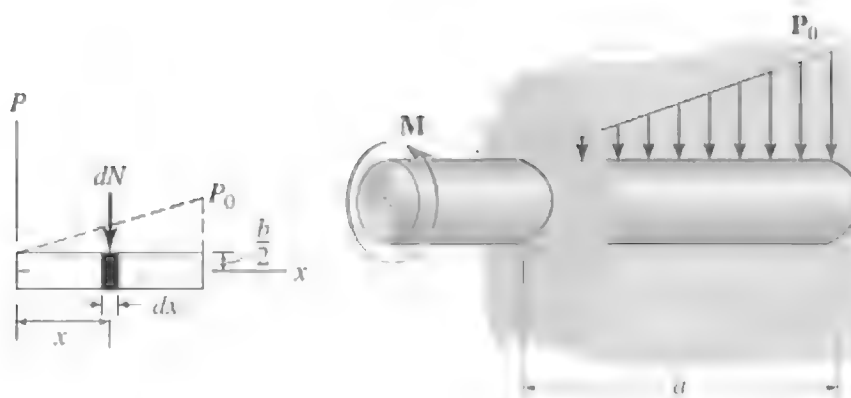
The shaft of diameter b is held in the hole such that the normal pressure acting around the shaft varies linearly with its depth as shown. Determine the frictional torque that must be overcome to rotate the shaft.

Given:

$a = 6$ in

$p_0 = 60 \frac{\text{lb}}{\text{in}^2}$

$b = 4$ in

$\mu_s = 0.2$

Solution:

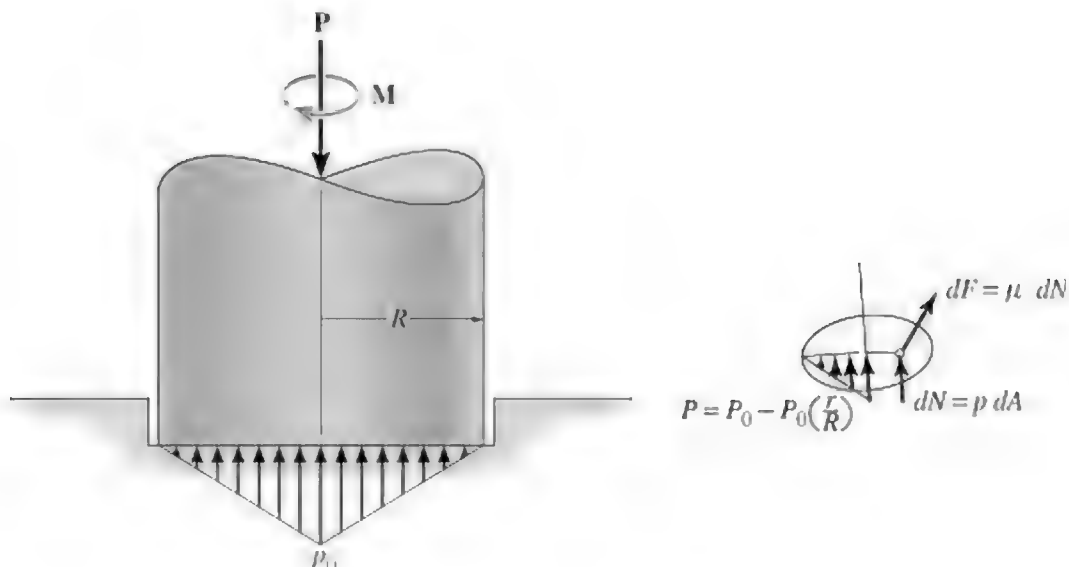
$$N = \int_0^a \left(\frac{x}{a}\right) p_0 2\pi\left(\frac{b}{2}\right) \text{dx} \qquad T = \mu_s N\left(\frac{b}{2}\right) \qquad T = 905 \text{ lb}\cdot\text{in}$$

Problem 8-114

Because of wearing at the edges, the pivot bearing is subjected to a conical pressure distribution at its surface of contact. Determine the torque **M** required to overcome friction and turn the shaft, which supports an axial force **P**. The coefficient of static friction is μ_s. For the solution, it is necessary to determine the peak pressure p_0 in terms of P and the bearing radius R.

Solution:

$$P = \int_0^{2\pi}\int_0^R \left(p_0 - p_0\frac{r}{R}\right) r \,\text{d}r\,\text{d}\theta = \frac{\pi p_0 R^2}{3} \qquad p_0 = \frac{3P}{\pi R^2}$$

$$\text{d}M = r\text{d}F = r\mu\text{d}N = r\mu p\text{d}A = r\mu p\, r\text{d}\theta\text{d}r$$

$$M = \int_0^R \mu\left(p0 - \frac{p0}{R}r\right)r^2\,dr\int_0^{2\pi} 1\,d\theta = \frac{\pi}{6}\mu p_0 R^3 = \frac{\pi}{6}\mu\left(\frac{3P}{\pi R^2}\right)R^3 = \frac{\mu P R}{2}$$

Thus, $M = \frac{\mu P R}{2}$

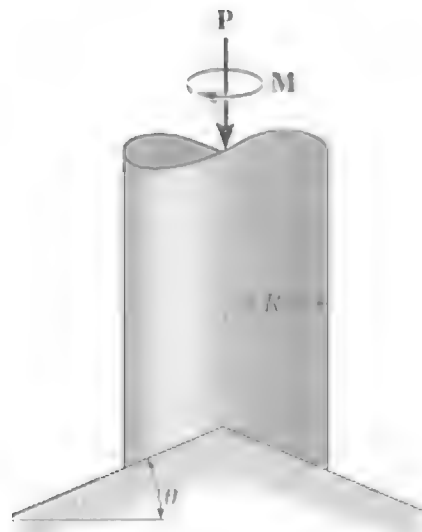
Problem 8-115

The conical bearing is subjected to a constant pressure distribution at its surface of contact. If the coefficient of static friction is μ_s, determine the torque **M** required to overcome friction if the shaft supports an axial force **P**.

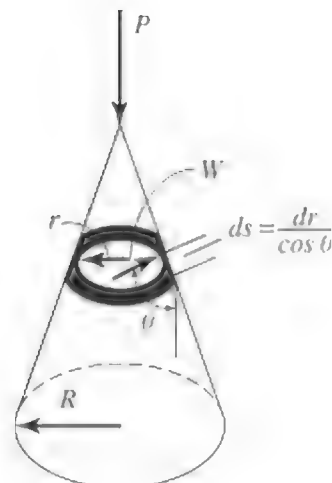
Solution:

The differential Area (shaded)

$$dA = 2\pi r\left(\frac{dr}{\cos(\theta)}\right) = \frac{2\pi r dr}{\cos(\theta)}$$

$$P = \int p\cos(\theta)\,dA = \int p\cos(\theta)\left(\frac{2\pi r}{\cos(\theta)}\right)dr = 2\pi p\int_0^R r\,dr = \pi p R^2 \qquad p = \frac{P}{\pi R^2}$$

$$dN = p\,dA = \frac{P}{\pi R^2}\left(\frac{2\pi r dr}{\cos(\theta)}\right) = \frac{2P}{R^2\cos(\theta)} r dr$$

$$M = \int r\,dF = \int r\mu_s\,dN = \left(\frac{2\mu_s P}{R^2\cos(\theta)}\right)\int_0^R r^2\,dr$$

$$M = \left(\frac{2\mu_s P}{R^2\cos(\theta)}\right)\frac{R^3}{3} = \frac{2\mu_s P R}{3\cos(\theta)}$$

Problem 8-116

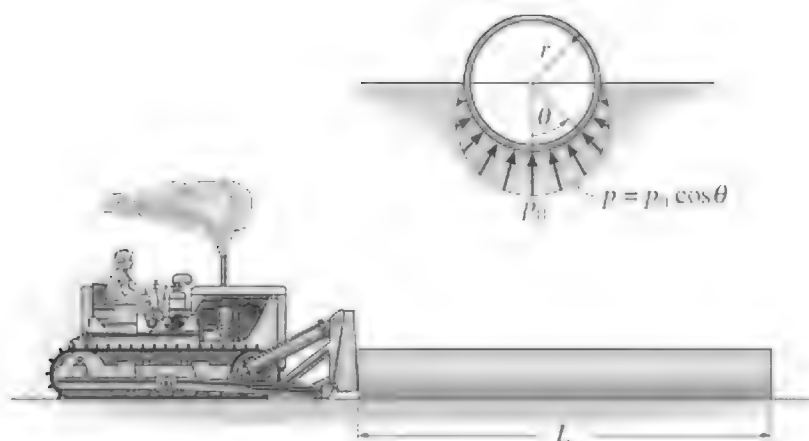
The tractor is used to push the pipe of weight W. To do this it must overcome the frictional forces at the ground, caused by sand. Assuming that the sand exerts a pressure on the bottom of the pipe as shown, and the coefficient of static friction between the pipe and the sand is μ_s, determine the force required to push the pipe forward. Also, determine the peak pressure p_0.

Given:

$W = 1500$ lb

$\mu_s = 0.3$

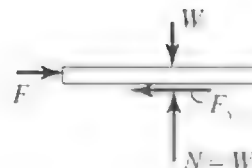
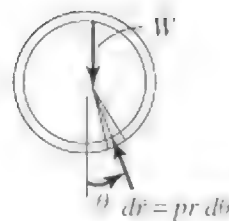
$L = 12$ ft

$r = 15$ in

Solution:

$$+\uparrow \Sigma F_y = 0; \quad 2L\int_0^{\frac{\pi}{2}} p_0\cos(\theta)r\,d\theta\cos(\theta) - W = 0$$

$$W = 2p_0Lr\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = p_0Lr\left(\frac{\pi}{2}\right)$$

$$p_0 = 2\frac{W}{rL\pi} \qquad p_0 = 0.442\,\frac{\text{lb}}{\text{in}^2}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \mu_s p_0 Lr\cos(\theta)\,d\theta \qquad F = 573\text{ lb}$$

Problem 8-117

Assuming that the variation of pressure at the bottom of the pivot bearing is defined as $p = p_0(R_2/r)$, determine the torque **M** needed to overcome friction if the shaft is subjected to an axial force **P.** The coefficient of static friction is μ_s. For the solution, it is necessary to determine p_0 in terms of P and the bearing dimensions R_1 and R_2.

Solution:

$$P = \int_0^{2\pi}\int_{R_1}^{R_2} p\,r\,dr\,d\theta$$

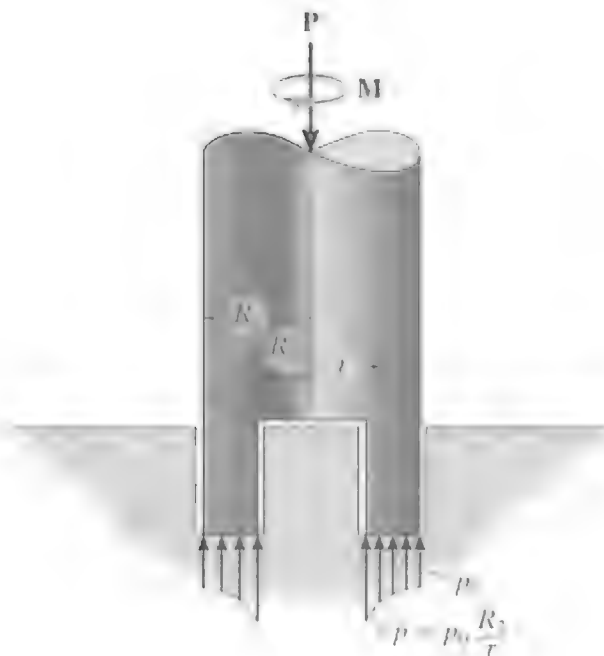
$$P = \int_0^{2\pi}\int_{R_1}^{R_2} p_0\left(\frac{R_2}{r}\right) r\, dr\, d\theta$$

$$P = 2\pi p_0 R_2 (R_2 - R_1)$$

$$p_0 = \frac{P}{2\pi R_2 (R_2 - R_1)}$$

$$M = \int_A r\, dF = \int_0^{2\pi}\int_{R_1}^{R_2} r\mu_s p_0 r\, dr\, d\theta$$

$$M = \int_0^{2\pi}\int_0^{2\pi} \mu_s p_0\left(\frac{R_2}{r}\right) r^2\, dr\, d\theta = \pi\mu_s p_0 R_2 \left(R_2^{\ 2} - R_1^{\ 2}\right)$$

$$M = \pi\mu_s\left[\frac{P}{2\pi R_2 (R_2 - R_1)}\right] R_2 \left(R_2^{\ 2} - R_1^{\ 2}\right) \qquad M = \frac{\mu_s P (R_2 + R_1)}{2}$$

Problem 8-118

A disk having an outer diameter a fits loosely over a fixed shaft having a diameter b. If the coefficient of static friction between the disk and the shaft is μ_s, determine the smallest vertical force **P**, acting on the rim, which must be applied to the disk to cause it to slip over the shaft. The disk weighs W.

Given:

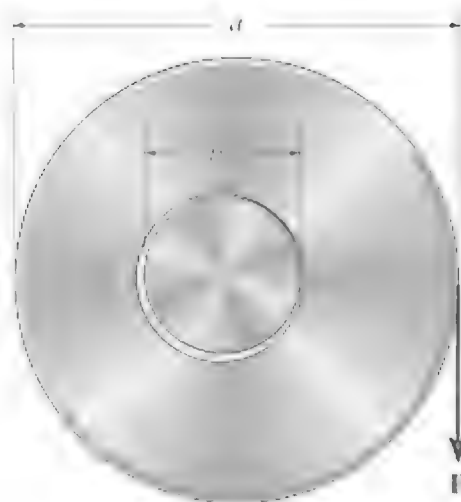
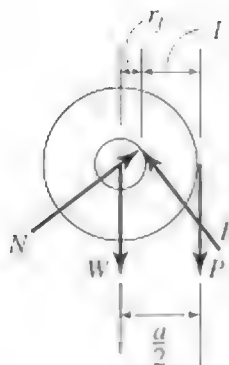
$a = 8$ in

$b = 3$ in

$\mu_s = 0.15$

$W = 10$ lb

Solution:

$\phi_s = \text{atan}(\mu_s)$

$$r_f = \left(\frac{b}{2}\right)\sin(\phi_s)$$

$$l = \left[\left(\frac{a}{2}\right) - r_f\right]$$

$\Sigma M_O = 0;$

$$W r_f - l P = 0 \qquad P = W\left(\frac{r_f}{l}\right) \qquad P = 0.59\,\text{lb}$$

Problem 8-119

The pulley has a radius r and fits loosely on the shaft of diameter d. If the loadings acting on the belt cause the pulley to rotate with constant angular velocity, determine the frictional force between the shaft and the pulley and compute the coefficient of kinetic friction. The pulley has weight W.

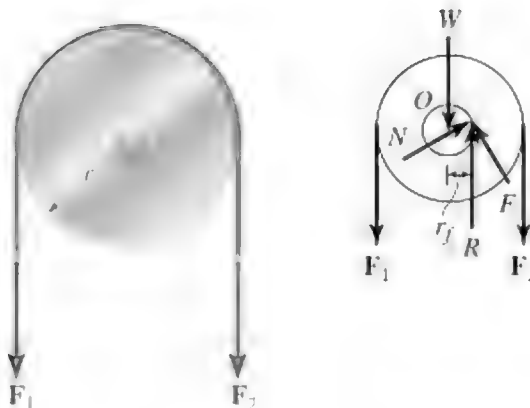
Given:

$r = 3$ in

$d = 0.5$ in

$W = 18$ lb

$F_1 = 5$ lb

$F_2 = 5.5$ lb

Solution:

$+\uparrow \Sigma F_y = 0;\quad R - W - F_1 - F_2 = 0 \qquad R = W + F_1 + F_2 \qquad R = 28.5\,\text{lb}$

$\Sigma M_O = 0;\quad -F_2 r + F_1 r + R r_f = 0 \qquad r_f = r\left(\frac{F_2 - F_1}{R}\right) \qquad r_f = 0.05263\,\text{in}$

$r_f = \frac{d}{2}\sin(\phi_k) \qquad \phi_k = \operatorname{asin}\left(2\frac{r_f}{d}\right) \qquad \phi_k = 12.15\,\text{deg}$

$\mu = \tan(\phi_k) \qquad \mu = 0.215$

Also,

$\Sigma M_O = 0;\quad -F_2 r + F_1 r + F\left(\frac{d}{2}\right) = 0 \qquad F = 2r\left(\frac{F_2 - F_1}{d}\right) \qquad F = 6\,\text{lb}$

Problem 8-120

The pulley has a radius r and fits loosely on the shaft of diameter d. If the loadings acting on the belt cause the pulley to rotate with constant angular velocity, determine the frictional force between the shaft and the pulley and compute the coefficient of kinetic friction. Neglect the weight of the pulley.

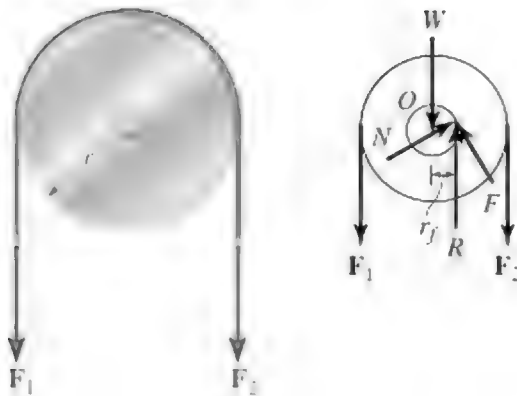
Given:

$r = 3$ in

$d = 0.5$ in

$W = 0$ lb

$F_1 = 5$ lb

$F_2 = 5.5$ lb

Solution:

$+\uparrow \Sigma F_y = 0; \quad R - W - F_1 - F_2 = 0 \qquad R = W + F_1 + F_2 \qquad R = 10.5\text{ lb}$

$\Sigma M_O = 0; \quad -F_2 r + F_1 r + R r_f = 0 \qquad r_f = r\left(\frac{F_2 - F_1}{R}\right) \qquad r_f = 0.14286\text{ in}$

$r_f = \frac{d}{2}\sin(\phi_k) \qquad \phi_k = \operatorname{asin}\left(2\frac{r_f}{d}\right) \qquad \phi_k = 34.85\text{ deg}$

$\mu = \tan(\phi_k) \qquad \mu = 0.696$

Also,

$\Sigma M_O = 0; \quad -F_2 r + F_1 r + F\frac{d}{2} = 0 \qquad F = 2r\left(\frac{F_2 - F_1}{d}\right) \qquad F = 6\text{ lb}$

Problem 8-121

A pulley of mass M has radius a and the axle has a diameter D. If the coefficient of kinetic friction between the axle and the pulley is μ_k determine the vertical force **P** on the rope required to lift the block of mass M_B at constant velocity.

Given:

$a = 120$ mm

$M = 5$ kg

$D = 40$ mm

$\mu_k = 0.15$

$M_B = 80 \text{ kg}$

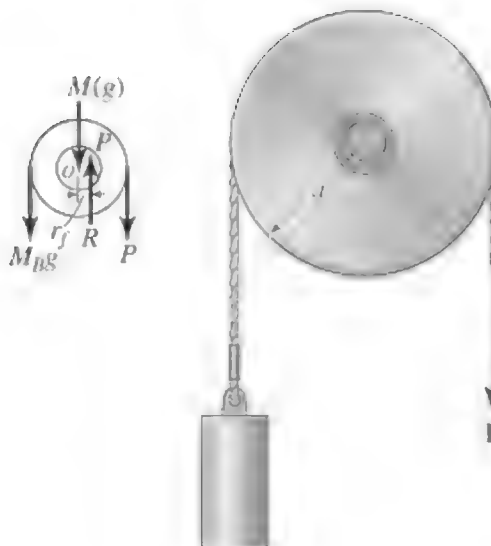
Solution:

$\phi_k = \operatorname{atan}(\mu_k)$

$r_f = \left(\frac{D}{2}\right)\sin(\phi_k)$

$\Sigma M_p = 0;$

$M_B g(a + r_f) + M g r_f - P(a - r_f) = 0$

$P = \frac{M_B g(a + r_f) + M g r_f}{a - r_f}$ $P = 826 \text{ N}$

Problem 8-122

A pulley of mass M has radius a and the axle has a diameter D. If the coefficient of kinetic friction between the axle and the pulley is μ_k determine the force **P** on the rope required to lift the block of mass M_B at constant velocity. Apply the force P horizontally to the right (not as shown in the figure).

Given:

$a = 120 \text{ mm}$

$M = 5 \text{ kg}$

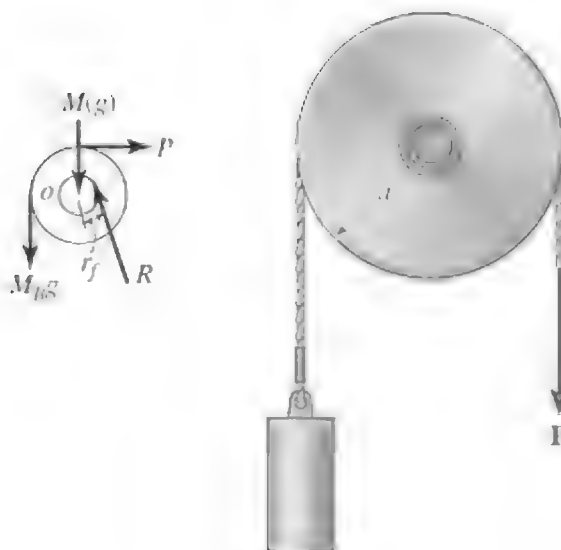
$D = 40 \text{ mm}$

$\mu_k = 0.15$

$M_B = 80 \text{ kg}$

Solution: $g = 9.81 \frac{\text{m}}{\text{s}^2}$

$\phi_k = \operatorname{atan}(\mu_k)$

$r_f = \frac{D}{2}\sin(\phi_k)$

Guesses

$P = 1\ \text{N} \qquad R = 1\ \text{N} \qquad \alpha = 1\ \text{deg}$

Given

$R\cos(\alpha) - M_B g - M g = 0$

$P - R\sin(\alpha) = 0$

$M_B g a - P a + R r_f = 0$

$\begin{pmatrix} P \\ R \\ \alpha \end{pmatrix} = \text{Find}(P, R, \alpha) \qquad P = 814\ \text{N}$

Problem 8-123

A wheel on a freight car carries a load W. If the axle of the car has a diameter D, determine the horizontal force P that must be applied to the axle to rotate the wheel. The coefficient of kinetic friction is μ_k.

Units Used:

$\text{kip} = 1000\ \text{lb}$

Given:

$W = 20\ \text{kip}$

$D = 2\ \text{in}$

$\mu_k = 0.05$

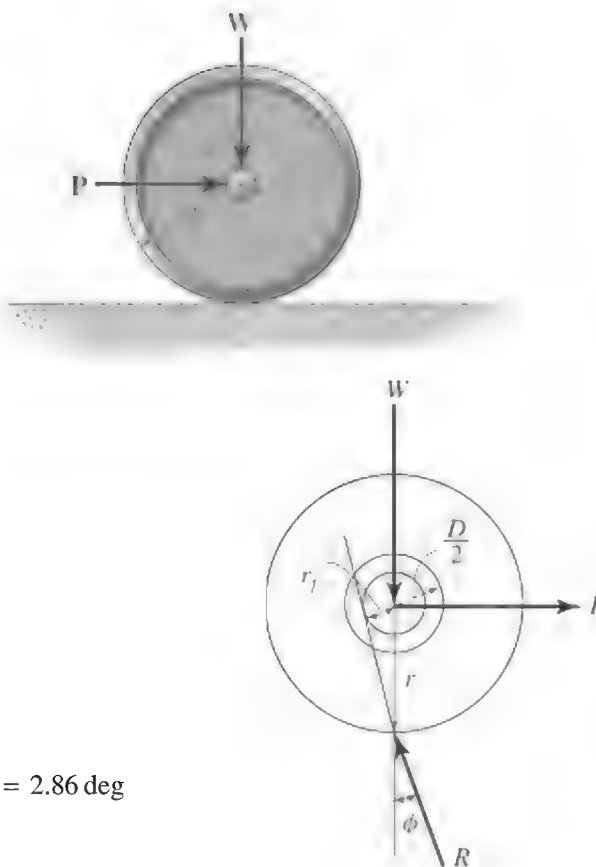
$r = 16\ \text{in}$

Solution:

$\Sigma F_x = 0; \qquad P - R\sin(\phi) = 0$

$\Sigma F_y = 0; \qquad R\cos(\phi) - W = 0$

Thus, $\qquad P = W\tan(\phi)$

$\phi_k = \text{atan}(\mu_k) \qquad \phi_k = 2.86\ \text{deg}$

$r_f = \left(\frac{D}{2}\right)\sin(\phi_k)$ $r_f = 0.04994\ \text{in}$

$\phi = \text{asin}\left(\frac{r_f}{r}\right)$ $\phi = 0.1788\ \text{deg}$

$P = W\tan(\phi)$

$P = 62.4\ \text{lb}$

Problem 8-124

The trailer has a total weight W and center of gravity at G which is directly over its axle. If the axle has a diameter D, the radius of the wheel is r, and the coefficient of kinetic friction at the bearing is μ_k, determine the horizontal force **P** needed to pull the trailer.

Given:

$W = 850\ \text{lb}$

$r = 1.5\ \text{ft}$

$\mu_k = 0.08$

$D = 1\ \text{in}$

Solution:

$\Sigma F_x = 0;$ $P - R\sin(\phi) = 0$

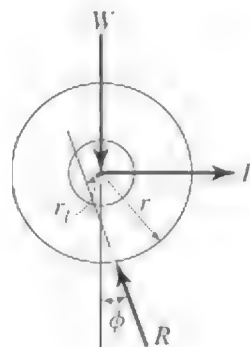
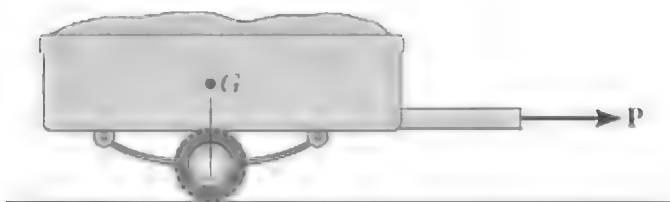
$\Sigma F_y = 0;$ $R\cos(\phi) - W = 0$

Thus, $P = W\tan(\phi)$

$\phi_k = \text{atan}(\mu_k)$ $\phi_k = 4.57\ \text{deg}$

$r_f = \left(\frac{D}{2}\right)\sin(\phi_k)$ $r_f = 0.03987\ \text{in}$

$\phi = \text{asin}\left(\frac{r_f}{r}\right)$ $\phi = 0.1269\ \text{deg}$

$P = W\tan(\phi)$ $P = 1.88\ \text{lb}$

Problem 8-125

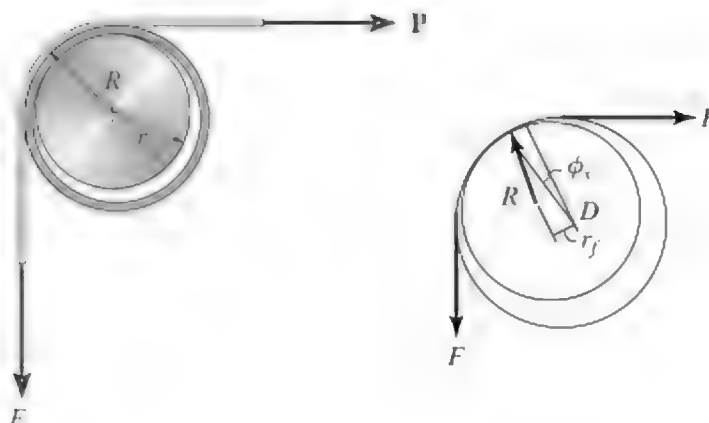
The collar fits *loosely* around a fixed shaft that has radius r. If the coefficient of kinetic friction between the shaft and the collar is μ_k, determine the force **P** on the horizontal segment of the belt so that the collar rotates counterclockwise with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt is R.

Given:

$r = 2$ in

$\mu_k = 0.3$

$R = 2.25$ in

$F = 20$ lb

Solution: $\phi_k = \text{atan}(\mu_k)$ $\phi_k = 16.699\,\text{deg}$

$r_f = r\sin(\phi_k)$ $r_f = 0.5747\,\text{in}$

Equilibrium:

$+\uparrow \Sigma F_y = 0;$ $R_y - F = 0$ $R_y = F$ $R_y = 20.00\,\text{lb}$

$\xrightarrow{+} \Sigma F_x = 0;$ $P - R_x = 0$ $R_x = P$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{P^2 + F^2}$$

Guess $P = 1$ lb

Given $-\left(\sqrt{P^2 + F^2}\right) r_f + F R - P R = 0$ $P = \text{Find}(P)$ $P = 13.79\,\text{lb}$

Problem 8-126

The collar fits *loosely* around a fixed shaft that has radius r. If the coefficient of kinetic friction between the shaft and the collar is μ_k, determine the force **P** on the horizontal segment

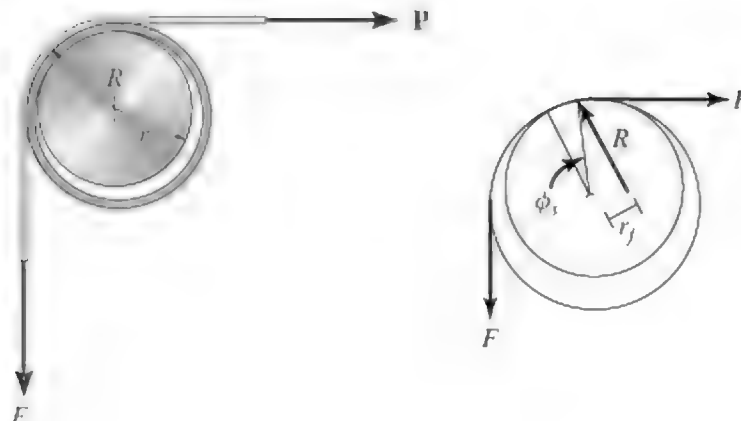
of the belt so that the collar rotates clockwise with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt is R.

Given:

$r = 2$ in

$\mu_k = 0.3$

$R = 2.25$ in

$F = 20$ lb

Solution: $\phi_k = \text{atan}(\mu_k)$ $\phi_k = 16.699$ deg

$r_f = r\sin(\phi_k)$ $r_f = 0.5747$ in

Equilibrium:

$+\uparrow \Sigma F_y = 0;$ $R_y - F = 0$ $R_y = F$ $R_y = 20.00$ lb

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $P - R_x = 0$ $R_x = P$

$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2}} = \sqrt{P^2 + F^2}$$

Guess $P = 1$ lb

Given $\left(\sqrt{P^2 + F^2}\right) r_f + F R - P R = 0$ $P = \text{Find}(P)$ $P = 29.00$ lb

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Problem 8-127

The connecting rod is attached to the piston by a pin at B of diameter d_1 and to the crank shaft by a bearing A of diameter d_2. If the piston is moving downwards, and the coefficient of static friction at these points is μ_s, determine the radius of the friction circle at each connection.

Given:

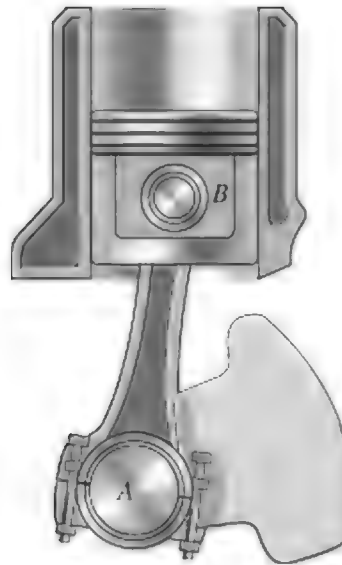
$d_1 = 0.75$ in

$d_2 = 2\ \text{in}$

$\mu_s = 0.2$

Solution:

$r_{fA} = \frac{1}{2} d_2\ \mu_s \qquad r_{fA} = 0.2\ \text{in}$

$r_{fB} = \frac{1}{2} d_1\ \mu_s \qquad r_{fB} = 0.075\ \text{in}$

Problem 8-128

The connecting rod is attached to the piston by a pin at B of diameter d_1 and to the crank shaft by a bearing A of diameter d_2. If the piston is moving upwards, and the coefficient of static friction at these points is μ_s, determine the radius of the friction circle at each connection.

Given:

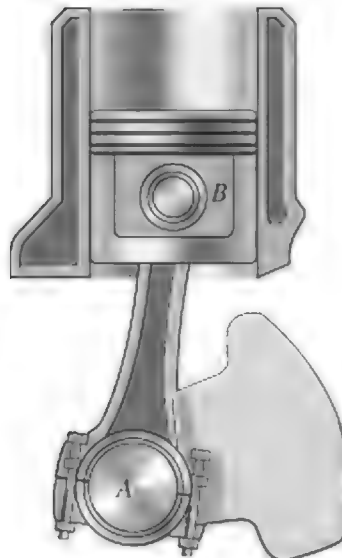
$d_1 = 20\ \text{mm}$

$d_2 = 50\ \text{mm}$

$\mu_s = 0.3$

Solution:

$r_{fA} = \frac{1}{2} d_2\ \mu_s \qquad r_{fA} = 7.50\ \text{mm}$

$r_{fB} = \frac{1}{2} d_1\ \mu_s \qquad r_{fB} = 3\ \text{mm}$

Problem 8-129

The lawn roller has mass M. If the arm BA is held at angle θ from the horizontal and the coefficient of rolling resistance for the roller is r, determine the force **P** needed to push the roller at constant speed. Neglect friction developed at the axle, A, and assume that the resultant force **P** acting on the handle is applied along arm BA.

Given:

$M = 80 \text{ kg}$

$\theta = 30 \text{ deg}$

$a = 250 \text{ mm}$

$r = 25 \text{ mm}$

Solution:

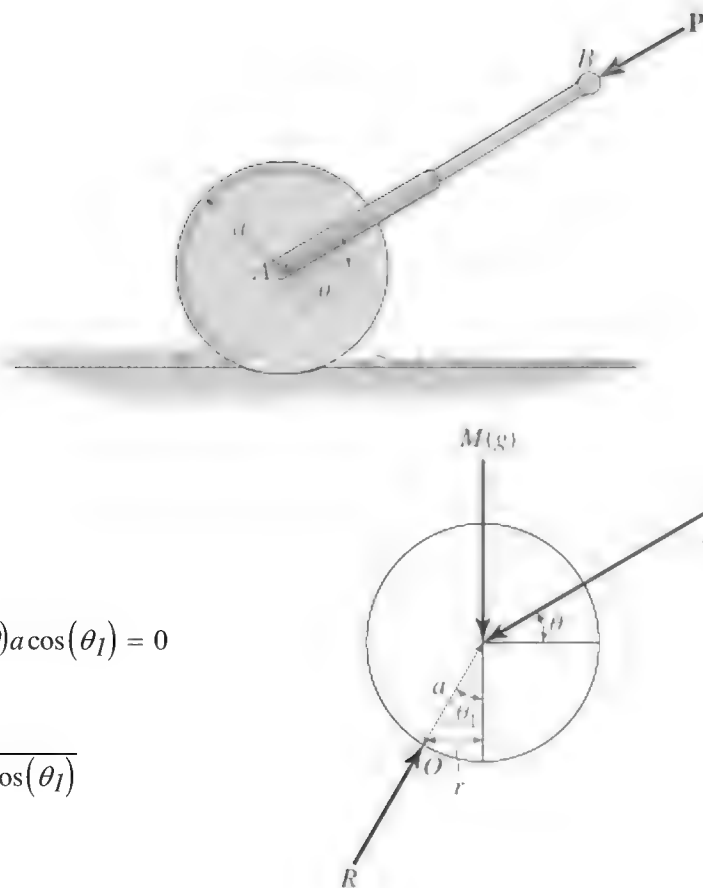
$$\theta_1 = \operatorname{asin}\left(\frac{r}{a}\right)$$

$$\theta_1 = 5.74 \text{ deg}$$

$\Sigma M_O = 0;$

$$-r M g - P \sin(\theta) r + P \cos(\theta) a \cos(\theta_1) = 0$$

$$P = \frac{r M g}{-\sin(\theta)\, r + \cos(\theta)\, a \cos(\theta_1)}$$

$$P = 96.7 \text{ N}$$

Problem 8-130

The handcart has wheels with a diameter D. If a crate having a weight W is placed on the cart, determine the force **P** that must be applied to the handle to overcome the rolling resistance. The coefficient of rolling resistance is μ. Neglect the weight of the cart.

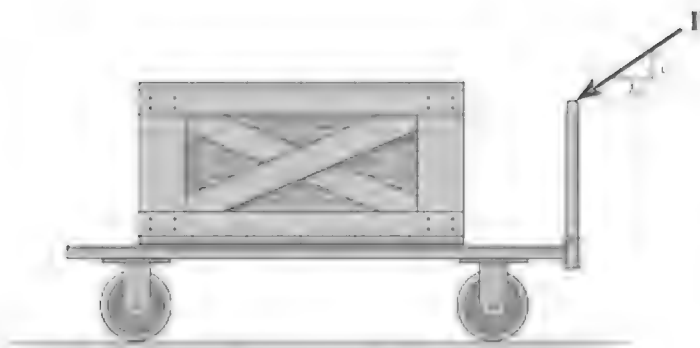
Given:

$W = 1500\ \text{lb}$

$D = 6\ \text{in}$

$a = 0.04\ \text{in}$

$c = 3$

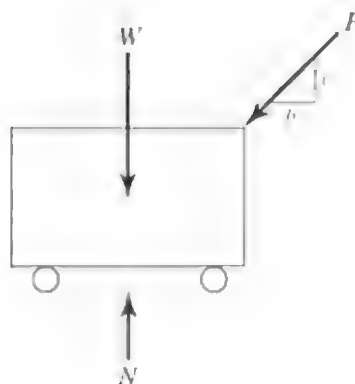
$b = 4$

Solution:

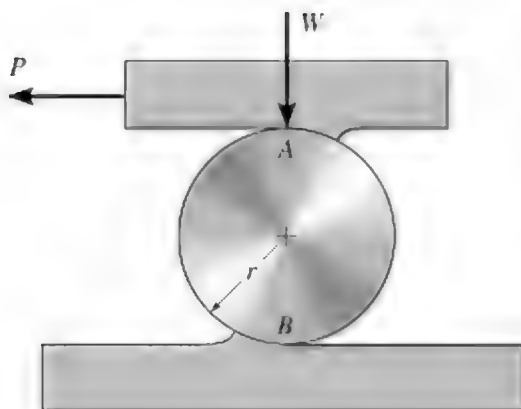
Guesses $N = 1\ \text{lb}$ $P = 1\ \text{lb}$

Given

$$N - W - P\left(\frac{c}{\sqrt{c^2 + b^2}}\right) = 0$$

$$\left(\frac{b}{\sqrt{b^2 + c^2}}\right)P = N\left(\frac{2a}{D}\right)$$

$$\begin{pmatrix} N \\ P \end{pmatrix} = \text{Find}(N, P) \qquad N = 1515\ \text{lb} \qquad P = 25.3\ \text{lb}$$

Problem 8-131

The cylinder is subjected to a load that has a weight W. If the coefficients of rolling resistance for the cylinder's top and bottom surfaces are a_A and a_B respectively, show that a force having a magnitude of $P = [W(a_A + a_B)]/2r$ is required to move the load and thereby roll the cylinder forward. Neglect the weight of the cylinder.

Solution:

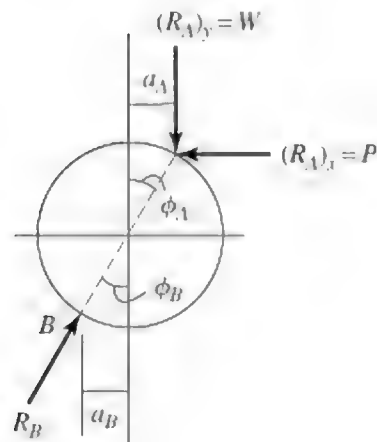
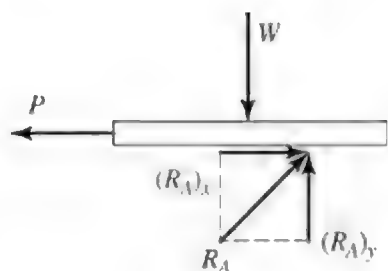
$\overset{+}{\rightarrow}\ \Sigma F_x = 0; \quad R_{Ax} - P = 0 \qquad R_{Ax} = P$

$+\uparrow \Sigma F_y = 0; \quad R_{Ay} - W = 0 \qquad R_{Ay} = \mathrm{P}$

$\Sigma M_B = 0; \quad P\left(r\cos(\phi_A) + r\cos(\phi_B)\right) - W(a_A + a_B) = 0$

Since ϕ_A and ϕ_B are very small,

$\cos(\phi_A) = \cos(\phi_B) = 1$ Hence from Eq.(1)

$$P = \frac{W(a_A + a_B)}{2r} \qquad \text{(QED)}$$

Problem 8-132

A steel beam of mass M is moved over a level surface using a series of rollers of diameter D for which the coefficient of rolling resistance is a_g at the ground and a_s at the bottom surface of the beam. Determine the horizontal force **P** needed to push the beam forward at a constant speed. *Hint:* Use the result of Prob. 8–131.

Units Used:

$\mathrm{Mg} = 1000\ \mathrm{kg}$

Given:

$M = 1.2\ \text{Mg}$

$D = 30\ \text{mm}$

$a_g = 0.4\ \text{mm}$

$a_s = 0.2\ \text{mm}$

Solution:

$$P = \frac{M g(a_g + a_s)}{2\left(\frac{D}{2}\right)} \qquad P = 235\,\text{N}$$

Problem 8-133

A machine of mass M is to be moved over a level surface using a series of rollers for which the coefficient of rolling resistance is a_g at the ground and a_m at the bottom surface of the machine. Determine the appropriate diameter of the rollers so that the machine can be pushed forward with a horizontal force **P**. *Hint:* Use the result of Prob. 8-131.

Units Used:

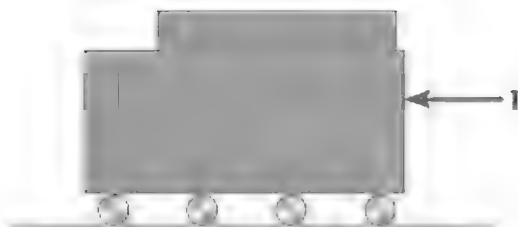
$\text{Mg} = 1000\ \text{kg}$

Given:

$M = 1.4\ \text{Mg}$

$a_g = 0.5\ \text{mm}$

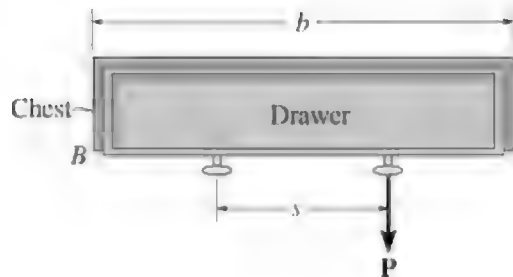
$a_m = 0.2\ \text{mm}$

$P = 250\ \text{N}$

Solution:

$$P = \frac{M\ g(a_g + a_m)}{2\ r}$$

$$r = M g\left(\frac{a_g + a_m}{2\ P}\right) \qquad r = 19.2\,\text{mm} \qquad d = 2\ r \qquad d = 38.5\,\text{mm}$$

Problem 8-134

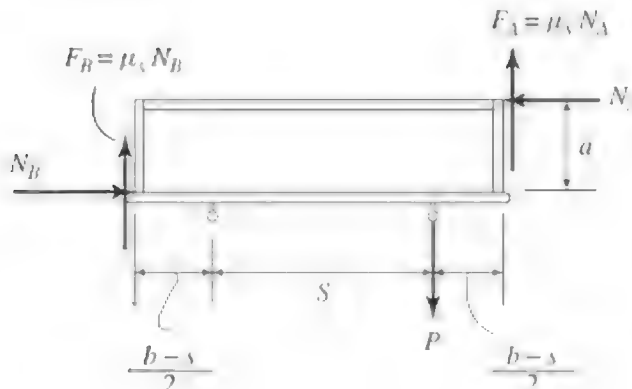
A single force **P** is applied to the handle of the drawer. If friction is neglected at the bottom and the coefficient of static friction along the sides is μ_s determine the largest spacing s between the symmetrically placed handles so that the drawer does not bind at the corners A and B when the force **P** is applied to one of the handles.

Given:

$\mu_s = 0.4$

$a = 0.3 \text{ m}$

$b = 1.25 \text{ m}$

Solution:

Equation of Equilibrium and Friction : If the drawer does not bind at corners A and B, slipping would have to occur at points A and B. Hence, $F_A = \mu N_A$ and $F_B = \mu N_B$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_B - N_A = 0$

$N_A = N_B = N$

$+\uparrow \Sigma F_y = 0; \qquad \mu_s N_A + \mu_s N_B - P = 0 \qquad P = 2\mu_s N$

$\Sigma M_B = 0; \qquad Na + \mu_s Nb - P\left(\frac{s+b}{2}\right) = 0$

$$\left[a + \mu_s\, b - 2\mu_s\left(\frac{s+b}{2}\right)\right] N = 0$$

$$a + \mu_s\, b - \mu_s\,(s+b) = 0 \qquad s = \frac{a}{\mu_s} \qquad s = 0.750 \text{ m}$$

Problem 8-135

The truck has mass M and a center of mass at G. Determine the greatest load it can pull if (a) the truck has rear-wheel drive while the front wheels are free to roll, and (b) the truck has

four-wheel drive. The coefficient of static friction between the wheels and the ground is μ_{st} and between the crate and the ground, it is μ_{sc}.

Units Used:

$kN = 10^3\ N$

$Mg = 1000\ kg$

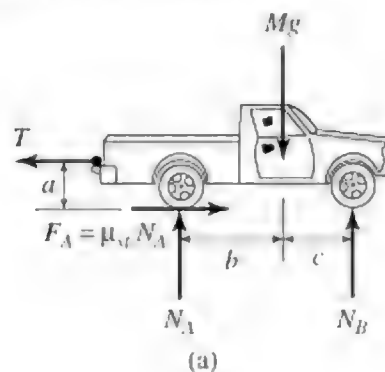
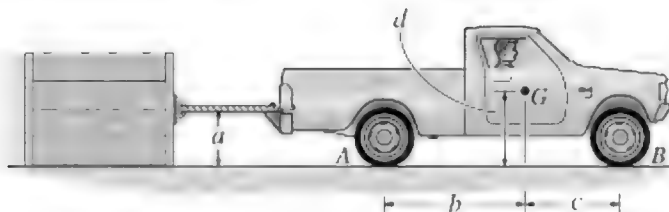
Given:

$M = 1.25\ Mg$

$\mu_{st} = 0.5$ $a = 600\ mm$

$\mu_{sc} = 0.4$ $b = 1.5\ m$

$c = 1\ m$

$g = 9.81\ \frac{m}{s^2}$ $d = 800\ mm$

Solution:

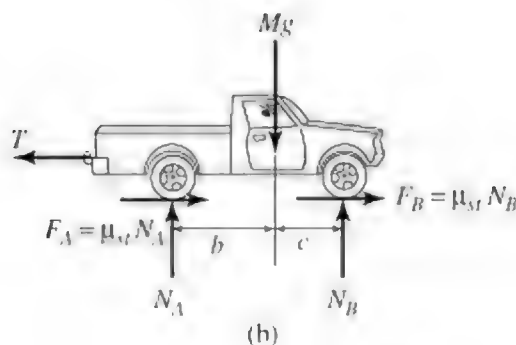
Guesses $N_A = 1\ N$ $N_B = 1\ N$

$T = 1\ N$ $N_C = 1\ N$ $W = 1\ N$

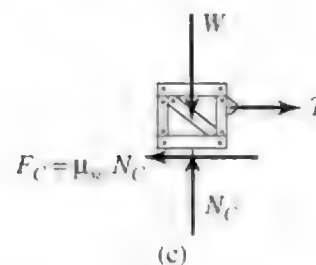
(a) Rear wheel drive

Given $-T + \mu_{st} N_A = 0$

$N_A + N_B - M g = 0$

$-M g b + N_B(b + c) + T a = 0$

$T - \mu_{sc} N_C = 0$

$N_C - W = 0$

$$\begin{pmatrix} N_A \\ N_B \\ T \\ N_C \\ W \end{pmatrix} = \text{Find}(N_A, N_B, T, N_C, W) \qquad W = 6.97\,\text{kN}$$

(b) Four wheel drive

Given $-T + \mu_{st} N_A + \mu_{st} N_B = 0$

$$N_A + N_B - M g = 0$$

$$-M g b + N_B(b + c) + T a = 0$$

$$T - \mu_{sc} N_C = 0$$

$$N_C - W = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ T \\ N_C \\ W \end{pmatrix} = \text{Find}(N_A, N_B, T, N_C, W) \qquad W = 15.33\ \text{kN}$$

Problem 8-136

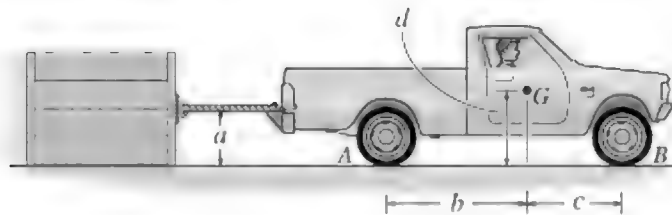
The truck has M and a center of mass at G. The truck is traveling up an incline of angle θ. Determine the greatest load it can pull if (a) the truck has rear-wheel drive while the front wheels are free to roll, and (b) the truck has four-wheel drive. The coefficient of static friction between the wheels and the ground is μ_{st} and between the crate and the ground, it is μ_{sc}.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

$$\text{Mg} = 1000\ \text{kg}$$

Given:

$$\theta = 10\ \text{deg}$$

$M = 1.25\ \text{Mg}$ $a = 600\ \text{mm}$

$\mu_{st} = 0.5$ $b = 1.5\ \text{m}$

$\mu_{sc} = 0.4$ $c = 1\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$ $d = 800\ \text{mm}$

Solution:

Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$

$T = 1\ \text{N}$ $N_C = 1\ \text{N}$ $W = 1\ \text{N}$

(a) Rear wheel drive

Given

$$-T + \mu_{st} N_A - M g \sin(\theta) = 0$$

$$N_A + N_B - M g \cos(\theta) = 0$$

$$-M g b \cos(\theta) + M g d \sin(\theta) + N_B(b + c) + T a = 0$$

$$T - \mu_{sc} N_C - W \sin(\theta) = 0$$

$$N_C - W \cos(\theta) = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ T \\ N_C \\ W \end{pmatrix} = \text{Find}(N_A, N_B, T, N_C, W) \qquad W = 1.25\ \text{kN}$$

(b) Four wheel drive

Given

$$-T + \mu_{st} N_A + \mu_{st} N_B - M g \sin(\theta) = 0$$

$$N_A + N_B - M g \cos(\theta) = 0$$

$$-M g b \cos(\theta) + M g d \sin(\theta) + N_B(b + c) + T a = 0$$

$$T - \mu_{sc} N_C - W \sin(\theta) = 0$$

$$N_C - W \cos(\theta) = 0$$

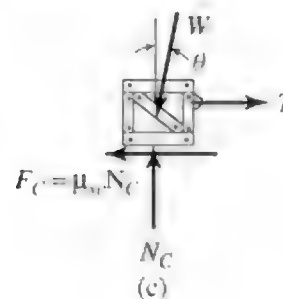
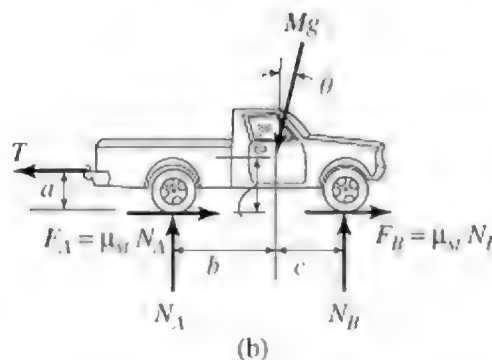
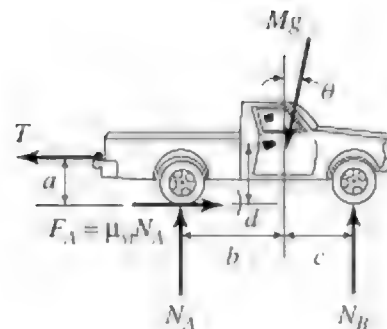
Mg θ T a $F_A = \mu_{st} N_A$ d b c N_A N_B

(a)

Mg θ T a $F_A = \mu_{st} N_A$ $F_B = \mu_{st} N_B$ b c N_A N_B

(b)

W θ T $F_C = \mu_{sc} N_C$ N_C

(c)

$$\begin{pmatrix} N_A \\ N_B \\ T \\ N_C \\ W \end{pmatrix} = \text{Find}(N_A, N_B, T, N_C, W) \qquad W = 6.89\,\text{kN}$$

Problem 8-137

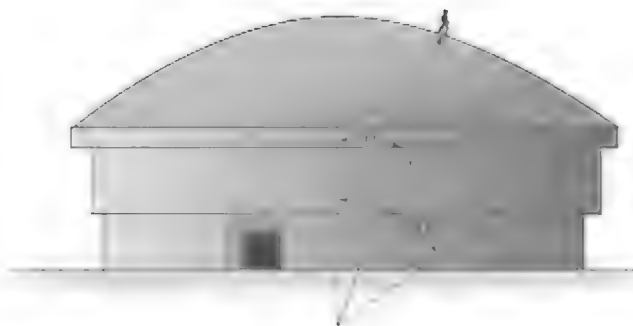
A roofer, having a mass M, walks slowly in an upright position down along the surface of a dome that has a radius of curvature ρ. If the coefficient of static friction between his shoes and the dome is μ_s determine the angle θ at which he first begins to slip.

Given:

$M = 70$ kg

$\rho = 20$ m

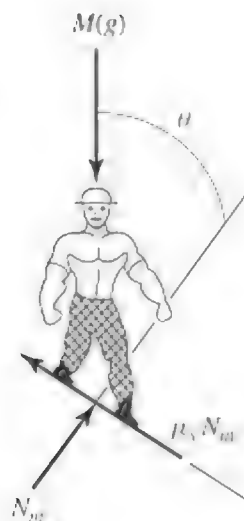
$\mu_s = 0.7$

Solution:

$\Sigma F_{y'} = 0;\qquad N_m - M g\cos(\theta) = 0$

$\Sigma F_{x'} = 0;\qquad M g\sin(\theta) - \mu_s N_m = 0$

$$\mu_s = \tan(\theta)$$

$$\theta = \text{atan}(\mu_s)$$

$$\theta = 35.0\,\text{deg}$$

Problem 8-138

A man attempts to lift the uniform ladder of weight W to an upright position by applying a force **P** perpendicular to the ladder at rung R. Determine the coefficient of static friction between the ladder and the ground at A if the ladder begins to slip on the ground when his

hands reach height c.

Given:

$a = 2\ \text{ft}$

$b = 14\ \text{ft}$

$c = 6\ \text{ft}$

$W = 40\ \text{lb}$

Solution:

$$\theta = \operatorname{asin}\left(\frac{c}{b}\right)$$

Initial guesses

$P = 10\ \text{lb}$ $\quad N_A = 100\ \text{lb}$ $\quad \mu_A = 100$

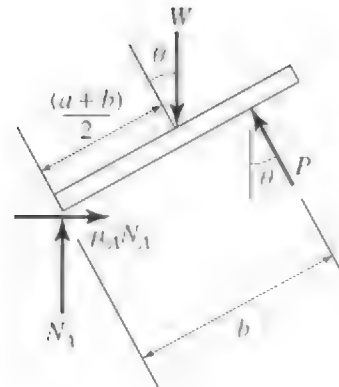
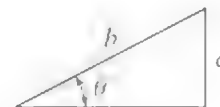
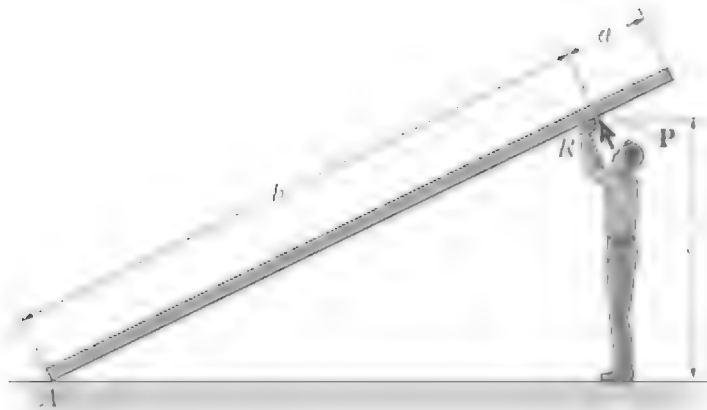
Given

$$\Sigma F_x = 0; \quad \mu_A N_A - P\sin(\theta) = 0$$

$$\Sigma F_y = 0; \quad N_A - W + P\cos(\theta) = 0$$

$$\Sigma M_A = 0; \quad -W\left(\frac{b+a}{2}\right)\cos(\theta) + P\,b = 0$$

$$\begin{pmatrix} P \\ N_A \\ \mu_A \end{pmatrix} = \text{Find}(P, N_A, \mu_A) \qquad \begin{pmatrix} P \\ N_A \end{pmatrix} = \begin{pmatrix} 20.7 \\ 21.3 \end{pmatrix}\text{lb}$$

$$\mu_A = 0.41$$

Problem 8-139

Column D is subjected to a vertical load W. It is supported on two identical wedges A and B for which the coefficient of static friction at the contacting surfaces between A and B and between B and C is μ_s. Determine the force **P** needed to raise the column and the equilibrium force **P'** needed to hold wedge A stationary. The contacting surface between A and D is smooth.

Units Used:

$kip = 10^3\ lb$

Given:

$W = 8000\ lb$

$\theta = 10\ deg$

$\mu_s = 0.4$

Solution:

wedge A:

$\Sigma F_y = 0;\qquad N\cos(\theta) - \mu_s N\sin(\theta) - W = 0$

$$N = \frac{W}{\cos(\theta) - \mu_s \sin(\theta)}$$

$N = 8739.8\ lb$

$\Sigma F_x = 0;\qquad \mu_s N\cos(\theta) + N\sin(\theta) - P' = 0$

$P' = \mu_s\ N\ \cos(\theta) + N\ \sin(\theta)$

$P' = 4.96\ kip$

Wedge B:

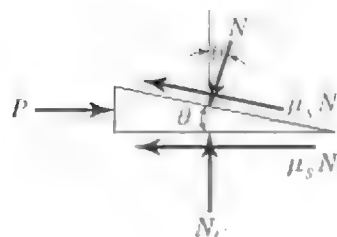
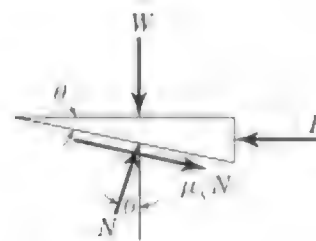
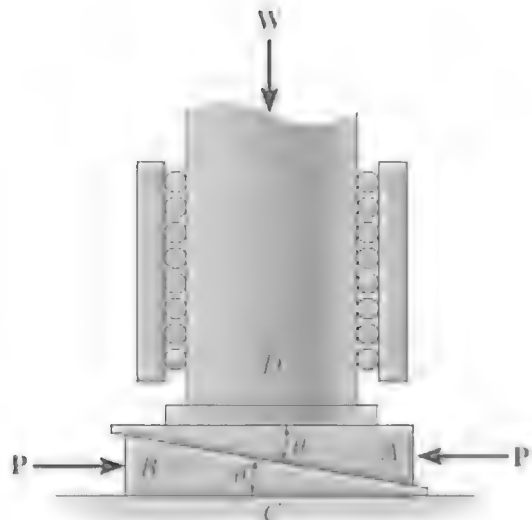
$\Sigma F_y = 0;\qquad N_C + \mu_s N\sin(\theta) - N\cos(\theta) = 0$

$N_C = -\mu_s\ N\ \sin(\theta) + N\ \cos(\theta)$

$N_C = 8000\ lb$

$\Sigma F_x = 0;\qquad P - \mu_s N_C - N\sin(\theta) - \mu_s N\cos(\theta) = 0$

$P = \mu_s\ N_C + N\ \sin(\theta) + \mu_s N\ \cos(\theta)$

$P = 8.16\ kip$

Problem 8-140

Column D is subjected to a vertical load W. It is supported on two identical wedges A and B for which the coefficient of static friction at the contacting surfaces between A and B and between B and C is μ_s. If the forces **P** and **P'** are removed, are the wedges self-locking? The contacting surface between A and D is smooth.

Given:

$W = 8000 \text{ lb}$

$\theta = 10 \text{ deg}$

$\mu_s = 0.4$

Solution:

Wedge A:

$\Sigma F_y = 0;$ $N - W\cos(\theta) = 0$

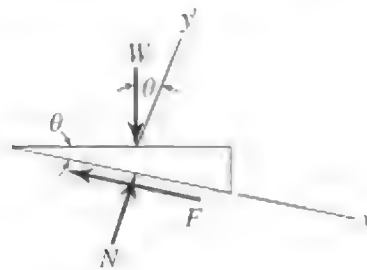
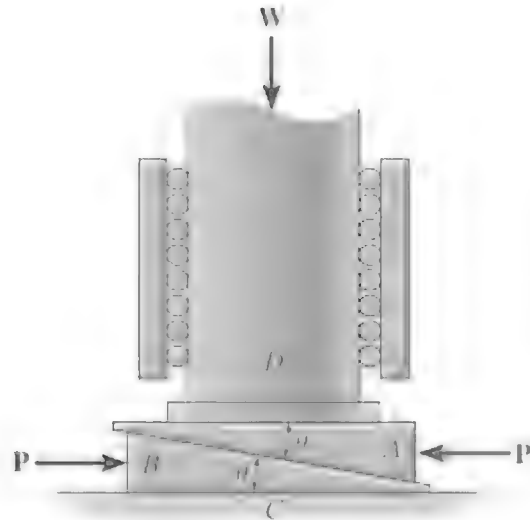
$N = W\cos(\theta)$

$N = 7878.5 \text{ lb}$

$\Sigma F_x = 0;$ $W\sin(\theta) - F = 0$

$F = W\sin(\theta)$

$F = 1389.2 \text{ lb}$

Friction $F_{max} = \mu_s N$

Since $F = 1389 \text{ lb} < F_{max} = 3151 \text{ lb}$ then the wedges do not slip at the contact surface AB.

Wedge B:

$\Sigma F_y = 0;$ $N_C - F\sin(\theta) - N\cos(\theta) = 0$

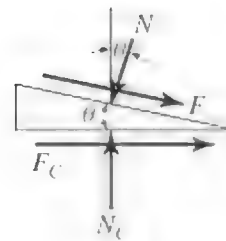
$N_C = F\sin(\theta) + N\cos(\theta)$

$N_C = 8000 \text{ lb}$

$\Sigma F_x = 0;$ $F_C + F\cos(\theta) - N\sin(\theta) = 0$

$F_C = -F\cos(\theta) + N\sin(\theta)$

$F_C = 0 \text{ lb}$

Friction $F_{Cmax} = \mu_s N_C$

Since $F_C = 0 \text{ lb} < F_{Cmax} = 3200 \text{ lb}$ then the wedges do not slip at the contact surface BC.

Therefore the wedges are self-locking.

Problem 9-1

Locate the center of mass of the homogeneous rod bent in the form of a parabola.

Given:

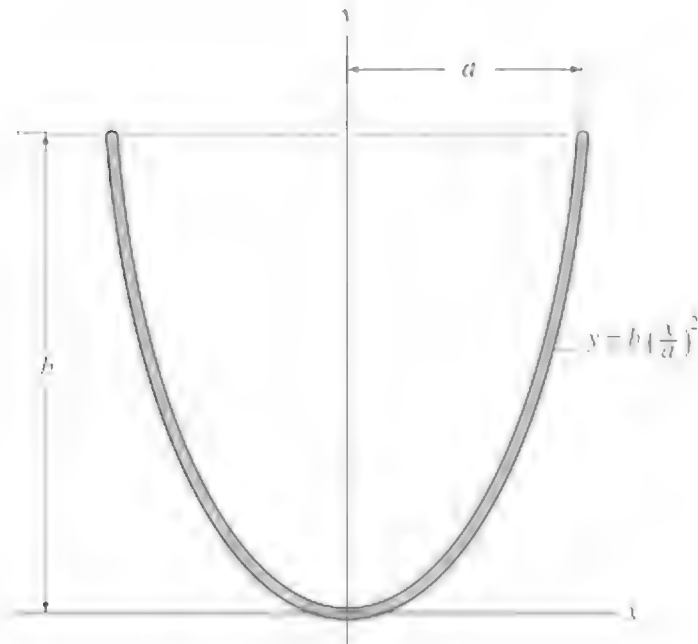
$a = 1\ \text{m}$

$b = 2\ \text{m}$

Solution:

$$y = b\left(\frac{x}{a}\right)^2$$

$$\frac{d\,y}{d\,x} = \frac{2b}{a^2}x$$

$$y_c = \frac{\displaystyle\int_0^a b\left(\frac{x}{a}\right)^2\sqrt{1+\left(\frac{2b}{a^2}x\right)^2}\,dx}{\displaystyle\int_0^a \sqrt{1+\left(\frac{2b}{a^2}x\right)^2}\,dx}$$

$y_c = 0.912\ \text{m}$ $\quad x_c = 0\ \text{m}$

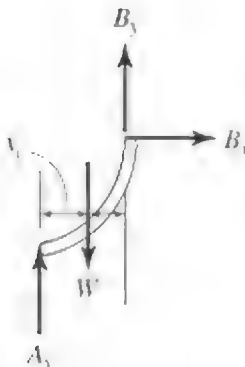
Problem 9-2

Locate the center of gravity x_c of the homogeneous rod. If the rod has a weight per unit length γ, determine the vertical reaction at A and the x and y components of reaction at the pin B.

Given:

$\gamma = 0.5\ \frac{\text{lb}}{\text{ft}}$

$a = 1\ \text{ft}$

$b = 2\ \text{ft}$

Solution:

$$y = b\left(\frac{x}{a}\right)^2 \qquad \frac{dy}{dx} = \frac{2b}{a^2}x$$

$$L = \int_0^a \sqrt{1+\left(\frac{2b}{a^2}x\right)^2}\,dx \qquad W = \gamma L \qquad W = 1.162\,\text{lb}$$

$$x_c = \frac{1}{L}\int_0^a x\sqrt{1+\left(\frac{2b}{a^2}x\right)^2}\,dx \qquad x_c = 0.620\,\text{ft}$$

Guesses $A_y = 1\text{lb}$ $B_x = 1\text{lb}$ $B_y = 1\text{lb}$

Given $B_x = 0$ $A_y + B_y - W = 0$ $-A_y\,a + W\left(a - x_c\right) = 0$

$$\begin{pmatrix} B_x \\ B_y \\ A_y \end{pmatrix} = \text{Find}\left(B_x, B_y, A_y\right) \qquad \begin{pmatrix} B_x \\ B_y \\ A_y \end{pmatrix} = \begin{pmatrix} 0.000 \\ 0.720 \\ 0.442 \end{pmatrix}\text{lb}$$

Problem 9-3

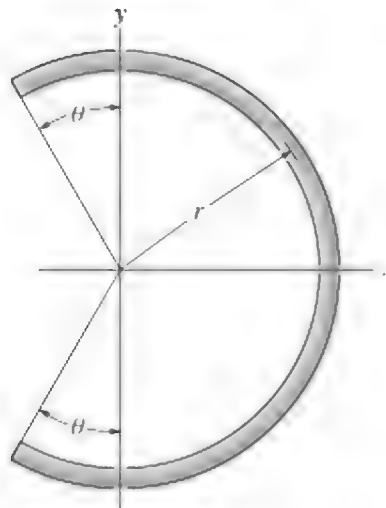
Locate the center of mass of the homogeneous rod bent into the shape of a circular arc.

Given:

$r = 300$ mm

$\theta = 30$ deg

Solution:

$y_c = 0$ Symmetry

$$x_c = \frac{\int_{\frac{-\pi}{2}-\theta}^{\frac{\pi}{2}+\theta} r\cos(\alpha)\, r\,d\alpha}{\int_{\frac{-\pi}{2}-\theta}^{\frac{\pi}{2}+\theta} r\,d\alpha} \qquad x_c = 124.049\text{ mm}$$

Problem 9-4

Locate the center of gravity x_c of the homogeneous rod bent in the form of a semicircular arc. The rod has a weight per unit length γ. Also, determine the horizontal reaction at the smooth support B and the x and y components of reaction at the pin A.

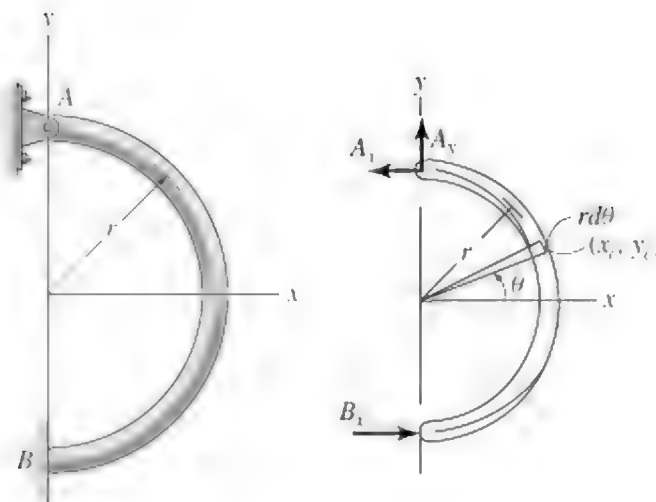
Given:

$$\gamma = 0.5\ \frac{\text{lb}}{\text{ft}}$$

$$r = 2\text{ ft}$$

Solution:

$$x_c = \frac{\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} r\cos(\theta) r\,d\theta}{\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} r\,d\theta} \qquad x_c = 1.273\text{ ft}$$

$\Sigma M_A = 0;\qquad -\pi\, r\,\gamma\, x_c + B_x\,(2\,r) = 0 \qquad B_x = \frac{\pi\, r\gamma x_c}{2r} \qquad B_x = 1\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + B_x = 0 \quad A_x = B_x \quad A_x = 1\,\text{lb}$

$+\uparrow \Sigma F_y = 0; \quad A_y - \pi r \gamma = 0 \quad A_y = \pi r \gamma \quad A_y = 3.14\,\text{lb}$

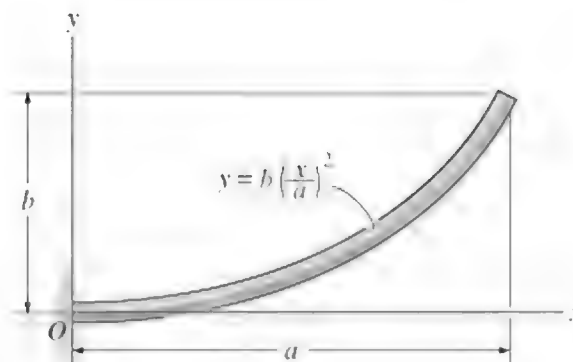
Problem 9-5

Determine the distance x_c to the center of gravity of the homogeneous rod bent into the parabolic shape. If the rod has a weight per unit length γ determine the reactions at the fixed support O.

Given:

$$\gamma = 0.5\ \frac{\text{lb}}{\text{ft}}$$

$$a = 1\ \text{ft}$$

$$b = 0.5\ \text{ft}$$

Solution:

$$y = b\left(\frac{x}{a}\right)^2$$

$$\frac{dy}{dx} = \frac{2bx}{a^2}$$

$$L = \int_0^a \sqrt{1 + \left(\frac{2bx}{a^2}\right)^2}\,dx \qquad L = 1.148\,\text{ft}$$

$$x_c = \frac{1}{L}\int_0^a x\sqrt{1 + \left(\frac{2bx}{a^2}\right)^2}\,dx \qquad x_c = 0.531\,\text{ft}$$

$\xrightarrow{+} \Sigma F_x = 0; \quad O_x = 0 \quad O_x = 0\ \text{lb} \quad O_x = 0\,\text{lb}$

$+\uparrow \Sigma F_y = 0;$ $O_y - \gamma L = 0$ $O_y = \gamma L$ $O_y = 0.574 \text{ lb}$

$\Sigma M_O = 0;$ $M_O - \gamma L x_c = 0$ $M_O = \gamma L x_c$ $M_O = 0.305 \text{ lb}\cdot\text{ft}$

Problem 9-6

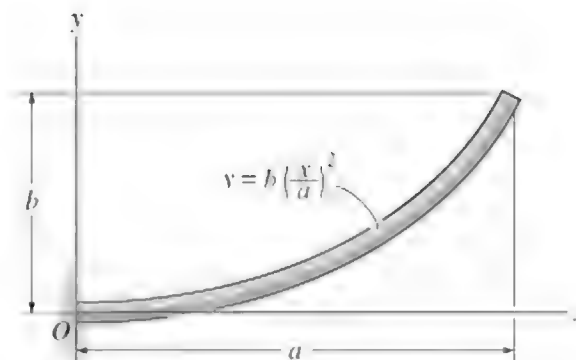
Determine the distance y_c to the center of gravity of the homogeneous rod bent into the parabolic shape.

Given:

$a = 1 \text{ ft}$

$b = 0.5 \text{ ft}$

Solution:

$$y = b\left(\frac{x}{a}\right)^2$$

$$\frac{dy}{dx} = \frac{2bx}{a^2}$$

$$L = \int_0^a \sqrt{1 + \left(\frac{2bx}{a^2}\right)^2}\, dx \qquad L = 1.148 \text{ ft}$$

$$y_c = \frac{1}{L}\left[\int_0^a b\left(\frac{x}{a}\right)^2 \sqrt{1 + \left(\frac{2bx}{a^2}\right)^2}\, dx\right] \qquad y_c = 0.183 \text{ ft}$$

Problem 9-7

Locate the centroid of the parabolic area.

Solution: $a = \dfrac{h}{b^2}$

$$dA = x\,dy$$

$$x_c = \frac{x}{2}$$

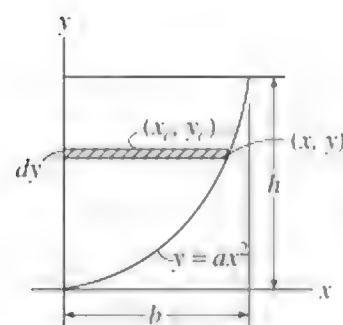
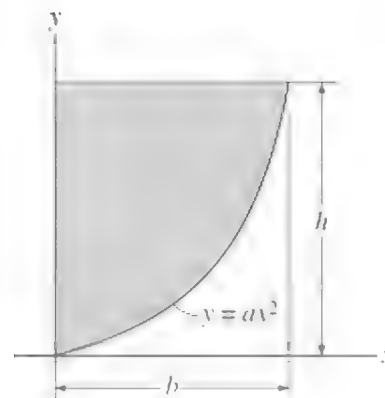
$$y_c = y$$

$$A = \int_0^h b\sqrt{\frac{y}{h}}\,dy = h\,b\left(\frac{y}{h}\right)^{\frac{1}{2}}$$

$$x_c = \frac{3}{2hb}\int_0^h \frac{1}{2}\left(b\sqrt{\frac{y}{h}}\right)^2 dy = \frac{3}{8\,h^2}\,b\,h^2$$

$$x_c = \frac{3}{8}b$$

$$y_c = \frac{3}{2hb}\int_0^h yb\sqrt{\frac{y}{h}}\,dy = \frac{3}{5}\,h\left(\frac{h}{h}\right)^{\frac{5}{2}}$$

$$y_c = \frac{3}{5}h$$

Problem 9-8

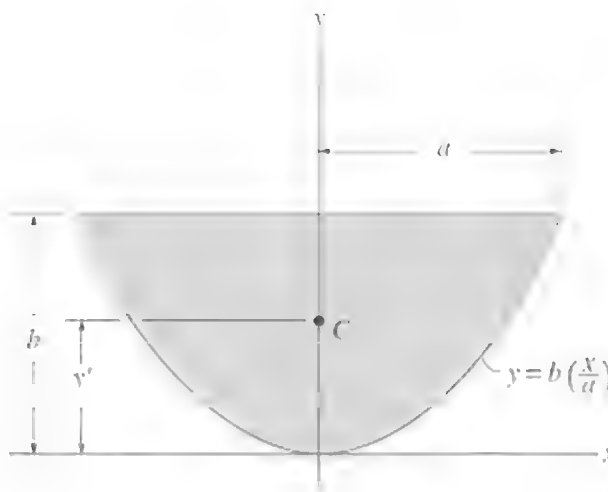
Locate the centroid y_c of the shaded area.

Given:

$a = 100$ mm

$b = 100$ mm

Solution:

$$y = b\left(\frac{x}{a}\right)^2$$

$$x = a\sqrt{\frac{y}{b}}$$

$$y_c = \frac{\int_0^b y2a\sqrt{\frac{y}{b}}\,dy}{\int_0^b 2a\sqrt{\frac{y}{b}}\,dy} \qquad y_c = 60\,\text{mm}$$

Problem 9-9

Locate the centroid x_c of the shaded area.

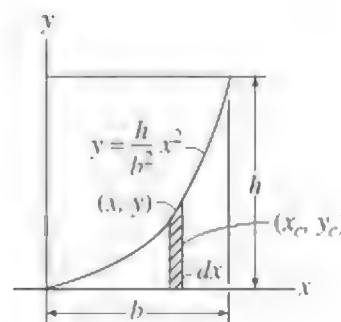
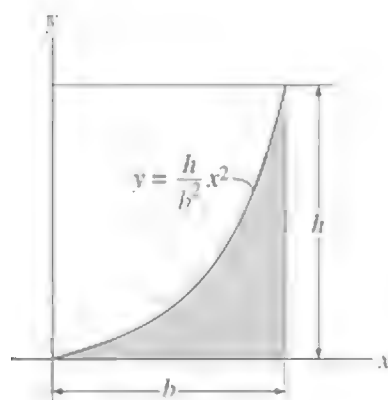
Solution:

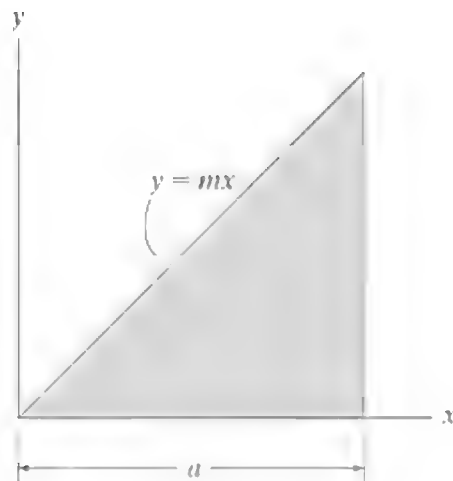
$$dA = y\,dx$$

$$x_c = x$$

$$y_c = \frac{y}{2}$$

$$x_c = \frac{\int_0^b x\frac{h}{b^2}x^2\,dx}{\int_0^b h\frac{x^2}{b^2}\,dx} = \frac{3}{4}\frac{b^4}{b^3} \qquad x_c = \frac{3}{4}b$$

$$y_c = \frac{\int_0^b \frac{1}{2}\left(\frac{h}{b^2}x^2\right)^2 dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{3}{10}\,b^5\,\frac{h^2}{b^5\,h} \qquad y_c = \frac{3}{10}h$$

Problem 9-10

Determine the location (x_c, y_c) of the centroid of the triangular area.

Solution:

$$A = \int_0^a m x \, dx = \frac{1}{2} a^2 m$$

$$x_c = \frac{2}{ma^2} \int_0^a x m x \, dx = \frac{2}{3} a \qquad x_c = \frac{2}{3}a$$

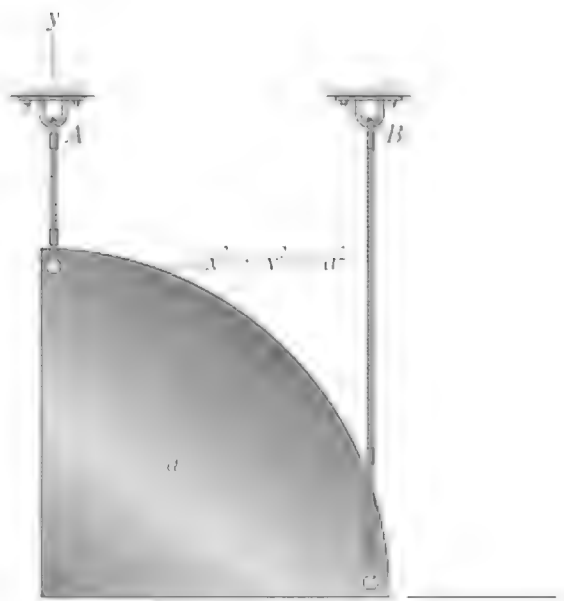
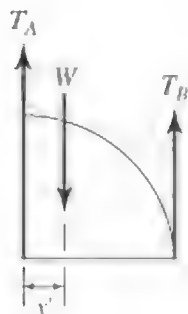
$$y_c = \frac{2}{ma^2} \int_0^a \frac{1}{2}(mx)^2 \, dx = \frac{1}{3} a\, m \qquad y_c = \frac{m}{3}a$$

Problem 9-11

Determine the location (x_c, y_c) of the center of gravity of the quartercircular plate. Also determine the force in each of the supporting wires.The plate has a weight per unit area of γ.

Given:

$$\gamma = 5 \frac{\text{lb}}{\text{ft}^2}$$

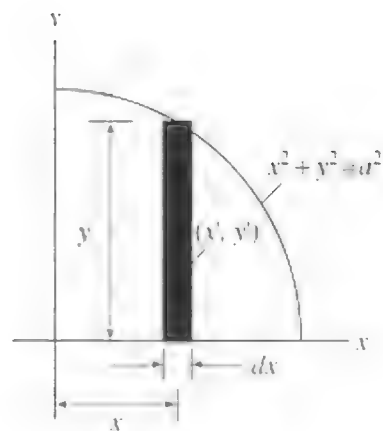
$$a = 4 \text{ ft}$$

Solution:

$$x^2 + y^2 = a^2$$

$$y = \sqrt{a^2 - x^2}$$

$$A = \pi\frac{a^2}{4} \qquad W = A\gamma \qquad W = 62.832\,\text{lb}$$

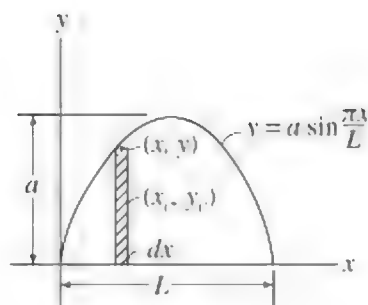
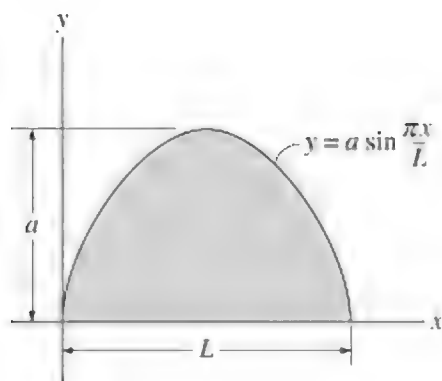
$$x_c = \frac{1}{A}\int_0^a x\sqrt{a^2 - x^2}\,dx \qquad x_c = 1.698\,\text{ft}$$

$$y_c = \frac{1}{A}\int_0^a \frac{1}{2}\left(a^2 - x^2\right)dx \qquad y_c = 1.698\,\text{ft}$$

Guesses $T_A = 1\text{lb}$ $T_B = 1\text{lb}$

Given $T_A + T_B - W = 0$ $T_B a - W x_c = 0$

$$\begin{pmatrix} T_A \\ T_B \end{pmatrix} = \text{Find}(T_A, T_B) \qquad \begin{pmatrix} T_A \\ T_B \end{pmatrix} = \begin{pmatrix} 36.2 \\ 26.7 \end{pmatrix}\text{lb}$$

Problem *9-12

Locate the centroid of the shaded area.

Solution:

$$dA = y\,dx \qquad x_C = x \qquad y_C = \frac{y}{2}$$

$$A = \int_0^L a \sin\left(\frac{\pi x}{L}\right) dx = \frac{2}{\pi} L\, a$$

$$x_C = \frac{\pi}{2La} \int_0^L x\, a \sin\left(\frac{\pi x}{L}\right) dx = \frac{1}{2}\, L \qquad x_C = \frac{1}{2} L$$

$$y_C = \frac{\pi}{2La} \int_0^L \frac{1}{2}\left(a \sin\left(\frac{\pi x}{L}\right)\right)^2 dx = \frac{1}{8}\,\pi a \qquad y_C = \frac{1}{8}\pi a$$

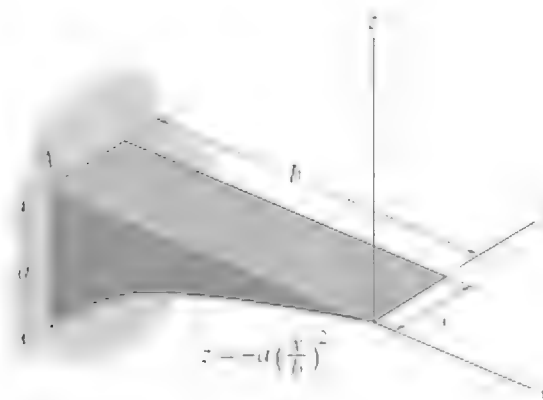
Problem 9-13

Locate the center of gravity of the homogeneous cantilever beam and determine the reactions at the fixed support.The material has a density of ρ.

Units Used: $\quad \text{Mg} = 10^3 \text{ kg} \qquad \text{kN} = 10^3 \text{ N}$

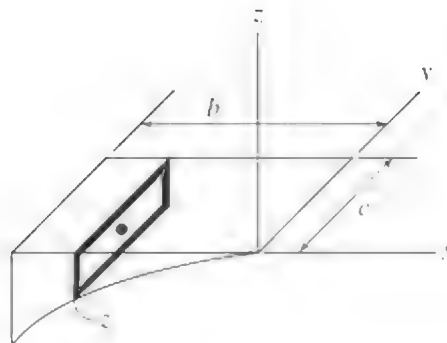
Given: $\quad \rho = 8 \dfrac{\text{Mg}}{\text{m}^3} \qquad a = 1 \text{ m}$

$b = 4 \text{ m}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2} \qquad c = 0.5 \text{ m}$

Solution:

$$V = \int_{-b}^{0} c\,a\left(\frac{x}{b}\right)^2 dx \qquad W = \rho g V$$

$$x_C = \frac{1}{V}\int_{-b}^{0} x\,c\,a\left(\frac{x}{b}\right)^2 dx$$

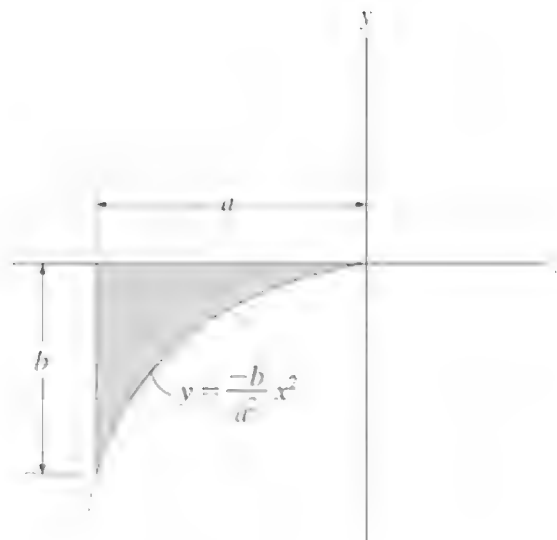
$$y_C = \frac{1}{V}\int_{-b}^{0} \frac{c}{2}\,c\,a\left(\frac{x}{b}\right)^2 dx$$

$$z_c = \frac{1}{V}\int_{-b}^{0} \frac{-c}{2}\left[a\left(\frac{x}{b}\right)^2\right]^2 dx \qquad \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = \begin{pmatrix} -3.00 \\ 0.25 \\ -0.30 \end{pmatrix} \text{m}$$

Guesses $A_x = 1\text{ N}$ $A_y = 1\text{ N}$ $A_z = 1\text{ N}$ $M_A = 1\text{ N}\cdot\text{m}$

Given $A_x = 0$ $A_y = 0$ $A_z - W = 0$ $M_A - W(b + x_c) = 0$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ M_A \end{pmatrix} = \text{Find}(A_x, A_y, A_z, M_A) \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \\ 52.32 \end{pmatrix} \text{kN} \qquad M_A = 52.32\,\text{kN}\cdot\text{m}$$

Problem 9-14

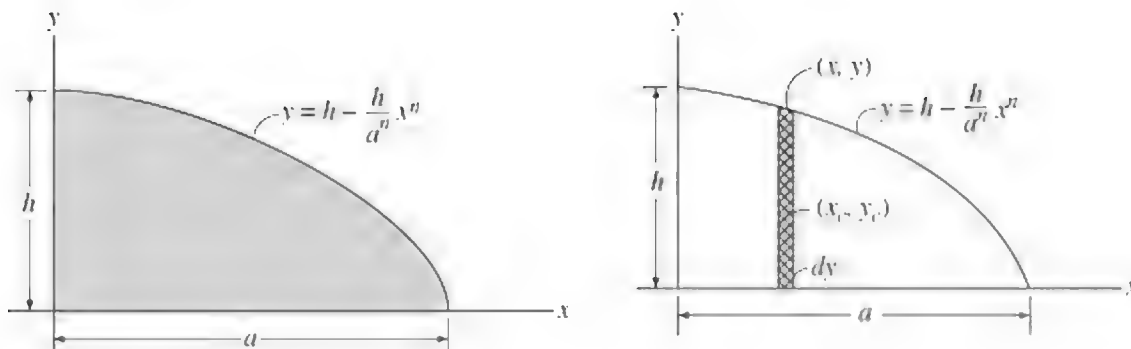
Locate the centroid (x_c, y_c) of the exparabolic segment of area.

Solution:

$$A = \int_{-a}^{0} \frac{b}{a^2}x^2\, dx = \frac{1}{3}\, a\, b$$

$$x_c = \frac{3}{a\,b}\int_{-a}^{0} x\frac{b}{a^2}x^2\, dx = \frac{-3}{4}\, a \qquad x_c = \frac{-3}{4}a$$

$$y_c = \frac{3}{a\,b}\int_{-a}^{0} -\frac{1}{2}\left(\frac{b}{a^2}x^2\right)^2 dx = \frac{-3}{10}b \qquad y_c = \frac{-3}{10}b$$

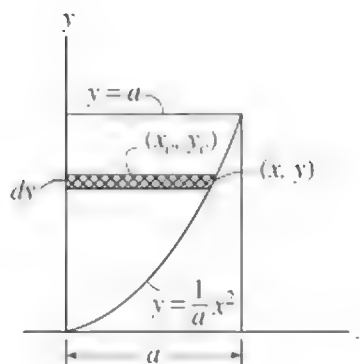
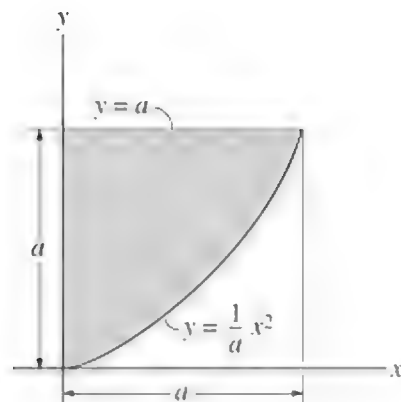
Problem 9-15

Locate the centroid of the shaded area.

Solution:

$$x_c = \frac{\int_0^a x\left(h - \frac{h}{a^n}x^n\right) dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right) dx} \qquad x_c = \frac{n+1}{2(n+2)}a \quad \text{provided that } n \neq -2$$

$$y_c = \frac{\frac{1}{2}\int_0^a \left(h - \frac{h}{a^n}x^n\right)^2 dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right) dx} \qquad y_c = \frac{n}{2n+1}h \quad \text{provided that } n \neq \frac{-1}{2}$$

Problem *9-16

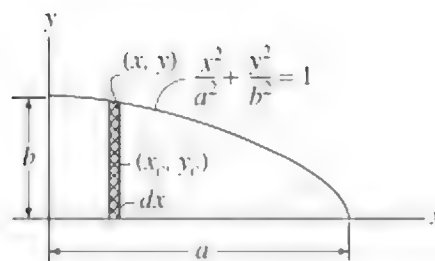
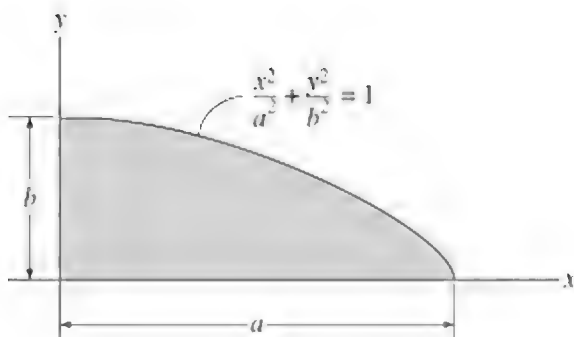
Locate the centroid of the shaded area bounded by the parabola and the line $y = a$.

Solution:

$$A = \int_0^a \sqrt{a\,y}\,dy = \frac{2}{3}\frac{\left(a^2\right)^{\frac{3}{2}}}{a} \qquad A = \frac{2a^2}{3}$$

$$x_c = \frac{3}{2a^2}\int_0^a \frac{1}{2}a\,y\,dy = \frac{3}{8}\,a \qquad x_c = \frac{3}{8}a$$

$$y_c = \frac{3}{2a^2}\int_0^a y\sqrt{a\,y}\,dy = \frac{3}{5\,a^4}\left(a^2\right)^{\frac{5}{2}} \qquad y_c = \frac{3}{5}a$$

Problem 9-17

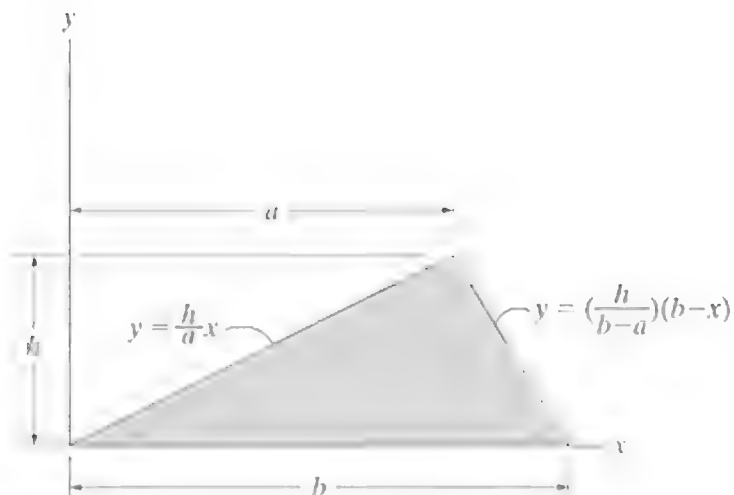
Locate the centroid of the quarter elliptical area.

Solution:

$$A = \int_0^a b\sqrt{1-\left(\frac{x}{a}\right)^2}\,dx \qquad A = \frac{\pi a b}{4}$$

$$x_c = \frac{4}{\pi a b}\int_0^a x b\sqrt{1-\left(\frac{x}{a}\right)^2}\,dx = \frac{4}{3\pi}a \qquad x_c = \frac{4}{3\pi}a$$

$$y_c = \frac{4}{\pi a b}\int_0^a \frac{1}{2}\left[b\sqrt{1-\left(\frac{x}{a}\right)^2}\right]^2 dx = \frac{4}{3\pi}b \qquad y_c = \frac{4}{3\pi}b$$

Problem 9-18

Locate the centroid x_c of the triangular area.

Solution:

$$A = \frac{bh}{2}$$

$$x_c = \frac{2}{bh}\left[\int_0^a x\frac{h}{a}x\,dx + \int_a^b x\frac{h}{b-a}(b-x)\,dx\right] \qquad x_c = \frac{a+b}{3}$$

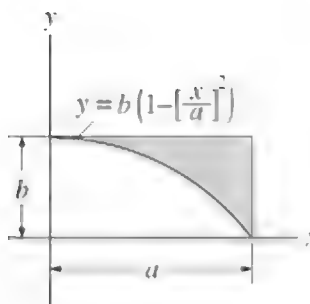
Problem 9-19

Locate the centroid of the shaded area.

Given:

$a = 8 \text{ m}$

$b = 4 \text{ m}$

Solution:

$$A = \int_0^a \left[b - b\left[1 - \left(\frac{x}{a}\right)^2\right]\right] dx \qquad A = 10.667 \text{ m}^2$$

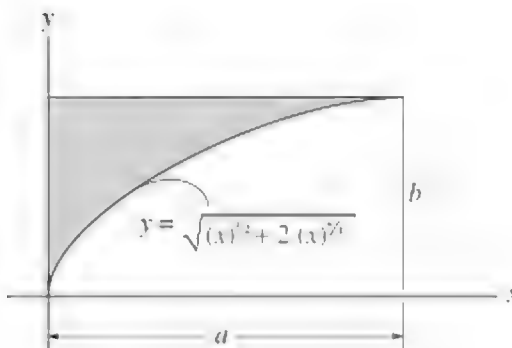
$$x_c = \frac{1}{A}\int_0^a x\left[b - b\left[1 - \left(\frac{x}{a}\right)^2\right]\right] dx \qquad x_c = 6 \text{ m}$$

$$y_c = \frac{1}{A}\int_0^a \frac{1}{2}\left[b + b\left[1 - \left(\frac{x}{a}\right)^2\right]\right]\left[b - b\left[1 - \left(\frac{x}{a}\right)^2\right]\right] dx \qquad y_c = 2.8 \text{ m}$$

Problem 9-20

Locate the centroid x_c of the shaded area. Solve the problem by evaluating the integrals using Simpson's rule.

Given:

$a = 2 \text{ ft}$

$$b = \sqrt{a^{\frac{1}{2}} + 2a^{\frac{5}{3}}}$$

Solution:

$$A = \int_0^a \left(b - \sqrt{x^{\frac{1}{2}} + 2x^{\frac{5}{3}}}\right) dx \qquad A = 2.177 \text{ ft}^2$$

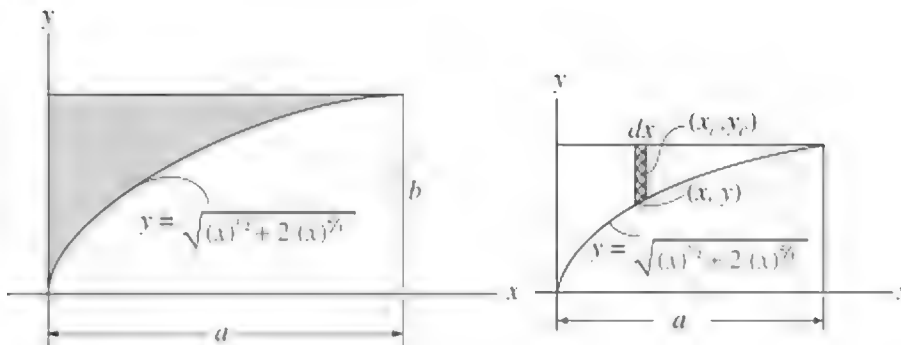
$$x_c = \frac{1}{A}\int_0^a x\left(b - \sqrt{x^{\frac{1}{2}} + 2x^{\frac{5}{3}}}\right) dx \qquad x_c = 0.649 \text{ ft}$$

Problem 9-21

Locate the centroid y_c of the shaded area. Solve the problem by evaluating the integrals using Simpson's rule.

Given:

$a = 2$ ft

$$b = \sqrt{a^{\frac{1}{2}} + 2a^{\frac{5}{3}}}$$

Solution:

$$A = \int_0^a \left(b - \sqrt{x^{\frac{1}{2}} + 2x^{\frac{5}{3}}}\right) dx \qquad A = 2.177 \text{ ft}^2$$

$$y_c = \frac{1}{A}\int_0^a \frac{1}{2}\left(b + \sqrt{x^{\frac{1}{2}} + 2x^{\frac{5}{3}}}\right)\left(b - \sqrt{x^{\frac{1}{2}} + 2x^{\frac{5}{3}}}\right) dx \qquad y_c = 2.04 \text{ ft}$$

Problem 9-22

The steel plate has thickness t and density ρ. Determine the location of its center of mass. Also compute the reactions at the pin and roller support.

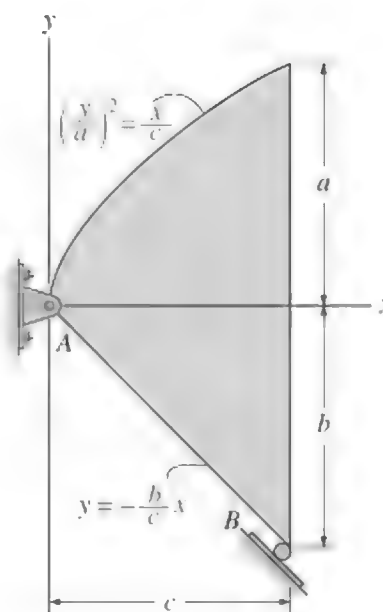
Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$t = 0.3$ m $\qquad a = 2$ m

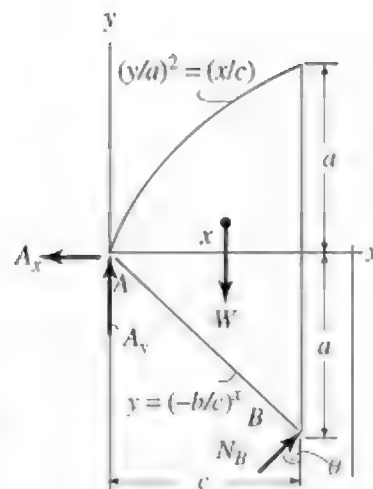
$\rho = 7850 \dfrac{\text{kg}}{\text{m}^3} \qquad b = 2$ m

$c = 2$ m

$g = 9.81 \frac{m}{s^2}$ $\theta = \text{atan}\left(\frac{b}{c}\right)$

Solution:

$A = \int_0^c \left(a\sqrt{\frac{x}{c}} + \frac{bx}{c}\right) dx$ $A = 4.667 \text{ m}^2$

$x_c = \frac{1}{A}\int_0^c x\left(a\sqrt{\frac{x}{c}} + \frac{bx}{c}\right) dx$ $x_c = 1.257 \text{ m}$

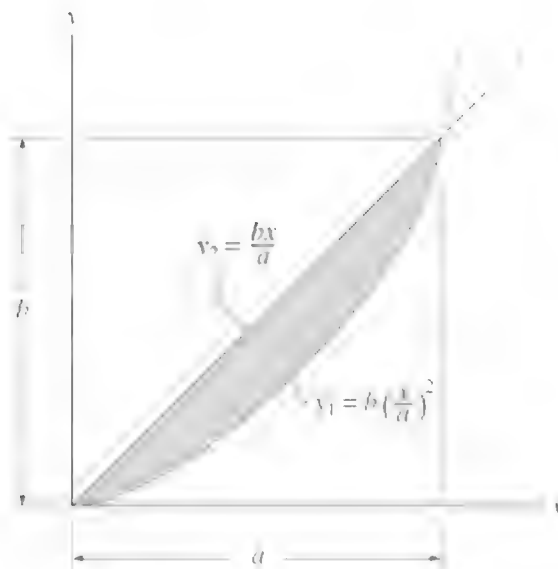
$y_c = \frac{1}{A}\int_0^c \frac{1}{2}\left(a\sqrt{\frac{x}{c}} + \frac{bx}{c}\right)\left(a\sqrt{\frac{x}{c}} - \frac{bx}{c}\right) dx$ $y_c = 0.143 \text{ m}$

Equilibrium $W = \rho A t g$

Guesses $A_x = 1 \text{ N}$ $A_y = 1 \text{ N}$ $N_B = 1 \text{ N}$

Given $A_y - W + N_B\cos(\theta) = 0$ $-A_x + N_B\sin(\theta) = 0$ $N_B\sqrt{b^2 + c^2} - W x_c = 0$

$\begin{pmatrix} A_x \\ A_y \\ N_B \end{pmatrix} = \text{Find}(A_x, A_y, N_B)$ $\begin{pmatrix} A_x \\ A_y \\ N_B \end{pmatrix} = \begin{pmatrix} 33.9 \\ 73.9 \\ 47.9 \end{pmatrix} \text{kN}$

Problem 9-23

Locate the centroid x_c of the shaded area.

Given:

$a = 4 \text{ ft}$

$b = 4 \text{ ft}$

Solution:

$$A = \int_0^a \left[\frac{b\,x}{a} - b\left(\frac{x}{a}\right)^2\right] dx$$

$$x_c = \frac{1}{A}\int_0^a x\left[\frac{b\,x}{a} - b\left(\frac{x}{a}\right)^2\right] dx \qquad x_c = 2.00\ \text{ft}$$

Problem 9-24

Locate the centroid y_c of the shaded area.

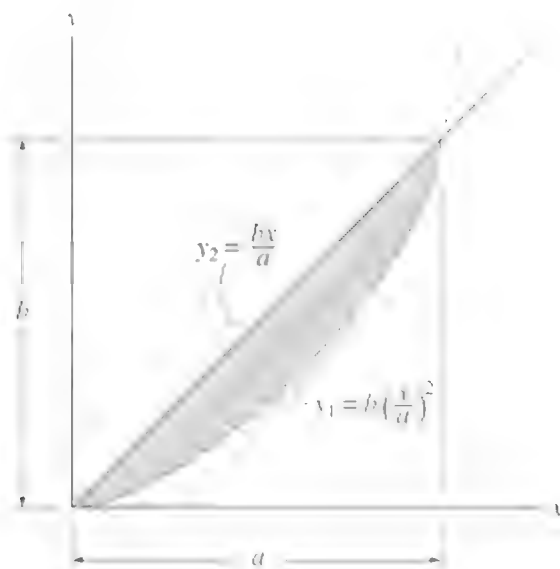
Given:

$a = 4$ ft

$b = 4$ ft

Solution:

$$A = \int_0^a \left[\frac{b\,x}{a} - b\left(\frac{x}{a}\right)^2\right] dx$$

$$y_c = \frac{1}{A}\int_0^a \frac{1}{2}\left[b\frac{x}{a} + b\left(\frac{x}{a}\right)^2\right]\left[\frac{b\,x}{a} - b\left(\frac{x}{a}\right)^2\right] dx \qquad y_c = 1.60\ \text{ft}$$

Problem 9-25

Locate the centroid x_c of the shaded area.

Given:

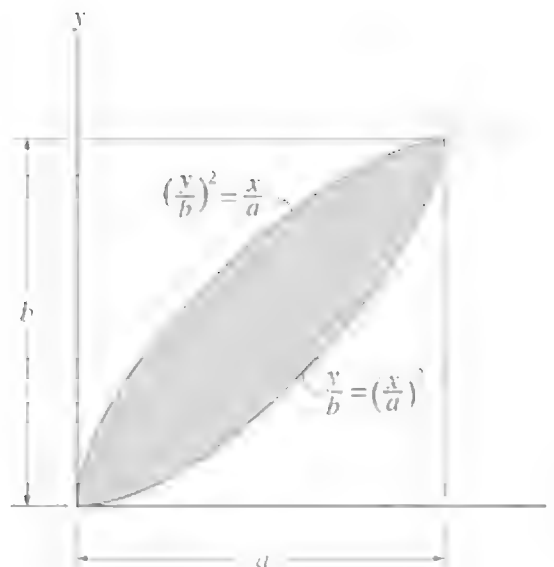
$a = 4$ m

$b = 4\text{ m}$

Solution:

$$A = \int_0^a \left[b\sqrt{\frac{x}{a}} - b\left(\frac{x}{a}\right)^2 \right] dx$$

$$x_c = \frac{1}{A}\int_0^a x\left[b\sqrt{\frac{x}{a}} - b\left(\frac{x}{a}\right)^2 \right] dx$$

$x_c = 1.80\text{ m}$

Problem 9-26

Locate the centroid y_c of the shaded area.

Given:

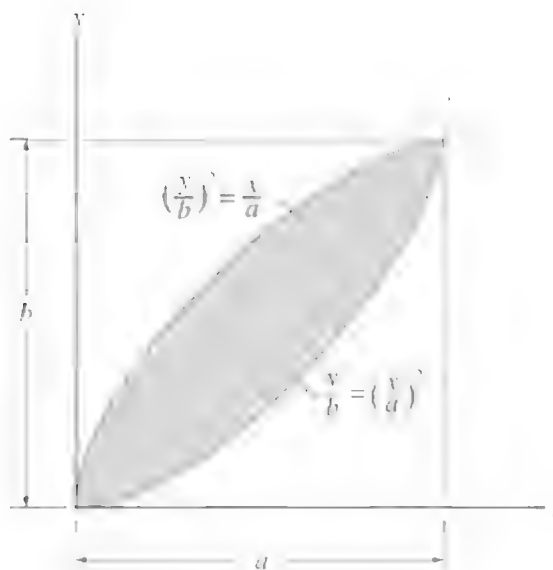
$a = 4\text{ m}$

$b = 4\text{ m}$

Solution:

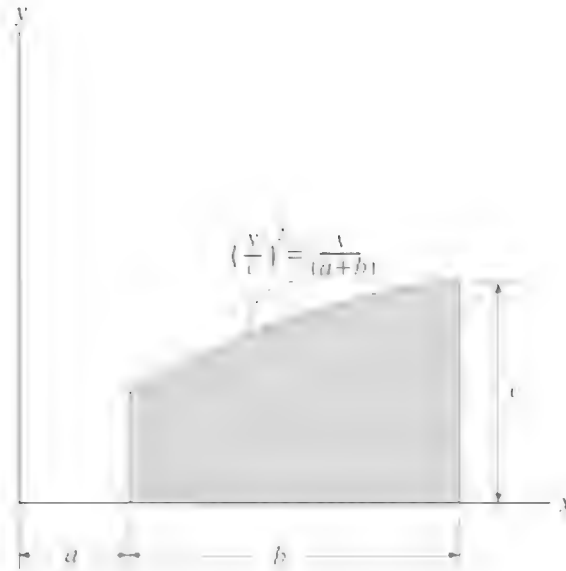
$$A = \int_0^a \left[b\sqrt{\frac{x}{a}} - b\left(\frac{x}{a}\right)^2 \right] dx$$

$$y_c = \frac{1}{A}\int_0^a \frac{1}{2}\left[b\sqrt{\frac{x}{a}} + b\left(\frac{x}{a}\right)^2 \right]\left[b\sqrt{\frac{x}{a}} - b\left(\frac{x}{a}\right)^2 \right] dx$$

$y_c = 1.80\text{ m}$

Problem 9-27

Locate the centroid x_c of the shaded area.

Given:

$a = 1\ \text{in}$

$b = 3\ \text{in}$

$c = 2\ \text{in}$

Solution:

$$A = \int_a^{a+b} c\sqrt{\frac{x}{a+b}}\, dx$$

$$x_C = \frac{1}{A}\int_a^{a+b} x\,c\sqrt{\frac{x}{a+b}}\, dx \qquad x_C = 2.66\ \text{in}$$

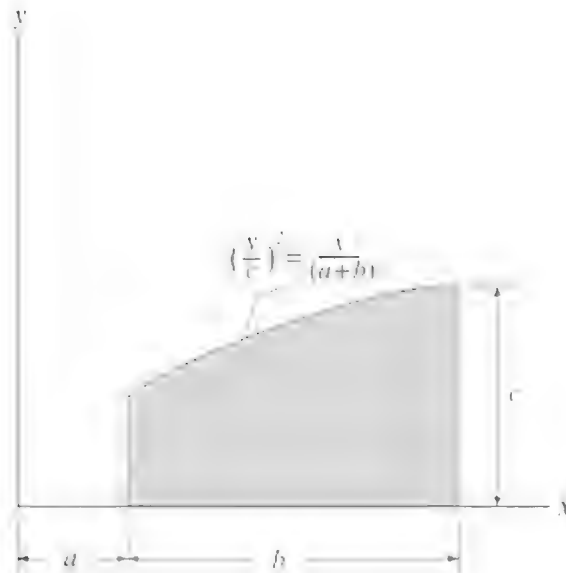
Problem 9-28

Locate the centroid y_c of the shaded area.

Given:

$a = 1\ \text{in}$

$b = 3\ \text{in}$

$c = 2\ \text{in}$

Solution:

$$A = \int_{a}^{a+b} c\sqrt{\frac{x}{a+b}}\,dx$$

$$y_c = \frac{1}{A}\int_{a}^{a+b} \frac{1}{2}\left(c\sqrt{\frac{x}{a+b}}\right)^2 dx \qquad y_c = 0.804\text{ in}$$

Problem 9-29

Locate the centroid x_c of the shaded area.

Given:

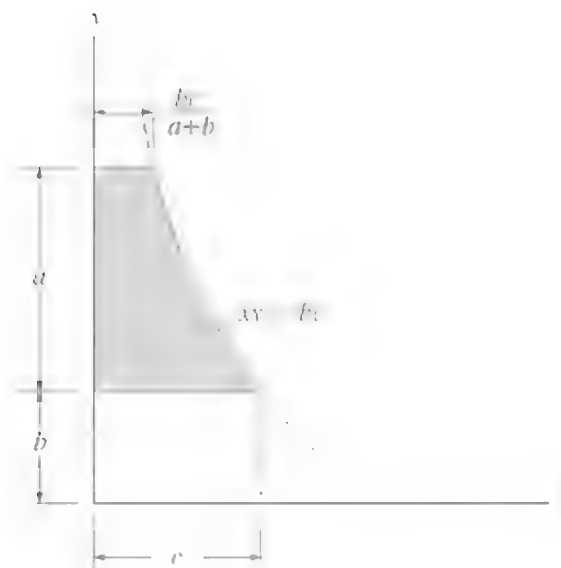
$a = 4$ in

$b = 2$ in

$c = 3$ in

Solution:

$$A = \int_{b}^{a+b} \frac{bc}{y}\,dy \qquad A = 6.592\text{ in}^2$$

$$x_c = \frac{1}{A}\int_{b}^{a+b} \frac{1}{2}\left(\frac{bc}{y}\right)^2 dy \qquad x_c = 0.910\text{ in}$$

Problem 9-30

Locate the centroid y_c of the shaded area.

Given:

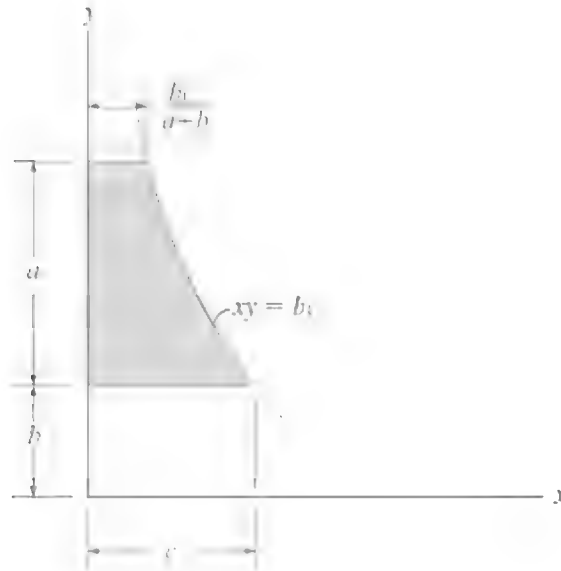
$a = 4$ in

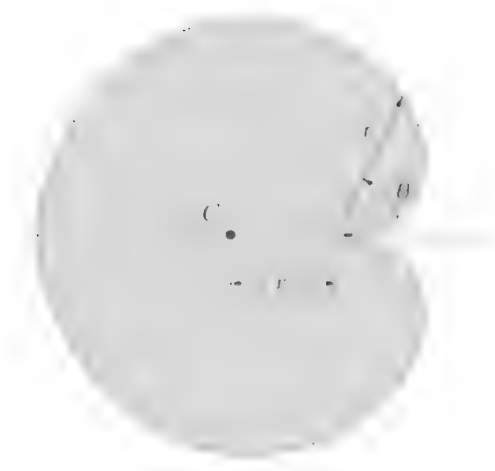
$b = 2$ in

$c = 3$ in

Solution:

$$A = \int_{b}^{a+b} \frac{b\,c}{y}\,dy \qquad A = 6.592\,\text{in}^2$$

$$y_c = \frac{1}{A}\int_{b}^{a+b} y\left(\frac{b\,c}{y}\right)dy \qquad y_c = 3.64\,\text{in}$$

Problem 9-31

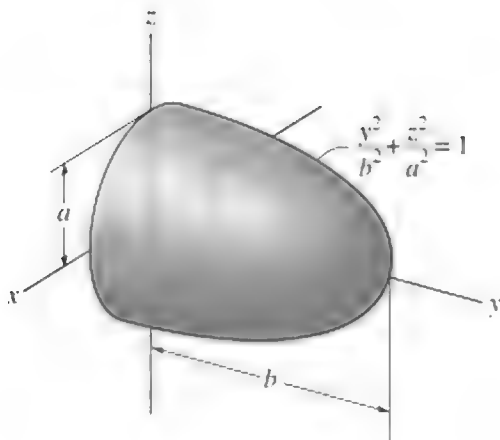
Determine the location r_c of the centroid C of the cardioid, $r = a(1 - \cos\theta)$.

Solution:

$$A = \int_{0}^{2\pi}\int_{0}^{a(1-\cos(\theta))} r\,dr\,d\theta = \frac{3}{2}\,a^2\,\pi$$

$$x_c = \frac{2}{3a^2\pi}\int_0^{2\pi}\int_0^{a(1-\cos(\theta))} r\cos(\theta) r\,dr\,d\theta = \frac{-5}{6}\,a \qquad r_c = \frac{5a}{6}$$

Problem 9-32

Locate the centroid of the ellipsoid of revolution.

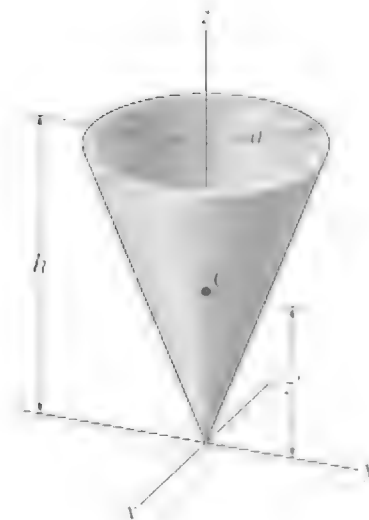
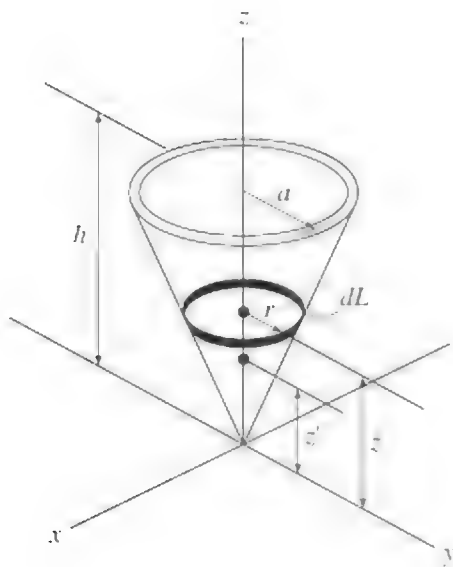
$\frac{y^2}{b^2} + \frac{z^2}{a^2} = 1$

Solution: $dV = \pi z^2\,dy \qquad z^2 = a^2\left(1 - \frac{y^2}{b^2}\right)$

$$V = \int_0^b \pi a^2\left(1 - \frac{y^2}{b^2}\right) dy = \frac{1}{3}\,b\,\frac{3\,b^2 - b^2}{b^2}\,a^2\,\pi$$

$$y_c = \frac{3}{2b\,a^2\,\pi}\int_0^b y\pi a^2\left(1 - \frac{y^2}{b^2}\right) dy = \frac{3}{8\,b}\,b^2 \qquad y_c = \frac{3b}{8}$$

By symmetry $\qquad x_c = z_c = 0$

Problem 9-33

Locate the centroid z_c of the very thin conical shell. *Hint:* Use thin ring elements having a center at (0, 0, z), radius y, and width $dL = \sqrt{(dy)^2 + (dz)^2}$

Solution:

$$dL = \sqrt{dy^2 + dz^2} = \sqrt{1 + \left(\frac{dy}{dz}\right)^2}\,dz = \sqrt{1 + \left(\frac{a}{h}\right)^2}\,dz \qquad r = \frac{a\,z}{h}$$

$$A = \int_0^h 2\pi\frac{a\,z}{h}\sqrt{1 + \left(\frac{a}{h}\right)^2}\,dz = h^2\,\pi\,\frac{a}{h}\left(\sqrt{\frac{h^2 + a^2}{h^2}}\right)$$

$$z_C = \frac{1}{\pi a\sqrt{a^2 + h^2}}\int_0^h z2\pi\frac{a\,z}{h}\sqrt{1 + \left(\frac{a}{h}\right)^2}\,dz = \frac{2}{3\,\pi\,a\left(\sqrt{h^2 + a^2}\right)}\,h^2\,\pi\,a\left(\sqrt{\frac{h^2 + a^2}{h^2}}\right)$$

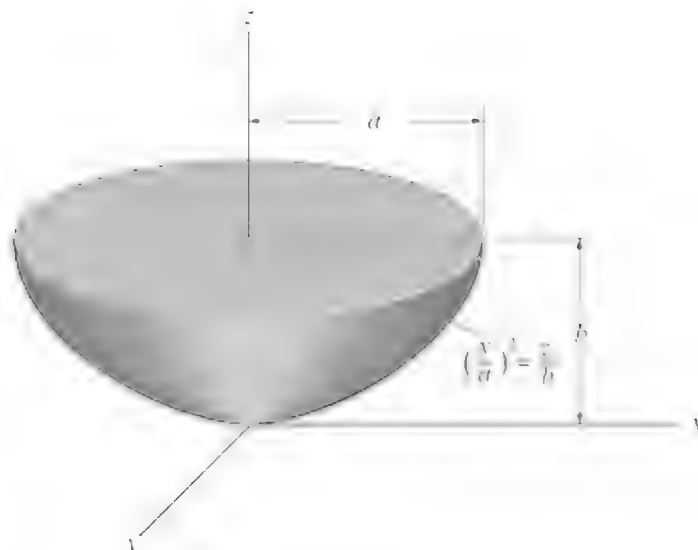
$$z_C = \frac{2h}{3}$$

Problem 9-34

Locate the centroid z_c of the volume.

Given:

$a = 2\ \text{ft}$

$b = 2\ \text{ft}$

Solution:

$$V = \int_0^b \pi \frac{a^2 z}{b}\, dz \qquad V = 12.566\ \text{ft}^3$$

$$z_c = \frac{1}{V}\int_0^b z\pi \frac{a^2 z}{b}\, dz \qquad z_c = 1.333\ \text{ft}$$

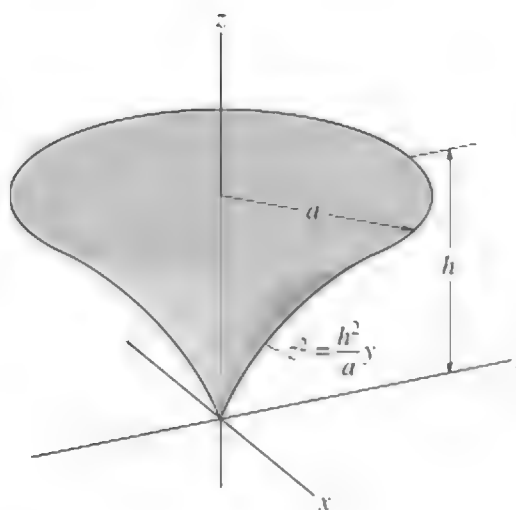
Problem 9-35

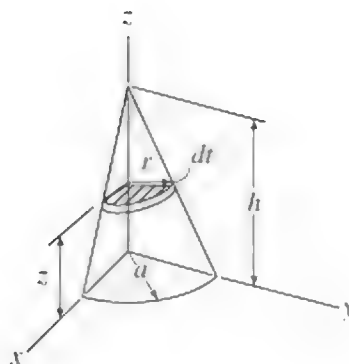
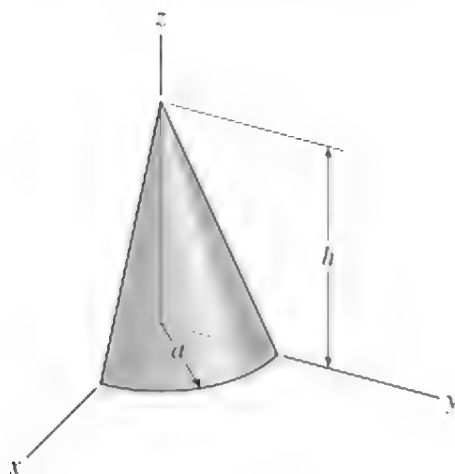
Locate the centroid of the solid.

Solution:

$$z^2 = \frac{h^2}{a} y \qquad y = a\left(\frac{z}{h}\right)^2$$

$x_c = y_c = 0$ By symmetry

$$z_c = \frac{\int_0^h z\pi\left[a\left(\frac{z}{h}\right)^2\right]^2 dz}{\int_0^h \pi\left[a\left(\frac{z}{h}\right)^2\right]^2 dz} = \frac{5}{6}\,\frac{h^6}{h^5} \qquad z_c = \frac{5h}{6}$$

Problem 9-36

Locate the centroid of the quarter-cone.

Solution:

$r = \frac{a}{h}(h - z) \qquad z_c = z \qquad x_c = y_c = \frac{4r}{3\pi}$

$$V = \int_0^h \frac{\pi}{4}\left[\frac{a}{h}(h-z)\right]^2 dz = \frac{1}{12}\, h\, a^2\, \pi$$

$$z_c = \frac{12}{h a^2 \pi}\int_0^h z\frac{\pi}{4}\left[\frac{a}{h}(h-z)\right]^2 dz = \frac{1}{4}\, h$$

$$x_c = \frac{12}{h a^2 \pi}\left[\int_0^h \frac{4}{3\pi}\left[\frac{a}{h}(h-z)\right]\frac{\pi}{4}\left[\frac{a}{h}(h-z)\right]^2 dz\right] = \frac{a}{\pi}$$

$$x_c = y_c = \frac{a}{\pi} \qquad z_c = \frac{h}{4}$$

Problem 9-37

Locate the center of mass x_c of the hemisphere. The density of the material varies linearly from zero at the origin O to ρ_o at the surface. *Hint*: Choose a hemispherical shell element for integration

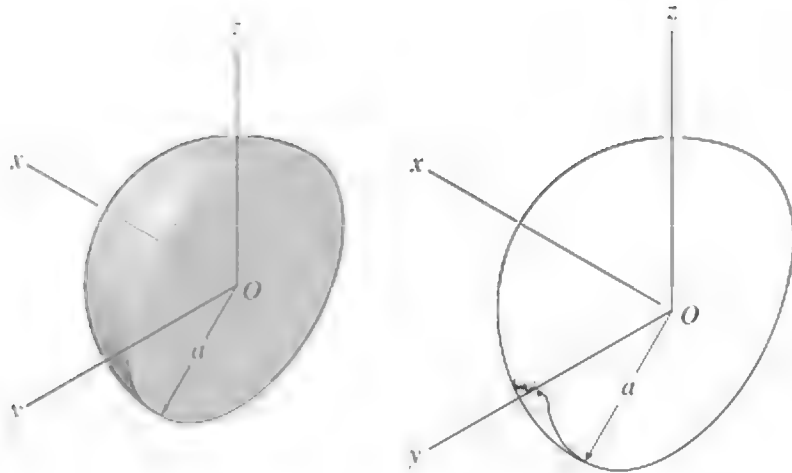
Solution:

for a spherical shell

$$x_C = \frac{x}{2}$$

$$\rho = \rho 0\left(\frac{x}{a}\right)$$

$$dV = 2\pi x^2 \, dx$$

$$x_C = \frac{\int_0^a \rho 0\left(\frac{x}{a}\right)\frac{x}{2} 2\pi x^2 \, dx}{\int_0^a \rho 0\left(\frac{x}{a}\right) 2\pi x^2 \, dx} = \frac{2}{5}\cdot a \qquad x_C = \frac{2}{5}a$$

Problem 9-38

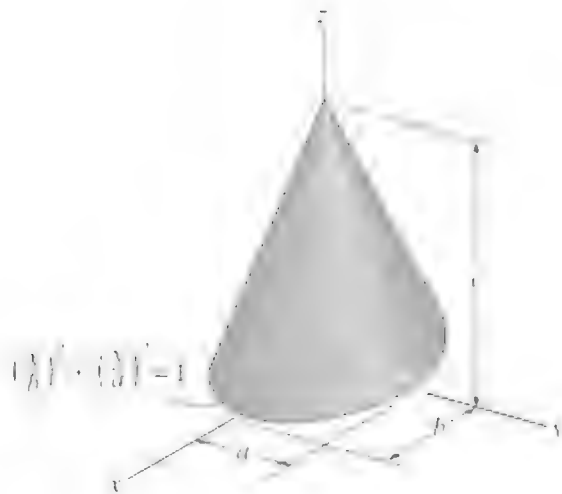
Locate the centroid z_c of the right-elliptical cone.

Given:

$a = 3$ ft

$b = 4$ ft

$c = 10$ ft

$$\left(\frac{x}{b}\right)^2 + \left(\frac{y}{a}\right)^2 = 1$$

Solution:

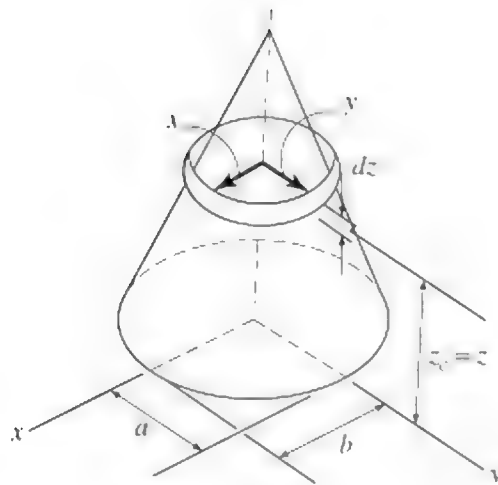
Volume and Moment Arm : From the geometry,

$$\frac{x}{c-z} = \frac{b}{c} \qquad x = \frac{b}{c}(c-z)$$

$$\frac{y}{c-z} = \frac{a}{c} \qquad y = \frac{a}{c}(c-z)$$

The volume of the thin disk differential element is

$$dV = \pi\frac{b}{c}(c-z)\frac{a}{c}(c-z)\,dz$$

$$z_C = \frac{\int_0^c z\pi\frac{b}{c}(c-z)\frac{a}{c}(c-z)\,dz}{\int_0^c \pi\frac{b}{c}(c-z)\frac{a}{c}(c-z)\,dz} \qquad z_C = 2.5\text{ ft}$$

Problem 9-39

Locate the center of gravity z_c of the frustum of the paraboloid.The material is homogeneous.

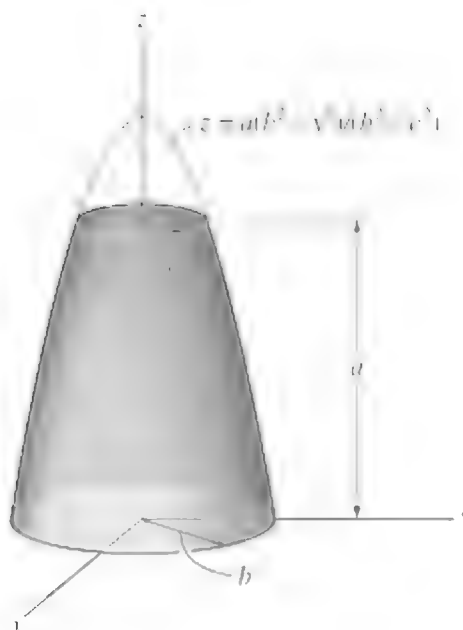
Given:

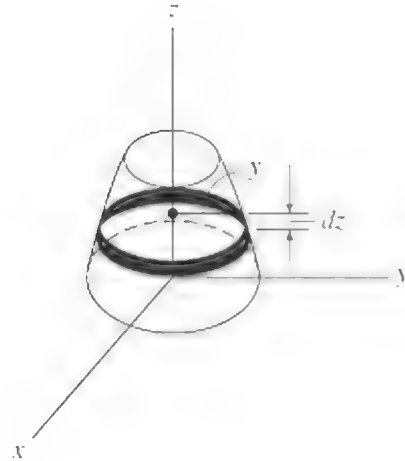
$a = 1\text{ m}$

$b = 0.5\text{ m}$

$c = 0.3\text{ m}$

Solution

$$V = \int_0^a \pi\left[b^2 - \frac{z}{a}\left(b^2 - c^2\right)\right]dz$$

$$z_c = \frac{1}{V}\int_0^a z\pi\left[b^2 - \frac{z}{a}\left(b^2 - c^2\right)\right] dz$$

$$z_c = 0.422 \text{ m}$$

Problem 9-40

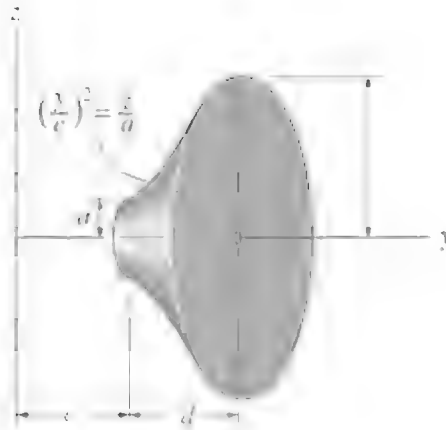
Locate the center of gravity y_c of the volume. The material is homogeneous.

Given:

$a = 25$ mm

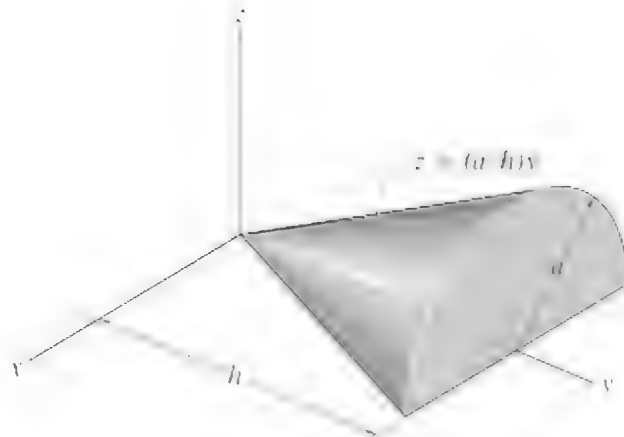
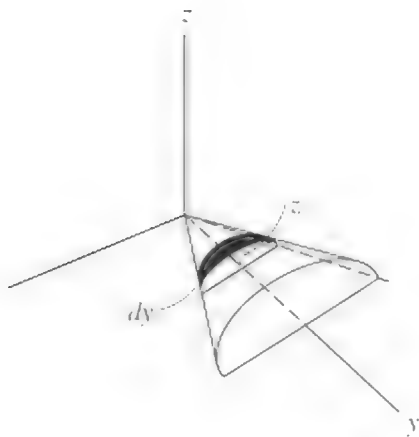
$c = 50$ mm

$d = 50$ mm

Solution:

$$V = \int_c^{c+d} \pi\left[a\left(\frac{y}{c}\right)^2\right]^2 dy$$

$$y_c = \frac{1}{V}\int_c^{c+d} y\pi\left[a\left(\frac{y}{c}\right)^2\right]^2 dy \qquad y_c = 84.7 \text{ mm}$$

Problem 9-41

Locate the center of gravity for the homogeneous half-cone.

Solution:

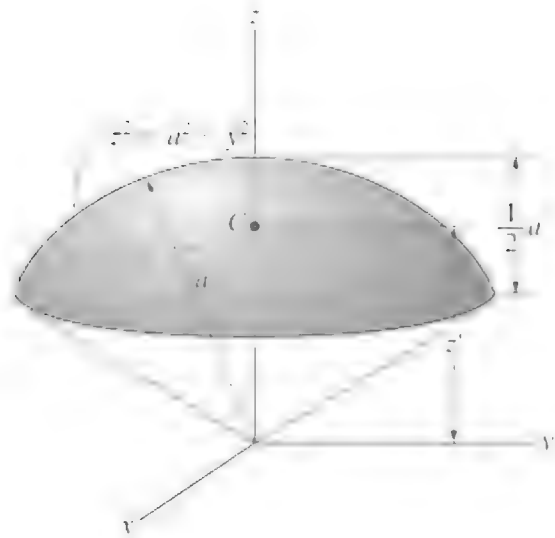
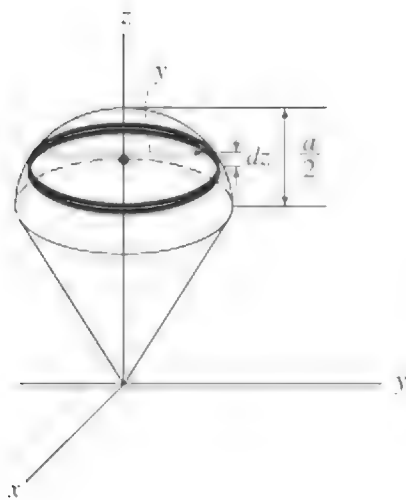
$$V = \int_0^h \frac{\pi}{2}\left(\frac{a\,y}{h}\right)^2 \mathrm{d}y = \frac{1}{6} h\,\pi a^2$$

$$y_c = \frac{6}{h\,a^2\,\pi}\int_0^h y\frac{\pi}{2}\left(\frac{a\,y}{h}\right)^2 \mathrm{d}y = \frac{3}{4}h \qquad y_c = \frac{3}{4}h$$

$$z_c = \frac{6}{h\,a^2\,\pi}\int_0^h \left(\frac{4a\,y}{3h\pi}\right)\frac{\pi}{2}\left(\frac{a\,y}{h}\right)^2 \mathrm{d}y = \frac{1}{\pi}a \qquad z_c = \frac{a}{\pi}$$

$$x_c = \frac{6}{h\,a^2\,\pi}\int_0^h 0\frac{\pi}{2}\left(\frac{a\,y}{h}\right)^2 \mathrm{d}y \qquad x_c = 0$$

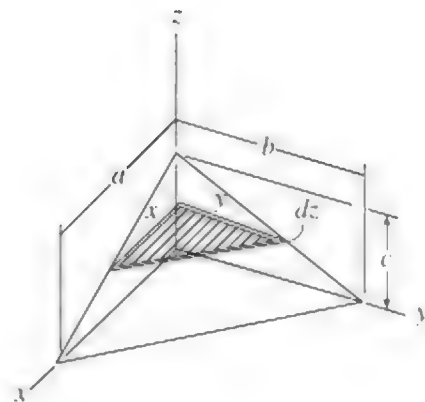
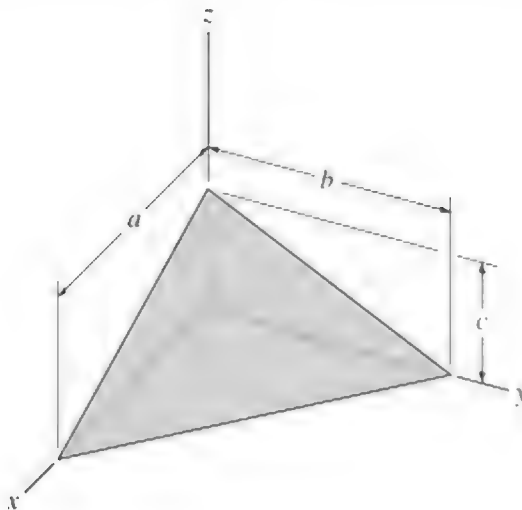
Problem 9-42

Locate the centroid z_c of the spherical segment.

Solution:

$$V = \int_{\frac{a}{2}}^{a} \pi\left(a^2 - z^2\right)\,dz = \frac{5}{24}\,a^3\,\pi$$

$$z_c = \frac{24}{5\pi a^3}\int_{\frac{a}{2}}^{a} z\pi\left(a^2 - z^2\right)\,dz = \frac{27}{40}\,a \qquad z_c = \frac{27}{40}\,a$$

Problem 9-43

Determine the location z_c of the centroid for the tetrahedron. *Suggestion:* Use a triangular "plate" element parallel to the *x-y* plane and of thickness d*z*.

Solution:

$$\frac{c-z}{c} = \frac{x}{a} = \frac{y}{b} \qquad x = \frac{a}{c}(c-z) \qquad y = \frac{b}{c}(c-z)$$

$$z_C = \frac{\displaystyle\int_0^c z\frac{a\,b}{c^2}(c-z)^2\,dz}{\displaystyle\int_0^c \frac{a\,b}{c^2}(c-z)^2\,dz} = \frac{1}{4}\,c \qquad z_C = \frac{1}{4}c$$

Problem 9-44

Determine the location (x, y) of the particle M_1 so that the three particles, which lie in the x–y plane, have a center of mass located at the origin O.

Given:

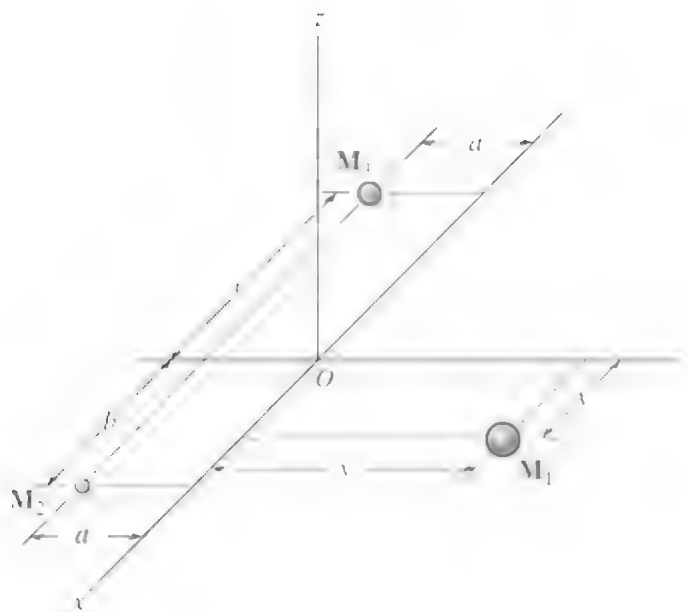
$M_1 = 7$ kg

$M_2 = 3$ kg

$M_3 = 5$ kg

$a = 2$ m

$b = 3$ m

$c = 4$ m

Solution:

Guesses $x = 1$ m $y = 1$ m

Given $M_1 x + M_2 b - M_3 c = 0$ $\qquad M_1 y - M_2 a - M_3 a = 0$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \text{Find}(x, y) \qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.57 \\ 2.29 \end{pmatrix} \text{ m}$$

Problem 9-45

Locate the center of gravity (x_c, y_c, z_c) of the four particles.

Given:

$M_1 = 2\ \text{lb}$ $a = 2\ \text{ft}$

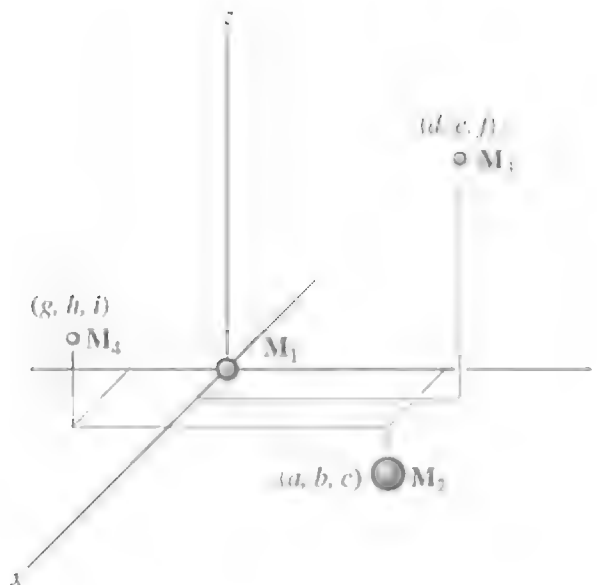
$M_2 = 3\ \text{lb}$ $b = 3\ \text{ft}$

$M_3 = 1\ \text{lb}$ $c = -1\ \text{ft}$

$M_4 = 1\ \text{lb}$ $d = 1\ \text{ft}$

$f = 4\ \text{ft}$ $e = 4\ \text{ft}$

$h = -2\ \text{ft}$ $g = 2\ \text{ft}$

$i = 2\ \text{ft}$

Solution:

$$x_c = \frac{M_1 0\text{ft} + M_2 a + M_3 d + M_4 g}{M_1 + M_2 + M_3 + M_4} \qquad x_c = 1.29\ \text{ft}$$

$$y_c = \frac{M_1 0\text{ft} + M_2 b + M_3 e + M_4 h}{M_1 + M_2 + M_3 + M_4} \qquad y_c = 1.57\ \text{ft}$$

$$z_c = \frac{M_1 0\text{ft} + M_2 c + M_3 f + M_4 i}{M_1 + M_2 + M_3 + M_4} \qquad z_c = 0.429\ \text{ft}$$

Problem 9-46

A rack is made from roll-formed sheet steel and has the cross section shown. Determine the location (x_c, y_c) of the centroid of the cross section. The dimensions are indicated at the center thickness of each segment.

Given:

$a = 15\ \text{mm}$

$c = 80\ \text{mm}$

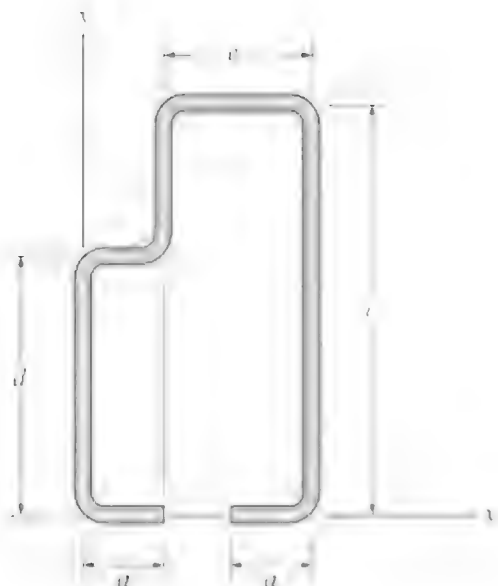
$d = 50 \text{ mm}$

$e = 30 \text{ mm}$

Solution:

$L = 3a + 2c + e$

$$x_c = \frac{2a\frac{a}{2} + a\left(e + \frac{a}{2}\right) + c(a+e) + e\left(a + \frac{e}{2}\right) + (c-d)a}{L} \qquad x_c = 24.4 \text{ mm}$$

$$y_c = \frac{d\frac{d}{2} + c\frac{c}{2} + (c-d)\frac{d+c}{2} + ad + ec}{L} \qquad y_c = 40.6 \text{ mm}$$

Problem 9-47

The steel and aluminum plate assembly is bolted together and fastened to the wall. Each plate has a constant width w in the z direction and thickness t. If the density of A and B is ρ_s, and the density of C is ρ_{al}, determine the location x_c, the center of mass. Neglect the size of the bolts.

Units Used:

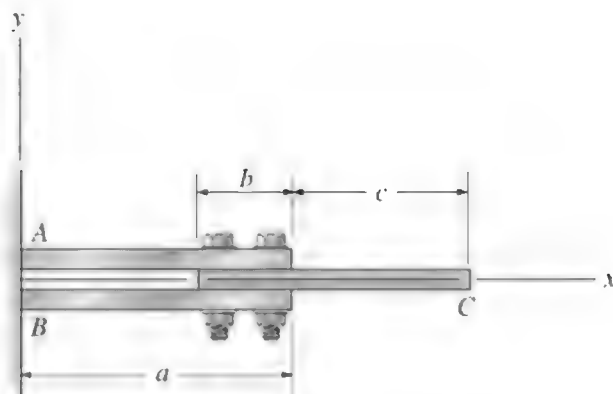
$\text{Mg} = 10^3 \text{ kg}$

Given:

$w = 200 \text{ mm}$ $\qquad a = 300 \text{ mm}$

$t = 20 \text{ mm}$ $\qquad b = 100 \text{ mm}$

$\rho_s = 7.85 \frac{\text{Mg}}{\text{m}^3}$ $\qquad c = 200 \text{ mm}$

$\rho_{al} = 2.71 \frac{\text{Mg}}{\text{m}^3}$

Solution:

$$x_c = \frac{2(\rho_s a t w)\frac{a}{2} + [\rho_{al}(b+c)tw]\left(a - b + \frac{b+c}{2}\right)}{2\rho_s a t w + \rho_{al}(b+c)tw}$$

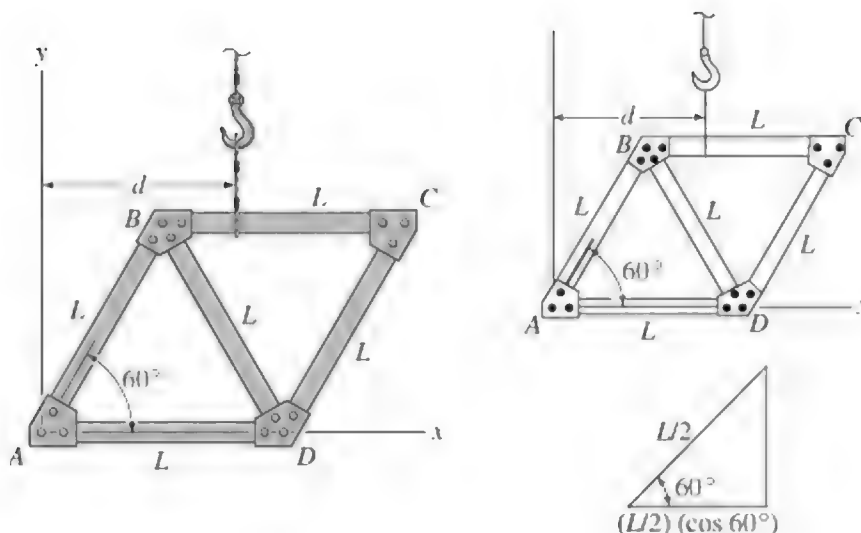
$x_c = 179\ \text{mm}$

Problem 9-48

The truss is made from five members, each having a length L and a mass density ρ. If the mass of the gusset plates at the joints and the thickness of the members can be neglected, determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.

Given:

$L = 4\ \text{m}$

$\rho = 7\ \frac{\text{kg}}{\text{m}}$

Solution:

$$d = \frac{\rho L\left(\frac{L}{2} + \frac{L}{4} + \frac{3L}{4} + L + \frac{5L}{4}\right)}{5\rho L}$$

$d = 3\ \text{m}$

Problem 9-49

Locate the center of gravity (x_c, y_c, z_c) of the homogeneous wire.

Given:

$a = 300 \text{ mm}$

$b = 400 \text{ mm}$

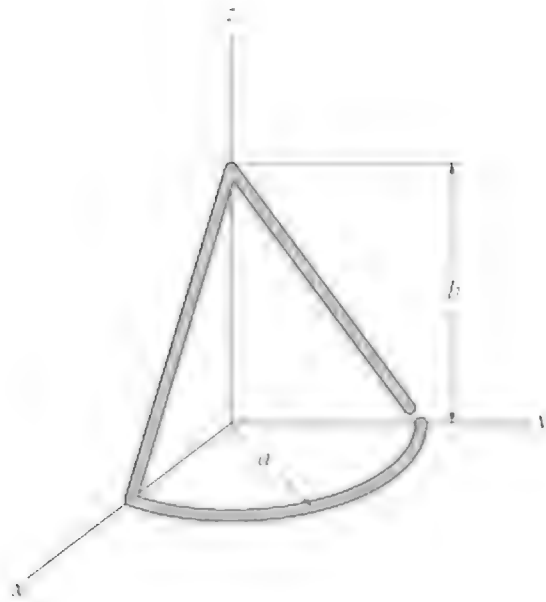
Solution:

$$L = \frac{\pi a}{2} + 2\sqrt{a^2 + b^2}$$

$$x_c = \frac{1}{L}\left[\sqrt{a^2 + b^2}\,\frac{a}{2} + \frac{\pi a}{2}\left(\frac{2a}{\pi}\right)\right]$$

$$y_c = \frac{1}{L}\left[\sqrt{a^2 + b^2}\,\frac{a}{2} + \frac{\pi a}{2}\left(\frac{2a}{\pi}\right)\right]$$

$$z_c = \frac{1}{L}\left[2\sqrt{a^2 + b^2}\left(\frac{b}{2}\right)\right]$$

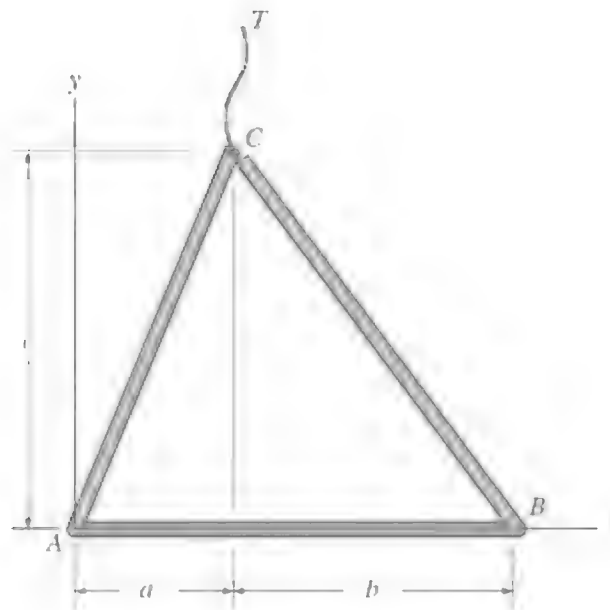
$$\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = \begin{pmatrix} 112.2 \\ 112.2 \\ 135.9 \end{pmatrix} \text{mm}$$

Problem 9-50

Determine the location (x_c, y_c) of the center of gravity of the homogeneous wire bent in the form of a triangle. Neglect any slight bends at the corners. If the wire is suspended using a thread T attached to it at C, determine the angle of tilt AB makes with the horizontal when the wire is in equilibrium.

Given:

$a = 5 \text{ in}$

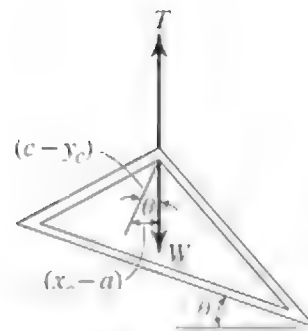
$b = 9 \text{ in}$

$c = 12 \text{ in}$

Solution:

$$L = a + b + \sqrt{a^2 + c^2} + \sqrt{b^2 + c^2}$$

$$x_c = \frac{1}{L}\left[(a+b)\frac{a+b}{2} + \sqrt{a^2+c^2}\,\frac{a}{2} + \sqrt{b^2+c^2}\left(a + \frac{b}{2}\right)\right] \qquad x_c = 6.50 \text{ in}$$

$$y_c = \frac{1}{L}\left(\sqrt{a^2+c^2}\,\frac{c}{2} + \sqrt{b^2+c^2}\,\frac{c}{2}\right) \qquad y_c = 4.00 \text{ in}$$

$$\theta = \operatorname{atan}\left(\frac{x_c - a}{c - y_c}\right) \qquad \theta = 10.6 \text{ deg}$$

Problem 9-51

The three members of the frame each have weight density γ. Locate the position (x_c, y_c) of the center of gravity. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the fixed support A.

Given:

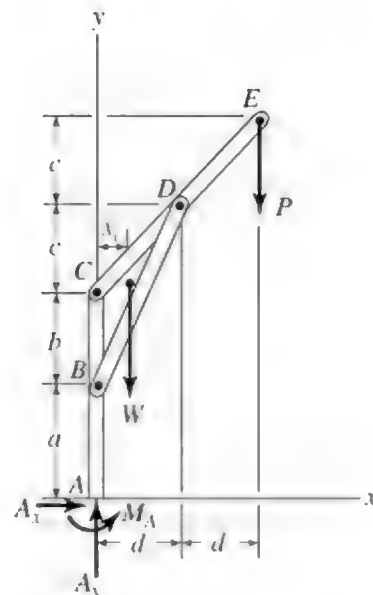
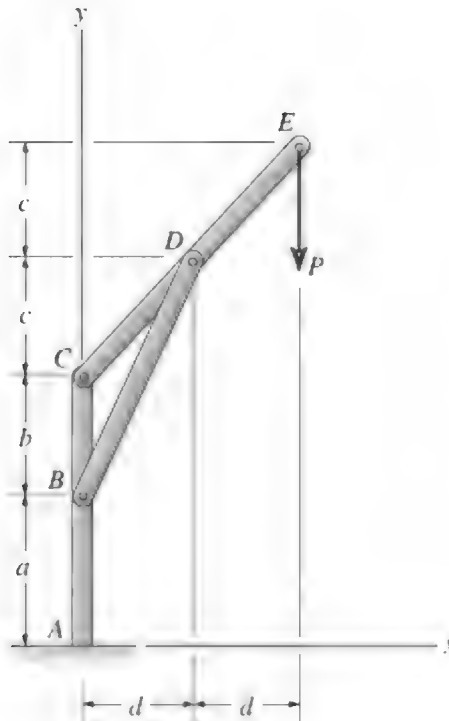
$\gamma = 4 \ \frac{\text{lb}}{\text{ft}}$

$P = 60$ lb

$a = 4$ ft

$b = 3$ ft

$c = 3$ ft

$d = 3$ ft

Solution:

$$W = \gamma\sqrt{d^2 + (b+c)^2} + \gamma 2\sqrt{d^2 + c^2} + \gamma(a+b) \qquad W = 88.774\ \text{lb}$$

$$x_c = \frac{\gamma\sqrt{d^2 + (b+c)^2}\,\frac{d}{2} + \gamma 2\sqrt{d^2 + c^2}\,d}{W} \qquad x_c = 1.6\ \text{ft}$$

$$y_c = \frac{\gamma(a+b)\left(\frac{a+b}{2}\right) + \gamma\sqrt{d^2 + (b+c)^2}\left(a + \frac{b+c}{2}\right) + \gamma 2\sqrt{d^2 + c^2}(a+b+c)}{W}$$

$$y_c = 7.043\ \text{ft}$$

Equilibrium

$A_x = 0 \qquad A_x = 0\text{lb} \qquad A_x = 0\ \text{lb}$

$A_y - W - P = 0 \qquad A_y = W + P \qquad A_y = 148.8\ \text{lb}$

$M_A - W x_c - P2d = 0 \qquad M_A = W x_c + P2d \qquad M_A = 502\ \text{lb ft}$

Problem 9-52

Locate the center of gravity G(x_c, y_c) of the streetlight. Neglect the thickness of each segment. The mass per unit length of each segment is given.

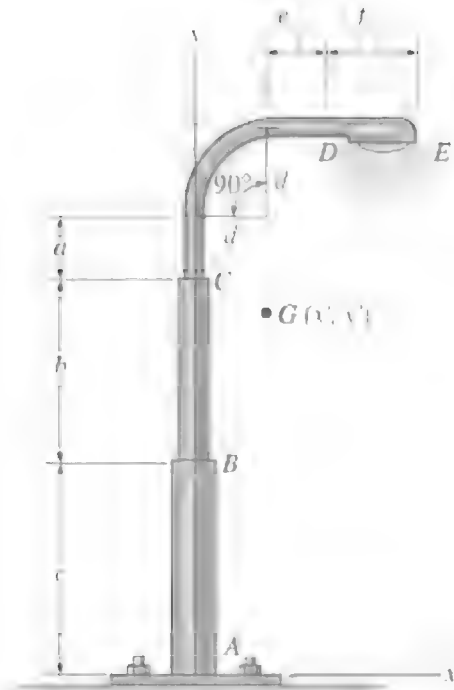
Given:

$a = 1\ \text{m} \qquad \rho_{AB} = 12\ \frac{\text{kg}}{\text{m}}$

$b = 3\ \text{m} \qquad \rho_{BC} = 8\ \frac{\text{kg}}{\text{m}}$

$c = 4\ \text{m} \qquad \rho_{CD} = 5\ \frac{\text{kg}}{\text{m}}$

$d = 1\ \text{m} \qquad \rho_{DE} = 2\ \frac{\text{kg}}{\text{m}}$

$e = 1\ \text{m} \qquad f = 1.5\ \text{m}$

Solution:

$$M = \rho_{AB}c + \rho_{BC}b + \rho_{CD}\left(a + e + \frac{\pi d}{2}\right) + \rho_{DE}f$$

$$x_c = \frac{1}{M}\left[\rho_{CD}\frac{\pi d}{2}\left(d - \frac{2d}{\pi}\right) + \rho_{CD}e\left(d + \frac{e}{2}\right) + \rho_{DE}f\left(d + e + \frac{f}{2}\right)\right]$$

$$y_c = \frac{1}{M}\left[\rho_{CD}\left[a\left(c + b + \frac{a}{2}\right) + \frac{\pi d}{2}\left(c + b + a + \frac{2d}{\pi}\right) + e(c + b + a + d)\right] \dots \\ + \rho_{DE}f(c + b + a + d) + \rho_{BC}b\left(c + \frac{b}{2}\right) + \rho_{AB}c\frac{c}{2}\right]$$

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} 0.200 \\ 4.365 \end{pmatrix} \text{ m}$$

Problem 9-53

Determine the location y_c of the centroid of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.

Given:

$d_1 = 50$ mm

$d_2 = 35$ mm

$h = 110$ mm

$t = 15$ mm

Solution:

$$y_c = \frac{\pi\left(\frac{d_1}{2}\right)^2 \frac{d_1}{2} + ht\left(d_1 + \frac{h}{2}\right) + \pi\left(\frac{d_2}{2}\right)^2\left(d_1 + h + \frac{d_2}{2}\right)}{\pi\left(\frac{d_1}{2}\right)^2 + ht + \pi\left(\frac{d_2}{2}\right)^2}$$

$y_c = 85.9$ mm

Problem 9-54

The gravity wall is made of concrete. Determine the location (x_c, y_c) of the center of gravity G for the wall.

Given:

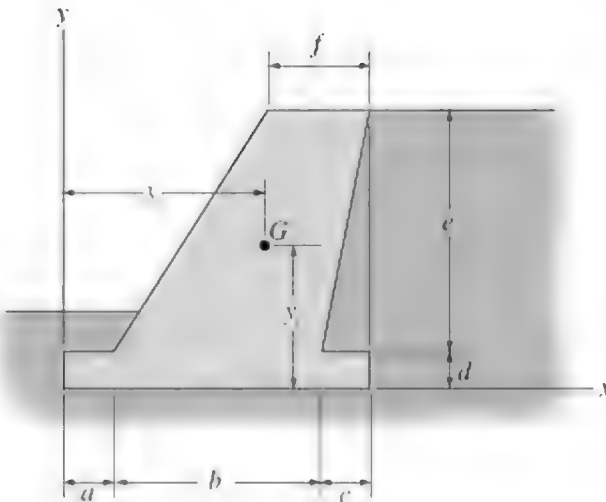
$a = 0.6$ m

$b = 2.4$ m

$c = 0.6$ m

$d = 0.4$ m

$e = 3$ m

$f = 1.2$ m

Solution:

$$A = (a+b+c)d + (b+c)e - \frac{ce}{2} - (b+c-f)\frac{e}{2} \qquad A = 6.84 \text{ m}^2$$

$$x_c = \frac{1}{A}\left[(a+b+c)d\left(\frac{a+b+c}{2}\right) + (b+c)e\left(a + \frac{b+c}{2}\right) - \frac{ce}{2}\left(a+b+\frac{2c}{3}\right) \ldots \right.$$
$$\left. + -(b+c-f)\frac{e}{2}\left(a + \frac{b+c-f}{3}\right)\right]$$

$$y_c = \frac{1}{A}\left[(a+b+c)d\frac{d}{2} + (b+c)e\left(d+\frac{e}{2}\right) - \frac{ce}{2}\left(d+\frac{e}{3}\right) \ldots \right.$$
$$\left. + -(b+c-f)\frac{e}{2}\left(d+\frac{2e}{3}\right)\right]$$

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} 2.221 \\ 1.411 \end{pmatrix} \text{ m}$$

Problem 9-55

Locate the centroid (x_c, y_c)of the shaded area.

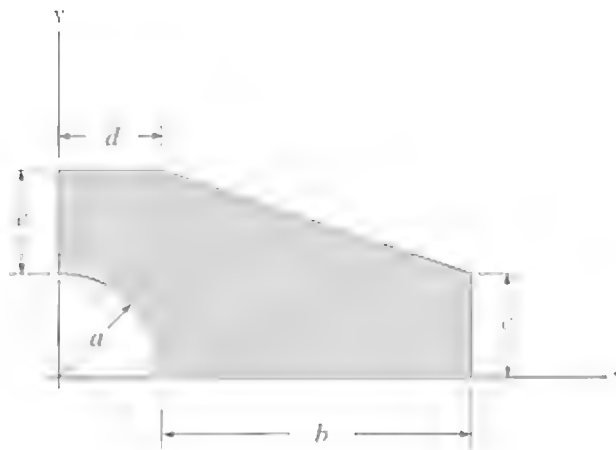
Given:

$a = 1$ in

$b = 3$ in

$c = 1$ in

$d = 1$ in

$e = 1$ in

Solution:

$$A = (a+b)(a+e) - \frac{\pi a^2}{4} - \frac{1}{2}(a+b-d)(a+e-c)$$

$$x_c = \frac{1}{A}\left[\frac{(a+b)^2}{2}(a+e) - \frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right) - \frac{1}{2}(a+b-d)(a+e-c)\left(a+b-\frac{a+b-d}{3}\right)\right]$$

$$y_c = \frac{1}{A}\left[(a+b)\frac{(a+e)^2}{2} - \frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right) - \frac{1}{2}(a+b-d)(a+e-c)\left(a+e-\frac{a+e-c}{3}\right)\right]$$

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} 1.954 \\ 0.904 \end{pmatrix} \text{in}$$

Problem 9-56

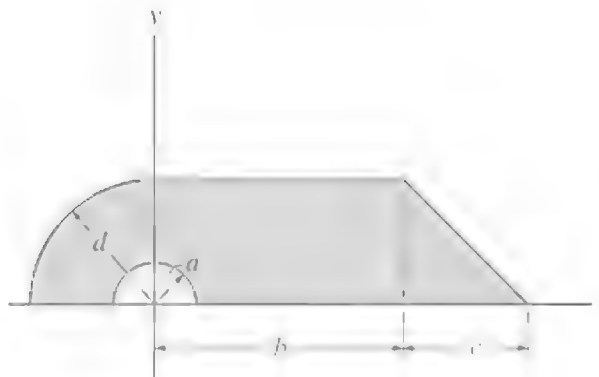
Locate the centroid (x_c, y_c) of the shaded area.

Given:

$a = 1$ in

$b = 6$ in

$c = 3$ in

$d = 3$ in

Solution:

$$A = bd + \frac{\pi d^2}{4} - \frac{\pi a^2}{2} + \frac{1}{2}(dc)$$

$$x_C = \frac{1}{A}\left[b\,d\frac{b}{2} - \frac{\pi d^2}{4}\left(\frac{4d}{3\pi}\right) + \frac{1}{2}d\,c\left(b + \frac{c}{3}\right)\right]$$ $x_C = 2.732$ in

$$y_C = \frac{1}{A}\left[b\,d\left(\frac{d}{2}\right) + \frac{\pi d^2}{4}\left(\frac{4d}{3\pi}\right) - \frac{\pi a^2}{2}\left(\frac{4a}{3\pi}\right) + \frac{1}{2}d\,c\left(\frac{d}{3}\right)\right]$$ $y_C = 1.423$ in

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Problem 9-57

Determine the location y_c of the centroidal axis $x_c x_c$ of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.

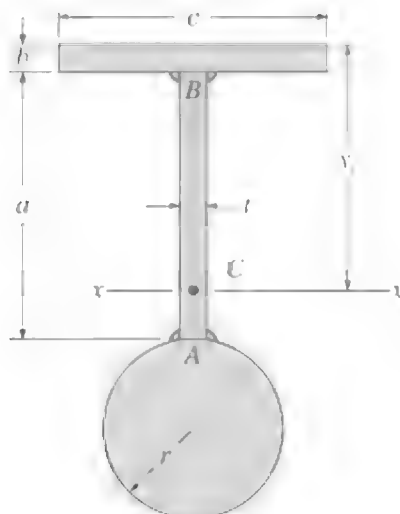
Given:

$r = 50$ mm

$t = 15$ mm

$a = 150$ mm

$b = 15$ mm

$c = 150$ mm

Solution:

$$y_C = \frac{b\,c\left(\frac{b}{2}\right) + a\,t\left(b + \frac{a}{2}\right) + \pi r^2 (b + a + r)}{b\,c + a\,t + \pi r^2}$$ $y_C = 154.443$ mm

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Problem 9-58

Determine the location (x_c, y_c) of the centroid C of the area.

Given:

$a = 6$ in

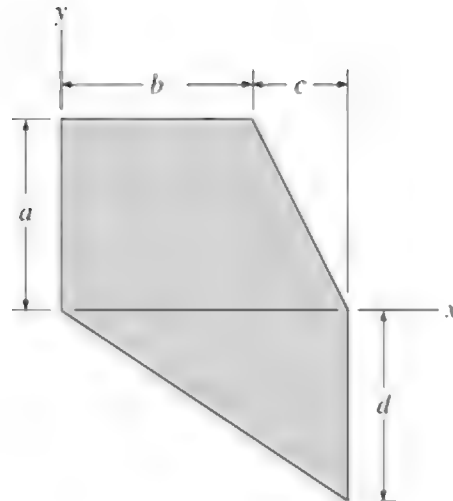
$b = 6$ in

$c = 3 \text{ in}$

$d = 6 \text{ in}$

Solution:

$$x_c = \frac{ab\left(\frac{b}{2}\right) + \frac{1}{2}ac\left(b + \frac{c}{3}\right) + \frac{1}{2}(b+c)d\frac{2}{3}(b+c)}{ab + \frac{1}{2}ca + \frac{1}{2}(b+c)d} \qquad x_c = 4.625 \text{ in}$$

$$y_c = \frac{ab\left(\frac{a}{2}\right) + \frac{1}{2}ac\left(\frac{a}{3}\right) - \frac{1}{2}(b+c)d\left(\frac{d}{3}\right)}{ab + \frac{1}{2}ca + \frac{1}{2}(b+c)d} \qquad y_c = 1 \text{ in}$$

Problem 9-59

Determine the location y_c of the centroid C for a beam having the cross-sectional area shown. The beam is symmetric with respect to the y axis.

Given:

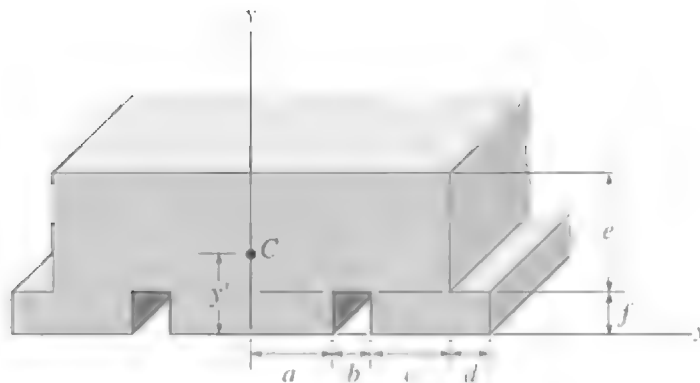
$a = 2 \text{ in}$

$b = 1 \text{ in}$

$c = 2 \text{ in}$

$d = 1 \text{ in}$

$e = 3 \text{ in}$

$f = 1 \text{ in}$

Solution:

$$A = 2[(a+b+c+d)(e+f) - bf - de] \qquad A = 40 \text{ in}^2$$

$$y_c = \frac{2}{A}\left[(a+b+c+d)\frac{(e+f)^2}{2} - b\frac{f^2}{2} - de\left(f+\frac{e}{2}\right)\right] \qquad y_c = 2.00 \text{ in}$$

Problem 9-60

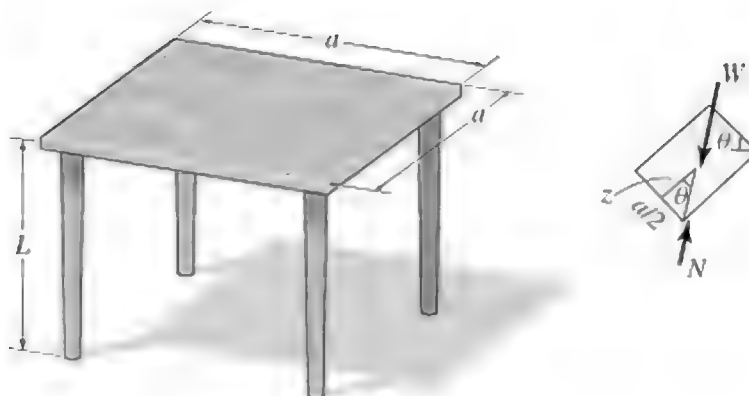
The wooden table is made from a square board having weight W. Each of the legs has wieght W_{leg} and length L. Determine how high its center of gravity is from the floor. Also, what is the angle, measured from the horizontal, through which its top surface can be tilted on two of its legs before it begins to overturn? Neglect the thickness of each leg.

Given:

$W = 15 \text{ lb}$

$W_{leg} = 2 \text{ lb}$

$L = 3 \text{ ft}$

$a = 4 \text{ ft}$

Solution:

$$z_c = \frac{WL + 4W_{leg}\left(\frac{L}{2}\right)}{W + 4W_{leg}} \qquad z_c = 2.478 \text{ ft}$$

$$\theta = \operatorname{atan}\left(\frac{\frac{a}{2}}{z_c}\right) \qquad \theta = 38.9 \text{ deg}$$

Problem 9-61

Locate the centroid y_c for the beam's cross-sectional area.

Given:

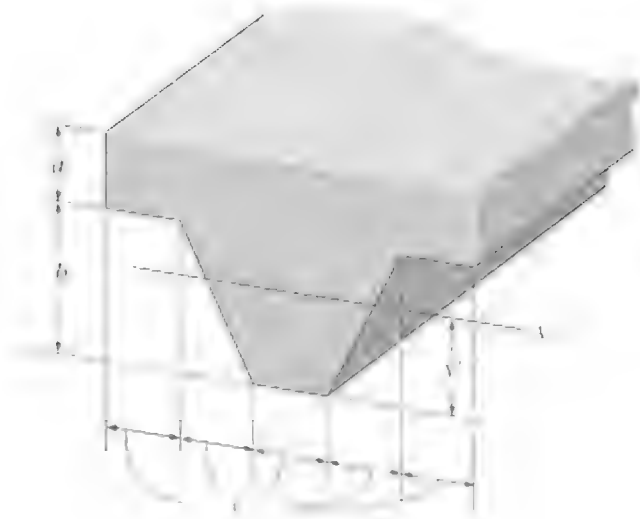
$a = 120 \text{ mm}$

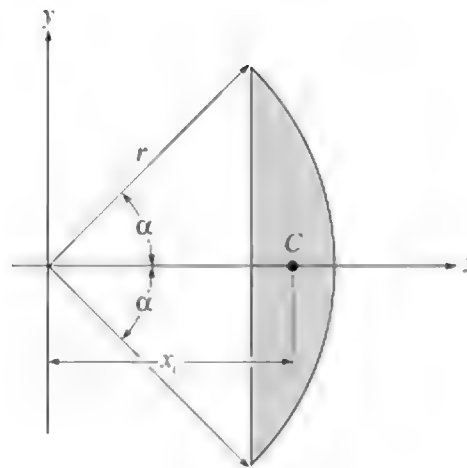
$b = 240\text{ mm}$

$c = 120\text{ mm}$

Solution:

$$A = (a+b)5c - 3bc$$

$$y_C = \frac{1}{A}\left[\frac{(a+b)^2}{2}5c - 2bc\left(\frac{b}{2}\right) - bc\left(\frac{b}{3}\right)\right] \qquad y_C = 229\text{ mm}$$

Problem 9-62

Determine the location x_c of the centroid C of the shaded area which is part of a circle having a radius r.

Solution:

$$A = \alpha r^2 - r^2\sin(\alpha)\cos(\alpha)$$

$$x_C = \frac{1}{A}\left(\alpha r^2\frac{2r\sin(\alpha)}{3\alpha} - r^2\sin(\alpha)\cos(\alpha)\frac{2}{3}r\cos(\alpha)\right)$$

$$x_c = 2r\frac{\sin(\alpha)}{3}\left(\frac{1-\cos(\alpha)^2}{\alpha-\sin(\alpha)\cos(\alpha)}\right)$$

$$x_c = \frac{2r}{3}\frac{\sin(\alpha)^3}{\alpha-\frac{\sin(2\alpha)}{2}}$$

Problem 9-63

Locate the centroid y_c for the strut's cross-sectional area.

Given:

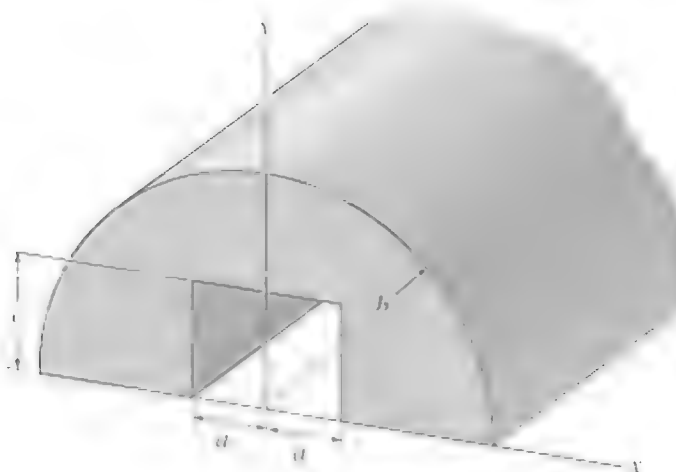
$a = 40$ mm

$b = 120$ mm

$c = 60$ mm

Solution:

$$A = \frac{\pi b^2}{2} - 2ac$$

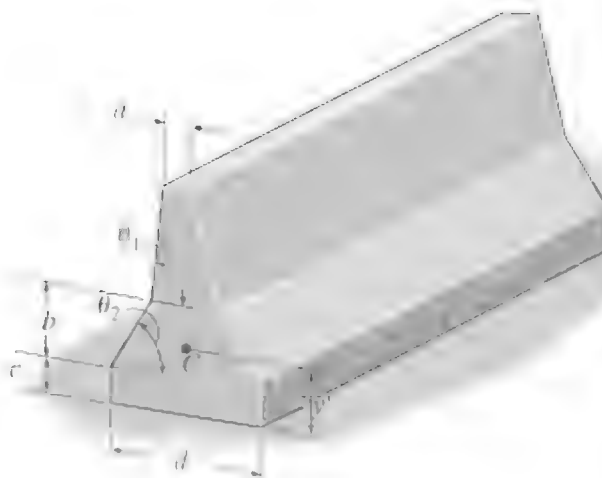
$$y_c = \frac{1}{A}\left[\frac{\pi b^2}{2}\left(\frac{4b}{3\pi}\right) - 2ac\left(\frac{c}{2}\right)\right] \qquad y_c = 56.6 \text{ mm}$$

Problem 9-64

The "New Jersey" concrete barrier is commonly used during highway construction. Determine the location y_c of its centroid.

Given:

$a = 4$ in

$b = 12$ in

$c = 6$ in

$d = 24\ \text{in}$

$\theta_1 = 75\ \text{deg}$

$\theta_2 = 55\ \text{deg}$

Solution:

$e = b\cot(\theta_2)$ $\quad f = d - 2e$ $\quad h = \dfrac{f-a}{2}\tan(\theta_1)$

$$A = d(c+b+h) - be - 2he - \frac{1}{2}(f-a)h$$

$$y_c = \frac{1}{A}\left[d\frac{(c+b+h)^2}{2} - be\left(c+\frac{2b}{3}\right) - 2he\left(c+b+\frac{h}{2}\right) - \frac{1}{2}(f-a)h\left(c+b+\frac{2h}{3}\right)\right]$$

$y_c = 8.69\ \text{in}$

Problem 9-65

The composite plate is made from both steel (A) and brass (B) segments. Determine the mass and location (x_c, y_c, z_c) of its mass center G.

Units Used:

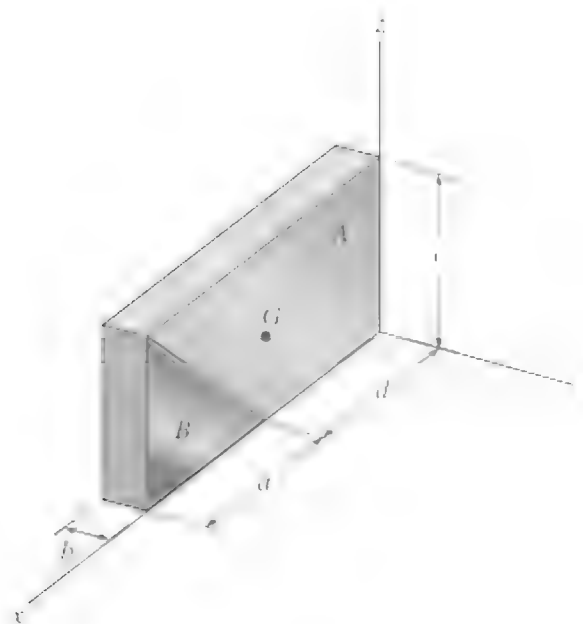
$\text{Mg} = 1000\ \text{kg}$

Given:

$\rho_{st} = 7.85\ \dfrac{\text{Mg}}{\text{m}^3}$ $\quad a = 150\ \text{mm}$

$b = 30\ \text{mm}$

$\rho_{br} = 8.74\ \dfrac{\text{Mg}}{\text{m}^3}$ $\quad c = 225\ \text{mm}$

$d = 150\ \text{mm}$

Solution:

$$M = \rho_{st}\left(dbc + \frac{1}{2}abc\right) + \rho_{br}\frac{1}{2}abc$$

$$x_c = \frac{1}{M}\left[\rho_{st}\left[dcb\frac{d}{2} + \frac{1}{2}abc\left(d+\frac{a}{3}\right)\right] + \rho_{br}\frac{1}{2}abc\left(d+\frac{2a}{3}\right)\right]$$

$$y_c = \frac{-1}{M}\left[\rho_{st}\left(d\,c\,b\frac{b}{2} + \frac{1}{2}a\,b\,c\frac{b}{2}\right) + \rho_{br}\frac{1}{2}a\,b\,c\frac{b}{2}\right]$$

$$z_c = \frac{1}{M}\left[\rho_{st}\left(d\,c\,b\frac{c}{2} + \frac{1}{2}a\,b\,c\frac{2c}{3}\right) + \rho_{br}\frac{1}{2}a\,b\,c\frac{c}{3}\right]$$

$$\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = \begin{pmatrix} 152.8 \\ -15.0 \\ 111.5 \end{pmatrix} \text{mm} \qquad M = 16.347 \text{ kg}$$

Problem 9-66

Locate the centroid y_c of the concrete beam having the tapered cross section shown.

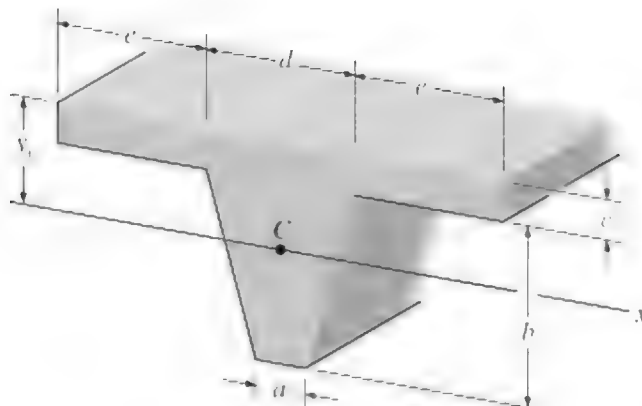
Given:

$a = 100$ mm

$b = 360$ mm

$c = 80$ mm

$d = 300$ mm

$e = 300$ mm

Solution:

$$y_c = \frac{(d+2e)c\left(\frac{c}{2}\right) + \frac{1}{2}(d-a)b\left(c+\frac{b}{3}\right) + a\,b\left(c+\frac{b}{2}\right)}{(d+2e)c + \frac{1}{2}(d-a)b + a\,b} \qquad y_c = 135\,\text{mm}$$

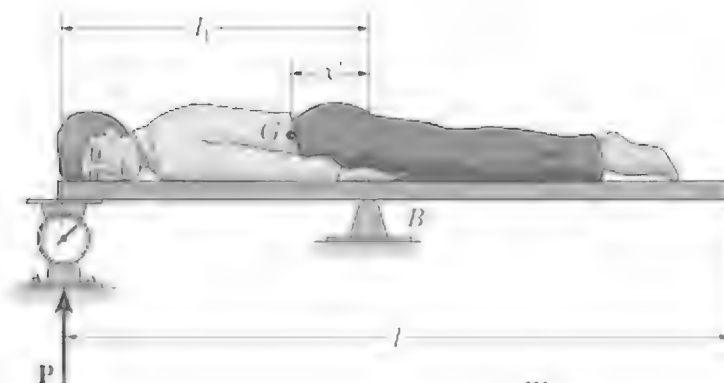
Problem 9-67

The anatomical center of gravity G of a person can be determined by using a scale and a rigid board having a uniform weight W_1 and length l. With the person's weight W known, the person lies down on the board and the scale reading $\mathbf{P}$ is recorded. From this show how to calculate the location x_c of the center of mass. Discuss the best place l_1 for the smooth support at B in order to improve the accuracy of this experiment.

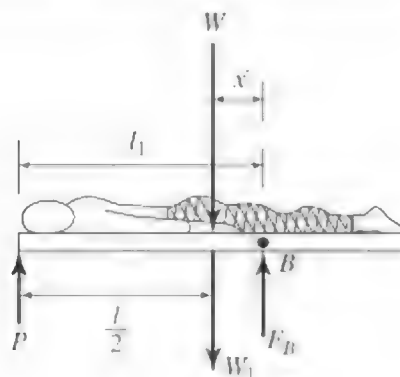
Given:

$a = 120$ mm

$b = 240$ mm

$c = 120$ mm

Solution:

$\Sigma M_B = 0;\qquad W x_c - P l_1 + W_1\left(l_1 - \frac{1}{2}\right) = 0$

$$x_c = \frac{P l_1 - W_1\left(l_1 - \frac{1}{2}\right)}{W}$$

Put B as close as possible to the center of gravity of the board, i.e., $l_1 = \frac{1}{2}$, then $W_1\left(l_1 - \frac{1}{2}\right) = 0$ and the effect of the board's weight will not be a large factor in the measurement.

Problem 9-68

The tank and compressor have a mass M_T and mass center at G_T and the motor has a mass M_M and a mass center at G_M. Determine the angle of tilt, θ, of the tank so that the unit will be on the verge of tipping over.

Given:

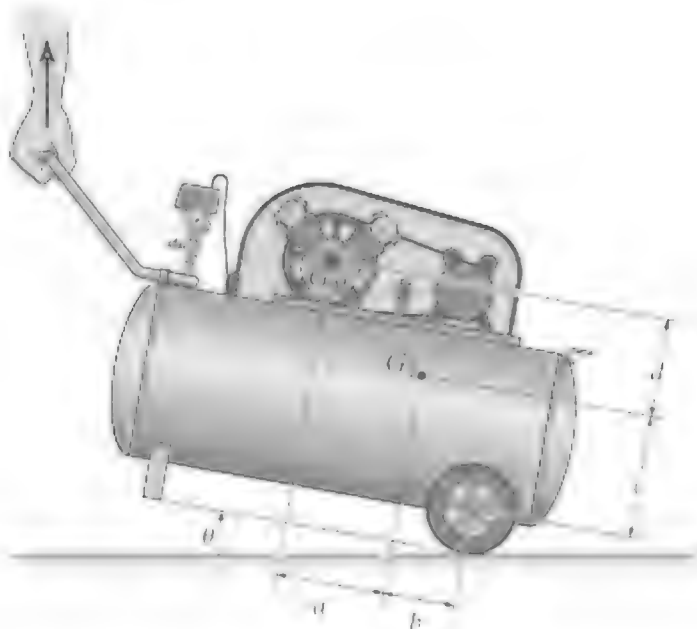
$a = 300$ mm

$b = 200$ mm

$c = 350$ mm

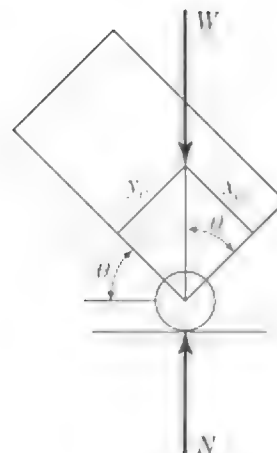
$d = 275$ mm

$M_T = 15$ kg

$M_M = 70$ kg

Solution:

$$x_c = \frac{bM_T + (a+b)M_M}{M_T + M_M} \qquad x_c = 0.4471\ \text{m}$$

$$y_c = \frac{cM_T + (c+d)M_M}{M_T + M_M} \qquad y_c = 0.57647\ \text{m}$$

$$\theta = \operatorname{atan}\left(\frac{x_c}{y_c}\right) \qquad \theta = 37.8\ \text{deg}$$

Problem 9-69

Determine the distance h to which a hole of diameter d must be bored into the base of the cone so that the center of mass of the resulting shape is located at z_c. The material has a density ρ.

Given:

$d = 100\ \text{mm}$

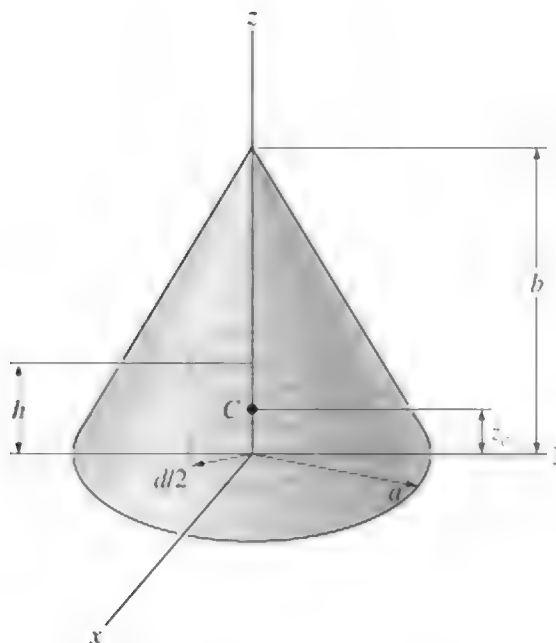
$z_c = 115\ \text{mm}$

$\rho = 8\ \frac{\text{mg}}{\text{m}^3}$

$a = 150\ \text{mm}$

$b = 500\ \text{mm}$

Solution:

Guess $\quad h = 200\ \text{mm}$

Given

$$z_c = \frac{\frac{1}{3}\pi a^2 b\left(\frac{b}{4}\right) - \pi\left(\frac{d}{2}\right)^2 h\left(\frac{h}{2}\right)}{\frac{1}{3}\pi a^2 b - \pi\left(\frac{d}{2}\right)^2 h} \qquad h = \text{Find}(h) \qquad h = 323\ \text{mm}$$

Problem 9-70

Determine the distance to the centroid of the shape which consists of a cone with a hole of height h bored into its base.

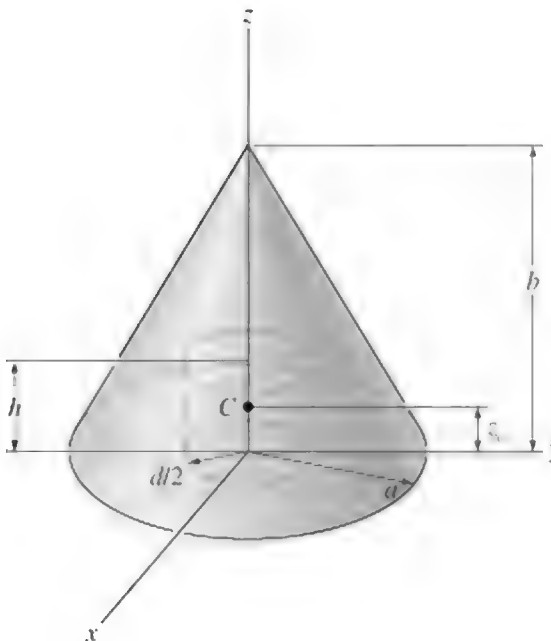
Given:

$d = 100 \text{ mm}$

$h = 50 \text{ mm}$

$\rho = 8 \frac{\text{mg}}{\text{m}^3}$

$a = 150 \text{ mm}$

$b = 500 \text{ mm}$

Solution:

$$z_c = \frac{\frac{1}{3}\pi a^2 b\left(\frac{b}{4}\right) - \pi\left(\frac{d}{2}\right)^2 h\left(\frac{h}{2}\right)}{\frac{1}{3}\pi a^2 b - \pi\left(\frac{d}{2}\right)^2 h} \qquad z_c = 128.4 \text{ mm}$$

Problem 9-71

The sheet metal part has the dimensions shown. Determine the location (x_c, y_c, z_c) of its centroid.

Given:

$a = 3 \text{ in}$

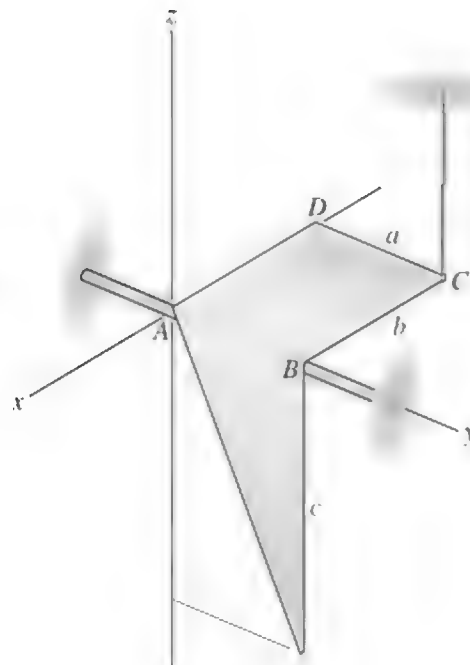
$b = 4 \text{ in}$

$c = 6 \text{ in}$

Solution:

$$x_c = \frac{-a b\left(\frac{b}{2}\right)}{a b + \frac{1}{2} a c} \qquad x_c = -1.143 \text{ in}$$

$$y_c = \frac{a b\left(\frac{a}{2}\right) + \frac{1}{2} a c\left(\frac{2a}{3}\right)}{a b + \frac{1}{2} a c} \qquad y_c = 1.714 \text{ in}$$

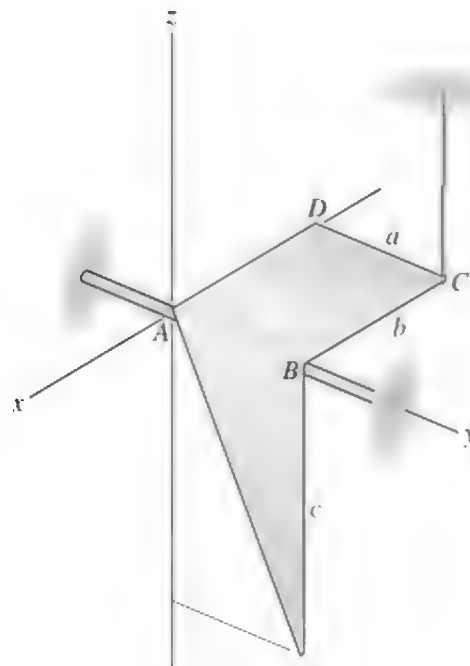
$$z_c = \frac{\frac{-1}{2} a c\left(\frac{c}{3}\right)}{a b + \frac{1}{2} a c} \qquad z_c = -0.857 \text{ in}$$

Problem 9-72

The sheet metal part has a weight per unit area of and is supported by the smooth rod and at C. If the cord is cut, the part will rotate about the y axis until it reaches equilibrium. Determine the equilibrium angle of tilt, measured downward from the negative x axis, that AD makes with the $-x$ axis.

Given:

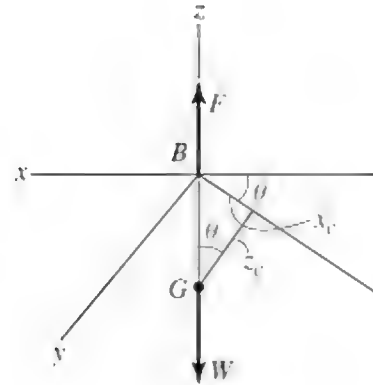
$a = 3 \text{ in}$

$b = 4 \text{ in}$

$c = 6 \text{ in}$

Solution:

$$x_c = \frac{ab\left(\frac{b}{2}\right)}{ab + \frac{1}{2}ac} \qquad x_c = 1.143 \text{ in}$$

$$z_c = \frac{\frac{1}{2}ac\left(\frac{c}{3}\right)}{ab + \frac{1}{2}ac} \qquad z_c = 0.857 \text{ in}$$

$$\theta = \operatorname{atan}\left(\frac{x_c}{z_c}\right) \qquad \theta = 53.13 \text{ deg}$$

Problem 9-73

A toy skyrocket consists of a solid conical top of density ρ_t, a hollow cylinder of density ρ_c, and a stick having a circular cross section of density ρ_s. Determine the length of the stick, x, so that the center of gravity G of the skyrocket is located along line aa.

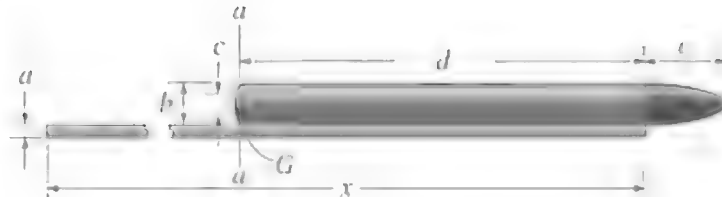
Given:

$a = 3$ mm $\qquad \rho_t = 600 \frac{\text{kg}}{\text{m}^3}$

$b = 10$ mm

$c = 5$ mm $\qquad \rho_c = 400 \frac{\text{kg}}{\text{m}^3}$

$d = 100$ mm

$e = 20$ mm $\qquad \rho_s = 300 \frac{\text{kg}}{\text{m}^3}$

Solution:

Guess $\qquad x = 200$ mm

Given

$$\rho_t \pi \left(\frac{b}{2}\right)^2 \frac{e}{3}\left(d + \frac{e}{4}\right) + \rho_c \frac{\pi}{4}\left(b^2 - c^2\right) d \left(\frac{d}{2}\right) + \rho_s \pi \left(\frac{a^2}{4}\right) x \left(d - \frac{x}{2}\right) = 0$$

$x = \text{Find}(x) \qquad x = 490$ mm

Problem 9-74

Determine the location (x_c, y_c) of the center of mass of the turbine and compressor assembly. The mass and the center of mass of each of the various components are indicated below.

Given:

$a = 0.75$ m $M_1 = 25$ kg

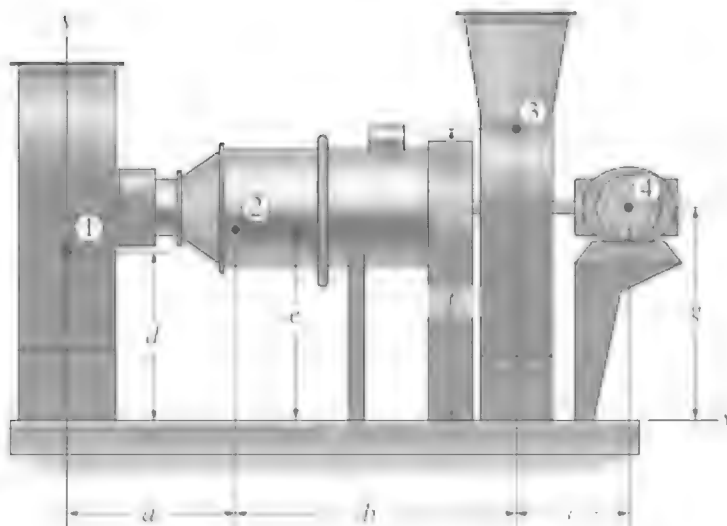
$b = 1.25$ m $M_2 = 80$ kg

$c = 0.5$ m $M_3 = 30$ kg

$d = 0.75$ m $M_4 = 105$ kg

$e = 0.85$ m

$f = 1.30$ m

$g = 0.95$ m

1. Intake housing M_1
2. Turbine M_2
3. Exhaust housing M_3
4. Compressor M_4

Solution:

$$M = M_1 + M_2 + M_3 + M_4$$

$$x_c = \frac{1}{M}\left[M_2 a + M_3(a+b) + M_4(a+b+c)\right] \qquad x_c = 1.594 \text{ m}$$

$$y_c = \frac{1}{M}\left(M_1 d + M_2 e + M_3 f + M_4 g\right) \qquad y_c = 0.940 \text{ m}$$

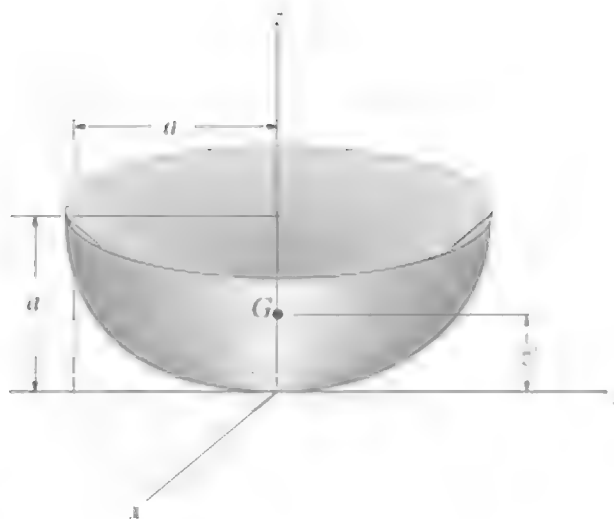
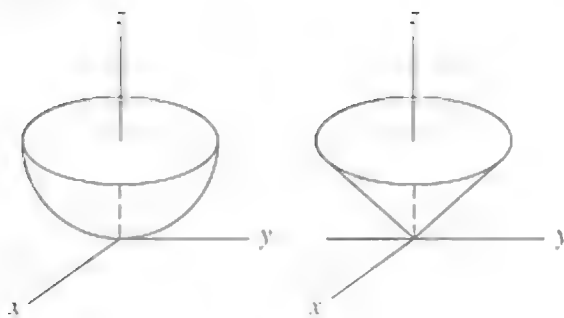
Problem 9-75

The solid is formed by boring a conical hole into the hemisphere. Determine the distance z_c to the center of gravity.

Solution:

$$V = \frac{2}{3}\pi a^3 - \frac{\pi}{3}a^2 a = \frac{\pi}{3}a^3$$

$$z_C = \frac{1}{V}\left[\frac{5a}{8}\left(\frac{2}{3}\pi a^3\right) - \frac{3}{4}a\left(\frac{\pi}{3}a^3\right)\right]$$

$$z_C = \frac{a}{2}$$

Problem 9-76

Determine the location x_c of the centroid of the solid made from a hemisphere, cylinder, and cone.

Given:

$a = 80$ mm

$b = 60$ mm

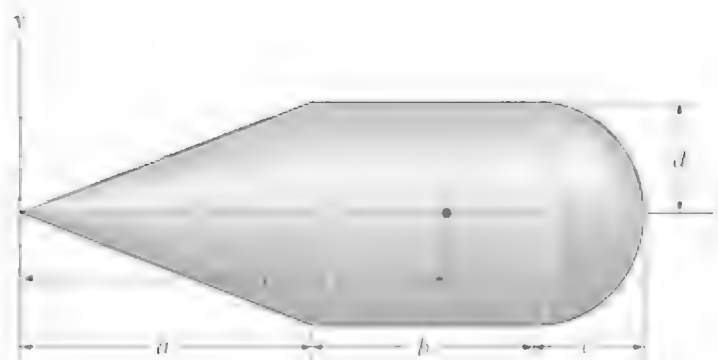
$c = 30$ mm

$d = 30$ mm

Solution:

$$V = \frac{1}{3}\pi d^2 a + \pi d^2 b + \frac{2}{3}\pi d^3$$

$$x_C = \frac{1}{V}\left[\frac{1}{3}\pi d^2 a\left(\frac{3a}{4}\right) + \pi d^2 b\left(a + \frac{b}{2}\right) + \frac{2}{3}\pi d^3\left(a + b + \frac{3c}{8}\right)\right]$$

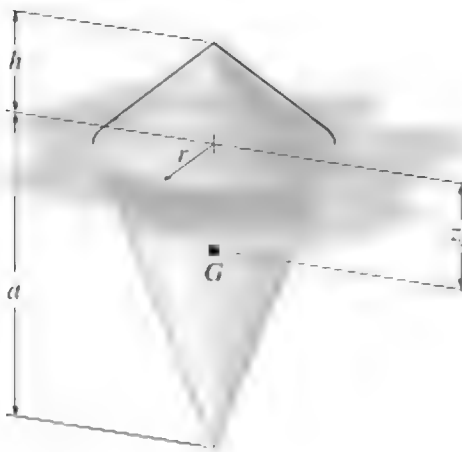
$$x_C = 105.2\text{ mm}$$

Problem 9-77

The buoy is made from two homogeneous cones each having radius r. Find the distance z_c to the buoy's center of gravity G.

Given:

$r = 1.5\ \text{ft}$

$h = 1.2\ \text{ft}$

$a = 4\ \text{ft}$

Solution:

$$z_c = \frac{\frac{\pi}{3}r^2 a\left(\frac{a}{4}\right) - \frac{\pi}{3}r^2 h\left(\frac{h}{4}\right)}{\frac{\pi}{3}r^2(a+h)} \qquad z_c = 0.7\ \text{ft}$$

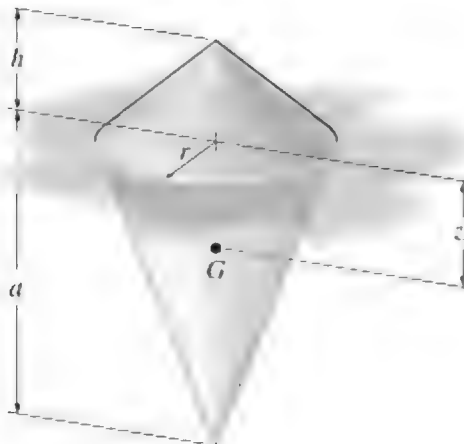
Problem 9-78

The buoy is made from two homogeneous cones each having radius r. If it is required that the buoy's center of gravity G be located at z_c,determine the height h of the top cone.

Given:

$z_c = 0.5\ \text{ft}$

$r = 1.5\ \text{ft}$

$a = 4\ \text{ft}$

Solution:

Guess $\quad h = 1\ \text{ft}$

Given $$z_c = \frac{\frac{\pi}{3}r^2 a\left(\frac{a}{4}\right) - \frac{\pi}{3}r^2 h\left(\frac{h}{4}\right)}{\frac{\pi}{3}r^2(a+h)}$$ $h = \text{Find}(h)$ $h = 2\,\text{ft}$

Problem 9-79

Locate the center of mass z_c of the forked lever, which is made from a homogeneous material and has the dimensions shown.

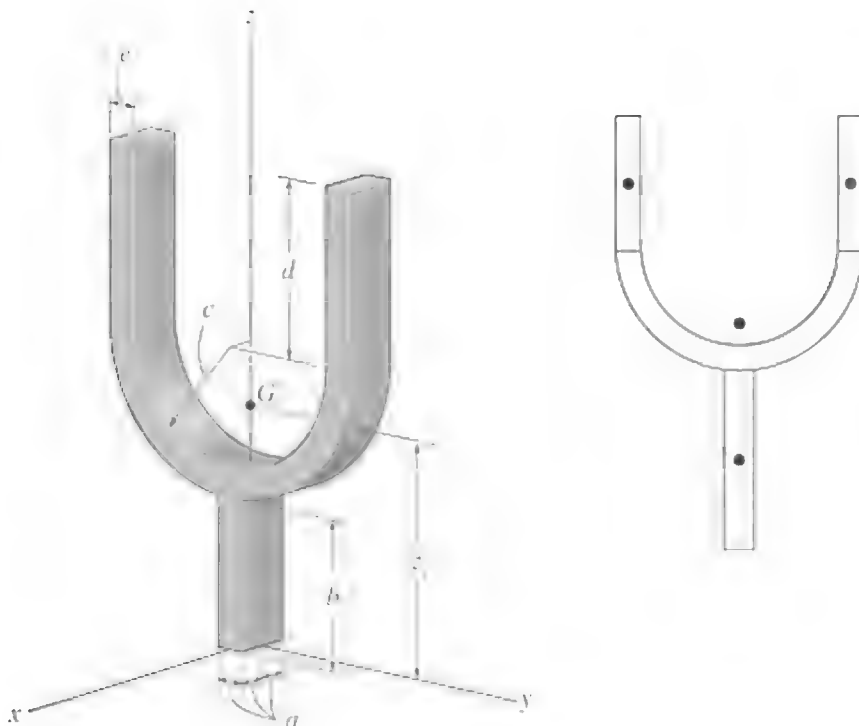
Given:

$a = 0.5\ \text{in}$

$b = 2.5\ \text{in}$

$c = 2\ \text{in}$

$d = 3\ \text{in}$

$e = 0.5\ \text{in}$

Solution:

$$V = b a^2 + 2e a d + \frac{\pi}{2}\left[(c+e)^2 - c^2\right]a$$

$$z_c = \frac{1}{V}\left[b a^2\left(\frac{b}{2}\right) + 2e a d\left(b+e+c+\frac{d}{2}\right) + \left(\frac{\pi a}{2}\right)(c+e)^2\left[b+c+e-4\left(\frac{c+e}{3\pi}\right)\right] \dots + \left(\frac{-\pi a}{2}\right)c^2\left[b+c+e-4\left(\frac{c}{3\pi}\right)\right]\right]$$

$z_c = 4.32\ \text{in}$

Problem 9-80

A triangular plate made of homogeneous material has a constant thickness which is very small. If it is folded over as shown, determine the location y_c of the plate's center of gravity G.

Given:

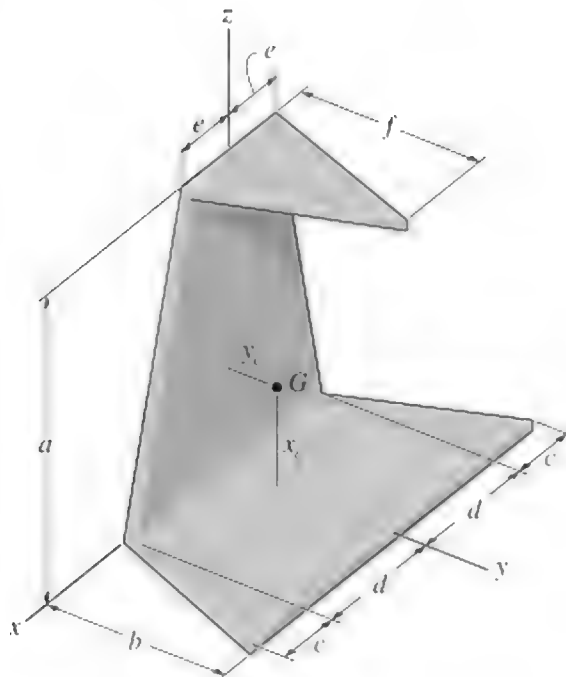
$a = 6 \text{ in}$

$b = 3 \text{ in}$

$c = 1 \text{ in}$

$d = 3 \text{ in}$

$e = 1 \text{ in}$

$f = 3 \text{ in}$

Solution:

$$y_c = \frac{2db\left(\frac{b}{2}\right) + \frac{1}{2}(2cb)\left(\frac{2b}{3}\right) + \frac{1}{2}(2ef)\left(\frac{f}{3}\right)}{2db + \frac{1}{2}(2cb) + \frac{1}{2}(2d)(a+f)} \qquad y_c = 0.75 \text{ in}$$

Problem 9-81

A triangular plate made of homogeneous material has a constant thickness which is very small. If it is folded over as shown, determine the location z_c of the plate's center of gravity G.

Given

$a = 6 \text{ in}$

$b = 3 \text{ in}$

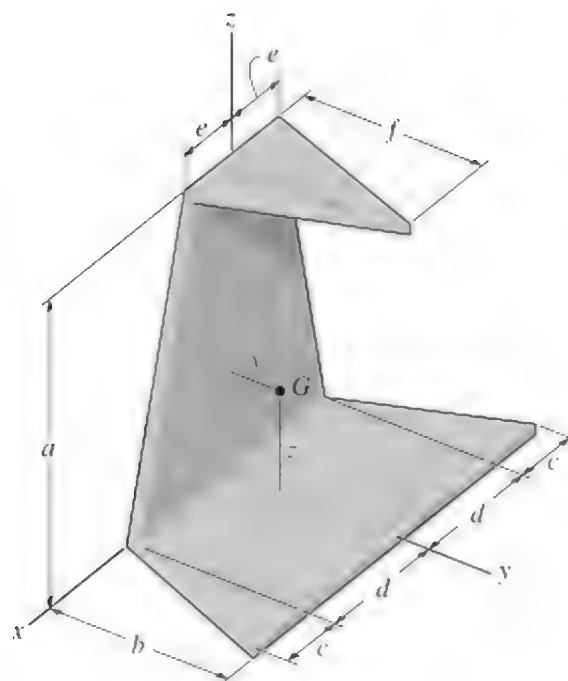
$c = 1 \text{ in}$

$d = 3 \text{ in}$

$e = 1 \text{ in}$

$f = 3 \text{ in}$

Solution:

$$z_c = \frac{\frac{1}{2}(2efa) + 2ea\left(\frac{a}{2}\right) + \frac{1}{2}2(d-e)a\left(\frac{a}{3}\right)}{2db + \frac{1}{2}(2cb) + \frac{1}{2}2d(a+f)} \qquad z_c = 1.625 \text{ in}$$

Problem 9-82

Each of the three homogeneous plates welded to the rod has a density ρ and a thickness a. Determine the length l of plate C and the angle of placement, θ, so that the center of mass of the assembly lies on the y axis. Plates A and B lie in the x–y and z–y planes, respectively.

Units Used:

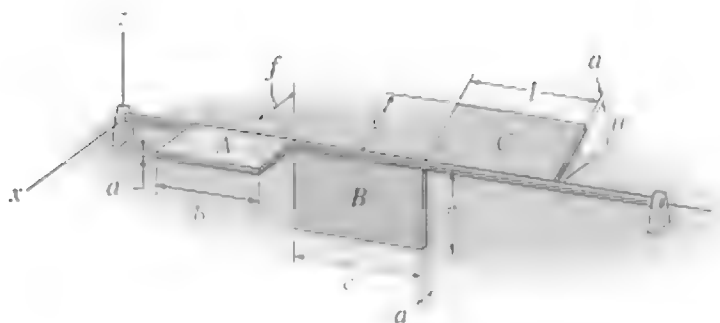
$\text{Mg} = 1000 \text{ kg}$

Given:

$a = 10 \text{ mm}$ $\qquad f = 100 \text{ mm}$

$b = 200 \text{ mm}$ $\qquad g = 150 \text{ mm}$

$c = 250 \text{ mm}$ $\qquad e = 150 \text{ mm}$

$\rho = 6 \frac{\text{Mg}}{\text{m}^3}$

Solution: The thickness and density are uniform

Guesses $\theta = 10 \text{ deg}$ $\qquad l = 10 \text{ mm}$

Given

$$b f\left(\frac{f}{2}\right) - g l\left(\frac{g}{2}\right)\cos(\theta) = 0 \qquad -c e\left(\frac{e}{2}\right) + g l\left(\frac{g}{2}\right)\sin(\theta) = 0$$

$$\begin{pmatrix} l \\ \theta \end{pmatrix} = \text{Find}(l, \theta) \qquad l = 265\,\text{mm} \qquad \theta = 70.4\,\text{deg}$$

Problem 9-83

The assembly consists of a wooden dowel rod of length L and a tight-fitting steel collar. Determine the distance x_c to its center of gravity if the specific weights of the materials are γ_w and γ_{st}. The radii of the dowel and collar are shown.

Given:

$L = 20\text{ in}$

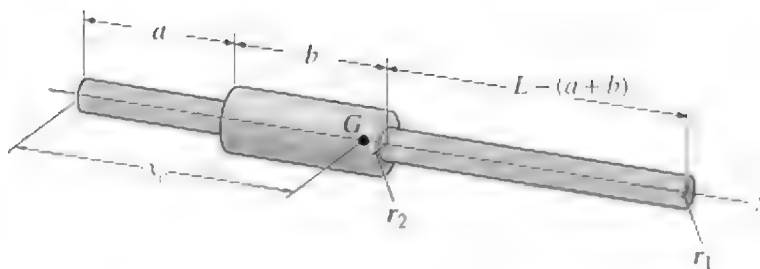
$\gamma_w = 150\,\frac{\text{lb}}{\text{ft}^3}$

$\gamma_{st} = 490\,\frac{\text{lb}}{\text{ft}^3}$

$a = 5\text{ in}$

$b = 5\text{ in}$

$r_1 = 1\text{ in}$

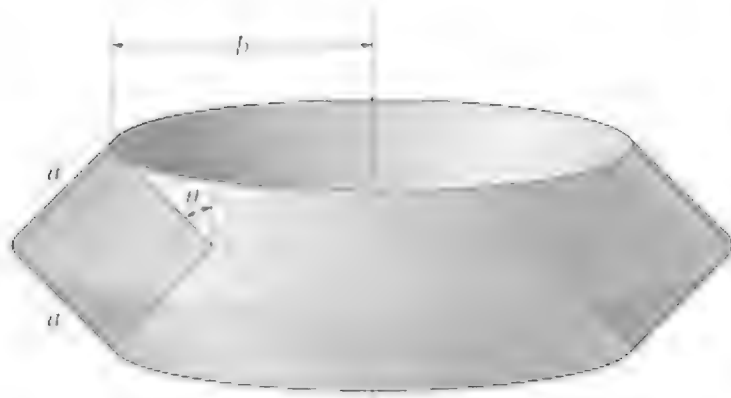
$r_2 = 2\text{ in}$

Solution:

$$x_c = \frac{\gamma_w \pi r_1^2 L \frac{L}{2} + \gamma_{st}\pi\left(r_2^2 - r_1^2\right) b\left(a + \frac{b}{2}\right)}{\gamma_w \pi r_1^2 L + \gamma_{st}\pi\left(r_2^2 - r_1^2\right) b} \qquad x_c = 8.225\text{ in}$$

Problem 9-84

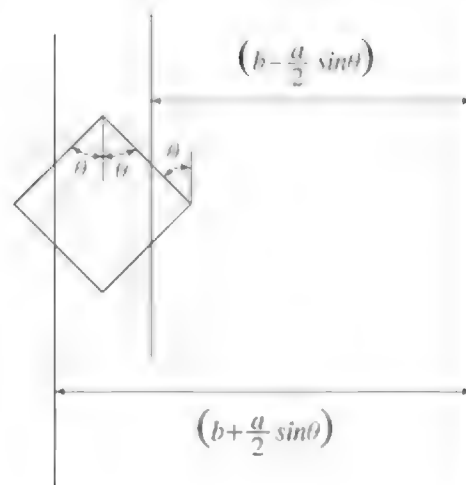
Determine the surface area and the volume of the ring formed by rotating the square about the vertical axis.

Given:

$\theta = 45 \text{ deg}$

Solution:

$$A = 2\left[2\pi\left(b - \frac{a}{2}\sin(\theta)\right)a\right] \dots$$
$$+ 2\left[2\pi\left(b + \frac{a}{2}\sin(\theta)\right)a\right]$$

$$A = 8\pi b a$$

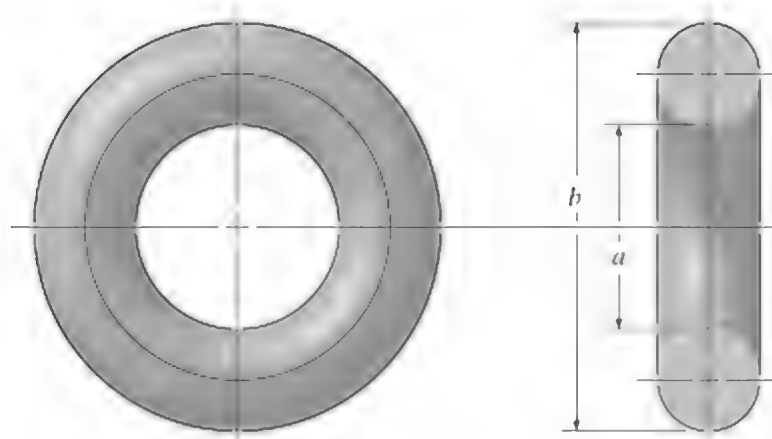
$$V = 2\pi b a^2$$

Problem 9-85

The anchor ring is made of steel having specific weight γ_{st}. Determine the surface area of the ring. The cross section is circular as shown.

Given:

$$\gamma_{st} = 490 \frac{\text{lb}}{\text{ft}^3}$$

$$a = 4 \text{ in}$$

$$b = 8 \text{ in}$$

Solution:

$$A = 2\pi\left(\frac{a}{2} + \frac{b-a}{4}\right)2\pi\left(\frac{b-a}{4}\right) \qquad A = 118\,\text{in}^2$$

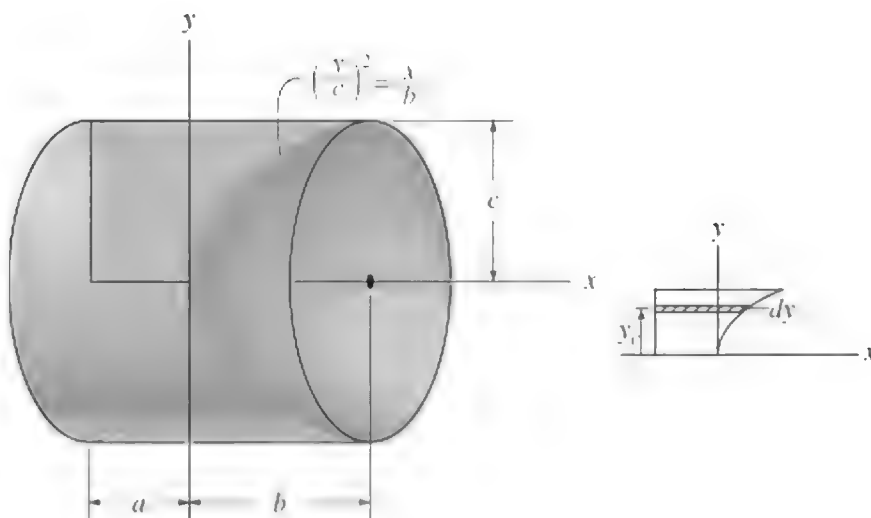
Problem 9-86

Using integration, determine both the area and the distance y_c to the centroid of the shaded area. Then using the second theorem of PappusGuldinus, determine the volume of the solid generated by revolving the shaded area about the x axis.

Given:

$a = 1$ ft

$b = 2$ ft

$c = 2$ ft

Solution:

$$A = \int_0^c \left[a + \left(\frac{y}{c}\right)^2 b\right] dy \qquad A = 3.333\,\text{ft}^2$$

$$y_c = \frac{1}{A}\int_0^c y\left[a + \left(\frac{y}{c}\right)^2 b\right] dy \qquad y_c = 1.2\,\text{ft}$$

$$V = 2\pi y_c A \qquad V = 25.1\,\text{ft}^3$$

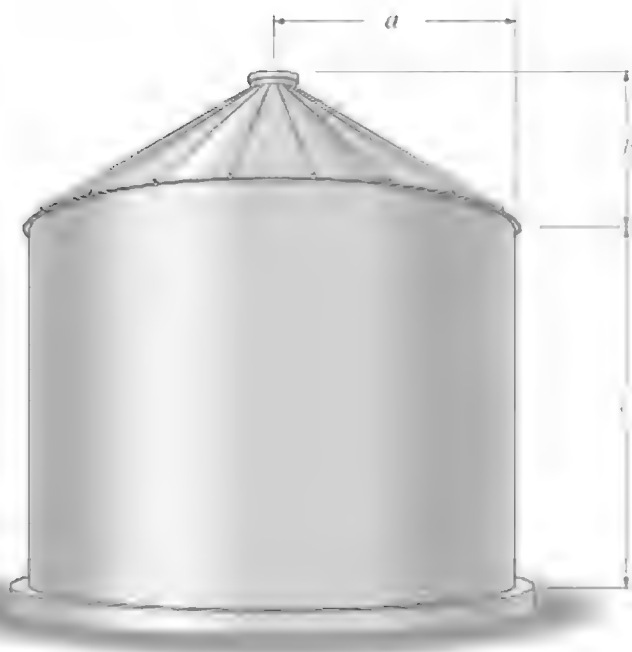
Problem 9-87

The grain bin of the type shown is manufactured by Grain Systems, Inc. Determine the required square footage of the sheet metal needed to form it, and also the maximum storage capacity (volume) within it.

Given:

$a = 30 \text{ ft}$

$b = 20 \text{ ft}$

$c = 45 \text{ ft}$

Solution:

$$A = 2\pi a c + 2\pi \frac{a}{2}\sqrt{a^2 + b^2}$$

$$A = 11.9 \times 10^3 \text{ ft}^2$$

$$V = 2\pi a c\left(\frac{a}{2}\right) + 2\pi \frac{a}{3}\left(\frac{1}{2} a b\right)$$

$$V = 146 \times 10^3 \text{ ft}^3$$

Problem 9-88

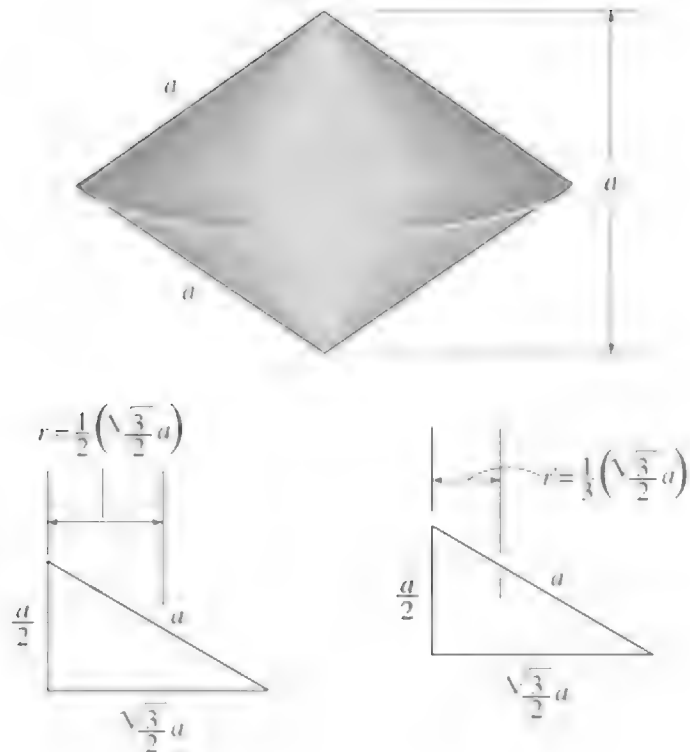
Determine the surface area and the volume of the conical solid.

Solution:

$$A = 2a\frac{\sqrt{3}}{2}\frac{a}{2}2\pi$$

$$A = \sqrt{3}\,\pi a^2$$

$$V = 2\left(\frac{1}{2}\frac{a}{2}\right)\left(\frac{\sqrt{3}}{2}a\right)\left(\frac{\sqrt{3}}{6}a2\pi\right)$$

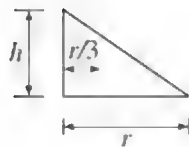
$$V = \frac{\pi}{4}a^3$$

Problem 9-89

Sand is piled between two walls as shown. Assume the pile to be a quarter section of a cone and that ratio p of this volume is voids (air space). Use the second theorem of Pappus-Guldinus to determine the volume of sand.

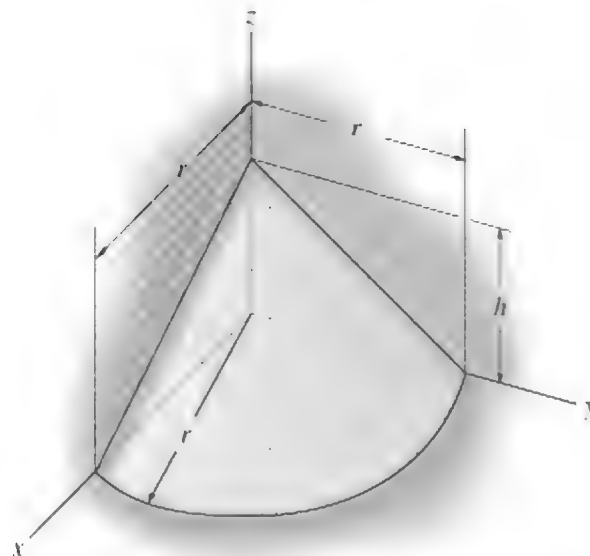
Given:

$r = 3\text{ m}$

$h = 2\text{ m}$

$p = 0.26$

Solution:

$$V = (1-p)\left(\frac{\pi}{2}\right)\left(\frac{r}{3}\right)\left(\frac{hr}{2}\right)$$

$$V = 3.487\text{ m}^3$$

Problem 9-90

The *rim* of a flywheel has the cross section *A-A* shown. Determine the volume of material needed for its construction.

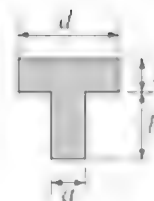
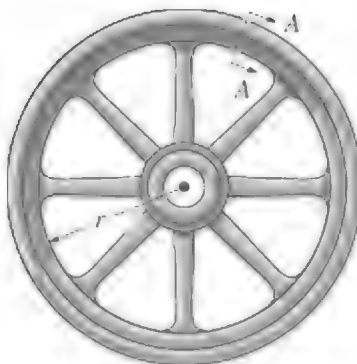
Given:

$r = 300\ \text{mm}$

$a = 20\ \text{mm}$

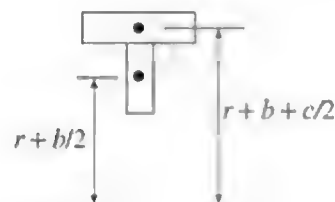
$b = 40\ \text{mm}$

$c = 20\ \text{mm}$

$d = 60\ \text{mm}$

Section A-A

Solution:

$$V = 2\pi\left(r + b + \frac{c}{2}\right)d\,c + 2\pi\left(r + \frac{b}{2}\right)b\,a$$

$$V = 4.25 \times 10^6\ \text{mm}^3$$

Problem 9-91

The Gates Manufacturing Co. produces pulley wheels such as the one shown. Determine the weight of the wheel if it is made from steel having a specific weight γ.

Given:

$a = 1\ \text{in}$

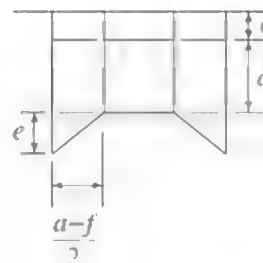
$c = 0.5\ \text{in}$

$d = 1\ \text{in}$

$e = 1\ \text{in}$

$f = 0.25\ \text{in}$

$b = 2(c + d + e)$

$\gamma = 490\ \frac{\text{lb}}{\text{ft}^3}$

Solution:

$$W = \gamma 2\pi \left[d\,a\left(c + \frac{d}{2}\right) + \left(c + d + \frac{a}{3}\right)\left(\frac{a-f}{2}\right)e \right] \qquad W = 3.01 \text{ lb}$$

Problem 9-92

The Gates Manufacturing Co. produces pulley wheels such as the one shown. Determine the total surface area of the wheel in order to estimate the amount of paint needed to protect its surface from rust.

Given:

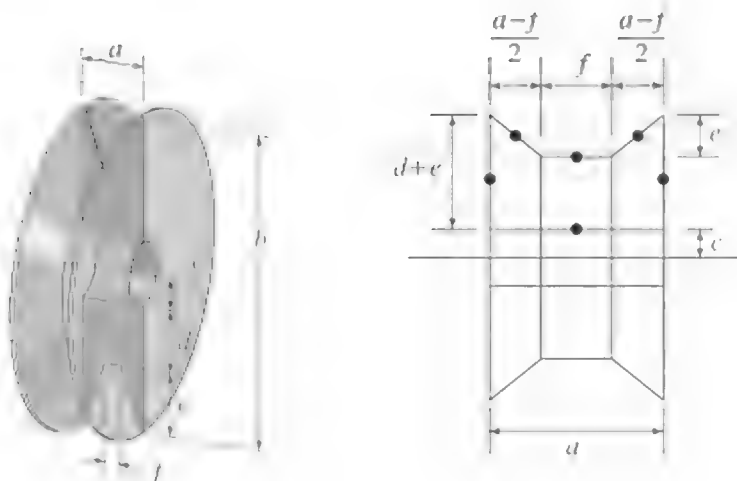
$a = 1$ in

$c = 0.5$ in

$d = 1$ in

$e = 1$ in

$f = 0.25$ in

$b = 2(c + d + e)$

Solution:

$$A = 2\pi \left[f(c + d) + a\,c + 2(d + e)\left(c + \frac{d + e}{2}\right) + 2\sqrt{e^2 + \left(\frac{a-f}{2}\right)^2}\left(c + d + \frac{e}{2}\right) \right]$$

$A = 70\,\text{in}^2$

Problem 9-93

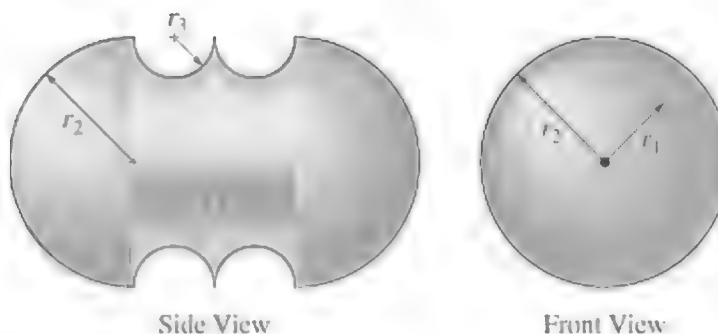
Determine the volume of material needed to make the casting.

Given:

$r_1 = 4$ in

$r_2 = 6$ in

Side View

Front View

$r_3 = r_2 - r_1$

Solution:

$$V = 2\pi\left[2\left(\frac{\pi}{4}\right)r_2^2\left(\frac{4r_2}{3\pi}\right) + 2r_2(2r_3)\left(\frac{r_2}{2}\right) - 2\left(\frac{\pi}{2}\right)r_3^2\left(r_2 - \frac{4r_3}{3\pi}\right)\right]$$

$V = 1.40 \times 10^3\ \text{in}^3$

Problem 9-94

A circular sea wall is made of concrete. Determine the total weight of the wall if the concrete has a specific weight γ_c.

Given:

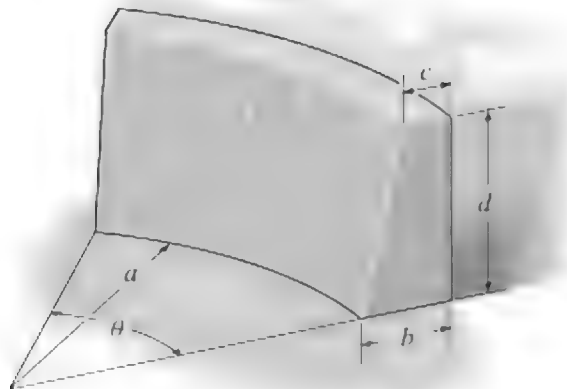
$\gamma_c = 150\ \frac{\text{lb}}{\text{ft}^3}$

$a = 60\ \text{ft}$

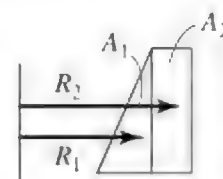
$b = 15\ \text{ft}$

$c = 8\ \text{ft}$

$d = 30\ \text{ft}$

$\theta = 50\ \text{deg}$

Solution:

$$W = \gamma_c\ \theta\left[a\left[\frac{1}{2}d(b-c)\right] + \frac{2}{3}(b-c)\left[\frac{1}{2}d(b-c)\right] + \left(a + b - \frac{c}{2}\right)d\,c\right]$$

$W = 3.12 \times 10^6\ \text{lb}$

Problem 9-95

Determine the surface area of the tank, which consists of a cylinder and hemispherical cap.

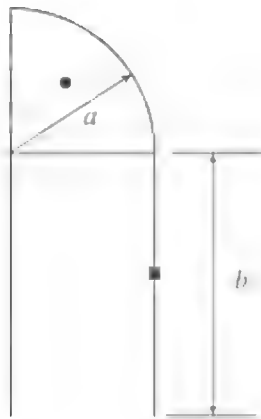
Given:

$a = 4\ \text{m}$

$b = 8\ \text{m}$

Solution:

$$A = 2\pi\left(a\,b + \frac{2a}{\pi}\frac{\pi a}{2}\right)$$

$A = 302\ \text{m}^2$

Problem 9-96

Determine the volume of the tank, which consists of a cylinder and hemispherical cap.

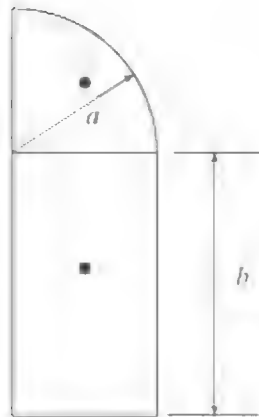
Given:

$a = 4\ \text{m}$

$b = 8\ \text{m}$

Solution:

$$V = 2\pi\left[\frac{4a}{3\pi}\left(\frac{\pi a^2}{4}\right) + \frac{a}{2}(b\,a)\right]$$

$V = 536\ \text{m}^3$

Problem 9-97

Determine the surface area of the silo which consists of a cylinder and hemispherical cap. Neglect the thickness of the plates.

Given:

$a = 10$ ft

$b = 10$ ft

$c = 80$ ft

Solution:

$$A = 2\pi\left[\frac{2a}{\pi}\left(\frac{\pi a}{2}\right) + a\,c\right]$$

$$A = 5.65 \times 10^3\ \text{ft}^2$$

Problem 9-98

Determine the volume of the silo which consists of a cylinder and hemispherical cap. Neglect the thickness of the plates.

Given:

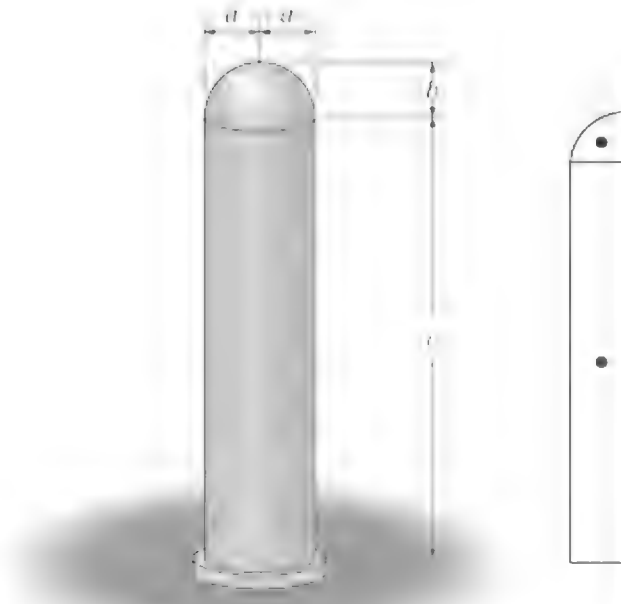
$a = 10$ ft

$b = 10$ ft

$c = 80$ ft

Solution:

$$V = 2\pi\left[\frac{4a}{3\pi}\left(\frac{\pi a^2}{4}\right) + c\,a\left(\frac{a}{2}\right)\right]$$

$$V = 27.2 \times 10^3\ \text{ft}^3$$

Problem 9-99

The process tank is used to store liquids during manufacturing. Estimate both the volume of the tank and its surface area. The tank has a flat top and the plates from which the tank is made have negligible thickness.

Given:

$a = 4\ \text{m}$

$b = 6\ \text{m}$

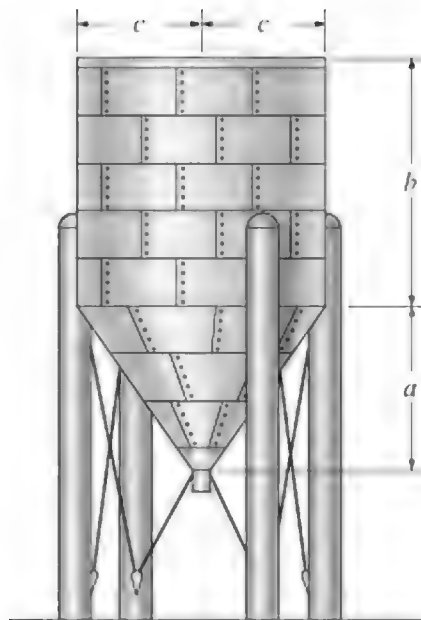
$c = 3\ \text{m}$

Solution:

$$V = 2\pi\left[\frac{c}{3}\left(\frac{c\,a}{2}\right) + \frac{c}{2}(c\,b)\right]$$

$$V = 207\,\text{m}^3$$

$$A = 2\pi\left(\frac{c}{2}c + c\,b + \frac{c}{2}\sqrt{a^2 + c^2}\right)$$

$$A = 188\ \text{m}^2$$

Problem 9-100

Determine the height h to which liquid should be poured into the cup so that it contacts half the surface area on the inside of the cup. Neglect the cup's thickness for the calculation.

Given:

$a = 30\ \text{mm}$

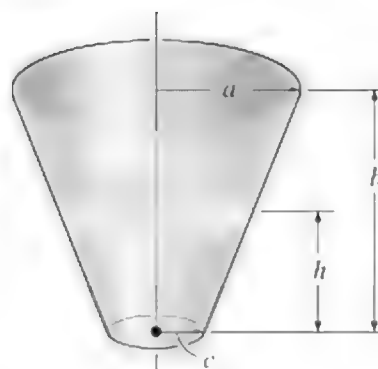
$b = 50\ \text{mm}$

$c = 10\ \text{mm}$

Solution:

Total area

$$A_{total} = 2\pi\left[c\frac{c}{2} + \frac{a+c}{2}\sqrt{b^2 + (a-c)^2}\right]$$

Guess $\quad h = 1\ \text{mm} \qquad e = 1\ \text{mm}$

Given

$$\frac{a-c}{b} = \frac{e-c}{h}$$

$$\frac{A_{total}}{2} = 2\pi\left[c\frac{c}{2} + \frac{e+c}{2}\sqrt{h^2 + (e-c)^2}\right]$$

$$\begin{pmatrix} e \\ h \end{pmatrix} = \text{Find}(e,h) \qquad e = 21.942\text{ mm} \qquad h = 29.9\text{ mm}$$

Problem 9-101

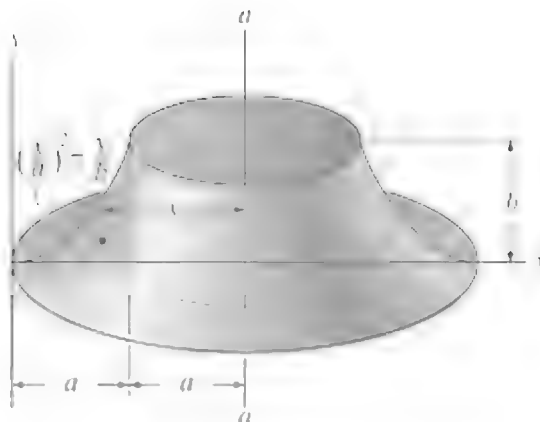
Using integration, compute both the area and the centroidal distance x_c of the shaded region. Then, using the second theorem of Pappus–Guldinus, compute the volume of the solid generated by revolving the shaded area about the aa axis.

Given:

$a = 8$ in

$b = 8$ in

Solution:

$$A = \int_0^a b\left(\frac{x}{a}\right)^2 dx$$

$$x_c = 2a - \frac{1}{A}\int_0^a x b\left(\frac{x}{a}\right)^2 dx \qquad A = 21.333\text{ in}^2 \qquad x_c = 10\text{ in}$$

$$V = 2\pi A x_c \qquad V = 1.34 \times 10^3\text{ in}^3$$

Problem 9-102

Using integration, determine the area and the centroidal distance y_c of the shaded area. Then, using the second theorem of Pappus–Guldinus, determine the volume of a solid formed by revolving the area about the x axis.

Given:

$a = 0.5 \text{ ft}$

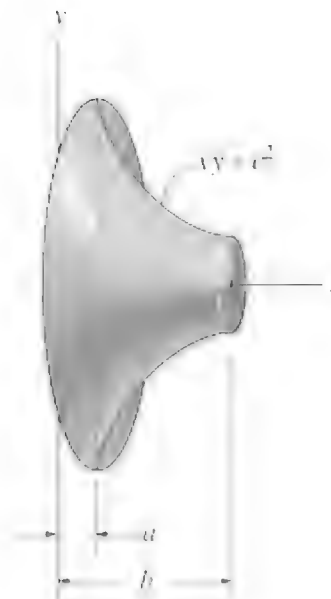
$b = 2 \text{ ft}$

$c = 1 \text{ ft}$

Solution:

$$A = \int_a^b \frac{c^2}{x}\, dx \qquad A = 1.386\,\text{ft}^2$$

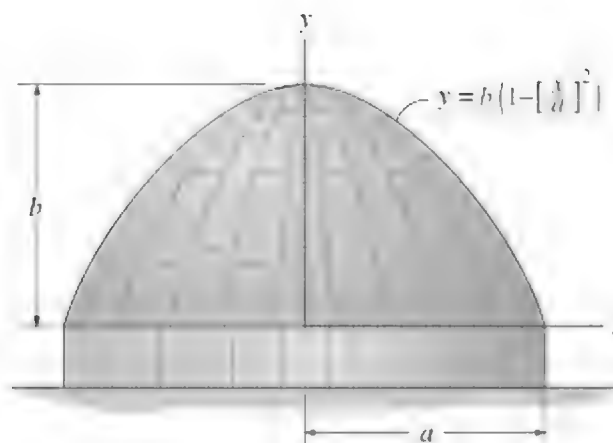
$$y_c = \frac{1}{A}\int_a^b \frac{1}{2}\left(\frac{c^2}{x}\right)^2 dx \qquad y_c = 0.541\,\text{ft}$$

$$V = 2\pi A\, y_c \qquad V = 4.71\,\text{ft}^3$$

Problem 9-103

Determine the surface area of the roof of the structure if it is formed by rotating the parabola about the y axis.

Given:

$a = 16 \text{ m}$

$b = 16 \text{ m}$

Solution:

Centroid : The length of the differential element is

$$dL = \sqrt{dx^2 + dy^2} = \left[\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right]dx$$

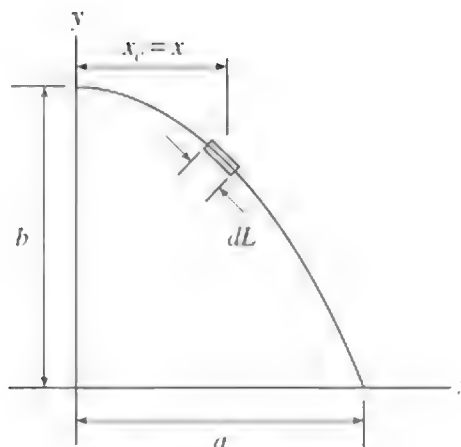
and its centroid is $x_c = x$

Here, $\frac{dy}{dx} = -2\frac{bx}{a^2}$

Evaluating the integrals, we have

$$L = \int_0^a \sqrt{1 + \frac{4b^2x^2}{a^4}}\,dx \qquad L = 23.663 \text{ m}$$

$$x_c = \frac{1}{L}\int_0^a x\sqrt{1 + \frac{4b^2x^2}{a^4}}\,dx \qquad x_c = 9.178 \text{ m}$$

$$A = 2\pi x_c L \qquad A = 1.365 \times 10^3 \text{ m}^2$$

Problem 9-104

The suspension bunker is made from plates which are curved to the natural shape which a completely flexible membrane would take if subjected to a full load of coal.This curve may be approximated by a parabola, $y/b = (x/a)^2$. Determine the weight of coal which the bunker would contain when completely filled. Coal has a specific weight of γ, and assume there is a fraction loss p in volume due to air voids. Solve the problem by integration to determine the cross-sectional area of *ABC*; then use the second theorem of Pappus–Guldinus to find the volume.

Units Used:

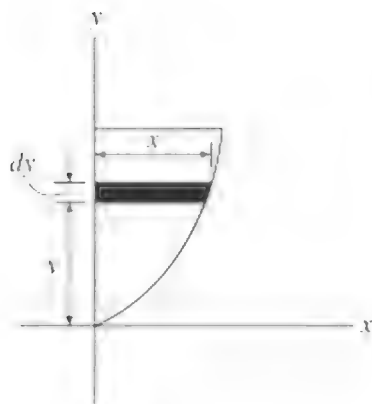
$$\text{kip} = 10^3 \text{ lb}$$

Given:

$$a = 10 \text{ ft}$$

$$b = 20 \text{ ft}$$

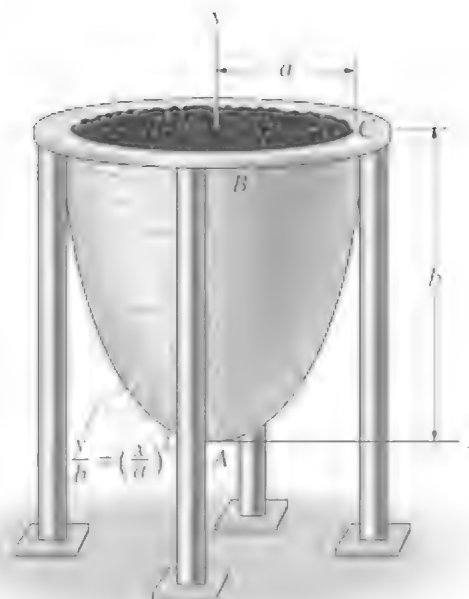
$\gamma = 50 \frac{\text{lb}}{\text{ft}^3}$

$p = 0.2$

Solution:

$A = \int_0^b a\sqrt{\frac{y}{b}}\, dy$ $\qquad A = 133.3\,\text{ft}^2$

$x_c = \frac{1}{A}\int_0^b \frac{1}{2}\left(a\sqrt{\frac{y}{b}}\right)^2 dy$ $\qquad x_c = 3.75\,\text{ft}$

$V = 2\pi A\, x_c$ $\qquad V = 3.142 \times 10^3\,\text{ft}^3$

$W = (1 - p)\gamma V$ $\qquad W = 125.7\,\text{kip}$

Problem 9-105

Determine the interior surface area of the brake piston. It consists of a full circular part. Its cross section is shown in the figure.

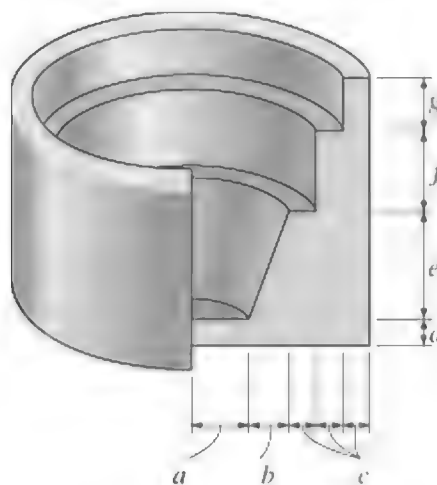
Given:

$a = 40$ mm

$b = 30$ mm

$c = 20$ mm

$d = 20$ mm

$e = 80$ mm

$f = 60 \text{ mm}$

$g = 40 \text{ mm}$

Solution:

$$A = 2\pi \left[\frac{a}{2} a + \left(a + \frac{b}{2} \right) \sqrt{b^2 + e^2} + c \left(a + b + \frac{c}{2} \right) + (a + b + c) f + \left(a + b + \frac{3c}{2} \right) c \ ... \right. \\ \left. + (a + b + 2c) g \right]$$

$$A = 119 \times 10^3 \text{ mm}^2$$

Problem 9-106

Determine the magnitude of the resultant hydrostatic force acting on the dam and its location H, measured from the top surface of the water. The width of the dam is w; the mass density is ρ_w.

Units Used:

$\text{Mg} = 10^3 \text{ kg}$

$\text{MN} = 10^6 \text{ N}$

Given:

$w = 8 \text{ m}$

$\rho_w = 1 \dfrac{\text{Mg}}{\text{m}^3}$

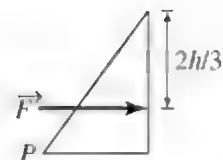
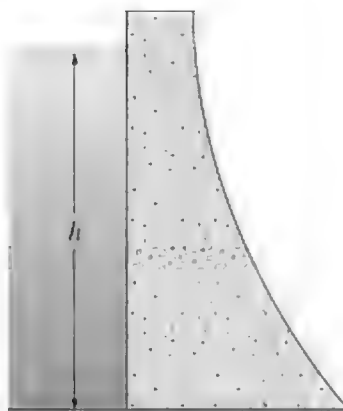
$h = 6 \text{ m}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

Solution:

$p = h\rho_w g \qquad p = 58860 \dfrac{\text{N}}{\text{m}^2}$

$F = \dfrac{1}{2} h w p \qquad F = 1.41 \text{ MN}$

$H = \left(\dfrac{2}{3}\right) h \qquad H = 4 \text{ m}$

Problem 9-107

The tank is filled with water to a depth d. Determine the resultant force the water exerts on side A and side B of the tank. If oil instead of water is placed in the tank, to what depth d should it reach so that it creates the same resultant forces? The densities are ρ_0 and ρ_w.

Given: $\text{kN} = 10^3\ \text{N}$

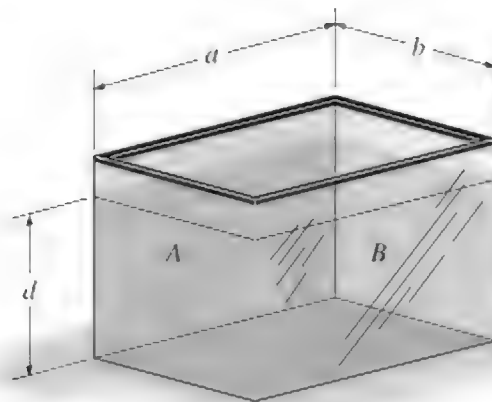
$d = 4\ \text{m}$

$a = 3\ \text{m}$

$b = 2\ \text{m}$

$\rho_o = 900\ \frac{\text{kg}}{\text{m}^3}$

$\rho_w = 1000\ \frac{\text{kg}}{\text{m}^3}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

For water

At side A: $W_A = b\ \rho_w\ g\ d$ $\qquad W_A = 78480\ \frac{\text{N}}{\text{m}}$

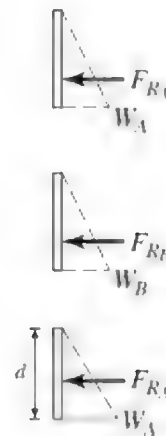
$F_{RA} = \frac{1}{2} W_A d$ $\qquad F_{RA} = 157\,\text{kN}$

At side B: $W_B = a \rho_w g d$ $\qquad W_B = 117720\ \frac{\text{N}}{\text{m}}$

$F_{RB} = \frac{1}{2} W_B d$ $\qquad F_{RB} = 235\,\text{kN}$

For oil

At side A: $F_{RA} = \frac{1}{2} b \rho_o g d_1 d_1$

$d_1 = \sqrt{\frac{2F_{RA}}{b\rho_o g}}$ $\qquad d_1 = 4.216\ \text{m}$

Problem 9-108

The factor of safety for tipping of the concrete dam is defined as the ratio of the stabilizing moment about O due to the dam's weight divided by the overturning moment about O due to the water pressure. Determine this factor if the concrete has specific weight γ_{conc} and water has specific weight γ_w.

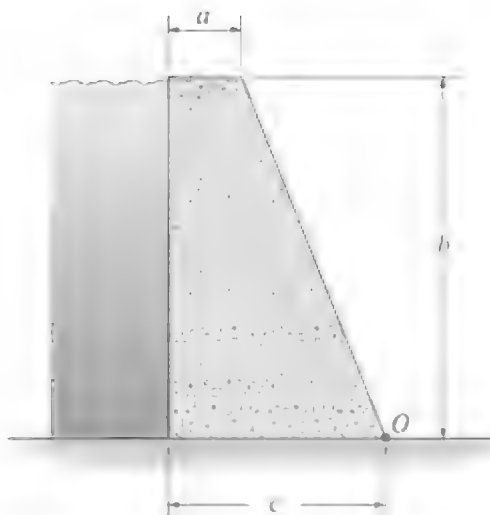
Given:

$a = 3\ \text{ft}$

$b = 15\ \text{ft}$

$c = 9\ \text{ft}$

$\gamma_w = 62.4\ \frac{\text{lb}}{\text{ft}^3}$

$\gamma_{conc} = 150\ \frac{\text{lb}}{\text{ft}^3}$

Solution:

For a 1-ft thick section:

$W = \gamma_w b(1\text{ft})$ $\quad W = 936\ \frac{\text{lb}}{\text{ft}}$

$F = \frac{1}{2} W b$ $\quad F = 7020\ \text{lb}$

$W_1 = \gamma_{conc}(1\text{ft}) a b$ $\quad W_1 = 6750\ \text{lb}$

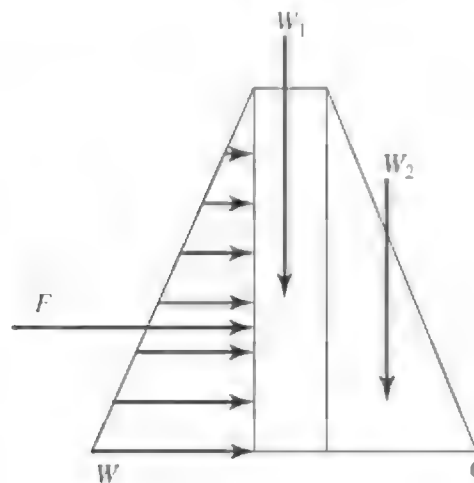
$W_2 = \gamma_{conc} \frac{1}{2}(c - a) b (1\text{ft})$ $\quad W_2 = 6750\ \text{lb}$

Moment to overturn:

$M_O = F \frac{1}{3} b$ $\quad M_O = 35100\ \text{lb ft}$

Moment to stabilize:

$M_S = W_1\left[(c-a) + \frac{a}{2}\right] + W_2\left[\frac{2}{3}(c-a)\right]$ $\quad M_S = 77625\ \text{lb}\cdot\text{ft}$

$F_s = \frac{M_S}{M_O}$ $\quad F_s = 2.21$

Problem 9-109

The concrete "gravity" dam is held in place by its own weight. If the density of concrete is ρ_c and water has a density ρ_w, determine the smallest dimension d that will prevent the dam from overturning about its end A.

Units Used:

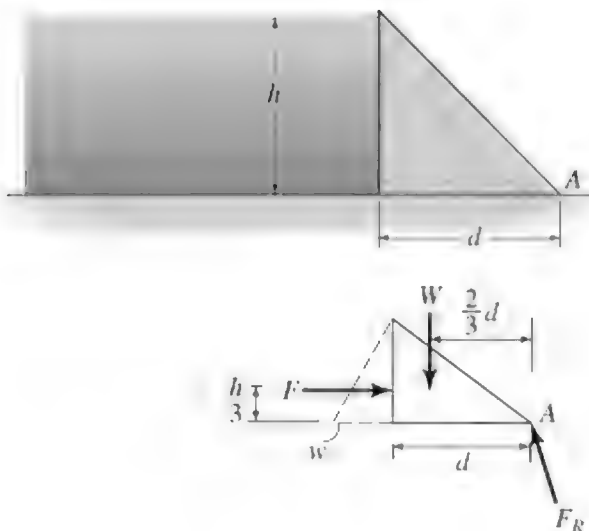
$$Mg = 10^3 \text{ kg}$$

Given:

$$\rho_c = 2.5 \frac{Mg}{m^3}$$

$$\rho_w = 1.0 \frac{Mg}{m^3}$$

$$h = 6 \text{ m}$$

$$g = 9.81 \frac{m}{s^2}$$

Solution:

Consider a dam of width $a = 1$ m.

$$w = \rho_w g h a \qquad w = 58860 \frac{N}{m} \qquad F = \frac{1}{2} w h \qquad F = 176580 \text{N}$$

$$W = \frac{1}{2}\rho_c g d h a$$

Equilibrium $\quad W\frac{2d}{3} - F\frac{h}{3} = 0$

$$\frac{1}{2}\rho_c g d h a \frac{2d}{3} = \frac{F h}{3}$$

$$d = \sqrt{\frac{F}{\rho_c g a}} \qquad d = 2.683 \text{ m}$$

Problem 9-110

The concrete dam is designed so that its face AB has a gradual slope into the water as shown. Because of this, the frictional force at the base BD of the dam is increased due to the hydrostatic force of the water acting on the dam. Calculate the hydrostatic force acting on the face AB of the dam. The dam has width w, the water density is γ_w.

Units Used:

$$kip = 10^3 \text{ lb}$$

Given:

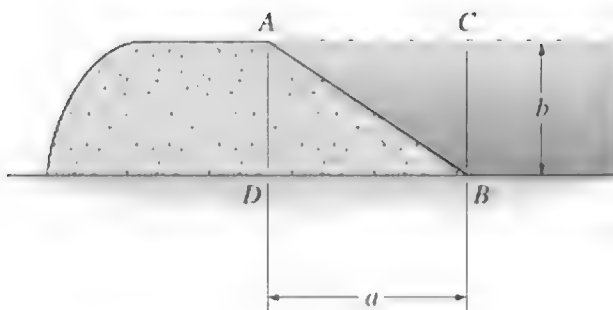
$$w = 60 \text{ ft}$$

$$\gamma_w = 62.4 \frac{\text{lb}}{\text{ft}^3}$$

$$a = 18 \text{ ft}$$

$$b = 12 \text{ ft}$$

Solution:

$$F_{AB} = \frac{1}{2} w \gamma_w b \sqrt{a^2 + b^2} \qquad F_{AB} = 486 \text{ kip}$$

Problem 9-111

The symmetric concrete "gravity" dam is held in place by its own weight. If the density of concrete is ρ_c and water has a density ρ_w, determine the smallest distance d at its base that will prevent the dam from overturning about its end A. The dam has a width w.

Units Used:

$$Mg = 10^3 \text{ kg} \qquad MN = 10^6 \text{ N}$$

Given:

$$a = 1.5 \text{ m} \qquad \rho_c = 2.5 \frac{\text{Mg}}{\text{m}^3}$$

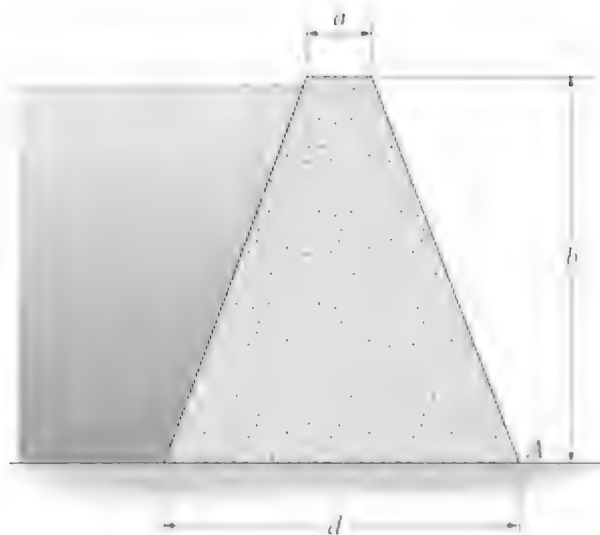
$$b = 9 \text{ m}$$

$$w = 8 \text{ m} \qquad \rho_w = 1.0 \frac{\text{Mg}}{\text{m}^3}$$

Solution:

Guesses

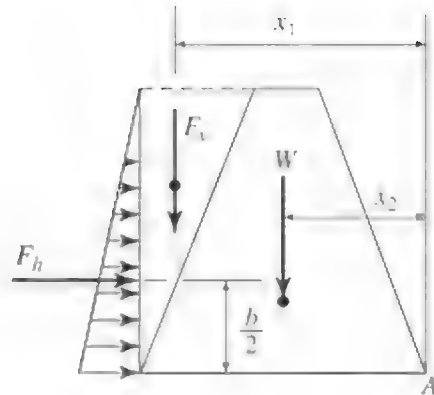
$$d = 3 \text{ m} \qquad F_h = 1 \text{ MN}$$

$$F_v = 1 \text{ MN} \qquad W = 1 \text{ MN}$$

Given

$$F_v = \frac{d-a}{4} b w \rho_w g$$

$$F_h = \frac{1}{2} \rho_w g b w b$$

$$W = \rho_c g w \left[a b + \left(\frac{d-a}{2} \right) b \right]$$

$$W \frac{d}{2} + F_v \left(d - \frac{d-a}{6} \right) - F_h \frac{b}{3} = 0$$

$$\begin{pmatrix} F_v \\ F_h \\ W \\ d \end{pmatrix} = \text{Find}(F_v, F_h, W, d) \qquad \begin{pmatrix} F_v \\ F_h \\ W \end{pmatrix} = \begin{pmatrix} 0.379 \\ 3.178 \\ 4.545 \end{pmatrix} \text{MN} \qquad d = 3.65 \text{ m}$$

Problem 9-112

The tank is used to store a liquid having a specific weight γ. If it is filled to the top, determine the magnitude of force the liquid exerts on each of its two sides *ABDC* and *BDFE*.

Units used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

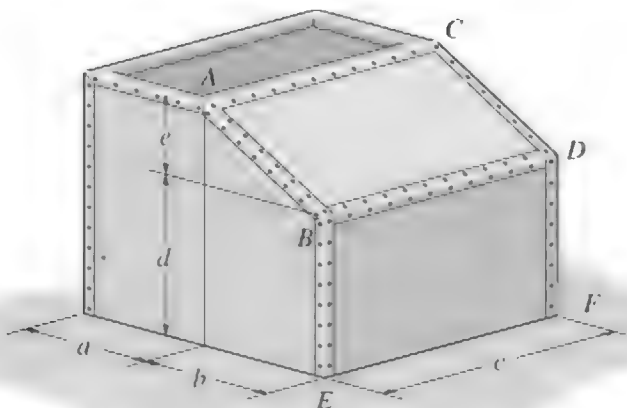
$$\gamma = 80 \frac{\text{lb}}{\text{ft}^3}$$

$$a = 6 \text{ ft}$$

$$b = 6 \text{ ft}$$

$$c = 12 \text{ ft}$$

$$d = 8 \text{ ft}$$

$$e = 4 \text{ ft}$$

Solution:

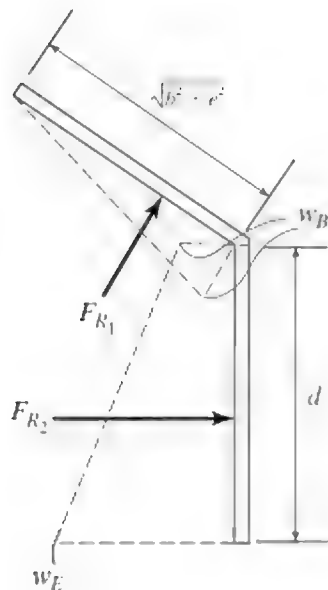
Fluid Pressure: The fluid pressure at points B and E can be determined using

$$p_B = \gamma e \qquad p_B = 320 \frac{\text{lb}}{\text{ft}^2}$$

$$p_E = \gamma(e + d) \qquad p_E = 960 \frac{\text{lb}}{\text{ft}^2}$$

Thus

$$w_B = p_B c \qquad w_B = 3.84 \frac{\text{kip}}{\text{ft}}$$

$$w_E = p_E c \qquad w_E = 11.52 \frac{\text{kip}}{\text{ft}}$$

Resultant Forces: The resultant Force acts on surface ABCD is

$$F_{R1} = \frac{1}{2} w_B \sqrt{e^2 + b^2} \qquad F_{R1} = 13.8\,\text{kip}$$

and on surface $BDFE$ is

$$F_{R2} = \frac{1}{2}(w_B + w_E)d \qquad F_{R2} = 61.4\,\text{kip}$$

Problem 9-113

The rectangular gate of width w is pinned at its center A and is prevented from rotating by the block at B. Determine the reactions at these supports due to hydrostatic pressure.

Units Used:

$$\text{Mg} = 10^3\ \text{kg} \qquad \text{kN} = 10^3\ \text{N}$$

Given:

$$a = 1.5\ \text{m} \qquad \rho_w = 1.0 \frac{\text{Mg}}{\text{m}^3}$$

$$b = 6\ \text{m}$$

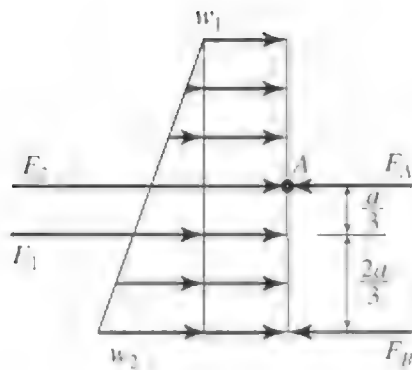
$w = 2\ \text{m}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$w_1 = \rho_w g(b - 2a)w$ $\quad w_1 = 59\ \frac{\text{kN}}{\text{m}}$

$w_2 = \rho_w g 2a w$ $\quad w_2 = 59\ \frac{\text{kN}}{\text{m}}$

$F_1 = \frac{1}{2} 2a w_1$ $\quad F_1 = 88\ \text{kN}$

$F_2 = w_2 2a$ $\quad F_2 = 177\ \text{kN}$

$\Sigma M_A = 0;\quad F_1 \frac{a}{3} - F_B a = 0 \qquad F_B = \frac{1}{3} F_1 \qquad F_B = 29.4\ \text{kN}$

$\Sigma F_x = 0;\quad F_1 + F_2 - F_B - F_A = 0 \qquad F_A = F_1 + F_2 - F_B \qquad F_A = 235\ \text{kN}$

Problem 9-114

The gate AB has width w. Determine the horizontal and vertical components of force acting on the pin at B and the vertical reaction at the smooth support A. The density of water is ρ_w.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

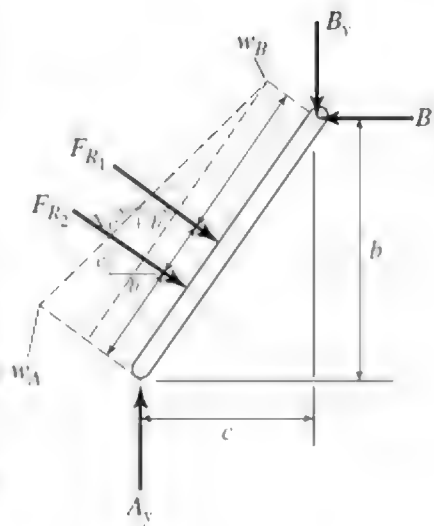
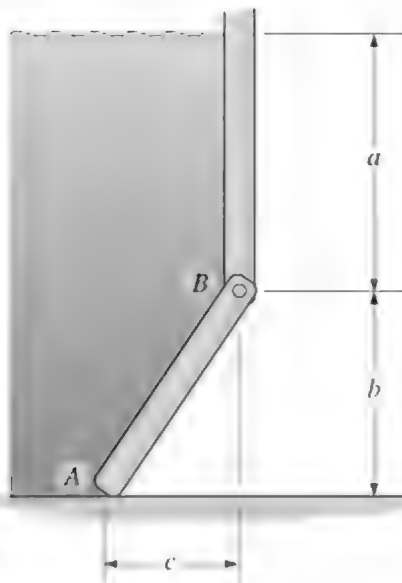
$\text{MN} = 10^6\ \text{N}$

Given:

$w = 8\ \text{m}$

$\rho_w = 1.0\ \frac{\text{Mg}}{\text{m}^3}$

$a = 5\ \text{m}$

$b = 4\ \text{m}$

$c = 3\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

Fluid Pressure: The fluid pressure at points A and B can be determined using Eq. 9-15,

$p_A = \rho_w g(a + b)$ $\quad p_A = 88.29\ \frac{\text{kN}}{\text{m}^2}$ $\quad w_A = p_A w$ $\quad w_A = 706.32\ \frac{\text{kN}}{\text{m}}$

$p_B = \rho_w g a$ $\quad p_B = 49.05\ \frac{\text{kN}}{\text{m}^2}$ $\quad w_B = p_B w$ $\quad w_B = 392.4\ \frac{\text{kN}}{\text{m}}$

Equilibrium

$$w_B \frac{b^2 + c^2}{2} + \frac{1}{2}(w_A - w_B) \frac{b^2 + c^2}{3} - A_y c = 0$$

$$A_y = \frac{w_B \frac{b^2 + c^2}{2} + \frac{1}{2}(w_A - w_B) \frac{2(b^2 + c^2)}{3}}{c} \qquad A_y = 2.507\ \text{MN}$$

$$A_y - w_B c - \frac{1}{2}(w_A - w_B)c - B_y = 0$$

$$B_y = A_y - w_B c - \frac{1}{2}(w_A - w_B)c \qquad B_y = 858.92\ \text{kN}$$

$$-B_x + w_B b + \frac{1}{2}(w_A - w_B)b = 0$$

$$B_x = w_B b + \frac{1}{2}(w_A - w_B)b \qquad B_x = 2.197\ \text{MN}$$

Problem 9-115

The storage tank contains oil having a specific weight γ. If the tank has width w, calculate the resultant force acting on the inclined side BC of the tank, caused by the oil, and specify its location along BC, measured from B. Also compute the total resultant force acting on the bottom of the tank.

Units Used:

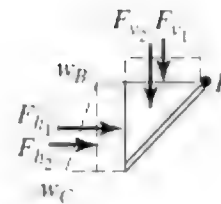
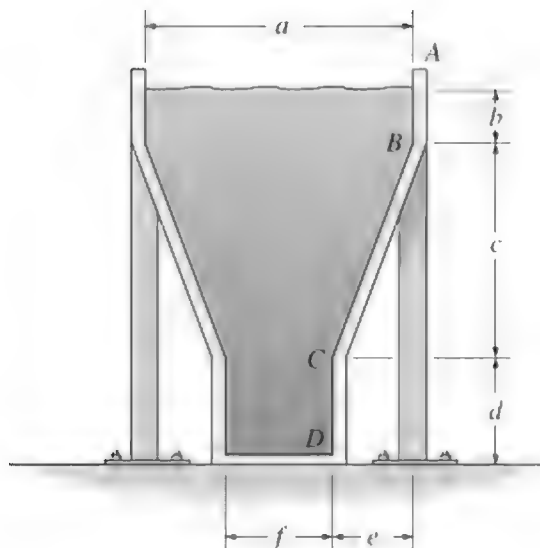
$kip = 10^3\ lb$

Given:

$\gamma = 56\ \frac{lb}{ft^3}$ $c = 8\ ft$

$w = 6\ ft$ $d = 4\ ft$

$a = 10\ ft$ $e = 3\ ft$

$b = 2\ ft$ $f = 4\ ft$

Solution:

$w_B = w\gamma b$ $w_B = 672\ \frac{lb}{ft}$ $w_C = w\gamma(b+c)$ $w_C = 3360\ \frac{lb}{ft}$

$F_{h1} = w_B c$ $F_{h2} = \frac{1}{2}(w_C - w_B)c$ $F_{v1} = \gamma w b e$ $F_{v2} = \frac{1}{2}\gamma w c e$

The resultant force

$F_{Rx} = F_{h1} + F_{h2}$ $F_{Ry} = F_{v1} + F_{v2}$ $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ $F_R = 17.225\ kip$

The location h measured from point B Guess $h = 1\ ft$ Given

$$F_{v1}\frac{e}{2} + F_{v2}\frac{2e}{3} + F_{h1}\frac{c}{2} + F_{h2}\frac{2c}{3} = F_{Rx}\frac{ch}{\sqrt{c^2+e^2}} + F_{Ry}\frac{eh}{\sqrt{c^2+e^2}}$$

$h = \text{Find}(h)$ $h = 5.221\ ft$

On the bottom of the tank

$F_{bot} = \gamma w f(b+c+d)$ $F_{bot} = 18.816\ kip$

Problem 9-116

The arched surface AB is shaped in the form of a quarter circle. If it has a length L, determine the horizontal and vertical components of the resultant force caused by the water acting on the surface. The density of water is ρ_w.

Units Used:

$Mg = 10^3$ kg

$kN = 10^3$ N

Given:

$L = 8$ m

$\rho_w = 1.0 \frac{Mg}{m^3}$

$a = 3$ m

$b = 2$ m

$g = 9.81 \frac{m}{s^2}$

Solution:

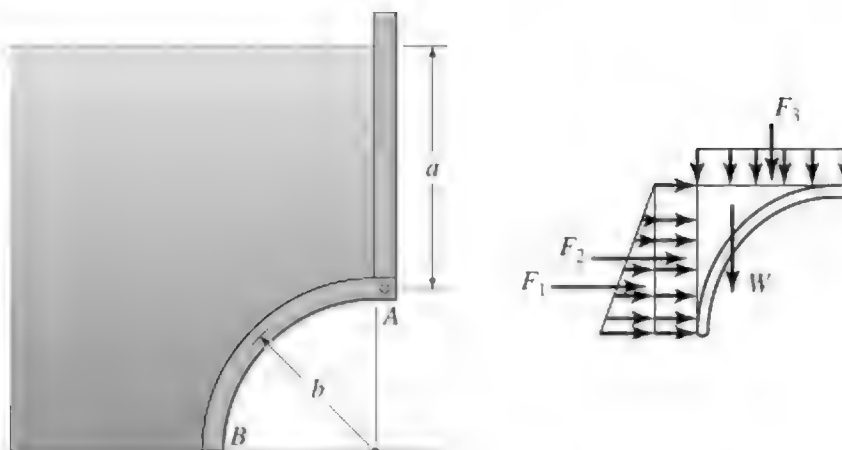
$F_3 = \rho_w g a b L$ $F_3 = 470.88$ kN

$F_2 = \rho_w g a b L$ $F_2 = 470.88$ kN

$F_1 = \rho_w g \frac{b}{2} b L$ $F_1 = 156.96$ kN

$W = \left(b^2 - \frac{\pi b^2}{4}\right) L \rho_w g$ $W = 67.368$ kN

$F_x = F_1 + F_2$ $F_x = 628$ kN

$F_y = F_2 + W$ $F_y = 538$ kN

Problem 9-117

The rectangular bin is filled with coal, which creates a pressure distribution along wall A that varies as shown, i.e. $p = p_0(z/b)^3$. Determine the resultant force created by the coal and specify its location measured from the top surface of the coal.

Units used:

$kip = 10^3$ lb

Given:

$a = 4\ \text{ft}$

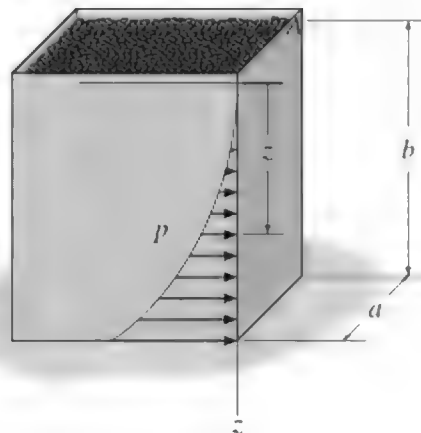
$b = 10\ \text{ft}$

$p_0 = 4000\ \frac{\text{lb}}{\text{ft}^2}$

Solution:

Resultant Force and its location:

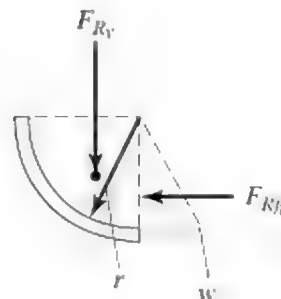
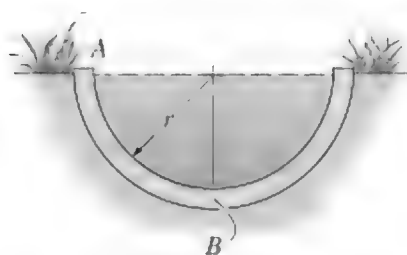
$$F = \int_0^b p_0 \left(\frac{z}{b}\right)^3 a\, dz \qquad F = 40\,\text{kip}$$

$$z_c = \frac{1}{F} \int_0^b z\, p_0 \left(\frac{z}{b}\right)^3 a\, dz \qquad z_c = 8\,\text{ft}$$

Problem 9-118

The semicircular drainage pipe is filled with water. Determine the resultant horizontal and vertical force components that the water exerts on the side AB of the pipe per foot of pipe length; water has density γ.

Given:

$\gamma = 62.4\ \frac{\text{lb}}{\text{ft}^3}$

$r = 2\ \text{ft}$

Solution:

$w = \gamma r \qquad w = 124.8\ \frac{\text{lb}}{\text{ft}^2}$

Resultant forces (per unit foot):

$$F_{Rh} = \frac{1}{2} w r \qquad F_{Rh} = 124.8\ \frac{\text{lb}}{\text{ft}}$$

$$F_{Rv} = \gamma \frac{\pi r^2}{4} \qquad F_{Rv} = 196.0 \frac{\text{lb}}{\text{ft}}$$

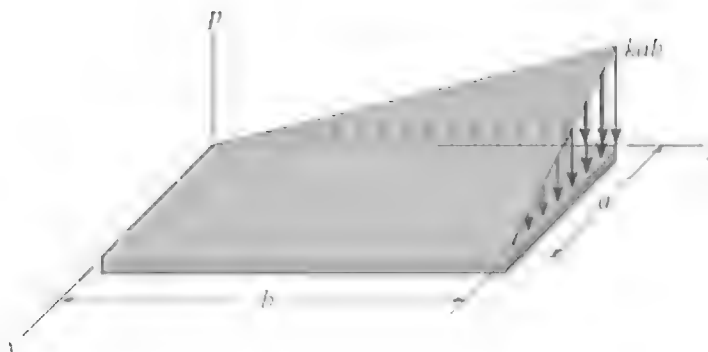
Problem 9-119

The load over the plate varies linearly along the sides of the plate such that $p = k\, y\, (a\text{-}x)$. Determine the magnitude of the resultant force and the coordinates (x_c, y_c) of the point where the line of action of the force intersects the plate.

Given:

$a = 2$ ft

$b = 6$ ft

$k = 10 \dfrac{\text{lb}}{\text{ft}^4}$

Solution:

$$p(x,y) = k\,y(a - x)$$

$$F_R = \int_0^a \int_0^b p(x,y)\,dy\,dx \qquad F_R = 360\,\text{lb}$$

$$x_c = \frac{1}{F_R}\int_0^a \int_0^b x\,p(x,y)\,dy\,dx \qquad x_c = 0.667\,\text{ft}$$

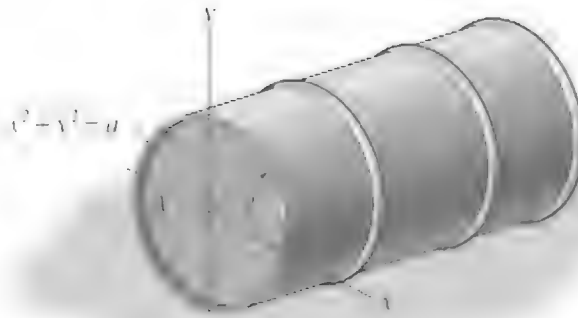
$$y_c = \frac{1}{F_R}\int_0^a \int_0^b y\,p(x,y)\,dy\,dx \qquad y_c = 4\,\text{ft}$$

Problem 9-120

The drum is filled to its top ($y = a$) with oil having a density γ. Determine the resultant force of the oil pressure acting on the flat end of plate A of the drum and specify its location measured from the top of the drum.

Given:

$a = 1.5\ \text{ft}$

$\gamma = 55\ \frac{\text{lb}}{\text{ft}^3}$

Solution:

$$F_R = \int_{-a}^{a} \gamma 2\sqrt{a^2 - y^2}(a - y)\ dy \qquad F_R = 583\,\text{lb}$$

$$d = a - \frac{1}{F_R}\int_{-a}^{a} y\gamma 2\sqrt{a^2 - y^2}(a - y)\ dy \qquad d = 1.875\,\text{ft}$$

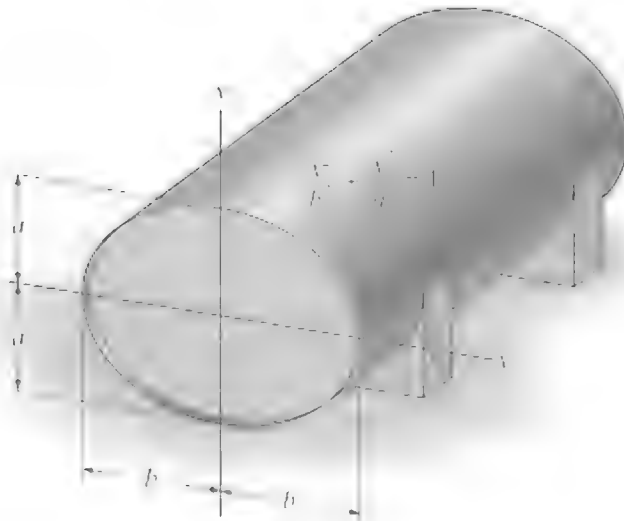
Problem 9-121

The gasoline tank is constructed with elliptical ends on each side of the tank. Determine the resultant force and its location on these ends if the tank is half full.

Given:

$a = 3\ \text{ft}$

$b = 4\ \text{ft}$

$\gamma = 41\ \frac{\text{lb}}{\text{ft}^3}$

Solution:

$$F_R = \int_{-a}^{0} -\gamma y 2\left(\frac{b}{a}\sqrt{a^2 - y^2}\right) dy \qquad F_R = 984\,\text{lb}$$

$$y_c = \frac{1}{F_R}\int_{-a}^{0} y\left[-\gamma y 2\left(\frac{b}{a}\sqrt{a^2 - y^2}\right)\right] dy$$

$y_c = -1.767 \text{ ft}$ $\quad x_c = 0 \text{ ft}$

Problem 9-122

The loading acting on a square plate is represented by a parabolic pressure distribution. Determine the magnitude of the resultant force and the coordinates (x_c, y_c) of the point where the line of action of the force intersects the plate. Also, what are the reactions at the rollers B and C and the ball-and-socket joint A? Neglect the weight of the plate.

Units Used:

$\text{kPa} = 10^3 \text{ Pa}$

$\text{kN} = 10^3 \text{ N}$

Given:

$a = 4 \text{ m}$

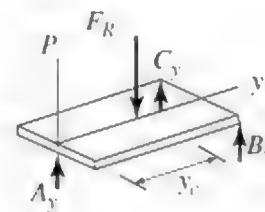
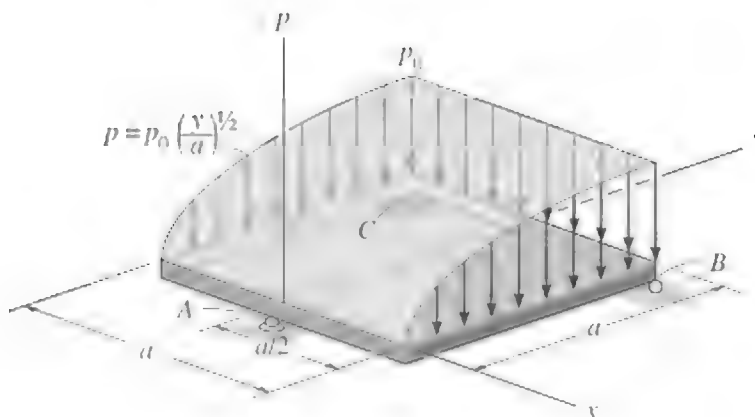
$p_0 = 4 \text{ kPa}$

Solution:

Due to symmetry

$x_c = 0$

$$F_R = \int_0^a p_0\sqrt{\frac{y}{a}}\, a \, dy \qquad F_R = 42.667 \text{ kN}$$

$$y_c = \frac{1}{F_R}\int_0^a y\, p_0\sqrt{\frac{y}{a}}\, a \, dy \qquad y_c = 2.4 \text{ m}$$

Equilibrium Guesses $A_y = 1 \text{ kN}$ $\quad B_y = 1 \text{ kN}$ $\quad C_y = 1 \text{ kN}$

Given $A_y + B_y + C_y - F_R = 0$

$$(B_y + C_y)a - F_R y_c = 0$$

$$B_y\frac{a}{2} - C_y\frac{a}{2} = 0$$

$$\begin{pmatrix} A_y \\ B_y \\ C_y \end{pmatrix} = \text{Find}(A_y, B_y, C_y) \qquad \begin{pmatrix} A_y \\ B_y \\ C_y \end{pmatrix} = \begin{pmatrix} 17.067 \\ 12.8 \\ 12.8 \end{pmatrix} \text{kN}$$

Problem 9-123

The tank is filled with a liquid which has density ρ. Determine the resultant force that it exerts on the elliptical end plate, and the location of the center of pressure, measured from the x axis.

Units Used:

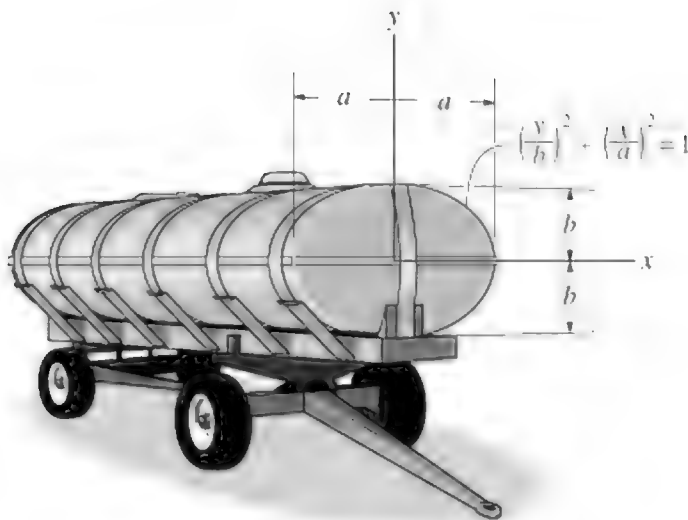
$$\text{kN} = 10^3\ \text{N}$$

Given:

$$a = 1\ \text{m}$$

$$b = 0.5\ \text{m}$$

$$\rho = 900\ \frac{\text{kg}}{\text{m}^3}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

Solution:

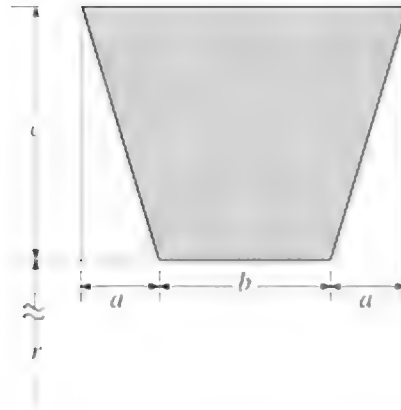
$$F_R = \int_{-b}^{b} \rho g 2a \sqrt{1 - \left(\frac{y}{b}\right)^2}\,(b - y)\ dy \qquad F_R = 6.934\ \text{kN}$$

$$y_c = \frac{1}{F_R}\int_{-b}^{b} y \rho g 2a \sqrt{1 - \left(\frac{y}{b}\right)^2}\,(b - y)\ dy \qquad y_c = -0.125\ \text{m}$$

Problem 9-124

A circular V-belt has an inner radius r and a cross-sectional area as shown. Determine the volume of material required to make the belt.

Given:

$r = 600\ \text{mm}$

$a = 25\ \text{mm}$

$b = 50\ \text{mm}$

$c = 75\ \text{mm}$

Solution:

$$V = 2\pi\left[\left(r + \frac{c}{3}\right)2\left(\frac{1}{2}\right)a\,c + \left(r + \frac{c}{2}\right)b\,c\right] \qquad V = 22.4 \times 10^{-3}\ \text{m}^3$$

Problem 9-125

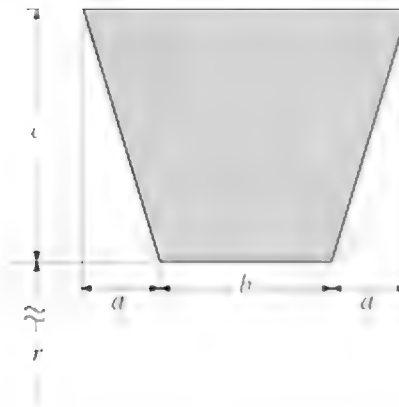
A circular V-belt has an inner radius r and a cross-sectional area as shown. Determine the surface area of the belt.

Given:

$r = 600\ \text{mm}$

$a = 25\ \text{mm}$

$b = 50\ \text{mm}$

$c = 75\ \text{mm}$

Solution:

$$A = 2\pi\left[rb + 2\left(r + \frac{c}{2}\right)\sqrt{a^2 + c^2} + (r + c)(b + 2a)\right] \qquad A = 1.246\ \text{m}^2$$

Problem 9-126

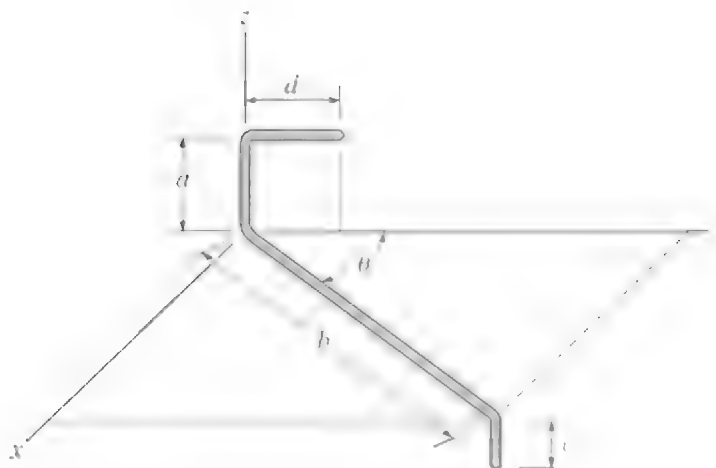
Locate the center of mass of the homogeneous rod.
Given:

$a = 200\ \text{mm}$

$b = 600\ \text{mm}$

$c = 100\ \text{mm}$

$d = 200\ \text{mm}$

$\theta = 45\ \text{deg}$

Solution:

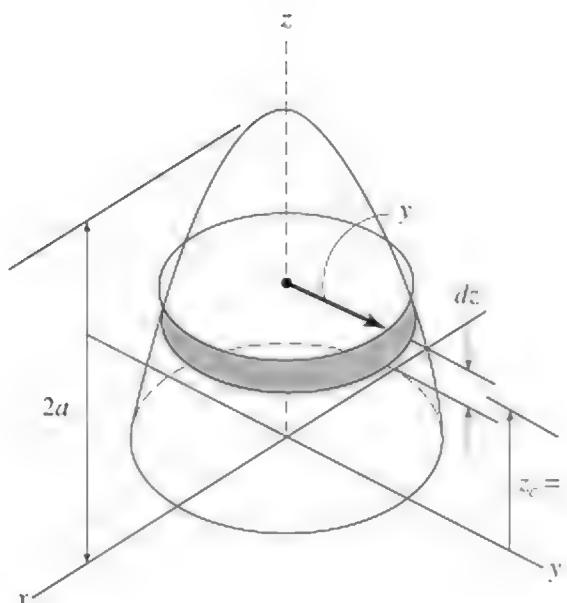
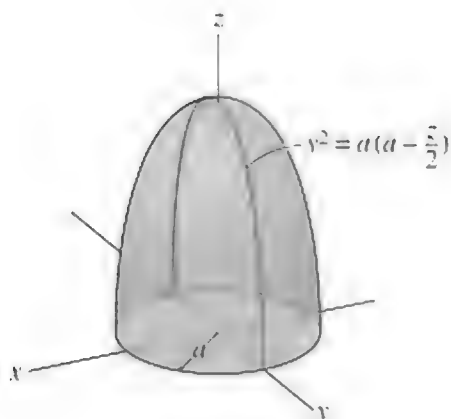
$L = a + b + c + d$

$$x_c = \frac{1}{L}\left(b\frac{b}{2}\sin(\theta) + c\,b\sin(\theta)\right) \qquad x_c = 154.3\ \text{mm}$$

$$y_c = \frac{1}{L}\left(d\frac{d}{2} + b\frac{b}{2}\cos(\theta) + c\,b\cos(\theta)\right) \qquad y_c = 172.5\ \text{mm}$$

$$z_c = \frac{1}{L}\left(a\frac{a}{2} + d\,a - c\frac{c}{2}\right) \qquad z_c = 50.0\ \text{mm}$$

Problem 9-127

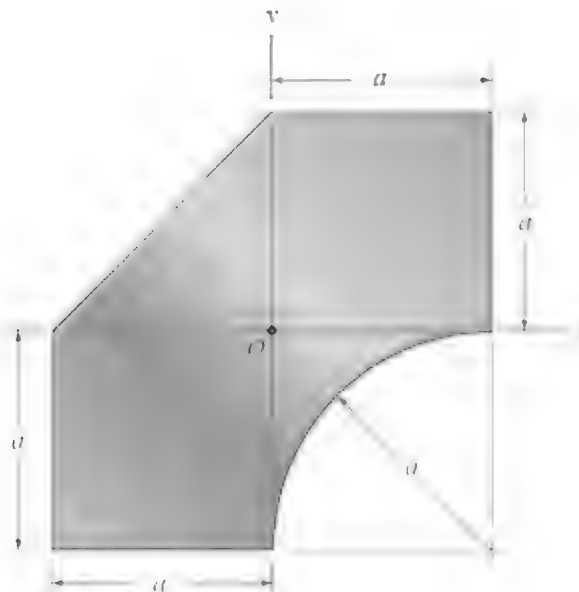
Locate the centroid of the solid

Solution:

$$y_c = \frac{\int_0^{2a} z\pi a\left(a - \frac{z}{2}\right) dz}{\int_0^{2a} \pi a\left(a - \frac{z}{2}\right) dz} = \frac{2\ a^4\ \pi - a^3\ \pi\ z}{\pi\ a^3} \qquad y_c = \frac{2}{3}\ a$$

Problem 9-128

Locate the centroid (x_c, y_c) of the thin plate.

Given:

$a = 6\ \text{in}$

Solution:

$$A = 4a^2 - \frac{a^2}{2} - \frac{\pi a^2}{4} \qquad A = 97.7\ \text{in}^2$$

$$x_c = \frac{1}{A}\left[\frac{-a^2}{2}\left(\frac{-2}{3}a\right) - \frac{\pi a^2}{4}\left(a - \frac{4a}{3\pi}\right)\right] \qquad x_c = -0.262\ \text{in}$$

$$y_c = \frac{1}{A}\left[\frac{-a^2}{2}\frac{2a}{3} - \frac{\pi a^2}{4}\left(\frac{4a}{3\pi} - a\right)\right] \qquad y_c = 0.262\ \text{in}$$

Problem 9-129

Determine the weight and location (x_c, y_c) of the center of gravity G of the concrete retaining wall. The wall has a length L, and concrete has a specific gravity of γ.

Units Used:

$kip = 10^3$ lb

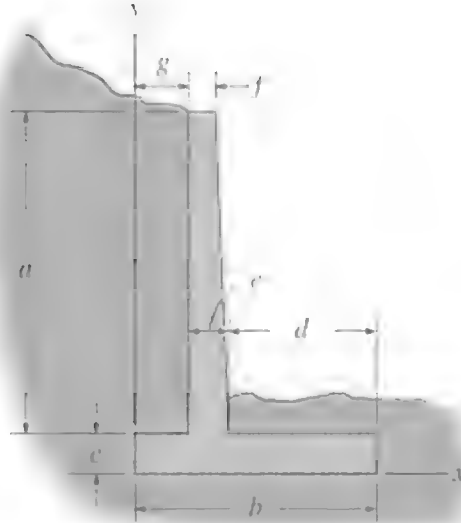
Given:

$a = 12$ ft $f = 1$ ft

$b = 9$ ft $g = 2$ ft

$c = 1.5$ ft $L = 10$ ft

$d = 5.5$ ft $\gamma = 150 \frac{lb}{ft^3}$

$e = 1.5$ ft

Solution:

$$A = b\,c + a\,f + \frac{1}{2}a(e-f) \qquad W = \gamma A L \qquad W = 42.8\,\text{kip}$$

$$x_c = \frac{1}{A}\left[b\,c\frac{b}{2} + a\,f\left(g+\frac{f}{2}\right) + \frac{1}{2}a(e-f)\left(g+f+\frac{e-f}{3}\right)\right] \qquad x_c = 3.52\,\text{ft}$$

$$y_c = \frac{1}{A}\left[b\,c\frac{c}{2} + a\,f\left(c+\frac{a}{2}\right) + \frac{1}{2}a(e-f)\left(c+\frac{a}{3}\right)\right] \qquad y_c = 4.09\,\text{ft}$$

Problem 9-130

The hopper is filled to its top with coal. Determine the volume of coal if the voids (air space) are a fraction p of the volume of the hopper.

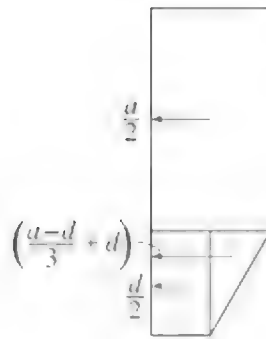
Given:

$a = 1.5$ m

$b = 4$ m

$c = 1.2$ m

$d = 0.2\ \text{m}$

$p = 0.35$

Solution:

$$V = (1-p)2\pi\left[d\,c\frac{d}{2} + \frac{1}{2}c(a-d)\left(d + \frac{a-d}{3}\right) + a\,b\frac{a}{2}\right]$$

$V = 20.5\ \text{m}^3$

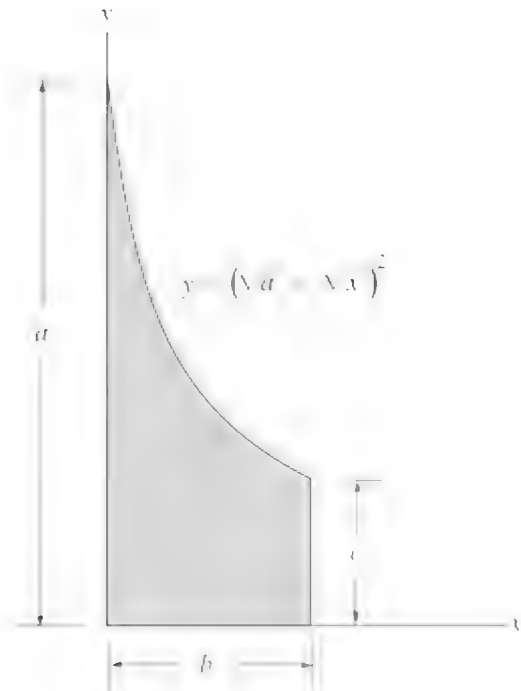
Problem 9-131

Locate the centroid (x_c, y_c) of the shaded area.

Given:

$a = 16\ \text{ft}$

$b = 4\ \text{ft}$

$c = \left(\sqrt{a} - \sqrt{b}\right)^2$

Solution:

$$A = \int_0^b \left(\sqrt{a} - \sqrt{x}\right)^2 \,dx \qquad A = 29.3\ \text{ft}^2$$

$$x_C = \frac{1}{A}\int_0^b x\left(\sqrt{a}-\sqrt{x}\right)^2 \, dx \qquad x_C = 1.6\text{ ft}$$

$$y_C = \frac{1}{A}\int_0^b \frac{\left[\left(\sqrt{a}-\sqrt{x}\right)^2\right]^2}{2} \, dx \qquad y_C = 4.15\text{ ft}$$

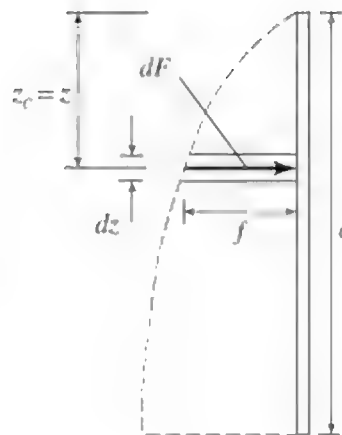
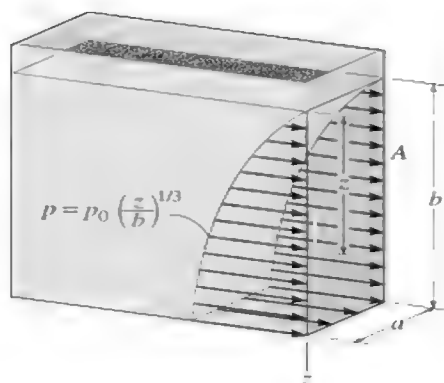
Problem 9-132

The rectangular bin is filled with coal, which creates a pressure distribution along wall A that varies as shown, i.e., $p = p_0(z/b)^{1/3}$. Compute the resultant force created by the coal, and its location, measured from the top surface of the coal.

Given:

$$p_0 = 8 \frac{\text{lb}}{\text{ft}^2}$$

$$a = 3 \text{ ft}$$

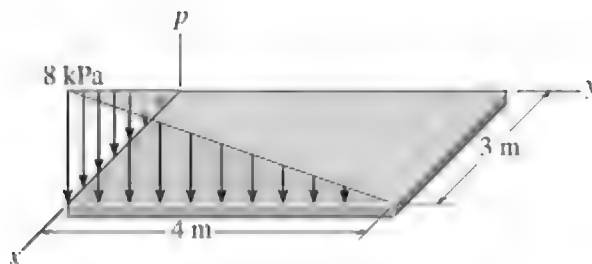
$$b = 8 \text{ ft}$$

Solution:

$$F = \int_0^b p_0\left(\frac{z}{b}\right)^{\frac{1}{3}} a \, dz \qquad F = 144\text{ lb}$$

$$z_C = \frac{1}{F}\int_0^b z\, p_0\left(\frac{z}{b}\right)^{\frac{1}{3}} a \, dz \qquad z_C = 4.57\text{ ft}$$

Problem 9-133

The load over the plate varies linearly along the sides of the plate such that $p = \frac{2}{3}x(4 - y)$ kPa

Determine the resultant force and its position (x_c, y_c) on the plate.

Solution:

$$F = \int_0^3 \int_0^4 \frac{2}{3}x(4-y)\, dy\, dx \qquad F = 24 \text{ kN}$$

$$x_c = \frac{1}{F}\int_0^3 \int_0^4 x\frac{2}{3}x(4-y)\, dy\, dx \qquad x_c = 2 \text{ m}$$

$$y_c = \frac{1}{F}\int_0^3 \int_0^4 y\frac{2}{3}x(4-y)\, dy\, dx \qquad y_c = 1.333 \text{ m}$$

Problem 9-134

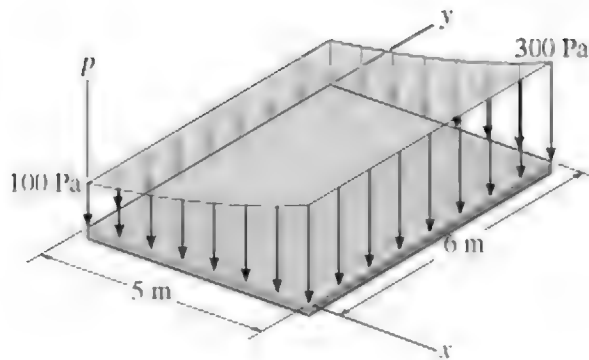
The pressure loading on the plate is described by the function $p = \{ -240/(x + 1) + 340 \}$ Pa. Determine the magnitude of the resultant force and coordinates of the point where the line of action of the force intersects the plate.

Solution:

Due to symmetry

$y_c = 3$ m

$$F = \int_0^5 \left(\frac{-240}{x+1} + 340\right) 6\, dx$$

$F = 7.62 \times \ \text{N}^3$

$$x_c = \frac{1}{F}\int_0^5 x\left(\frac{-240}{x+1} + 340\right)6\ dx$$

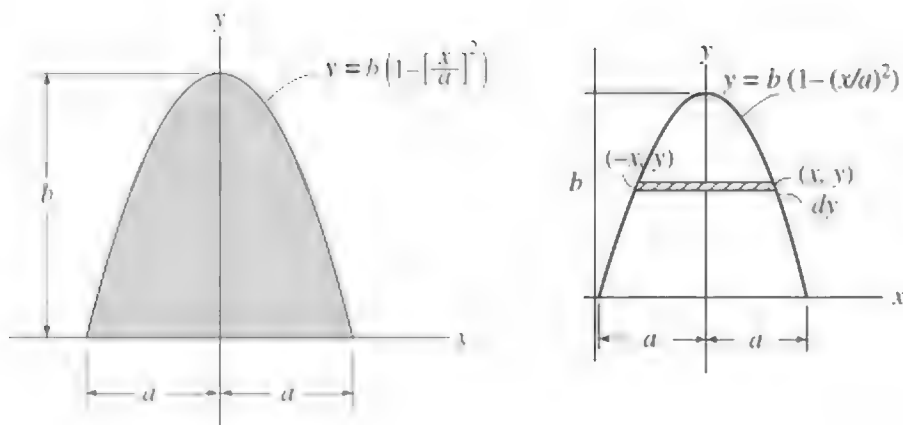
$x_c = 2.74\,\text{m}$

Problem 10-1

Determine the moment of inertia for the shaded area about the x axis.

Given:

$a = 2$ m

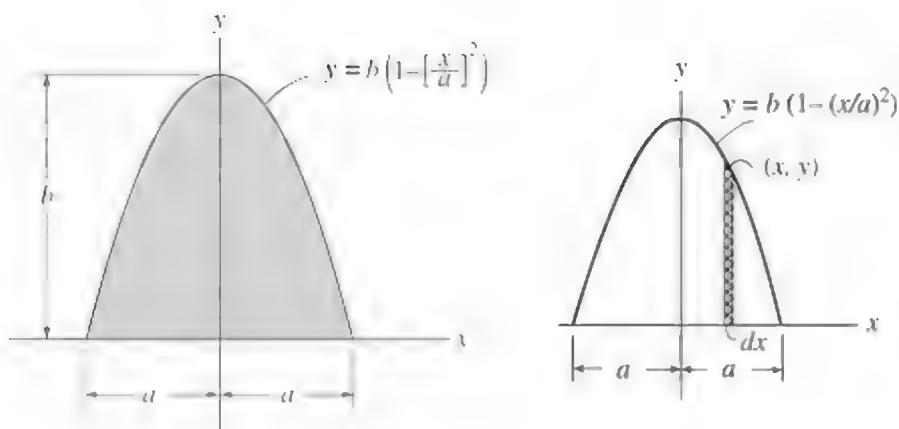
$b = 4$ m

Solution: $$I_x = 2\int_0^b y^2 a\sqrt{1 - \frac{y}{b}}\, dy$$ $I_x = 39.0\,\text{m}^4$

Problem 10-2

Determine the moment of inertia for the shaded area about the y axis.

Given:

$a = 2$ m

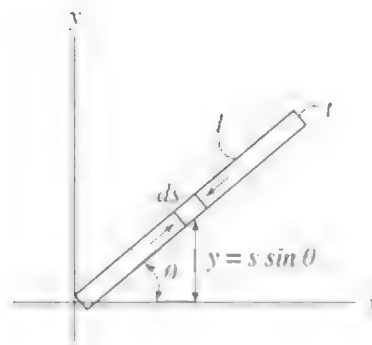
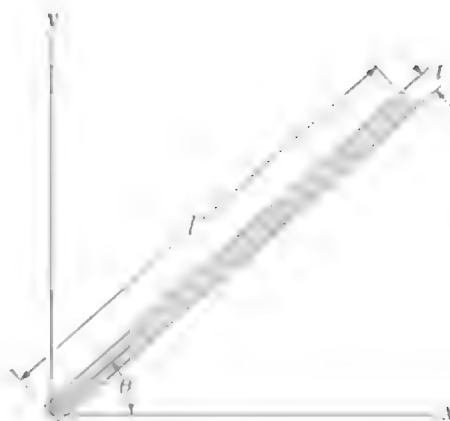
$b = 4$ m

Solution: $$I_y = 2\int_0^a x^2 b\left[1 - \left(\frac{x}{a}\right)^2\right] dx$$ $I_y = 8.53\ \text{m}^4$

Problem 10-3

Determine the moment of inertia for the thin strip of area about the x axis. The strip is oriented at an angle θ from the x axis. Assume that $t << l$.

Solution:

$$I_x = \int_A y^2\, dA = \int_0^l s^2 \sin^2(\theta) t\, ds$$

$$I_x = \frac{1}{3} t\, l^3 \sin^2(\theta)$$

Problem 10-4

Determine the moment for inertia of the shaded area about the x axis.

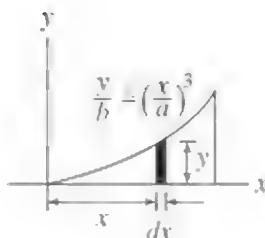
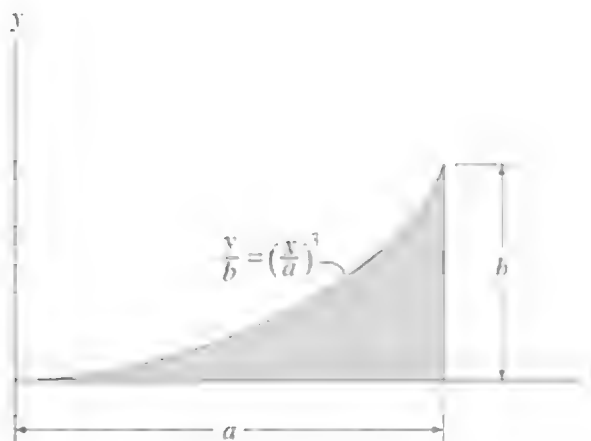
Given:

$a = 4$ in

$b = 2$ in

Solution:

$$I_x = \int_0^a \frac{1}{3}\left[b\left(\frac{x}{a}\right)^3\right]^3 dx$$

$$I_x = 1.07\ \text{in}^4$$

Problem 10-5

Determine the moment for inertia of the shaded area about the y axis.

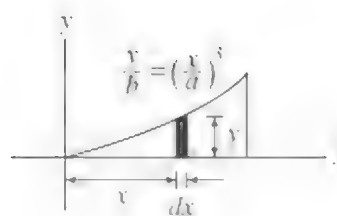
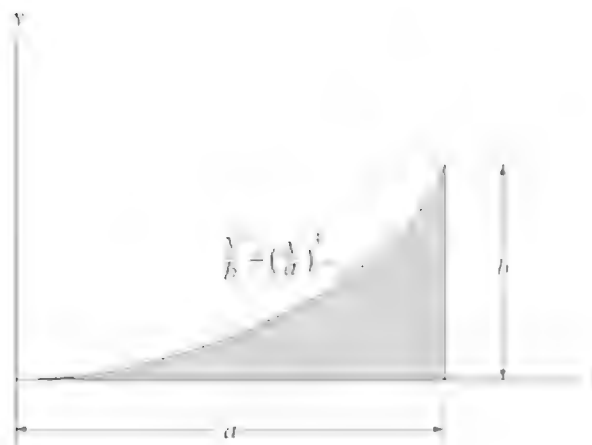
Given:

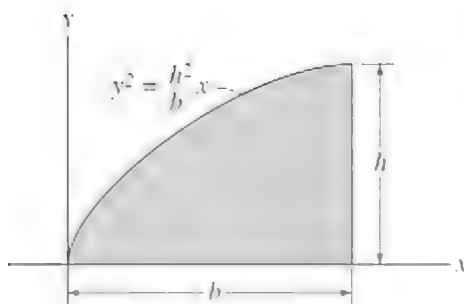
$a = 4$ in

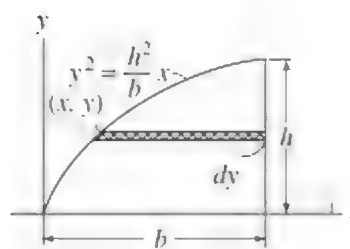
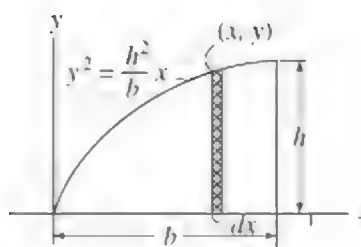
$b = 2$ in

Solution:

$$I_y = \int_0^a x^2 b\left(\frac{x}{a}\right)^3 dx$$

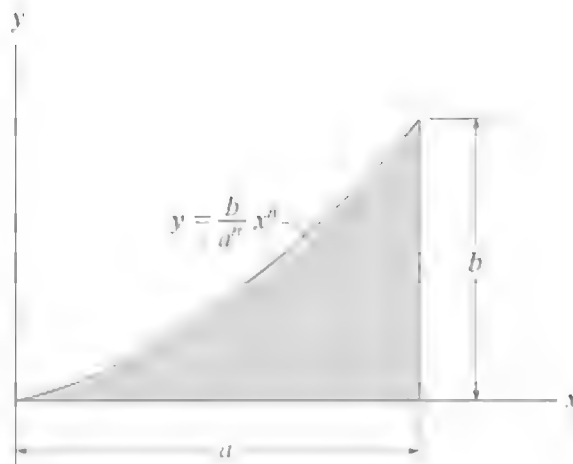
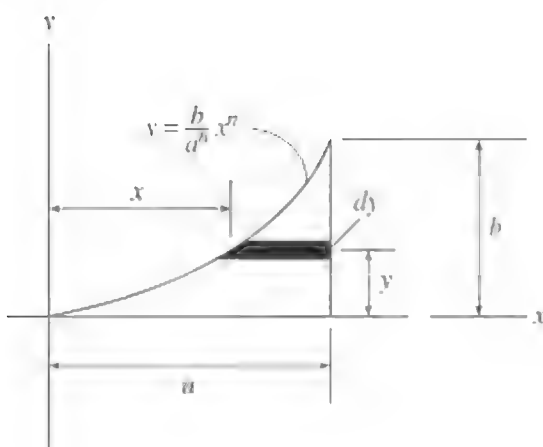
$$I_y = 21.33 \text{ in}^4$$

Problem 10-6

Determine the moment of inertia for the shaded area about the x axis.

Solution:

$$I_x = \int_0^b \frac{\left(h\sqrt{\frac{x}{b}}\right)^3}{3} dx = \frac{2}{15} b\, h^3 \qquad I_x = \frac{2}{15} b h^3$$

Alternatively

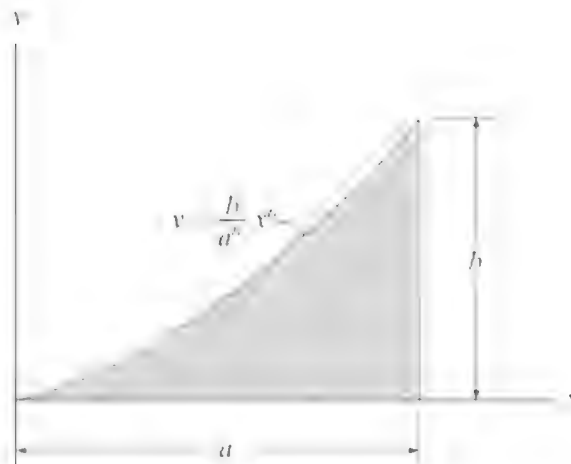
$$I_x = \int_0^h y^2\left(b - b\frac{y^2}{h^2}\right) dy = \frac{2}{15}\, b\, h^3 \qquad I_x = \frac{2}{15} b h^3$$

Problem 10-7

Determine the moment of inertia for the shaded area about the x axis.

Solution:

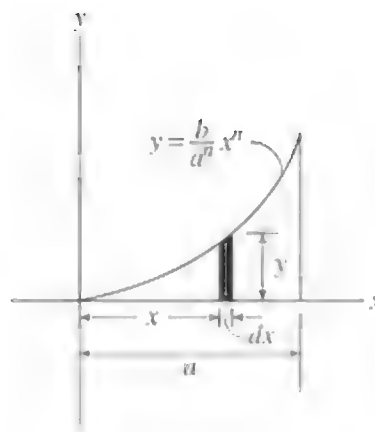
$$I_x = \int_0^b A\, y^2 \left[a - a\left(\frac{y}{b}\right)^{\frac{1}{n}}\right] dy \qquad I_x = \frac{a b^3}{3(1+3n)}$$

Problem 10-8

Determine the moment of inertia for the shaded area about the y axis.

Solution:

$$I_y = \int x^2\, dA = \int_0^a x^2 y\, dx$$

$$I_y = \frac{b}{a^n}\int_0^a x^{n+2}\, dx = \left[\left(\frac{b}{a^n}\right)\frac{x^{n+3}}{n+3}\right]_0^a$$

$$I_y = \frac{b a^3}{n+3}$$

Problem 10-9

Determine the moment of inertia for the shaded area about the x axis.

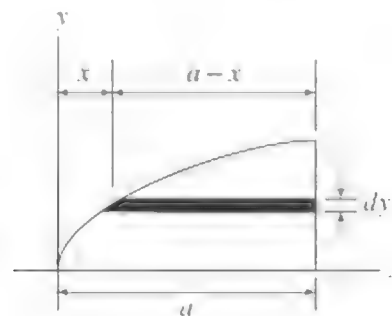
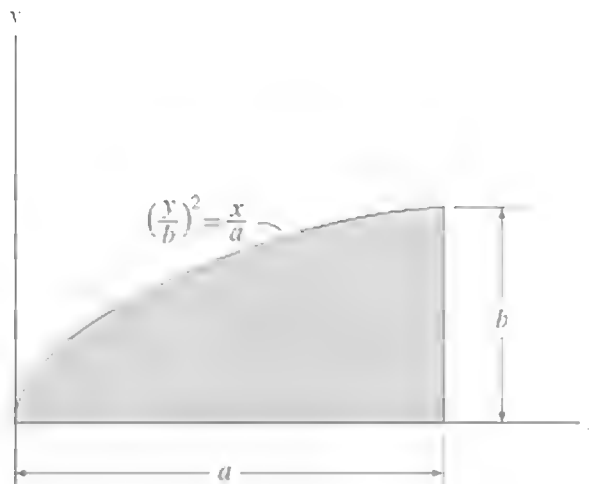
Given:

$a = 4$ in

$b = 2$ in

Solution:

$$I_x = \int_0^b y^2\left[a - a\left(\frac{y}{b}\right)^2\right] dy$$

$$I_x = 4.27\ \text{in}^4$$

Problem 10-10

Determine the moment of inertia for the shaded area about the y axis.

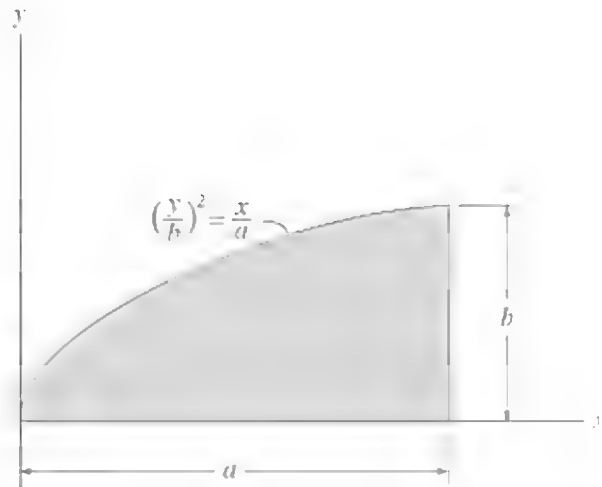
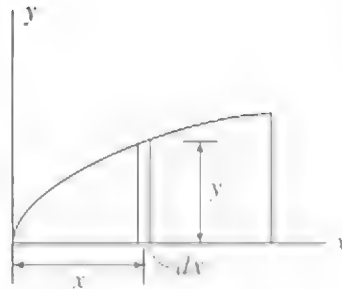
Given:

$a = 4\text{ in}$

$b = 2\text{ in}$

Solution:

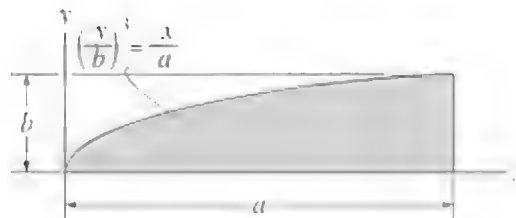
$$I_y = \int_0^a x^2 b\sqrt{\frac{x}{a}}\,dx$$

$$I_y = 36.6\text{ in}^4$$

Problem 10-11

Determine the moment of inertia for the shaded area about the x axis

Given:

$a = 8\text{ in}$

$b = 2\text{ in}$

Solution:

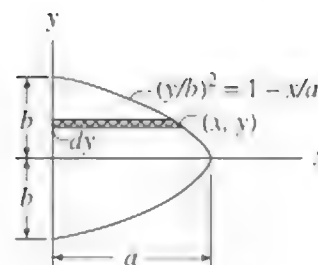
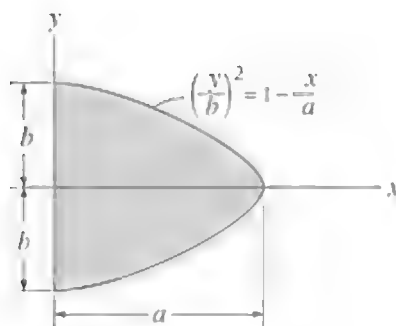
$$I_x = \int_0^b y^2\left(a - a\frac{y^3}{b^3}\right)dy \qquad I_x = 10.67\text{ in}^4$$

Problem 10-12

Determine the moment of inertia for the shaded area about the x axis

Given:

$a = 2 \text{ m}$

$b = 1 \text{ m}$

Solution:

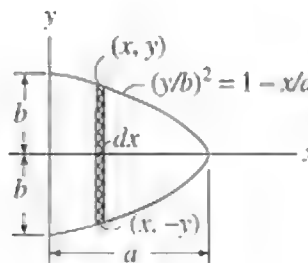
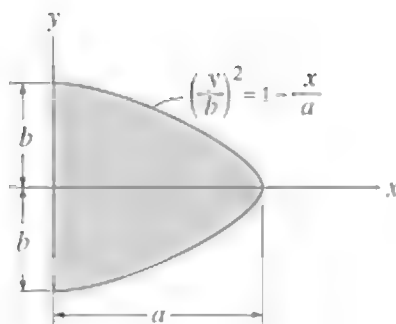
$$I_x = \int_{-b}^{b} y^2 a\left(1 - \frac{y^2}{b^2}\right) dy \qquad I_x = 0.53 \text{ m}^4$$

Problem 10-13

Determine the moment of inertia for the shaded area about the y axis

Given:

$a = 2 \text{ m}$

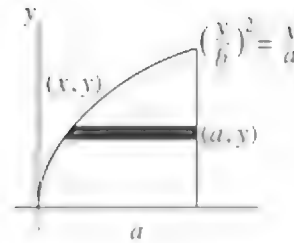
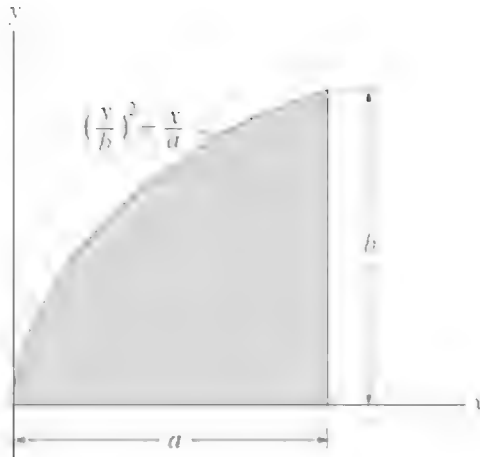
$b = 1 \text{ m}$

Solution:

$$I_y = \int_{0}^{a} x^2 2b\sqrt{1 - \frac{x}{a}}\, dx \qquad I_y = 2.44 \text{ m}^4$$

Problem 10-14

Determine the moment of inertia for the shaded area about the x axis.

Given:

$a = 4 \text{ in}$ $\qquad b = 4 \text{ in}$

Solution:

$$I_x = \int_0^b y^2\left[a - a\left(\frac{y}{b}\right)^2\right] dy$$

$I_x = 34.1 \text{ in}^4$

Problem 10-15

Determine the moment of inertia for the shaded area about the y axis.

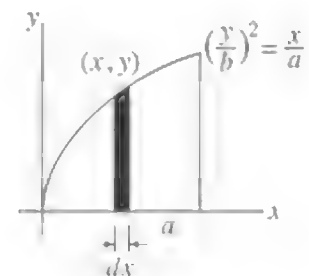
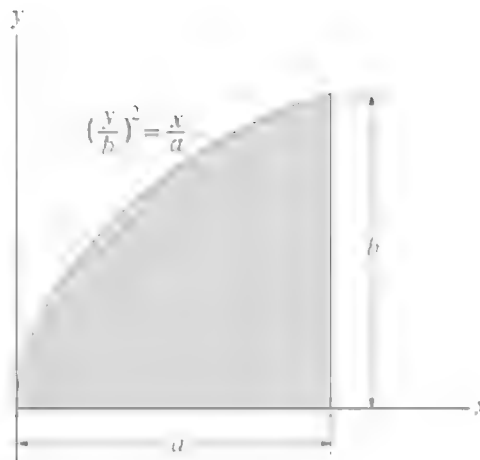
Given:

$a = 4 \text{ in}$

$b = 4 \text{ in}$

Solution:

$$I_y = \int_0^a x^2 b \sqrt{\frac{x}{a}}\, dx$$

$I_y = 73.1 \text{ in}^4$

Problem 10-16

Determine the moment of inertia of the shaded area about the x axis.

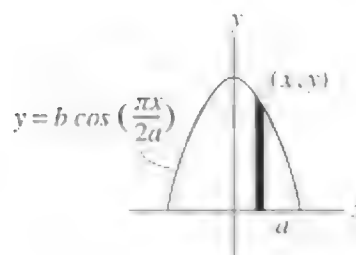
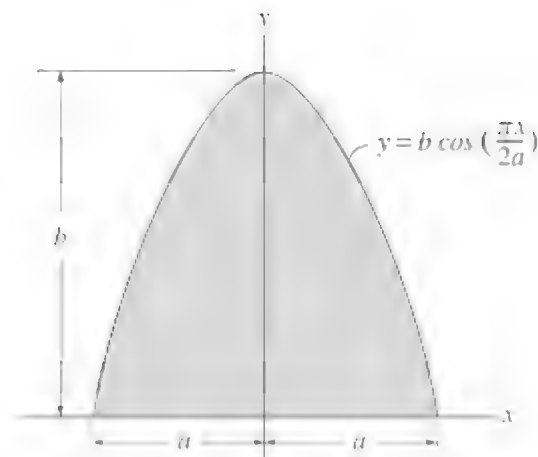
Given:

$a = 2\text{ in}$

$b = 4\text{ in}$

Solution:

$$I_x = \int_{-a}^{a} \frac{1}{3}\left(b\cos\left(\frac{\pi x}{2a}\right)\right)^3 dx$$

$$I_x = 36.2\text{ in}^4$$

Problem 10-17

Determine the moment of inertia for the shaded area about the y axis.

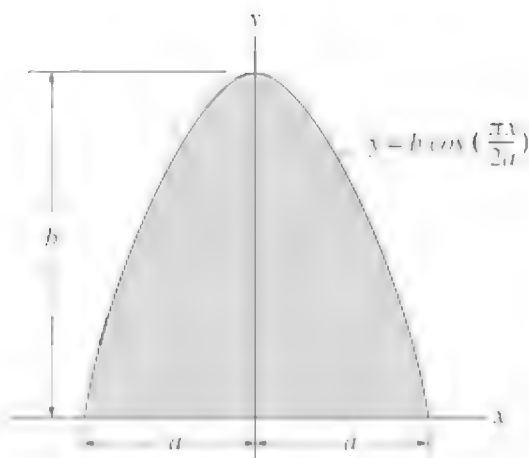
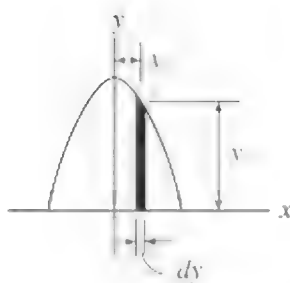
Given:

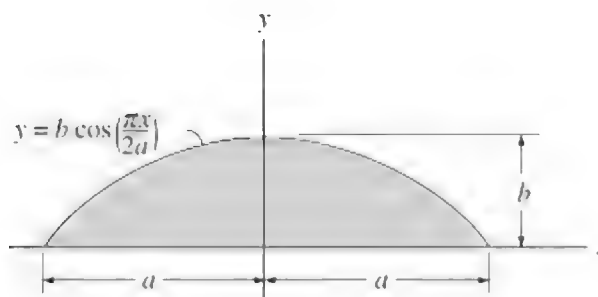
$a = 2\text{ in}$

$b = 4\text{ in}$

Solution:

$$I_y = \int_{-a}^{a} x^2 b\cos\left(\frac{\pi x}{2a}\right) dx$$

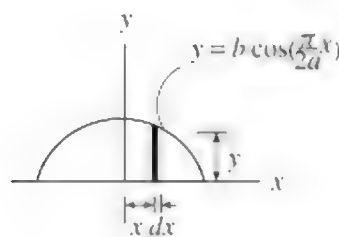
$$I_y = 7.72\text{ in}^4$$

Problem 10-18

Determine the moment of inertia for the shaded area about the x axis.

Given:

$a = 4 \text{ in}$

$b = 2 \text{ in}$

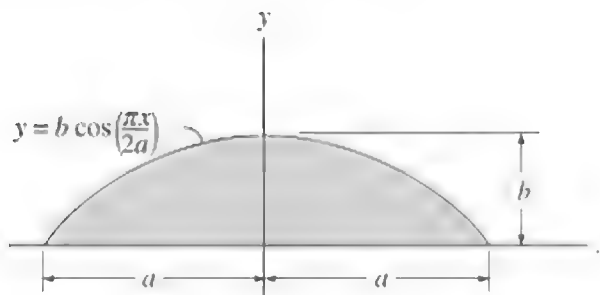
Solution:

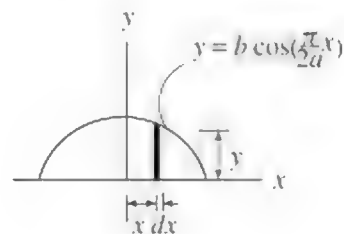
$$I_x = \int_{-a}^{a} \frac{\left(b\cos\left(\frac{\pi x}{2a}\right)\right)^3}{3}\, dx \qquad I_x = 9.05 \text{ in}^4$$

Problem 10-19

Determine the moment of inertia for the shaded area about the y axis.

Given:

$a = 4 \text{ in}$

$b = 2 \text{ in}$

Solution:

$$I_y = \int_{-a}^{a} x^2 b\cos\left(\frac{\pi x}{2a}\right) dx \qquad I_y = 30.9 \text{ in}^4$$

Problem 10-20

Determine the moment for inertia of the shaded area about the x axis.

Given:

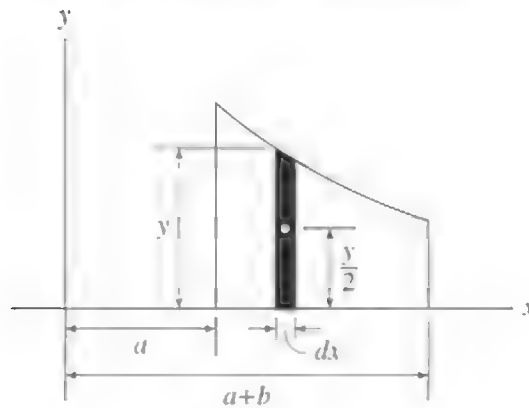
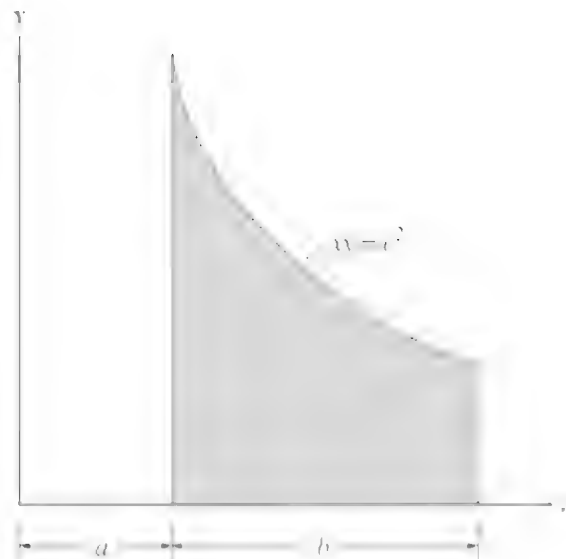
$a = 2$ in

$b = 4$ in

$c = \sqrt{12}$ in

Solution:

$$I_x = \int_a^{a+b} \frac{1}{3}\left(\frac{c^2}{x}\right)^3 dx$$

$I_x = 64.0\,\text{in}^4$

Problem 10-21

Determine the moment of inertia of the shaded area about the y axis.

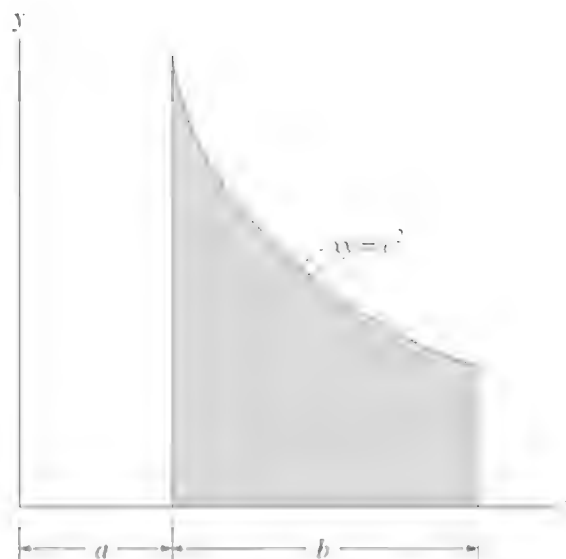
Given:

$a = 2$ in

$b = 4$ in

$c = \sqrt{12}$ in

Solution:

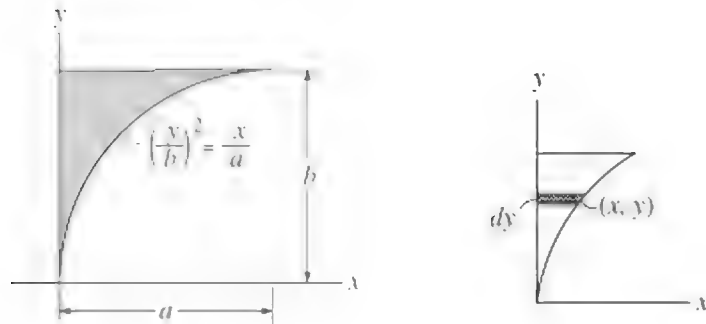
$$I_y = \int_a^{a+b} x^2\left(\frac{c^2}{x}\right) dx$$

$I_y = 192.00\ \text{in}^4$

Problem 10-22

Determine the moment of inertia for the shaded area about the x axis.

Given:

$a = 2\ \text{m}$

$b = 2\ \text{m}$

Solution:

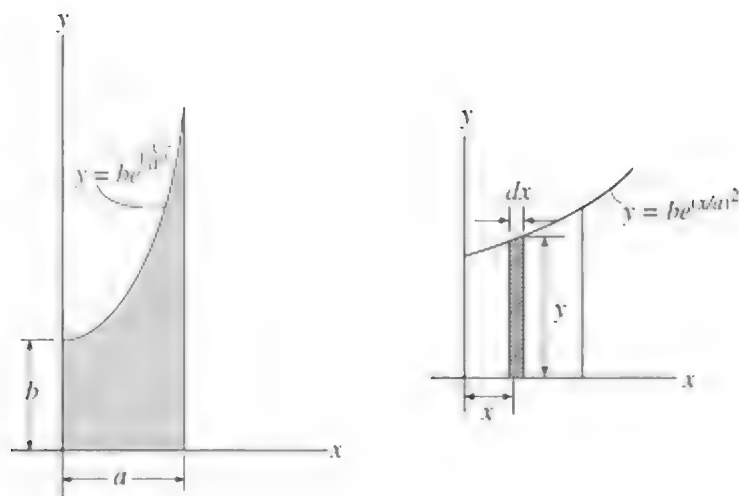
$$I_x = \int_0^b y^2 a\left(\frac{y^2}{b^2}\right) \text{d}y \qquad I_x = 3.20\ \text{m}^4$$

Problem 10-23

Determine the moment of inertia for the shaded area about the y axis. Use Simpson's rule to evaluate the integral.

Given:

$a = 1\ \text{m}$

$b = 1\ \text{m}$

Solution:

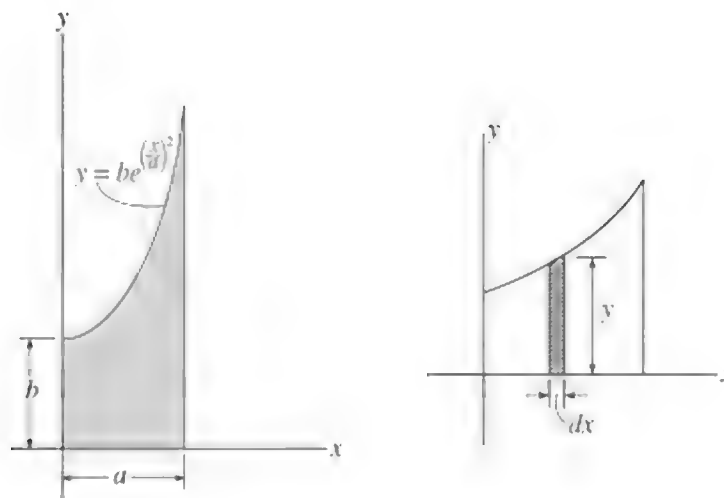
$$I_y = \int_0^a x^2 b e^{\left(\frac{x}{a}\right)^2} \, dx \qquad I_y = 0.628 \text{ m}^4$$

Problem 10-24

Determine the moment of inertia for the shaded area about the x axis. Use Simpson's rule to evaluate the integral.

Given:

$a = 1 \text{ m}$

$b = 1 \text{ m}$

Solution:

$$I_y = \int_0^a \frac{\left[b e^{\left(\frac{x}{a}\right)^2}\right]^3}{3} \, dx \qquad I_y = 1.41 \text{ m}^4$$

Problem 10-25

The polar moment of inertia for the area is I_C about the z axis passing through the centroid C. The moment of inertia about the x axis is I_x and the moment of inertia about the y' axis is $I_{y'}$. Determine the area A.

Given:

$I_C = 28 \text{ in}^4$

$I_x = 17\ \text{in}^4$

$I_{y'} = 56\ \text{in}^4$

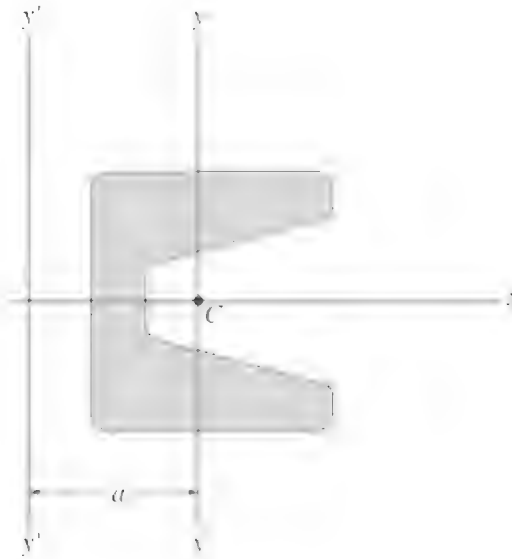
$a = 3\ \text{in}$

Solution:

$I_C = I_x + I_y$

$I_y = I_C - I_x$

$I_{y'} = I_y + A\,a^2$

$A = \dfrac{I_{y'} - I_y}{a^2}$ $\quad A = 5.00\ \text{in}^2$

Problem 10-26

The polar moment of inertia for the area is J_{cc} about the z' axis passing through the centroid C. If the moment of inertia about the y' axis is $I_{y'}$ and the moment of inertia about the x axis is I_x. Determine the area A.

Given:

$J_{cc} = 548 \times 10^6\ \text{mm}^4$

$I_{y'} = 383 \times 10^6\ \text{mm}^4$

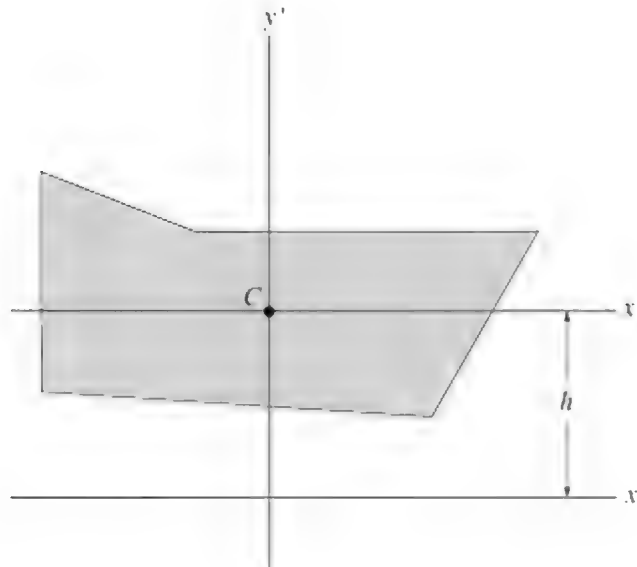
$I_x = 856 \times 10^6\ \text{mm}^4$

$h = 250\ \text{mm}$

Solution:

$I_{x'} = I_x - A\,h^2$

$J_{cc} = I_{x'} + I_{y'}$

$J_{cc} = I_x - A\,h^2 + I_{y'}$

$$A = \frac{I_x + I_{y'} - J_{cc}}{h^2}$$

$$A = 11.1 \times 10^3 \text{ mm}^2$$

Problem 10-27

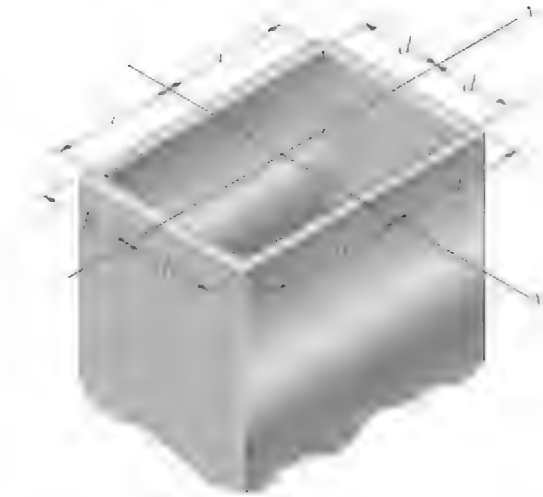
Determine the radius of gyration k_x of the column's cross-sectional area.

Given:

$a = 100$ mm

$b = 75$ mm

$c = 90$ mm

$d = 65$ mm

Solution:

Cross-sectional area:

$$A = (2b)(2a) - (2d)(2c)$$

Moment of inertia about the x axis:

$$I_x = \frac{1}{12}(2b)(2a)^3 - \frac{1}{12}(2d)(2c)^3$$

Radius of gyration about the x axis:

$$k_x = \sqrt{\frac{I_x}{A}} \qquad k_x = 74.7 \text{ mm}$$

Problem 10-28

Determine the radius of gyration k_y of the column's cross-sectional area.

Given:

$a = 100$ mm

$b = 75$ mm

$c = 90\ \text{mm}$

$d = 65\ \text{mm}$

Solution:

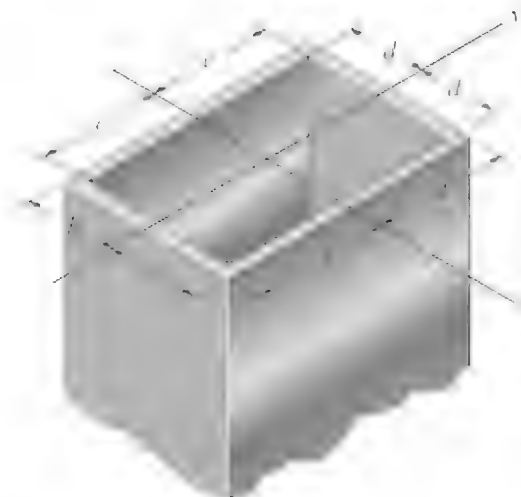
Cross-sectional area:

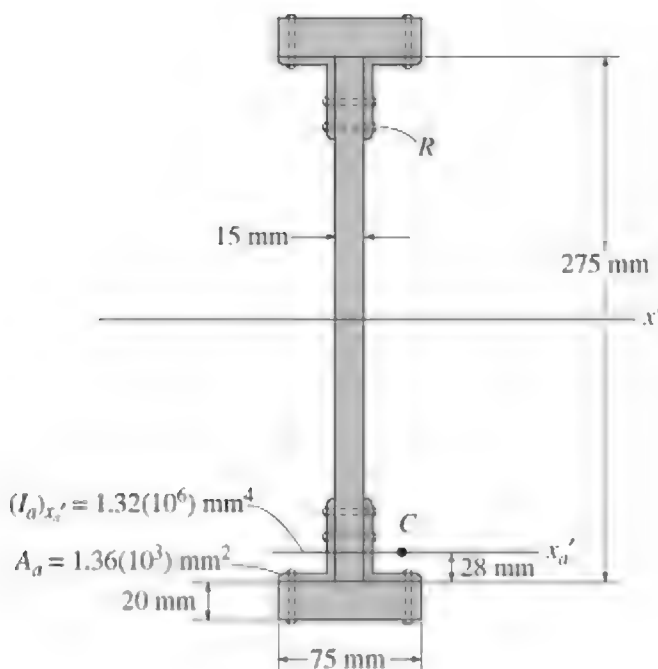
$$A = (2b)(2a) - (2d)(2c)$$

Moment of inertia about the *y* axis:

$$I_y = \frac{1}{12}(2a)(2b)^3 - \frac{1}{12}(2c)(2d)^3$$

Radius of gyration about the *y* axis:

$$k_y = \sqrt{\frac{I_y}{A}} \qquad k_y = 59.4\ \text{mm}$$

Problem 10-29

Determine the moment of inertia for the beam's cross-sectional area with respect to the x' centroidal axis. Neglect the size of all the rivet heads, R, for the calculation. Handbook values for the area, moment of inertia, and location of the centroid C of one of the angles are listed in the figure.

Solution:

$$I_E = \frac{1}{12}(15\ \text{mm})(275\ \text{mm})^3 + 4\left[1.32\left(10^6\right)\text{mm}^4 + 1.36\left(10^3\right)\text{mm}^2\left(\frac{275\ \text{mm}}{2} - 28\ \text{mm}\right)^2\right] \ldots$$
$$+ 2\left[\frac{1}{12}(75\ \text{mm})(20\ \text{mm})^3 + (75\ \text{mm})(20\ \text{mm})\left(\frac{275\ \text{mm}}{2} + 10\text{mm}\right)^2\right]$$

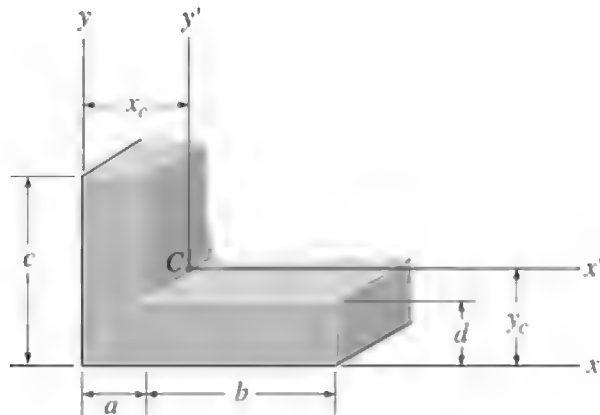
$I_E = 162 \times 10^6 \text{ mm}^4$

Problem 10-30

Locate the centroid y_c of the cross-sectional area for the angle. Then find the moment of inertia $I_{x'}$ about the x' centroidal axis.

Given:

$a = 2 \text{ in}$

$b = 6 \text{ in}$

$c = 6 \text{ in}$

$d = 2 \text{ in}$

Solution:

$$y_c = \frac{a\,c\left(\frac{c}{2}\right) + b\,d\left(\frac{d}{2}\right)}{a\,c + b\,d} \qquad y_c = 2.00 \text{ in}$$

$$I_{x'} = \frac{1}{12}a\,c^3 + a\,c\left(\frac{c}{2} - y_c\right)^2 + \frac{1}{12}b\,d^3 + b\,d\left(y_c - \frac{d}{2}\right)^2 \qquad I_{x'} = 64.00 \text{ in}^4$$

Problem 10-31

Locate the centroid x_c of the cross-sectional area for the angle. Then find the moment

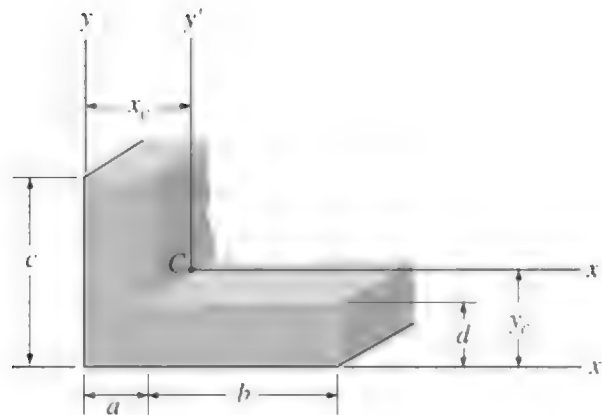
of inertia $I_{y'}$ about the centroidal y' axis.

Given:

$a = 2$ in

$b = 6$ in

$c = 6$ in

$d = 2$ in

Solution:

$$x_C = \frac{a\,c\left(\frac{a}{2}\right) + b\,d\left(a + \frac{b}{2}\right)}{a\,c + b\,d} \qquad x_C = 3.00\text{ in}$$

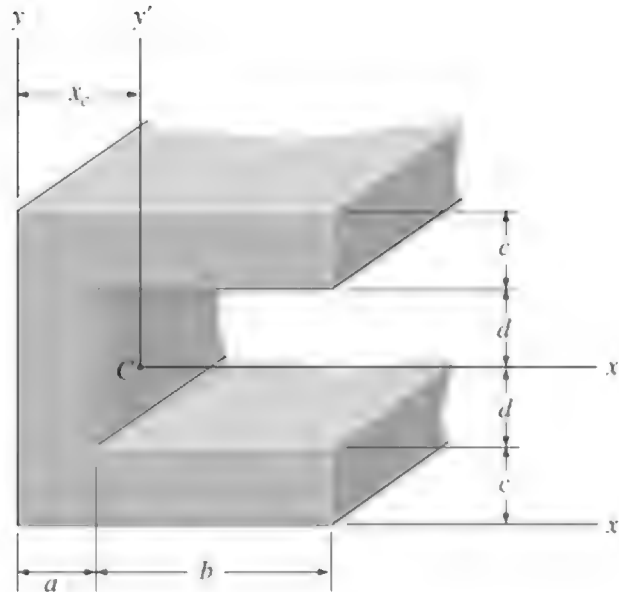
$$I_{y'} = \frac{1}{12}c\,a^3 + c\,a\left(x_C - \frac{a}{2}\right)^2 + \frac{1}{12}d\,b^3 + d\,b\left(a + \frac{b}{2} - x_C\right)^2 \qquad I_{y'} = 136.00\text{ in}^4$$

Problem 10-32

Determine the distance x_c to the centroid of the beam's cross-sectional area: then find the moment of inertia about the y' axis.

Given:

$a = 40\ \text{mm}$

$b = 120\ \text{mm}$

$c = 40\ \text{mm}$

$d = 40\ \text{mm}$

Solution:

$$x_c = \frac{2(a+b)c\left(\frac{a+b}{2}\right) + 2ad\frac{a}{2}}{2(a+b)c + 2da} \qquad x_c = 68.00\ \text{mm}$$

$$I_{y'} = 2\left[\frac{1}{12}c(a+b)^3 + c(a+b)\left(\frac{a+b}{2} - x_c\right)^2\right] + \frac{1}{12}2da^3 + 2da\left(x_c - \frac{a}{2}\right)^2$$

$$I_{y'} = 36.9 \times 10^6\ \text{mm}^4$$

Problem 10-33

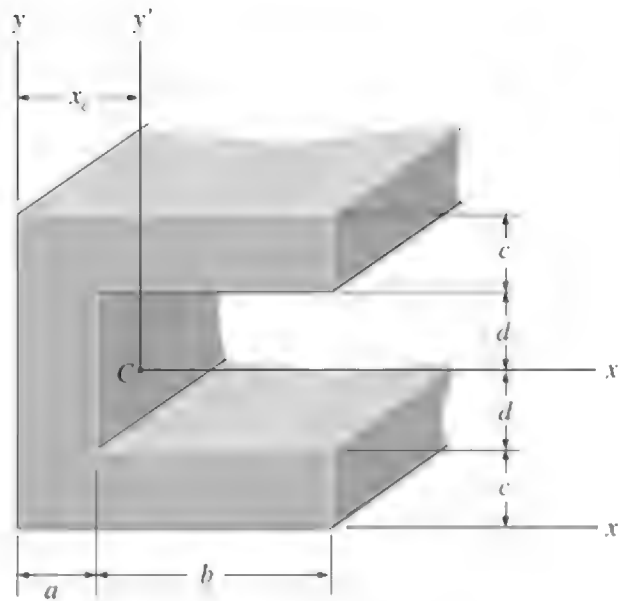
Determine the moment of inertia of the beam's cross-sectional area about the x' axis.

Given:

$a = 40\ \text{mm}$

$b = 120\ \text{mm}$

$c = 40\ \text{mm}$

$d = 40\ \text{mm}$

Solution:

$$I_{x'} = \frac{1}{12}(a+b)(2c+2d)^3 - \frac{1}{12}b(2d)^3 \qquad I_{x'} = 49.5 \times 10^6\ \text{mm}^4$$

Problem 10-34

Determine the moments of inertia for the shaded area about the x and y axes.

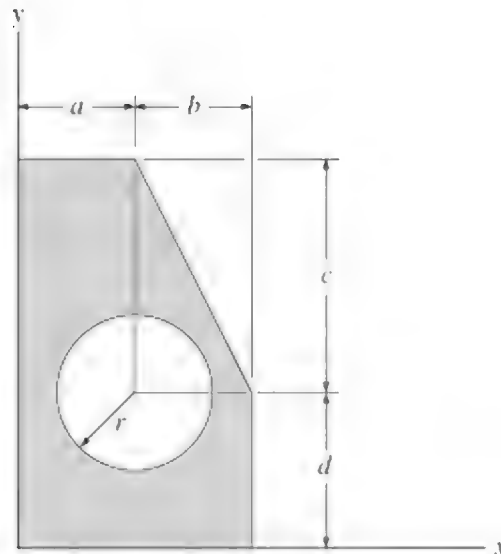
Given:

$a = 3\ \text{in}$

$b = 3\ \text{in}$

$c = 6\ \text{in}$

$d = 4\ \text{in}$

$r = 2\ \text{in}$

Solution:

$$I_x = \frac{1}{3}(a+b)(c+d)^3 - \left[\frac{1}{36}bc^3 + \frac{1}{2}bc\left(d + \frac{2c}{3}\right)^2\right] - \left(\frac{\pi r^4}{4} + \pi r^2 d^2\right)$$

$I_x = 1192\,\text{in}^4$

$$I_y = \frac{1}{3}(c+d)(a+b)^3 - \left[\frac{1}{36}cb^3 + \frac{1}{2}bc\left(a + \frac{2b}{3}\right)^2\right] - \left(\frac{\pi r^4}{4} + \pi r^2 a^2\right)$$

$I_y = 364.84\,\text{in}^4$

Problem 10-35

Determine the location of the centroid y' of the beam constructed from the two channels and the cover plate. If each channel has a cross-sectional area A_c and a moment of inertia about a horizontal axis passing through its own centroid C_c, of $I_{x'c}$, determine the moment of inertia of the beam's cross-sectional area about the x' axis.

Given:

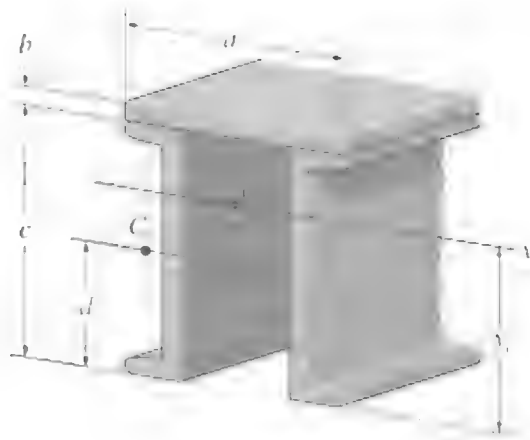
$a = 18$ in

$b = 1.5$ in

$c = 20$ in

$d = 10$ in

$A_c = 11.8\ \text{in}^2$

$I_{x'c} = 349\ \text{in}^4$

Solution:

$$y_c = \frac{2A_c d + ab\left(c + \frac{b}{2}\right)}{2A_c + ab} \qquad y_c = 15.74\,\text{in}$$

$$I_{x'} = \left[I_{x'c} + A_c\left(y_c - d\right)^2\right]2 + \frac{1}{12}ab^3 + ab\left(c + \frac{b}{2} - y_c\right)^2 \qquad I_{x'} = 2158\,\text{in}^4$$

Problem 10-36

Compute the moments of inertia I_x and I_y for the beam's cross-sectional area about

the x and y axes.

Given:

$a = 30$ mm

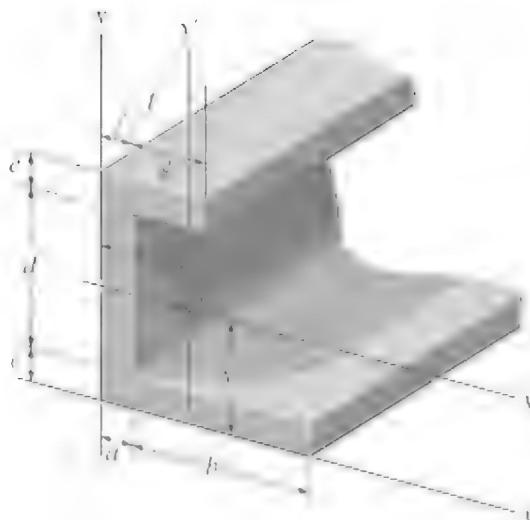
$b = 170$ mm

$c = 30$ mm

$d = 140$ mm

$e = 30$ mm

$f = 30$ mm

$g = 70$ mm

Solution:

$$I_x = \frac{1}{3}a(c+d+e)^3 + \frac{1}{3}bc^3 + \frac{1}{12}ge^3 + ge\left(c+d+\frac{e}{2}\right)^2$$

$I_x = 154 \times 10^6\ \text{mm}^4$

$$I_y = \frac{1}{3}c(a+b)^3 + \frac{1}{3}df^3 + \frac{1}{3}c(f+g)^3$$

$I_y = 91.3 \times 10^6\ \text{mm}^4$

Problem 10-37

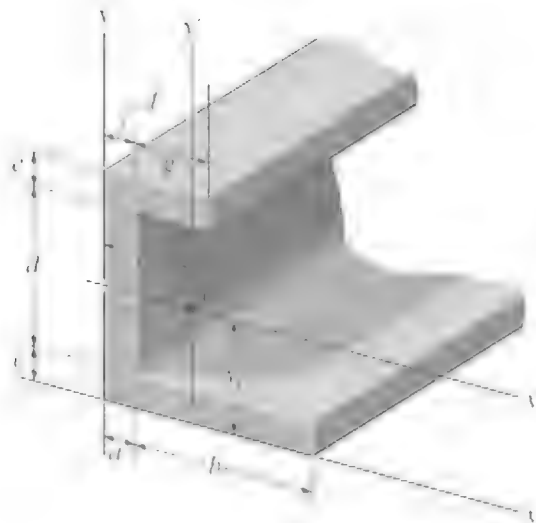
Determine the distance y_c to the centroid C of the beam's cross-sectional area and then compute the moment of inertia $I_{cx'}$ about the x' axis.

Given:

$a = 30$ mm $e = 30$ mm

$b = 170$ mm $f = 30$ mm

$c = 30$ mm $g = 70$ mm

$d = 140$ mm

Solution:

$$y_C = \frac{(a+b)c\left(\frac{c}{2}\right) + df\left(c + \frac{d}{2}\right) + (f+g)e\left(c + d + \frac{e}{2}\right)}{(a+b)c + df + (f+g)e}$$

$$y_C = 80.7\ \text{mm}$$

$$I_{x'} = \frac{1}{12}(a+b)\,c^3 + (a+b)c\left(y_C - \frac{c}{2}\right)^2 + \frac{1}{12}fd^3 + fd\left(c + \frac{d}{2} - y_C\right)^2 \ldots$$
$$+ \frac{1}{12}(f+g)\,e^3 + (f+g)e\left(c + d + \frac{e}{2} - y_C\right)^2$$

$$I_{x'} = 67.6 \times 10^6\ \text{mm}^4$$

Problem 10-38

Determine the distance x_c to the centroid C of the beam's cross-sectional area and then compute the moment of inertia $I_{y'}$ about the y' axis.

Given:

$a = 30\ \text{mm}$

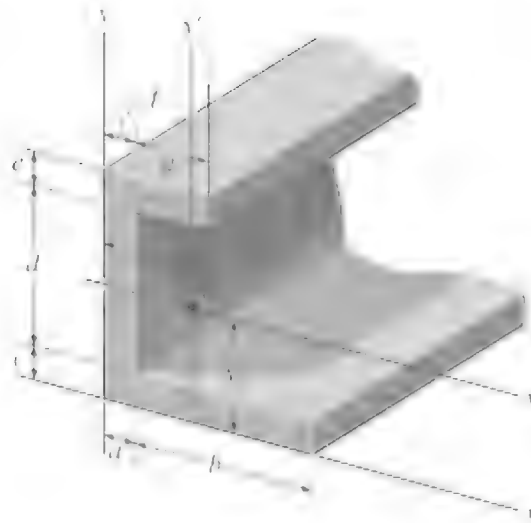
$b = 170\ \text{mm}$

$c = 30\ \text{mm}$

$d = 140\ \text{mm}$

$e = 30\ \text{mm}$

$f = 30\ \text{mm}$

$g = 70\ \text{mm}$

Solution:

$$x_C = \frac{bc\left(\frac{b}{2} + a\right) + (c+d)f\left(\frac{f}{2}\right) + (f+g)e\frac{f+g}{2}}{bc + bc + (f+g)e}$$

$$x_C = 61.6\ \text{mm}$$

$$I_{y'} = \frac{1}{12}c(a+b)^3 + c(a+b)\left(\frac{a+b}{2} - x_c\right)^2 + \frac{1}{12}df^3 + df\left(x_c - \frac{f}{2}\right)^2 \ldots$$
$$+ \frac{1}{12}e(f+g)^3 + e(f+g)\left(x_c - \frac{f+g}{2}\right)^2$$

$$I_{y'} = 41.2 \times 10^6\ \text{mm}^4$$

Problem 10-39

Determine the location y_c of the centroid C of the beam's cross-sectional area. Then compute the moment of inertia of the area about the x' axis

Given:

$a = 20\ \text{mm}$

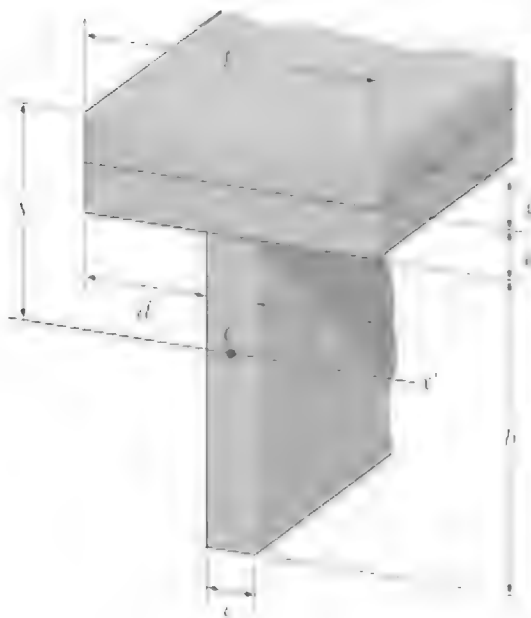
$b = 125\ \text{mm}$

$c = 20\ \text{mm}$

$f = 120\ \text{mm}$

$g = 20\ \text{mm}$

$d = \dfrac{f-c}{2}$

$e = \dfrac{f-c}{2}$

Solution:

$$y_c = \frac{(a+g)f\left(\frac{a+g}{2}\right) + cb\left(a+g+\frac{b}{2}\right)}{(a+g)f + cb}$$

$$y_c = 48.25\ \text{mm}$$

$$I_{x'} = \frac{1}{12}f(a+g)^3 + (f)(a+g)\left(y_c - \frac{a+g}{2}\right)^2 + \frac{1}{12}cb^3 + cb\left(\frac{b}{2} + a + g - y_c\right)^2$$

$$I_{x'} = 15.1 \times 10^6\ \text{mm}^4$$

Problem 10-40

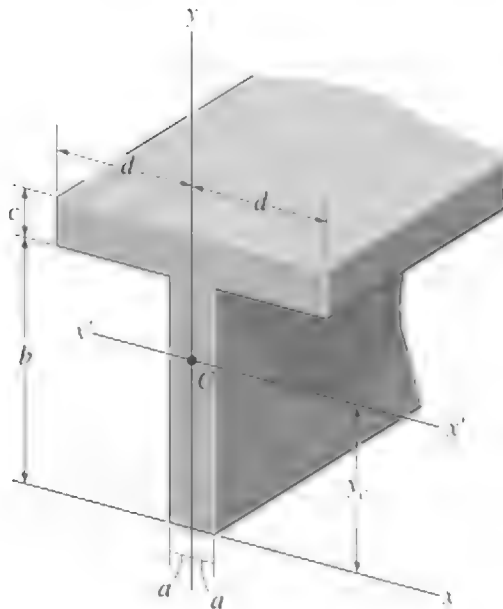
Determine y_c, which locates the centroidal axis x' for the cross-sectional area of the T-beam, and then find the moments of inertia $I_{x'}$ and $I_{y'}$.

Given:

$a = 25\text{ mm}$

$b = 250\text{ mm}$

$c = 50\text{ mm}$

$d = 150\text{ mm}$

Solutuion:

$$y_c = \frac{\left(\frac{b}{2}\right)b2a + \left(b + \frac{c}{2}\right)2dc}{b2a + c2d}$$

$$y_c = 207\text{ mm}$$

$$I_{x'} = \frac{1}{12}2ab^3 + 2ab\left(y_c - \frac{b}{2}\right)^2 + \frac{1}{12}2dc^3 + c2d\left(b + \frac{c}{2} - y_c\right)^2$$

$$I_{x'} = 222 \times 10^6\text{ mm}^4$$

$$I_{y'} = \frac{1}{12}b(2a)^3 + \frac{1}{12}c(2d)^3$$

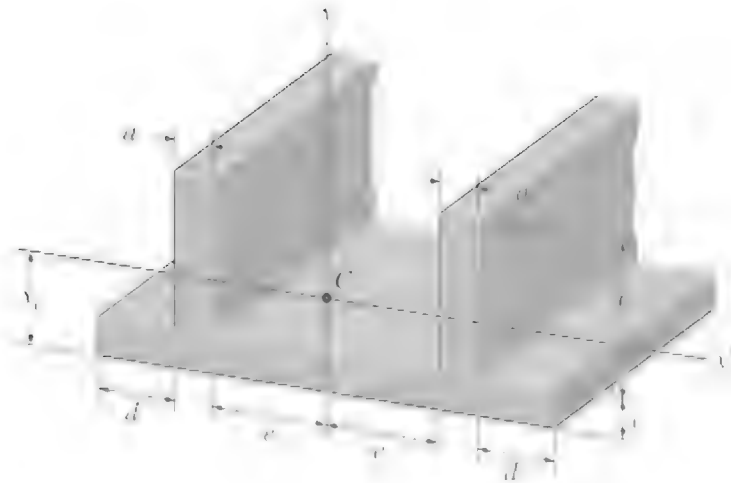
$$I_{y'} = 115 \times 10^6\text{ mm}^4$$

Problem 10-41

Determine the centroid y' for the beam's cross-sectional area; then find $I_{x'}$.

Given:

$a = 25\text{ mm}$

$b = 100\text{ mm}$

$c = 25\text{ mm}$

$d = 50\text{ mm}$

$e = 75\text{ mm}$

Solution:

$$y_c = \frac{2(a+e+d)c\left(\frac{c}{2}\right) + 2ab\left(c + \frac{b}{2}\right)}{2(a+e+d)c + 2ab} \qquad y_c = 37.50\text{ mm}$$

$$I_{x'} = \frac{2}{12}(a+e+d)\,c^3 + 2(a+e+d)c\left(y_c - \frac{c}{2}\right)^2 \ldots$$
$$+ 2\left[\frac{1}{12}ab^3 + ab\left(c + \frac{b}{2} - y_c\right)^2\right]$$

$$I_{x'} = 16.3 \times 10^6\text{ mm}^4$$

Problem 10-42

Determine the moment of inertia for the beam's cross-sectional area about the y axis.

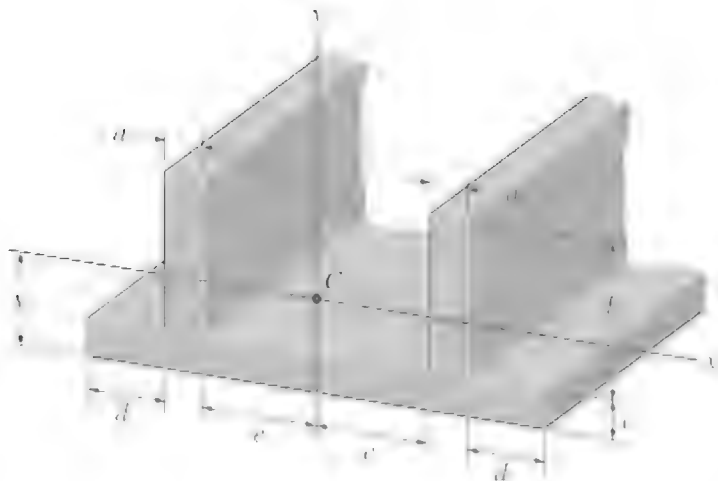
Given:

$a = 25\text{ mm}$

$b = 100\text{ mm}$

$c = 25\text{ mm}$

$d = 50\text{ mm}$

$e = 75\text{ mm}$

Solution:

$$I_y = \frac{1}{12} 2^3 (a+d+e)^3 c + 2\left[\frac{1}{12} b a^3 + a b \left(e + \frac{a}{2}\right)^2\right]$$

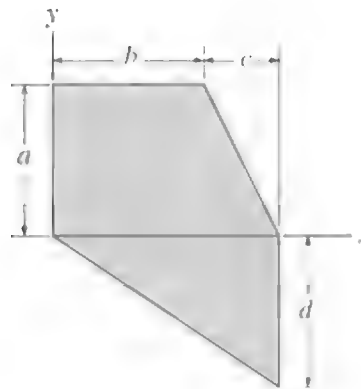
$$I_y = 94.8 \times 10^6 \text{ mm}^4$$

Problem 10-43

Determine the moment for inertia I_x of the shaded area about the x axis.

Given:

$a = 6$ in

$b = 6$ in

$c = 3$ in

$d = 6$ in

Solution:

$$I_x = \frac{b a^3}{3} + \frac{1}{12} c a^3 + \frac{1}{12}(b+c) d^3 \qquad I_x = 648 \text{ in}^4$$

Problem 10-44

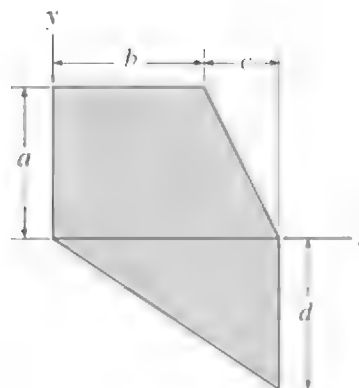
Determine the moment for inertia I_y of the shaded area about the y axis.

Given:

$a = 6$ in

$b = 6$ in

$c = 3$ in

$d = 6$ in

Solution:

$$I_y = \frac{a b^3}{3} + \frac{1}{36} a c^3 + \frac{1}{2} a c \left(b + \frac{c}{3} \right)^2 + \frac{1}{36} d (b + c)^3 + \frac{1}{2} d (b + c) \left[\frac{2(b + c)}{3} \right]^2$$

$I_y = 1971 \text{ in}^4$

Problem 10-45

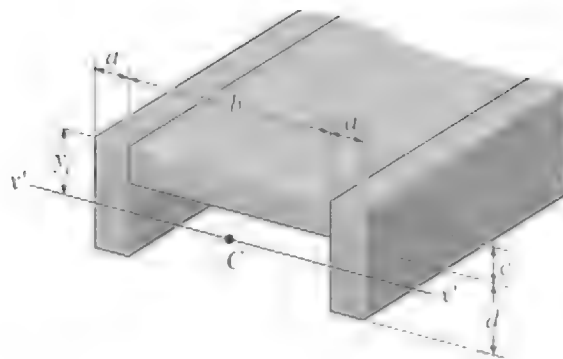
Locate the centroid y_c of the channel's cross-sectional area, and then determine the moment of inertia with respect to the x' axis passing through the centroid.

Given:

$a = 2 \text{ in}$

$b = 12 \text{ in}$

$c = 2 \text{ in}$

$d = 4 \text{ in}$

Solution:

$$y_c = \frac{\frac{c}{2} b c + 2 \left(\frac{c + d}{2} \right) (c + d) a}{b c + 2(c + d) a}$$

$y_c = 2 \text{ in}$

$$I_x = \frac{1}{12} b c^3 + b c \left(y_c - \frac{c}{2} \right)^2 + \frac{2}{12} a (c + d)^3 + 2a(c + d) \left(\frac{c + d}{2} - y_c \right)^2$$

$I_x = 128 \text{ in}^4$

Problem 10-46

Determine the moments for inertia I_x and I_y of the shaded area.

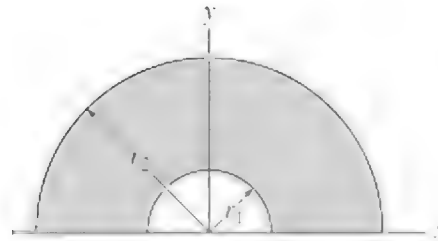
Given:

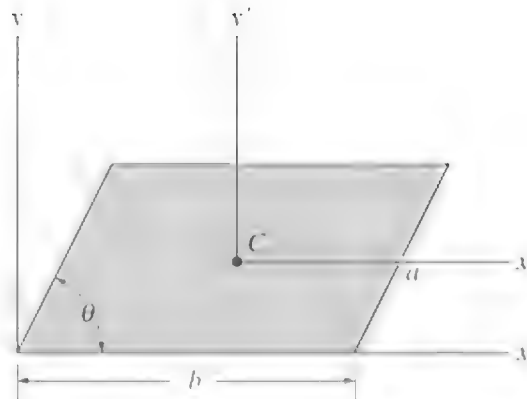
$r_1 = 2 \text{ in}$

$r_2 = 6 \text{ in}$

Solution:

$$I_x = \left(\frac{\pi r_2^4}{8} - \frac{\pi r_1^4}{8}\right) \qquad I_x = 503 \text{ in}^4$$

$$I_y = \left(\frac{\pi r_2^4}{8} - \frac{\pi r_1^4}{8}\right) \qquad I_y = 503 \text{ in}^4$$

Problem 10-47

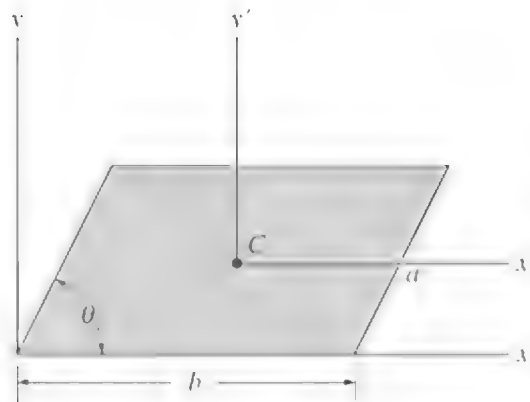
Determine the moment of inertia for the parallelogram about the x' axis, which passes through the centroid C of the area.

Solution:

$$h = (a)\sin(\theta)$$

$$I_{xc} = \frac{1}{12} b h^3 = \frac{1}{12} b\left[(a)\sin(\theta)\right]^3 = \frac{1}{12} a^3 b \sin(\theta)^3$$

$$I_{xc} = \frac{1}{12} a^3 b \sin(\theta)^3$$

Problem 10-48

Determine the moment of inertia for the parallelogram about the y' axis, which passes through the centroid C of the area.

Solution:

$$A = b(a)\sin(\theta)$$

$$x_c = \frac{1}{b(a)\sin(\theta)}\left[\left[b(a)\sin(\theta)\frac{b}{2} - \frac{1}{2}(a)\cos(\theta)(a)\sin(\theta)\frac{(a)\cos(\theta)}{3}\right]\ \ldots \atop + \frac{1}{2}(a)\cos(\theta)(a)\sin(\theta)\left[b + \frac{(a)\cos(\theta)}{3}\right]\right] = \frac{b + (a)\cos(\theta)}{2}$$

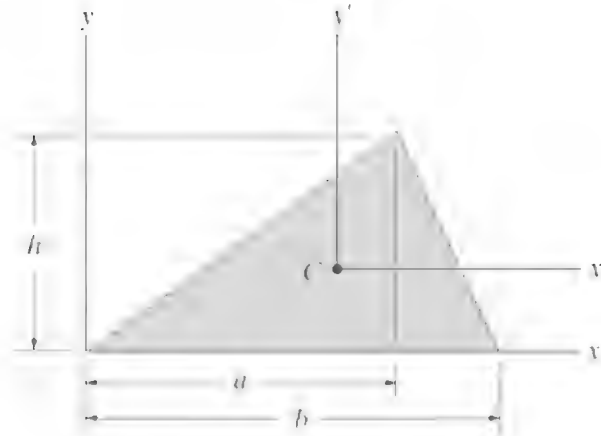
$$I_{y'} = \frac{1}{12}(a)\sin(\theta)b^3 + (a)\sin(\theta)b\left(\frac{b}{2} - x_c\right)^2 \ \ldots$$
$$+ -\left[\frac{1}{36}(a)\sin(\theta)\left[(a)\cos(\theta)\right]^3 + \frac{1}{2}(a)\sin(\theta)(a)\cos(\theta)\left[x_c - \frac{(a)\cos(\theta)}{3}\right]^2\right] \ldots$$
$$+ \frac{1}{36}(a)\sin(\theta)\left[(a)\cos(\theta)\right]^3 + \frac{1}{2}(a)\sin(\theta)(a)\cos(\theta)\left[b + \frac{(a)\cos(\theta)}{3} - x_c\right]^2$$

Simplifying we find.

$$I_{y'} = \frac{ab}{12}\left(b^2 + a^2\cos(\theta)^2\right)\sin(\theta)$$

Problem 10-49

Determine the moments of inertia for the triangular area about the x' and y' axes, which pass through the centroid C of the area.

Solution:

$$I_{x'} = \frac{1}{36} b h^3$$

$$x_C = \frac{\frac{2}{3} a \frac{1}{2} h a + \left(a + \frac{b-a}{3}\right) \frac{1}{2} h (b-a)}{\frac{1}{2} h a + \frac{1}{2} h (b-a)} = \frac{b+a}{3}$$

$$I_{y'} = \frac{1}{36} h a^3 + \frac{1}{2} h a \left(\frac{b+a}{3} - \frac{2}{3} a\right)^2 + \frac{1}{36} h (b-a)^3 + \frac{1}{2} h (b-a) \left(a + \frac{b-a}{3} - \frac{b+a}{3}\right)^2$$

$$I_{y'} = \frac{1}{36} h b \left(b^2 - a b + a^2\right)$$

Problem 10-50

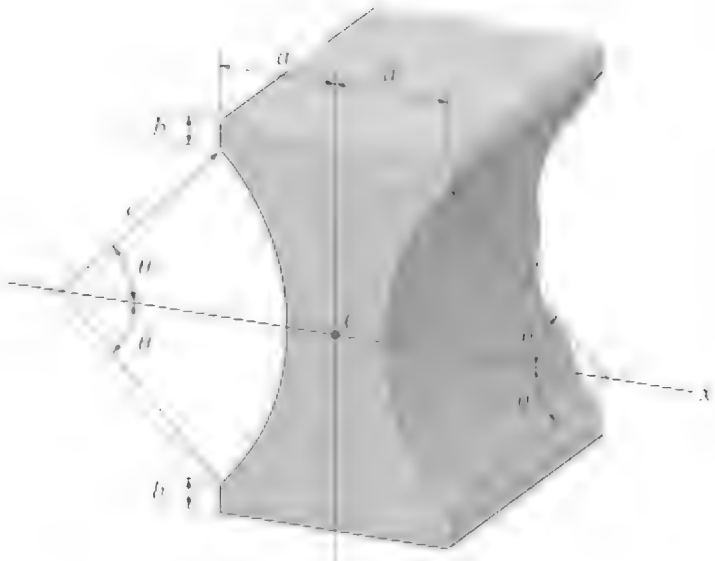
Determine the moment of inertia for the beam's cross-sectional area about the x' axis passing through the centroid C of the cross section.

Given:

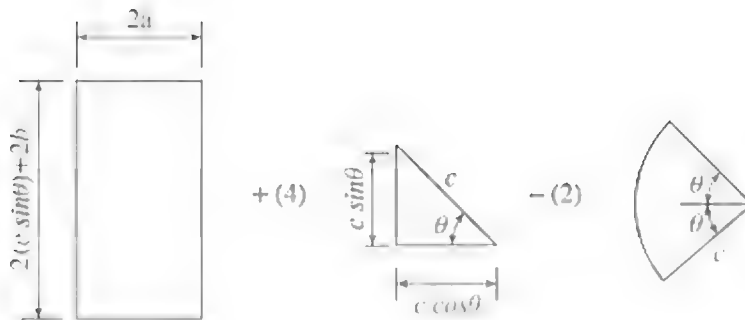
$a = 100$ mm

$b = 25$ mm

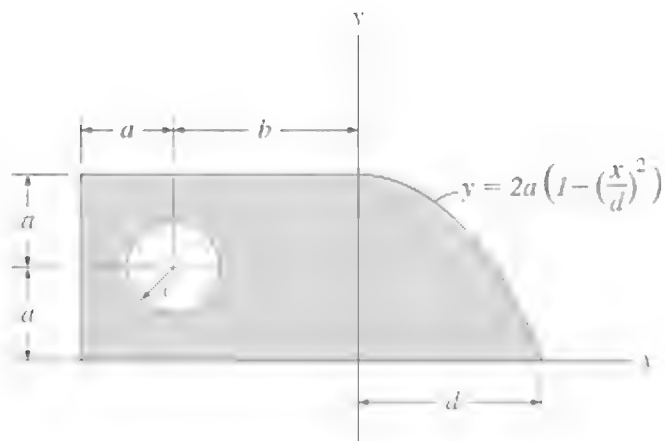
$c = 200$ mm

$\theta = 45$ deg

Solution:

$$I_{x'} = \frac{1}{12}\left[2a\left[2(c\sin(\theta) + b)\right]^3\right] \; ...$$
$$+ 4\left[\frac{1}{12}(c\cos(\theta))(c\sin(\theta))^3\right] - 2\left[\frac{1}{4}c^4\left(\theta - \frac{1}{2}\sin(2\theta)\right)\right]$$

$$I_{x'} = 520 \times 10^6 \text{ mm}^4$$

Problem 10-51

Determine the moment of inertia of the composite area about the x axis.

Given:

$a = 2$ in

$b = 4$ in

$c = 1$ in

$d = 4$ in

Solution:

$$I_x = \frac{1}{3}(a+b)(2a)^3 - \left(\frac{\pi c^4}{4} + \pi c^2 a^2\right) + \int_0^d \frac{1}{3}\left[2a\left[1 - \left(\frac{x}{d}\right)^2\right]\right]^3 \, dx$$

$$I_x = 153.7 \text{ in}^4$$

Problem 10-52

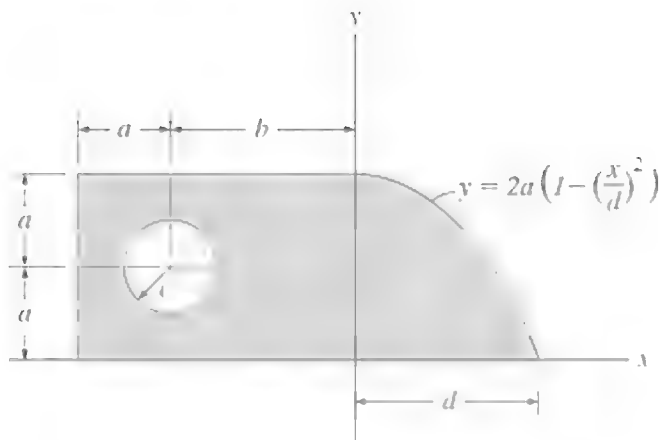
Determine the moment of inertia of the composite area about the y axis.

Given:

$a = 2$ in

$b = 4$ in

$c = 1$ in

$d = 4$ in

Solution:

$$I_y = \frac{1}{3}(2a)(a+b)^3 - \left(\frac{\pi c^4}{4} + \pi c^2 b^2\right) + \int_0^d x^2 2a\left[1 - \left(\frac{x}{d}\right)^2\right] dx$$

$I_y = 271.1 \text{ in}^4$

Problem 10-53

Determine the radius of gyration k_x for the column's cross-sectional area.

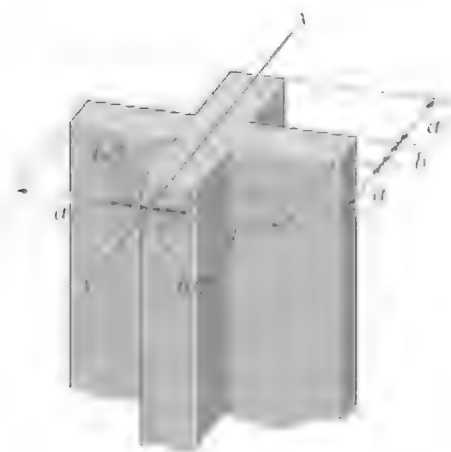
Given:

$a = 200$ mm

$b = 100$ mm

Solution:

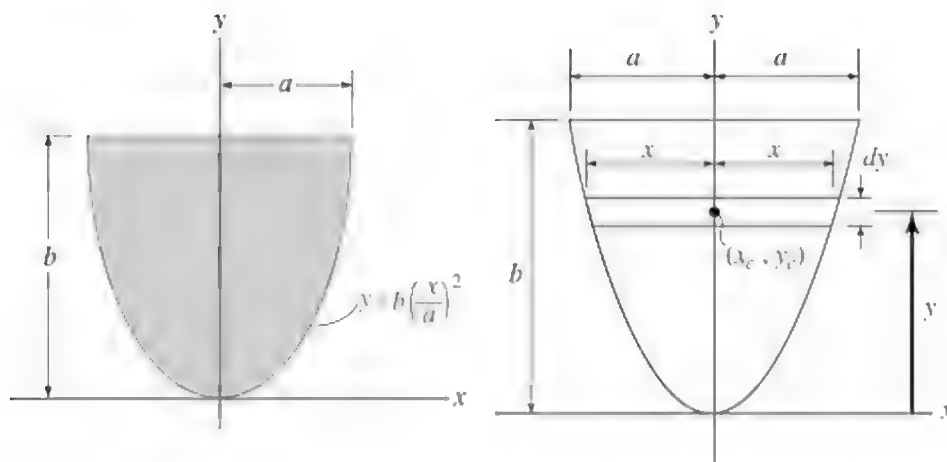
$$I_x = \frac{1}{12}(2a+b)b^3 + 2\left[\frac{1}{12}ba^3 + ba\left(\frac{a}{2} + \frac{b}{2}\right)^2\right]$$

$$k_x = \sqrt{\frac{I_x}{b(2a+b) + 2ab}} \qquad k_x = 109 \text{ mm}$$

Problem 10-54

Determine the product of inertia for the shaded portion of the parabola with respect to the x and y axes.

Given:

$a = 2$ in

$b = 1$ in

$$I_{xy} = \int_{-a}^{a} \int_{b\left(\frac{x}{a}\right)^2}^{b} x\,y\,dy\,dx \qquad I_{xy} = 0.00\ \mathrm{m}^4$$

Also because the area is symmetric about the y axis, the product of inertia must be zero.

Problem 10-55

Determine the product of inertia for the shaded area with respect to the x and y axes.

Solution:

$$I_{xy} = \int_{0}^{b} \int_{0}^{h\left(\frac{x}{b}\right)^{\frac{1}{3}}} x\,y\,dy\,dx = \frac{3}{16}\,b^2\,h^2$$

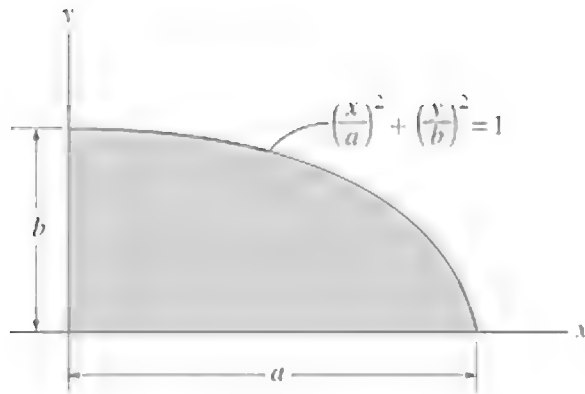
$$I_{xy} = \frac{3}{16} b^2 h^2$$

Problem 10-56

Determine the product of inertia of the shaded area of the ellipse with respect to the x and y axes.

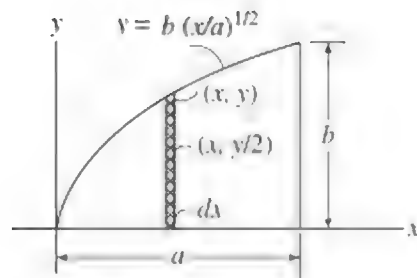
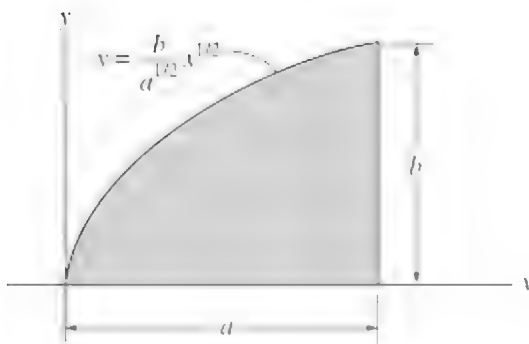
Given:

$a = 4\ \text{in}$

$b = 2\ \text{in}$

Solution:

$$I_{xy} = \int_0^a x\left[\frac{b\sqrt{1-\left(\frac{x}{a}\right)^2}}{2}\right] b\sqrt{1-\left(\frac{x}{a}\right)^2}\,dx \qquad I_{xy} = 8.00\ \text{in}^4$$

Problem 10-57

Determine the product of inertia of the parabolic area with respect to the x and y axes.

Solution:

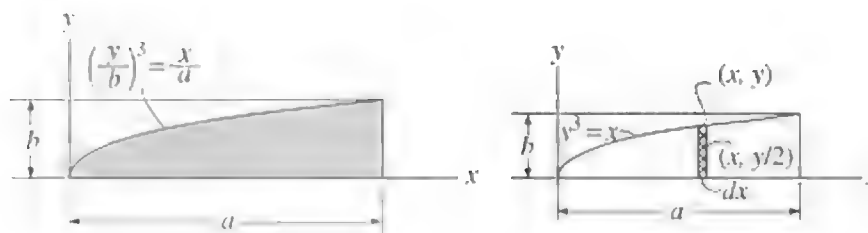
$$I_{xy} = \int_0^a x\left(\frac{b\sqrt{\frac{x}{a}}}{2}\right)b\sqrt{\frac{x}{a}}\,dx = \frac{1}{6}a^3\frac{b^2}{a} \qquad I_{xy} = \frac{1}{6}a^2b^2$$

Problem 10-58

Determine the product of inertia for the shaded area with respect to the x and y axes.

Given:

$a = 8$ in

$b = 2$ in

Solution:

$$I_{xy} = \int_0^a x\frac{b\left(\frac{x}{a}\right)^{\frac{1}{3}}}{2}b\left(\frac{x}{a}\right)^{\frac{1}{3}}\,dx \qquad I_{xy} = 48.00\text{ in}^4$$

Problem 10-59

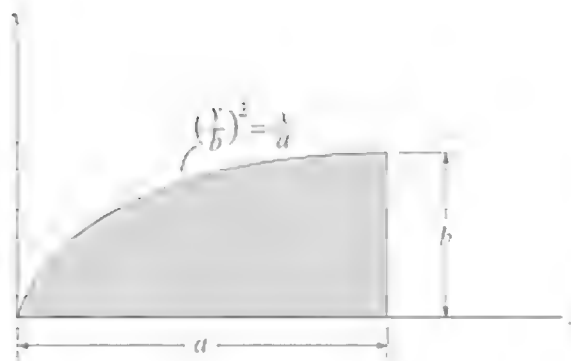
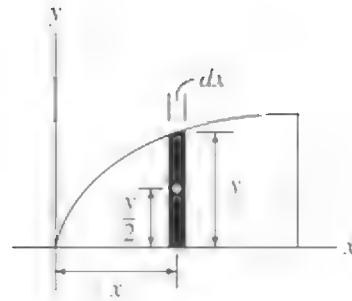
Determine the product of inertia for the shaded parabolic area with respect to the x and y axes.

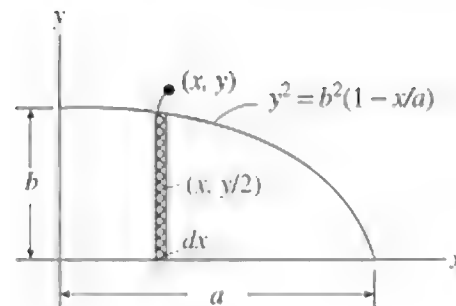
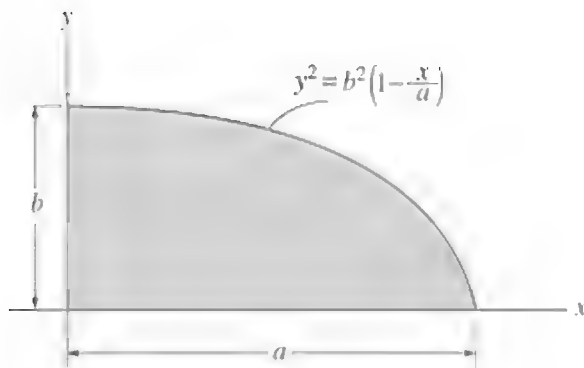
Given:

$a = 4$ in

$b = 2$ in

Solution:

$$I_{xy} = \int_0^a x\frac{b}{2}\sqrt{\frac{x}{a}}b\sqrt{\frac{x}{a}}\,dx$$

$I_{xy} = 10.67\ \text{in}^4$

Problem 10-60

Determine the product of inertia for the shaded area with respect to the x and y axes.

Given:

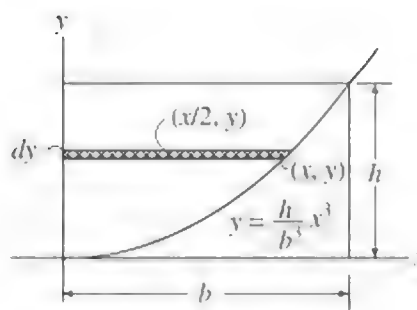
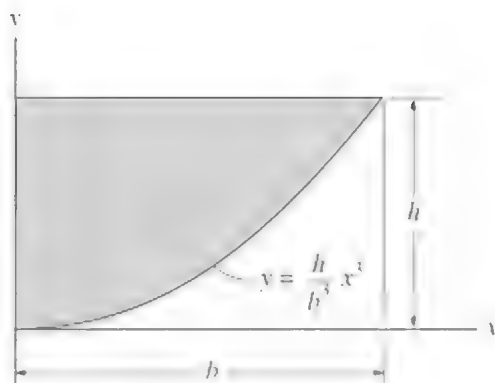
$a = 2\ \text{m}$

$b = 1\ \text{m}$

Solution:

$$I_{xy} = \int_0^a x\left(\frac{b}{2}\sqrt{1-\frac{x}{a}}\right)b\sqrt{1-\frac{x}{a}}\ dx \qquad I_{xy} = 0.333\ \text{m}^4$$

Problem 10-61

Determine the product of inertia for the shaded area with respect to the x and y axes.

Solution:

$$I_{xy} = \int_0^h y\frac{1}{2}\left[b\left(\frac{y}{h}\right)^{\frac{1}{3}}\right]^2 dy = \frac{3}{16}\, b^2\, h^2 \qquad I_{xy} = \frac{3}{16} h^2 b^2$$

Problem 10-62

Determine the product of inertia of the shaded area with respect to the x and y axes.

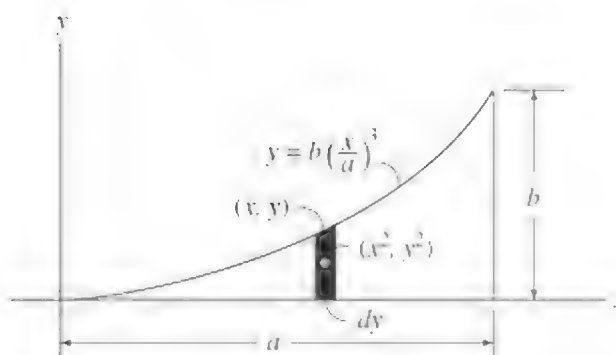
Given:

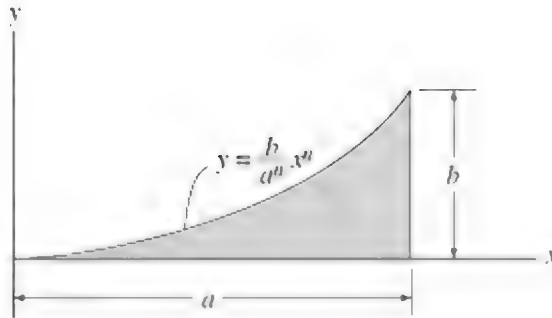
$a = 4$ in

$b = 2$ in

Solution:

$$I_{xy} = \int_0^a x\left(\frac{b}{2}\right)\left(\frac{x}{a}\right)^3 b\left(\frac{x}{a}\right)^3 dx$$

$$I_{xy} = 4.00\,\text{in}^4$$

Problem 10-63

Determine the product of inertia for the shaded area with respect to the x and y axes.

Solution:

$$I_{xy} = \int_0^a x\left(\frac{b}{2}\frac{x^n}{a^n}\right)b\frac{x^n}{a^n}\,dx \qquad I_x = \frac{a^2 b^2}{4(n+1)} \qquad \text{provided } n \neq -1$$

Problem 10-64

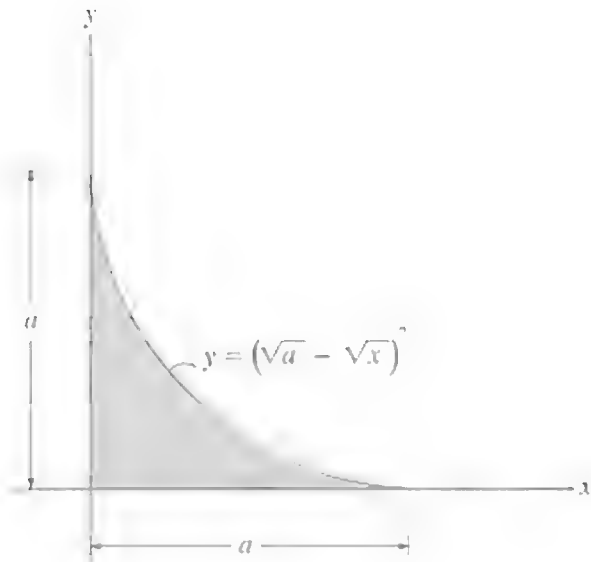
Determine the product of inertia for the shaded area with respect to the x and y axes.

Given:

$a = 4$ ft

Solution:

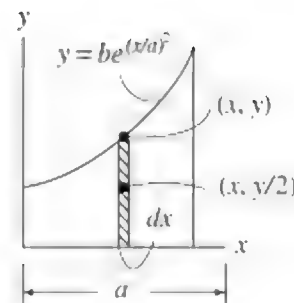
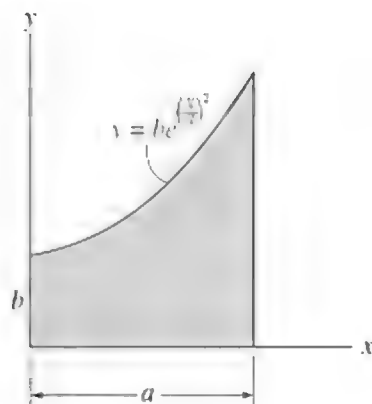
$$I_{xy} = \int_0^a x\frac{\left(\sqrt{a}-\sqrt{x}\right)^2}{2}\left(\sqrt{a}-\sqrt{x}\right)^2\,dx$$

$I_{xy} = 0.91\ \text{ft}^4$

Problem 10-65

Determine the product of inertia for the shaded area with respect to the x and y axes. Use Simpson's rule to evaluate the integral.

Given:

$a = 1\ \text{m}$

$b = 0.8\ \text{m}$

Solution:

$$I_{xy} = \int_0^a x\left(\frac{b}{2}\right)e^{\left(\frac{x}{a}\right)^2} b\,e^{\left(\frac{x}{a}\right)^2}\,dx \qquad I_{xy} =$$

Problem 10-66

Determine the product of inertia for the parabolic area with respect to the x and y axes.

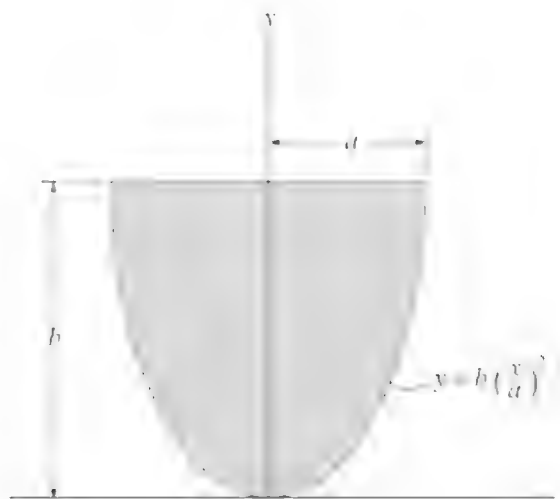
Given:

$a = 1\ \text{in}$

$b = 2\ \text{in}$

Solution:

Due to symmetry about y axis

$I_{xy} = 0$

$$I_{xy} = \int_{-a}^{a} x\,\frac{b + b\frac{x^2}{a^2}}{2}\left(b - b\frac{x^2}{a^2}\right)dx \qquad I_{xy} = 0.00\ \text{m}^4$$

Problem 10-67

Determine the product of inertia for the cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.

Given:

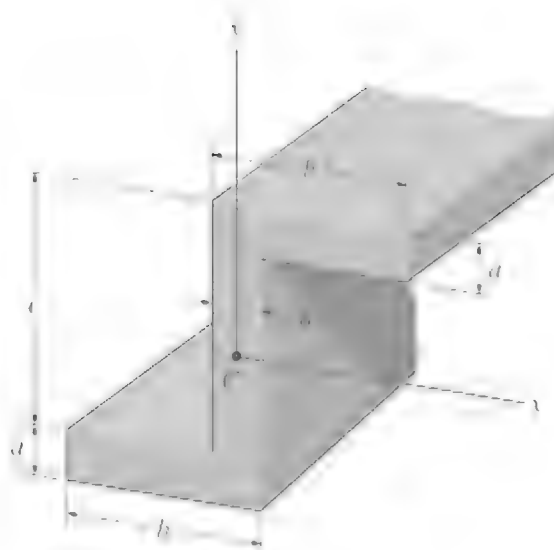
$$a = 20\ \text{mm}$$

$$b = 80\ \text{mm}$$

$$c = 100\ \text{mm}$$

Solution:

$$I_{xy} = 2ba\frac{c}{2}\left(\frac{b}{2} - \frac{a}{2}\right)$$

$$I_{xy} = 4800000.00\ \text{mm}^4$$

Problem 10-68

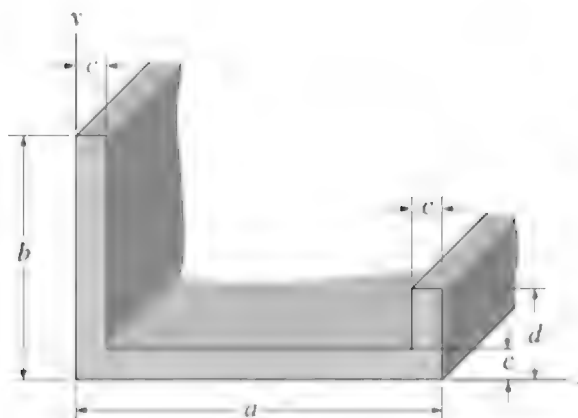
Determine the product of inertia for the beam's cross-sectional area with respect to the x and y axes.

Given:

$$a = 12\ \text{in}$$

$$b = 8\ \text{in}$$

$$c = 1\ \text{in}$$

$$d = 3\ \text{in}$$

Solution:

$$I_{xy} = \left(\frac{c}{2}\right)\left(\frac{b}{2}\right)cb + \left(\frac{a}{2}\right)\left(\frac{c}{2}\right)(a - 2c)c + dc\left(a - \frac{c}{2}\right)\left(\frac{d}{2}\right) \qquad I_{xy} = 97.75\ \text{in}^4$$

Problem 10-69

Determine the location (x_c, y_c) of the centroid C of the angle's cross-sectional area, and then

compute the product of inertia with respect to the x' and y' axes.

Given:

$a = 18$ mm

$b = 150$ mm

Solution:

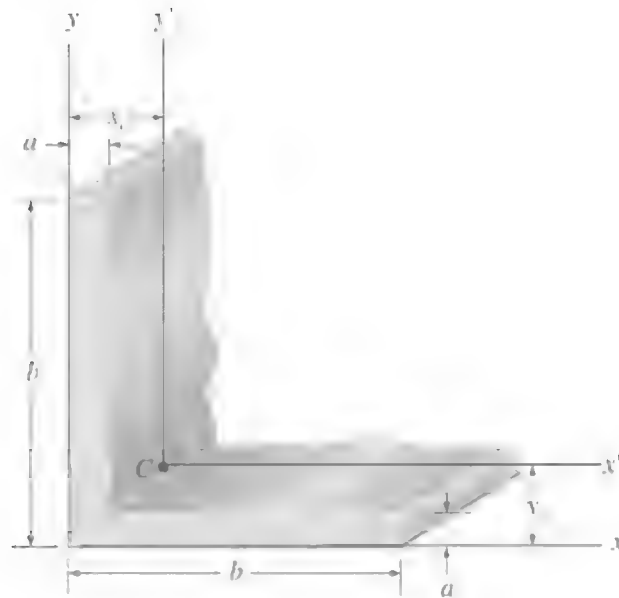
$$x_c = \frac{\left(\frac{a}{2}\right)a\,b + a(b-a)\left(\frac{a+b}{2}\right)}{a\,b + a(b-a)}$$

$x_c = 44.1$ mm

$$y_c = \frac{\left(\frac{b}{2}\right)a\,b + \left(\frac{a}{2}\right)a(b-a)}{a\,b + a(b-a)}$$

$y_c = 44.1$ mm

$$I_{x'y'} = a\,b\cdot -\left(x_c - \frac{a}{2}\right)\left(\frac{b}{2} - y_c\right) + a(b-a)\cdot -\left(y_c - \frac{a}{2}\right)\left(\frac{b}{2} + \frac{a}{2} - x_c\right)$$

$I_{x'y'} = -6.26 \times 10^{6}\ \text{mm}^{4}$

Problem 10-70

Determine the product of inertia of the beam's cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.

Given:

$a = 5\ \text{mm}$

$b = 30\ \text{mm}$

$c = 50\ \text{mm}$

Solution:

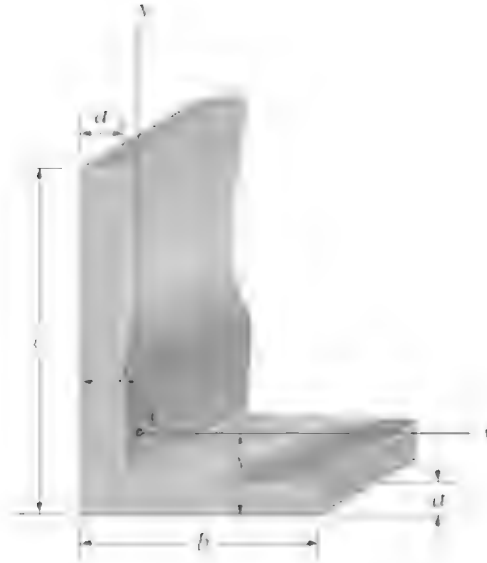
$$x_c = \frac{a(b-a)\left(\frac{a+b}{2}\right) + c\,a\left(\frac{a}{2}\right)}{a(b-a) + a\,c}$$

$x_c = 7.50\ \text{mm}$

$$y_c = \frac{a(b-a)\left(\frac{a}{2}\right) + c\,a\left(\frac{c}{2}\right)}{a(b-a) + c\,a}$$

$y_c = 17.50\ \text{mm}$

$$I_{xy} = (b-a)a\left(\frac{a}{2} - y_c\right)\left(\frac{a+b}{2} - x_c\right) + a\,c\left(\frac{a}{2} - x_c\right)\left(\frac{c}{2} - y_c\right)$$

$I_{xy} = -28.1 \times 10^3\ \text{mm}^4$

Problem 10-71

Determine the product of inertia for the shaded area with respect to the x and y axes.

Given:

$a = 2$ in

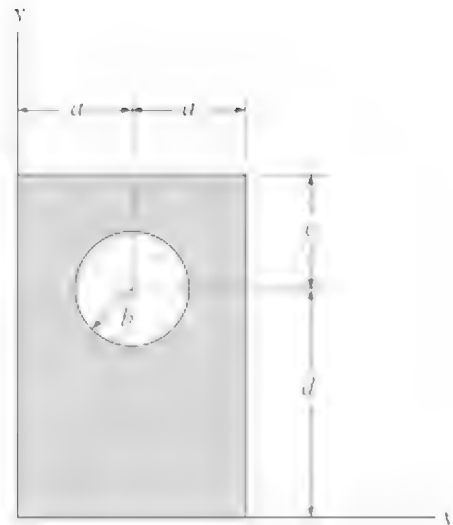
$b = 1$ in

$c = 2$ in

$d = 4$ in

Solution:

$$I_{xy} = 2a(c+d)a\left(\frac{c+d}{2}\right) - \pi b^2 a\, d$$

$$I_{xy} = 119\,\text{in}^4$$

Problem 10-72

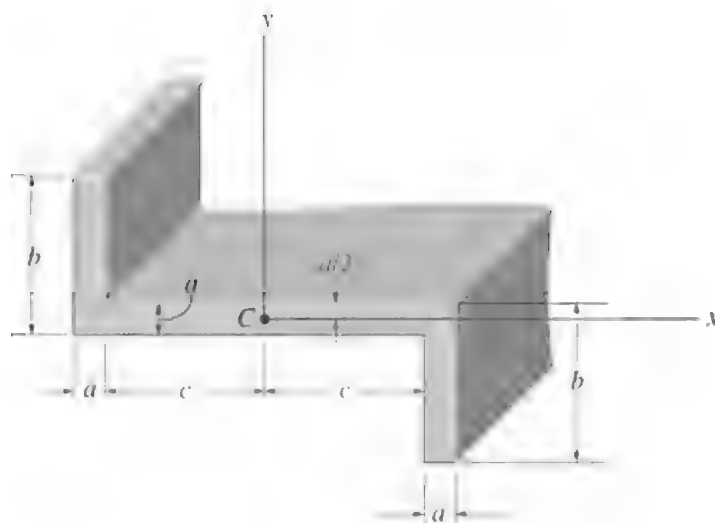
Determine the product of inertia for the beam's cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.

Given:

$a = 1$ in $\quad b = 5$ in $\quad c = 5$ in

Solution:

$$I_{xy} = 2b\,a\left(\frac{a}{2} - \frac{b}{2}\right)\left(c + \frac{a}{2}\right)$$

$$I_{xy} = -110\ \text{in}^4$$

Problem 10-73

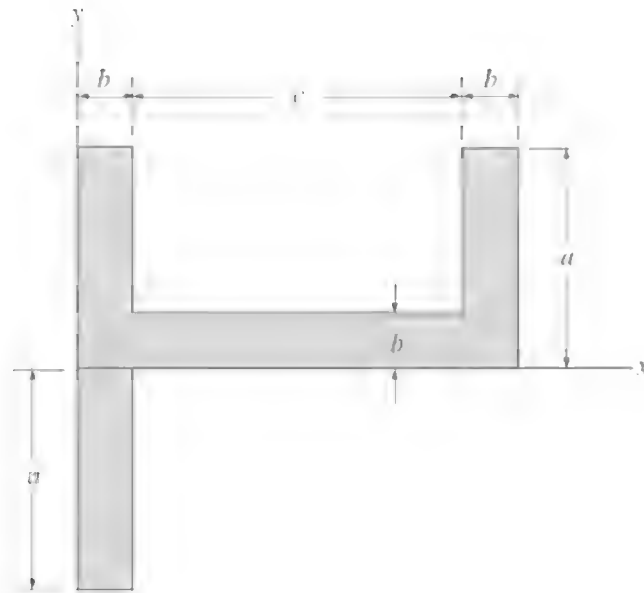
Determine the product of inertia for the cross-sec-tional area with respect to the x and y axes.

Given:

$a = 4 \text{ in}$

$b = 1 \text{ in}$

$c = 6 \text{ in}$

Solution:

$$I_{xy} = b\,a\left(\frac{a}{2}\right)\left(c + \frac{3b}{2}\right) + c\,b\left(b + \frac{c}{2}\right)\left(\frac{b}{2}\right) \qquad I_{xy} = 72 \text{ in}^4$$

Problem 10-74

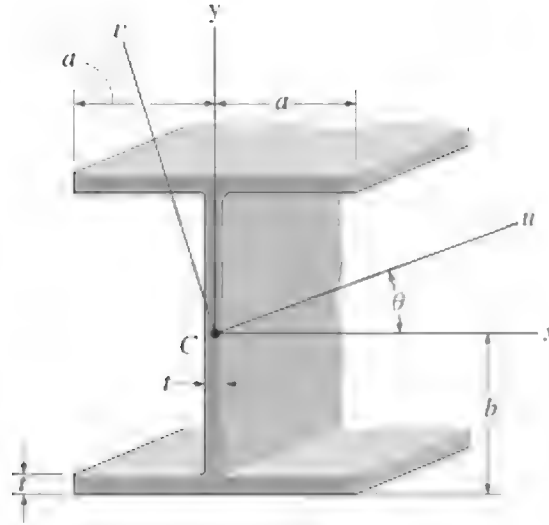
Determine the product of inertia for the beam's cross-sectional area with respect to the u and v axes.

Given:

$a = 150\ \text{mm}$

$b = 200\ \text{mm}$

$t = 20\ \text{mm}$

$\theta = 20\ \text{deg}$

Solution:

Moments of inertia I_x and I_y:

$$I_x = \frac{1}{12}2a(2b)^3 - \frac{1}{12}(2a - t)(2b - 2t)^3 \qquad I_x = 511.36 \times 10^6\ \text{mm}^4$$

$$I_y = \frac{2}{12}t(2a)^3 + \frac{2}{12}(b - t)t^3 \qquad I_y = 90240000.00\ \text{mm}^4$$

The section is symmetric about both x and y axes; therefore $I_{xy} = 0$. $\qquad I_{xy} = 0\text{mm}^4$

$$I_{uv} = \left(\frac{I_x - I_y}{2}\right)\sin(2\theta) + I_{xy}\cos(2\theta) \qquad I_{uv} = 135 \times 10^6\ \text{mm}^4$$

Problem 10-75

Determine the moments of inertia I_u and I_v and the product of inertia I_{uv} for the rectangular area. The u and v axes pass through the centroid C.

Given:

$a = 40\ \text{mm}$

$b = 160\ \text{mm}$

$\theta = 30\ \text{deg}$

Solution:

$$I_x = \frac{1}{12}a b^3 \qquad I_y = \frac{1}{12}b a^3 \qquad I_{xy} = 0 \text{ mm}^4$$

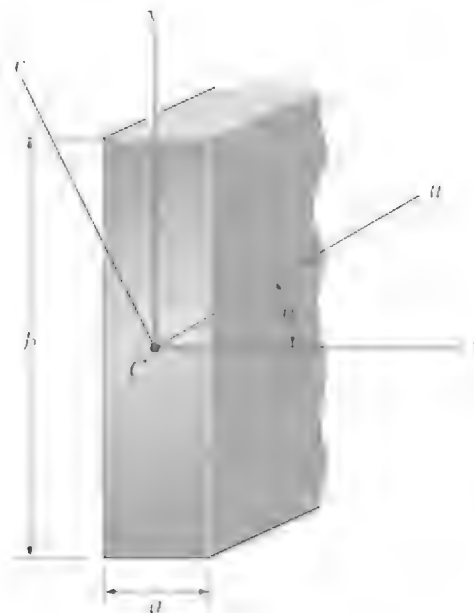
$$I_u = \frac{I_x + I_y}{2} + \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin(2\theta)$$

$$I_u = 10.5 \times 10^6 \text{ mm}^4$$

$$I_v = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin(2\theta)$$

$$I_v = 4.05 \times 10^6 \text{ mm}^4$$

$$I_{uv} = \left(\frac{I_x - I_y}{2}\right)\sin(2\theta) + I_{xy}\cos(2\theta)$$

$$I_{uv} = 5.54 \times 10^6 \text{ mm}^4$$

Problem 10-76

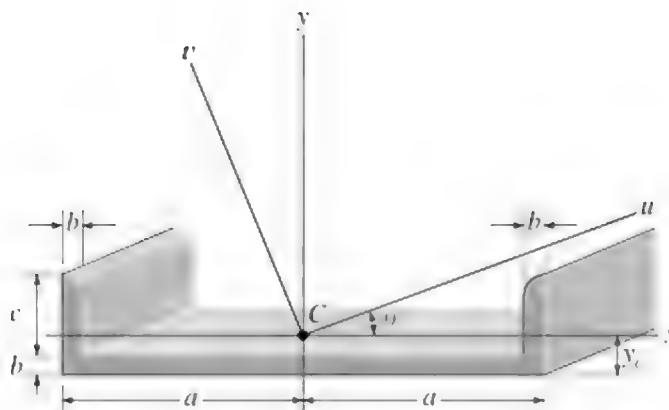
Determine the distance y_c to the centroid of the area and then calculate the moments of inertia I_u and I_v for the channel`s cross-sectional area. The u and v axes have their origin at the centroid C. For the calculation, assume all corners to be square.

Given:

$a = 150$ mm

$b = 10$ mm

$c = 50$ mm

$\theta = 20$ deg

Solution:

$$y_c = \frac{2ab\frac{b}{2} + 2cb\left(b + \frac{c}{2}\right)}{2ab + 2cb}$$ $y_c = 12.50\text{ mm}$

$$I_x = \frac{1}{12}2ab^3 + 2ab\left(y_c - \frac{b}{2}\right)^2 + 2\left[\frac{1}{12}bc^3 + bc\left(b + \frac{c}{2} - y_c\right)^2\right]$$

$I_x = 908.3 \times 10^3\text{ mm}^4$

$$I_y = \frac{1}{12}b(2a)^3 + 2\left[\frac{1}{12}cb^3 + cb\left(a - \frac{b}{2}\right)^2\right]$$ $I_y = 43.53 \times 10^6\text{ mm}^4$

(By symmetry) $I_{xy} = 0\text{ mm}^4$

$$I_u = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin(2\theta)$$ $I_u = 5.89 \times 10^6\text{ mm}^4$

$$I_v = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) + I_{xy}\sin(2\theta)$$ $I_v = 38.5 \times 10^6\text{ mm}^4$

Problem 10-77

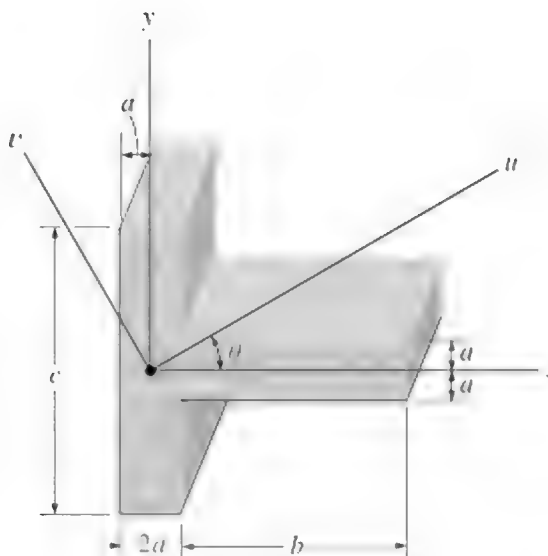
Determine the moments of inertia for the shaded area with respect to the u and v axes.

Given:

$a = 0.5\text{ in}$

$b = 4\text{ in}$

$c = 5\text{ in}$

$\theta = 30\text{ deg}$

Solution:

Moment and Product of Inertia about x and y Axes: Since the shaded area is symmetrical about the x axis,

$I_{xy} = 0\ \text{in}^4$

$$I_x = \frac{1}{12}2ac^3 + \frac{1}{12}b(2a)^3 \qquad I_x = 10.75\ \text{in}^4$$

$$I_y = \frac{1}{12}2ab^3 + 2ab\left(a + \frac{b}{2}\right)^2 + \frac{1}{12}c(2a)^3 \qquad I_y = 30.75\ \text{in}^4$$

Moment of Inertia about the Inclined u and v Axes

$$I_u = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin(2\theta) \qquad I_u = 15.75\ \text{in}^4$$

$$I_v = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) + I_{xy}\sin(2\theta) \qquad I_v = 25.75\ \text{in}^4$$

Problem 10-78

Determine the directions of the principal axes with origin located at point O, and the principal moments of inertia for the rectangular area about these axes.

Given:

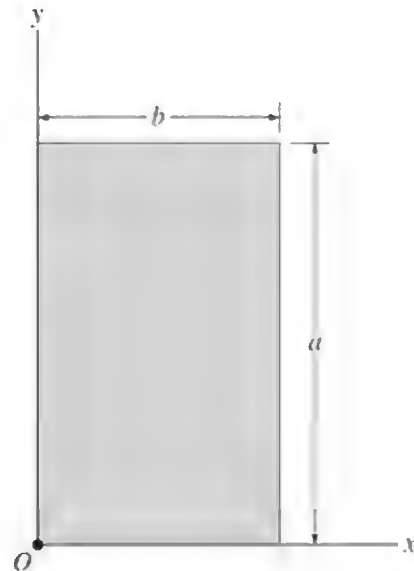
$a = 6\ \text{in}$

$b = 3\ \text{in}$

Solution:

$$I_x = \frac{1}{3}ba^3 \qquad I_x = 216\ \text{in}^4$$

$$I_y = \frac{1}{3}ab^3 \qquad I_y = 54\ \text{in}^4$$

$$I_{xy} = \frac{a}{2}\frac{b}{2}ab \qquad I_{xy} = 81\ \text{in}^4$$

$$\tan(2\theta) = \frac{-2I_{xy}}{I_x - I_y} \qquad \theta = \frac{1}{2}\text{atan}\left(2\frac{I_{xy}}{-I_x + I_y}\right) \qquad \theta = -22.5 \text{ deg}$$

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{max} = 250 \text{ in}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{min} = 20.4 \text{ in}^4$$

Problem 10-79

Determine the moments of inertia I_u, I_v and the product of inertia I_{uv} for the beam's cross-sectional area.

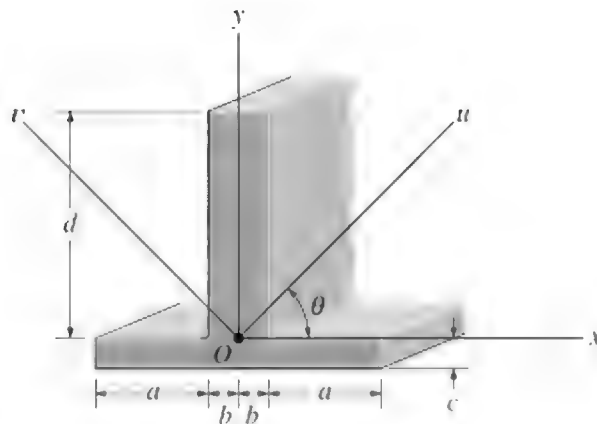
Given:

$\theta = 45$ deg

$a = 8$ in

$b = 2$ in

$c = 2$ in

$d = 16$ in

Solution:

$$I_x = \frac{2}{3}(a+b)c^3 + \frac{1}{12}2bd^3 + 2bd\left(\frac{d}{2}\right)^2 \qquad I_x = 5.515 \times 10^3 \text{ in}^4$$

$$I_y = \frac{1}{12}[2(a+b)]^3 c + \frac{1}{12}(2b)^3 d \qquad I_y = 1.419 \times 10^3 \text{ in}^4$$

$$I_{xy} = 0 \text{ in}^4$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos(2\theta) - I_{xy}\sin(2\theta) \qquad I_u = 3.47 \times 10^3 \text{ in}^4$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos(2\theta) + I_{xy}\sin(2\theta) \qquad I_v = 3.47 \times 10^3 \text{ in}^4$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin(2\theta) + I_{xy}\cos(2\theta) \qquad I_{uv} = 2.05 \times 10^3 \text{ in}^4$$

Problem 10-80

Determine the directions of the principal axes with origin located at point O, and the principal moments of inertia for the area about these axes.

y d c b r a O x

Given:

$a = 4$ in

$b = 2$ in

$c = 2$ in

$d = 2$ in

$r = 1$ in

Solution:

$$I_x = \frac{1}{3}(c+d)(a+b)^3 - \left(\frac{\pi r^4}{4} + \pi r^2 a^2\right) \qquad I_x = 236.95 \text{ in}^4$$

$$I_y = \frac{1}{3}(a+b)(c+d)^3 - \left(\frac{\pi r^4}{4} + \pi r^2 d^2\right) \qquad I_y = 114.65 \text{ in}^4$$

$$I_{xy} = \left(\frac{a+b}{2}\right)\left(\frac{d+c}{2}\right)(a+b)(d+c) - d\,a\pi r^2 \qquad I_{xy} = 118.87 \text{ in}^4$$

$$\tan(2\,\theta_p) = \frac{-I_{xy}}{\frac{I_x - I_y}{2}} \qquad \theta_p = \frac{1}{2}\operatorname{atan}\left(2\,\frac{I_{xy}}{-I_x + I_y}\right) \qquad \theta_p = -31.39 \text{ deg}$$

$$\theta_{p1} = \theta_p \qquad \theta_{p1} = -31.39 \text{ deg}$$

$$\theta_{p2} = 90 \text{ deg} + \theta_{p1} \qquad \theta_{p2} = 58.61 \text{ deg}$$

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{max} = 309 \text{ in}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{min} = 42.1\ \text{in}^4$$

Problem 10-81

Determine the principal moments of inertia for the beam's cross-sectional area about the principal axes that have their origin located at the centroid C. Use the equations developed in Section 10.7. For the calculation, assume all corners to be square.

Given: $a = 4$ in $\quad b = 4$ in $\quad t = \frac{3}{8}$ in

Solution:

$$I_x = 2\left[\frac{1}{12}at^3 + at\left(b - \frac{t}{2}\right)^2\right] + \frac{1}{12}t(2b - 2t)^3$$

$$I_x = 55.55\ \text{in}^4$$

$$I_y = 2\left[\frac{1}{12}t(a-t)^3 + t(a-t)\left(\frac{a-t}{2} + \frac{t}{2}\right)^2\right] + \frac{1}{12}2bt^3 \qquad I_y = 13.89\ \text{in}^4$$

$$I_{xy} = -2\left[\frac{a-t}{2} + \left(\frac{t}{2}\right)\right]\left(b - \frac{t}{2}\right)t(a-t) \qquad I_{xy} = -20.73\ \text{in}^4$$

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{max} = 64.1\ \text{in}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{min} = 5.33\ \text{in}^4$$

Problem 10-82

Determine the principal moments of inertia for the angle's cross-sectional area with respect to a set of principal axes that have their origin located at the centroid *C*. Use the equation developed in Section 10.7. For the calculation, assume all corners to be square.

Given:

$a = 100 \text{ mm}$

$b = 100 \text{ mm}$

$t = 20 \text{ mm}$

Solution:

$$x_c = \frac{t b \frac{t}{2} + (a - t) t \left(t + \frac{a - t}{2}\right)}{t b + (a - t) t} \qquad x_c = 32.22 \text{ mm}$$

$$y_c = \frac{t b \frac{b}{2} + (a - t) t \frac{t}{2}}{t b + (a - t) t} \qquad y_c = 32.22 \text{ mm}$$

$$I_x = \frac{1}{12} t^3 (a - t) + t(a - t)\left(x_c - \frac{t}{2}\right)^2 + \frac{1}{12} t b^3 + t b \left(\frac{b}{2} - x_c\right)^2 \qquad I_x = 3.142 \times 10^6 \text{ mm}^4$$

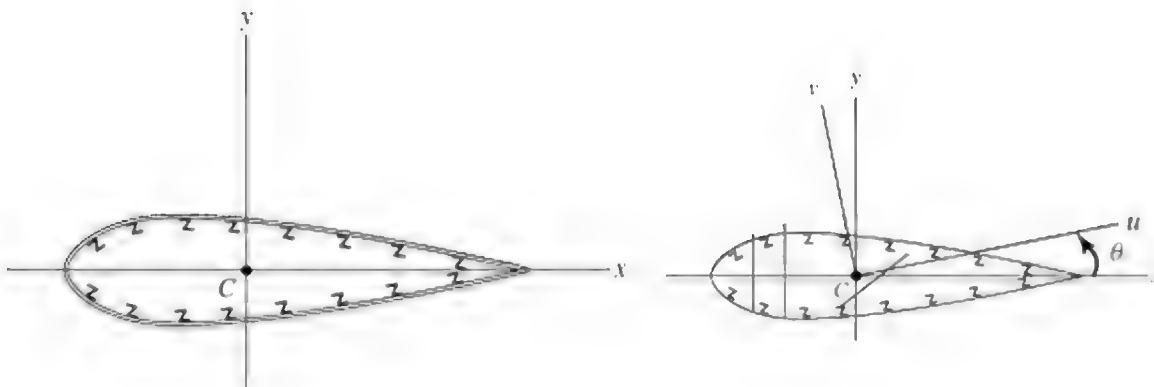
$$I_y = \frac{1}{12} b t^3 + b t \left(x_c - \frac{t}{2}\right)^2 + \frac{1}{12} t (a - t)^3 + t(a - t)\left(t + \frac{a - t}{2} - x_c\right)^2 \qquad I_y = 3.142 \times 10^6 \text{ mm}^4$$

$$I_{xy} = -\left(x_c - \frac{t}{2}\right)\left(\frac{b}{2} - y_c\right) b t - \left(\frac{a - t}{2} + t - x_c\right)\left(y_c - \frac{t}{2}\right)(a - t) t \qquad I_{xy} = -1.778 \times 10^6 \text{ mm}^4$$

$$I_{max} = \left(\frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\right) - I_{xy} \qquad I_{max} = 4.92 \times 10^6 \text{ mm}^4$$

$$I_{min} = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right) + I_{xy} \qquad I_{min} = 2.22 \times 10^6 \text{ mm}^4$$

Problem 10-83

The area of the cross section of an airplane wing has the listed properties about the x and y axes passing through the centroid C. Determine the orientation of the principal axes and the principal moments of inertia.

Given: $I_x = 450 \text{ in}^4$ $I_y = 1730 \text{ in}^4$ $I_{xy} = 138 \text{ in}^4$

Solution:

$$\tan(2\theta) = \frac{-2I_{xy}}{I_x - I_y} \qquad \theta = \frac{1}{2}\operatorname{atan}\left(2\frac{I_{xy}}{-I_x + I_y}\right) \qquad \theta = 6.08 \text{ deg}$$

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{max} = 1745 \text{ in}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \qquad I_{min} = 435 \text{ in}^4$$

Problem 10-84

Using Mohr's circle, determine the principal moments of inertia for the triangular area and the orientation of the principal axes of inertia having an origin at point O.

Given:

$a = 30 \text{ mm}$

$b = 40 \text{ mm}$

Solution:

Moment of inertia I_x and I_y:

$$I_x = \frac{1}{12} b a^3 \qquad I_x = 90 \times 10^3 \text{ mm}^4$$

$$I_y = \frac{1}{12} a b^3 \qquad I_y = 160 \times 10^3 \text{ mm}^4$$

Product of inertia I_{xy}:

$$I_{xy} = \int_0^b \frac{x}{2}\left(a - \frac{a}{b}x\right)^2 \text{d}x \qquad I_{xy} = 60 \times 10^3 \text{ mm}^4$$

Mohr's circle :

$$OA = \sqrt{\left(\frac{I_x + I_y}{2} - I_x\right)^2 + I_{xy}^2}$$

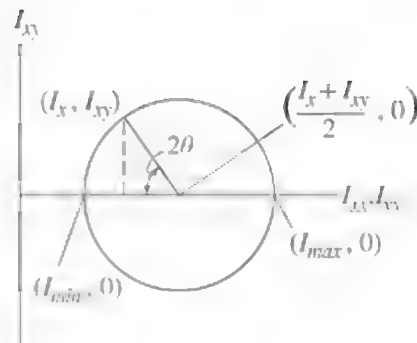
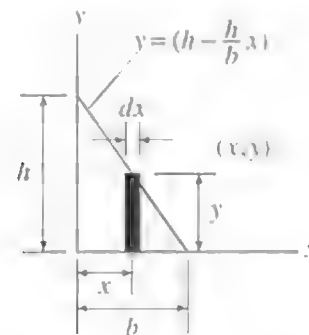
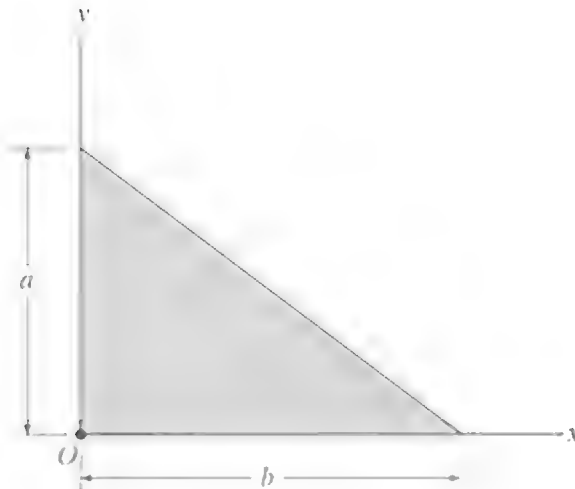
$$OA = 69.462 \times 10^3 \text{ mm}^4$$

$$I_{max} = \left(\frac{I_x + I_y}{2} + OA\right)$$

$$I_{max} = 194.462 \times 10^3 \text{ mm}^4$$

$$I_{min} = \left(\frac{I_x + I_y}{2} - OA\right)$$

$$I_{min} = 55.5 \times 10^3 \text{ mm}^4$$

$$\tan(2\theta) = \frac{I_{xy}}{\frac{I_x + I_y}{2} - I_x} \qquad \theta = \frac{1}{2}\text{atan}\left(2\frac{I_{xy}}{-I_x + I_y}\right) \qquad \theta = 29.9 \text{ deg}$$

Problem 10-85

Determine the directions of the principal axes with origin located at point O, and the principal moments of inertia for the rectangular area about these axes.
Solve using Mohr's circle.

Given:

$a = 6 \text{ in}$

$b = 3 \text{ in}$

Solution:

$I_x = \frac{1}{3} b a^3 \qquad I_x = 216 \text{ in}^4$

$I_y = \frac{1}{3} a b^3 \qquad I_y = 54 \text{ in}^4$

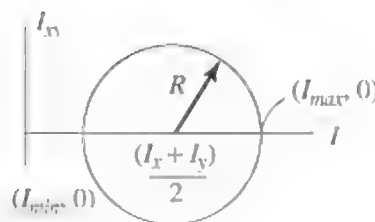
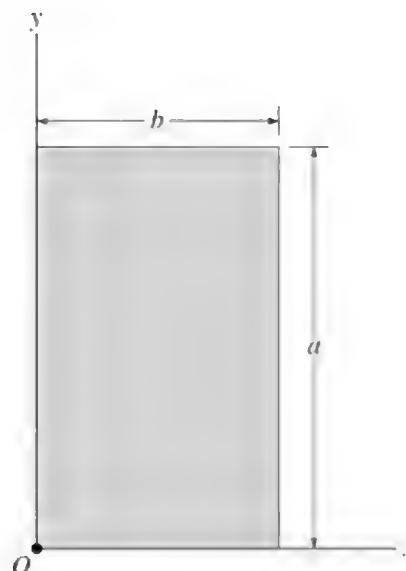
$I_{xy} = \frac{a}{2} \frac{b}{2} a b \qquad I_{xy} = 81 \text{ in}^4$

$R = \sqrt{\left[I_x - \left(\frac{I_x + I_y}{2}\right)\right]^2 + I_{xy}^2} \qquad R = 114.55 \text{ in}^4$

$I_{max} = \frac{I_x + I_y}{2} + R \qquad I_{max} = 250 \text{ in}^4$

$I_{min} = \frac{I_x + I_y}{2} - R \qquad I_{min} = 20.4 \text{ in}^4$

$\theta_{p1} = \frac{-1}{2} \operatorname{asin}\left(\frac{I_{xy}}{R}\right) \qquad \theta_{p1} = -22.50 \text{ deg}$

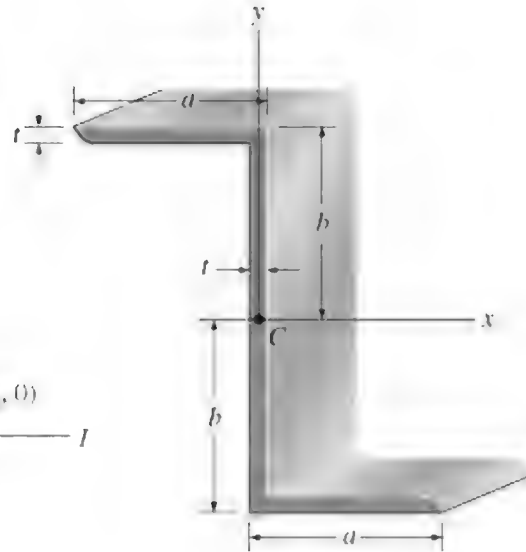
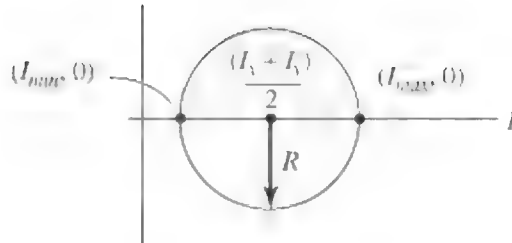
$\theta_{p2} = \theta_{p1} + 90 \text{ deg} \qquad \theta_{p2} = 67.50 \text{ deg}$

Problem 10-86

Determine the principal moments of inertia for the beam's cross-sectional area about the principal axes that have their origin located at the centroid *C*. For the calculation, assume all corners to be square. Solve using Mohr's circle.

Given:

$a = 4\ \text{in}$

$b = 4\ \text{in}$

$t = \frac{3}{8}\ \text{in}$

Solution:

$$I_x = 2\left[\frac{1}{12}a t^3 + a t\left(b - \frac{t}{2}\right)^2\right] + \frac{1}{12}t(2b - 2t)^3 \qquad I_x = 55.55\ \text{in}^4$$

$$I_y = 2\left[\frac{1}{12}t(a - t)^3 + t(a - t)\left(\frac{a - t}{2} + \frac{t}{2}\right)^2\right] + \frac{1}{12}2b t^3 \qquad I_y = 13.89\ \text{in}^4$$

$$I_{xy} = -2\left[\frac{a - t}{2} + \left(\frac{t}{2}\right)\right]\left(b - \frac{t}{2}\right)t(a - t) \qquad I_{xy} = -20.73\ \text{in}^4$$

$$R = \sqrt{\left(I_x - \frac{I_x + I_y}{2}\right)^2 + I_{xy}^2} \qquad R = 29.39\ \text{in}^4$$

$$I_{max} = \frac{I_x + I_y}{2} + R \qquad I_{max} = 64.1\ \text{in}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - R \qquad I_{min} = 20.45\ \text{in}^4$$

Problem 10-87

Determine the principal moments of inertia for the angle's cross-sectional area with respect to a set of principal axes that have their origin located at the centroid *C*. For the calculation, assume all corners to be square. Solve using Mohr's ciricle.

Given: $a = 100\text{ mm}$ $b = 100\text{ mm}$ $t = 20\text{ mm}$

Solution:

$$x_c = \frac{tb\left(\frac{t}{2}\right) + (a-t)t\left(t + \frac{a-t}{2}\right)}{tb + (a-t)t}$$ $x_c = 32.22\text{ mm}$

$$y_c = \frac{tb\left(\frac{b}{2}\right) + (a-t)t\left(\frac{t}{2}\right)}{tb + (a-t)t}$$ $y_c = 32.22\text{ mm}$

$$I_x = \frac{1}{12}t^3(a-t) + t(a-t)\left(x_c - \frac{t}{2}\right)^2 + \frac{1}{12}tb^3 + tb\left(\frac{b}{2} - x_c\right)^2$$ $I_x = 3.142 \times 10^6\text{ mm}^4$

$$I_y = \frac{1}{12}bt^3 + bt\left(x_c - \frac{t}{2}\right)^2 + \frac{1}{12}t(a-t)^3 + t(a-t)\left(t + \frac{a-t}{2} - x_c\right)^2$$

$I_y = 3.142 \times 10^6\text{ mm}^4$

$$I_{xy} = -\left(x_c - \frac{t}{2}\right)\left(\frac{b}{2} - y_c\right)bt - \left(\frac{a-t}{2} + t - x_c\right)\left(y_c - \frac{t}{2}\right)(a-t)t$$ $I_{xy} = -1.778 \times 10^6\text{ mm}^4$

$$R = \sqrt{\left(I_x - \frac{I_x + I_y}{2}\right)^2 + I_{xy}^2}$$ $R = 1.78 \times 10^6\text{ mm}^4$

$$I_{max} = \frac{I_x + I_y}{2} + R \qquad I_{max} = 4.92 \times 10^6 \text{ mm}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - R \qquad I_{min} = 1364444.44 \text{ mm}^4$$

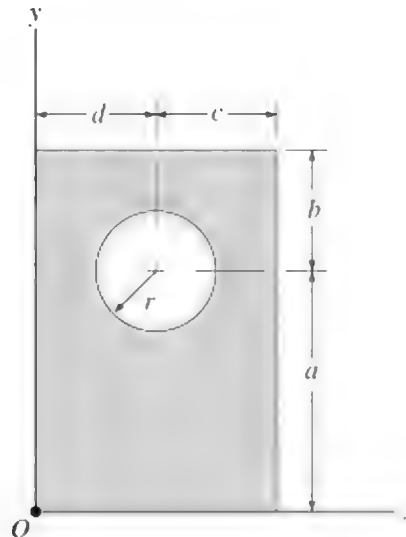
Problem 10-88

Determine the directions of the principal axes with origin located at point O, and the principal moments of inertia for the area about these axes. Solve using Mohr's circle

Given:

$a = 4$ in

$b = 2$ in

$c = 2$ in

$d = 2$ in

$r = 1$ in

Solution:

$$I_x = \frac{1}{3}(c + d)(a + b)^3 - \left(\frac{\pi r^4}{4} + \pi r^2 a^2\right) \qquad I_x = 236.95 \text{ in}^4$$

$$I_y = \frac{1}{3}(a + b)(c + d)^3 - \left(\frac{\pi r^4}{4} + \pi r^2 d^2\right) \qquad I_y = 114.65 \text{ in}^4$$

$$I_{xy} = \left(\frac{a + b}{2}\right)\left(\frac{d + c}{2}\right)(a + b)(d + c) - d a \pi r^2 \qquad I_{xy} = 118.87 \text{ in}^4$$

$$R = \sqrt{\left[I_x - \left(\frac{I_x + I_y}{2}\right)\right]^2 + I_{xy}^2} \qquad R = 133.67 \text{ in}^4$$

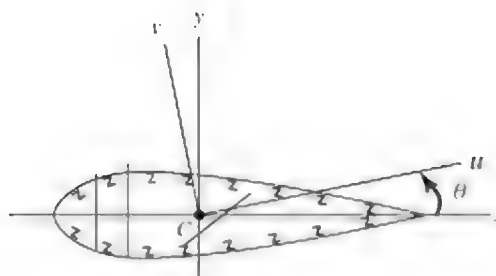
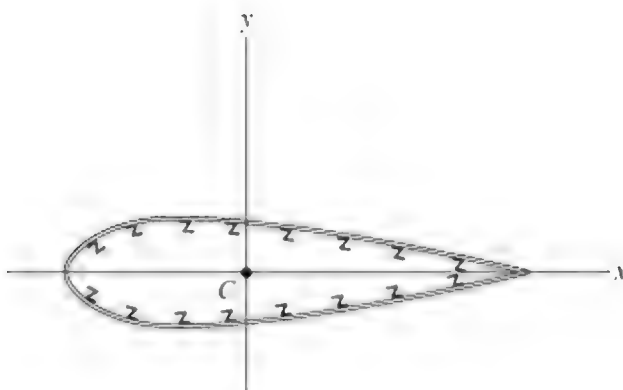
$$I_{max} = \frac{I_x + I_y}{2} + R \qquad I_{max} = 309 \text{ in}^4$$

$$I_{min} = \frac{I_x + I_y}{2} - R \qquad I_{min} = 42.1 \text{ in}^4$$

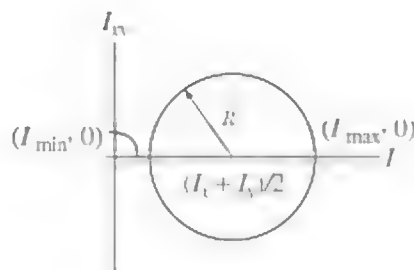
$$\theta_{p1} = \frac{-1}{2}\text{asin}\left(\frac{I_{xy}}{R}\right) \qquad \theta_{p1} = -31.39\text{ deg}$$

$$\theta_{p2} = \theta_{p1} + \frac{\pi}{2} \qquad \theta_{p2} = 58.61\text{ deg}$$

Problem 10-89

The area of the cross section of an airplane wing has the listed properties about the x and y axes passing through the centroid C. Determine the orientation of the principal axes and the principal moments of inertia. Solve using Mohr's circle.

Given: $I_x = 450\text{ in}^4$

$I_y = 1730\text{ in}^4$

$I_{xy} = 138\text{ in}^4$

Solution:

$$R = \sqrt{\left[I_x - \left(\frac{I_x + I_y}{2}\right)\right]^2 + I_{xy}^2} \qquad R = 654.71\text{ in}^4$$

$$I_{max} = \left(\frac{I_x + I_y}{2} + R\right) \qquad I_{max} = 1.74 \times 10^3\text{ in}^4$$

$$I_{min} = \left(\frac{I_x + I_y}{2} - R\right) \qquad I_{min} = 435\text{ in}^4$$

$$\theta_{p1} = \frac{1}{2}\operatorname{asin}\left(\frac{I_{xy}}{R}\right) \qquad \theta_{p1} = 6.08\,\text{deg}$$

$$\theta_{p2} = \theta_{p1} + 90\text{ deg} \qquad \theta_{p2} = 96.08\,\text{deg}$$

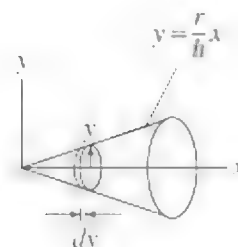
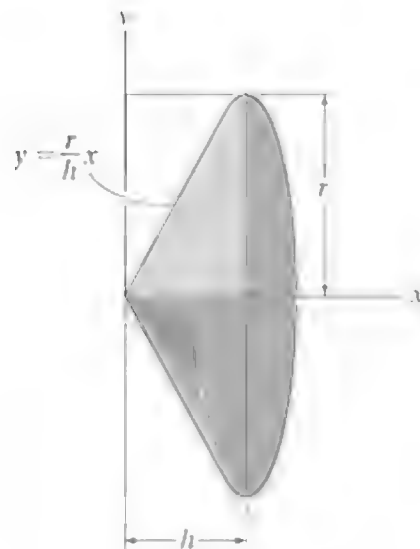
Problem 10-90

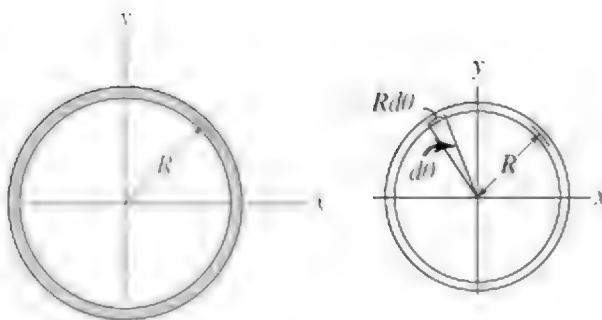
The right circular cone is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.

Solution:

$$m = \int_0^h \rho\,\pi\left(\frac{rx}{h}\right)^2 dx = \frac{1}{3}\,h\,\rho\,\pi\,r^2$$

$$I_x = \frac{3m}{\pi h r^2}\int_0^h \frac{1}{2}\pi\left(\frac{rx}{h}\right)^4 dx = \frac{3}{10}\,m\,r^2$$

$$I_x = \frac{3}{10}\,m\,r^2$$

Problem 10-91

Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.

Solution:

$$m = \rho 2\pi R \qquad \rho = \frac{m}{2\pi R}$$

$$I = \int_0^{2\pi} \left(\frac{m}{2\pi R}\right) R^2 R \, d\theta = m\, R^2 \qquad I = m\, R^2$$

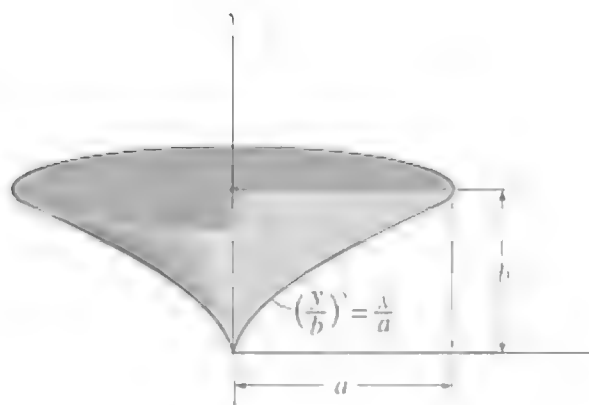
Problem 10-92

The solid is formed by revolving the shaded area around the y axis. Determine the radius of gyration k_y. The specific weight of the material is γ.

Given:

$a = 3 \text{ in}$

$b = 3 \text{ in}$

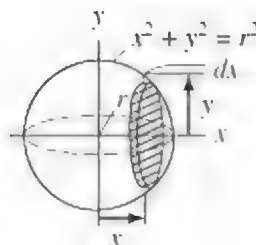
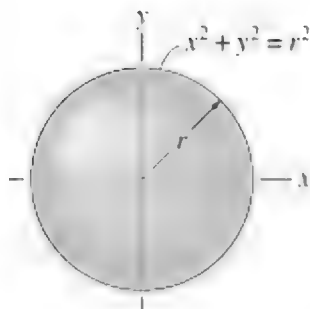
$\gamma = 380 \, \frac{\text{lb}}{\text{ft}^3}$

Solution:

$$m = \int_0^b \gamma \pi \left[a\left(\frac{y}{b}\right)^3\right]^2 dy \qquad m = 2.66 \text{ lb}$$

$$I_y = \int_0^b \gamma \pi \left[a\left(\frac{y}{b}\right)^3\right]^2 \frac{1}{2}\left[a\left(\frac{y}{b}\right)^3\right]^2 dy \qquad I_y = 6.46 \text{ lb}\cdot\text{in}^2$$

$$k_y = \sqrt{\frac{I_y}{m}} \qquad k_y = 1.56 \text{ in}$$

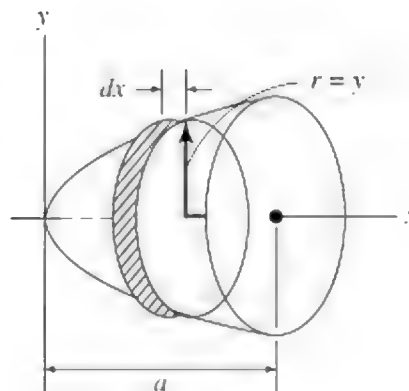
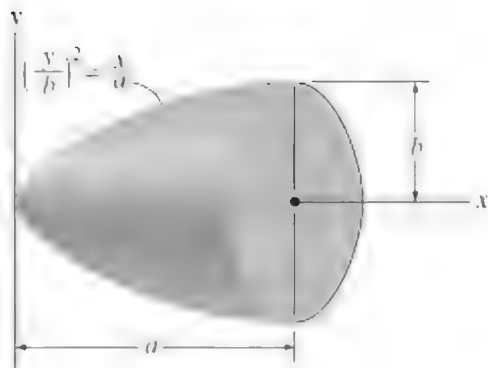
Problem 10-93

Determine the moment of inertia I_x for the sphere and express the result in terms of the total mass m of the sphere. The sphere has a constant density ρ.

Solution:

$$m = \rho \frac{4\pi r^3}{3} \qquad \rho = \frac{3m}{4\pi r^3}$$

$$I_x = \int_{-r}^{r} \frac{1}{2}\left(\frac{3m}{4\pi r^3}\right)\pi\left(r^2 - x^2\right)\left(r^2 - x^2\right)\,dx = \frac{2}{5}\,m\,r^2 \qquad I_x = \frac{2}{5}\,m\,r^2$$

Problem 10-94

Determine the radius of gyration k_x of the paraboloid. The density of the material is ρ.

Units Used: $\quad \text{Mg} = 1000\text{ kg}$

Given: $\quad \rho = 5\,\dfrac{\text{Mg}}{\text{m}^3} \qquad a = 200\text{ mm} \qquad b = 100\text{ mm}$

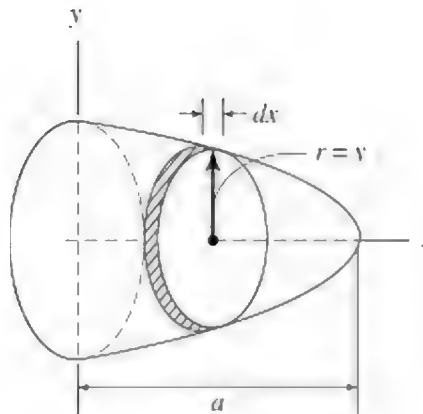
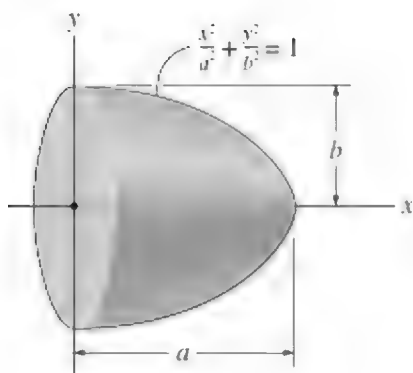
Solution:

$$m_p = \int_0^a \rho\,\pi\left(\frac{b^2 x}{a}\right) dx \qquad m_p = 15.71\ \text{kg}$$

$$I_x = \int_0^a \frac{1}{2}\rho\,\pi\left(\frac{b^2 x}{a}\right)\left(\frac{b^2 x}{a}\right) dx \qquad I_x = 52.36 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$k_x = \sqrt{\frac{I_x}{m_p}} \qquad k_x = 57.7\ \text{mm}$$

Problem 10-95

Determine the moment of inertia of the semi-ellipsoid with respect to the x axis and express the result in terms of the mass m of the semiellipsoid. The material has a constant density ρ.

Solution:

$$m = \int_0^a \rho\,\pi b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{2}{3}\,a\,\rho\,\pi\,b^2 \qquad \rho = \frac{3m}{2\pi a b^2}$$

$$I_x = \int_0^a \frac{1}{2}\left(\frac{3m}{2\pi a b^2}\right)\pi b^2\left(1 - \frac{x^2}{a^2}\right) b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{2}{5}\,m\,b^2 \qquad I_x = \frac{2}{5}\,m\,b^2$$

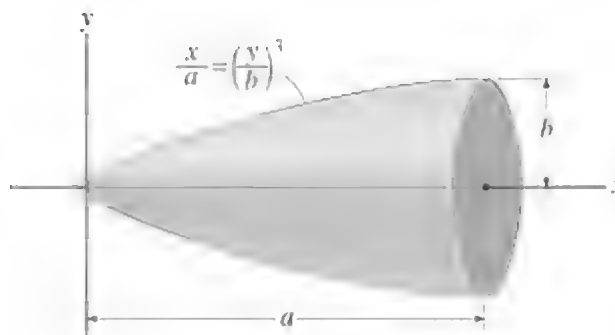
Problem 10-96

Determine the radius of gyration k_x of the body. The specific weight of the material is γ.

Given:

$$\gamma = 380\ \frac{\text{lb}}{\text{ft}^3}$$

$$a = 8\ \text{in}$$

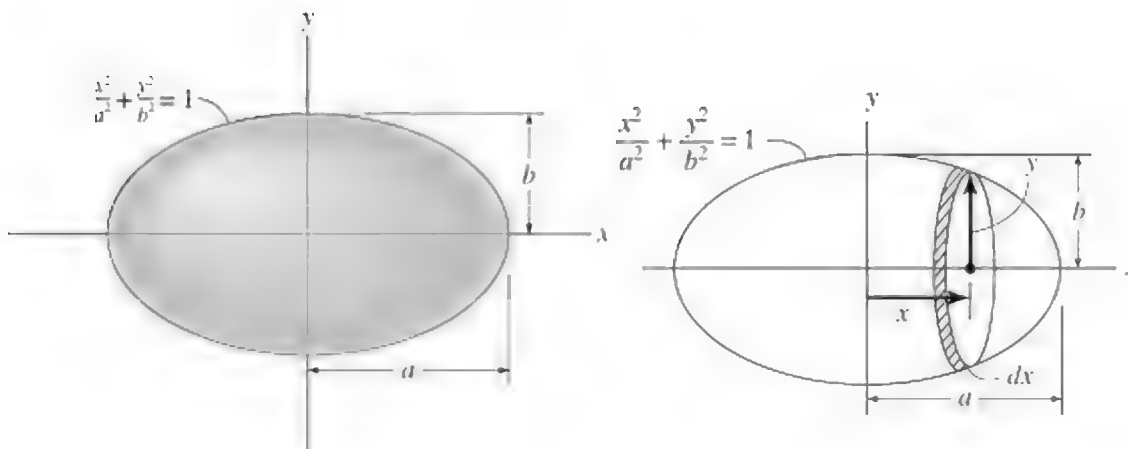
$$b = 2\ \text{in}$$

Solution:

$$m_b = \int_0^a \gamma \pi b^2 \left(\frac{x}{a}\right)^{\frac{2}{3}} dx \qquad m_b = 13.26\ \text{lb}$$

$$I_x = \int_0^a \frac{1}{2}\gamma \pi b^2 \left(\frac{x}{a}\right)^{\frac{2}{3}} b^2 \left(\frac{x}{a}\right)^{\frac{2}{3}} dx \qquad I_x = 0.59\ \text{slug}\cdot\text{in}^2$$

$$k_x = \sqrt{\frac{I_x}{m_b}} \qquad k_x = 1.20\ \text{in}$$

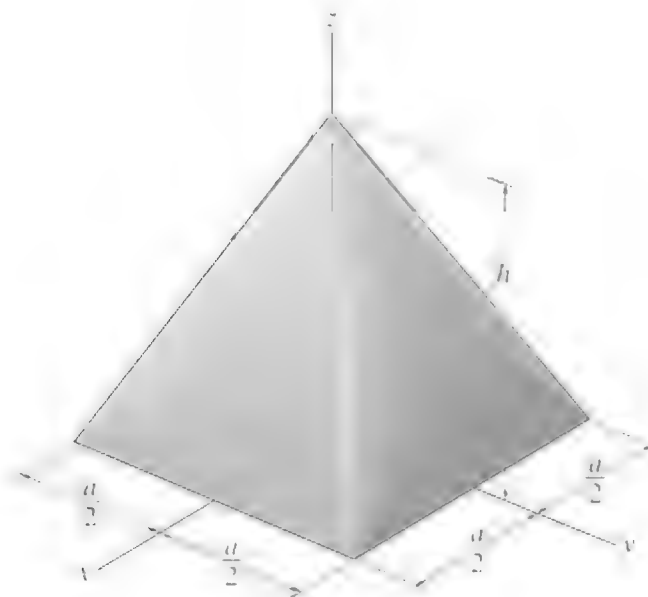
Problem 10-97

Determine the moment of inertia for the ellipsoid with respect to the x axis and express the result in terms of the mass m of the ellipsoid. The material has a constant density ρ.

Solution:

$$m = \int_{-a}^{a} \rho \pi b^2 \left(1 - \frac{x^2}{a^2}\right) dx = \frac{4}{3} a \rho \pi b^2 \qquad \rho = \frac{3m}{4\pi a b^2}$$

$$I_x = \int_{-a}^{a} \frac{1}{2} \frac{3m}{4\pi a b^2} \pi b^2 \left(1 - \frac{x^2}{a^2}\right) b^2 \left(1 - \frac{x^2}{a^2}\right) dx = \frac{2}{5} m b^2 \qquad I_x = \frac{2}{5} m b^2$$

Problem 10-98

Determine the moment of inertia of the homogeneous pyramid of mass m with respect to the z axis. The density of the material is ρ. *Suggestion:* Use a rectangular plate element having a volume of $dV = (2x)(2y)\ dz$.

Solution:

$$V = \int_0^h \left[a\left(1 - \frac{z}{h}\right)\right]^2 dz = \frac{1}{3} h a^2 \qquad \rho = \frac{m}{V} = \frac{3m}{a^2 h}$$

$$I_z = \frac{3m}{a^2 h} \int_0^h \frac{1}{6}\left[a\left(1 - \frac{z}{h}\right)\right]^4 dz = \frac{1}{10} m a^2 \qquad I_z = \frac{1}{10} m a^2$$

Problem 10-99

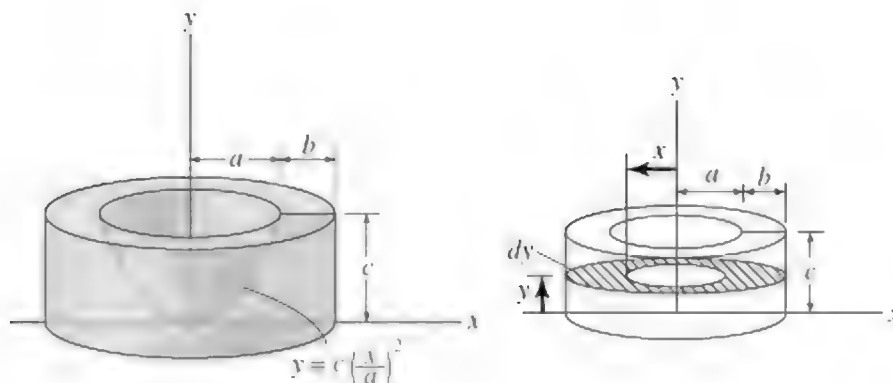
The concrete shape is formed by rotating the shaded area about the y axis. Determine the moment of inertia I_y. The specific weight of concrete is γ.

Given:

$$\gamma = 150\ \frac{\text{lb}}{\text{ft}^3}$$

$$a = 6\ \text{in}$$

$$b = 4\ \text{in}$$

$$c = 8\ \text{in}$$

Solution:

$$I_y = \frac{1}{2}\gamma\pi(a+b)^2 c(a+b)^2 - \int_0^c \frac{1}{2}\gamma\left(\pi\frac{a^2 y}{c}\right)\frac{a^2 y}{c}\,dy \qquad I_y = 2.25\ \text{slug}\cdot\text{ft}^2$$

Problem 10-100

Determine the moment of inertia of the thin plate about an axis perpendicular to the page and passing through the pin at O. The plate has a hole in its center. Its thickness is c, and the material has a density of ρ

Given:

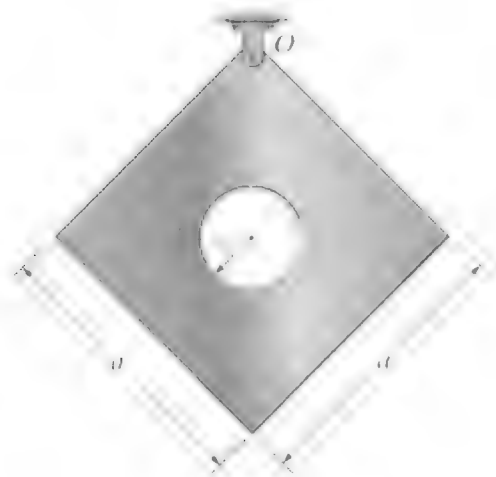
$$a = 1.40\ \text{m} \qquad c = 50\ \text{mm}$$

$$b = 150\ \text{mm} \qquad \rho = 50\ \frac{\text{kg}}{\text{m}^3}$$

Solution:

$$I_G = \frac{1}{12}\rho a^2 c\left(a^2 + a^2\right) - \frac{1}{2}\rho\pi b^2 c b^2$$

$$I_G = 1.60\ \text{kg}\cdot\text{m}^2$$

$$I_O = I_G + m d^2$$

$$m = \rho a^2 c - \rho \pi b^2 c$$

$$m = 4.7233 \text{ kg}$$

$$I_O = I_G + m(a \sin(45 \text{ deg}))^2$$

$$I_O = 6.23 \text{ kg}\cdot\text{m}^2$$

Problem 10-101

Determine the moment of inertia I_z of the frustum of the cone which has a conical depression. The material has a density ρ.

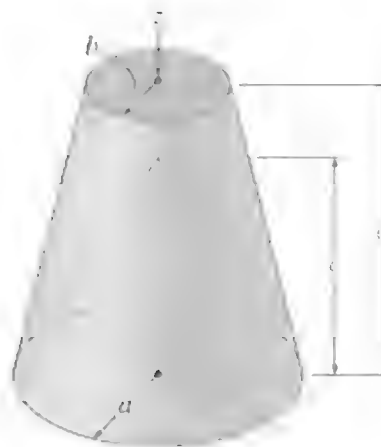
Given:

$$\rho = 200 \frac{\text{kg}}{\text{m}^3}$$

$$a = 0.4 \text{ m}$$

$$b = 0.2 \text{ m}$$

$$c = 0.6 \text{ m}$$

$$d = 0.8 \text{ m}$$

Solution:

$$h = \frac{d a}{a - b}$$

$$I_z = \frac{3}{10}\left[\rho\left(\frac{1}{3}\pi a^2 h\right)\right]a^2 - \frac{3}{10}\left[\rho\left(\frac{1}{3}\pi a^2 c\right)\right]a^2 - \frac{3}{10}\left[\rho\left[\frac{1}{3}\pi b^2 (h - d)\right]\right]b^2$$

$$I_z = 1.53 \text{ kg}\cdot\text{m}^2$$

Problem 10-102

Determine the moment of inertia for the assembly about an axis which is perpendicular to the page and passes through the center of mass G. The material has a specific weight γ.

Given:

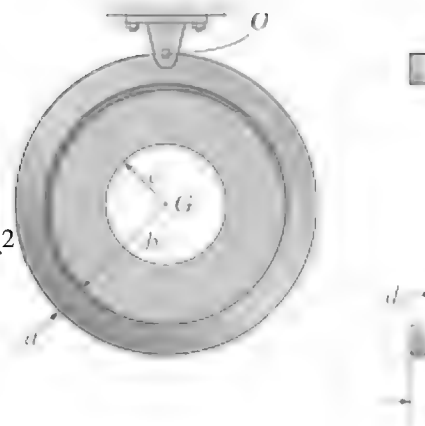
$a = 0.5 \text{ ft}$ $\quad d = 0.25 \text{ ft}$

$b = 2\ \text{ft}$ $e = 1\ \text{ft}$

$c = 1\ \text{ft}$ $\gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$

Solution:

$$I_G = \frac{1}{2}\gamma\pi(a+b)^2 e(a+b)^2 - \frac{1}{2}\gamma\pi b^2(e-d)\,b^2 - \frac{1}{2}\gamma\pi c^2 d c^2$$

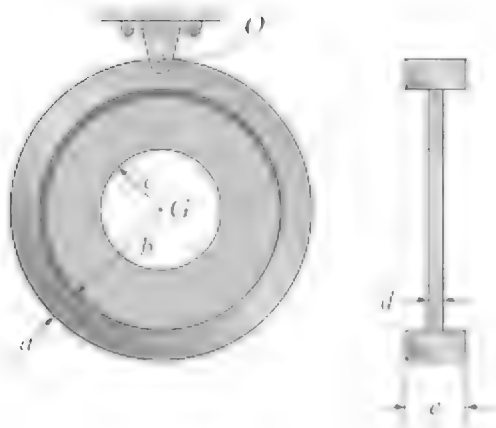
$$I_G = 118\ \text{slug}\cdot\text{ft}^2$$

Problem 10-103

Determine the moment of inertia for the assembly about an axis which is perpendicular to the page and passes through point *O*. The material has a specific weight γ.

Given:

$a = 0.5\ \text{ft}$ $d = 0.25\ \text{ft}$

$b = 2\ \text{ft}$ $e = 1\ \text{ft}$

$c = 1\ \text{ft}$ $\gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$

Solution:

$$I_G = \frac{1}{2}\gamma\pi(a+b)^2 e(a+b)^2 - \frac{1}{2}\gamma\pi b^2(e-d)\,b^2 - \frac{1}{2}\gamma\pi c^2 d c^2$$

$$I_G = 118\ \text{slug}\cdot\text{ft}^2$$

$$M = \gamma\pi(a+b)^2 e - \gamma\pi b^2(e-d) - \gamma\pi c^2 d \qquad M = 848.23\ \text{lb}$$

$$I_O = I_G + M(a+b)^2 \qquad I_O = 283\ \text{slug}\cdot\text{ft}^2$$

Problem 10-104

The wheel consists of a thin ring having a mass M_1 and four spokes made from slender rods, each having a mass M_2. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.

Given:

$M_1 = 10\ \text{kg}$

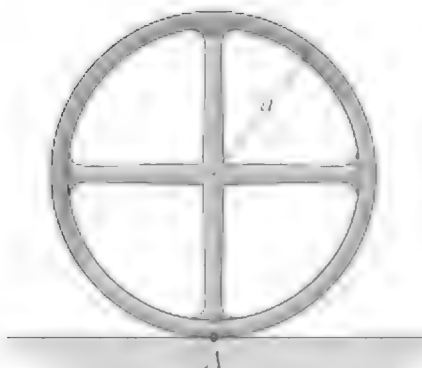
$M_2 = 2\ \text{kg}$

$a = 500\ \text{mm}$

Solution:

$$I_G = M_1 a^2 + 4\frac{1}{3}M_2 a^2$$

$$I_A = I_G + (M_1 + 4M_2)a^2$$

$$I_A = 7.67\ \text{kg}\cdot\text{m}^2$$

Problem 10-105

The slender rods have a weight density γ. Determine the moment of inertia for the assembly about an axis perpendicular to the page and passing through point A.

Given:

$\gamma = 3\ \dfrac{\text{lb}}{\text{ft}}$

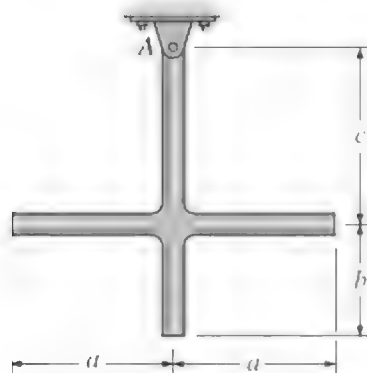
$a = 1.5\ \text{ft}$

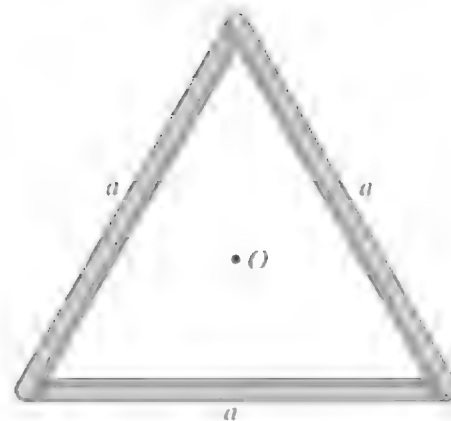
$b = 1\ \text{ft}$

$c = 2\ \text{ft}$

Solution:

$$I = \frac{1}{3}\gamma(b + c)(b + c)^2 + \frac{1}{12}\gamma 2a(2a)^2 + \gamma 2a c^2$$

$$I = 2.17\ \text{slug}\cdot\text{ft}^2$$

Problem 10-106

Each of the three rods has a mass m. Determine the moment of inertia for the assembly about an axis which is perpendicular to the page and passes through the center point O.

Solution:

$$I_O = 3\left[\frac{1}{12}m a^2 + m\left(\frac{a\sin(60\text{ deg})}{3}\right)^2\right]$$

$$I_O = \frac{1}{2}m a^2$$

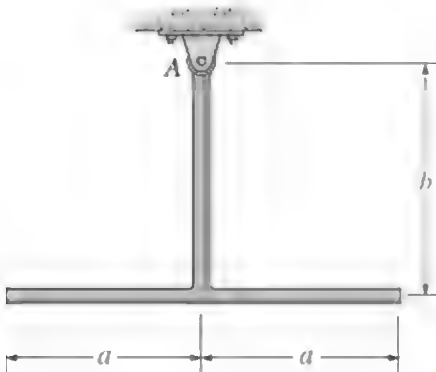
Problem 10-107

The slender rods have weight density γ. Determine the moment of inertia for the assembly about an axis perpendicular to the page and passing through point A

Given:

$$\gamma = 3\ \frac{\text{lb}}{\text{ft}}$$

$$a = 1.5\text{ ft}$$

$$b = 2\text{ ft}$$

Solution:

$$I_A = \frac{1}{3}\gamma b b^2 + \frac{1}{12}\gamma 2a(2a)^2 + \gamma(2a)\,b^2 \qquad I_A = 1.58\text{ slug}\cdot\text{ft}^2$$

Problem 10-108

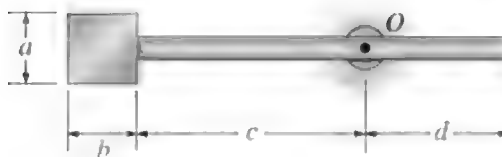
The pendulum consists of a plate having weight W_p and a slender rod having weight W_r. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.

Given:

$W_p = 12 \text{ lb}$ $a = 1 \text{ ft}$

$W_r = 4 \text{ lb}$ $b = 1 \text{ ft}$

$c = 3 \text{ ft}$

$d = 2 \text{ ft}$

Solution:

$$I_O = \frac{1}{12}W_r(c+d)^2 + W_r\left(\frac{c+d}{2} - c\right)^2 + \frac{1}{12}W_p\left(a^2 + b^2\right) + W_p\left(c + \frac{b}{2}\right)^2$$

$$k_O = \sqrt{\frac{I_O}{W_p + W_r}} \qquad k_O = 3.15 \text{ ft}$$

Problem 10-109

Determine the moment of inertia for the overhung crank about the x axis. The material is steel having density ρ.

Units Used:

$\text{Mg} = 1000 \text{ kg}$

Given:

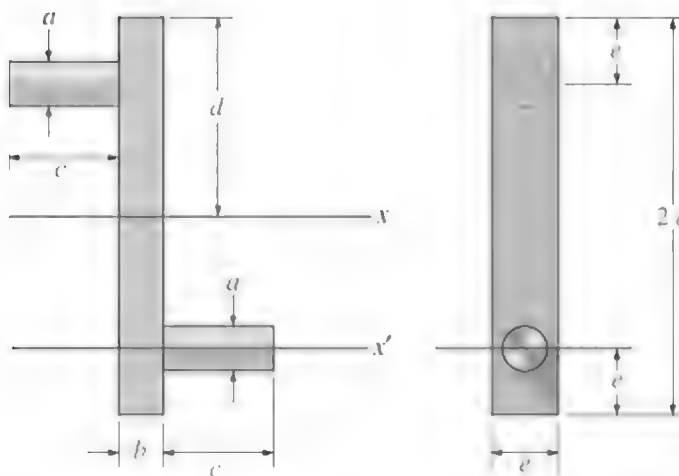
$\rho = 7.85 \frac{\text{Mg}}{\text{m}^3}$

$a = 20 \text{ mm}$

$b = 20 \text{ mm}$

$c = 50 \text{ mm}$

$d = 90 \text{ mm}$

$e = 30 \text{ mm}$

Solution:

$$m = \rho\pi\left(\frac{a}{2}\right)^2 c \qquad m = 0.12 \text{ kg}$$

$$M = \rho 2dbe \qquad M = 0.85 \text{ kg}$$

$$I_x = 2\left[\frac{1}{2}m\left(\frac{a}{2}\right)^2 + m(d-e)^2\right] + \frac{1}{12}M\left[(2d)^2 + e^2\right]$$

$$I_x = 3.25 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Problem 10-110

Determine the moment of inertia for the overhung crank about the x' axis. The material is steel having density ρ.

Units used:

$$\text{Mg} = 1000\ \text{kg}$$

Given:

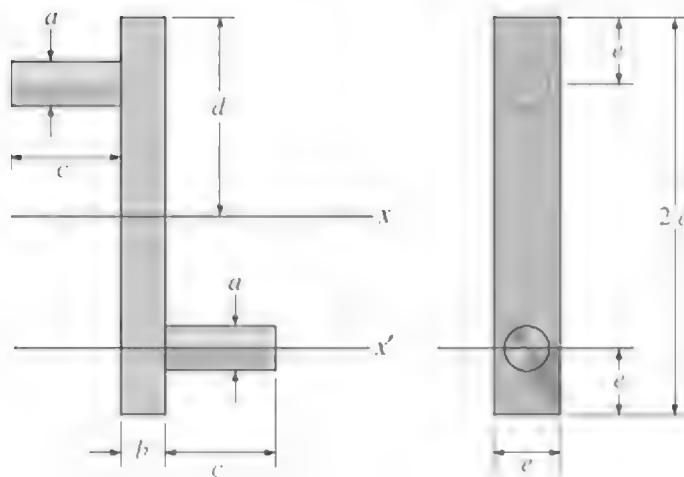
$$\rho = 7.85\ \frac{\text{Mg}}{\text{m}^3}$$

$$a = 20\ \text{mm}$$

$$b = 20\ \text{mm}$$

$$c = 50\ \text{mm}$$

$$d = 90\ \text{mm}$$

$$e = 30\ \text{mm}$$

Solution:

$$m = \rho\pi\left(\frac{a}{2}\right)^2 c \qquad m = 0.12\ \text{kg}$$

$$M = \rho 2dbe \qquad M = 0.85\ \text{kg}$$

$$I_x = 2\left[\frac{1}{2}m\left(\frac{a}{2}\right)^2 + m(d-e)^2\right] + \frac{1}{12}M\left[(2d)^2 + e^2\right]$$

$$I_{x'} = I_x + (M + 2m)(d-e)^2 \qquad I_{x'} = 7.19 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Problem 10-111

Determine the moment of inertia for the solid steel assembly about the x axis. Steel has a specific weight γ_{st}.

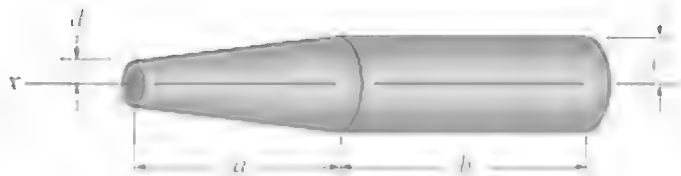
Given:

$a = 2\ \text{ft}$

$b = 3\ \text{ft}$

$c = 0.5\ \text{ft}$

$d = 0.25\ \text{ft}$

$\gamma_{st} = 490\ \frac{\text{lb}}{\text{ft}^3}$

Solution:

$$h = \frac{c\,a}{c-d}$$

$$I_x = \gamma_{st}\left[\pi c^2 b\left(\frac{c^2}{2}\right) + \frac{\pi}{3}c^2 h\left(\frac{3c^2}{10}\right) - \frac{\pi}{3}d^2(h-a)\left(\frac{3d^2}{10}\right)\right]$$

$$I_x = 5.64\ \text{slug}\cdot\text{ft}^2$$

Problem 10-112

The pendulum consists of two slender rods AB and OC which have a mass density ρ_r. The thin plate has a mass density ρ_p. Determine the location y_c of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.

Given:

$\rho_r = 3\ \frac{\text{kg}}{\text{m}}$

$\rho_s = 12\ \frac{\text{kg}}{\text{m}^2}$

$a = 0.4\ \text{m}$

$b = 1.5\ \text{m}$

$c = 0.1 \text{ m}$

$d = 0.3 \text{ m}$

Solution:

$$y_c = \frac{b\rho_r \frac{b}{2} + \pi d^2 \rho_s (b+d) - \pi c^2 \rho_s (b+d)}{b\rho_r + \pi d^2 \rho_s - \pi c^2 \rho_s + \rho_r 2a} \qquad y_c = 0.888 \text{ m}$$

$$I_G = \frac{1}{12} 2a\rho_r (2a)^2 + 2a\rho_r y_c^2 + \frac{1}{12} b\rho_r b^2 \ldots$$
$$+ b\rho_r \left(\frac{b}{2} - y_c\right)^2 + \frac{1}{2}\pi d^2 \rho_s d^2 + \pi d^2 \rho_s \left(b + d - y_c\right)^2 \ldots$$
$$+ \frac{1}{2}\pi c^2 \rho_s c^2 - \pi c^2 \rho_s \left(b + d - y_c\right)^2$$

$$I_G = 5.61 \text{ kg}\cdot\text{m}^2$$

Problem 10-113

Determine the moment of inertia for the shaded area about the x axis.

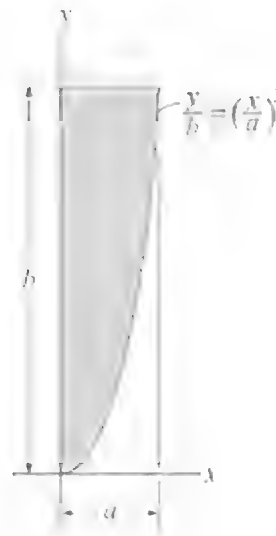
Given:

$a = 2 \text{ in}$

$b = 8 \text{ in}$

Solution:

$$I_x = \int_0^b y^2 a \left(\frac{y}{b}\right)^{\frac{1}{3}} \, dy$$

$$I_x = 307 \text{ in}^4$$

Problem 10-114

Determine the moment of inertia for the shaded area about the y axis.

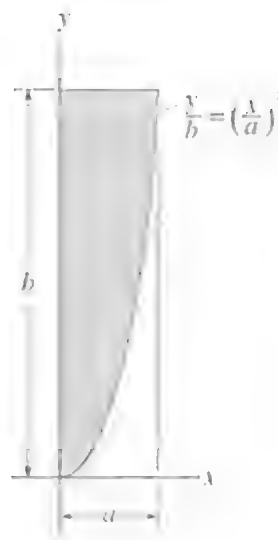
Given:

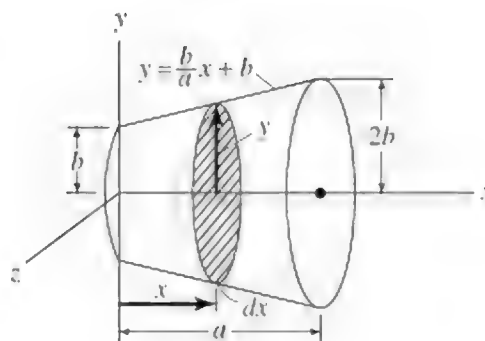
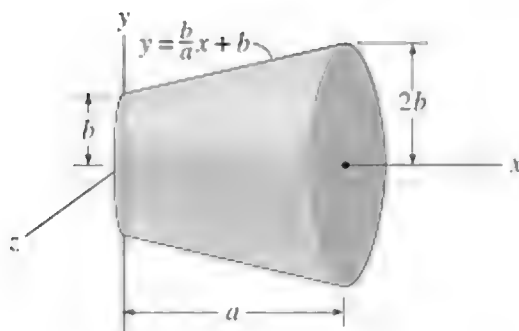
$a = 2 \text{ in}$

$b = 8 \text{ in}$

Solution:

$$I_y = \int_0^a x^2 \left[b - b\left(\frac{x}{a}\right)^3 \right] dx$$

$$I_y = 10.67 \text{ in}^4$$

Problem 10-115

Determine the mass moment of inertia I_x of the body and express the result in terms of the total mass m of the body. The density is constant.

Solution:

$$m = \int_0^a \rho\,\pi \left(\frac{bx}{a} + b \right)^2 dx = \frac{7}{3} a\,\rho\,\pi b^2 \qquad \rho = \frac{3m}{7\pi a b^2}$$

$$I_x = \int_0^a \frac{1}{2}\left(\frac{3m}{7\pi a b^2}\right)\pi\left(\frac{b\,x}{a}+b\right)^2\left(\frac{b\,x}{a}+b\right)^2 dx = \frac{93}{70}m\,b^2$$

$$I_x = \frac{93}{70}m\,b^2$$

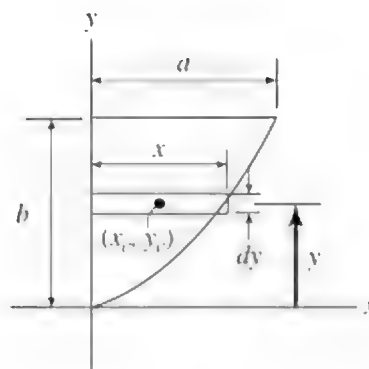
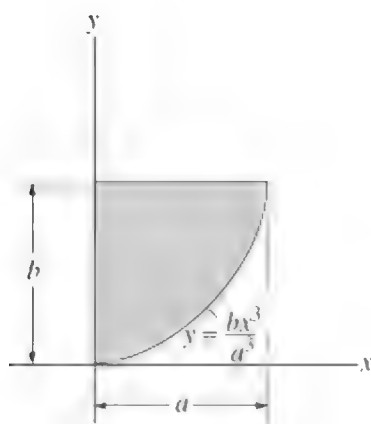
Problem 10-116

Determine the product of inertia for the shaded area with respect to the x and y axes.

Given:

$a = 1\ \text{m}$

$b = 1\ \text{m}$

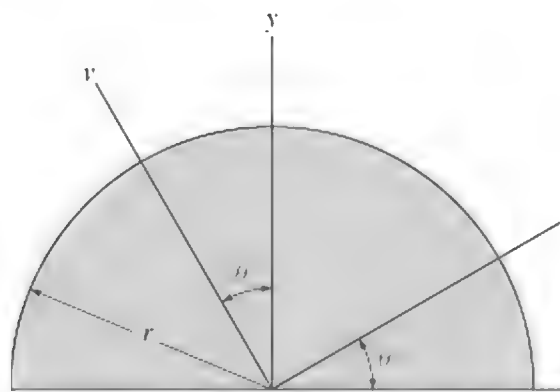
Solution:

$$I_{xy} = \int_0^b \frac{1}{2}y\,a\left(\frac{y}{b}\right)^{\frac{1}{3}} a\left(\frac{y}{b}\right)^{\frac{1}{3}} dy \qquad I_{xy} = 0.1875\ \text{m}^4$$

Problem 10-117

Determine the area moments of inertia I_u and I_v and the product of inertia I_{uv} for the semicircular area.

Given:

$r = 60\ \text{mm}$

$\theta = 30\ \text{deg}$

Solution:

$$I_x = \frac{\pi r^4}{8} \qquad I_y = I_x$$

$$I_{xy} = 0 \text{ mm}^4$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos(2\theta) - I_{xy}\sin(2\theta) \qquad I_u = 5.09 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos(2\theta) - I_{xy}\sin(2\theta) \qquad I_v = 5.09 \times 10^6 \text{ mm}^4$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin(2\theta) + I_{xy}\cos(2\theta) \qquad I_{uv} = 0 \text{ m}^4$$

Problem 10-118

Determine the moment of inertia for the shaded area about the x axis.

Given:

$a = 3$ in

$b = 9$ in

Solution:

$$I_x = \int_0^b y^2 a\sqrt{1 - \frac{y}{b}}\, dy$$

$$I_x = 333 \text{ in}^4$$

Problem 10-119

Determine the moment of inertia for the shaded area about the y axis.

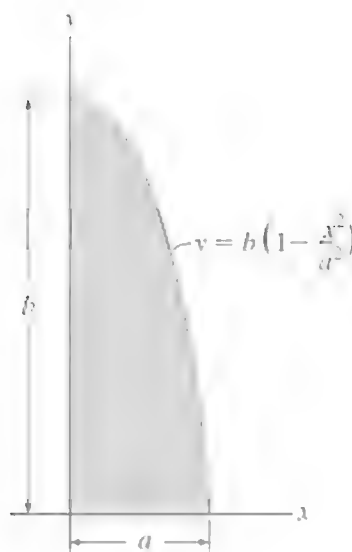
Given:

$a = 3\ \text{in}$

$b = 9\ \text{in}$

Solution:

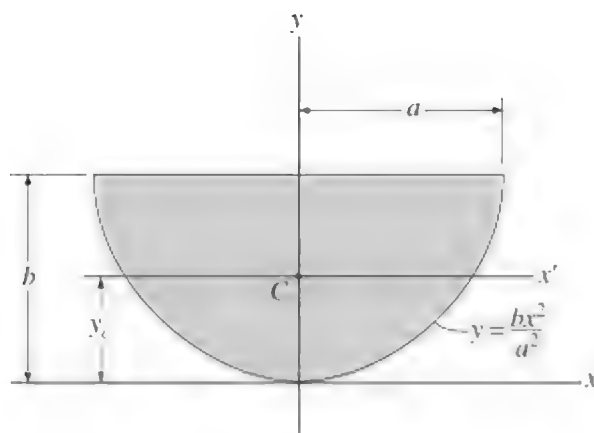
$$I_y = \int_0^a x^2 b\left(1 - \frac{x^2}{a^2}\right) dx$$

$$I_y = 32.4\ \text{in}^4$$

Problem 10-120

Determine the area moment of inertia of the area about the x axis. Then, using the parallel-axis theorem, find the area moment of inertia about the x' axis that passes through the centroid C of the area.

Given:

$a = 200\ \text{mm}$

$b = 200\ \text{mm}$

Solution:

$$I_x = \int_0^b y^2 2a\sqrt{\frac{y}{b}}\, dy \qquad I_x = 914 \times 10^6\ \text{mm}^4$$

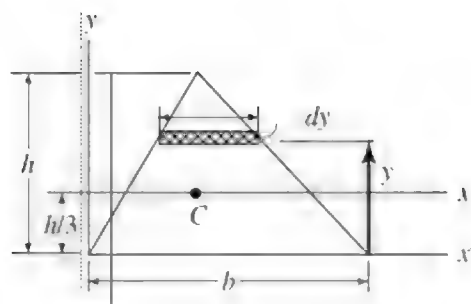
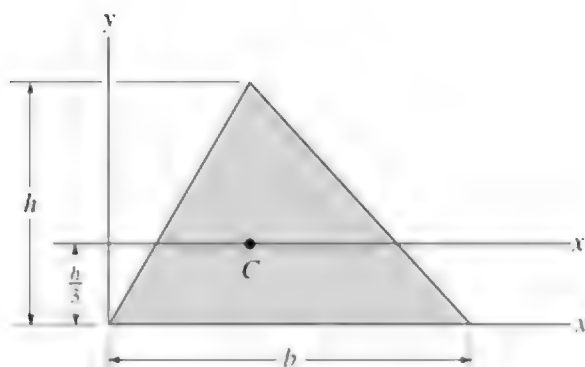
Find the area and the distance to the centroid

$$A = \int_0^b 2a\sqrt{\frac{y}{b}}\, dy \qquad A = 53.3 \times 10^3\ \text{mm}^2$$

$$y_c = \frac{1}{A}\int_0^b y2a\sqrt{\frac{y}{b}}\,dy \qquad y_c = 120.0\,\text{mm}$$

$$I_{x'} = I_x - A\,y_c^2 \qquad I_{x'} = 146 \times 10^6\,\text{mm}^4$$

Problem 10-121

Determine the area moment of inertia for the triangular area about (a) the x axis, and (b) the centroidal x' axis.

Solution:

$$I_x = \int_0^h y^2\,\frac{b}{h}(h-y)\,dy = \frac{1}{12}\cdot h^3\cdot b \qquad I_x = \frac{1}{12}b\,h^3$$

$$I_{x'} = \frac{b\,h^3}{12} - \frac{1}{2}b\,h\left(\frac{h}{3}\right)^2 = \frac{1}{36}\cdot h^3\cdot b \qquad I_{x'} = \frac{1}{36}b\,h^3$$

Problem 10-122

Determine the product of inertia of the shaded area with respect to the x and y axes.

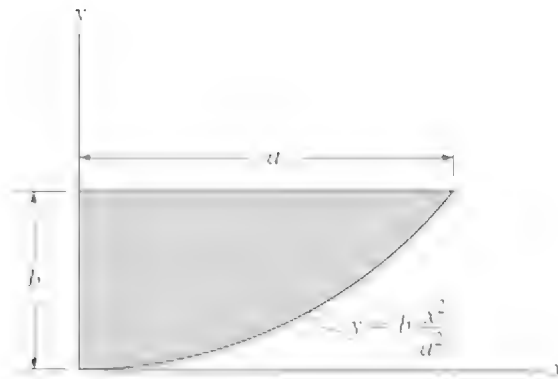
Given:

$a = 2\ \text{in}$

$b = 1\ \text{in}$

Solution:

$$I_{xy} = \int_0^b \frac{a}{2}\sqrt{\frac{y}{b}}\,y\,a\sqrt{\frac{y}{b}}\,dy$$

$$I_{xy} = 0.667\ \text{in}^4$$

Problem 11-1

The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force **P** needed to hold it in equilibrium.

Solution:

$$x_p = L\cos(\theta) \qquad \delta x_p = -L\sin(\theta)\delta\theta$$

$$y_W = \left(\frac{L}{2}\right)\sin(\theta) \qquad \delta y_W = \left(\frac{L}{2}\right)\cos(\theta)\delta\theta$$

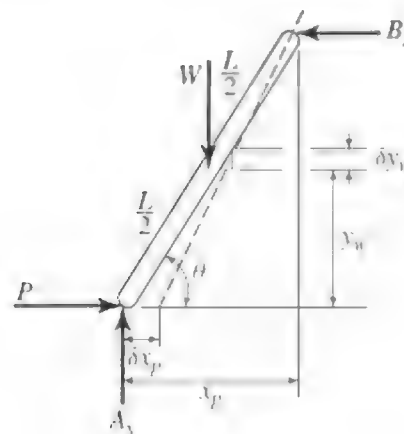
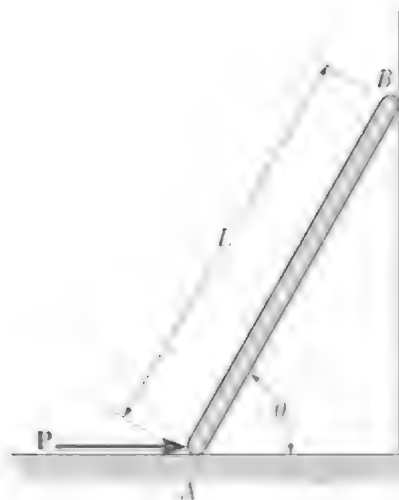
$\delta U = 0;$ $$-P\,\delta x_p - W\delta y_W = 0$$

$$-P(-L\sin(\theta)\delta\theta) - W\left(\frac{L}{2}\cos(\theta)\delta\theta\right) = 0$$

$$\delta\theta\left[P L\sin(\theta) - \left(\frac{W L}{2}\right)\cos(\theta)\right] = 0$$

Since $\delta\theta \neq 0$

$$P\,L\sin(\theta) - \left(\frac{W L}{2}\right)\cos(\theta) = 0$$

$$P = \frac{W}{2}\cot(\theta)$$

Problem 11-2

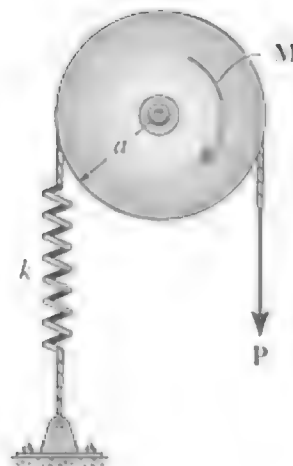
The disk has a weight W and is subjected to a vertical force **P** and a couple moment M. Determine the disk's rotation θ if the end of the spring wraps around the periphery of the disk as the disk turns. The spring is originally unstretched.

Given:

$W = 10$ lb

$P = 8$ lb

$M = 8$ lb·ft

$a = 1.5$ ft

$$k = 12\ \frac{\text{lb}}{\text{ft}}$$

Solution:

$$\delta U = P\ a\delta\ \theta + M\ \delta\theta - ka\theta a\delta\theta = \left(P\ a + M - ka^2\theta\right)\ \delta\ \theta = 0$$

$$P\ a + M - ka^2\theta = 0 \qquad \theta = \frac{P\ a + M}{ka^2} \qquad \theta = 42.4\ \text{deg}$$

Problem 11-3

The platform supports a load W. Determine the horizontal force **P** that must be supplied by the screw in order to support the platform when the links are at the arbitrary angle θ.

Solution:

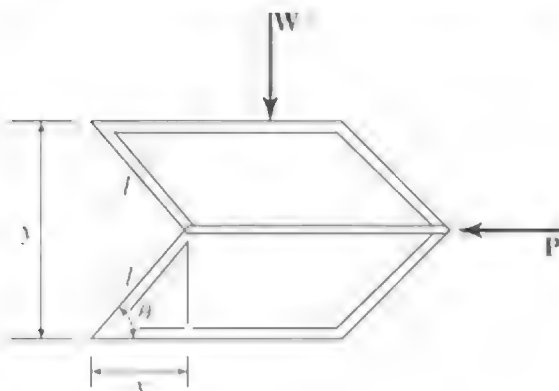
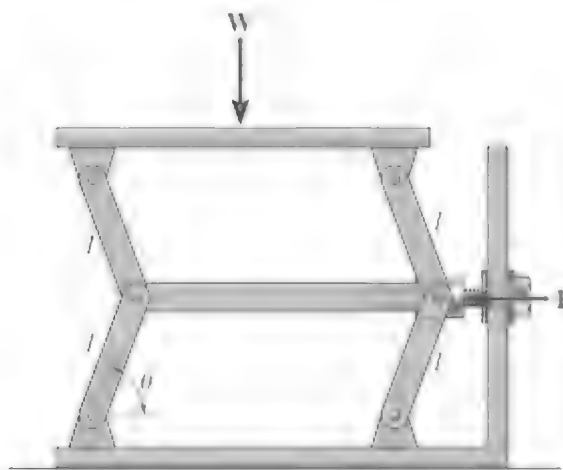
$$x = l\cos(\theta) \qquad \delta x = -l\sin(\theta)\delta\theta$$

$$y = 2\ l\sin(\theta) \qquad \delta y = 2\ l\cos(\theta)\delta\theta$$

$$\delta\ U = -W\ \delta\ y - P\ \delta\ x = 0$$

$$-W(2\ l\cos(\theta)\delta\theta) - P(-l\sin(\theta)\delta\ \theta) = 0$$

$$-2\ W\cos(\theta) + P\sin(\theta) = 0$$

$$P = 2\ W\cot(\theta)$$

Problem 11-4

Each member of the pin-connected mechanism has mass m_l. If the spring is unstretched when $\theta = 0°$, determine the angle θ for equilibrium.

Given:

$m_1 = 8$ kg

$k = 2500 \frac{\text{N}}{\text{m}}$

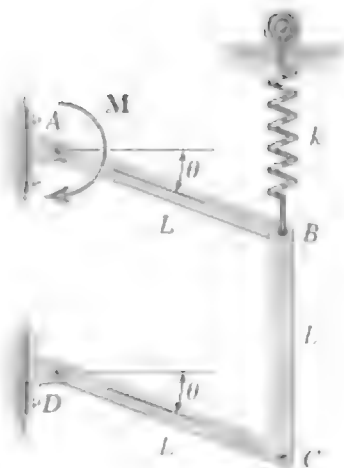
$L = 300$ mm

$M = 50$ N m

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

$y_1 = \left(\frac{L}{2}\right)\sin(\theta) \qquad \delta y_1 = \left(\frac{L}{2}\right)\cos(\theta)\delta\theta \qquad y_2 = L\sin(\theta) \qquad \delta y_2 = L\cos(\theta)\delta\theta$

$$\delta U = 2\, m_1 g \delta y_1 + m_1 g \delta y_2 - k y_2 \delta y_2 + M \delta\theta = 0$$

$$\delta U = \left[m_1 g L\left[2\left(\frac{1}{2}\right)\cos(\theta) + \cos(\theta)\right] - kL\sin(\theta)L\cos(\theta) + M\right]\delta\theta = 0$$

There are 2 solutions found by starting with different guesses

Guess $\theta = 10$ deg Given

$m_1\, g L\, 2\cos(\theta) - kL^2\sin(\theta)\cos(\theta) + M = 0 \qquad \theta = \text{Find}(\theta) \qquad \theta = 27.4$ deg

Guess $\theta = 60$deg Given

$m_1\, g\, L\, 2\cos(\theta) - kL^2\sin(\theta)\cos(\theta) + M = 0 \qquad \theta = \text{Find}(\theta) \qquad \theta = 72.7$ deg

Problem 11-5

Each member of the pin-connected mechanism has mass m_1. If the spring is unstretched when $\theta = 0°$, determine the required stiffness k so that the mechanism is in equilibrium when $\theta = \theta_0$.

Units Used:

$$kN = 10^3\ N$$

Given:

$$m_1 = 8\ kg$$

$$\theta = 30\ deg$$

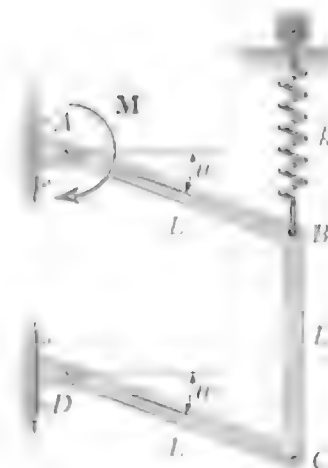
$$L = 300\ mm$$

$$M = 0\ N \cdot m$$

$$g = 9.81\ \frac{m}{s^2}$$

Solution:

$$y_1 = \left(\frac{L}{2}\right)\sin(\theta) \qquad \delta y_1 = \left(\frac{L}{2}\right)\cos(\theta)\delta\theta \qquad y_2 = L\sin(\theta) \qquad \delta y_2 = L\cos(\theta)\delta\theta$$

$$\delta U = 2m_1 g\delta y_1 + m_1 g\delta y_2 - k y_2 \delta y_2 + M\ \delta\theta = 0$$

$$\delta U = \left[m_1 g L\left[2\left(\frac{1}{2}\right)\cos(\theta) + \cos(\theta)\right] - kL\sin(\theta)L\cos(\theta) + M\right]\delta\theta = 0$$

Guess $k = 1\ \frac{kN}{m}$ Given

$$m_1 g L 2\cos(\theta) - kL^2\sin(\theta)\cos(\theta) + M = 0 \qquad k = \text{Find}(k) \qquad k = 1.046\ \frac{kN}{m}$$

Problem 11-6

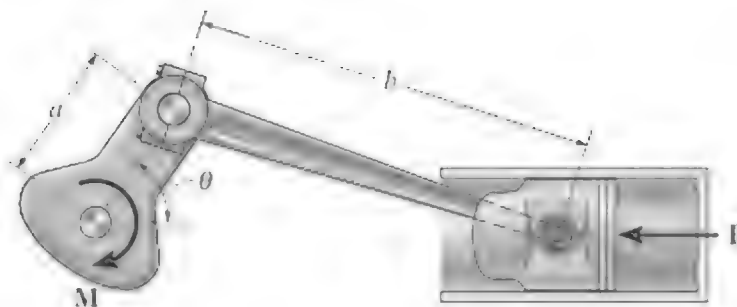
The crankshaft is subjected to torque M. Determine the horizontal compressive force **F** applied to the piston for equilibrium when $\theta = \theta_0$

Given:

$M = 50 \text{ N m}$

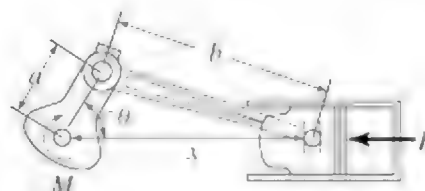
$\theta_0 = 60 \text{ deg}$

$a = 100 \text{ mm}$

$b = 400 \text{ mm}$

Solution: $\theta = \theta_0$

$$b^2 = a^2 + x^2 - 2\,a\,x\cos(\theta)$$

$$0 = 2\,x\,\delta x - 2\,a\cos(\theta)\delta x + 2\,a x\sin(\theta)\,\delta\theta$$

$$\delta x = \left(\frac{a x\sin(\theta)}{x - a\cos(\theta)}\right)\delta\theta$$

$$\delta U = -F\,\delta x + M\,\delta\theta = \left[-F\left(\frac{a x\sin(\theta)}{x - a\cos(\theta)}\right) + M\right]\delta\theta = 0$$

Guesses $x = 1 \text{ m}$ $F = 1 \text{ N}$

Given $b^2 = a^2 + x^2 - 2ax\cos(\theta)$ $-F\left(\frac{a x\sin(\theta)}{x - a\cos(\theta)}\right) + M = 0$

$\begin{pmatrix} F \\ x \end{pmatrix} = \text{Find}(F, x)$ $x = 440.512 \text{ mm}$ $F = 512 \text{ N}$

Problem 11-7

The crankshaft is subjected to torque **M**. Determine the horizontal compressive force **F** and plot the result of **F** (ordinate) versus θ(abscissa) for 0° <= θ <= 90°.

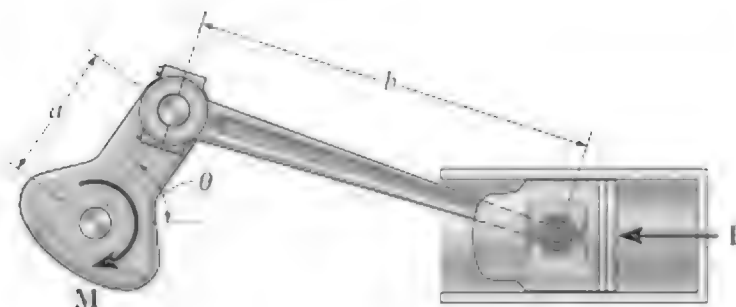
Units Used:

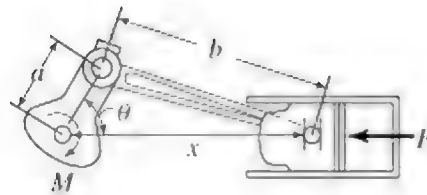
$\text{kN} = 10^3 \text{ N}$

Given:

$M = 0.05 \text{ kN·m}$

$a = 0.1 \text{ m}$

$b = 0.4 \text{ m}$

Solution:

$$b^2 = a^2 + x^2 - 2ax\cos(\theta)$$

Solving

$$x = a\cos(\theta) + \sqrt{a^2\cos(\theta)^2 + b^2 + a^2}$$

Virtual displacements

$$b^2 = a^2 + x^2 - 2ax\cos(\theta)$$

$$0 = 2x\delta x - 2a\cos(\theta)\delta x + 2ax\sin(\theta)\delta\theta \qquad \delta x = \left(\frac{ax\sin(\theta)}{x - a\cos(\theta)}\right)\delta\theta$$

$$\delta U = -F\,\delta x + M\delta\theta = \left[-F\left(\frac{ax\sin(\theta)}{x - a\cos(\theta)}\right) + M\right]\delta\theta = 0 \qquad F = M\left(\frac{x - a\cos(\theta)}{ax\sin(\theta)}\right)$$

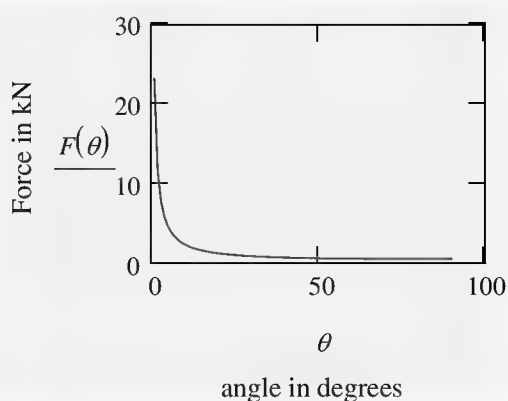
$$\theta = 0..90 \qquad x(\theta) = a\cos(\theta\text{deg}) + \sqrt{a^2\cos(\theta\,\text{deg})^2 + b^2 + a^2}$$

$$F(\theta) = M\left(\frac{x(\theta) - a\cos(\theta\text{deg})}{a\,x(\theta)\sin(\theta\text{deg})}\right)$$

Force in kN

$\frac{F(\theta)}{}$

θ

angle in degrees

$$F(60) = 0.515$$

Problem 11-8

If a force **P** is applied perpendicular to the handle of the toggle press, determine the compressive force developed at *C*.

Given:

$P = 30 \text{ lb}$

$\theta = 30 \text{ deg}$

$a = 12 \text{ in}$

$b = 2 \text{ in}$

Solution:

$\delta s = a\ \delta\ \theta$

$y = 2\ b\cos(\theta)$

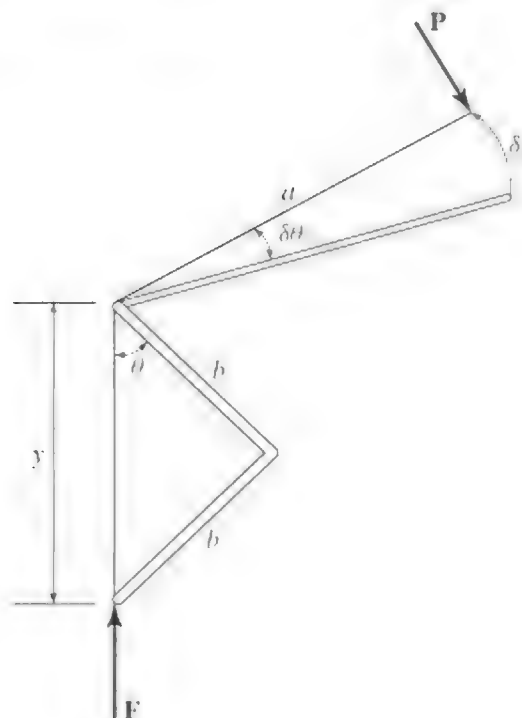
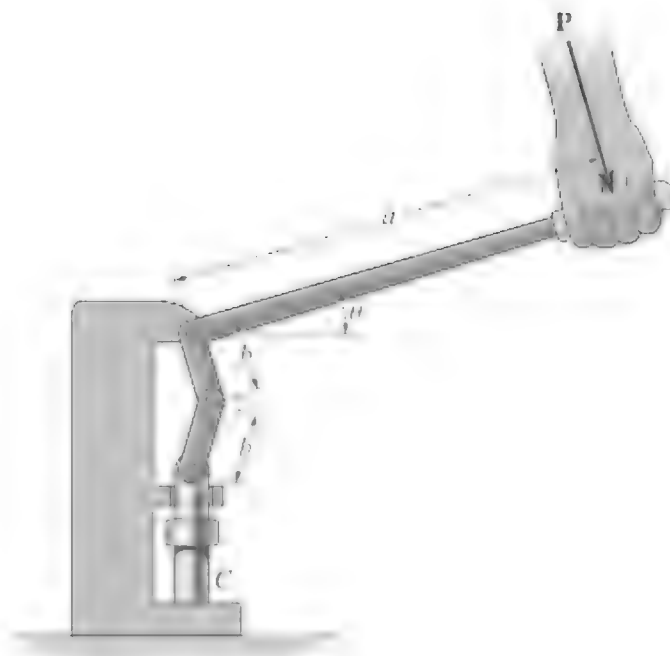
$\delta y = -2b\sin(\theta)$

$\delta U = -P\ \delta_s + -F\ \delta_y = 0$

$-P\,a\delta\theta + F2b\sin(\theta)\delta\theta = 0$

$F2b\sin(\theta) = P\,a$

$F = \frac{1}{2}\ P\left(\frac{a}{b\ \sin(\theta)}\right)$

$F = 180\,\text{lb}$

Problem 11-9

A force **P** is applied to the end of the lever. Determine the horizontal force **F** on the piston for equilibrium.

Solution:

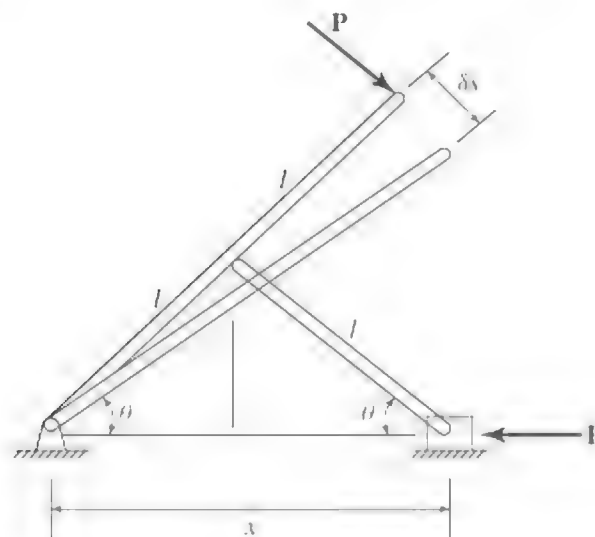
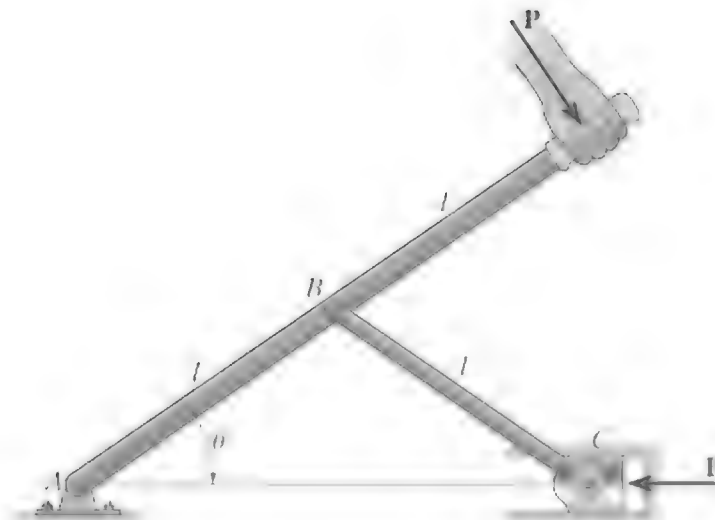
$$\delta s = 2\ l\ \delta\theta$$

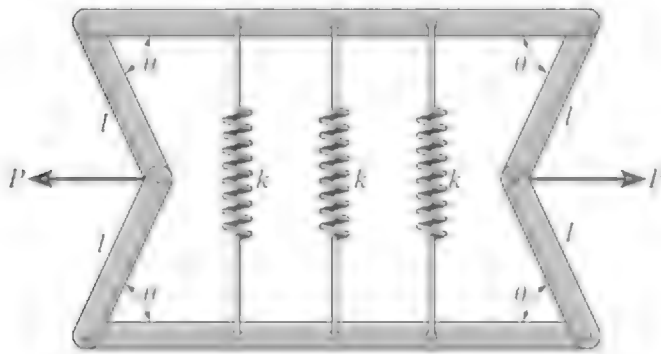
$$x = 2\ l\ \cos(\theta)$$

$$\delta x = -2\ l\ \sin(\theta)\ \delta\theta$$

$$\delta U = -P\ \delta s - F\,\delta x = 0$$

$$-P\,2l\delta\theta + F2l\sin(\theta)\delta\theta = 0$$

$$F = P\csc(\theta)$$

Problem 11-10

The mechanism consists of the four pin-connected bars and three springs, each having a stiffness k and an unstretched length l_0 Determine the horizontal forces **P** that must be applied to the pins in order to hold the mechanism in the horizontal position for equilibrium.

Solution:

$x = l\cos(\theta)$ $\qquad \delta x = -l\sin(\theta)\,\delta\theta$

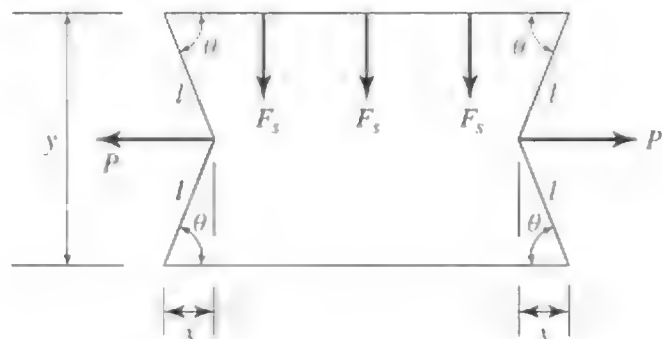
$y = 2\,l\sin(\theta)$ $\qquad \delta y = 2\,l\cos(\theta)\,\delta\theta$

$\delta U = 0;\quad -2\,P\delta x - 3\,F_S = 0$

$$2\,P\,l\sin(\theta)\delta\theta - 3\,F_S\,2l\cos(\theta)\delta\theta = 0$$

$$P\sin(\theta) = 3\,F_S\cos(\theta)$$

Since $F_S = k\left(2\,l\sin(\theta) - l_0\right)$,then

$$P = 3\,k\cot(\theta)\left(2\,l\sin(\theta) - l_0\right)$$

Problem 11-11

When $\theta = \theta_0$, the uniform block of weight W_b compresses the two vertical springs a distance δ. If the uniform links *AB* and *CD* each weigh W_L, determine the magnitude of the applied couple moments **M** needed to maintain equilibrium.

Given:

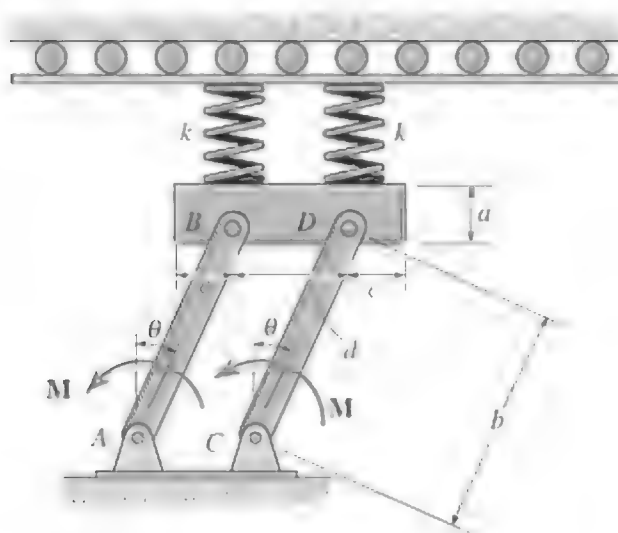
$\theta_0 = 20 \text{ deg}$ $a = 1 \text{ ft}$

$W_b = 50 \text{ lb}$ $b = 4 \text{ ft}$

$\delta = 4 \text{ in}$ $c = 1 \text{ ft}$

$W_L = 10 \text{ lb}$ $d = 2 \text{ ft}$

$k = 2 \frac{\text{lb}}{\text{in}}$

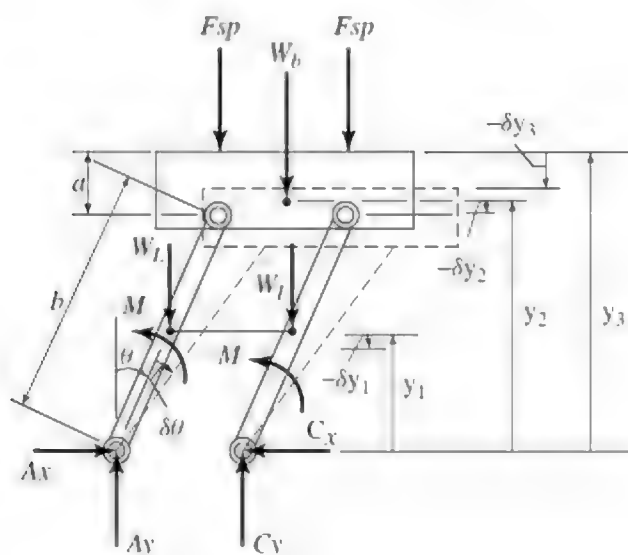
Solution: $\theta = \theta_0$

$$y_1 = \frac{b}{2}\cos(\theta)$$

$$\delta\, y_1 = \frac{-b}{2}\sin(\theta)\delta\theta$$

$$y_2 = \frac{a}{2} + b\cos(\theta)$$

$$\delta y_2 = -b\sin(\theta)\delta\theta$$

$$y_3 = y_2 + \frac{a}{2}$$

$$\delta y_3 = \delta y_2$$

$$\delta U = -2W_L\delta y_1 - W_b\delta y_2 - 2k\delta\,\delta y_3 - 2M\delta\theta = 0$$

$$\delta U = \left[2W_L\left(\frac{b}{2}\right)\sin(\theta) + W_b b\sin(\theta) + 2k\delta b\sin(\theta) - 2M\right]\delta\theta = 0$$

$$M = \left[\left(\frac{W_L + W_b}{2}\right)b + k\delta b\right]\sin(\theta) \qquad M = 52.0\,\text{lb}\cdot\text{ft}$$

Problem 11-12

The spring is unstretched when $\theta = 0$. Determine the angle θ for equilibrium. Due to the roller guide, the spring always remains vertical. Neglect the weight of the links.

Given:

$P = 8\ \text{lb}$

$k = 50\ \frac{\text{lb}}{\text{ft}}$

$a = 2\ \text{ft}$

$b = 2\ \text{ft}$

Solution:

$y_1 = a\sin(\theta)$

$\delta y_1 = a\cos(\theta)\delta\theta$

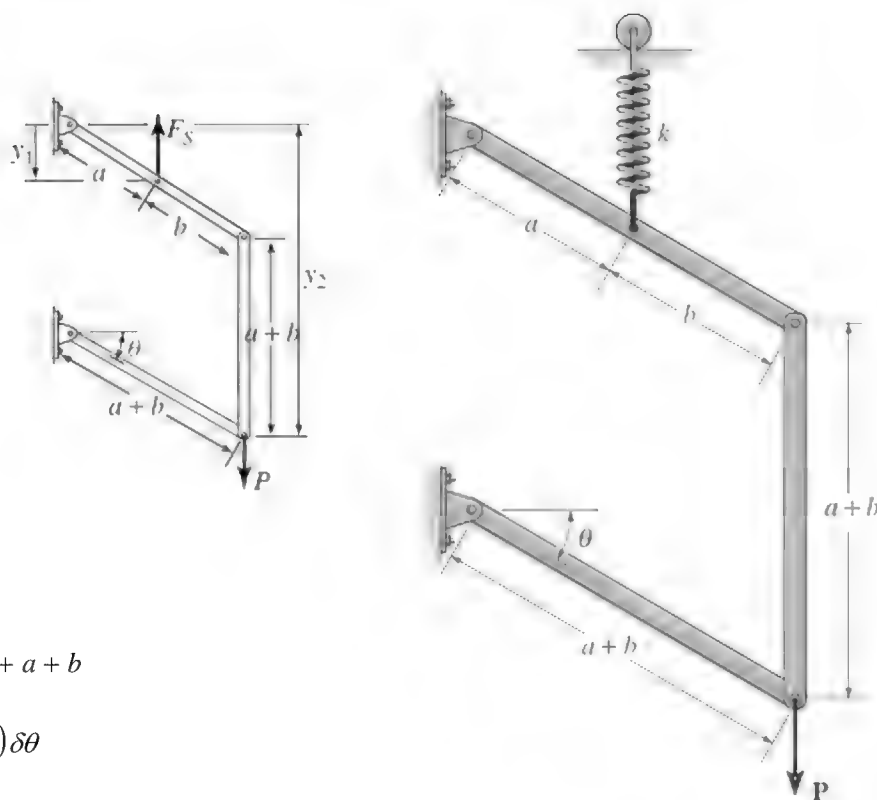
$y_2 = (a + b)\sin(\theta) + a + b$

$\delta y_2 = (a + b)\cos(\theta)\delta\theta$

$$\delta U = -k\,y_1\delta y_1 + P\delta y_2 = \left[-k a\sin(\theta)a\cos(\theta) + P(a + b)\cos(\theta)\right]\delta\theta = 0$$

$$\cos(\theta)\left[P(a + b) - k a^2\sin(\theta)\right] = 0$$

There are 2 answers $\theta_1 = \text{acos}(0)$ $\theta_1 = 90\,\text{deg}$

$\theta_2 = \text{asin}\left[\frac{P(a + b)}{k a^2}\right]$ $\theta_2 = 9.207\,\text{deg}$

Problem 11-13

Determine the force **P** required to lift the block of mass M using the differential hoist. The lever arm is fixed to the upper pulley and turns with it.

Given:

$a = 800\ \text{mm}$

$M = 15$ kg

$b = 150$ mm

$c = 300$ mm

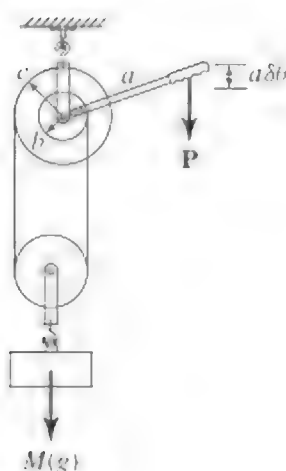
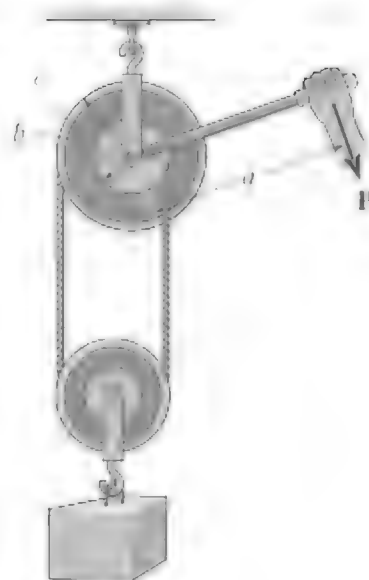
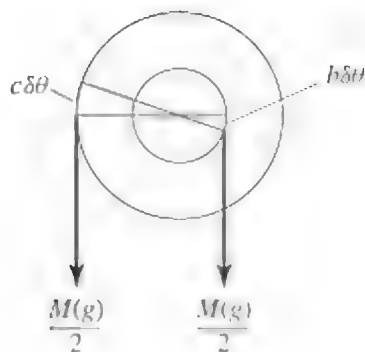
$g = 9.81 \frac{m}{s^2}$

Solution:

$\delta U = 0; \qquad P\ a\ \delta\theta + \frac{M g}{2}(-c\ \delta\theta + b\delta\theta) = 0$

$$P\ a + \frac{M g}{2}(b - c) = 0$$

$$P = \frac{M\ g}{2}\left(\frac{c - b}{a}\right)$$

$$P = 13.8\,\text{N}$$

Problem 11-14

Determine the magnitude of the applied couple moments **M** needed to maintain equilibrium at θ. The plate E has a weight W. Neglect the weight of the links AB and CD.

Given:

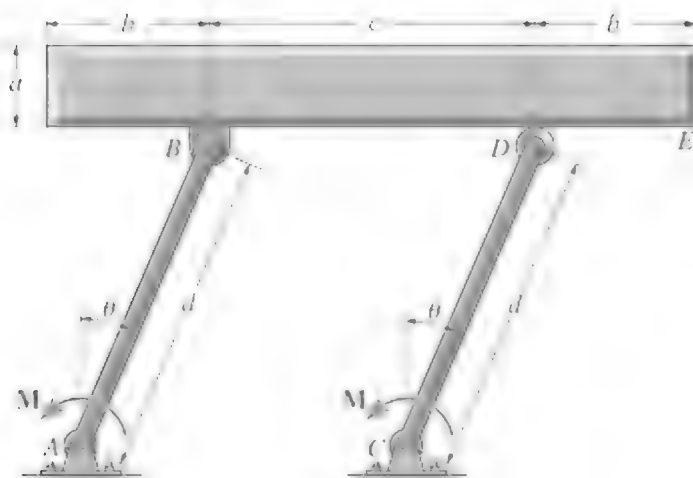
$a = 0.5$ ft

$d = 2$ ft

$b = 1$ ft

$c = 2$ ft

$W = 50$ lb

$\theta = 20$ deg

Solution:

$$y_m = d\cos(\theta) + \frac{a}{2} \qquad \delta y_m = -d\sin(\theta)\delta\theta$$

$$\delta U = -2\,M\,\delta\theta - W[-d\sin(\theta)(\delta\theta)] = (W\,d\sin(\theta) - 2\,M)\delta\theta = 0$$

$$M = \frac{1}{2}\,W\,d\sin(\theta)$$

$$M = 17.1\ \text{lb}\cdot\text{ft}$$

Problem 11-15

The members of the mechanism are pin connected. If a horizontal force **P** acts at *A*, determine the angle θ for equilibrium. The spring is unstretched when $\theta = 90°$.

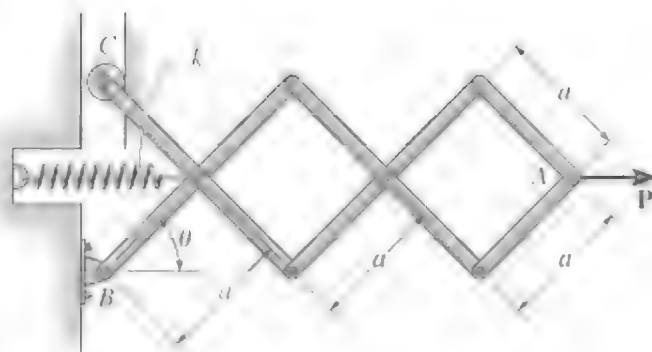
Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$a = 0.5\ \text{m}$$

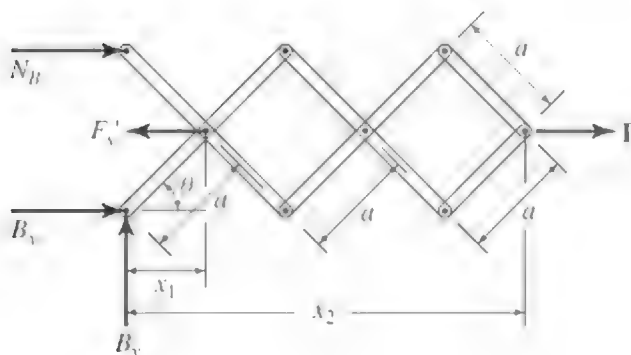
$$k = 20\ \frac{\text{kN}}{\text{m}}$$

$$P = 400\ \text{N}$$

Solution:

$$x_1 = a\cos(\theta)$$

$$\delta x_1 = -a\sin(\theta)\delta\theta$$

$$x_2 = 5a\cos(\theta)$$

$$\delta x_2 = -5\ a\sin(\theta)\,\delta\theta$$

$$\delta U = [P(-5\ a\sin(\theta)) - k\,a\cos(\theta)(-a\sin(\theta))]\delta\theta = 0$$

$$\sin(\theta)(-5\ P + k\,a\cos(\theta)) = 0$$

There are 2 equilibrium angles.

$\theta_1 = \text{asin}(0)$ $\theta_1 = 0\,\text{deg}$

$\theta_2 = \text{acos}\left(\frac{5P}{ka}\right)$ $\theta_2 = 78.5\,\text{deg}$

Problem 11-16

Determine the force F needed to lift the block having weight W. *Hint:* Note that the coordinates S_A and S_B can be related to the constant vertical length l of the cord.

Given:

$W = 100\ \text{lb}$

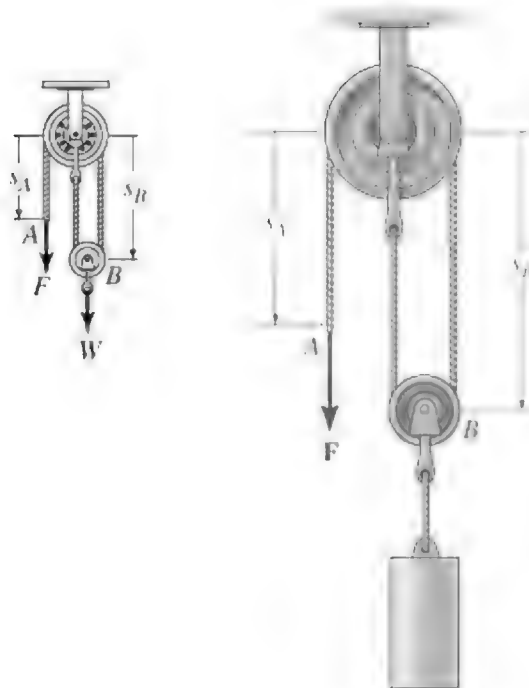
Solution:

$l = s_A + 2\ s_B$

$0 = \delta s_A + 2\ \delta s_B$

$\delta s_A = -2\ \delta s_B$

$\delta U = F\delta s_A + W\delta s_B = (-2\ F + W)\delta s_B = 0$

$F = \frac{W}{2}$ $F = 50\,\text{lb}$

Problem 11-17

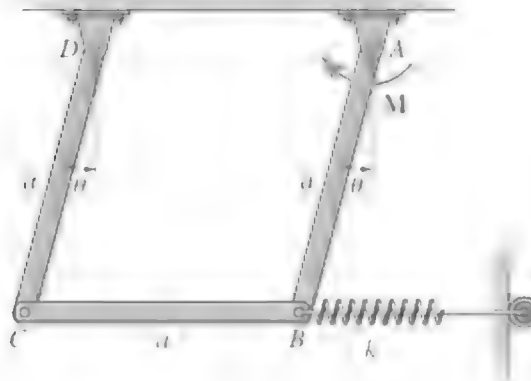
Each member of the pin-connected mechanism has a mass m_1. If the spring is unstretched when $\theta = 0°$ determine the angle θ for equilibrium.

Given:

$a = 300 \text{ mm}$

$k = 2500 \frac{\text{N}}{\text{m}}$

$m_1 = 8 \text{ kg}$

$M = 50 \text{ N}\cdot\text{m}$

Solution:

$x = a\sin(\theta) \qquad \delta x = a\cos(\theta)\delta\theta$

$y = a\cos(\theta) \qquad \delta y = -a\sin(\theta)\delta\theta$

$F_s = kx \qquad F_s = ka\sin(\theta)$

$$\delta U = m_1 g\left[2\left(\frac{\delta y}{2}\right) + \delta y\right] - F_s\delta x + M\delta\theta = 0$$

$$\delta U = \left(-2m_1\, g a\sin(\theta) - ka\sin(\theta)a\cos(\theta) + M\right)\delta U = 0$$

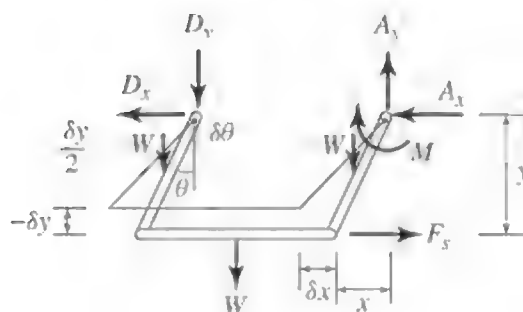
Initial Guesses: $\theta = 10 \text{ deg}$

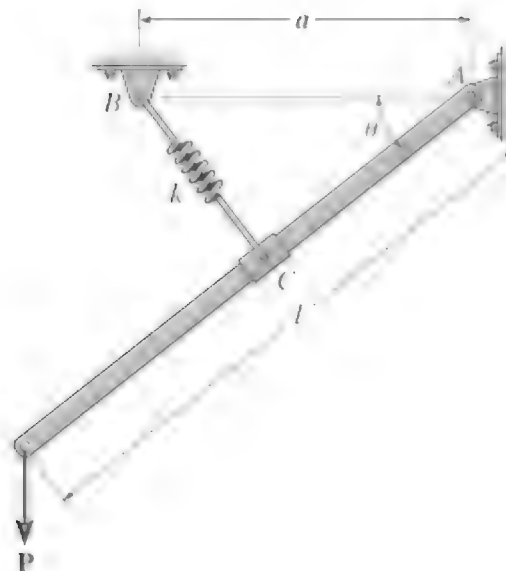
Given

$-2\, m_1\, g a\sin(\theta) - ka\sin(\theta)a\cos(\theta) + M = 0 \qquad \theta = \text{Find}(\theta) \qquad \theta = 10.7 \text{ deg}$

Now starting with a different guess we find anothe answer. $\theta = 90 \text{ deg}$

Given

$-2\, m_1\, g a\sin(\theta) - ka\sin(\theta)a\cos(\theta) + M = 0 \qquad \theta = \text{Find}(\theta) \qquad \theta = 89.3 \text{ deg}$

Problem 11-18

The bar is supported by the spring and smooth collar that allows the spring to be always perpendicular to the bar for any angle θ. If the unstretched length of the spring is l_0, determine the force **P** needed to hold the bar in the equilibrium position θ. Neglect the weight of the bar.

Solution:

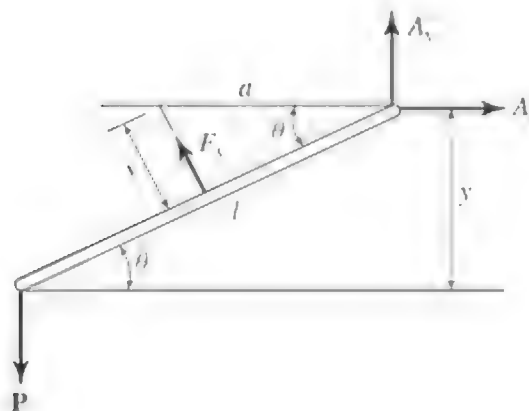
$s = a\sin(\theta) \qquad \delta_s = a\cos(\theta)\delta\theta$

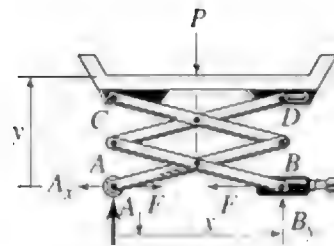
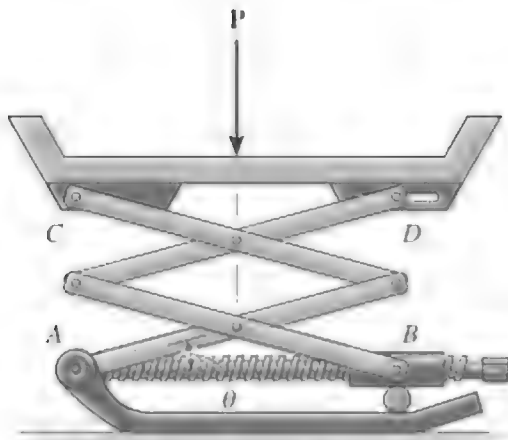
$y = l\sin(\theta) \qquad \delta_y = l\cos(\theta)\delta\theta$

$F_s = k(a\sin(\theta) - l_0)$

$\delta U = P\delta y - F_s\delta s = 0$

$\delta U = P\,l\cos(\theta)\delta\theta - k(a\sin(\theta) - l_0)a\cos(\theta)\delta\theta = 0$

$P = \frac{ka}{l}(a\sin(\theta) - l_0)$

Problem 11-19

The scissors jack supports a load **P**. Determine the axial force in the screw necessary for equilibrium when the jack is in the position θ. Each of the four links has a length L and is pin-connected at its center. Points B and D can move horizontally.

Solution:

$x = L\cos(\theta)$ $\qquad \delta x = -L\sin(\theta)\delta\theta$

$y = 2L\sin(\theta)$ $\qquad \delta y = 2L\cos(\theta)\delta\theta$

$\delta U = -P\,\delta y - F\,\delta x = (-P\,2L\cos(\theta) + F\,L\sin(\theta))\delta\theta = 0$ $\qquad F = 2P\cot(\theta)$

Problem 11-20

Determine the masses m_A and m_B of A and B required to hold the desk lamp of mass M in balance for any angles θ and ϕ. Neglect the weight of the mechanism and the size of the lamp.

Given:

$M = 400$ gm

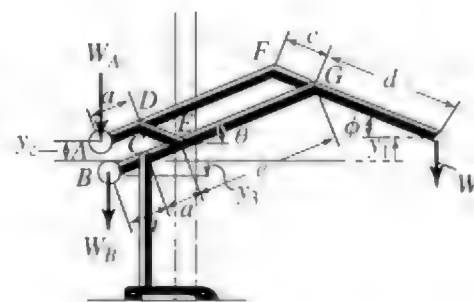
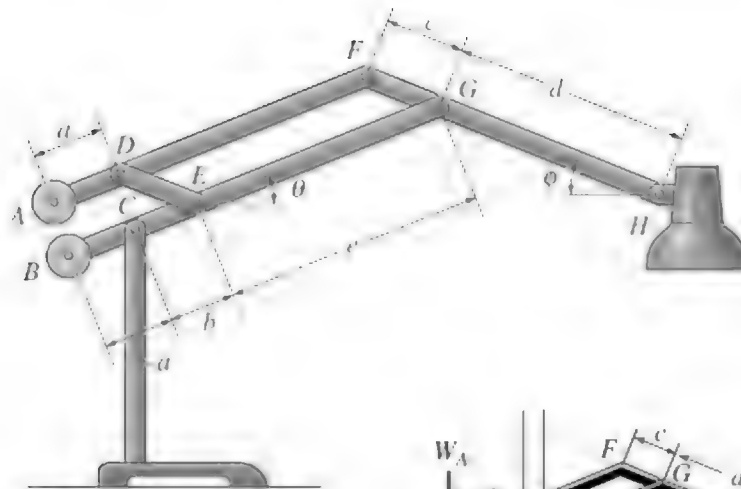
$a = 75$ mm

$b = 75$ mm

$c = 75$ mm

$d = 300$ mm

$e = 300$ mm

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$y_1 = (b+e)\sin(\theta) - d\sin(\phi)$ $\delta y_1 = (b+e)\cos(\theta)\,\delta\theta - d\cos(\phi)\,\delta\phi$

$y_2 = b\sin(\theta) + c\sin(\phi) - a\sin(\theta)$ $\delta y_2 = (b-a)\cos(\theta)\delta\theta + c\cos(\phi)\delta\phi$

$y_3 = -a\sin(\theta)$ $\delta y_3 = -a\cos(\theta)\delta\theta$

$\delta U = -M g\delta y_1 - m_A g\delta y_2 - m_B g\delta y_2 = 0$

$\delta U = g\left[\left[-M(b+e) - m_A(b-a) + m_B a\right]\cos(\theta)\delta\theta + \left(M d - m_A c\right)\cos(\phi)\delta\phi\right] = 0$

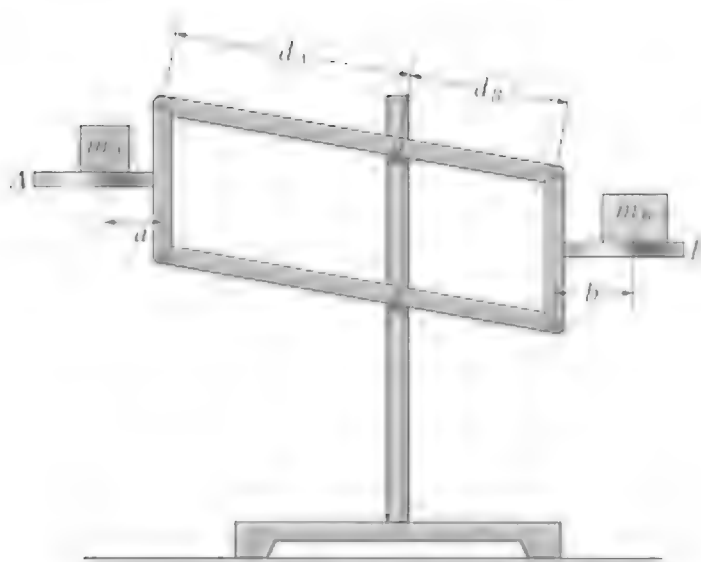
We now solve by setting both coefficients to zero.

Guesses $m_A = 1\text{ kg}$ $m_B = 1\text{ kg}$

Given $-M(b+e) - m_A(b-a) + m_B a = 0$ $M d - m_A c = 0$

$\begin{pmatrix} m_A \\ m_B \end{pmatrix} = \text{Find}(m_A, m_B)$ $\begin{pmatrix} m_A \\ m_B \end{pmatrix} = \begin{pmatrix} 1.6 \\ 2 \end{pmatrix}\text{ kg}$

Problem 11-21

The *Roberval balance* is in equilibrium when no weights are placed on the pans *A* and *B*. If two masses m_A and m_B are placed at *any* location *a* and *b* on the pans, show that equilibrium is maintained if $m_A\, d_A = m_B\, d_B$.

Solution:

$y_A = d_A \sin(\theta)$

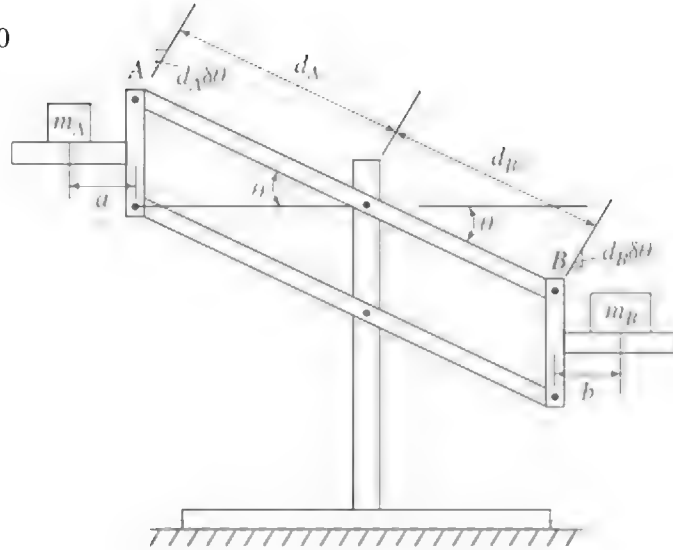
$\delta y_A = d_A \cos(\theta)\delta\theta$

$y_B = -d_B \sin(\theta)$

$\delta y_B = -d_B \cos(\theta)\delta\theta$

$\delta U = -m_A\, g\, d_A \cos(\theta)\delta\theta - m_B\, g\left(-d_B \cos(\theta)\right)\delta\theta = 0$

$$\delta U = (m_B d_B - m_A d_A)\, g \cos(\theta) \delta\theta = 0$$

$m_A d_A = m_B d_B$ Q.E.D

Problem 11-22

The chain puller is used to draw two ends of a chain together in order to attach the "master link." The device is operated by turning the screw S, which pushes the bar AB downward, thereby drawing the tips C and D towards one another. If the sliding contacts at A and B are smooth, determine the force **F** maintained by the screw at E which is required to develop a drawing tension **T** in the chains.

Given:

$T = 120$ lb

$\theta = 60 \text{ deg}$

$a = 3 \text{ in}$

$b = 1 \text{ in}$

Solution:

$$\delta s = \frac{b\ \delta\theta}{\cos(\theta)}$$

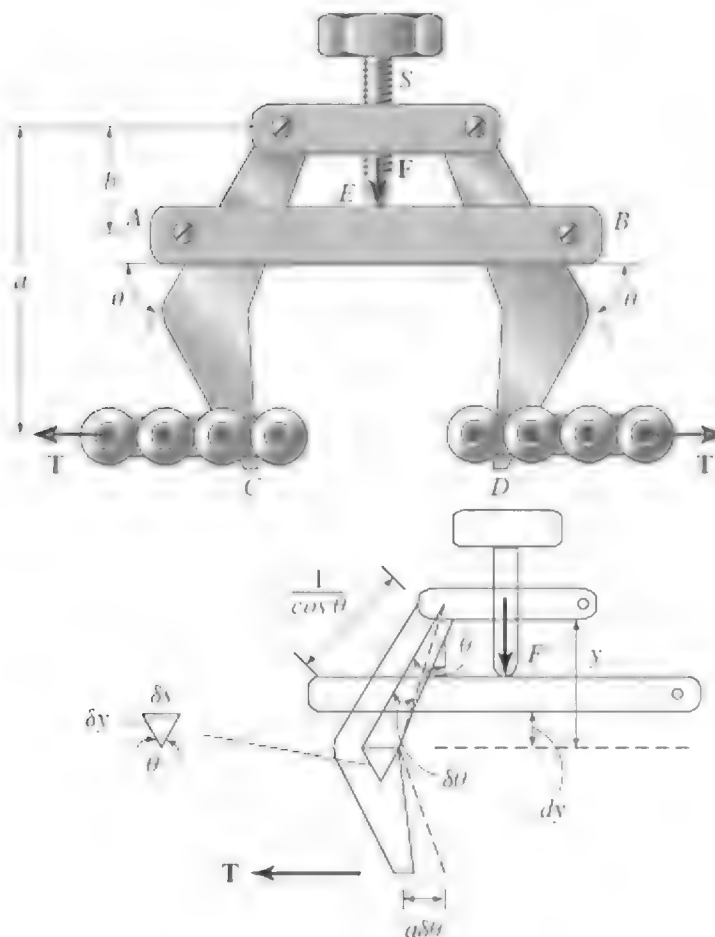
$$\delta y = \left(\frac{b}{\cos(\theta)}\right)\left(\frac{1}{\sin(\theta)}\right)\delta\theta$$

$$\delta U = 0$$

$$F\delta y - 2T(a\delta\theta) = 0$$

$$\frac{F\ b}{\cos(\theta)\sin(\theta)} = 2\ T\ a$$

$$F = 2\ T\ a\ \cos(\theta)\left(\frac{\sin(\theta)}{b}\right)$$

$$F = 312\,\text{lb}$$

Problem 11-23

The service window at a fast-food restaurant consists of glass doors that open and close automatically using a motor which supplies a torque **M** to each door. The far ends, *A* and *B*, move along the horizontal guides. If a food tray becomes stuck between the doors as shown, determine the horizontal force the doors exert on the tray at the position θ.

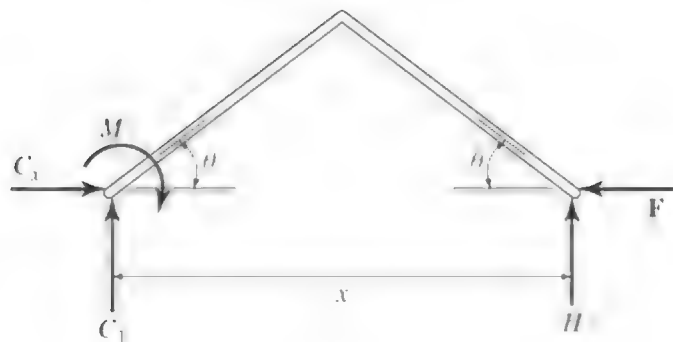
Solution:

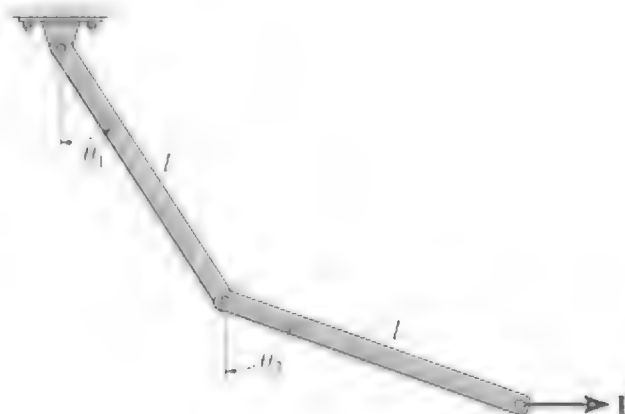
$$x = 2a\cos(\theta)$$

$$\delta x = -2a\sin(\theta)\delta\theta$$

$$\delta U = 0; \quad -M\,\delta\theta - F\,\delta x = 0$$

$$-M\,\delta\theta + F\;2\;a\;\sin(\theta)\delta\theta = 0$$

$$F = \frac{M}{2\;a\;\sin(\theta)}$$

Problem 11-24

A horizontal force acts on the end of the link as shown. Determine the angles θ_1 and θ_2 for equilibrium of the two links. Each link is uniform and has a mass m.

Solution:

$$x = l\sin(\theta_1) + l\sin(\theta_2)$$

$$y_1 = \frac{l}{2}\cos(\theta_1)$$

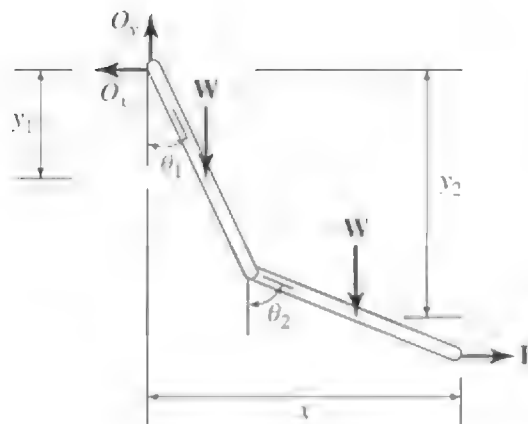
$$y_2 = l\cos(\theta_1) + \frac{l}{2}\cos(\theta_2)$$

$$\delta x = l\cos(\theta_1)\delta\theta_1 + l\cos(\theta_2)\delta\theta_2$$

$$\delta y_1 = -\frac{l}{2}\sin(\theta_1)\delta\theta_1$$

$$\delta y_2 = -l\sin(\theta_1)\delta\theta_1 - \frac{l}{2}\sin(\theta_2)\delta\theta_2$$

$$\delta U = P\,\delta_x + m\,g\,\delta y_1 + m\,g\,\delta y_2 = 0$$

$$\delta U = P\,l\left(\cos(\theta_1)\,\delta\theta_1 + \cos(\theta_2)\,\delta\theta_2\right) - m\,g\left(\frac{l}{2}\right)\left(3\sin(\theta_1)\delta\theta_1 + \sin(\theta_2)\,\delta\theta_2\right) = 0$$

$$\delta U = \left(P\,l\cos(\theta_1) - \frac{3}{2} m\,g\,l\sin(\theta_1)\right)\delta\theta_1 + \left(P\,l\cos(\theta_2) - \frac{1}{2} m\,g\,l\sin(\theta_2)\right)\delta\theta_2 = 0$$

Thus we have 2 equations:

$$P\ l\ \cos(\theta_1) - \frac{3}{2}\ m\ g\ l\ \sin(\theta_1) = 0 \qquad \theta_1 = \text{atan}\left(\frac{2\ P}{3\ m\ g}\right)$$

$$P\ l\ \cos(\theta_2) - \frac{1}{2}\ m\ g\ l\ \sin(\theta_2) = 0 \qquad \theta_2 = \text{atan}\left(\frac{2\ P}{m\ g}\right)$$

Problem 11-25

Rods AB and BC have centers of mass located at their midpoints. If all contacting surfaces are smooth and BC has mass m_{BC} determine the appropriate mass m_{AB} of AB required for equilibrium.

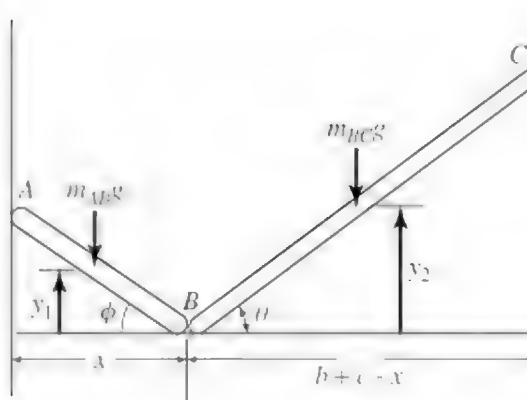
Given:

$m_{BC} = 100$ kg

$a = 0.75$ m

$b = 1$ m

$c = 2$ m

$d = 1.5$ m

Solution:

Use θ as the independent variable

Define $L_1 = \sqrt{a^2 + b^2}$ $\quad L_2 = \sqrt{c^2 + d^2}$ $\quad \theta = \text{atan}\left(\frac{d}{c}\right)$ $\quad \phi = \text{atan}\left(\frac{a}{b}\right)$

Then $L_1\cos(\phi) + L_2\cos(\theta) = b + c$ $\qquad -L_1\sin(\phi)\delta\phi - L_2\sin(\theta)\delta\theta = 0$

Thus $\delta\phi = \left(\frac{L_2 \sin(\theta)}{L_1 \sin(\phi)}\right)\delta\theta$

Also $y_1 = \frac{L_1}{2}\sin(\phi)$ $\delta y_1 = \frac{L_1}{2}\cos(\phi)\delta\phi = \left(\frac{-L_2\sin(\theta)\cot(\phi)}{2}\right)\delta\theta$

$y_2 = \frac{L_2}{2}\sin(\theta)$ $\delta y_2 = \frac{L_2}{2}\cos(\theta)\delta\theta$

$$\delta U = -m_{AB}\, g\, \delta y_1 - m_{BC}\, g\, \delta y_2 = g\left[m_{AB}\left(\frac{L_2\sin(\theta)\cot(\phi)}{2}\right) - m_{BC}\left(\frac{L_2}{2}\cos(\theta)\right)\right]\delta\theta = 0$$

$m_{AB} = m_{BC}\tan(\phi)\cot(\theta)$ $m_{AB} = 100 \text{ kg}$

Problem 11-26

If the potential energy for a conservative two-degree-of-freedom system is expressed by the relation $V = ay^2 + bx^2$, where y and x, determine the equilibrium positions and investigate the stability at each position.

Given:

$a = 3\,\frac{\text{N}}{\text{m}}$ $b = 2\,\frac{\text{N}}{\text{m}}$

Solution:

$$V = a\,y^2 + b\,x^2$$

Equilibrium position:

$\frac{\partial}{\partial x}V = 2b\,x = 0$ $x = 0$

$\frac{\partial}{\partial y}V = 2a\,y = 0$ $y = 0$

Stability:

At $(0,0)$ $\frac{\partial^2}{\partial x^2}V = 2b$ $2b = 4\,\frac{\text{N}}{\text{m}}$ > 0

At $(0,0)$ $\frac{\partial^2}{\partial y^2}V = 2a$ $2a = 6\,\frac{\text{N}}{\text{m}}$ > 0

At $(0,0)$ $\frac{\partial}{\partial x}\frac{\partial}{\partial y}V = 0$

At $(0,0)$ $\left[\left(\frac{\partial}{\partial x}\frac{\partial}{\partial y}V\right)^2 - \left(\frac{\partial^2}{\partial x^2}V\right)\left(\frac{\partial^2}{\partial y^2}V\right)\right] = -4ab$ $\qquad -4ab = -24\frac{\text{N}^2}{\text{m}^2}$ $\qquad < 0$

Stable at (0,0)

Problem 11-27

If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = (ax^3 + bx^2 + cx + d)$, determine the equilibrium positions and investigate the stability at each position.

Given: $a = 4\frac{\text{lb}}{\text{ft}^2}$ $\quad b = -1\frac{\text{lb}}{\text{ft}}$ $\quad c = -3\ \text{lb}$ $\quad d = 10\ \text{ft}\cdot\text{lb}$

Solution:

$$V = ax^3 + bx^2 + cx + d$$

Required Position:

$$\frac{d}{dx}V = 3ax^2 + 2bx + c = 0$$

$$x_1 = \frac{-2b + \sqrt{4b^2 - 4(3a)c}}{2(3a)}$$

$$x_1 = 0.59\ \text{ft}$$

$$x_2 = \frac{-2b - \sqrt{4b^2 - 4(3ac)}}{2\ (3a)}$$

$$x_2 = -0.424\ \text{ft}$$

Stability:

$$\frac{d^2}{dx^2}V = V'' = 6ax + 2b$$

At $x = x_1$ $\quad V''_1 = 6ax_1 + 2b$ $\quad V''_1 = 12.2\frac{\text{lb}}{\text{ft}}$ $\quad V''_1 > 0$ $\quad$ Stable

At $x = x_2$ $\quad V''_2 = 6ax_2 + 2b$ $\quad V''_2 = -12.2\frac{\text{lb}}{\text{ft}}$ $\quad V''_2 < 0$ $\quad$ Unstable

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Problem 11-28

If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = a\sin(\theta) + b\cos(2\theta)$, $0 \text{ deg} \le \theta \le 180 \text{ deg}$, determine the equilibrium positions and investigate the stability at each position.

Given:

$a = 24 \text{ ft}\cdot\text{lb}$ $\quad b = 10 \text{ ft}\cdot\text{lb}$

Solution:

$$V = a\sin(\theta) + b\cos(2\theta)$$

$$\frac{d}{d\theta}V = V' = a\cos(\theta) - 2b\sin(2\theta) = a\cos(\theta) - 4b\sin(\theta)\cos(\theta)$$

$$V' = \cos(\theta)(a - 4b\sin(\theta)) = 0$$

$$\frac{d^2}{dx^2}V = V'' = -a\sin(\theta) - 4b\cos(2\theta)$$

Equilibirium Positions:

$\theta_1 = \text{acos}(0)$ $\quad \theta_1 = 90 \text{ deg}$

$\theta_2 = \text{asin}\left(\frac{a}{4b}\right)$ $\quad \theta_2 = 36.87 \text{ deg}$

$\theta_3 = \pi - \theta_2$ $\quad \theta_3 = 143.13 \text{ deg}$

Check Stability

$V''_1 = -a\sin(\theta_1) - 4b\cos(2\theta_1)$ $\quad V''_1 = 16 \text{ lb ft}$ $\quad$ Stable

$V''_2 = -a\sin(\theta_2) - 4b\cos(2\theta_2)$ $\quad V''_2 = -25.6 \text{ lb ft}$ $\quad$ Unstable

$V''_3 = -a\sin(\theta_3) - 4b\cos(2\theta_3)$ $\quad V''_3 = -25.6 \text{ lb ft}$ $\quad$ Unstable

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Problem 11-29

If the potential energy for a conservative two-degree-of-freedom system is expressed by the relation $V = ay^2 + bx^2$, where y and x, determine the equilibrium positions and investigate the

stability at each position.

Given:

$$a = 6\,\frac{\text{N}}{\text{m}} \qquad b = 2\,\frac{\text{N}}{\text{m}}$$

Solution:

$$V = a\,y^2 + b\,x^2$$

Equilibrium position:

$$\frac{\partial}{\partial x}V = 2bx = 0 \qquad x = 0$$

$$\frac{\partial}{\partial y}V = 2a\,y = 0 \qquad y = 0$$

Stability:

At $(0,0)$ $\quad \dfrac{\partial^2}{\partial x^2}V = 2b \qquad 2b = 4\,\dfrac{\text{N}}{\text{m}} \qquad > 0$

At $(0,0)$ $\quad \dfrac{\partial^2}{\partial y^2}V = 2a \qquad 2a = 12\,\dfrac{\text{N}}{\text{m}} \qquad > 0$

At $(0,0)$ $\quad \dfrac{\partial}{\partial x}\dfrac{\partial}{\partial y}V = 0$

At $(0,0)$ $\quad \left[\left(\dfrac{\partial}{\partial x}\dfrac{\partial}{\partial y}V\right)^2 - \left(\dfrac{\partial^2}{\partial x^2}V\right)\left(\dfrac{\partial^2}{\partial y^2}V\right)\right] = -4a\,b \qquad -4a\,b = -48\,\dfrac{\text{N}^2}{\text{m}^2} \qquad < 0$

Stable at (0,0)

Problem 11-30

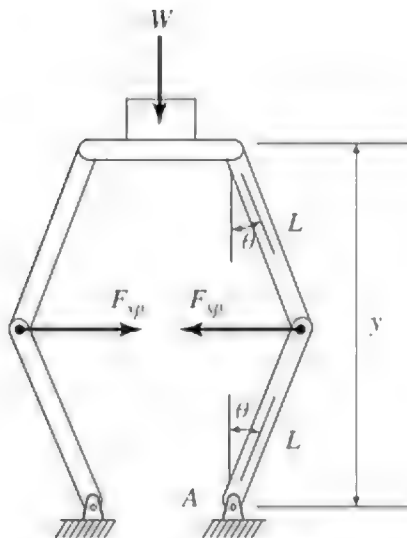
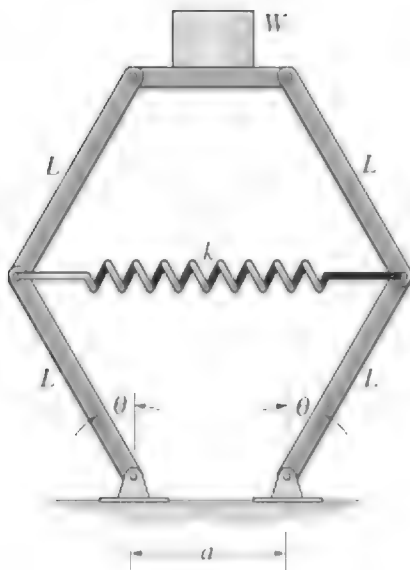
The spring of the scale has an unstretched length a. Determine the angle θ for equilibrium when a weight W is supported on the platform. Neglect the weight of the members. What value W would be required to keep the scale in neutral equilibrium when $\theta = 0°$?

Solution:

Potential Function: The datum is established at point A. Since the weight W is above the datum, its potential energy is positive.

$$V = \frac{1}{2}k(2L\sin(\theta))^2 + W2L\cos(\theta) = 2kL^2\sin(\theta)^2 + 2WL\cos(\theta)$$

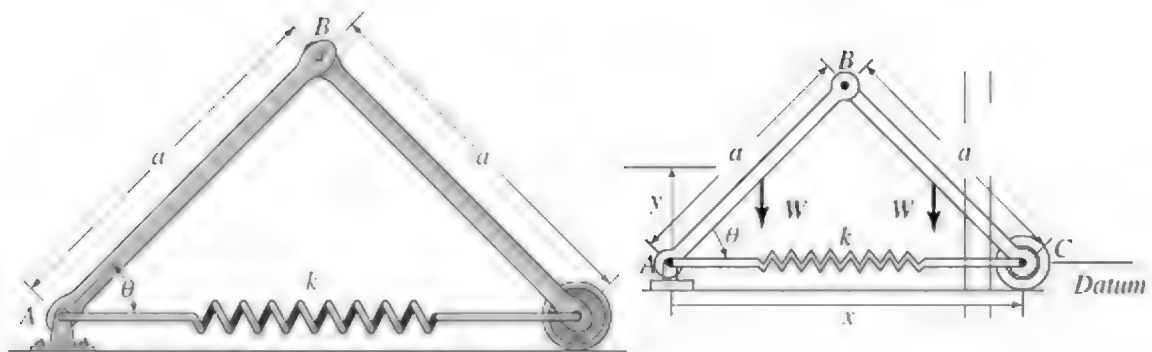
Equilibrium Position: The system is in equilibrium if

$$\frac{d}{d\theta}V = 4kL^2\sin(\theta)\cos(\theta) - 2WL\sin(\theta) = 2L\sin(\theta)(2kL\cos(\theta) - W) = 0$$

Solving, $\theta = 0°$ or $\theta = \operatorname{acos}\left(\frac{W}{2kL}\right)$

To have neutral stability at $\theta = 0$, we require that

$$\frac{d^2}{d\theta^2}V = 4kL^2\cos(2\times 0) - 2WL\cos(0) = 4kL^2 - 2WL = 0 \qquad W = 2kL$$

Problem 11-31

The two bars each have weight W. Determine the required stiffness k of the spring so that the two bars are in equilibrium at $\theta = \theta_0$. The spring has an unstretched length δ.

Given: $W = 8\ \text{lb}$ $\theta_0 = 30\ \text{deg}$ $\delta = 1\ \text{ft}$ $a = 2\ \text{ft}$

Solution: $\theta = \theta_0$

$$V = 2W\left(\frac{a}{2}\right)\sin(\theta) + \frac{1}{2}k(2a\cos(\theta) - \delta)^2$$

$$\frac{dV}{d\theta} = Wa\cos(\theta) - k(2a\cos(\theta) - \delta)\,2a\sin(\theta) = 0$$

$$k = \frac{Wa\cos(\theta)}{(2a\cos(\theta) - \delta)2a\sin(\theta)} \qquad k = 2.812\ \frac{\text{lb}}{\text{ft}}$$

Problem 11-32

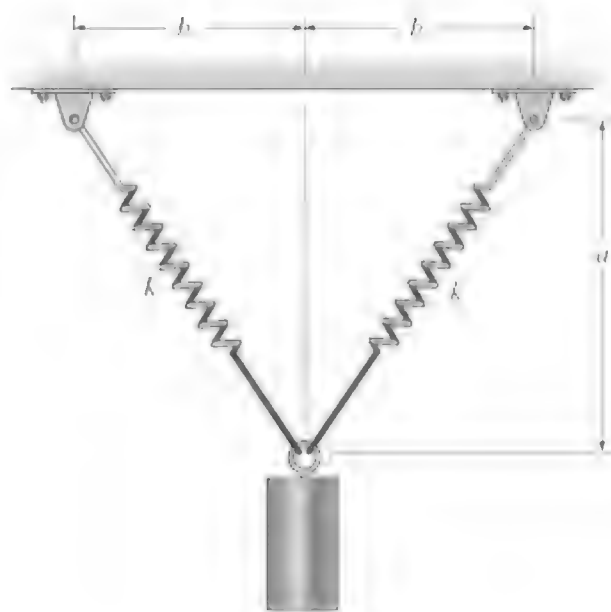
Each of the two springs has an unstretched length δ. Determine the mass M of the cylinder when it is held in the equilibrium position shown, i.e., $y = a$.

Given:

$a = 1\ \text{m}$

$b = 500\ \text{mm}$

$\delta = 500\ \text{mm}$

$k = 200\ \frac{\text{N}}{\text{m}}$

Solution:

$$V = 2\frac{k}{2}\left(\sqrt{y^2 + b^2} - \delta\right)^2 - M\,g\,y$$

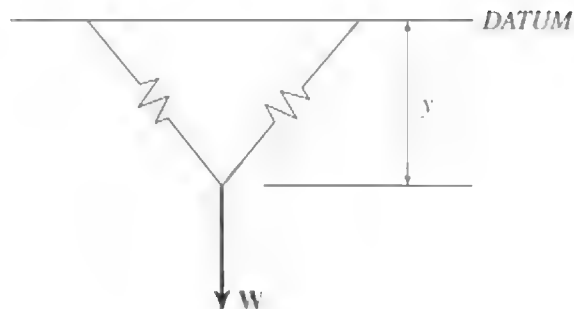
$$\frac{dV}{dy} = 2k\left(\sqrt{y^2 + b^2} - \delta\right)\left(\frac{y}{\sqrt{y^2 + b^2}}\right) - M\,g$$

Set $y = a$

Guess $M = 1\ \text{kg}$

Given

$$2k\left(\sqrt{y^2 + b^2} - \delta\right)\left(\frac{y}{\sqrt{y^2 + b^2}}\right) - M\,g = 0$$

$M = \text{Find}(M)$ $\quad M = 22.5\ \text{kg}$

Problem 11-33

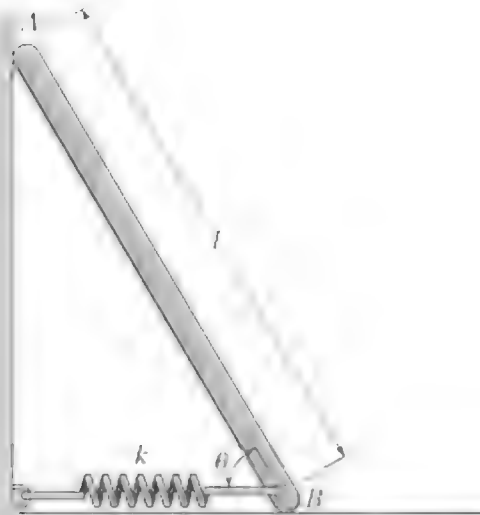
The uniform beam has mass M. If the contacting surfaces are smooth, determine the angle θ for equilibrium and investigate the stability of the beam when it is in this position. The spring has an unstretched length of δ.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$M = 200\ \text{kg}$ $\quad k = 1.2\ \frac{\text{kN}}{\text{m}}$

$\delta = 0.5\ \text{m}$ $\quad l = 2\ \text{m}$

Solution:

$$V = M\,g\left(\frac{l}{2}\right)\sin(\theta) + \frac{1}{2}k\left(l\cos(\theta) - \delta\right)^2$$

$$\frac{d}{d\theta}V = V' = M\,g\left(\frac{l}{2}\right)\cos(\theta) - k\left(l\cos(\theta) - \delta\right)l\sin(\theta) = 0$$

$$\frac{d^2}{d\theta^2}V = V'' = -M\,g\left(\frac{l}{2}\right)\sin(\theta) + k\,l^2\sin(\theta)^2 - k\left(l\cos(\theta) - \delta\right)l\cos(\theta)$$

There are 2 equilibrium points

Guess $\theta = 30\ \text{deg}$ Given $M g\left(\frac{l}{2}\right)\cos(\theta) - k(l\cos(\theta) - \delta)l\sin(\theta) = 0$

$\theta_1 = \text{Find}(\theta)$ $\theta_1 = 36.4\,\text{deg}$

Guess $\theta = 60\ \text{deg}$ Given $M g\left(\frac{l}{2}\right)\cos(\theta) - k(l\cos(\theta) - \delta)l\sin(\theta) = 0$

$\theta_2 = \text{Find}(\theta)$ $\theta_2 = 62.3\,\text{deg}$

Check Stability

$$V''_1 = -M g\left(\frac{l}{2}\right)\sin(\theta_1) + k l^2\sin(\theta_1)^2 - k(l\cos(\theta_1) - \delta)l\cos(\theta_1)$$

$$V''_2 = -M g\left(\frac{l}{2}\right)\sin(\theta_2) + k l^2\sin(\theta_2)^2 - k(l\cos(\theta_2) - \delta)l\cos(\theta_2)$$

$V''_1 = -1.624\,\text{kN}\cdot\text{m}$ Unstable $V''_2 = 1.55\,\text{kN}\cdot\text{m}$ Stable

---

Problem 11-34

The bar supports a weight W at its end. If the springs are originally unstretched when the bar is vertical, determine the required stiffness $k_1 = k_2 = k$ of the springs so that the bar is in neutral equilibrium when it is vertical.

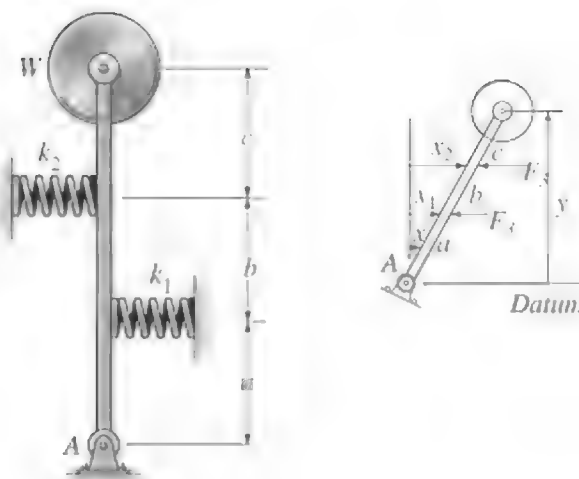
Given:

$W = 500\ \text{lb}$

$a = 3\ \text{ft}$

$b = 3\ \text{ft}$

$c = 3\ \text{ft}$

Solution:

$$V = W(a + b + c)\cos(\theta) + \frac{1}{2}k(a\sin(\theta))^2 + \frac{1}{2}k[(a + b)\sin(\theta)]^2$$

$$\frac{d}{d\theta}V = -W(a + b + c)\sin(\theta) + \frac{k}{2}\left[a^2 + (a + b)^2\right]\sin(2\theta)$$

$$\frac{d^2}{d\theta^2}V = -W(a+b+c)\cos(\theta) + k\left[a^2 + (a+b)^2\right]\cos(2\theta)$$

at $\theta = 0$

$$\frac{d^2}{d\theta^2}V = -W(a+b+c) + k\left[a^2 + (a+b)^2\right] = 0$$ for neutral stabiility

$$k = \frac{W(a+b+c)}{a^2 + (a+b)^2} \qquad k = 100\frac{\text{lb}}{\text{ft}}$$

Problem 11-35

The uniform rod AB has a mass M. If spring DC is unstretched at $\theta = 90$ deg, determine the angle θ for equilibrium and investigate the stability at the equilibrium position. The spring always acts in the horizontal position due to the roller guide at D.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

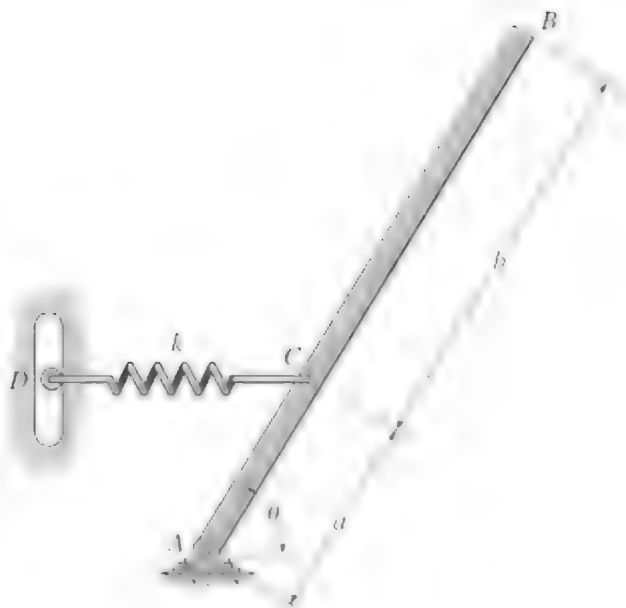
$$M = 80 \text{ kg} \qquad a = 1 \text{ m}$$

$$k = 2\frac{\text{kN}}{\text{m}} \qquad b = 2 \text{ m}$$

Solution:

$$V = Mg\left(\frac{a+b}{2}\right)\sin(\theta) + \frac{1}{2}k(a\cos(\theta))^2$$

$$V' = \frac{d}{d\theta}V = Mg\left(\frac{a+b}{2}\right)\cos(\theta) - \frac{k}{2}a^2\sin(2\theta)$$

$$V'' = \frac{d^2}{d\theta^2}V = -Mg\left(\frac{a+b}{2}\right)\sin(\theta) - ka^2\cos(2\theta)$$

Equilibrium

Guess $\theta = 30\ \text{deg}$ Given $M g\left(\frac{a+b}{2}\right)\cos(\theta) - \frac{k}{2}a^2\sin(2\theta) = 0$ $\theta_1 = \text{Find}(\theta)$

Guess $\theta = 70\ \text{deg}$ Given $M g\left(\frac{a+b}{2}\right)\cos(\theta) - \frac{k}{2}a^2\sin(2\theta) = 0$ $\theta_2 = \text{Find}(\theta)$

Check Staibility

$$V''_1 = -M g\left(\frac{a+b}{2}\right)\sin(\theta_1) - k a^2\cos(2\theta_1)$$

$$V''_2 = -M g\left(\frac{a+b}{2}\right)\sin(\theta_2) - k a^2\cos(2\theta_2)$$

$\theta_1 = 36.1\ \text{deg}$ $V''_1 = -1.3\ \text{kN}\cdot\text{m}$ Unstable

$\theta_2 = 90.0\ \text{deg}$ $V''_2 = 0.82\ \text{kN}\cdot\text{m}$ Stable

---

Problem 11-36

Determine the angle θ for equilibrium and investigate the stability at this position. The bars each have mass m_b and the suspended block D has mass m_D Cord DC has a total length of L.

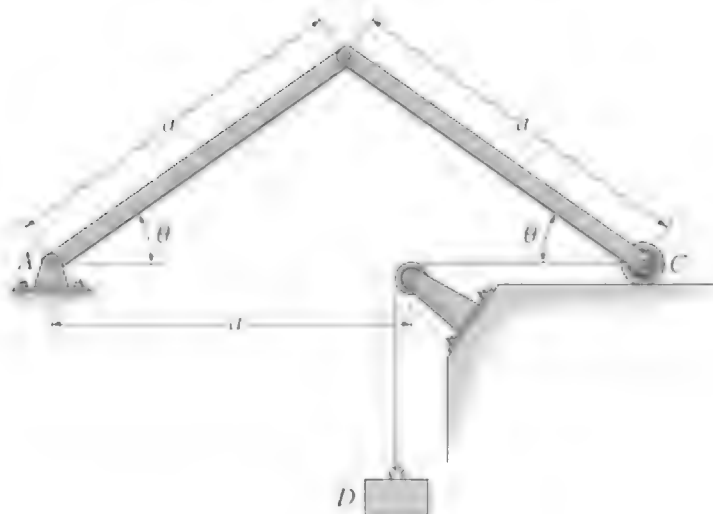
Given:

$m_b = 3\ \text{kg}$

$m_D = 7\ \text{kg}$

$a = 500\ \text{mm}$

$L = 1\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Equilibrium

$$V = 2m_b g\frac{a}{2}\sin(\theta) - m_D g(L + a - 2a\cos(\theta)) = 0$$

$$V = m_b g a\sin(\theta) + 2m_D g a\cos(\theta) - m_D g(L + a)$$

$$\frac{d}{d\theta}V = m_b g a \cos(\theta) - 2m_D g a \sin(\theta) = 0$$

$$\tan(\theta) = \frac{m_b}{2m_D} \qquad \theta = \operatorname{atan}\left(\frac{m_b}{2m_D}\right) \qquad \theta = 12.095 \text{ deg}$$

Stability

$$V'' = \frac{d^2}{d\theta^2}V = -m_b g a \sin(\theta) - 2m_D g a \cos(\theta)$$

$$V'' = -m_b g a \sin(\theta) - 2m_D g a \cos(\theta) \qquad V'' = -70.229 \text{ N}\cdot\text{m}$$

Since $V'' < 0$ the equilibrium point is unstable.

Problem 11-37

The bar supports a weight of W at its end. If the springs are originally unstretched when the bar is vertical, investigate the stability of the bar when it is in the vertical position.

Given:

$$k_1 = 300 \frac{\text{lb}}{\text{ft}}$$

$$k_2 = 500 \frac{\text{lb}}{\text{ft}}$$

$$W = 500 \text{ lb}$$

$$a = 3 \text{ ft}$$

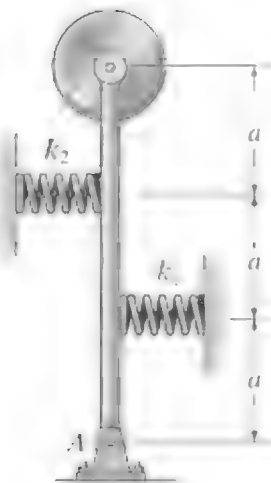
Solution:

$$V = W3a\cos(\theta) + \frac{1}{2}k_1(a\sin(\theta))^2 + \frac{1}{2}k_2(2a\sin(\theta))^2$$

$$V = 3Wa\cos(\theta) + \frac{a^2}{2}(k_1 + 4k_2)\sin(\theta)^2$$

$$V' = \frac{d}{d\theta}V = -3Wa\sin(\theta) + \frac{a^2}{2}(k_1 + 4k_2)\sin(2\theta)$$

$$V'' = \frac{d^2}{d\theta^2}V = -3Wa\cos(\theta) + a^2(k_1 + 4k_2)\cos(2\theta)$$

At $\theta = 0$ deg

$$V'' = -3Wa\cos(\theta) + a^2(k_1 + 4k_2)\cos(2\theta)$$

Since $V'' = 1.62 \times 10^4$ lb ft > 0, then the vertical position is stable.

Problem 11-38

If each of the three links of the mechanism has a weight W, determine the angle θ for equilibrium. The spring, which always remains vertical, is unstretched when $\theta = 0°$.

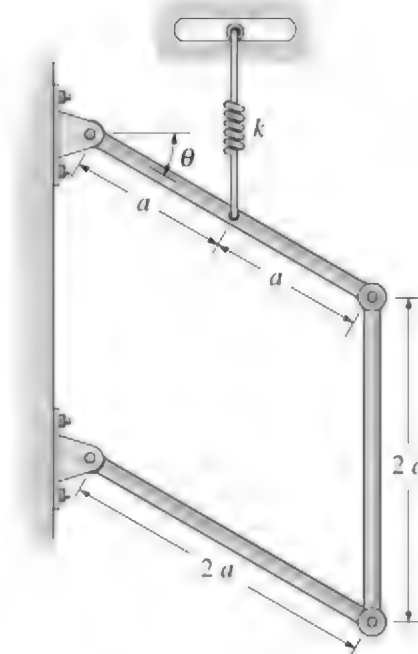
Solution:

$$V = \frac{1}{2}k(a\sin(\theta))^2 - 2Wa\sin(\theta) - W(2a)\sin(\theta)$$

$$V = \frac{ka^2}{2}\sin(\theta)^2 - 4Wa\sin(\theta)$$

$$\frac{d}{d\theta}V = ka^2\sin(\theta)\cos(\theta) - 4Wa\cos(\theta) = 0$$

$\cos(\theta) = 0$ $\qquad \theta = 90$ deg

$\sin(\theta) = \dfrac{4W}{ka}$ $\qquad \theta = \operatorname{asin}\left(\dfrac{4W}{ka}\right)$

Problem 11-39

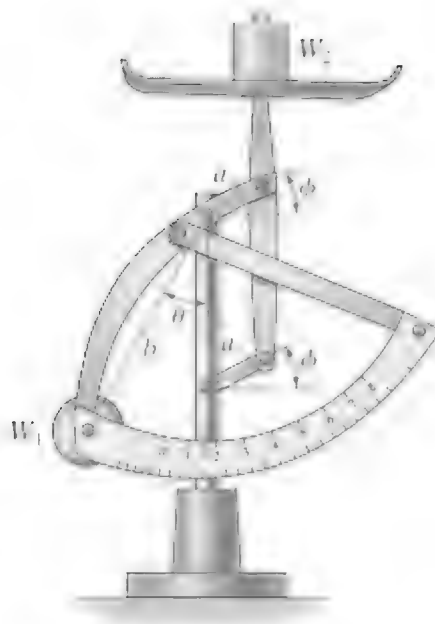
The small postal scale consists of a counterweight W_1 connected to the members having negligible weight. Determine the weight W_2 that is on the pan in terms of the angles θ and ϕ and the dimensions shown. All members are pin connected.

Solution:

$$\phi = -\theta + \text{constant}$$

$$V = W_2 a \sin(\phi) - W_1 b \cos(\theta)$$

$$V' = \frac{d}{d\theta}V = -W_2 a \cos(\phi) + W_1 b \sin(\theta) = 0$$

$$W_2 = W_1\left(\frac{b\sin(\theta)}{a\cos(\phi)}\right)$$

Problem 11-40

The uniform right circular cone having a mass m is suspended from the cord as shown. Determine the angle θ at which it hangs from the wall for equilibrium. Is the cone in stable equilibrium?

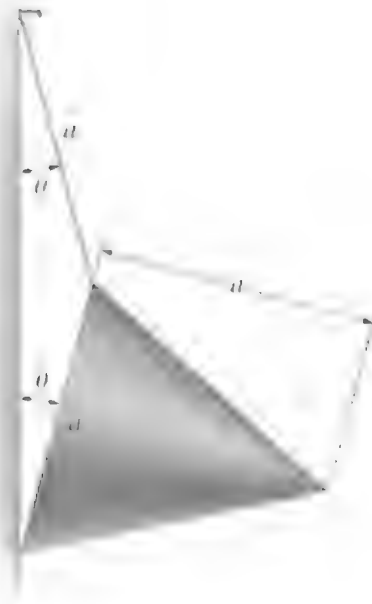
Solution:

$$V = -\left(\frac{3a}{2}\cos(\theta) + \frac{a}{4}\sin(\theta)\right)mg$$

$$V' = \frac{dV}{d\theta} = -\left(\frac{-3a}{2}\sin(\theta) + \frac{a}{4}\cos(\theta)\right)mg$$

$$V'' = \frac{d^2V}{d\theta^2} = -\left(\frac{-3a}{2}\cos(\theta) - \frac{a}{4}\sin(\theta)\right)mg$$

Equilibrium

$V' = 0 \qquad \frac{3}{2}\sin(\theta) = \frac{1}{4}\cos(\theta) \qquad \tan(\theta) = \frac{1}{6} \qquad \theta = \text{atan}\left(\frac{1}{6}\right) \qquad \theta = 9.462\,\text{deg}$

$$V'' = -\left(\frac{-3}{2}\cos(\theta) - \frac{1}{4}\sin(\theta)\right) \; amg \qquad V'' = 1.5 \, amg \qquad \text{Stable}$$

Problem 11-41

The homogeneous cylinder has a conical cavity cut into its base as shown. Determine the depth d of the cavity so that the cylinder balances on the pivot and remains in neutral equilibrium.

Given:

$a = 50 \text{ mm}$

$b = 150 \text{ mm}$

Solution:

$$y_c = \frac{\frac{b}{2}a^2\pi b - \frac{d}{4}\left(\frac{1}{3}\pi a^2 d\right)}{\pi a^2 b - \frac{1}{3}\pi a^2 d}$$

$$V = (y_c - d)\cos(\theta)W$$

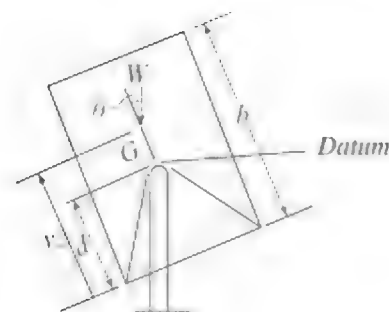
$$\frac{d}{d\theta}V = -W\sin(\theta)(y_c - d)$$

$\theta = 0 \text{ deg}$ (equilibrium position)

$$\frac{d^2}{d\theta^2}V = -W\cos(\theta)(y_c - d) = 0$$

$$d = y_c$$

Guess $d = 10 \text{ mm}$

Given $$d = \frac{\frac{b}{2}a^2\pi b - \frac{d}{4}\left(\frac{1}{3}\pi a^2 d\right)}{\pi a^2 b - \frac{1}{3}\pi a^2 d} \qquad d = \text{Find}(d) \qquad d = 87.868 \text{ mm}$$

Problem 11-42

The conical manhole cap is made of concrete and has the dimensions shown. Determine the critical location $h = h_{cr}$ of the pick-up connectors at A and B so that when hoisted with constant velocity the cap is in neutral equilibrium. Explain what would happen if the connectors were placed at a point $h > h_{cr}$.

Given:

$a = 2 \text{ ft}$

$b = 2.5 \text{ ft}$

$c = 3 \text{ ft}$

$d = 5 \text{ ft}$

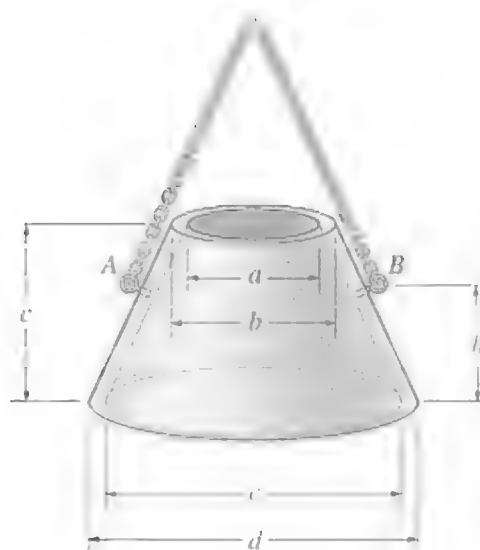
$e = d - (b - a)$

Solution:

$$V = W(y_c - h)\cos(\theta)$$

$$\frac{d}{d\theta}V = W(h - y_c)\sin(\theta) = 0$$

Equilibrium at

$\sin(\theta) = 0 \qquad \theta = 0\text{deg}$

For neutral equilibrium require

$$\frac{d^2}{d\theta^2}V = W(h - y_c)\cos(\theta) = 0 \qquad \text{Thus} \qquad y_c = h$$

Thus, A and B must be at the elevation of the center of gravity of the cap. $c_1 = \dfrac{cd}{d-b}$ $\qquad c_2 = \dfrac{ce}{e-a}$

$$y_c = \frac{\left(\frac{d}{2}\right)^2\left(\frac{c_1}{3}\right)\left(\frac{c_1}{4}\right) - \left(\frac{b}{2}\right)^2\left(\frac{c_1 - c}{3}\right)\left(\frac{c_1 + 3c}{4}\right) - \left(\frac{e}{2}\right)^2\left(\frac{c_2}{3}\right)\left(\frac{c_2}{4}\right) + \left(\frac{a}{2}\right)^2\left(\frac{c_2 - c}{3}\right)\left(\frac{c_2 + 3c}{4}\right)}{\left(\frac{d}{2}\right)^2\left(\frac{c_1}{3}\right) - \left(\frac{b}{2}\right)^2\left(\frac{c_1 - c}{3}\right) - \left(\frac{e}{2}\right)^2\left(\frac{c_2}{3}\right) + \left(\frac{a}{2}\right)^2\left(\frac{c_2 - c}{3}\right)}$$

$h_{cr} = y_c \qquad h_{cr} = 1.32 \text{ ft} \qquad$ If $h > h_{cr}$ then stable.

Problem 11-43

Each bar has a mass per length of m_0. Determine the angles θ and ϕ at which they are suspended in equilibrium. The contact at A is smooth, and both are pin con-nected at B.

Solution:

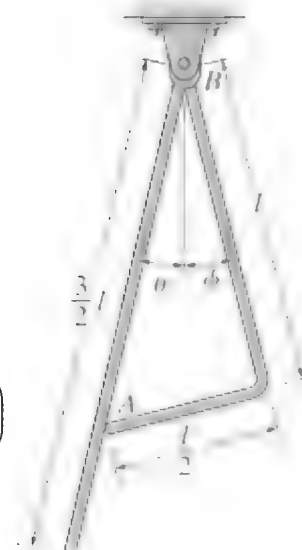
$$\theta + \phi = \operatorname{atan}\left(\frac{1}{2}\right)$$

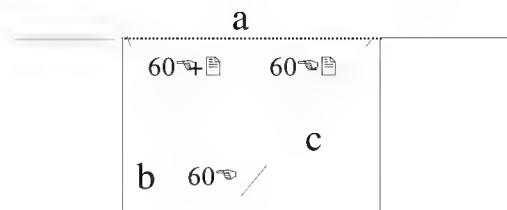
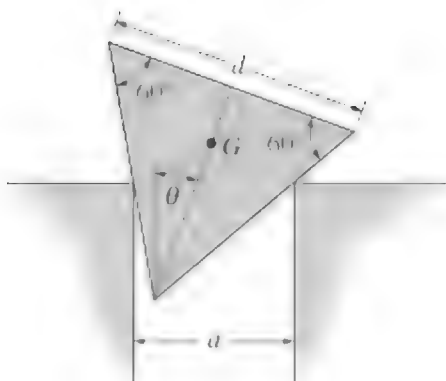
$$V = -\frac{3l}{2} m_0 \left(\frac{3l}{4}\right) \cos(\theta) - l\, m_0 \left(\frac{l}{2}\right) \cos(\phi) - \frac{l}{2} m_0 \left(l \cos(\phi) + \frac{l}{4} \sin(\phi)\right)$$

$$\frac{d}{d\theta} V = \frac{9 m_0 l^2}{8} \sin(\theta) - m_0 l^2 \sin(\phi) + \frac{m_0 l^2}{8} \cos(\phi) = 0$$

Guess $\theta = 10 \text{ deg}$ $\phi = 10 \text{ deg}$

Given $\theta + \phi = \operatorname{atan}\left(\frac{1}{2}\right)$ $\frac{9}{8} \sin(\theta) - \sin(\phi) + \frac{1}{8} \cos(\phi) = 0$

$$\begin{pmatrix} \theta \\ \phi \end{pmatrix} = \operatorname{Find}(\theta, \phi) \qquad \begin{pmatrix} \theta \\ \phi \end{pmatrix} = \begin{pmatrix} 9.18 \\ 17.38 \end{pmatrix} \text{deg}$$

Problem 11-44

The triangular block of weight W rests on the smooth corners which are a distance a apart. If the block has three equal sides of length d, determine the angle θ for equilibrium.

Solution:

$$\frac{a}{\sin(60\ \text{deg})} = \frac{b}{\sin(60\ \text{deg} - \theta)} \qquad b = a\frac{\sin(60\ \text{deg} - \theta)}{\sin(60\ \text{deg})}$$

$$V = W\left(\frac{2}{3}d\sin(60\ \text{deg})\cos(\theta) - b\cos(30\ \text{deg} - \theta)\right)$$

$$V = \frac{W}{2\sqrt{3}}(2d\cos(\theta) - 2a\cos(2\theta) - a)$$

$$\frac{d}{d\theta}V = \frac{W}{2\sqrt{3}}(-2d\sin(\theta) + 8a\sin(\theta)\cos(\theta)) = 0$$

$$\theta_1 = \operatorname{asin}(0) \qquad \theta_1 = 0\,\text{deg}$$

$$\theta_2 = \operatorname{acos}\left(\frac{d}{4a}\right)$$

Problem 11-45

A homogeneous cone rests on top of the cylindrical surface. Derive a relationship between the radius r of the cylinder and the height h of the cone for neutral equilibrium. *Hint:* Establish the potential function for a *small* angle θ of tilt of the cone, i.e., approximate $\sin\theta \approx 0$ and $\cos\theta \approx 1-\theta^2/2$.

Solution:

$$V = \left[\left(r + \frac{h}{4}\right)\cos(\theta) + r\theta\sin(\theta)\right]W$$

$$V_{app} = \left[\left(r + \frac{h}{4}\right)\left(1 - \frac{\theta^2}{2}\right) + r\theta^2\right]W$$

$$\frac{d}{d\theta}V_{app} = \left[-\left(r + \frac{h}{4}\right)\theta + 2r\theta\right]W = 0$$

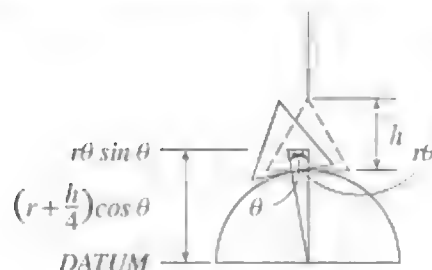
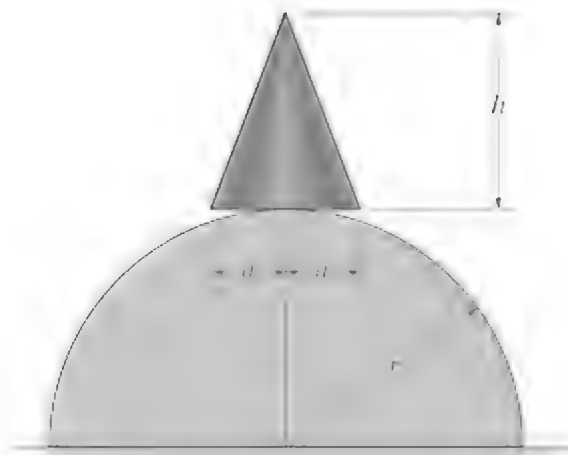
$$\frac{dV_{app}}{d\theta} = \left(r - \frac{h}{4}\right)\theta W = 0$$

$$\frac{d^2}{d\theta^2}V_{app} = r - \frac{h}{4} = 0$$

Equilibrium

$\theta = 0$ deg

For neutral equilibrium:

$$r = \frac{h}{4}$$

Problem 11-46

The door has a uniform weight W_1. It is hinged at A and is held open by the weight W_2 and the pulley. Determine the angle θ for equilibrium.

Given:

$W_1 = 50$ lb

$W_2 = 30$ lb

$a = 6\ \text{ft}$

$b = 6\ \text{ft}$

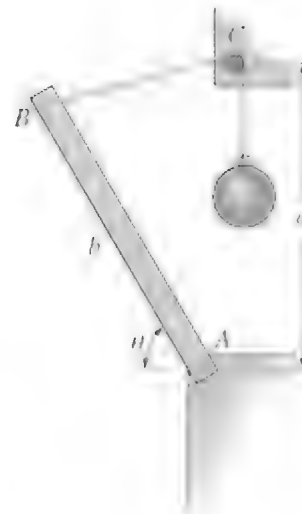
Solution:

$$V = W_1\left(\frac{b}{2}\right)\sin(\theta) + W_2\sqrt{a^2 + b^2 - 2ab\sin(\theta)}$$

$$\frac{d}{d\theta}V = W_1\left(\frac{b}{2}\right)\cos(\theta) - W_2\left(\frac{ab\cos(\theta)}{\sqrt{a^2 + b^2 - 2ab\sin(\theta)}}\right) = 0$$

Guess $\theta = 10\ \text{deg}$

Given $W_1\left(\frac{b}{2}\right)\cos(\theta) - W_2\left(\frac{ab\cos(\theta)}{\sqrt{a^2 + b^2 - 2ab\sin(\theta)}}\right) = 0$ $\theta = \text{Find}(\theta)$

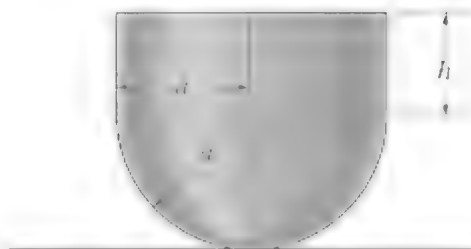
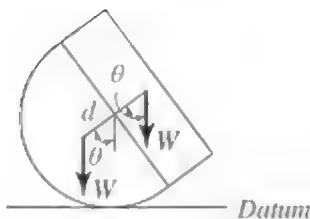
$\theta = 16.26\ \text{deg}$

Problem 11-47

The hemisphere of weight W supports a cylinder having a specific weight γ. If the radii of the cylinder and hemisphere are both a., determine the height h of the cylinder which will produce neutral equilibrium in the position shown.

Given:

$W = 60\ \text{lb}$

$a = 5\ \text{in}$

$\gamma = 311\ \frac{\text{lb}}{\text{ft}^3}$

Solution:

$$V = -W\left(\frac{3a}{8}\right)\cos(\theta) + \gamma\pi a^2 h\left(\frac{h}{2}\right)\cos(\theta)$$

$$V = \left(\frac{\gamma \pi a^2 h^2}{2} - \frac{W3a}{8}\right)\cos(\theta)$$

$$\frac{d}{d\theta}V = -\left(\frac{\gamma \pi a^2 h^2}{2} - \frac{W3a}{8}\right)\sin(\theta)$$

$$\frac{d^2}{d\theta^2}V = -\left(\frac{\gamma \pi a^2 h^2}{2} - \frac{W3a}{8}\right)\cos(\theta)$$

For neutral equilibrium we must have

$$\frac{\gamma \pi a^2 h^2}{2} - \frac{W3a}{8} = 0 \qquad h = \sqrt{\frac{W3}{4\pi \gamma a}} \qquad h = 3.99\ \text{in}$$

Problem 11-48

Compute the force developed in the spring required to keep the rod of mass M_{rod} in equilibrium at θ. The spring remains horizontal due to the roller guide.

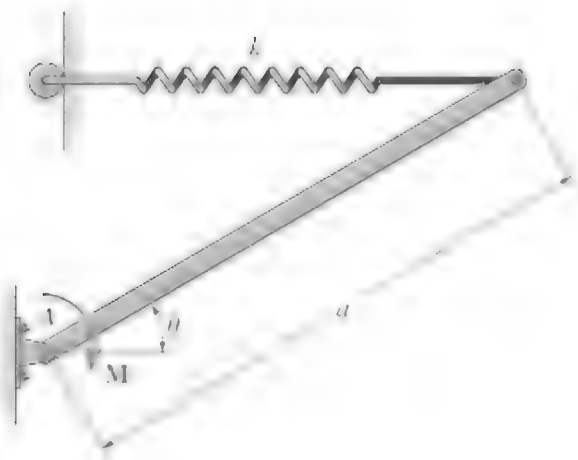
Given:

$$k = 200\ \frac{\text{N}}{\text{m}}$$

$$M = 40\ \text{N}\cdot\text{m}$$

$$a = 0.5\ \text{m}$$

$$\theta = 30\ \text{deg}$$

$$M_{rod} = 6\ \text{kg}$$

Solution:

$$V = M\theta + M_{rod}g\left(\frac{a}{2}\right)\sin(\theta) + \frac{1}{2}k(a\cos(\theta) - \delta)^2$$

$$\frac{d}{d\theta}V = M + M_{rod}g\left(\frac{a}{2}\right)\cos(\theta) - k(a\cos(\theta) - \delta)a\sin(\theta) = 0$$

Guess $\delta = 100$ mm

Given $M + M_{rod} g \left(\frac{a}{2}\right) \cos(\theta) - k(a\cos(\theta) - \delta) a \sin(\theta) = 0$

$\delta = \text{Find}(\delta)$ $\delta = -0.622$ m $F = k(a\cos(\theta) - \delta)$ $F = 211.0\,\text{N}$

Problem 11-49

Determine the force **P** acting on the cord which is required to maintain equilibrium of the horizontal bar *CB* of mass M. *Hint:* First show that the coordinates s_A and s_B are related to the *constant* vertical length l of the cord by the equation $5s_B - s_A = L$.

Given:

$M = 20$ kg

Solution:

$L = 4s_B + (s_B - s_A)$

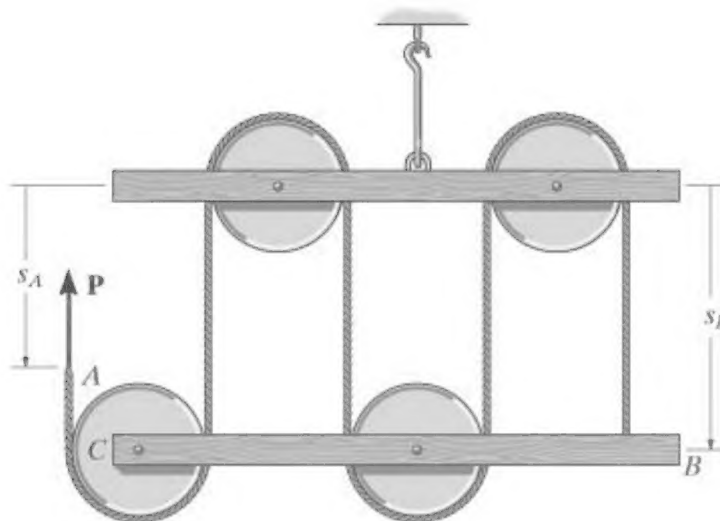
$L = 5s_B - s_A$

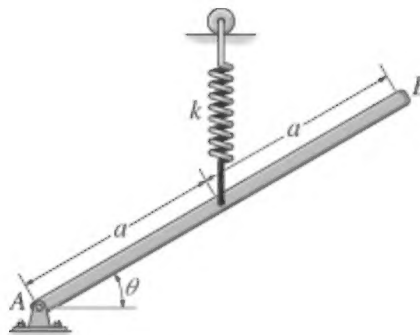
$\Delta L = 5\Delta s_B - \Delta s_A = 0$

$\Delta s_A = 5\Delta s_B$

$V = -M g s_B + P s_A$

$\Delta V = -M g \Delta s_B + P\Delta s_A = (-M g + 5P)\Delta s_B = 0$

$P = \frac{M g}{5}$ $P = 39.2\,\text{N}$

Problem 11-50

The uniform bar *AB* has weight *W*. If the attached spring is unstretched when $\theta = 90$ deg, use the method of virtual work and determine the angle θ for equilibrium. Note that the spring always remains in the vertical position due to the roller guide.

Given:

$$W = 10\ \text{lb}$$

$$k = 5\ \frac{\text{lb}}{\text{ft}}$$

$$a = 4\ \text{ft}$$

Solution:

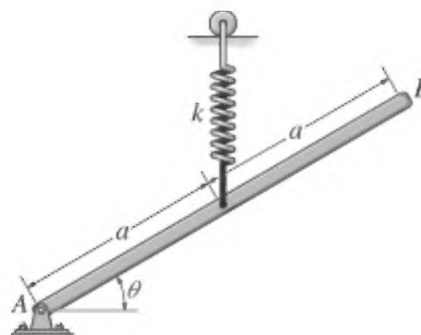
$$y = a\sin(\theta) \qquad \delta y = a\cos(\theta)\delta\theta$$

$$\delta U = (-W + F_s)\delta_y = [k(a - a\sin(\theta)) - W]a\cos(\theta)\delta\theta = 0$$

$$\cos(\theta_1) = 0 \qquad \theta_1 = \text{acos}(0) \qquad \theta_1 = 90\,\text{deg}$$

$$\sin(\theta_2) = 1 - \frac{W}{ka} \qquad \theta_2 = \text{asin}\left(1 - \frac{W}{ka}\right) \qquad \theta_2 = 30\,\text{deg}$$

Problem 11-51

The uniform bar *AB* has weight *W*. If the attached spring is unstretched when $\theta = 90$ deg, use the principle of potential energy and determine the angle θ for equilibrium. Investigate the stability of the equilibrium positions. Note that the spring always remains in the vertical position due to the roller guide.

Given:

$$W = 10\ \text{lb}$$

$$k = 5\ \frac{\text{lb}}{\text{ft}}$$

$$a = 4\ \text{ft}$$

Solution:

$$V = W a \sin(\theta) + \frac{1}{2}k(a - a\sin(\theta))^2 = W a \sin(\theta) + \frac{1}{2}k a^2 (1 - \sin(\theta))^2$$

Equilibrium

$$\frac{d}{d\theta}V = W a \cos(\theta) - k a^2 (1 - \sin(\theta))\cos(\theta) = 0$$

$\cos(\theta_1) = 0$ $\quad\quad$ $\theta_1 = \text{acos}(0)$ $\quad\quad$ $\theta_1 = 90\,\text{deg}$

$\sin(\theta_2) = 1 - \dfrac{W}{k a}$ $\quad\quad$ $\theta_2 = \text{asin}\left(1 - \dfrac{W}{k a}\right)$ $\quad\quad$ $\theta_2 = 30\,\text{deg}$

Check Stability $\quad$ If $V'' > 0$ the equilibrium point is stable. If $V'' < 0$, then unstable

$$V'' = \frac{d^2 V}{d\theta^2} = -W a \sin(\theta) + k a^2 \sin(\theta) + k a^2 \cos(2\theta)$$

$V''_1 = -W a \sin(\theta_1) + k a^2 \sin(\theta_1) + k a^2 \cos(2\theta_1)$ $\quad\quad$ $V''_1 = -40\,\text{lb}\cdot\text{ft}$

$V''_2 = -W a \sin(\theta_2) + k a^2 \sin(\theta_2) + k a^2 \cos(2\theta_2)$ $\quad\quad$ $V''_2 = 60\,\text{lb}\cdot\text{ft}$

Problem 11-52

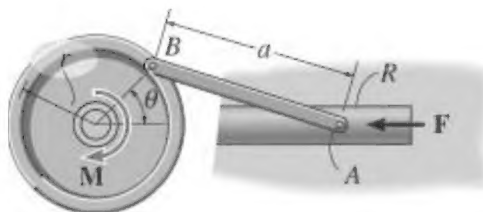
The punch press consists of the ram *R*, connecting rod *AB*, and a flywheel. If a torque *M* is applied to the flywheel, determine the force **F** applied at the ram to hold the rod in the position $\theta = \theta_0$.

Given:

$M = 50\ \text{N}\cdot\text{m}$

$\theta_0 = 60\ \text{deg}$

$r = 0.1\ \text{m}$

$a = 0.4\ \text{m}$

Solution: $\quad \theta = \theta_0$

Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only force **F** and Moment *M* do work.

$$a^2 = x^2 + r^2 - 2 x r \cos(\theta)$$

$0 = 2x\delta x - 2r\cos(\theta)\delta x + 2 x r \sin(\theta)\delta\theta$ $\quad\quad$ $\delta x = \left(\dfrac{x r \sin(\theta)}{r\cos(\theta) - x}\right)\delta\theta$

$$\delta U = -F\,\delta x - M\delta\theta = \left[-F\left(\frac{x r \sin(\theta)}{r\cos(\theta) - x}\right) - M\right]\delta\theta = 0$$

Guesses $F = 1\ \text{N}$ $x = 0.1\ \text{m}$

Given $a^2 = x^2 + r^2 - 2xr\cos(\theta)$ $-F\left(\dfrac{xr\sin(\theta)}{r\cos(\theta) - x}\right) - M = 0$

$\begin{pmatrix} x \\ F \end{pmatrix} = \text{Find}(x, F)$ $x = 0.441\ \text{m}$ $F = 512\ \text{N}$